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A CLARIFICATION OF MULTI REGGE THEORY

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A CLARIFICATION OF MULTI-REGGE THEORY*

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ABSTRACT

We are concerned here with the amplitude for the reaction $a + b \to 1 + 2 + \cdots + N$. We assert that the prevalent notion of adding multi-Regge diagrams, corresponding to the different ordering of final particles, has no basis. Arguments supporting this assertion are followed by a list of rules for calculating cross sections. A sample of the literature that motivated this paper is briefly discussed.

I. INTRODUCTION

Many models have been proposed on the basis of a generalization of Regge theory from $2 \to 2$ reactions to $2 \to N$ reactions. We are concerned here with two concepts that seem widespread.

<u>Concept A</u>: The amplitude M, for the $2 \to N$ process, is a sum of amplitudes corresponding to all the multi-Regge diagrams related by a permutation of final particle legs.

<u>Concept B</u>: If A is accepted, the question of interference terms between the different terms arises. One finds arguments that either emphasize their insignificance or exploit their importance.

We argue here that concept A has no place in any theory that generalizes $2 \to 2$ Regge theory, by seeking asymptotic expansions of M in certain special regions of phase space. We shall, however, work within the framework of the Bali, Chew, and Pignotti (BCP)[1,2] multi-Regge hypothesis, which seems to be the natural generalization of the "J plane" analyticity of $2 \to 2$ reactions. We shall show that concept A has no place in the implementation of this hypothesis. Concept A seems to be a result of the superficial resemblance that multi-Regge diagrams bear to Feynman diagrams.

In Sec. II we see how, and <u>in what sense</u>, multi-Regge diagrams approximate the actual amplitude, M. We dilate on those aspects that distinguish an asymptotic expansion within an S-matrix framework from perturbative expansions of field theory. Rules for calculating cross sections are discussed in Sec. III. In Sec. IV we discuss a sample of the literature where concepts A and B are employed. We have not specified whether the final particles are distinguishable, identical, or a mixture of both, since our assertion regarding concept

A is independent of this question. In what follows, however, it must be born in mind that we use the word "phase space" to mean ϕ_M, the mathematical phase space, in which the final particle momenta go over all the values allowed by energy momentum conservation. (We contrast ϕ_M with ϕ_O, the observable phase space, in which the momenta of the final particles are restricted so that each distinguishable final state occurs just once).

II. THE MULTI-REGGE HYPOTHESIS OF BCP

We assume familiarity with Toller variables[1,2] and deal only with certain special aspects that are germane to the issue. For concreteness, the reader may consider the $N = 2$ case, in what follows.

(i) Consider the amplitude M, for the process $a + b \rightarrow 1 + 2 + \cdots + N$, involving spinless particles. Bali, Chew, and Pignotti[1,2] explain how, by ordering the N particles in any arbitrary way, we can define the Toller variables. Figure 1 is the Toller diagram employed for this purpose. We emphasize that

(a) It is kinematical in nature and merely establishes a convention for the Toller variables.

(b) The ordering of particles in Fig. 1 is not their ordering in rapidity. The latter is decided by the values of the ω's, ξ's, and t's. Thus, one Toller diagram and the set of variables defined by it, are all we need to span the entire phase space ϕ.

(c) No factorization of M is intended or implied.

We have then

$$M = M(t_{12},t_{23},\cdots;\ \xi_{12},\xi_{23},\cdots;\ \omega_2,\omega_3,\cdots) = M(t_{12},\cdots;\ g_{12},\cdots)$$

where $g_{i,i+1}$ stands for the group variables of the ith link. We now expand the amplitude over the $O(2,1)$ group functions. In symbolic form (for brevity) we have

$$M(t_{12},\cdots;\ g_{12},\cdots) = \int d\ell^{12}\, d\ell^{23} \cdots \times d^{\ell_{12}}(g_{12}) d^{\ell_{23}}(g_{23}) \cdots \times B^{\ell_{12},\ell_{23}\cdots}(t_{12},t_{23},\cdots) \qquad (1)$$

where, in Eq. (1), $\ell_{i,i+1}$ stands for the label of the irreducible representations of $O(2,1)$, the d's are the group functions, and B is the "partial wave amplitude." (We are aware that the above symbolic form has suppressed the m,n indices, the contours in the ℓ planes etc.)

