Title
ITD formulation for the currents on a plane angular sector

Journal
IEEE ANTENNAS AND PROPAGATION SOCIETY INTERNATIONAL SYMPOSIUM 1997, VOLS 1-4, 3(9)

ISSN
0272-4693

Authors
Albani, M
Capolino, F
Maci, S
et al.

Publication Date
1997

ITD FORMULATION FOR THE CURRENTS ON A PLANE ANGULAR SECTOR

M. ALBANI[1], F. CAPOLINO[1], S. MACI[2], R. TIBERIO[1], F. MOLINET[3]

[1]*College of Engineering, Univ. of Siena, Via Roma 56, 53100, Siena, Italy*
[2]*Dept. of Elect. Eng., Univ. of Florence, Via S.Marta 3, 50139, Florence, Italy*
[3]*MOTHESIM, Centre d'affaires La Boursidier, Les Plessis Robinson, France*

1. INTRODUCTION

The electromagnetic scattering of a corner at the interconnection of two straight edges joined by a plane angular sector is important in the framework of Geometrical Theory of Diffraction (GTD) and of its uniform extension (UTD) [1]. The same canonical problem is also useful in order to obtain fringe currents, namely the currents that are induced on the face by the total diffraction mechanism; their radiation provides the improvement of the Physical Optics (PO) field in the framework of the Physical Theory of Diffraction (PTD) [2]. The exact electromagnetic solution of the plane angular sector is due to Satterwhite and Kouyoumjian [3], but the series expansion of this solution is very slowly convergent for increasing distance from the tip and not well-suited for practical asymptotic evaluation. The solution in [3] has been used in [4] for obtaining the currents close to the tip.

Recently, vertex diffraction coefficients have been derived in the plane-wave far-field regime by using the induction theorem [5]. These coefficients account for second order interactions between the two edges and provide the physical property to satisfy reciprocity. The same formulation has been used in [6] for deriving a uniform solution to apply at finite distance from the tip.

In this paper, asymptotic, closed form expressions of the fringe currents of the plane angular sector are derived by using the Incremental Theory of Diffraction (ITD) [7][8]. The application of this theory in deriving currents has been found particularly attractive, as demonstrated in [9] for the case of a circular ground plane illuminated by a vertical dipole. This is due to the fact that the ITD diffraction coefficients satisfy the boundary conditions (BC) of their relevant canonical problem, so that they warrant a reasonably accurate prediction of the currents.

2. FORMULATION

The geometry at a vertex interconnecting the two edges (denoted by 1 and 2) of a plane angular sector is shown in Fig. 1a, in which Ω denotes the angle between the two edges. Both a spherical coordinate systems (r, β_i, ϕ_i) with the relevant unitary vectors $(\hat{r}, \hat{\beta}_i, \hat{\phi}_i)$, and a cylindrical coordinate system (ρ_i, ϕ_i, z_i) are defined at each edge i=1,2 with their origin at the tip (Fig.1a). A plane wave illumination is assumed; the direction of incidence is denoted by (β_i', ϕ_i') and its polarisation by the unit vector $\hat{\beta}_i$, $\hat{\phi}_i$. In the following, all the current contributions relevant to diffraction mechanism at the first and second edge are denoted with the suffix 1 and 2, respectively.

In our description the total current $\vec{J}$ is represented as

$$\vec{J} = \vec{J}^{PO} + \vec{J}_1^f + \vec{J}_2^f \qquad (1)$$

where $\vec{J}^{PO}$ denotes PO current, and $\vec{J}_1^f$, $\vec{J}_2^f$ are fringe current contributions relevant to edge 1 , 2 respectively. The latter are represented as the sum of those induced by the ray contributions represented in Fig. 1b. Singly diffracted rays from edges 1 and 2 arise at Q_1' and Q_2', respectively. A vertex contribution arises from the tip and a double diffraction (DD) contribution

0-7803-4178-3/97/$10.00

from a point Q_2'' on edge 2 after diffracting at the point Q_1'' on edge 1 (the other DD term $2 \rightarrow 1$ is not depicted in the same figure). In the following, we will discuss only the contributions in which the first diffraction occurs at the edge 1; indeed $\vec{J}_2^f$ can be easily obtained from $\vec{J}_1^f$ by changing 1 with 2.

