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J. Mahoney, M. C. Mermaz and D. K. Scott

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the shapes of the inclusive energy spectra as well as for the isotope yields at both energies. The relative cross sections are target independent (i.e., factorize) for the data at 315 MeV incident energy, but limiting fragmentation (i.e. isotope yields independent of energy) applies only at higher energies. The differential cross sections peak at the grazing angle or decrease exponentially, depending on the incident energy and the mass of the ejectile; the distributions are described by classical trajectories and diffraction models for which the reaction time is estimated to be typical of direct reactions. The kinetic energies of the reaction products depend primarily on their charge and only slightly on their mass number. For the 315 MeV results, the velocities of the reaction products at the maximum of the spectrum and at the grazing angle are slightly less than the beam velocity and decrease rapidly for larger scattering angles. The results are interpreted with simple friction and fragmentation models, and are compared to observations at 33.6 GeV.

ENERGY DEPENDENCE OF PERIPHERAL
REACTIONS INDUCED BY HEAVY IONS*

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ABSTRACT

Collisions of ^{16}O ions on targets of ^{94}Zr , ^{197}Au , ^{208}Pb , and ^{232}Th are investigated at incident energies of 140 and 315 MeV, and compared to published data on peripheral collisions at 33.6 GeV. At 140 MeV, the isotope-production cross sections resemble the ground-state Q-value systematics characteristic of a partially-equilibrated, di-nuclear system formed in deeply-inelastic scattering. The yields are independent of shell and pairing effects in the target and residual nuclei. These Q-value systematics fail at 315 MeV, where the relative element yields are similar to those at 33.6 GeV. A theory of projectile fragmentation accounts for

1. INTRODUCTION

With the availability of beams as heavy as uranium up to energies of 10 MeV/A and of lighter ions up to energies of 2 GeV/A, it has become feasible to study the characteristics of nucleus-nucleus interactions over a wide range of incident energies and for a large variety of colliding nuclei. A substantial amount of data is now available for heavy-ion collisions at energies of a few MeV/A above the Coulomb barrier [for recent references see e.g., (Refs. 1-4)], and at relativistic energies of a few hundred MeV/A to 2 GeV/A. The intermediate energy range of a few tens to a few hundreds of MeV/A is still largely inaccessible with existing accelerators.

The significance of this energy region is illustrated in fig. 1, which relates the relative velocities (v) in heavy-ion collisions at touching distance, and the associated interaction times (t), to the kinetic energies per nucleon above the Coulomb barrier.⁵ Most heavy-ion research has been performed at relative particle velocities of $v/c < 0.1$. Since these velocities are smaller than the average nucleon velocity inside nuclei (the average Fermi velocity v_F/c is approximately 0.2, corresponding to a transit time for nuclear dimensions of $t = \tau_N$), the macroscopic features of reactions may be characterized by a variety of equilibrating phenomena ranging from non-equilibrating direct, quasi-elastic and deeply-inelastic

collisions^{3,4,6} with various degrees of equilibration⁷⁻¹² to complete fusion reactions^{13,14} in which statistical equilibration of the composite system is finally attained. Recent experiments have been performed¹⁵⁻¹⁸ at incident energies between 1-2 GeV/A. Here the relative particle velocities ($v/c \approx 0.9$) considerably exceed the Fermi velocity, and peripheral collisions are characterized by rapid processes such as the abrasion of nucleons or projectile fragmentation. Because of the short interaction times, equilibration phenomena are considered to be unimportant during the primary interaction of the two nuclei. Nevertheless the subsequent statistical decay of the primary reaction products can still be of major importance for understanding the observed particle yields.¹⁸⁻²⁴ The incident energy of 20 MeV/A marks a transition between these two regimes, since the corresponding velocity ($v/c \approx 0.2$) is close to the Fermi velocity. At such a boundary, qualitative changes in the characteristic heavy-ion interactions might be expected to occur.^{18,25} This energy also defines the threshold of the supersonic region, where particle velocities exceed the velocity of sound in nuclear matter.^{18,26,27} A second transition region may be defined^{18,25} at energies of 140 MeV/A, corresponding to the meson threshold. In this domain only a few experimental results are available²⁸ and indicate that the reaction yields closely obey two asymptotic limits, originally developed for elementary-particle interactions at relativistic energies.^{29,30} The "limiting fragmentation" hypothesis predicts that spectra and yields of fragments will become independent

of bombarding energy at sufficiently high energies, and the "factorization" hypothesis asserts that spectra and yields of projectile (or target) fragments will depend on the target (or projectile) only via trivial geometrical factors.

In this paper we report on experiments performed with ^{16}O ions of 140 and 315 MeV beam energy on heavy targets ($A > 90$), which provide a link between low-energy ($[E-V]/A \leq 5 \text{ MeV/A}$) equilibration phenomena and high-energy ($[E-V]/A \geq 1 \text{ GeV/A}$) projectile fragmentation phenomena. Our initial results^{31,32} indicated that the particle-production cross sections are significantly different at 140 and 315 MeV but rather similar at 315 MeV and 33.6 GeV. The data emphasize the need for a theory incorporating aspects of both deeply-inelastic and fragmentation phenomena. In the absence of such a theory, we approach the data from several existing qualitative viewpoints.

Experimental details are given in Section 2. In Section 3 the energy spectra are presented and discussed in terms of a semiclassical theory of direct reactions, and of a fragmentation model developed for similar reactions at relativistic energies. The most probable kinetic energy losses are compared with various classical models used for the interpretation of quasi-elastic and deeply-inelastic reactions, and with experimental results^{15,16} obtained at relativistic energies. The angular distributions are presented in Section 4 and are discussed in terms of classical trajectories as well as diffraction and Regge-pole

parameterizations. From the shapes of the angular distributions, the reaction time scale is estimated. In Section 5 the peripheral nature of the reactions in relation to central collisions is treated. The characteristics of the energy spectra and angular distributions are compared in Section 6 to deeply-inelastic scattering phenomena observed at energies only a few MeV/A above the Coulomb barrier. In Section 7 the ground-state Q-value systematics^{33,34} for isotope-production cross sections observed in deeply-inelastic scattering are applied to the cross sections at 140 MeV, and evidence is presented that shell effects and pairing energies of the residual nuclei are of minor importance for the understanding of relative particle yields. The energy dependence of the cross sections is discussed in Section 8. The isotope yields are compared with the predictions both of the ground-state Q-value systematics and of a simple ablation model.²² Whereas the ground-state Q-value systematics fail to reproduce the trends of the isotope-production cross sections at 315 MeV and 33.6 GeV, the ablation model accounts rather well for the experimental particle yields. The similarity of cross sections at 315 MeV and 33.6 GeV is emphasized, together with the implications regarding the hypothesis of limiting fragmentation. In Section 9 the target dependence of the cross sections is discussed and it is shown that, even at incident energies as low as 315 MeV, the concept of factorization applies over the range of target nuclei investigated. A summary and conclusions are given in Section 10.

2. EXPERIMENTAL DETAILS

The experiments were performed with 140 and 315 MeV ^{16}O beams produced by the 88-inch cyclotron at the Lawrence Berkeley Laboratory. Cross sections were obtained for targets of ^{94}Zr , ^{197}Au , ^{208}Pb , and ^{232}Th of 0.5, 1.8, 0.7 and 10 mg/cm^2 thickness, respectively. Isotope identification was achieved by means of triple ΔE - ΔE -E solid-state detector telescopes of 17, 26, 3000 μm and 40, 80, 3000 μm thickness at the energies of 140 and 315 MeV, respectively. Some spectra at 315 MeV were also taken with the 17, 26, 3000 μm detector telescope in order to increase the dynamic range; for these spectra, however, the isotope separation in the high kinetic energy region of the energy spectra was insufficient for a quantitative analysis of isotope-production cross sections (see Section 3.1). The data were stored as multiparameter events on magnetic tape and analyzed off-line.

Absolute cross sections were obtained from the integrated beam current, the target thickness and the solid angle ($\Delta\Omega = 0.17$ msr) of the detector telescope. From a comparison of this absolute normalization with that derived by measuring ^{16}O elastic scattering on ^{197}Au , ^{208}Pb and ^{232}Th at small angles, where the deviations from Rutherford scattering are only a few percent, the associated uncertainties in this normalization are estimated to be less than 20%. The absolute error of the cross sections for the ^{94}Zr target is estimated to be smaller than 25%.

In order to reduce the effect of different angular distributions for different reaction products (see Section 4), cross sections at 140 MeV incident energy were integrated over angular intervals of 15°-30° for the ^{94}Zr target and 20°-45° for the ^{197}Au and ^{208}Pb targets. At 315 MeV, the cross sections were measured between 10° and 20° for the ^{197}Au and ^{232}Th targets and between 6° and 50° for the ^{208}Pb target; for the ^{94}Zr target, the differential cross sections at the laboratory angle of 10° were measured. The angle-integrated particle yields are listed in Tables 1 and 2. In Table 2, the cross sections for ^{208}Pb were obtained by extrapolating the distribution over the entire angular range. The cross sections for particles with $Z > 9$ were estimated to be less than 5% of the cross sections for smaller values of Z .

The cross sections at 33.6 GeV used for the discussion of the energy dependence of the particle yields were taken from Ref. 16.

3. ENERGY SPECTRA

3.1 Characteristic Features of the Energy Spectra

Since the energy spectra obtained from the different targets are qualitatively very similar, only spectra resulting from the reaction of ^{16}O on ^{208}Pb will be discussed in detail. The yields from the various targets are compared later in Section 9.

The energy spectra of nitrogen, carbon, boron, beryllium and lithium ions measured at the laboratory angles $\theta_L = 40^\circ$ for $E_L = 140$ MeV, and $\theta_L = 15^\circ$ for $E_L = 315$ MeV are shown in figs. 2 and 3, respectively. These angles are close to the "grazing" angle where the elastic scattering starts to deviate from Rutherford scattering (see, for example, fig. 15). For orientation, we denote some characteristic energies on the figures by arrows: the energy corresponding to the ground-state (g.s.) of a two-body final state transfer reaction; the energy (E_p) corresponding to a final fragment velocity equal to the initial velocity of the projectile (as will be shown later, this energy is almost equal to the optimum final energy expected from semiclassical matching conditions in a two-body transfer reaction); the energy (E_F) expected from a simple fragmentation process; the exit-channel Coulomb barrier (V_c), calculated as

$$V_c = e^2 Z_1 Z_2 / [r_0 (A_1^{1/3} + A_2^{1/3})], \quad (1)$$

where $Z_{1,2}$ and $A_{1,2}$ denote the charge and mass numbers of the final nuclei, and the parameter $r_0 = 1.44$ fm has been used. These quantities have been calculated for the isotopes with the largest measured cross sections assuming a two-body reaction mechanism. The energy spectra are broad and bell-shaped, and their maxima systematically correspond to smaller energies for lighter elements. At both 140 and 315 MeV incident energy, the spectra near the grazing angle are peaked at energies expected from simple fragmentation processes. For particles close to the projectile mass (nitrogen and carbon isotopes), these peaks also correspond to velocities which are almost equal to the beam velocity. For the lightest particles (lithium isotopes), the energy spectra peak close to the exit-channel Coulomb barrier for the incident energy of 140 MeV; at 315 MeV incident energy, however, they are peaked at energies significantly higher than the exit-channel Coulomb barrier. The energy corresponding to the exit-channel Coulomb barrier is characteristic of complete equilibration of the kinetic energy of relative motion into internal excitation of the fragments as observed in deeply-inelastic scattering.

Whereas the energy spectra close to the grazing angle show only one peak, two peaks are observed in the nitrogen and carbon spectra at 315 MeV incident energy for angles larger than the grazing angle. This effect is illustrated in fig. 4 for the angle of $\theta_L = 25^\circ$, where the low-energy components, peaked near the exit-channel Coulomb barrier, and the high-energy components are of nearly equal strength. These low-energy components will not be discussed in detail, since they constitute a small fraction of the total cross section, and furthermore they may be distorted by reaction products ($\approx 30\%$) from light contaminants in the heavy targets.

Energy spectra showing two broad peaks are well known from studies of deeply-inelastic scattering⁶ and have been interpreted as the result of partial orbiting of the colliding nuclei which leads to large energy losses due to a longer action of frictional forces³⁵ (see also the discussions given in Sections 4 and 6).

A comparison of the shapes of the spectra shown in figs. 3 and 4 might suggest the presence of two unresolved components in the boron spectra, such that the high-energy component dominates at forward angles and the low-energy component dominates at backward angles. Since the separation of these components is arbitrary, the entire spectrum is integrated for the discussion of the isotope-production cross sections. Contributions from light contaminants amount to less than 10% in the boron spectra and to less than 3% in the beryllium and lithium spectra.

