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Authors
Barletta, W A
Sessler, A M

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W.A. Barletta and A.M. Sessler

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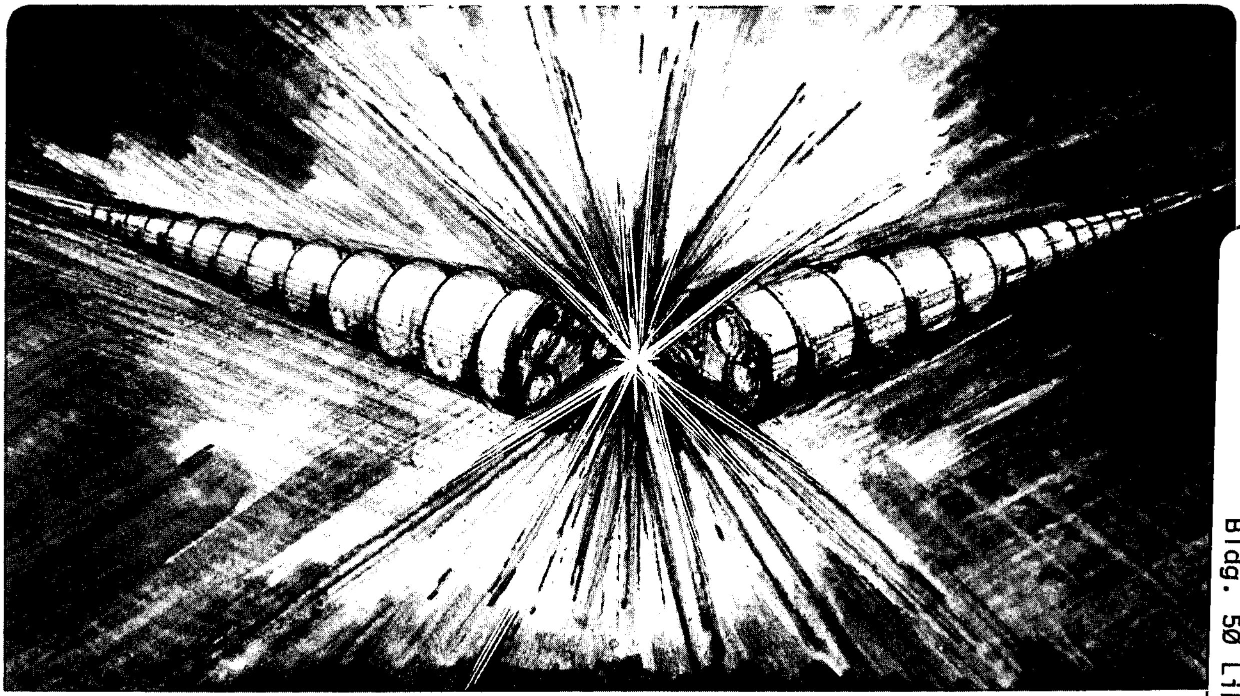

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William A. Barletta

and

Andrew M. Sessler

Accelerator and Fusion Research Division
Lawrence Berkeley Laboratory
University of California
Berkeley, CA 94720

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STOCHASTIC COOLING IN MUON COLLIDERS

William A. Barletta and Andrew M. Sessler
Lawrence Berkeley Laboratory, Berkeley CA 94720

ABSTRACT

Analysis of muon production techniques for high energy colliders indicates the need for rapid and effective beam cooling in order that one achieve luminosities > 10^{30} cm^{-2}s^{-1} as required for high energy physics experiments. This paper considers stochastic cooling to increase the phase space density of the muons in the collider. Even at muon energies greater than 100 GeV, the number of muons per bunch must be limited to ~10^3 for the cooling rate to be less than the muon lifetime. With such a small number of muons per bunch, the final beam emittance implied by the luminosity requirement is well below the thermodynamic limit for beam electronics at practical temperatures. Rapid bunch stacking after the cooling process can raise the number of muons per bunch to a level consistent with both the luminosity goals and with practical temperatures for the stochastic cooling electronics. A major advantage of our stochastic cooling/stacking scheme over scenarios that employ only ionization cooling is that the power on the production target can be reduced below 1 MW.