We denote by $\vec{J}_1^d$ and $\vec{J}_1^{dd}$ the currents induced by the singly and doubly diffracted rays, respectively; these contributions are discontinuous when, the observation is such that Q_1' or Q_n'' (n=1,2) disappears from the vertex, respectively, i.e. at the shadow boundary lines (SBLs) defined by $\beta_1=\beta_1'$ and $\beta_2=\beta_1'-\Omega$, respectively. These discontinuities are compensated by the vertex contribution, that is conveniently subdivided into two different contributions $\vec{J}_1^v$ and $\vec{J}_1^{vv}$, providing the continuity of the total current.

Thus our representation of the fringe current $\vec{J}_1^f$ is

$$\vec{J}_1^f = \vec{J}_1^{f'} + \vec{J}_1^{f''} \qquad (2)$$

where

$$\vec{J}_1^{f'} = \vec{J}_1^d + \vec{J}_1^v \qquad \text{and} \qquad \vec{J}_1^{f''} = \vec{J}_1^{dd} + \vec{J}_1^{vv} \qquad (3)$$

In the following subsections, the two contributions $\vec{J}_1^{f'}$ and $\vec{J}_1^{f''}$ are discussed separately.

2.a Contribution $\vec{J}_1^{f'}$

Let us consider an infinite half-plane with its edge aligned with edge 1 of the plane angular sector. The sum $\vec{J}_{1\infty}^d(\beta_1')$ of the fringe currents on the two faces of this infinite structure, , is exactly expressed in terms of Fresnel integral. By means of the ITD localisation process, incremental diffraction contribution $\vec{i}_1^d(z_1')$ associated to the incremental diffracted current distribution may be derived. These contributions exhibits a spherical spreading outgoing from the point z_1'. The fringe current contribution $\vec{J}_1^{f'}$ relevant to the semi-infinite edge may be constructed by superimposing the incremental contributions relevant to all the points on the semi-infinite extent $z_1' \geq 0$ of the edge, that are properly weighted by the incident wave phase factor; i.e.,

$$\vec{J}_1^{f'} = \int_0^\infty \vec{i}_1^d(z_1')\, e^{-jkz_1'\cos\beta_1'}\, dz_1' \qquad (4)$$

The asymptotic evaluation of this integral provides two ray contributions. The first one is relevant to the stationary phase point of the integrand, which occurs at the point Q_1'. This contribution may be recognize as that produced by the singly diffracted rays. The second contribution ($\vec{J}_1^v$) is relevant to the end-point and provides the desired continuity of the currents $\vec{J}_1^{f'}$ when the point Q_1' disappears from the vertex.

In order to asymptotically evaluate $\vec{J}_1^{f'}$ for kr large, the spectral representation $\vec{i}_1^d(z_1')$ is introduced in (4) yielding

$$\vec{J}_1^{f'} = \frac{1}{2\pi j}\int_C \vec{J}_{1\infty}^d(\theta_1)\,\frac{\sin\theta_1}{(\cos\beta_1'-\cos\theta_1)}\, d\theta_1 \qquad (5)$$

where $C \equiv (-j\infty, \pi+j\infty)$ with a counterclockwise indentation around the pole $\theta_1=\beta_1'$. $\vec{J}_{1\infty}^d(\theta_1)$ exibits a saddle point at $\theta_1=\beta_1$. By deforming C onto a pertinent steepest descent path (SDP) through the saddle point $\theta_1=\beta_1$, the pole residue is captured when $\beta_1<\beta_1'$; thus, leading to the contributions

$$\vec{J}_1^d = U(\beta_1'-\beta_1)\; \vec{J}_{1\infty}^d(\beta_1') \qquad (6)$$

Applying the modified Pauli-Clemmow method [1] to the SDP integral, leads to an asymptotic, closed form expression for the vertex contribution $\vec{J}_1^v$.

