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Authors

Fomel, S.

Sethian, J.A.

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S. Fomel and J.A. Sethian

Computing Sciences Directorate

Mathematics Department

April 2001

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Multiple Arrivals using Liouville Equations

S. Fomel and J.A. Sethian

Computing Sciences Directorate
Mathematics Department
Ernest Orlando Lawrence Berkeley National Laboratory
University of California
Berkeley, California 94720

April 2001

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Multiple Arrivals using Liouville Equations

S. Fomel and J.A. Sethian

Mathematics Department

Lawrence Berkeley National Laboratory

University of California, Berkeley

April 16, 2001¹

Traveltime from a fixed source $\tau(\mathbf{x})$ in an isotropic medium ($\mathbf{x} \in \mathbf{R}^N$) is governed by the eikonal equation

$$|\nabla \tau|^2 v^2(\mathbf{x}) = 1, \quad (1)$$

where $v(\mathbf{x})$ is the velocity distribution.

The rays [characteristics of equation (1)] are defined by the system of Hamilton-Jacobi ordinary differential equations:

$$\frac{d\mathbf{x}}{dt} = v^2(\mathbf{x}) \mathbf{p}; \quad (2)$$

$$\frac{d\mathbf{p}}{dt} = -\frac{1}{v(\mathbf{x})} \nabla v, \quad (3)$$

where t has the meaning of the traveltime along the ray, and $\mathbf{p}$ corresponds to $\nabla \tau$ and is constrained by the Hamilton equation

$$|\mathbf{p}|^2 v^2(\mathbf{x}) = 1, \quad (4)$$

equivalent to (1).

In the two-dimensional case, where $\mathbf{x} = \{x, z\}$, it is convenient to introduce the angle θ between the vertical and the slowness vector $\mathbf{p}$ such that $\mathbf{p} = \{\sin \theta / v(x, z), \cos \theta / v(x, z)\}$. Equation (4) is then automatically satisfied, and we can rewrite system (2-3) in the form

$$\frac{dx}{dt} = v(x, z) \sin \theta; \quad (5)$$

$$\frac{dz}{dt} = v(x, z) \cos \theta; \quad (6)$$

$$\frac{d\theta}{dt} = \frac{\partial v}{\partial z} \sin \theta - \frac{\partial v}{\partial x} \cos \theta. \quad (7)$$

The initial conditions for solving system (5-7) consist of the initial point $\{x_0, z_0\}$ and the take-off angle θ_0 .

The solution of system (5-7) as a function of time t and the initial conditions x_0, z_0 , and θ_0 satisfies the Liouville partial differential equations:

$$\frac{\partial x}{\partial t} + v \sin \theta_0 \frac{\partial x}{\partial x_0} + v \cos \theta_0 \frac{\partial x}{\partial z_0} + \left(\frac{\partial v}{\partial z_0} \sin \theta_0 - \frac{\partial v}{\partial x_0} \cos \theta_0 \right) \frac{\partial x}{\partial \theta_0} = 0 \quad (8)$$

¹This Lawrence Berkeley National Laboratory Technical Report was written on April 16, 2001. Backup files listing from Department of Energy computers are included at the end of the report, as well as dated e-mail containing the correspondence between the two authors.

$$\frac{\partial z}{\partial t} + v \sin \theta_0 \frac{\partial z}{\partial x_0} + v \cos \theta_0 \frac{\partial z}{\partial z_0} + \left(\frac{\partial v}{\partial z_0} \sin \theta_0 - \frac{\partial v}{\partial x_0} \cos \theta_0 \right) \frac{\partial z}{\partial \theta_0} = 0 \quad (9)$$

$$\frac{\partial \theta}{\partial t} + v \sin \theta_0 \frac{\partial \theta}{\partial x_0} + v \cos \theta_0 \frac{\partial \theta}{\partial z_0} + \left(\frac{\partial v}{\partial z_0} \sin \theta_0 - \frac{\partial v}{\partial x_0} \cos \theta_0 \right) \frac{\partial \theta}{\partial \theta_0} = 0 \quad (10)$$

where the velocity v is evaluated at $\{x_0, z_0\}$. The appropriate initial conditions for system (8-10) are $\{x, z, \theta\} = \{x_0, z_0, \theta_0\}$ at $t = 0$.