1. INTRODUCTION

In a previous paper we have analyzed the design of a muon collider with a center-of-mass energy of 200 to 400 GeV with a time-averaged luminosity > 10^{30} cm^{-2}s^{-1} [1]. In that scheme, the muons are generated as secondary beams from an electron beam striking a production target. The muons which emerge from the target are gathered and accelerated rapidly to an intermediate energy and cooled, at which point they can be accelerated to the final, high energy and injected into a storage ring collider with superconducting magnets. The chief advantage of producing the muons with an electron beam from a high energy, linear accelerator is that the muons are naturally formed in short bunches (< 1 cm) for subsequent acceleration to the desired high energy in a linear accelerator. As the muons retain their short bunch length in the collider, a low β interaction region can be employed.

The analysis of ref. 1, based on the use of ionization cooling [2] of the muon bunches, concluded that even with optimistic assumptions, it is difficult to envision a 200 GeV $\mu^+\mu^-$ collider functioning with a luminosity > 10^{27} cm^{-2}s^{-1}. In the scenario of Ref. 1 achieving a luminosity $\approx 10^{30}$ cm^{-2}s^{-1} would require major advances in several of the constituent technologies: superconducting dipoles and quadrupoles, multi-kiloampere electron beam sources, and multi-megawatt muon production targets.

In this paper we consider the extent to which the inclusion of a stage of stochastic cooling after the ionization cooler can improve the prospects of designing muon colliders with luminosity exceeding 10^{30} cm^{-2}s^{-1}.

2. COLLIDER CONSIDERATIONS

The number of muons per bunch, $N_\mu{}^*$, that circulate in the collider will be determined by the production efficiency, A_μ, by the charge, N_e in the electron bunch that strikes the production target, and by the path length through the cooling lattice. The electron bunches will be produced in a macropulse of duration, τ_e, that is chosen to match the circulation period. If the average dipole field in the collider is B_{ave}, and if the muon energy is E_μ, the circulation period will be

$$\tau_e = 2\ \mu s \left(\frac{3\ T}{B_{ave}}\right)\left(\frac{E_\mu}{100\ GeV}\right). \tag{1}$$

For N_b bunches per macropulse, the collision frequency will be

$$f_{coll} = \frac{N_b}{\tau_e}. \tag{2}$$

The electron linac is pulsed at a rate, $\tau_\mu{}^{-1}$, where τ_μ is the muon lifetime as seen in the laboratory. As the duty factor of the linac is τ_e/τ_μ, the average power of the electron beam on the muon production target is

$$P_{beam} = \frac{N_b\, N_e}{\tau_e}\,\frac{\tau_e}{\tau_\mu}\,E_e = \frac{N_b\, N_e}{\tau_\mu}\,E_e. \tag{3}$$

The peak luminosity of the collider containing muons with a geometrical emittance, ε, can be written as

$$L = \frac{N_\mu^{*\ 2}\, f_{coll}}{4\pi\, \varepsilon\, \beta^*}\ , \tag{4}$$

where $\gamma = E_\mu/\mu$ and β^* is the value of the beta function at the collision point. Following the analysis of ref. 1, we obtain the average luminosity of a collider of repetition rate, R,

$$\langle L \rangle = \frac{A_\mu^2\, N_e^2\, N_b\, \gamma\, C_\mu}{4\pi\, r_{sh}\, \phi_{accept}\, \beta^*\, \tau_e}\left(\frac{\gamma}{\gamma_{prod}}\right)\exp\left(-\frac{2\, L_{cool}}{c\, \tau_\mu\, \gamma_c}\right)\left[1 - \exp\left(\frac{2}{\tau_\mu\, \gamma\, R}\right)\right]\left(\frac{\tau_\mu\, \gamma\, R}{2}\right). \tag{5}$$

In eq. (5) r_{sh} is the shower radius, ϕ_{accept} is the acceptance angle of the muons, and C_μ accounts for the possibility of cooling the muons; in the absence of transverse cooling, $C_\mu = 1$ and $L_{cool} = 0$.

3. STOCHASTIC COOLING SCENARIOS

The average luminosity of a muon collider will be far more sensitive to the decay lifetime of the muons than to the depopulation of the beams due to colisions. Therefore, for maximum luminosity, the beam emittance should be made as low as possible. In dealing with unstable particles such as muons, the technique of ionization cooling has the advantage that the rate of cooling the beam to the limiting emittance is independent of the number of particles. The limiting emittance of the beam is set by multiple scattering in the cooling cells. Our previous analysis suggests that it will be difficult to increase C_μ to > 1000 via ionization cooling.