2.b Contribution $\vec{J}_1^{f''}$

In order to define the wave that impinges on the second edge, only the field contribution relevant to the current $\vec{J}_1^d$ is considered. At each point z_2' of the second edge, these currents are related to the magnetic field component $\vec{H}_{t_1}(z_2')$ along the direction $\hat{t}_1 = \hat{s}_1 \times \hat{n}$ ($\hat{n}$ normal to the top face) transverse to the grazing ray direction $\hat{s}_1$. This magnetic field can be interpreted as a local H-polarized plane wave impinging on the second edge, so that a new set of incremental current distribution $\vec{i}_2^d(z_2')$ can be associated to each point z_2' of the second edge; thus, obtaining

$$\vec{J}_1^{f''} = \int_0^\infty H_{t_1}(z_2')\; \vec{i}_2^d(z_2')\, dz_2 \qquad (7)$$

The asymptotic evaluation of this integral provides two contributions. The First is associated to the stationary phase point, that occurs at Q_2'' (Fig. 1b), thus providing the term $\vec{J}_1^{dd}$ corresponding to the doubly diffracted rays. The second contribution is an end-point from the vertex $\vec{J}_1^{vv}$, that provides the desired continuity of the currents $\vec{J}_1^{f''}$ at the SBL $\beta_2 = \beta_1' - \Omega$, namely when both the points Q_1'' and Q_2'' disappear from the vertex.

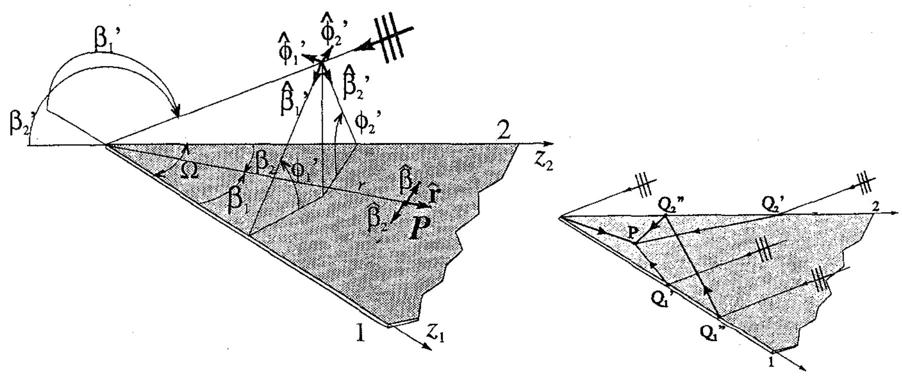

Fig.1 a) Coordinate systems at the edges. b) Ray contributions at a plane angular sector.

3. NUMERICAL RESULTS

Numerical results have been carried out in order to test the solution presented above. Figure 2a shows radial currents on a plane angular sector with $\Omega=90°$, illuminated by a plane wave as depicted in the inset. The polarization of the incident electric field is parallel to the incidence plane. The scan is performed versus β_1 coordinate from 0° to Ω at a distance $r=0.5\lambda$ from the tip. Our solution (continuous line) is compared with the exact solution (dashed line, from [4]) and the UTD solution (dotted line). This latter exhibits discontinuities at the two SBLs ($\beta_1 \simeq 38°$ and $\beta_1 \simeq 52°$), that are well compensated by the vertex contributions. In Fig. 2b radial currents are plotted for the case of a plane angular sector with $\Omega=90°$, illuminated by a plane wave as shown in the inset. The polarization of the E field is parallel to the incidence plane. The scan is performed versus β_1 coordinate from 0° to Ω at a distance $r=1.2\lambda$ from the tip. Even in this case our solution improves the UTD. At the singly diffracted field SBL ($\beta_1 \simeq 38°$) the compensation mechanism of $\bar{J}^f$ behaves as in Fig.2. Nevertheless, in the last part of the scan, the second order contributions play an important role in closer recovering the exact solution. Indeed a doubly diffracted field SBL occurs at $\beta_1 \simeq 52°$, at this aspect a doubly diffracted current contribution $\bar{J}^{dd}$ appears smoothly compensated by the relevant $\bar{J}^{vv}$ term.