3.2 Semiclassical Analysis of the Energy Spectra

For the case of the single-nucleon stripping, the low-excitation region of the energy spectra (shown in fig. 5) contains well-separated transitions to single-particle states which can be described within the framework of direct reaction theories.³⁶ However, only few attempts have been made to extend these theories to the description of the complete energy spectrum. First promising results have been obtained by incorporating statistical aspects into direct reaction theories.³⁷⁻³⁹ The present data could provide an interesting test case for these theories since the nuclear structure of the projectile and target nuclei are well known and a large range of incident energies is covered. However, it is not the aim of the present paper to attempt a microscopic description of the reaction mechanism and, therefore, only gross properties of the energy spectra will be discussed using a semiclassical model.

In a semiclassical model,^{40,41} the transition probability for the direct transfer of a cluster from an initial state, specified by orbital and magnetic quantum numbers $(\ell_1 \lambda_1)$, to a final state $(\ell_2 \lambda_2)$ is given by

$$P(\lambda_2, \lambda_1) \propto \exp \left[- \left(\frac{R \Delta k}{\sigma_1} \right)^2 - \left(\frac{\Delta L}{\sigma_2} \right)^2 \right] \quad (2)$$

where

$$\Delta k = k_0 - \lambda_1 / R_1 - \lambda_2 / R_2 \quad (3)$$

$$\Delta L = \lambda_2 - \lambda_1 + 1/2 k_0 (R_1 - R_2) + Q_{\text{eff}} / \hbar v \quad (4)$$

$$Q_{\text{eff}} = Q - (Z_1^f Z_2^f - Z_1^i Z_2^i) e^2 / R. \quad (5)$$

In these equations, $k_0 = \Delta M v / \hbar$, $R = R_1 + R_2$, ΔM is the mass of the transferred cluster, v is its velocity in the region of transfer, and R_1, R_2 are the radii of the projectile and target nuclei. The parameters σ_1, σ_2 measure the widths of the Δk and ΔL distributions. By including the details of angular-momentum coupling⁴¹ -- which we omit in eq. 2 and in the following discussion of the continuum -- this theory gives a satisfactory description of the transitions to discrete states for a wide variety of heavy-ion reactions.^{41, 42}

For transitions to the continuum, the maximum value of P occurs when $\Delta k = \Delta L = 0$, corresponding approximately to conservation of linear and angular momentum. Since the initial intrinsic spin is small in the reactions discussed here, we set $\lambda_1 = 0$, and then eq. 3 gives

$$\lambda_2 = k_0 R_2 = J_f \quad (6)$$

where J_f is the spin of the residual nucleus, assumed perpendicular to the reaction plane. Then eq. 4 leads to⁴⁰

$$Q_{\text{eff}} = - \Delta M v^2 / 2 = - \frac{\Delta M}{M_p} (E_{\text{c.m.,i}} - V_{\text{c,i}}) \quad (7)$$

where M_p is the projectile mass, $E_{\text{c.m.,i}}$ the incident energy, and $V_{\text{c,i}}$

the entrance-channel Coulomb barrier. Equation 7 gives the preferred energy loss for the reaction and the corresponding final energy of the fragment coincides closely with the arrows labelled E_p in figs. 2 and 3. In general, the experimental values of Q_{eff} are larger than predicted by eq. 7 due to additional energy dissipation (see Sections 3.4, 3.5). For our present purposes we adjust the optimum Q -value to agree with the experimental value, and use eq. 2 to calculate the probability distribution as a function of Q and L . The results for the ($^{16}\text{O}, ^{12}\text{C}$) reaction are shown in figs. 2 and 3; for both incident energies, the value of $\sigma_1 = 3.5$ was used, close to the theoretical value⁴⁰ of π . (For a calculation of the locus of optimum excitation, it is not necessary to specify σ_2 , since for each Q_{eff} the value of λ_2 to make ΔL in eq. 4 identically equal to zero is deduced). This simple calculation fails to reproduce the observed width of the distribution, although it does give rise to an approximately symmetrical distribution of excitation strength about an optimum value, as observed experimentally. In a more detailed treatment it might be necessary to incorporate a correction for the density of levels of simple configurations in the residual nucleus.⁴³ As later discussions in the paper will show, an appreciable contribution to the continuum cross sections at 315 MeV arises from a fragmentation process, rather than a two-body transfer reaction; however, there is a close similarity in the form of the transition

probability calculated for these two reaction mechanisms.⁴⁴ A simple treatment of the spectra at 315 MeV using the theory of projectile fragmentation follows in the next section.

3.3 Analysis of Energy Spectra by Projectile Fragmentation

A simple approach to projectile fragmentation has been applied to heavy-ion collisions at 2.1 GeV/A. In this theory,²⁰ the spectral distributions are governed by the nucleon momenta in the projectile; in the projectile frame of reference, the momentum distribution of the fragments has the form:

$$N(p) \propto \exp \left[- \frac{(p-p_0)^2}{2\sigma^2} \right] \quad (8)$$

where p_0 is the momentum corresponding to the peak of the distribution, of width σ^2 given by

$$\sigma^2 = \frac{\sigma_0^2 M_F (M_P - M_F)}{M_P - 1} \quad (9)$$

M_F and M_P are the masses of the observed fragment and the projectile, respectively. An analysis¹⁵ of reactions at 2.1 GeV/A leads to

$\sigma_0 = 86 \text{ MeV/c}$. (The associated Fermi momentum²⁰ of $P_F = \sqrt{5} \sigma_0 = 192 \text{ MeV/c}$, should be compared to $P_F = 230 \text{ MeV/c}$, obtained⁴⁵ from electron scattering on ^{16}O .)

If, instead of assuming a sudden liberation of virtual clusters, one assumes²⁰ that the projectile has come to thermal equilibrium at an excitation energy of temperature T , eq. 9 follows once more with

$$\sigma_o^2 = m_n T \frac{M_p - 1}{M_p} \quad (10)$$

where m_n is the nucleon mass. The results at 2.1 GeV/A lead to $T = 8.5 \text{ MeV}$.¹⁵

We shall show later in Section 8, that a plausible description of the isotope yields at 315 MeV can be obtained by the decay of an excited ^{16}O nucleus at a temperature of 7.3 MeV, from which $\sigma_o = 80 \text{ MeV/c}$ is obtained, a momentum surprisingly close to the value of 86 MeV/c in the relativistic region.

A consistent description of the energy spectra at 315 MeV can be obtained using eq. 8 with this value of σ_o . The calculations are shown by the dotted curves in fig. 3. In each case a value of p_o was chosen to locate the theoretical distribution (transformed to an energy distribution in the laboratory frame) at the experimental maximum. (As discussed in Section 3.4, the experimental peaks are close to the energies expected from a fragmentation model, labelled E_F in fig. 3). Although in a more detailed treatment, the energy distributions depend on the details of the fragmentation channel,⁴⁶ the above simple

model already gives a satisfactory account of the shapes of the energy distributions, apart from the low-energy tails which become most pronounced in the B and Be spectra. At lower incident energies (<20 MeV/A), this tail develops into the deeply-inelastic, equilibrated component. It is an interesting question at what energy this relaxed component disappears or ceases to be fully relaxed.⁴⁷ In some cases, the presence of a low-energy tail, deviating from the Gaussian momentum distribution, has also been observed in the relativistic region.⁴⁸

In its simplest form, the fragmentation model does not describe the energy distributions at 140 MeV incident energy, which are much narrower than the widths expected from the Fermi distribution or from a temperature of 7 to 8 MeV. The continuum at 140 MeV may still contain a large contribution from two-body transfer reactions. In a similar energy region, it has been shown, however, that for the α - particle final channel, fragmentation reactions play a significant role.^{49,50} The transition between these different approaches to the continuum in heavy-ion reactions is an interesting problem in the study of reaction mechanisms.

3.4 Dependence of Fragment Kinetic Energies on Mass and Scattering Angle

In the next two sections, the fragment kinetic energies are discussed. Initially both sets of data at 140 and 315 MeV are analyzed in terms of two-body transfer processes. Then, as in the case of the energy spectra, it is shown that the kinetic energies at 315 MeV can be treated consistently with the 2.1 GeV/A data using a fragmentation model.

For a discussion of the observed kinetic energies of reaction products, it is customary to assume a two-body reaction mechanism and to calculate the Q-value of the reaction from the kinetic energy measured for one particle. For relatively low incident energies, this assumption seems to be justified rather well^{51,52} and has, in fact, been proven to be correct for a few cases.^{53,54} However, in view of the above treatment of the energy spectra and as a result of the energy dependence of isotope yields (discussed in Section 8), it is unlikely that the assumption of a two-body reaction is still valid at 315 MeV. We therefore present in figs. 6-9 the most probable final kinetic energies T_f of the detected particle in the center-of-mass system, defined by the position of the maximum intensity of the experimental energy spectrum.

The dependence of T_f on the scattering angle is shown in figs. 6 and 7 for the elements N, C, B, Be, and Li at the incident energies of 140 and 315 MeV, respectively. Whereas the final kinetic energies

observed at $E_L = 140$ MeV depend only slightly on scattering angle (see fig. 6), the energies strongly decrease with increasing scattering angle at $E_L = 315$ MeV (fig. 7). The shift of T_f toward smaller values for increasing scattering angles could be associated with a flattening of the distributions for particles with lower final kinetic energies, as has already been pointed out in Ref. 55. The dependence of T_f on scattering angle is rather small in the vicinity of the grazing angle and, for most particles, T_f has its maximum value at the grazing angle.

In figs. 8 and 9, the most probable fragment kinetic energies are shown for the various isotopes detected at angles of 40° and 15° , corresponding to incident energies of 140 and 315 MeV, respectively. The solid lines correspond to constant energies per nucleon in both the entrance and exit channels. Two points should be noted: (i) For both incident energies, the most probable final fragment energies are consistently below the calculated curves. This observation suggests that a significant amount of energy is dissipated into internal energy of the unobserved reaction products (friction); (ii) The final kinetic energies are determined mainly by the nuclear charges of the reaction products and depend only slightly on their mass. Such a dependence is not expected from classical direct-reaction models,^{40,56} and cannot be explained by the differences between entrance-and exit-channel Coulomb barrier $V_{c,i}$ and $V_{c,f}$. This point is illustrated in figs. 10

and 11 where the effective Q-values (calculated by assuming a two-body reaction having the Q-value Q) are shown for isotopes observed near the grazing angle. The effective Q-values take into account the trivial effects of different Coulomb barriers in entrance and exit channels and are defined by⁴⁰ $Q_{\text{eff}} = Q + V_{C,i} - V_{C,f}$. As shown earlier in Section 3.1, from semiclassical considerations of optimum kinematic matching between entrance and exit channels,⁴⁰ maximum cross sections are expected if the effective Q-value is given by eq. 7. At lower incident energies (near the Coulomb barrier), an alternative prescription for the optimum Q-value, Q_{opt} , can be derived^{57,58} by matching the distances of closest approach before and after the transfer:

$$Q_{\text{opt}} = - \left(\frac{Z_1 Z_2 - Z_3 Z_4}{Z_1 Z_2} \right) E_{\text{c.m.,i.}} \quad (11)$$

For transfer reactions on heavy targets of the type discussed here, $Z_2 \approx Z_4$, $(Z_1 - Z_3)/Z_1 \approx \Delta M/M_P$, and therefore similar results to the solid curves in figs. 10 and 11 are obtained. A combination of eqs. 7 and 11 has also been derived^{59,60}, but these methods lead to similar final energies for the reactions discussed in this paper. It is evident that the experimentally observed effective Q-values are significantly below the ones predicted from eqs. 7 and 11. The results demonstrate, once again, the importance of energy dissipation processes.

which appear to depend mainly on the nuclear charge of the reaction products.

As has been pointed out above, large amounts of energy are dissipated during the collision process. It is interesting, therefore, to apply to our data a simple model for tangential friction.⁶¹ In this model, the reaction is assumed to proceed in three steps: energy dissipation in the entrance channel, followed by the transfer of nucleons, and further energy dissipation in the exit channel. Complete damping of the radial velocity in the first stage of the reaction and a weak velocity-dependent tangential frictional force in both entrance and exit channels are assumed.