(ii) The multi-Regge hypothesis: "The amplitude B is an analytic function of the ℓ's, with the rightmost singularity being a factorizable pole $\alpha_{i,i+1}(t_{i,i+1})$ in the $\ell_{i,i+1}$ plane." We are not interested in analyzing the validity of the above hypothesis, but rather in examining the consequences.

(iii) The above hypothesis, even if true, is useful only in special circumstances. For the ordering of particles in Fig. 1, there is one part of phase space where, as $s \rightarrow \infty$, we can have $t_{i,i+1}$ fixed, the subenergies $s_{i,i+1} \rightarrow \infty$; i.e., $\xi_{i,i+1} \rightarrow \infty$.[1,2] In this region the particles will be ordered <u>in rapidity</u> as they are in Fig. 1 (see Fig 2). <u>In such a region</u>, the contributions from the rightmost poles will dominate the ℓ integrals, and we can write the famous expansion:

$$M(t_{12},\cdots;\ \xi_{12},\cdots)\ \xrightarrow[\text{all } \xi\to\infty]{}\ \beta_{a1}(t_{12})(\cosh \xi_{12})^{\alpha_{12}(t_{12})}\cdots$$

$$\chi\ (\cosh \xi_{N-1,N})^{\alpha_{N-1,N}(t_{N-1,N})}\ \beta_{bN}(t_{N-1,N})\ .$$

+ terms coming from the nonleading singularities of the ℓ planes, whose effect is negligible in this part of space phase

$$= M_1^{(1)} + \text{neglected terms}. \qquad (2)$$

In $M_1^{(1)}$, the subscript refers to the region of phase space, ϕ_1, corresponding to this ordering of particles; while the superscript indicates that only the leading pole was retained in each expansion. We represent $M_1^{(1)}$ by a multi-Regge diagram (Fig. 3), (the origin of all this misunderstanding!), and remark that:

(a) It is a dynamical diagram.

(b) Factorization is implied.

(c) Rapidity ordering of particles is as in diagram.

To calculate any cross section in this part of ϕ_1, we can use $M_1^{(1)}$ instead of M, with little error. If we want, we can keep two poles, α and α' in each expansion (assuming the second leading singularity is a pole) to get $M_1^{(2)}$, which will be a sum of 2^{N-1} terms, each with its own diagram. Here the addivity is a consequence of Cauchy's theorem and not the superposition principle.

(iv) Consider now the part of phase space where the rapidity plot is as in Fig. 4. It is clear that the physics here is as simple as in Fig. 2. However, $t_{12} = (p_a - p_1)^2$ cannot be held fixed as $s_{12} \to \infty$. Therefore the Toller variables defined in Fig. 1 are undesirable, despite their formal completeness. An expansion in those variables will, at best, have poor convergence properties. (We cannot, asymptotically, call a few terms of the expansion as "leading" and ignore the rest.) To exploit the dynamical simplification in the situation, we must draw a new Toller diagram with particle ordering $(2,1,3,4,5,\cdots N)$. Then $\tilde{t}_{12} = (p_a - p_2)^2$ can be held fixed as $\tilde{s}_{12} \to \infty$ (so that $\tilde{\xi}_{12} \to \infty$) to yield:

$$M = \beta_{a2}(\tilde{t}_{12})(\tilde{s}_{12})^{\tilde{\alpha}(\tilde{t}_{12})}\ \beta(\tilde{t}_{12},\tilde{\omega}_2,\tilde{t}_{23})\cdots\beta_{Nb}(\tilde{t}_{N-1,N})$$

$$+ \text{terms from neglected singularities}\ . \qquad (3)$$

By our convention, the leading term is $M_2^{(1)}$. To calculate cross sections in this neighborhood, we can use $M_2^{(1)}$ or $M_2^{(2)}$ instead of M.