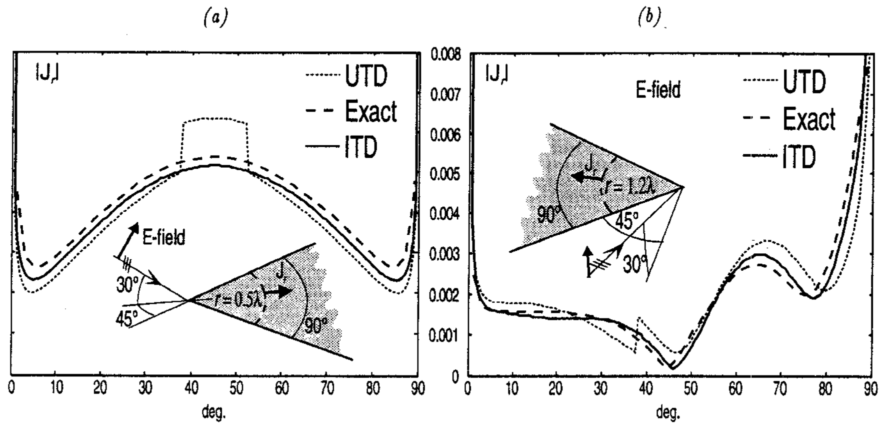

Fig. 2. Currents on a plane angular sector

REFERENCES

[1] Kouyoumjian, R. G., and P. H. Pathak, "A uniform geometrical theory of diffraction for an edge in a perfectly conducting surface," *Proc. IEEE*, 62 (11), 1448-1461, 1974.

[2] P. Ya. Ufimtsev, "Elementary edge waves and the physical theory of diffraction," *Electromagnetics*, Vol.11, no. 2, pp. 125-159, April-June 1991.

[3] Satterwhite, R. S., "Diffraction by a quarter plane, the exact solution and some numerical results", *IEEE Trans. Antennas Propagat.*, *AP*-22 (3), 500-503, 1974.

[4] T. J. Brinkley "current near the vertex of perfectly conducting angular sector" Ph.D. Thesis, Ohio state University, Dept. of Electrical eng., 1991.

[5] S. Maci, R. Tiberio, A. Toccafondi "Diffraction at a plane angular sector", *Jour. on e.m. Wave and Applic.*, vol.8, N. 9\10 pp. 1247-1276 September 1994

[6] F. Capolino, S. Maci "Uniform high-frequency description of singly, doubly, and vertex diffracted rays for a plane angular sector" to be published on *Jour. on e.m. Wave and Applic.*, 1996.

[7] R. Tiberio, S. Maci "Incremental Theory of Diffraction. Scalar formulation" *IEEE Trans on Antennas and Propagat.*, Vol. 42, n.5, pp. 600-612, May 1994

[8] R. Tiberio, S. Maci, A. Toccafondi "Incremental Theory of Diffraction. Electromagnetic formulation" *IEEE Trans on Antennas Propagat.*, Vol. 43, No. 1, pp. 87-96, Jan. 1995.

[9] S. Maci, R. Tiberio, A. Toccafondi " Incremental diffraction coefficients for source and observation at finite distance from an edge" *IEEE Trans. on Antennas Propagat.*, Vol. 44, N. 5, 1996.