Let us denote by $T(x, z, \theta)$ the time at which the ray that starts at point $\{x, z\}$ with the take-off angle θ first reaches the surface $z = 0$. Correspondingly, the emergence point and the emergence angle of this ray at the surface will be defined by functions $X(x, z, \theta)$ and $\Theta(x, z, \theta)$. Differentiating the condition

$$z(T(x_0, z_0, \theta_0), x_0, z_0, \theta_0) = 0, \quad (11)$$

where $z(t, x_0, z_0, \theta_0)$ is the solution of equation (9), we find that, in the region where $\frac{\partial z}{\partial t}$ is different from zero, the function T has to satisfy the partial differential equation

$$v(x, z) \sin \theta \frac{\partial T}{\partial x} + v(x, z) \cos \theta \frac{\partial T}{\partial z} + \left(\frac{\partial v}{\partial z} \sin \theta - \frac{\partial v}{\partial x} \cos \theta \right) \frac{\partial T}{\partial \theta} = 1. \quad (12)$$

with the boundary condition $T|_{z=0} = 0$. As follows from equations (12), (8), and (10), and the conditions

$$z(T(x_0, z_0, \theta_0), x_0, z_0, \theta_0) = X(x_0, z_0, \theta_0); \quad (13)$$

$$\theta(T(x_0, z_0, \theta_0), x_0, z_0, \theta_0) = \Theta(x_0, z_0, \theta_0), \quad (14)$$

the functions $X(x, z, \theta)$ and $\Theta(x, z, \theta)$ additionally satisfy the orthogonal equations

$$v(x, z) \sin \theta \frac{\partial X}{\partial x} + v(x, z) \cos \theta \frac{\partial X}{\partial z} + \left(\frac{\partial v}{\partial z} \sin \theta - \frac{\partial v}{\partial x} \cos \theta \right) \frac{\partial X}{\partial \theta} = 0 \quad (15)$$

$$v(x, z) \sin \theta \frac{\partial \Theta}{\partial x} + v(x, z) \cos \theta \frac{\partial \Theta}{\partial z} + \left(\frac{\partial v}{\partial z} \sin \theta - \frac{\partial v}{\partial x} \cos \theta \right) \frac{\partial \Theta}{\partial \theta} = 0 \quad (16)$$

with the boundary conditions $X|_{z=0} = x$ and $\Theta|_{z=0} = \theta$.

We propose to apply equations (12), (15) and (16) for a numerical computations of traveltimes on a fixed x, z grid. Although both $T(x, z, \theta)$ and $X(x, z, \theta)$ functions are strictly single-valued, we can extract from them the possibly multi-valued traveltimes from every grid point x, z to a surface point y at $z = 0$. The extraction would simply amount to evaluating $T(x, z, \theta)$ at the level set of $X(x, z, \theta) = y$.

Dated E-mail Correspondence containing Technical Memo

From fomel@math.lbl.gov Mon Apr 16 14:00 PDT 2001
 Received: from math.lbl.gov (math.lbl.gov [128.3.7.22])
 by math.berkeley.edu (8.9.3/8.9.3) with ESMTP id 0AA04060
 for <sethian@math.berkeley.edu>; Mon, 16 Apr 2001 14:00:25 -0700 (PDT)
 Received: from dnierpr (dnierpr.lbl.gov [128.3.3.153])
 by math.lbl.gov (8.10.2/8.10.2) with SMTP id f3GL0PD27256
 for <sethian@math.berkeley.edu>; Mon, 16 Apr 2001 14:00:25 -0700 (PDT)
 Message-Id: <200104162100.f3GL0PD27256@math.lbl.gov>
 Date: Mon, 16 Apr 2001 14:01:16 -0700 (PDT)
 From: Sergey Fomel <fomel@math.lbl.gov>
 Reply-To: Sergey Fomel <fomel@math.lbl.gov>
 Subject: Re: Your Message Sent on Tue, 10 Apr 2001 09:41:07 -0700 (PDT)
 To: sethian@math.berkeley.edu
 MIME-Version: 1.0
 X-Mailer: dtmail 1.3.0 CDE Version 1.3 SunOS 5.7 sun4u sparc
 Content-Type: MULTIPART/mixed; BOUNDARY=Band_of_Gorillas_852_000
 Content-Length: 7111
 Status: RO
 X-Status:
 X-Keywords:
 X-UID: 352