Friction in the entrance channel reduces the kinetic energy of relative motion (see fig. 12) to

$$E_1 = (E_i - V_{c,i}) \cos^2 \phi - A_t \cos \phi \left[\frac{2(E_i - V_{c,i})}{\mu_i} \right]^{1/2} \quad (12)$$

where ϕ is defined in fig. 12, E_i is the initial center-of-mass energy, μ denotes the reduced mass, and A_t is a parameter that describes the effect of the tangential friction.⁶¹ Transfer of nucleons reduces the kinetic energy to^{56,61}

$$E_2 = \frac{\mu_f}{\mu_i} E_1 \left[1 - \frac{\Delta M_1}{M_{f,2}} - \frac{\Delta M_2}{M_{f,1}} \right]^2 \quad (13)$$

where ΔM_1 and ΔM_2 are the masses transferred from the projectile to the target nuclei and vice versa. The masses of the projectile residue and the target residue are denoted by $M_{f,1}$ and $M_{f,2}$, respectively. (Note that eq. 13 differs slightly from the value of E_2 implied by eq. 7, which was accurate only to first order⁴⁰ in ΔM ; for small mass transfer and without friction, the effective Q-value derived from eqs. 12-14 reduces to the one given by eq. 7). Friction in the exit channel and Coulomb repulsion of the final reaction products result in the final kinetic energy

$$E_f = E_2 - A_t (2E_2/\mu_f)^{1/2} + V_{c,f} \quad (14)$$

from which we derive T_f , the kinetic energy of the ejectile in the center-of-mass frame:

$$T_f = \frac{M_{f,2}}{M_{f,1} + M_{f,2}} E_f \quad (15)$$

As can be readily seen from fig. 12, the scattering angle is given by $\theta = \theta_i + \theta_f + \phi$ where θ_i and θ_f are the entrance- and exit-channel deflections. Consequently, the scattering angle θ can be related to the incident angle ϕ only if particular assumptions about the entrance- and exit-channel trajectories are made. A particularly simple situation arises, however, for incident grazing trajectories where $\phi = 0$ and no radial

friction is necessary to damp the radial motion. In figs. 8 and 9, the corresponding kinetic energies are compared to the most probable final fragment energies observed at the grazing angle (dotted lines). The energy-independent friction parameters have been adjusted for each element, and are listed in Table 3. The dependence of the most probable final-fragment energies on the incident energy is rather well reproduced, although at 315 MeV the dependence on the mass of the reaction products tends to be slightly larger than is observed experimentally. The friction parameters have to be increased for elements further removed from the projectile. Since A_T is the product of the tangential component of the friction tensor and an effective path length of interaction, the path lengths implied by A_T in Table 3 can be calculated using an estimate of the friction tensor from data on complete fusion cross sections,⁶² viz., $10^{-22} \text{ MeV} \cdot \text{s} \cdot \text{fm}^{-2}$. The associated path lengths then vary from 0.4 to 7.6 fm.

We have investigated the dependence of T_f on scattering angle under two extreme assumptions: (i) no deflection of the particle trajectories in the entrance and exit channels, i.e., $\theta = \phi$; and (ii) only Coulomb forces acting in the entrance and exit channels, i.e. $\theta = \theta_{c,i} + \theta_{c,f} + \phi$ where θ_c is given by the classical relation $\sin \theta_c = (2E/V_c - 1)^{-1}$ and θ_c is half the angle for classical Rutherford scattering, $\theta_c = \theta_R/2$. Both assumptions predict decreasing kinetic energies for increasing scattering angles. Assumption (ii) leads to an underestimate of the angular variation of T_f by more than a factor of three at 315 MeV. Furthermore,

the predicted minimum scattering angles are larger than 40° at 140 MeV, indicating that the effects of nuclear attraction have to be included. Assumption (i), on the other hand, gives reasonable agreement with the angular dependence of T_f for nitrogen, carbon, and lithium element yields at 315 MeV. However, this assumption underestimates the angular variations of T_f for boron and beryllium yields by more than a factor of two. [This discrepancy could be due to an unresolved low-energy component in the boron and beryllium spectra leading to too strong an angular dependence of T_f (see also Section 3.1)]. At 140 MeV, assumption (i) predicts a slight decrease of T_f with increasing scattering angle (typical variations of T_f are of the order of 6 - 12 MeV between 20° and 40°). This trend, however, is not observed.

Although the present friction model can account for the observed final kinetic energies to some extent, the necessity for more complete calculations, which include the effects of the nuclear potential, is obvious. However, it has been shown that coupling to the nuclear collective degrees of freedom can also cause a significant loss in final kinetic energies.^{63,64} Therefore, it seems more appropriate to perform full dynamical calculations than to stress the present rather crude model. Calculations using a microscopic one-body frictional force are reported to be in progress.⁶⁵

3.5 Dependence of Fragment Kinetic Energies on Incident Energy

Since the energy spectra at 315 MeV were reasonably well accounted for with a projectile fragmentation model (Section 3.3), it is of interest to see whether this approach is successful also in explaining the kinetic energies at the peak of the spectra. Referring to the discussion of Section 3.3, the derived values of $T = 7.2$ MeV at 315 MeV, and 8.5 MeV at 33.6 GeV, for the temperature of the decaying ^{16}O projectile are surprisingly close to the average binding energy per nucleon in the ^{16}O nucleus. Although only a small fraction of the incident energy is transformed into internal excitation in such peripheral collisions, it is large enough for complete dissociation of the projectile by nucleon abrasion. Such a model has in fact been used²⁴ to account for the non-zero values of p_0 at 2.1 GeV/A. (The quantity p_0 is the mean momentum of the fragment in the projectile frame and would be identically zero if the fragment emerged with the beam velocity.) In order to see if this process consistently accounts for the p_0 values required for our data, we relate the two energy regions by the following kinematic model.⁶⁶ Denoting the rest masses of the projectile, target, excited projectile and excited target by M_p , M_T , M_p^* , M_T^* then, in the projectile frame, energy conservation gives:

$$M_p + \sqrt{M_T^2 + p_T^2} = \sqrt{M_p^{*2} + q^2} + \sqrt{M_T^{*2} + (p_T - q)^2} \quad (16)$$

where p_T is the target momentum and q is the momentum transfer. Ignoring terms of order q^2 , we find:

$$q = \frac{1}{v} \left[(M_p^* - M_p) + \sqrt{1 - v^2} (M_T^* - M_T) \right] \quad (17)$$

where $v = p_T / \sqrt{M_T^2 + p_T^2}$, which is also the beam velocity. For the fragment M_F produced in the decay of M_p^* ,

$$p_o = - \frac{M_F}{M_p} \frac{1}{v} (E_p^* + \frac{1}{\gamma} E_T^*) \quad (18)$$

where E_p^* and E_T^* are the excitation energies deposited in the projectile and target, respectively. The projectile excitation energy $E_p^* = E_S$ where E_S is the separation energy of the fragments and E_K is the kinetic energy, for which an approximate value can be deduced^{24,66} from the momentum distribution, viz. $E_K = 3T/2$. Since T is almost the same at 20 MeV/A and 2.1 GeV/A, eq. 18 predicts that the values of p_o for pure projectile excitation should vary as $1/v$, the reciprocal of the beam velocity. Expressed alternatively as the difference ΔT between the observed energy of the fragment and its energy when travelling with beam velocity, viz. $(\gamma-1)M_F$:

$$\Delta T = \gamma \beta p_0 + \gamma \left[\sqrt{M_F^2 + p_0^2} - M_F \right] \quad (19)$$

$$\approx \gamma \beta p_0 \approx -\gamma \frac{M_F}{M_P} E_P^* .$$

The experimental and theoretical values of p_0 are listed in Table 4, where the theoretical values were calculated on the assumption that the projectile is separated into the fragment and accompanying nucleons, except in the case of ^{12}C , for which we used the α -separation energy.

The model gives excellent overall agreement with the p_0 values observed at 2.1 GeV/A, and exact agreement for the average value. Apart from the N and C isotopes, the agreement is satisfactory also at 20 MeV/A. In particular, the model predicts a theoretical ratio of 4.8 for p_0 between 20 MeV and 2.1 GeV/A, compared to the average experimental value of 3.9 ± 1.4 , excluding the nitrogen isotopes. (The nitrogen, and also possibly the carbon isotopes, probably suffer a distortion of their energy spectra by a contribution from two-body transfer processes.) Expressed alternatively, by eq. 19, in terms of ΔT , the difference between the energy of the fragment and the energy when travelling with beam velocity, the predicted ratio is 3.1 between 2.1 GeV/A and 20 MeV/A, compared to the average experimental value (again excluding the N isotopes) of 3.8 (the energy deficit is larger at 2.1 GeV/A). The laboratory energies corresponding to the above

fragmentation process at 315 MeV are marked in fig. 3 with arrows E_F calculated for the isotopes produced with the largest cross section, neglecting the contribution of internal excitation E_K to E_p^* . For Li there are two arrows, since two adjacent isotopes were produced with comparable cross sections (${}^6, {}^7\text{Li}$). The above methods have not been applied to the data at 140 MeV because, as discussed in Section 3.3, these energy spectra are not compatible with a simple fragmentation process. It is interesting to note, however, that the energy predicted for the break-up of the projectile into the channel with the minimum Q-value agrees closely with the experimental peak (see the arrows labelled E_F in fig. 2).

In relation to our earlier treatment of the kinetic energies by a friction model, it is worth noting that the slowing down of a projectile during a fragmentation reaction at relativistic energies is due to the binding energy for removal of nucleons from the projectile, and thus is viewed as a friction phenomenon.^{24,66} This approach gave a satisfactory account of the observed values of p_0 . At low energies, an alternative approach has been used by equating the optimum Q-values in heavy-ion transfer reactions to the sum of the separation energies of the transferred nucleons.⁶⁷ Such a model would fail to account for the strong energy dependence of the optimum Q-value observed in our experiments.

4. ANGULAR DISTRIBUTIONS

The angular distributions of the energy-integrated element yields are shown in figs. 13 and 14 for 140 and 315 MeV incident energy, respectively. For these angular distributions only the high-energy component of the nitrogen and carbon spectra were included.

At 140 MeV incident energy, the angular distribution for nitrogen exhibits a maximum at the grazing angle of about 40° ; for particles further removed from the projectile the angular distributions do not show a maximum but decrease monotonically with increasing scattering angle. The angular distributions at 315 MeV in general do not show a maximum at the grazing angle of about 15° , but decrease exponentially with increasing scattering angle. However, the possibility of a grazing peak for the nitrogen isotopes cannot be ruled out. A systematic decrease of the slope of the angular distributions is observed for particles further removed from the projectile. The shapes of these angular distributions will be discussed in terms of classical trajectories and wave-mechanical concepts in Sections 4.2 and 4.3.

For comparison, fig. 15 and the insert in fig. 5 show the angular distributions of the elastic scattering and ground state transition of the one-proton transfer reaction $^{208}\text{Pb}(^{16}\text{O}, ^{15}\text{N})^{209}\text{Bi}$ at 312.6 MeV incident energy.³⁶ The widths of the transfer angular distributions, as well as the relative intensities of the observed transitions, can be rather well understood with a conventional optical-model DWBA analysis,³⁶ as is shown by the solid line in fig. 5 (the optical potential parameters are listed in Table 5). The angular distributions for transfer reactions to the low-lying states have a very pronounced maximum at the grazing angle in contrast to the majority of the angular distributions for the continuum shown in figs. 13 and 14.

4.1 Diffraction and Refractive Widths for One-Proton Transfer

By neglecting spin effects and by assuming a Gaussian distribution of the S-matrix elements in angular momentum space, a particularly simple parameterization has been derived for the angular distributions^{68,69} of heavy-ion reactions

$$\frac{d\sigma}{d\Omega}(\theta) \propto \frac{1}{\sin\theta} \left(\exp \left[- \left(\frac{\theta - \theta_0}{\Delta\theta} \right)^2 \right] + \exp \left[- \left(\frac{\theta + \theta_0}{\Delta\theta} \right)^2 \right] \right) \quad (20)$$

where θ_0 and $\Delta\theta$ describe the centroids and the widths of the Gaussian-shaped angular distributions centered around the angles $\theta = \pm\theta_0$. In

eq. 20, interference terms originating from scattering from opposite sides of the nucleus have been neglected. The width of the angular distribution has diffractive and refractive contributions^{68, 70-73} and may be written

$$\Delta\theta = [(\Delta\theta_d)^2 + (\Delta\theta_r)^2]^{1/2} \quad (21)$$

where the diffractive width is given by

$$\Delta\theta_d = \sqrt{2}/\Delta\ell \quad (22)$$

and the refractive width is given by

$$\Delta\theta_r = \frac{\Delta\ell}{\sqrt{2}} [d\theta_\ell/d\ell]_{\ell_0} \quad (23)$$

In eqs. 21-23, $\Delta\ell$ is the width of the Gaussian distribution of the modulus of the S-matrix; the deflection function θ_ℓ is the first derivative of the phase of the S-matrix with respect to the angular momentum, ℓ . By assuming Rutherford trajectories, we obtain⁷²

$$(d\theta_\ell/d\ell)_{\ell_0} = -\frac{2}{\eta} \sin^2(\theta_0/2) \quad (24)$$

For the case of single-nucleon transfer to low-lying states in the residual nucleus, bell-shaped angular distributions are observed both at 140 MeV⁷⁴ and at 315 MeV (see Ref. 36 and fig. 5). Since the distorted wave Born approximation (DWBA) accounts rather well for the shapes of these angular

distributions,³⁶ it is of interest to compare the results of DWBA calculations with the parameterizations of eqs. 20 - 24. At 140 MeV, the width of the S-matrix elements calculated with the DWBA is $\Delta l \approx 10$, and the width of the corresponding DWBA angular distribution is $\Delta \theta \approx 7^\circ$. Using the values $\theta_0 = 40^\circ$ and $\Delta l = 10$, the values $\Delta \theta_r = 2.7^\circ$, $\Delta \theta_d = 8.1^\circ$ and $\Delta \theta = 8.5^\circ$ are obtained from eqs. 20-24. At 315 MeV, the values $\Delta l \approx 23$ and $\Delta \theta \approx 3^\circ$ are obtained from the DWBA analysis. Inserting the values $\theta_0 = 15^\circ$ and $\Delta l = 23$ into eqs. 20-24 yields $\Delta \theta_r = 1.4^\circ$, $\Delta \theta_d = 3.5^\circ$, and $\Delta \theta = 3.8^\circ$. At both energies, eqs. 20-24 are consistent* with microscopic calculations. The widths of the angular distributions are dominated by diffractive effects; the refractive effects of different classical trajectories are of only minor importance. Consequently, the concept of classical trajectories can be used only to obtain some average quantities. For a quantitative analysis, quantum-mechanical calculations are required.