Rather than relying solely on ionization cooling, one might also consider using stochastic cooling in a linear array [3] or in a storage ring to reduce the emittance of the muon beams. The difficulties of this approach derive from the limited number of particles that can be effectively cooled stochasticly. In a single pass by the pickup and kicker electrodes of the ring the emittance of the beam can be reduced by an amount

$$\frac{\Delta\varepsilon}{\varepsilon} = \frac{g}{N_{bunch}} . \tag{6}$$

with optimized amplifiers $g = 1$.

In eq. (6) N_{bunch} is the number of particles that are within the resolution bandwidth of the electronics. For our purposes, a bandwidth of 3 GHz is a reasonable assumption that would match the use of an S-band rf-system in the ring. For a ring with revolution frequency, f_s, containing P pairs of pickups and kickers, the maximum rate at which the emittance can be damped is

$$\alpha_s = \frac{f_s P}{N_{bunch}} . \tag{7}$$

If the beam of initial, geometrical emittance, ε_o, is cooled over a period of ζ muon lifetimes, then the final emittance of the bunch will be

$$\varepsilon_f = \varepsilon_o \exp\left[- \alpha_s \zeta \gamma \tau_\mu \right] . \tag{8}$$

Consider that in the absence of cooling the luminosity of the collider is L_{nc}; then for the collider to operate with a required luminosity, L_{req}, the cooling rate and cooling time should satisfy

$$\zeta \gamma \tau_\mu (2 - \alpha_s) = \ln \left[\frac{L_{nc}}{L_{req}} \right] . \tag{9}$$

For rapid damping $\alpha >> 1$; hence, for stochastic cooling of a 100 GeV muon beam

$$\zeta \frac{f_s P}{N_{bunch}} \approx 455 \ln \left[\frac{L_{req}}{L_{nc}} \right] . \tag{10}$$

Using eq. (6), we can rewrite eq. (10) as

$$\zeta \frac{f_s P}{N_{bunch}} \approx 455 \ln \left[\frac{4\pi \beta^* \varepsilon_o L_{req}}{N_{bunch}^2 f_{coll}} \right] . \tag{11}$$

Solving of eq. (11) for a system to reach any desired luminosity is complicated by the implicit dependence of ε_o on N_{bunch}. The indentification of a starting point for numerical investigations is simplified by the slow variation of the right-hand-side of the equation. For 100 GeV muons in a stochastic cooling ring with an average dipole field of 6 T and with 5 feedback pickups and kickers per ring, $f_sP \approx 3 \times 10^6$. Further numerical experimentation with eq. (11) suggests that if stochastic cooling is to be useful, the number of particles per bunch must be limited to the order of 10^3. It follows that the number of bunches in the collider ring must be as large as possible and that the original emittance must be as low as possible. Applying the considerations described above to the case of electro-production of muons, we find that a very high luminosity muon collider with stochastic cooling might have the characteristics described in Table 1.

Table 1. Characteristics of a 100 GeV × 100 GeV muon collider using electro-production, ionization cooling and stochastic cooling. The quantities with daggers require technological inventions or are of dubious validity.

Production			**Stochastic Cooler**	
E_e (GeV)	30		E_{cool} (GeV)	100
P_{beam} (MW)	1		Number of rings	1
N_e (particles)	3.3×10^8		No.of feedbacks	8
$N_{bunches}$	3000		$\langle B_d \rangle$ (T) in arcs	$6^\dagger$
E_{accept}(GeV)	4.5		V_{ring} (GeV/turn)	< 0.05
$(\Delta p/p)_\mu$ (%)	± 2		C_{ring} (m)	260
N_μ(nC)	3.6×10^4		Muon lifetimes	1.55
ε_n (π m-rad)	4.3×10^{-5}		$\varepsilon_{n,eq}$ (π m-rad)	1.8×10^{-16} †
Ionization Cooler			C_μ-tot	2.3×10^{11}
E_{cool} (GeV)	60			
Number of rings	1		**Collider**	
F_{cool}	0.6		Repetition rate (Hz)	200
$\langle B_d \rangle$ (T) in arcs	4.5		N^*_μ (particles)	530
V_{ring} (GeV/turn)	0.3		N_{bunch}	3000
C_{ring} (m)	1110		B_{ave}(T)	4
(B_q (T), a_q (cm))	$(7^\dagger, 1)$		$C_{collider}$ (m)	520
β_{cool} (cm)	0.1		f_{coll} (GHz)	2
$\varepsilon_{n,eq}$ (π m-rad)	4.1×10^{-7}		β^* (cm)	0.4
N_μ-out (particles)	2500		$(\Delta E/E)_{collider}$ (%)	< 0.1
C_μ	103		$\langle L \rangle$ ($cm^{-2}s^{-1}$)	1.2×10^{30}