It is clear that in the $N!$ regions of phase space, $\phi_1,\phi_2,\cdots\phi_{N!}$, corresponding to the different orderings of final particles in rapidity, we must define $N!$ different Toller diagrams and $N!$ sets of Toller variables, in order to exploit the simplicity introduced by the multi-Regge hypothesis. The reason for permuting the legs is thus the need to set up new sets of Toller variables, and not the superposition or Bose principle. It is clear that nowhere does the theory require or admit the addition of one expansion, M_i, of ϕ_i, to another, M_j, of ϕ_j; of one and the same amplitude M. The different expansions are alternate and not additive. In ϕ_i we can use $M_i^{(1)}$ or $M_i^{(2)}$ but not $M_i^{(1)} + M_j^{(1)}$. Such an addition is an arbitrary recipe, and certainly not forced upon us by the superposition or Bose principles. In fact, these principles are not imposed on M by hand (as in perturbative field theories where M is built from little pieces), but are demanded of M in S-matrix Regge calculations, where one begins with the "complete" amplitude and seeks its asymptotic expansions.

These ideas are transparent in the $2 \to 2$, equal mass, case. The t- and u-channel expansions (not their leading pole approximations) M_t and M_u are each alternate, complete expansions of M. A choice between them is made when we wish to approximate M in some special regions of phase space. If we approximate M_t by the leading pole contribution $M_t^{(1)}$, we are assured that at any fixed t, as $s \to \infty$, $M_t^{(1)}$ will approach M to any given accuracy. In practice, when we work at fixed s, $M_t^{(1)}$ can be a poor approximation to M except for very small t. At larger t, if $M_t^{(1)}$ is a bad fit, we can try $M_t^{(2)}$ etc. While adding more t poles to M_t is not guaranteed to give better approximation, it is a legitimate process one can try. Similar results hold for M_u. By contrast, the process of adding some singularities of M_t to some of M_u, to get approximations for M, is a purely arbitrary recipe and not a consequence of the theory. The expansions, M_t and M_u, are dual and alternative, as M_s, the direct channel expansion (which may possibly be approximated by a few resonances) is dual to M_t, the cross channel Regge expansion (which may possibly be approximated by a few Regge poles). Fits to the data,

using an M constructed by adding t and u Regge poles, do not test the theory.

We similarly conclude that the following, oft-quoted recipe, for processes with identical particles in the final state, is also ad hoc, and not a consequence of the multi-Regge hypothesis:

Step 1: Calculate the multi-Regge amplitude M_i corresponding to one ordering of final particle momenta. (Then M_i approaches M in a sub-region of ϕ_i where the Regge limit is reached.)

Step 2: Set $M = \sum_i M_i$, where i runs through all the permutations of the identical particle momenta in the final state.

Though this recipe guarantees Bose statistics manifestly, the flaw in the argument is the following. Bose statistics merely requires that $M(A) = M(B)$, where A and B are two points in phase space, related by a permutation of identical bosons. There is, however, no requirement that M achieve this symmetry by the recipe $M = \sum M_i$. We illustrate this point by considering a Veneziano-like amplitude, $B(u,t)$, for a fictitious $2 \to 2$ process where the s channel has identical particles and no resonances. Bose symmetry requires that if

$$B(u,t) \xrightarrow[(\lim u \to a,\, t=b)]{} \frac{F(b)}{u - a}$$

then we must have

$$B(u,t) \xrightarrow[(\lim t \to a,\, u=b)]{} \frac{F(b)}{t - a}$$

This is certainly true of the beta function $B(u,t)$. However, when we expand it to display the pole structure, we have

$$B(u,t) = \sum_{N=0}^{\infty} \frac{g_N(u)}{t - \xi_N} \qquad \text{(exhibiting the } t \text{ poles)}$$

$$= \sum_{N=0}^{\infty} \frac{g_N(t)}{u - \xi_N} \qquad \text{(exhibiting the } u \text{ poles)} .$$

(The g_N and ξ_N are the same in both expansions.)