4.2 Angular Distributions at 140 MeV

Since at present no complete theory is available that could predict the cross sections and angular distributions shown in figs. 13 and 14, it is worthwhile describing the shapes of the angular distributions with the simple parameterizations of Section 4.1. Extensions of these methods have also been applied recently^{75,76} to deeply-inelastic collisions, involving

*The application of eqs. 20-24 yields slightly larger widths of the angular distributions than those calculated with the DWBA. This discrepancy is due to the fact that the S-matrix elements calculated with the DWBA do not have strictly Gaussian distributions.

heavier projectiles, and are useful for gaining physical insight into more elaborate theories.

The angular distributions obtained at 140 MeV are compared in fig. 13 with those calculated from eq. 20, using the parameters shown in Table 3. Although no effort has been made to obtain optimum fits, it is apparent that the main characteristics of the experimental angular distributions are rather well described. There is a definite tendency towards decreasing parameters θ_0 and increasing parameters $\Delta\theta$ for particles further removed from the projectile.

The decrease of the mean scattering angle θ_0 with increasing inelasticity of the reaction (compare also figs. 2-4) may have two simple physical interpretations: (1) More inelastic collisions correspond to smaller impact parameters and the particle trajectories are deflected to more forward angles by the attractive nuclear force. (2) More inelastic collisions correspond to longer lifetimes τ of the short-lived, rotating di-nuclear system,^{33,34} and smaller scattering angles correspond to larger "life-angles" $\theta_\tau = \theta_{gr} - \theta_0 = \omega\tau$, where θ_{gr} is the grazing angle and ω is the angular velocity of the rotating di-nuclear system.

According to eqs. 21-23, increasing widths $\Delta\theta$ can have two different physical origins: (1) The refractive contribution $\Delta\theta_r$ (eq. 23) could increase with increasing inelasticity. This effect can arise, on the one hand, from the contribution of a wider window of partial waves to the

reaction amplitude (and, correspondingly, a decrease of the diffractive width $\Delta\theta_d$, defined in eq. 22). On the other hand, the deflection function could be steeper for smaller impact parameters, i.e. for more inelastic collisions. These assumptions imply a rather dramatic increase of the refractive width compared to the value obtained in single-nucleon transfer reactions to low-lying states (see Section 4.1). (2) The reaction amplitude could be more sharply localized in angular momentum space for more inelastic collisions. This would imply an increase of the diffractive spreading of the wave packets with increasing inelasticity or, in a Regge-pole model,⁷⁷⁻⁸² larger "life angles" of the surface waves traveling around the nucleus. (For a discussion of the connection between the diffraction and Regge-pole models see Ref. 82; see also Section 4.3.)

The assumptions that the particles move on Rutherford trajectories (see eq. 24) and that the phase of the inelastic S-matrix elements varies with angular momentum in the same way as the elastic scattering phase shifts δ_ℓ (obtained from an optical model analysis) are inconsistent with the variation of the parameters θ_0 and $\Delta\theta$ given in Table 3. This conclusion may be understood by examining fig. 16 where the Coulomb deflection function θ_ℓ^C (dashed line), the moduli $|\eta_\ell|$, and the phase derivatives $\theta_\ell = 2 \frac{d}{d\ell} \delta_\ell$ of the elastic scattering S-matrix elements, obtained from optical-model calculations, are shown (dotted lines). The optical potential parameters used

for these calculations were taken from Ref. 36, and are listed in Table 5. These parameters yield a consistent analysis of elastic scattering and one-nucleon transfer reactions at 140 and 312.6 MeV incident ^{16}O energy. Both the Coulomb and the optical-model deflection functions increase for smaller partial waves (i.e., for smaller impact parameters). Therefore, the decrease of Θ_0 for more inelastic processes (see Table 3) would be consistent with these deflection functions only by assuming more inelasticity for larger impact parameters. This situation is, however, rather unlikely. Furthermore, for all partial waves that are subject to absorption, these deflection functions are always significantly larger than the value of $\Theta_0 = 20^\circ$ used for the angular distributions of Li and Be.

For small impact parameters ($\ell < \ell_{\text{gr}}$), the elastic scattering is dominated by the reflection of the incoming waves from the surface of the optical potential⁸³ and is of wave-mechanical origin. This contribution to the elastic scattering (denoted by the dot-dash curve in fig. 16) is seen to account reasonably well for the elastic scattering of the low partial waves. For $\ell < \ell_{\text{gr}}$, the contributions to elastic scattering corresponding to reflections from the classical turning point (calculated in the WKB approximation for real particle trajectories) are negligible, as is shown by the solid curve in fig. 16. It is, however, interesting to note that this classical deflection function is consistent with the variation of the parameter Θ_0 shown in Table 3. This deflection

function predicts deflection to negative angles, as well as to smaller angles for decreasing impact parameters. It is indeed possible that deeply-inelastic scattering corresponds to penetrating orbits and that reflections from the nuclear surface are unimportant for these processes, in contrast to the results for elastic scattering. The implication of such a description is that the potential for deeply-inelastic scattering cannot be determined by measuring the elastic scattering, which is sensitive only to the nuclear surface. At higher energies the elastic scattering is more sensitive to the nuclear interior and the potentials may be determined less ambiguously.⁸⁴ Several theoretical descriptions of macroscopic features of heavy-ion collisions nevertheless use classical equations of motion.^{63, 85-88}

4.3 Angular Distributions of 315 MeV

The angular distributions at 315 MeV fall off towards larger angles according to the functional form

$$\frac{d\sigma}{d\Omega}(\theta) \propto \frac{1}{\sin\theta} \exp(-\theta/\alpha) \quad (25)$$

as shown by the dashed lines in fig. 14. There is a definite tendency towards increasing "decay angle" α for particles further removed from the projectile.

Within the framework of the semiclassical theory, exponentially-decaying angular distributions of the form described by eq. 25 are

expected for orbiting collisions (see, e.g., Ref. 87, eq. 3.1) where α is a parameter measuring the strength of the frictional force. The smooth increase of α for lighter reaction products then corresponds to stronger frictional energy losses for particles further removed from the projectile. The interpretation of the angular distributions in terms of orbiting has two shortcomings: (i) The integrated cross sections for the reaction channels shown in fig. 14 represent more than 30% of the total reaction cross section (see Table 2 and Section 6). Since orbiting occurs only for a rather narrow region of angular momenta, it is not clear how this region could represent such a large fraction of the reaction cross section. A detailed calculation will be required to answer this question. (ii) Similar to the observations at the lower energy of 140 MeV, the elastic scattering deflection function does not give any indication for orbiting. This is demonstrated in fig. 17, where the Coulomb (dashed line) and optical model (dotted line) deflection functions, Θ_ℓ , as well as the magnitude of the elastic scattering S-matrix, $|\eta_\ell|$, are shown. These quantities were calculated with the optical potential (see Table 5) used for the analysis of elastic scattering and one-proton transfer reactions³⁶ (see figs. 5 and 15). For comparison, the contributions to the S-matrix from reflections at the classical turning point and at the potential surface are shown by the solid and dot-dashed lines. Classically, only rather deep potentials will

give rise to orbiting at energies as high as 315 MeV.* For this 50 MeV deep potential, orbiting does not occur and only a rather shallow nuclear rainbow is produced in the classical deflection function calculated with the WKB approximation (solid line).

An exponential angular distribution can also arise from a Lorentzian distribution of S-matrix elements in angular momentum space with the same approximations⁶⁸ as for the derivation of eq. 20. In that case an increase of α is associated with a decrease of the width of the Lorentzian. It is interesting to note that Lorentzian distributions of S-matrix elements are obtained in the simplest form of the Regge-pole description of heavy-ion transfer reactions.⁷⁹⁻⁸² In this description, which has been successfully applied to heavy-ion reactions,⁷⁷⁻⁸² the parameter α of eq. 25 is interpreted as half the "life angle" of the decaying surface wave traveling around the nucleus.^{77,78,81,82} Longer lifetimes of the surface wave (i.e., larger "life angles") are connected with narrower widths of the Lorentzian S-matrix distributions — an extreme case being orbiting resonances which occur only for a narrow window in ℓ -space. The parameter α given in fig. 14 implies that the removal of more nucleons or more energy from the projectile is connected with increasing sticking times of the di-nuclear complex, a trend which is physically reasonable.

* Orbiting can occur only if $E_{c.m.} < \max[V(r) + r/2 \cdot dV(r)/dr]$. [See W.H. Miller, J. Chem. Phys. 51 (1969) 3631]. For a Woods-Saxon potential of radius $R = 10\text{fm}$ and diffuseness $a = 0.5\text{fm}$, potential depths of $V > 97\text{ MeV}$ are necessary for orbiting to occur classically.

Note that the "life angles" of $2\alpha = 5^\circ - 40^\circ$ are of the same order of magnitude as those obtained from an analysis of the $^{26}\text{Mg}(^{16}\text{O}, ^{14}\text{C})^{28}\text{Si}$ ground-state transition over a wide range of incident energies.⁸²

The intuitive picture of decaying surface waves suggested by the Regge-pole model has the simple classical analog of a rotating di-nuclear system of lifetime τ and angular velocity ω . For such a system, the angular distribution of the decay products is given by¹²

$$d\sigma/d\Omega = c/\sin\theta \cdot \left[\exp(-\theta/\omega\tau) + \exp(-(2\pi-\theta)/\omega\tau) \right] \quad (26)$$

In eq. 26, c is a constant and $\theta=0$ has been assumed for the starting condition at $t=0$. For large lifetimes, $\tau \gg 2\pi/\omega$, the angular distributions are described by the $d\sigma/d\Omega \approx 1/\sin\theta$ dependence, characteristic of compound nuclear reactions. For very short lifetimes, $\tau \ll 2\pi/\omega$, the angular distributions are strongly forward-peaked and given by eq. 25, with $\alpha = \omega\tau$.

The angular velocity may be estimated from the classical expression

$$\omega = \hbar \ell_i / (\mu_i R_i^2) \quad (27)$$

Using the grazing angular momentum, $\ell_i = 140$, and the distance of closest approach, $R_i = 11.5$ fm., one obtains $\omega = 4.5 \times 10^{21} \text{ sec}^{-1}$. The

decay angles $\alpha = 2.5^\circ - 20^\circ$ then correspond to lifetimes $\tau = (0.1 - 0.8 \times 10^{-22})$ sec. The reaction, therefore, proceeds on a time scale comparable to that of direct reactions.⁸²

5. LOCALIZATION OF THE REACTIONS

At both low and high incident energies, it is customary and useful to divide heavy-ion reactions into two principal categories: central and peripheral collisions. At low energies, central collisions are believed to lead mainly to the amalgamation of the two colliding nuclei, followed by particle evaporation or fission.^{14,89} (This process has, however, not been observed for collisions between Kr ions and very heavy targets.⁸⁹⁻⁹³) At relativistic energies, little experimental information is available^{17,94} concerning central collisions or the existence of predicted shock-wave phenomena, pion condensates, and density isomers.^{26,27, 95-97} Peripheral collisions, on the other hand, mainly involve quasi-elastic and deeply-inelastic reactions³ at low energies, and projectile fragmentation at relativistic energies;^{15,16} as we show in the present work, the transition between these phenomena appears to set in at a few tens of MeV/A above the Coulomb barrier^{31,32,98} (see also Section 8).

Although the classification into central and peripheral collisions is a convenient operational procedure, it is necessary to discuss the extent to which such collisions can be associated with different impact

parameters or incoming angular momenta. The simplest procedure is to define a critical angular momentum ℓ_c by the prescription¹⁴

$$\sigma_c = \pi k^{-2} (\ell_c + 1)^2 \quad (28)$$

where σ_c is the cross section for central collisions and k is the wave number of relative motion in the incoming channel. The sharp cut-off model, used in deriving eq. 28, implies that all partial waves with angular momentum $\ell \leq \ell_c$ lead to central collisions and all higher partial waves lead to peripheral reactions or elastic scattering. Since the total reaction cross section σ_R is given by the sum of central and peripheral reaction cross sections,

$$\sigma_R = \sigma_c + \sigma_p = \sum_{\ell} \sigma_{R,\ell} \quad (29)$$

the values of ℓ_c can be determined either by measuring σ_c or by determining the partial-wave reaction cross section $\sigma_{R,\ell}$ and the peripheral reaction cross section σ_p .