The algebra suggests that the muon emittance can be reduced by eleven orders of magnitude to the incredible value of $1.8 \times 10^{-16}\ \pi$ m-rad. Why is this "solution" absurd? First, we should check that the beam-beam tune shift is a reasonable value. The head-on tune shift is given by

$$\Delta\nu \approx \frac{r_\mu N_\mu}{4\pi\varepsilon} . \qquad (12)$$

where r_μ is the classical radius of the muon. Applying eq. (12) to the case of Table 1, we find that $\Delta\nu >> 1$. Hence, the beam disruption will be so large that the beams cannot be made to collide many times.

Second, for the parameters of Table 1 the beam's transverse temperature is << 1 °K. However, the beam cannot be colder than the pick-ups, kickers, and electronics, which we can assume are all maintained at a temperature, T_f. In the best possible situation, the amplifiers do not magnify the thermal noise that is impressed on the beam. At pickups and kickers, assume the beta function is β_f. In that case minimum normalized emittance will be

$$\varepsilon_{n,\min} = \beta_f \frac{2\,kT_f}{m_\mu c^2} . \qquad (1$$

Let $T_f = 1$ °K and $\beta_f = 5$ cm, then $\varepsilon_{n,\min} \approx 10^{-12}\ \pi$ m-rad. Using this limiting emittance in the example of Table 1, we find that the tune shift is reduced dramatically to $\Delta\nu \approx 0.001$, but the time-average luminosity is likewisw reduced to only $\approx 10^{27}$ $cm^{-2}\ s^{-1}$. This value may seem discouragingly low; however, it is not so far from what is needed. Is some invention possible?

Noting that we are allowed to increase the tune shift by roughly two orders of magnitude, we look for a way to increase the number of particles per bunch without affecting the damping rate. The obvious solution is to cool a large number of bunches in an intermediate storage ring and then to stack the bunches in the collider.

A difficulty with simply stacking directly into the collider is that the number of revolutions to accomplish the stacking would be equal to the number of bunches in the stochastic cooling ring, $N_{b,st}$. Unfortunately, in a ring with average dipole field B_{ave} the lifetime of the muons is only 300 B_{ave} turns. Too many muons would decay during the stacking process. This limitation can be overcome by the use of one or more intermediate accumulator rings. For simpilcity of design the accumulator may be assumed to have the same size and average dipole field as the cooler rings. Such an approach is illustrated in figure 1. In an arrangement with one intermediate accumulator the complete stacking can be accomplished in as few as $2\sqrt{N_{b;st}}$ turns. The technique of bunched beam cooling followed by stacking should yield $\langle L \rangle$ as high as 10^{31} $cm^{-2}\ s^{-1}$ at 200 GeV.

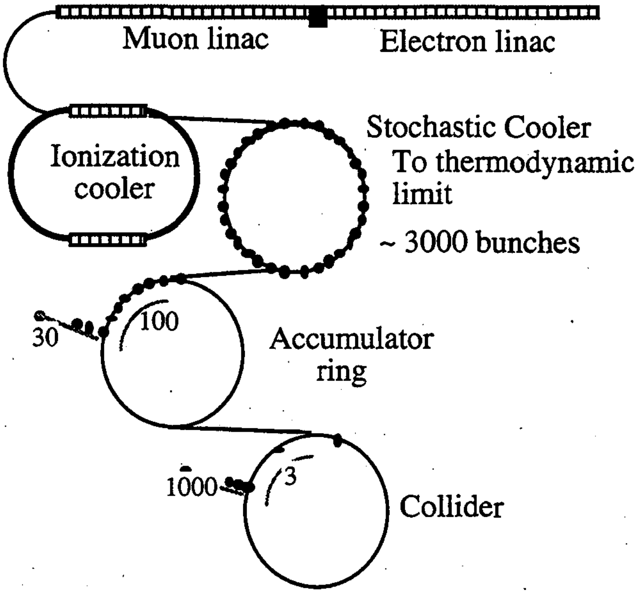

Figure 1. A schematic of the scenario of bunched beam cooling followed by stacking in a intermediate accumulator ring.