While <u>either</u> expansion has Bose symmetry <u>as defined above</u>, the symmetry is not achieved by the recipe. It is clear that, while

$$\sum_{N=0}^{\infty} \frac{g_N(u)}{t - \xi_N} + \frac{g_N(t)}{u - \xi_N}$$

is manifestly symmetric, it is not equal to the amplitude $B(u,t)$.

III. CROSS SECTION CALCULATIONS

For brevity, we restrict ourselves to total cross sections, σ_T, for $2 \to N$ processes. The rules for partial cross sections will be clear from this. <u>In principle</u>, to calculate σ_T, in the multi-Regge pole approximation, we must:

(a) Divide the phase space ϕ in $N!$ <u>distinct, nonoverlapping</u> regions ϕ_i, corresponding to the different orderings of final particles in rapidity.

(b) In each region ϕ_i, approximate M by $M_i^{(1)}$ or $M_i^{(2)}$ etc., integrate the approximate $|M_i|^2$ over ϕ_i to get the approximate contribution $\tilde{\sigma}_i$.

We then have, in the multi-Regge approximation, $\sigma_T \simeq \sum_i \tilde{\sigma}_i$.

(c) If identical particles are present, consider just the distinguishable orderings, i.e., $\sigma_T \simeq \sum_{\text{distinguishable}} \tilde{\sigma}_i$.

Such approximations to σ_T may, for example, be useful in bootstrap calculations that connect $2 \to 2$ absorptive parts to $2 \to 2$ total cross sections, via unitarity. In these calculations, it is hoped that the contributions to σ_T from the subregions of ϕ_i, where $M_i^{(1)}$ approximates M well, will dominate. The sharp fall off of residues with momentum transfers makes this plausible.

<u>In practice</u>, however, the conditions for "distinct, nonoverlapping regions" can only be achieved by restricting the Toller variables of each ordering by clumsy constraints. [In $2 \to 2$ equal mass scattering, the t channel $|M_t|^2$ is to be integrated over ϕ_t, the forward hemisphere, i.e., from $t = 0$ to $t = \frac{1}{2}(4m^2 - s)$; and the u channel $|M_u|^2$ over ϕ_u, from $u = 0$ to $u = \frac{1}{2}(4m^2 - s)$]. However, due to the rapid fall off of residues, in t, in the leading term $M_t^{(1)}$ of M_t, we can integrate $|M_t^{(1)}|^2$ over all t. The same goes for $|M_u^{(1)}|^2$. We then have symbolically (omitting flux factors),

$$\sigma^{e\ell}_{total} \simeq \tilde{\sigma}_t + \tilde{\sigma}_u = \int_{\phi_t} |M_t^{(1)}|^2 \, d\phi_t + \int_{\phi_u} |M_u^{(1)}|^2 \, d\phi_u$$

$$\simeq \int_{\phi} |M_t^{(1)}|^2 \, d\phi + \int_{\phi} |M_u^{(1)}|^2 \, d\phi \quad .$$

For $2 \to 2$ reactions, as $s \to \infty$, this will be an excellent approximation. If $N > 2$, largeness of s does not guarantee large subenergies s_{ij}. We must then use severe cuts on the data (and hence the phase space ϕ) to ensure large s_{ij}'s. Then, the assumed t dependence of the residues will allow us to perform free integrals in the t's without appreciable overcounting.

If we relax the constraints on the s_{ij}'s, we face the prospect of double counting, by doing free t integrals over phase space--we run through the same region of phase space several times, each time integrating a different approximation for $|M|^2$. When we do this, we must be cognizant of this error.

We urge the reader to read Ref. 3, where the author deals with the cross sections for the reactions $p\bar{p} \to m\pi^+ + m\pi^- + k\pi^0$. Apart from his remark on interference terms, we find that his paper adheres to the above rules.

IV. LITERATURE SAMPLING

We now discuss briefly, a sample (by no means exhaustive), of instances where concepts A and B, mentioned earlier, are encountered.