The partial-wave reaction cross sections shown in fig. 18 for $^{16}\text{O} + ^{208}\text{Pb}$ at 315 MeV incident energy have been obtained from optical-model calculations that fit the elastic-scattering angular distributions (see fig. 15). By summing the particle yields listed in Table 2, the peripheral cross section, $\sigma_p = 1295 \pm 194$ mb, is obtained. The corresponding critical angular momentum of $\ell_c = 118 \pm \frac{5}{5.5}$ is marked on fig. 18. This angular momentum corresponds to a turning point of the

classical trajectory (i.e., the distance of closest approach) of 9.8 fm, if the classical turning point is calculated from the real part of the optical potential only. This turning point corresponds to a touching distance of the charge density radii of approximately 30% of the central density.⁹⁹ In the sharp cut-off model, therefore, peripheral collisions correspond to interactions of the nuclei where the surfaces hardly overlap.

For the value $\sigma_R = 3.46b$, the cross section for central collisions at 315 MeV may be deduced to be $\sigma_C = 2.16 \pm 0.26b$. At 33.6 GeV, the corresponding value^{31, 100-102} is $\sigma_C = 2.26b$. Consequently, the cross section for central collisions hardly changes between these two energies and seems to be determined mainly by the overlap of nuclear matter.

Although the sharp cut-off model and the associated critical angular momentum have been very useful concepts in the analysis of heavy-ion fusion reactions,¹⁰³ it is clear that they represent an oversimplification¹⁰⁴ and that there is a transition region in angular-momentum space where the probability for the amalgamation of the colliding ions drops smoothly from 1 to 0. At present, there is hardly any information available concerning the extent of this transition region, and it has been suggested^{105,106} that fusion and transfer reactions might coexist for a rather large region of angular momenta. In order to emphasize this point we have included in fig. 18 a typical distribution of the dominant radial integrals, corresponding to the DWBA calculation for $^{208}\text{Pb}(^{16}\text{O}, ^{15}\text{N})^{209}\text{Bi}$ transitions to discrete states shown in fig. 5. These radial integrals extend rather far into

the region assigned to central collisions, thus demonstrating the shortcomings of the concept of a sharp critical angular momentum. Furthermore, it should be remembered that the width of the angular distribution shown in fig. 5 can be completely explained by diffraction due to the narrow width of the DWBA S-matrix elements. If diffraction effects are negligible and if a description in terms of classical trajectories is not adequate for the interpretation of the angular distributions shown in fig. 14, even broader distributions are required. If, on the other hand, the angular distributions are also to be interpreted in terms of diffractive phenomena, sharper localizations of the S-matrix elements for larger energy losses are required. The latter assumption would be in somewhat better agreement with the sharp cut-off model.

6. COMPARISON WITH OTHER EXPERIMENTS

In this section it will be shown that the qualitative features of the experimental results discussed so far are characteristic of a wide range of heavy-ion interactions. Depending on the choice of projectile and target nuclei, and the incident beam energy, certain features may be more pronounced and, hence, more readily studied by different reactions. However, qualitatively, there is a great similarity underlying heavy-ion reactions induced by very light and very heavy projectiles.¹⁰⁷ Only quantitative analyses of differential cross sections and detailed calculations will, perhaps, reveal deeper insights concerning the importance of nuclear structure effects.

Energy spectra similar to those shown in figs. 2-4 have been observed for a large variety of experiments. These spectra are generally bell-shaped and peaked at energies corresponding to large negative Q-values if the reaction proceeds via a two-body reaction mechanism (for a recent review, see e.g. Ref. 107). Reactions of this kind have, therefore, been termed deeply-inelastic reactions. In cases where two peaks are observed in the energy spectrum, as in fig. 4, the terminology is less clear and the high-energy component has been called quasi-elastic and the low-energy component deeply-inelastic,⁶ although both components may involve large negative Q-values. As shown in Ref. 35 and in figs. 2-4, there is a smooth transition from small to highly negative Q-values as a function of both mass transfer and scattering angle, and the terminologies of "quasi-elastic" and "deeply-inelastic" may not be used without some arbitrariness. We

therefore prefer to use the term "peripheral" interactions, since the angular distributions exhibit all the characteristics of a rapid reaction process located at the nuclear surface. This terminology is used at relativistic energies^{15,16} where rather similar processes are observed (see Sections 3.3, 3.5, 8 and 9).

The occurrence of two peaks in the energy spectra of reaction products close to the projectile (as observed in fig. 4) has been established for a large variety of entrance channels^{6,92,107-109} ranging from systems¹⁰⁸ as light as $^{14}\text{N} + ^{53}\text{Cr}$ to systems^{6,92} as heavy as $^{40}\text{Ar} + ^{232}\text{Th}$ and $^{86}\text{Kr} + ^{130}\text{La}$. An interesting interpretation of the low-energy components in terms of orbiting has been proposed in Ref. 35. However, an interpretation in terms of a deflection function exhibiting two rainbows has also been suggested.⁶⁴

A very thorough analysis of a most probable final-fragment kinetic energies (optimum Q-values) has been performed¹⁰⁸ for carbon- and nitrogen-induced reactions on light target nuclei ($A \leq 100$) and at incident energies up to about 100 MeV. It has been observed that the optimum Q-values are independent of the scattering angle and that the effective Q-values follow a relation similar to eq. 7, i.e. $Q_{\text{eff}} = \alpha \Delta M + \beta$, for $\Delta M \leq 5$ amu, where α and β are constants. This Q-value dependence is not observed at the higher incident energies and with the heavier targets used in the present work (see figs. 10 and 11). Although the most probable final fragment energies change only slightly over the angular range covered at 140 MeV, there is a pronounced dependence on scattering angle at 315 MeV (compare fig. 7). Such an angular dependence has also been observed in Refs. 6,55,110 and was explained by the observation that, for a given

particle, the angular distributions become flatter with increasing inelasticity of the reaction.^{55,110,111} In contrast to observations at lower energies,^{108,110,112} our data do not show a unique dependence of either the most probable kinetic energies (compare figs. 8 and 9) or the corresponding effective Q-values (compare figs. 10 and 11) on the transferred mass. Instead, the main parameter determining the most probable kinetic energies of the reactions products is their nuclear charge. This effect is already noticeable at 140 MeV incident energy and is very pronounced at 315 MeV incident energy.

Our observations are also different from the results of Ref. 113 where the final kinetic energies observed for the reaction $^{40}\text{Ar} + ^{232}\text{Th}$ at $E_L = 295$ MeV depend on the transferred mass at the grazing angle ($\theta_L = 40^\circ$) and on the nuclear charge of the reaction products at a more forward angle. The kinetic energies observed in Ref. 113 at $\theta_L = 18^\circ$ were already completely "relaxed," but in the present work, the final kinetic energies observed in the vicinity of the grazing angle (figs. 6-9) are well above the "relaxed" energies corresponding to the exit-channel Coulomb barriers. On the other hand, our data do follow the qualitative trend observed in Refs. 108, 114 that the velocities corresponding to the most probable final kinetic energies are closer to the projectile velocity for smaller mass transfer and decrease significantly for larger mass transfers (see figs. 2, 3, 8 and 9).

At 140 MeV the angular distribution for nitrogen exhibits a clear maximum in the vicinity of the grazing angle ("side peaking"¹¹⁵) whereas it decreases monotonically for beryllium and lithium. This change of shape is observed in a large variety of heavy-ion reactions^{107,115} and has attracted great interest. Here, we want to point out that such changes have been observed even for single-nucleon transfer reactions, depending on the excitation energy of the residual nucleus (see, e.g., Ref. 116).

It has been proposed¹⁰⁷ to classify heavy-ion reactions according to the magnitude of the parameter

$$\bar{\eta} = Z_1 Z_2 e^2 / (\hbar \bar{v}) \quad (30)$$

where

$$\bar{v} = [2(E_i - V_{C,i})/\mu_i]^{1/2} \quad (31)$$

Reactions with $\bar{\eta} \leq 150 - 200$ should have rather broad angular distributions with their maximum peaked well forward of the grazing angle (orbiting picture) and reactions with $\bar{\eta} \geq 250 - 300$ should exhibit a focussing of the cross section close to the grazing angle with an increase of the focussing phenomenon as $\bar{\eta}$ increases. For the case of $^{16}\text{O} + ^{208}\text{Pb}$, these parameters are $\bar{\eta}(140 \text{ MeV}) = 55$ and $\bar{\eta}(315 \text{ MeV}) = 27$. Whereas no grazing-type angular distributions are

observed at 315 MeV [except for transitions to low-lying single-particle states (see figs. 14 and 5)], they are observed at 140 MeV, which is not expected from the above classification scheme (see fig. 13). Although we confirm the trend of a decreasing tendency for "side-peaked" angular distributions as the bombarding energy is increased, it appears that the parameter $\bar{\eta}$ is not a general criterion for describing the shapes of angular distributions. Note that significant differences in the shapes of angular distributions corresponding to transfer reactions to low-lying states have been predicted¹¹⁷ and found experimentally^{118,119} for reactions proceeding mainly via one-step or two-step reaction mechanisms, the study of which may relate macroscopic and microscopic approaches to heavy-ion reactions.

7. DEPENDENCE OF CROSS SECTIONS ON GROUND STATE Q-VALUES

Systematic trends of isotope-production cross sections observed in heavy-ion reactions at incident energies of a few MeV/A above the Coulomb barrier have been investigated in great detail,^{33,34,76,112} and the important discovery has been made that the cross sections can be parameterized in the form

$$d\sigma/d\Omega(N,Z) = f(Z) \exp(Q_{gg}/T) \quad (32)$$

where $f(Z)$ depends only on the nuclear charge of the reaction products, Q_{gg} is the ground state Q-value of the two-body transfer reaction

producing the observed particle, and T is a constant. Such a dependence has been derived from a statistical model of a partially-equilibrated, di-nuclear system and the parameter T has been interpreted as an effective temperature.^{34,120,121} In this model, the effective temperature should vary as¹²⁰

$$T = (E^*/a_{\text{eff}})^{1/2} \quad (33)$$

where E^* is the excitation energy of the di-nuclear system and a_{eff} is an effective level density parameter. The dependence of a_{eff} on the mass of the composite system A^* is given by

$$a_{\text{eff}} \approx A^*/c \quad (34)$$

where it has been found empirically^{120,121} that $c \approx 20$. If statistical equilibrium is acquired for the di-nuclear system, the function $f(Z)$ has the form^{120,121}

$$f(Z) = \exp [(V_{c,i} - V_{c,f}) / T] \quad (35)$$

Although the above discussion derives the dependence of $(d\sigma/d\Omega)$ on the ground state Q -values from a statistical model of a partially-equilibrated,

di-nuclear system, we note that eq. 32 has also been obtained by the molecular wave-function method¹²² which does not use the concept of temperature.

Figures 19-21 show the experimental differential cross sections observed for the reactions induced by ^{16}O ions of 140 MeV incident energy on targets of ^{208}Pb , ^{197}Au , and ^{94}Zr at the scattering angles of 40° , 32.5° and 20° respectively. In parts (a) of the figures, these cross sections are compared with the predictions of eq. 32 by plotting the logarithms of the cross sections vs. Q_{gg} . According to these systematics, the points for different isotopes of a given element should lie on a straight line, and the lines for different elements should have the same slope, which varies only slowly with the mass of the composite system. For reactions on ^{208}Pb (fig. 19a) and on ^{197}Au (fig. 20a), these predicted trends are in reasonable agreement with the experimental cross sections, although significant deviations are apparent. The lines shown in figs. 19a and 20a correspond to effective temperatures of $T = 5.3$ MeV and $T = 3.2$ MeV, respectively. Such a shift in temperature is not expected from eqs. 33 and 35. Furthermore, no unique set of lines may be defined for the case of the ^{94}Zr target and, consequently, the Q_{gg} - systematics do not apply for these reactions (compare fig. 21a).

As has been discussed in Ref. 120, the major part of the excitation energy is expected to remain in the heavy target residue and eqs. 32-35 reflect, to a good degree of accuracy, the level density of the residual nucleus. Therefore, it seems justified³⁴ to take the effects of pairing

energies into account by substituting $Q_{gg} - \Delta_p - \Delta_n$ for Q_{gg} in eq. 32, where Δ_p and Δ_n are the proton and neutron pairing energies (see Ref. 116). A rather significant improvement is obtained from this modification of the Q_{gg} - systematics for the system $^{16}\text{O} + ^{94}\text{Zr}$, as is shown in fig. 21b. For $^{16}\text{O} + ^{208}\text{Pb}$ and $^{16}\text{O} + ^{197}\text{Au}$, the effects of pairing corrections are less dramatic and only slightly better agreement is obtained. These results agree with those of Ref. 34.