To assure a small beam size and a short bunch length we propose stacking the beams in "synchrotron space" as illustrated in Figure 2. If the 4-sigma energy spread in the cooler is Δp_{cooler}, then after cooling is complete, we must introduce an energy increment on every turn $\geq \Delta_1 = \Delta p_{cooler}$. Once the cooler is filled, we must then introduce an energy variation per turn of Δ_2, where

$$\Delta_2 \approx N_{stack}\, \Delta p_{cooler} \approx \sqrt{N_{bunch}}\, \Delta p_{cooler} \,. \tag{14}$$

Hence, the total energy spread will be

$$\Delta p_{tot} \approx N_{bunch}\, \Delta p_{cooler} \frac{E_{cooler}}{E_{collider}} \,. \tag{15}$$

If we require that the energy spread of the muon bunch in the collider, $\sigma_{E,coll}$ be 0.5%, then Δp_{cooler} should be $\approx 10^{-6}$. Such a small energy spread can be achieved only if the muons are originally selected in a rather tight momentum band ($\approx$1%) around a relatively low central energy (for example, 2 – 5 GeV). The small, final energy spread is achieved through a combination of adiabatic damping and longitudinal cooling in the ionization cooler. The consequence of this choice is a decreased conversion efficiency from electrons to muons.

The choice of stacking the bunches in synchrotron space also requires that the accumulator rings and collider be designed with a rf-system (and synchrotron frequency) carefully matched to the the phase space configuration of the stacked beams. Insufficient attention to this detail could lead to bunch lengthening and reduced luminosity.

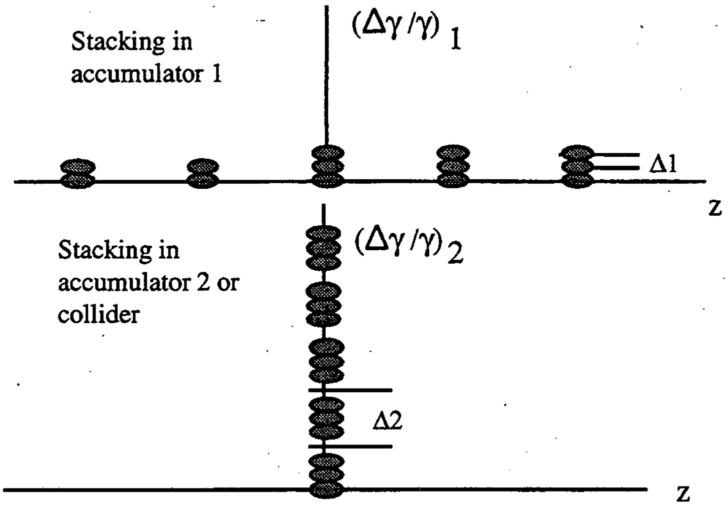

Figure 2. Schematic of stacking the muon bunches in synchrotron space

Contrary to scenarios employing ionization cooling only, the luminosity of the muon collider with stochastic cooling will be tune shift limited. In this case, the luminosity is proportional to $N_\mu \varepsilon^{-0.5}$ while the tune shift is proportional to $N_\mu \varepsilon^{-1}$. Hence, the tune shift limit can be avoided by simultaneously raising the emittance and the number of muons per bunch in the collider. Unfortunately, the number of muons per bunch in the cooler is limited to $\sim 10^3$. Hence, raising N_μ must be accomplished by increasing the number of bunches in the cooling rings.

Raising N_{bunch} in turn implies increasing the energy spread of the muon bunch in the collider. If the increase in energy spread due to two stage of stacking in synchrotron space is too large to be acceptable, the second stacking can be performed in the transverse plane with a consequent sacrifice of beam emittance and luminosity. This second approach has been adopted in the example of Table 2. Note that in our example the power on the production target is only 250 kW. The long bunch train (5000 bunches) would require the use of at least 2 rings for the first stage of stacking. In this example, the energy spread after cooling is $\approx 10^{-5}$; hence, the bunch-to-bunch energy variation that is required for the first stage of stacking can be accomplished with an rf-cavity.