Ref. 4,5: Theoretical papers that assume M is a sum of pieces from all diagrams obtained by permuting final particle legs. It is argued in Ref. 4 that the interference terms are negligible, while Ref. 5 exploits their importance.

Ref. 6: A double-Regge analysis of $\pi^+p \to \pi^+\rho^0 p$ at 13.1 GeV/c. Achieves a good fit by phase space overcounting, of the type discussed earlier (by admitting small s_{ij} regions). It is shown that a coherent addition of amplitudes obtained by permuting external legs is in disagreement with data.

Ref. 7: Fits data by coherent addition of permuted pieces in double-Regge analysis of $K^-n \to K^-\pi^-p$ at 5.5 GeV/c.

Ref. 8: A study of $pp \to pp + 2\pi^+ + 2\pi^-$ at 23 GeV/c. Gets M by

(a) Adding diagrams corresponding to different ordering of the protons in the chain (allowing them to go at the most one link from the ends).

(b) Symmetrizing by hand with respect to identical pions.

We find that a common trend in current phenomenology is to fit the Regge parameters of various diagrams in regions of phase space where they best approximate the amplitude, and then, to use their sum, coherent or incoherent, to get the cross sections in the rest of phase space. Since such fits involve multiple counting in the amplitude or phase space, they neither verify nor vilify the BCP multi-Regge hypothesis.

How then are we to test the above hypothesis? The heart of the multi-Regge hypothesis is that in certain special regions of phase space, the $2 \to N$ amplitude may be described by a few factorizable Regge poles. Factorizability implies that the trajectory and residue of a Regge pole, deduced in one situation, may be used in other situations where it occurs. We therefore suggest the following type of test of the hypothesis. For example, we could consider the region appropriate to the multi-Regge diagram of Fig. 5. The end couplings, $\beta_{\pi\pi\rho}(t_1)$ and $\beta_{ppP}(t_2)$ are known from pi-nucleon scattering. We can thus measure the middle coupling $\beta_{\rho\rho P}(t_1,t_2,\omega)$ (where P is the pomeron).

This residue, together with $\beta_{ddP}(t)$, measured from, say, πd scattering, must then fully determine M in the region corresponding to Fig. 6, if the multi-Regge hypothesis is correct.

It may be argued that the BCP hypothesis is not the, but a, multi-Regge hypothesis, and therefore, theorists and phenomenologists need not adhere to the rules it implies. Though we do not share such skepticism, we nevertheless wish to say this: Any multi-Regge theory, which is a natural generalization of $2 \to 2$ Regge theory, will likewise seek asymptotic expansions of M in certain special regions of phase space. Such expansions will be alternate and not additive, just as in $2 \to 2$ theory. Adding diagrams obtained by permuting external legs has a natural and legitimate place in perturbative field theory and in the reflexes of its expert practitioners, but not in any S-matrix calculation like $2 \to 2$ Regge theory or its generalization.

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FIGURE CAPTIONS

Fig. 1. Toller diagram for $a + b \to 1 + 2 + \cdots N$.

Fig. 2. Rapidity plot for multi-Regge region of Fig. 1.

Fig. 3. Multi-Regge diagram depicting $M_1^{(1)}$ of Eq. (2).

Fig. 4. Rapidity plot in multi-Regge region of ϕ_2.

Fig. 5. $\pi^+ p \to \pi^+ \rho^0 p$ in double Regge region.

Fig. 6. Double Regge region of $\pi^+ d \to \pi^+ \rho^0 d$.