Since the heavy target residue is expected to be highly excited, specific structure effects should not be apparent in its level density. The pairing corrections discussed above are a first step for removing nuclear structure effects, which are important for the ground-state binding energies but might be negligible at high excitation energies. In order to see whether nuclear structure effects of the target residues may be completely neglected in the present reactions, we have substituted the masses of target and residual nuclei by their liquid-drop masses, $^{124}M_{i,2}^{\text{LD}}$, $M_{f,2}^{\text{LD}}$, and have defined liquid-drop Q_{gg} - values by

$$Q_{gg}^{\text{LD}} = M_{i,1} + M_{i,2}^{\text{LD}} - M_{f,1} - M_{f,2}^{\text{LD}}, \quad (36)$$

where $M_{i,1}$ and $M_{f,1}$ are the ground-state masses of projectile and reaction product, respectively. By substituting Q_{gg}^{LD} in eq. 32, isotope-production cross sections are predicted in quite

remarkable agreement with the experimental data. This result is shown in figs. 19b, 20b, and 21c, where the logarithms of the isotope cross sections are plotted vs. Q_{gg}^{LD} . For all three cases, the Q_{gg}^{LD} - systematics are in better agreement with the experimental data than the Q_{gg} - systematics. Furthermore, it is quite noteworthy that very similar temperatures are obtained: $T = 2.7$ MeV for the ^{208}Pb and ^{197}Au targets and $T = 2.6$ MeV for ^{94}Zr .

The systematic dependence of isotope-production cross sections on the liquid-drop values Q_{gg}^{LD} might have some important implications:

(i) The isotope-production cross sections are determined mainly by the binding energies of the light projectile and reaction products, and therefore should vary rather smoothly for different target nuclei. Experimental evidence concerning this point has been presented in Ref. 32. (see also Section 9). However, more detailed studies of the target and energy dependence of the isotope yields are necessary for confirmation.

(ii) Shell effects might still be important for the light projectile and residual nuclei. However, it has been suggested in Ref. 125 that a significant fraction of the light reaction products might receive enough excitation energy during the collision process to decay in flight by particle emission. The resulting secondary particle yields could be rather similar to the experimental observations.

Recent experiments performed at lower energies seem to favor the hypothesis that the light reaction products are produced with rather low excitation

energies and that light particles are emitted mainly from the highly excited residual nucleus.⁵² Light particle emission has been shown to be important for more symmetric entrance channels at comparable energies.^{11,53} More detailed coincidence experiments are, however, necessary to investigate the important question of the decay-in-flight of light reaction products for very asymmetric entrance channels, since it is predicted to be of major importance at relativistic energies^{15,16, 19-24} and may already be significant at the energies of the present experiment^{31,32} (see also Section 8). (iii) Shell effects are not important for the target residues. If, indeed, the target residues behave like liquid drops, there is little hope of producing superheavy nuclei by these reactions since the corresponding liquid drops are unstable against spontaneous fission. Until now, there is no other experimental evidence available that would support this speculation. The question is important enough to merit careful experimental investigation.

We discuss finally the dependence of the particle yields on the nuclear charge of the reaction products. If statistical equilibrium is reached for the di-nuclear system, the charge dependence of the cross sections should be described by eq. 35. Since the exit-channel Coulomb barrier decreases for higher Z-values, eq. 35 predicts an enhancement of light-element production cross sections. More

quantitatively, for the system $^{16}\text{O} + ^{208}\text{Pb}$, the exit-channel Coulomb barrier decreases by about 8.5 MeV for each transferred charge. The temperature $T = 2.7$ MeV (see fig. 19b) then implies an enhancement factor of about 23. The experimentally observed enhancement factors are, however, only of the order of 4. Therefore, complete equilibrium is not reached for the charge-transfer degree of freedom. This result was already pointed out in Ref. 120, where it was argued that the exchange of protons is hindered by the penetration of the Coulomb barrier. Typical hindrance factors of 0.1 - 0.2 have been estimated¹²⁰ for similar reactions. The resulting enhancement of proton transfer by a factor of 2.3 - 4.6 for the ^{208}Pb and ^{197}Au targets is in quite good agreement with the observed enhancement. Note, however, that a similar argument cannot be applied for the system $^{16}\text{O} + ^{94}\text{Zr}$ for which no significant enhancements are observed between nitrogen and carbon yields or between boron and beryllium yields (compare figs. 21b,c).

8. ENERGY DEPENDENCE OF CROSS SECTIONS

In the preceding sections, we have stressed the similarities of our observations to results obtained from the investigation of deeply-inelastic scattering of heavy ions at energies only a few MeV/A above the Coulomb barrier. In the following sections, we show that there is a smooth transition between low-energy, deeply-inelastic

reactions and projectile fragmentation reactions occurring at relativistic energies. The transition between these phenomena appears to occur at energies of a few tens of MeV/A above the Coulomb barrier.^{31,32}

Isotope-production cross sections for the system $^{16}\text{O} + ^{208}\text{Pb}$ are available at the energies of 140 MeV (Table 1), 315 MeV (Table 2), and 33.6 GeV.¹⁶ The energy dependence of the relative particle yields is displayed in fig. 22 (taken from Ref. 32). Here the ratios of element and isotope yields measured for $^{16}\text{O} + ^{208}\text{Pb}$ at 140 and 315 MeV (part a) and at 315 MeV and 33.6 GeV (part b) are shown. The relative particle yields decrease systematically for particles further removed from the projectile at 140 MeV compared to 315 MeV (see fig. 22a). At 315 MeV and 33.6 GeV, on the other hand, the relative element yields are remarkably similar (see fig. 22b), although there is a general trend towards larger cross sections for the production of more neutron-deficient isotopes at 33.6 GeV compared to 315 MeV incident energy. (The differences of cross sections imply that the hypothesis of limiting fragmentation is not valid at 315 MeV incident energy.)

In fig. 23 the logarithms of the integrated cross sections are plotted as a function of the ground state Q-values Q_{gg} . With increasing projectile energy, more significant deviations from the Q_{gg} -systematics discussed in the preceding section become apparent. At 315 MeV, the lines drawn through various isotopes are not very well defined and have different slopes for the various elements. The breakdown of the

Q_{gg} systematics at 33.6 GeV is quite obvious and no temperature can be defined. The cross sections for the beryllium isotopes even increase with more negative Q -values, which cannot be explained using eq. 32 and positive values of T .

At relativistic energies, the formation of a partially-equilibrated, di-nuclear system is not expected to take place and the reaction has been proposed to proceed by an abrasion-ablation mechanism.²²⁻²⁴ If the reaction proceeds mainly by the excitation of the projectile and subsequent statistical decay of the excited projectile into the experimentally observed fragments,^{22,23} the particle yields may be described by an expression²² similar to eq. 32, i.e.,

$$\sigma(N,Z) = C \sum_i \exp (Q_i/T), \quad (37)$$

where C is a constant, the Q_i are threshold Q -values for the various (many-body) projectile fragmentation channels, and T is a parameter, interpreted as the effective temperature of the excited projectile.²² The sum includes all possible break-up channels producing the experimentally observed fragment. Since, for high incident energies, the excitation energy of the projectile, and therefore the effective temperature (T), should vary only slightly for different target nuclei, eq. 37 implies that the relative particle yields are target independent (see also Section 9).

Although eqs. 32 and 37 are formally quite similar, they imply quite different physical processes. For the derivation of eq. 32, it is assumed^{120,121} that nucleons are exchanged between the colliding nuclei and that at least partial statistical equilibrium is reached while the two nuclei are in contact. The cross sections are determined mainly by the level density of the heavy residual nucleus. For the derivation of eq. 37, on the other hand, it is assumed²² that the colliding ions are highly excited during the collision. However, no nucleons are exchanged between the two nuclei. After the collision, the excited reaction products decay in flight and the cross sections are determined by the level densities in the (many-body) final decay channels. The relative particle yields at 2.1 GeV/A have also been interpreted in terms of a direct fragmentation of the projectile into its cluster substructures.⁴⁶ Therefore the aspect of statistical decay still requires experimental verification from a measurement of the relative probabilities of the different fragmentation channels.

If eq. 37 is a valid description of the experimentally observed particle yields, σ_{expt} , the ratios

$$C(N,Z) = \sigma_{\text{expt}}(N,Z) / \left(\sum_i \exp(Q_i/T) \right) \quad (38)$$

should be constant for the appropriate choice of T . In fig. 24, these

ratios are shown for the system $^{16}\text{O} + ^{208}\text{Pb}$ at 140 and 315 MeV, and at 33.6 GeV. The temperatures shown in the figure have been chosen to give an optimum fit to the data at each energy.* The relative particle yields at 315 MeV and 33.6 GeV are rather well described by eq. 38. At 140 MeV, where projectile fragmentation is not expected to be the dominant reaction mechanism (see also the discussion of Section 7), the deviations are somewhat larger. However it is probably true to say that both formalisms of eq. 38 and 32 give comparable agreement with the 140 MeV data. For the system $^{16}\text{O} + ^{232}\text{Th}$, studied at a similar incident energy of 137 MeV,³³ however, the ratios $C(N,Z)$ vary by several orders of magnitude for any temperature between 1 and 10 MeV, demonstrating that eq. 38 is not relevant. Finally, it should be noted that eq. 37 can be applied only for stripping reactions, and not for pickup or charge-exchange reactions. At energies of a few MeV/A above the Coulomb barrier,^{33,34} these processes are observed to occur with significantly higher cross sections than at 315 MeV³¹ where they constitute less than 5% of the peripheral cross section. No evidence has been found for the occurrence of pickup reactions at relativistic energies¹⁶ (see also figs. 19-21; Tables 1,2). These observations

* Intuitively, increasing temperatures are expected with increasing incident energy. However, the effective temperature is not well enough defined by the data to reach definite conclusions about its energy dependence. In particular, the slightly higher value at 315 MeV compared to the one at 33.6 GeV is not well established. Clearly, measurements at different intermediate energies are necessary.

represent further evidence that the reaction mechanism changes at energies of only a few tens of MeV/A above the Coulomb barrier.

Recently experiments to detect coincidences between light and heavy fragments have been initiated⁵⁰ at 140 MeV and 315 MeV incident energy. Preliminary results show that fragmentation does appear to play a role at 315 MeV for a reaction channel such as $^{12}\text{C} + \alpha$. These measurements will be important for determining the details of the production mechanism of the fragments since, as we showed earlier in Section 3.5, it is difficult from the inclusive spectra to distinguish between a fast fragmentation process and a slow evaporation from the excited projectile. Coincidence studies of other systems suggest¹²⁶ that particles may be emitted from a localized region of high temperature between the colliding nuclei, similar to the "hot-spot" predicted in low¹²⁷ and high¹²⁸ energy collisions.

9. TARGET DEPENDENCE OF CROSS SECTIONS

At relativistic energies, the relative isotope-production cross sections have been observed¹⁶ to be target independent. The cross sections can be written in the form

$$\sigma(a + b \rightarrow x) = \Gamma(a, x) \cdot c(a, b). \quad (39)$$

Here, a and b denote the projectile and target nuclei, respectively, and x is the observed particle. The form of eq. 39 will be loosely referred to as factorization, and implies that, for fixed projectile and fixed incident energy, the relative particle cross sections are independent of the target nucleus, i.e., the ratios

$$R(x; b, b') = \sigma(a + b \rightarrow x) / \sigma(a + b' \rightarrow x) \quad (40)$$

are independent of x for different target nuclei b and b' . Only the absolute normalization of the cross sections depends on the properties of target and projectile.* At relativistic energies this normalization was shown to depend on the nuclear radii.¹⁶ It has been pointed out¹⁰ that factorization is, e.g., a direct consequence of the statistical model of nuclear decay. Indeed, the simple ablation model of eq. 37 predicts the factorization of the isotope cross sections if the parameter T does not depend on the target nucleus.

*If $R(x; b, b')$ does not depend on incident energy, the hypothesis of limiting fragmentation is valid.

(Note the similarity of factorization to the Bohr independence hypothesis of compound nuclear decay.¹⁹⁾)

On the other hand, the Q_{gg} -systematics (see eq. 32 of Section 7) predict that the cross sections do not factorize. Assuming that the effective temperature of the di-nuclear system is target independent, one obtains from eqs. 32 and 35

$$R(x;b,b') = f_b(Z)/f_{b'}(Z) \cdot \exp[\Delta Q(x;b,b')/T] \quad (41)$$

where

$$\Delta Q(x;b,b') = Q_{gg}(x,b) - Q_{gg}(x,b') \quad (42)$$

Since $\Delta Q(x;b,b')$ varies substantially for different isotopes (see fig. 25), the Q_{gg} -systematics are incompatible with factorization. More quantitatively, using typical values of $T \approx 3$ MeV, one expects, e.g., variations larger than one order of magnitude among the carbon yields from the reactions induced by ^{16}O ions of 140 MeV on targets of ^{208}Pb and ^{197}Au ; such variations are not observed in our experiments.

In order to investigate the validity of the factorization hypothesis, we show in figs. 26-28 the ratios $R(x;b,^{208}\text{Pb})$ for reactions induced by ^{16}O ions on different target nuclei b at 140 MeV, 315 MeV, and 33.6 GeV

incident energies, respectively.³² Several observations follow:

(1) Both at 33.6 GeV (fig. 28) and at 315 MeV (fig. 27), the cross sections factorize within the experimental accuracy,^{16,32} providing further evidence for similar reaction mechanisms in the energy range between 20 MeV/A and 2 GeV/A. The validity of factorization at these energies explains the breakdown of Q_{gg} -systematics (see fig. 23) and the satisfactory description of the relative cross sections in terms of the simple ablation model (eq. 37; see also fig. 24).