One of the chief advantages of the stochastic cooling scenarios vis á vis ionization cooling is that the electron beam power on the production target is not a critical parameter. To some extent this lack of sensitivity could be viewed as a disadvantage were one really able to design targets that could withstand 5 MW of beam power. In particular, it appears difficult to raise the luminosity by raising the power on target except by lengthening the pulse macropulse from the electron linac. One might, for example, view a 5 MW production target as an integral feature of any 5 TeV on 5 TeV e^+e^- or $\mu^+\mu^-$ collider. A cursory examination of scaling stochastic cooling schemes to such high energy suggests that it is difficult to take advantage of the faster muon production rate which a 5 MW target would provide. Specifically, the luminosity seems to increase slower than linearly with energy - much more slowly than needed to overcome the rapidly decreasing cross sections.

Table 2. The characteristics of a 100 GeV × 100 GeV muon collider using electro-production, ionization cooling and stochastic cooling. A single accumulator ring has the same circumference and average dipole field as the stochastic cooling ring. The quantities with daggers require technological innovation.

Production			**Stochastic Cooler**	
E_e (GeV)	50		E_{cool} (GeV)	100
P_{beam} (MW)	0.25		Number of rings	3
N_e (particles)	3×10^7		No.of feedbacks	12
$N_{bunches}$	5000		$\langle B_d \rangle$ (T) n arcs	6.4 †
E_{accept}(GeV)	10		V_{ring} (GeV/turn)	< 0.05
$(\Delta p/p)_\mu$ (%)	± 1		C_{ring} (m)	322
N_μ(nC)	7.3×10^3		Muon lifetimes	1.04
ε_n (π m-rad)	9.5×10^{-5}		$\varepsilon_{n,eq}$ (π m-rad)	1.05×10^{-12}
			C_{μ}-tot	9×10^7
Ionization Cooler			**Accumulator**	
E_{cool} (GeV)	50		$\langle B_d \rangle$ (T) n arcs	6.4 †
Number of rings	1		C_{ring} (m)	322
Frac. of cooling cells	0.6		**Collider**	
$\langle B_d \rangle$ (T) in arcs	4.5		Repetition rate (Hz)	200
V_{ring} (GeV/turn)	0.3		N^*_μ (particles/bunch)	4.6×10^6
C_{ring} (m)	922		N_{bunch}	1
(B_q (T), a_q (cm))	(5, 1.5)		$\varepsilon_{n,eq}$ (π m-rad)	7.1×10^{-11}
β_{cool} (cm)	0.1		B_{ave}(T)	6
$\varepsilon_{n,eq}$ (π m-rad)	9.5×10^{-6}		$C_{collider}$ (m)	346
N_μ-out (particles)	2700		f_{coll} (MHz)	1
C_μ	10		β^* (cm)	0.3
			$(\Delta E/E)_{collider}$ (%)	0.1
			$\langle L \rangle$ (cm^{-2} s^{-1})	1.2×10^{30}

4. PROSPECTS AND CONCLUSIONS

Adding a stochastic cooling stage between the ionization cooler and the main collider ring leads to designs very different from the scenarios outlined in Ref. 1. The collider would have to contain a large number of bunches with few particles per bunch. The low initial emittance desired prior to the stochastic cooling stage must be provided by making tight cuts on both the angular spread and the momentum spread of the muons accepted from the target. As the acceleration system upstream of the production target will accept muons of a fixed geometrical emittance, one should now choose to accept lower energy muons than would be optimum in the scenarios with ionization cooling only. Accelerating the bunches to the full energy of the collider before either ionization or stochastic cooling will adiabatically damp the momentum spread of the muons that the cooling lattices must accept to values of order of ±0.1%. The stage of ionization cooling that precedes the stochastic cooler both lowers the normalized emittance further and reduces the number of particles per bunch so that the stochastic cooling can proceed more rapidly.

While our proposed approach seems to have eliminated the need for a number of the technological inventions demanded by ionization cooling, the chief difficulty is that the incredibly small normalized emittance required will put extremely severe (and perhaps unrealizable) demands on noise levels permissible in the feedback electronics and for the magnet power supplies. Additional difficulties of the approach include the need for efficient, rapid beam transfer among the several storage rings. These issues requires detailed study before one can consider the stochastic cooling option as more than a mathematical construct.

ACKNOWLEDGMENTS:

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LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
TECHNICAL INFORMATION DEPARTMENT
BERKELEY, CALIFORNIA 94720