--Band_of_Gorillas_852_000
 Content-Type: TEXT/plain; charset=us-ascii
 Content-MD5: m/JCi/3d78hmCWQ9eIG0eg==

>I would think that we should indeed link them together. Can you send me
 >a very short latex file with the equations, pointing to the exact
 >point you mean - I want to think about it while I am traveling....

Jamie,

Please find the latex file enclosed. The explanation is very raw and will
 probably need some refining, but the main equations are there.

I have been reading some more literature on the subject. Apparently, the idea of
 computing traveltimes in the (x, z, θ) space is not new. This idea is the
 essence of Maslov's asymptotic ray theory. Asymptotic theoreticians (like
 Hormander) use it to construct uniform asymptotics of the ray-theoretical
 solution near the caustics. However, the idea to use Liouville's PDE and the
 corresponding numerical scheme look like an entirely new computational approach.

Have a nice trip. I plan to be in the office on Wednesday.

Sergey

--Band_of_Gorillas_852_000
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 Content-Description: theory.tex
 Content-MD5: s0qmS4du0ZQ0H1eCwIlkHQ==

Travelttime from a fixed source $\tau(\mathbf{x})$ in an isotropic medium
 $(\mathbf{x} \in \mathbb{R}^N)$ is governed by the eikonal equation

$$\|\nabla \tau(\mathbf{x})\|^2 = v^2(\mathbf{x})$$
 where $v(\mathbf{x})$ is the velocity distribution.

The rays [characteristics of equation (1)] are defined
 by the system of Hamilton-Jacobi ordinary differential equations:

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}(\mathbf{x}), \quad \frac{d\tau}{dt} = 1$$

$$\frac{d}{dt} \mathbf{p} = - \nabla v(\mathbf{x}),$$
 where t has the meaning of the traveltime along the ray, and $\mathbf{p}$ corresponds to $\nabla \tau$ and is constrained by the Hamilton equation

$$|\mathbf{p}|^2 = 1,$$
 equivalent to (eikonal).

In the two-dimensional case, where $\mathbf{x} = (x, z)$, it is convenient to introduce the angle θ between the vertical and the slowness vector $\mathbf{p}$ such that $\mathbf{p} = (\sin \theta / v(x, z), \cos \theta / v(x, z))$. Equation (Hamilton) is then automatically satisfied, and we can rewrite system (xray-pray) in the form

$$\begin{aligned} \frac{dx}{dt} &= v(x, z) \sin \theta, \\ \frac{dz}{dt} &= v(x, z) \cos \theta, \\ \frac{d\theta}{dt} &= - \frac{\partial v}{\partial z} \sin \theta - \frac{\partial v}{\partial x} \cos \theta. \end{aligned}$$

The initial conditions for solving system (xt-thetat) consist of the initial point (x_0, z_0) and the take-off angle θ_0 .

The solution of system (xt-thetat) as a function of time t and the initial conditions x_0 , z_0 , and θ_0 satisfies the Liouville partial differential equations:

$$\begin{aligned} \frac{\partial x}{\partial t} + v \sin \theta_0, \\ \frac{\partial x}{\partial x_0} + v \cos \theta_0, \\ \frac{\partial x}{\partial z_0} + \left(\frac{\partial v}{\partial z_0} \sin \theta_0 - \frac{\partial v}{\partial x_0} \cos \theta_0 \right), \\ \frac{\partial x}{\partial \theta_0} &= 0, \\ \frac{\partial z}{\partial t} + v \cos \theta_0, \\ \frac{\partial z}{\partial x_0} + v \sin \theta_0, \\ \frac{\partial z}{\partial z_0} + \left(\frac{\partial v}{\partial z_0} \cos \theta_0 - \frac{\partial v}{\partial x_0} \sin \theta_0 \right), \\ \frac{\partial z}{\partial \theta_0} &= 0, \\ \frac{\partial \theta}{\partial t} + v, \\ \frac{\partial \theta}{\partial x_0} + v \cos \theta_0, \\ \frac{\partial \theta}{\partial z_0} + \left(\frac{\partial v}{\partial z_0} \sin \theta_0 - \frac{\partial v}{\partial x_0} \cos \theta_0 \right), \\ \frac{\partial \theta}{\partial \theta_0} &= 0, \end{aligned}$$