(2) At 140 MeV incident energy, factorization does not apply. For the ^{94}Zr target, there is a systematic trend towards increasing yields for lighter elements (fig. 26b). Increasing yields for lighter elements are expected for increasing incident energy (see fig. 22a), and the kinetic energy above the Coulomb barrier increases from 3.5 MeV/A for $^{16}\text{O} + ^{208}\text{Pb}$ to 5.4 MeV/A for $^{16}\text{O} + ^{94}\text{Zr}$. Therefore the breakdown of factorization might be explained as partly due to the different Coulomb barriers for $^{16}\text{O} + ^{94}\text{Zr}$ and $^{16}\text{O} + ^{208}\text{Pb}$, and by a strong energy dependence of the particle yields at energies of only a few MeV/A above the Coulomb barrier. The isotope- and element-production cross sections for the systems $^{16}\text{O} + ^{208}\text{Pb}$ and $^{16}\text{O} + ^{197}\text{Au}$ are still remarkably similar at 140 MeV (see fig. 26a).

(3) The close similarity of isotope- and element-production cross sections for the systems $^{16}\text{O} + ^{197}\text{Au}$ and $^{16}\text{O} + ^{208}\text{Pb}$ at 140 MeV is not expected for the Q_{gg} -systematics (see discussion above). As shown in Section 7, a more consistent description of the isotope yields results from substituting the liquid-drop values Q_{gg}^{LD} (see eq. 36) for the

ground-state Q-values in eq. 32. Since the liquid-drop masses vary rather slowly with A and Z, the values $\Delta Q_{(x; {}^{197}\text{Au}, {}^{208}\text{Pb})}^{\text{LD}}$, defined in analogy with eq. 42, vary only between the extreme values of $\Delta Q_{\text{min}}^{\text{LD}} = -1.3 \text{ MeV}$ and $\Delta Q_{\text{max}}^{\text{LD}} = +0.9 \text{ MeV}$. Consequently, the application of the $Q_{\text{gg}}^{\text{LD}}$ -systematics predicts that the relative isotope yields should differ by not more than a factor of about two. This result is in significantly better agreement with the data than the application of the Q_{gg} -systematics.

10. SUMMARY AND CONCLUSIONS

In this paper we have discussed peripheral reactions induced by ${}^{16}\text{O}$ ions at incident energies up to 20 MeV/A on ${}^{94}\text{Zr}$, ${}^{197}\text{Au}$, ${}^{208}\text{Pb}$ and ${}^{232}\text{Th}$. We have compared the results with reactions studied at relativistic energies of 2.1 GeV/A. The energy region spanning from 20 MeV/A to 200 MeV/A has been experimentally inaccessible, but is likely to be the subject of increasing study in the future. Since the energy of 20 MeV/A is close to the average Fermi energy of a nucleon in nuclear matter, this energy was expected to define a transition between the phenomena characteristic of low- and high-energy heavy-ion reactions.

The present work indicates that, at an incident energy of 20 MeV/A, a transitional region is indeed encountered, and phenomena are observed which are reminiscent of heavy-ion reactions at low incident energies close to the Coulomb barrier, and of high, relativistic incident

energies. Below 20 MeV/A, reactions are generally discussed within the framework of two-body transfer processes, either by the direct transfer of one or more nucleons in quasi-elastic scattering, or by a partial equilibration and subsequent decay of a di-nuclear system in deeply-inelastic scattering. At relativistic incident energies, peripheral reactions are presently interpreted as fragmentation processes, in which more than two particles are produced in the exit channel. We found that many aspects of the data at 20 MeV/A and at 2.1 GeV/A are quite similar, such as the widths of the energy spectra, which are related to the Fermi momentum in the projectile, the shift of the kinetic energies of the fragments towards velocities smaller than the beam velocity, the production cross sections of elements and isotopes, and the applicability of the high-energy concept of factorization.

The existence of a transition occurring in the region of 20 MeV/A was demonstrated by describing the data in terms of two different systematics, which have been developed for the low-and high-energy extremes, and which embody the essential content of more elaborate theoretical approaches. The systematic exponential dependence of cross sections on two-body, ground-state Q-values begins to fail at 20 MeV/A. A more satisfactory description of the data is given by the exponential dependence on the Q-values for projectile fragmentation, which also applies at 2.1 GeV/A. Of course, similarities to reactions at lower incident energies remain. Both two-body, direct transfer reactions and equilibrated, deeply-inelastic scattering also take place at 20 MeV/A, although with small cross sections. Therefore, traditional approaches were followed in our discussion of energy dissipation

and of differential cross sections.

The classification of reactions into peripheral and central collisions appears to be a useful concept over the wide range of incident energies discussed in this work. At energies much lower than 20 MeV/A, central collisions can result in complete fusion, and at relativistic energies they may lead to many predicted exotic phenomena. Although the detailed processes are therefore radically different, our analysis showed that the cross section for central collisions hardly changes between the energies of 20 MeV/A and 2.1 GeV/A, and appears to be determined mainly by a critical overlap of nuclear matter. The peripheral reaction cross section is dominated at low energies by quasi-elastic and deeply-inelastic scattering, and at high energies by projectile fragmentation. With increasing energy, the time scale of the reaction changes. This change may account for the observation at low incident energies that equilibration of the excitation energy takes place over the intermediate complex. At high incident energies, the equilibration of excitation energy over the whole system becomes less significant compared with faster processes.

It would clearly be interesting to extend these studies of the evaluation of heavy-ion reaction mechanisms over a wide energy range to much heavier projectiles such as ^{40}Ar , for which the characteristic heavy-ion continuum processes at low energies are better developed. There is already evidence¹²⁹ in argon-induced reactions that the element production cross sections do not change significantly between 9.2 MeV/A⁶

and 1.8 GeV/A,¹³⁰ whereas there are substantial changes between 7.4 MeV/A and 9.2 MeV/A.⁶ The study of the interplay of the various contributing reaction mechanisms, and their relationship over a wide energy range in this transitional region, will become possible on the new heavy-ion accelerators under construction. These studies may lead to the development of a general theoretical framework, which will encompass the various aspects discussed in this paper, and from which the present approaches might emerge as extreme limits.

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Table 1. Isotope-production cross sections for ^{16}O -induced reactions at 140 MeV incident energy. The cross sections are given in mb and have been integrated over the angular ranges indicated. Errors include statistical errors, background from incomplete particle separation, and estimated effects of the finite number of angles measured over the angular interval.

Target	^{94}Zr	^{197}Au	^{208}Pb
Angular Range	$15^\circ - 30^\circ$	$20^\circ - 45^\circ$	$20^\circ - 45^\circ$
Reaction product			
^{16}N	--	--	$12.5 \pm 20\%$
^{15}N	$31.1 \pm 15\%$	$64.0 \pm 25\%$	$60.7 \pm 11\%$
^{14}N	$15.6 \pm 20\%$	--	$17.6 \pm 20\%$
^{13}N	--	--	$6.0 \pm 30\%$
^{15}C	--	--	$5.87 \pm 25\%$
^{14}C	$7.8 \pm 20\%$	$29.2 \pm 25\%$	$32.5 \pm 12\%$
^{13}C	$18.4 \pm 20\%$	$40.7 \pm 25\%$	$36.8 \pm 12\%$
^{12}C	$32.0 \pm 15\%$	$46.1 \pm 25\%$	$37.9 \pm 20\%$
^{11}C	--	$4.95 \pm 25\%$	$6.24 \pm 30\%$
^{12}B	$1.75 \pm 30\%$	$3.6 \pm 30\%$	$3.86 \pm 30\%$
^{11}B	$8.9 \pm 15\%$	$12.6 \pm 25\%$	$11.2 \pm 15\%$
^{10}B	$3.84 \pm 25\%$	$2.8 \pm 30\%$	$4.18 \pm 30\%$
^{10}Be	$1.73 \pm 40\%$	$3.32 \pm 30\%$	$3.53 \pm 20\%$
^9Be	$4.78 \pm 25\%$	$4.47 \pm 30\%$	$4.0 \pm 20\%$
^7Be	$0.61 \pm 35\%$	$0.47 \pm 50\%$	$0.95 \pm 30\%$
^7Li	$3.94 \pm 30\%$	$2.4 \pm 50\%$	$2.8 \pm 30\%$
^6Li	$3.55 \pm 30\%$	$2.36 \pm 50\%$	$3.1 \pm 30\%$

Table 2. Isotope-production cross sections for ^{16}O -induced reactions at 315 MeV incident energy. The cross sections are given in mb and have been integrated over the angular ranges indicated. The low-energy components of the nitrogen and carbon spectra (see fig. 4) have not been included. Errors include statistical errors, background from incomplete particle separation, and estimated effects of the finite number of angles measured over the angular interval.

Target	$^{94}\text{Zr}^*$	^{197}Au	^{208}Pb	^{232}Th
Angular range	10°	$10^\circ - 20^\circ$	$0^\circ - 180^\circ$	$10^\circ - 20^\circ$
REACTION PRODUCT				
^{19}F	--	--	$20 \pm 30\%$	--
^{15}O	--	--	$38 \pm 50\%$	--
^{15}N	$236 \pm 10\%$	$29.1 \pm 25\%$	$211 \pm 25\%$	$30.0 \pm 25\%$
^{14}N	$181 \pm 10\%$	$15.7 \pm 25\%$	$140 \pm 30\%$	$19.0 \pm 25\%$
^{13}N	$42 \pm 25\%$	$3.5 \pm 30\%$	$27 \pm \begin{smallmatrix} 20\% \\ 50\% \end{smallmatrix}$	$6.7 \pm \begin{smallmatrix} 25\% \\ 30\% \end{smallmatrix}$
^{14}C	$74 \pm 25\%$	$9.0 \pm 30\%$	$43 \pm \begin{smallmatrix} 15\% \\ 50\% \end{smallmatrix}$	$12.0 \pm 30\%$
^{13}C	$180 \pm 20\%$	$29.0 \pm 25\%$	$127 \pm \begin{smallmatrix} 15\% \\ 25\% \end{smallmatrix}$	$45.0 \pm 25\%$
^{12}C	$410 \pm 10\%$	$40.0 \pm 25\%$	$198 \pm \begin{smallmatrix} 15\% \\ 15\% \end{smallmatrix}$	$45.0 \pm \begin{smallmatrix} 25\% \\ 40\% \end{smallmatrix}$
^{11}C	$76 \pm 30\%$	$5.5 \pm 35\%$	$28 \pm \begin{smallmatrix} 15\% \\ 50\% \end{smallmatrix}$	$7.7 \pm 30\%$
^{12}B	$31 \pm 30\%$	$3.9 \pm 30\%$	$23 \pm \begin{smallmatrix} 34\% \\ 50\% \end{smallmatrix}$	$5.6 \pm 30\%$
^{11}B	$202 \pm 10\%$	$22.0 \pm 25\%$	$114 \pm \begin{smallmatrix} 35\% \\ 15\% \end{smallmatrix}$	$33.0 \pm 25\%$
^{10}B	$96 \pm 25\%$	$8.5 \pm 25\%$	$50 \pm 36\%$	$9.6 \pm 25\%$
^{10}Be	$47 \pm 20\%$	$8.3 \pm 25\%$	$39 \pm \begin{smallmatrix} 50\% \\ 25\% \end{smallmatrix}$	$10.2 \pm 25\%$
^9Be	$78 \pm 15\%$	$10.4 \pm 25\%$	$54 \pm \begin{smallmatrix} 50\% \\ 15\% \end{smallmatrix}$	$12.0 \pm 25\%$
^7Be	$36 \pm 15\%$	$2.8 \pm 25\%$	$15 \pm \begin{smallmatrix} 50\% \\ 20\% \end{smallmatrix}$	$2.5 \pm 25\%$
^9Li	--	--	$5 \pm \begin{smallmatrix} 36\% \\ 50\% \end{smallmatrix}$	--
^8Li	--	--	$16 \pm \begin{smallmatrix} 35\% \\ 50\% \end{smallmatrix}$	$3.8 \pm 30\%$
^7Li	$100 \pm \begin{smallmatrix} 25\% \\ 15\% \end{smallmatrix}$	$17.8 \pm 30\%$	$93 \pm \begin{smallmatrix} 35\% \\ 20\% \end{smallmatrix}$	$16.7 \pm 25\%$
^6Li	$86 \pm 25\%$	$10.2 \pm 30\%$	$54 \pm \begin{smallmatrix} 35\% \\ 30\% \end{smallmatrix}$	$11.4 \pm 35\%$

*Differential cross section in mb/sr.

Table 3. Parameters used for the description of the angular distributions shown in Fig. 13 [Eq. (20) of text] and the most probable kinetic energies shown in figs. 8 and 9 [Eqs. (12-14) of text].