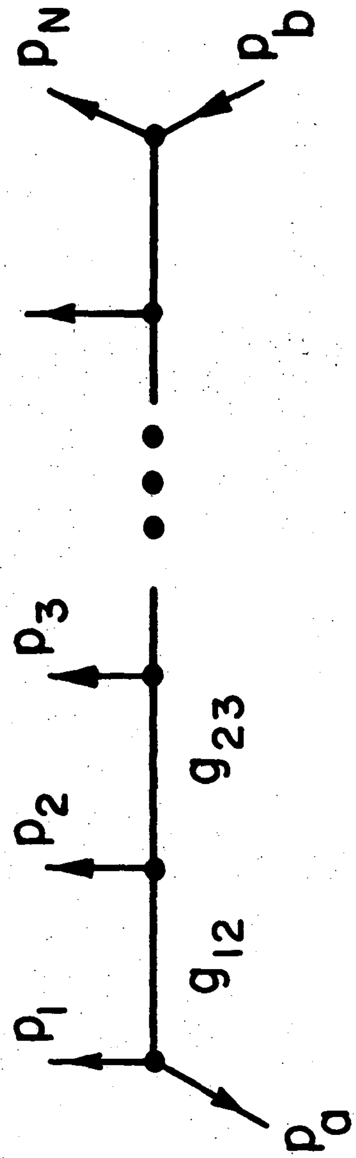

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Fig. 1

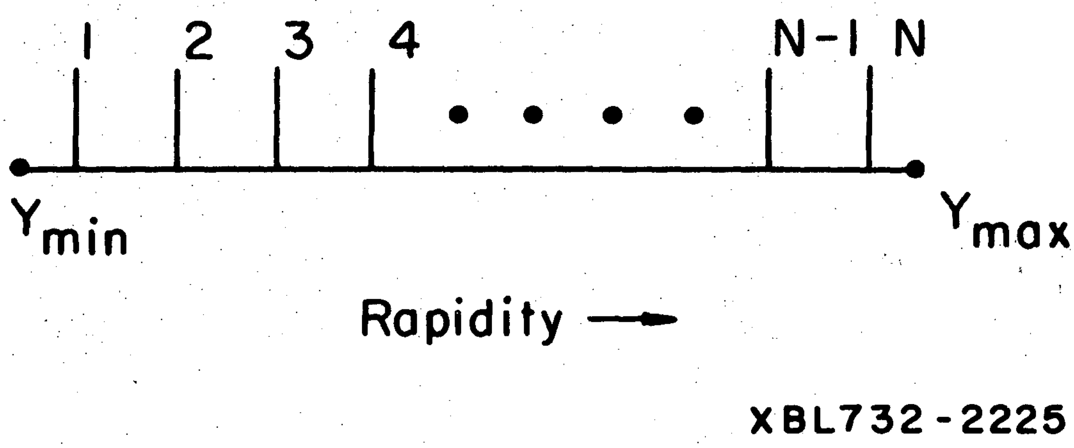

Fig. 2

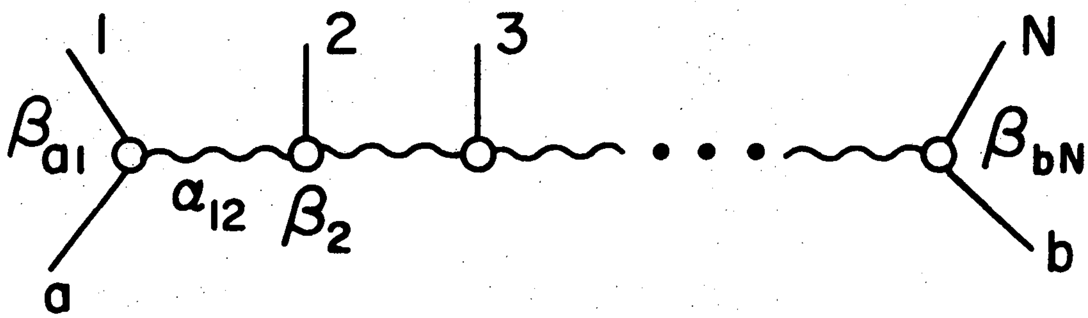

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Fig. 3

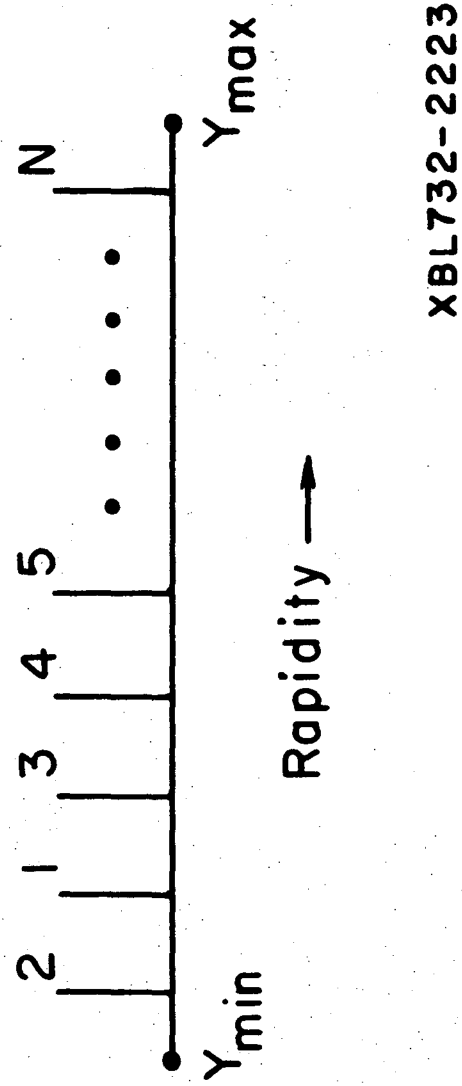
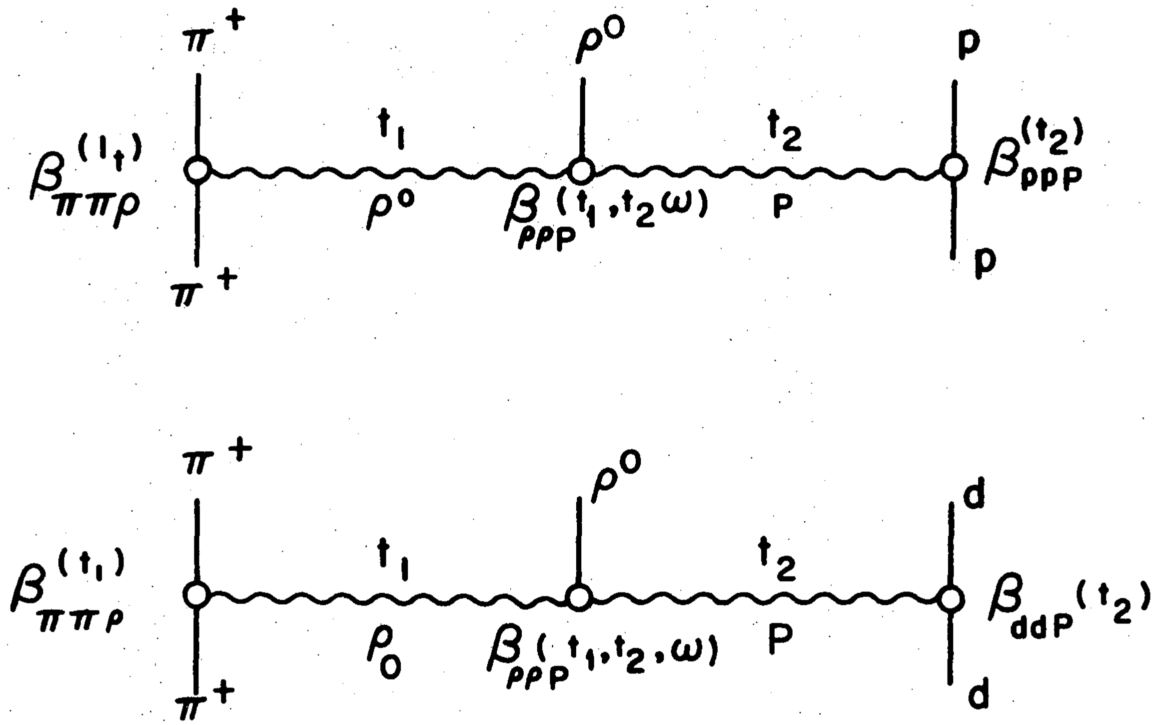


Fig. 4



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Figs. 5 and 6

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