Element	θ_0 (deg)	$\Delta\theta$ (deg)	$\Delta L \equiv \sqrt{Z}/\Delta\theta$	A_T [10^{-22} MeV·sec/fm]
N	40	10.8	7.5	0.4
C	35	18	4.5	1.4
B	30	18	4.5	4.4
Be	20	32.4	2.5	7.0
Li	20	54	1.5	7.6

Table 4. Experimental and theoretical values of p_0 , the momentum of the fragments in the projectile rest frame, for reactions induced by ^{16}O on ^{208}Pb at 315 MeV and 33.6 GeV incident energies (see eqs. 8,18).

FRAGMENT	$E_L = 33.6 \text{ GeV}$ $p_0 (\text{MeV}/c)$		$E_L = 315 \text{ MeV}$ $p_0 (\text{MeV}/c)$	
	EXPT.	THEORY	EXPT.	THEORY
^{15}N	-21 ± 6	-23	$+24 \pm 15$	-111
^{14}N	-27 ± 3	-30	$+10 \pm 30$	-141
^{13}N	-35 ± 4	-35	$+69 \pm 50$	-164
^{14}C	-38 ± 7	-31	-105 ± 50	-148
^{13}C	-33 ± 7	-36	-38 ± 30	-172
^{12}C	-25 ± 6	-15	$+7 \pm 15$	-70
^{11}C	-45 ± 13	-47	$+41 \pm 30$	-225
^{12}B	-59 ± 10	-47	-292 ± 80	-224
^{11}B	-53 ± 3	-45	-187 ± 50	-214
^{10}B	-40 ± 7	-49	$+102 \pm 50$	-233
^{10}Be	-65 ± 6	-49	-409 ± 50	-233
^9Be	-47 ± 7	-48	-278 ± 50	-227
^7Be	-45 ± 9	-47	-122 ± 50	-222
^7Li	-46 ± 6	-46	-266 ± 30	-219
^6Li	-33 ± 7	-42	-155 ± 30	-199
AVERAGE	-41 ± 8	-41	-159 (excluding nitrogen)	-195
			-120 (total)	

Table 5. Optical-potential parameters used for the analysis of elastic scattering and single-nucleon transfer induced by ^{16}O on ^{208}Pb .

E_{lab} (MeV)	V (MeV)	r_o (fm)	a_R (fm)	W (MeV)	r_I (fm)	a_I (fm)
140	51.1	1.21	0.68	51.5	1.21	0.63
312.6	51.1	1.11	0.80	51.5	1.11	0.74

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FIGURE CAPTIONS

Fig. 1. Schematic classification of time scales of heavy-ion collisions, velocities and MeV/nucleon as a function of mass and energy of the ions above the Coulomb barrier. The curves are constructed for fixed distances of 15 fm. The characteristic features of macroscopic reaction processes are divided into different regions: equilibration phenomena observed in deeply-inelastic scattering for energies less than 10 MeV/nucleon and fragmentation phenomena at relativistic energies. The bold line at 20 MeV/nucleon corresponds to energies close to the average Fermi energy of a nucleon, and to the characteristic nuclear time τ_N . Up to energies of 20 MeV/A, direct two-body, transfer reactions are also known to take place.

Fig. 2. Energy spectra of the elements nitrogen, carbon, boron, beryllium, and lithium produced in the reaction $^{16}\text{O} + ^{208}\text{Pb}$ at $\theta_L = 40^\circ$ and $E_L = 140$ MeV. The arrows marked by g.s., E_p , V_c and E_F correspond to the energy of the ground-state transition for a two-body transfer reaction, the energy of a reaction product with the projectile velocity, the exit-channel Coulomb barrier, and the energy predicted for a fragmentation of the projectile by the channel with minimum Q-value, respectively.

Fig. 3. Energy spectra of the elements nitrogen, carbon, boron, beryllium, and lithium produced in the reaction $^{16}\text{O} + ^{208}\text{Pb}$ at $\theta_L = 15^\circ$ and $E_L = 315$ MeV. The arrows marked by g.s., E_p , V_c and E_F correspond to the energy of the ground-state transition for a two-body transfer reaction, the energy of a product with the projectile velocity, the exit-channel Coulomb barrier, and the energy predicted for a fragmentation of the projectile into the observed fragment together with individual nucleons or alpha particles, as discussed in the text.

Fig. 4. Energy spectra of the elements nitrogen, carbon, boron, beryllium, and lithium produced by $^{16}\text{O} + ^{208}\text{Pb}$ at $\theta_L = 25^\circ$ and $E_L = 315$ MeV. The arrows marked by g.s. and V_c correspond to the energy of the ground-state transition for a two-body transfer reaction and the exit-channel Coulomb barrier, respectively.

Fig. 5. Energy spectrum for the reaction $^{208}\text{Pb}(^{16}\text{O}, ^{15}\text{N}) ^{209}\text{Bi}$ populating single-particle states in ^{209}Bi at $\theta_L = 15^\circ$ and $E_L = 312.6$ MeV. The insert shows a typical differential cross section, together with the distribution (solid curve) predicted by a distorted wave Born approximation (DWBA) calculation. The dashed lines are drawn to guide the eye.

Fig. 6. Dependence of most probable final fragment kinetic energies on scattering angle for elements produced by ^{16}O on ^{208}Pb at 140 MeV. The curves are drawn to guide the eye.

Fig. 7. Dependence of most probable final fragment kinetic energies on scattering angle for elements produced by ^{16}O on ^{208}Pb at 315 MeV. The curves are drawn to guide the eye.

Fig. 8. Most probable final fragment energies of isotopes detected near the grazing angle for reactions produced by ^{16}O on ^{208}Pb at 140 MeV. The solid curve corresponds to identical energies per nucleon of projectile M_i and reaction product M_f . The dotted curves represent the friction model calculations discussed in the text.

Fig. 9. Most probable final-fragment energies of isotopes detected near the grazing angle for reactions produced by ^{16}O on ^{208}Pb at 315 MeV. The solid curve corresponds to identical energies per nucleon of projectile M_i and reaction product M_f . The dotted curves represent the friction model calculations discussed in the text.

Fig. 10. Effective Q-values corresponding to the most probable final fragment kinetic energies shown in fig. 8. The theoretical curve is discussed in the text.

Fig. 11. Effective Q-values corresponding to the most probable final fragment kinetic energies shown in fig. 9. The theoretical curve is discussed in the text.

Fig. 12. Schematic diagram illustrating the influence of complete damping of radial velocity on classical trajectories. For the grazing trajectory, $\phi = 0$.

Fig. 13. Angular distributions of the integrated energy spectra for the elements nitrogen, carbon, boron, beryllium, and lithium produced by ^{16}O on ^{208}Pb at $E_L = 140$ MeV. The curves have been calculated with the parameterization of eq. 20.

Fig. 14. Angular distributions of the integrated energy spectra for the elements nitrogen, carbon, boron, beryllium, and lithium produced by the reaction of ^{16}O on ^{208}Pb at $E_L = 315$ MeV. The low-energy components of the carbon and nitrogen spectra are not included. The curves are exponential distributions $d\alpha/d\Omega \sim e^{-\theta/\alpha}/\sin\theta$, with the decay angles α shown.

Fig. 15. Angular distribution of the elastic scattering of ^{16}O on ^{208}Pb at the laboratory energy of 312.6 MeV. The curve represents an optical-model calculation with the potential parameters given in Table 5.

Fig. 16. Deflection function $\Theta_\ell = 2d\delta_\ell/d\ell$, and reflection coefficients, $|\eta_\ell| = S_\ell$, of elastic-scattering S-matrix elements calculated with the optical potential parameters of Table 5 (Ref. 36) for ^{16}O on ^{208}Pb at $E_L = 140$ MeV. The solid points correspond to the numerical integration of the Schrödinger equation; the solid lines are the WKB results using real trajectories, and the dot-dashed curve is the surface-reflection term discussed in Ref. 83. The dashed line is the Coulomb deflection function.

Fig. 17. Deflection function and reflection coefficients of elastic-scattering S-matrix elements calculated with the optical potential parameters of Table 5 (Ref. 36) for $^{16}\text{O} + ^{208}\text{Pb}$ at $E_L = 315$ MeV. The solid points correspond to the numerical integration of the Schrödinger equation, the solid lines are the WKB results using real trajectories, the dashed line is the Coulomb deflection function, and the dot-dashed curve is the surface-reflection term discussed in Ref. 83.

Fig. 18. Partial-wave reaction cross sections $\sigma_{R,\ell}$ as a function of the incoming angular momentum ℓ and distance of closest approach. The critical angular momentum ℓ_c is defined in the text. The dashed curve corresponds to the distribution of the dominating radial integrals of the DWBA calculation shown in fig. 5; the normalization of the radial integrals is arbitrary.

Fig. 19. Isotope-production cross sections observed for reactions induced by ^{16}O on ^{208}Pb at the laboratory angle $\theta_L = 40^\circ$ and 140 MeV incident energy. In (a) the dependence on the ground-state Q-value, Q_{gg} , of the corresponding transfer reaction is shown, and in (b) the dependence on the liquid-drop values Q_{gg}^{LD} (see eq. 36) is shown. The lines correspond to the parameters $T = 5.3$ MeV(a) and $T = 2.7$ MeV (b).

Fig. 20. Isotope-production cross sections observed for reactions induced by ^{16}O on ^{197}Au at the laboratory angle $\theta_L = 32.5^\circ$ and 140 MeV incident energy. In (a) the dependence on the ground-state Q-value, Q_{gg} , of the corresponding transfer reaction is shown, and in (b) the dependence on the liquid-drop values Q_{gg}^{LD} (see eq. 36) is shown. The lines correspond to the parameters $T = 3.2$ MeV (a) and $T = 2.7$ MeV (b).

Fig. 21. Isotope-production cross sections observed for reactions induced by ^{16}O on ^{94}Zr at the laboratory angle $\theta_L = 20^\circ$ and 140 MeV incident energy. In (a) the dependence on the ground-state Q-value, Q_{gg} , of the corresponding transfer reaction is shown; in (b) the pairing-energy corrections have been applied, and in (c) the liquid-drop values Q_{gg}^{LD} (see eq. 36) have been used. No unique set of straight lines can be found for a simple Q_{gg} dependence. The lines in (b) and (c) correspond to the parameters $T = 3.2$ MeV and $T = 2.6$ MeV, respectively.

Fig. 22. Ratios of isotope- and element-production cross sections measured for reactions induced by ^{16}O on ^{208}Pb at (a) 140 and 315 MeV (the vertical axis is in arbitrary units), and (b) at 315 MeV and 33.6 GeV (the vertical axis represents an absolute scale),

Fig. 23. Dependence of integrated isotope-production cross sections on ground-state Q-values, Q_{gg} , for reactions induced by ^{16}O on ^{208}Pb at 140 MeV, 315 MeV, and 33.6 GeV incident energies.

Fig. 24. Plot of the ratios $C(N,Z)$ (see eq. 38) for isotopes observed in the reactions induced by ^{16}O on ^{208}Pb at (a) 140 MeV, (b) 315 MeV and (c) 33.6 GeV incident energy. The effective temperatures of 5.1, 7.3, and 6.2 MeV, respectively, have been used.

Fig. 25. Differences of ground-state Q-values, Q_{gg} , for reactions induced by ^{16}O on ^{94}Zr , ^{197}Au , ^{208}Pb , and ^{232}Th targets.

Fig. 26. Comparison of isotope yields observed in the bombardment of ^{208}Pb , ^{197}Au , and ^{94}Zr targets by ^{16}O at 140 MeV laboratory energy. The vertical scale is in arbitrary units.

Fig. 27. Comparison of isotope yields observed in the bombardment of ^{232}Th , ^{208}Pb , ^{197}Au , and ^{94}Zr targets by ^{16}O ions at 315 MeV laboratory energy. The vertical scale is in arbitrary units.

Fig. 28. Comparison of isotope yields observed in the bombardment of Pb, Ag, and Au targets by ^{16}O ions of 33.6 GeV energy. Data have been taken from Ref. 15.

XBL7610-4154

Fig. 1

ML 772-444

Fig. 2

Fig. 3

Fig. 4

-93-

XBL 7610 4290A

Fig. 5

Fig. 6

-95-

XBL 769 4055

Fig. 7

XBL 7611-4352

Fig. 8

XBL 7611-4353

Fig. 9

XBL 7610 4112

Fig. 10

-99-

Fig. 11

XBL 769 4054

XBL772 - 441

Fig. 12

Fig. 13

Fig. 14

-103-

Fig. 15

XBL 7610-4313

XBL 772-443

Fig. 16

-105-

XBL 772-440

Fig. 17

Fig. 18

-107-

XBL 7610-4958

Fig. 19

Fig. 20

XBL 7610-4956

Fig. 21

Fig. 22

XBL 7610-4957

Fig. 23

XBL 7610 4242

Fig. 24

XBL 7610 4102

Fig. 25

XBL769-3949

Fig. 26

XBL769 - 3954

Fig. 27

XBL769-3953

Fig. 28

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