where the velocity v is evaluated at (x_0, z_0) . The appropriate initial conditions for system (xl-thetal) are $(x, z, \theta) = (x_0, z_0, \theta_0)$ at $t=0$.

Let us denote by $T(x, z, \theta)$ the time at which the ray that starts at point (x, z) with the take-off angle θ first reaches the surface $z=0$. Correspondingly, the emergence point and the emergence angle of this ray at the surface will be defined by functions $X(x, z, \theta)$ and $\Theta(x, z, \theta)$. Differentiating the condition

```

\begin{equation}
\label{eq:zsurface}
z(T(x_0,z_0,\theta_0),x_0,z_0,\theta_0) = 0\;,
\end{equation}
where  $z(t,x_0,z_0,\theta_0)$  is the solution of
equation~(\ref{eq:zl}), we find that, in the region where
 $\frac{\partial z}{\partial t}$  is different from zero, the function
 $T$  has to satisfy the partial differential equation
\begin{equation}
\label{eq:tmarch}
v(x,z)\sin\theta,
\frac{\partial T}{\partial x} + v(x,z)\cos\theta,
\frac{\partial T}{\partial z} + \left(
\frac{\partial v}{\partial z}\sin\theta -
\frac{\partial v}{\partial x}\cos\theta\right),
\frac{\partial T}{\partial \theta} = 1\;.
\end{equation}
with the boundary condition  $\left.T\right|_{z=0} = 0$ . As follows
from equations~(\ref{eq:tmarch}), (\ref{eq:x1}),
and~(\ref{eq:theta1}), and the conditions
\begin{equation}
\label{eq:xsurface}
x(T(x_0,z_0,\theta_0),x_0,z_0,\theta_0) = x
X(x_0,z_0,\theta_0)\;, \quad \backslash\backslash
\label{eq:thetasurface}
\theta(T(x_0,z_0,\theta_0),x_0,z_0,\theta_0) = \theta
\Theta(x_0,z_0,\theta_0)\;,
\end{equation}
the functions  $X(x,z,\theta)$  and  $\Theta(x,z,\theta)$  additionally
satisfy the orthogonal equations
\begin{equation}
\label{eq:xmarch}
v(x,z)\sin\theta,
\frac{\partial X}{\partial x} + v(x,z)\cos\theta,
\frac{\partial X}{\partial z} + \left(
\frac{\partial v}{\partial z}\sin\theta -
\frac{\partial v}{\partial x}\cos\theta\right),
\frac{\partial X}{\partial \theta} = 0\;, \quad \backslash\backslash
\label{eq:thetamarch}
v(x,z)\sin\theta,
\frac{\partial \Theta}{\partial x} + v(x,z)\cos\theta,
\frac{\partial \Theta}{\partial z} + \left(
\frac{\partial v}{\partial z}\sin\theta -
\frac{\partial v}{\partial x}\cos\theta\right),
\frac{\partial \Theta}{\partial \theta} = 0\;.
\end{equation}
with the boundary conditions  $\left.X\right|_{z=0} = x$  and
 $\left.\Theta\right|_{z=0} = \theta$ .

We propose to apply equations~(\ref{eq:tmarch}), (\ref{eq:xmarch}) and
(\ref{eq:thetamarch}) for a numerical computations of traveltimes on a
fixed  $\{x,z\}$  grid. Although both  $T(x,z,\theta)$  and  $X(x,z,\theta)$ 
functions are strictly single-valued, we can extract from them the
possibly multi-valued traveltimes from every grid point  $\{x,z\}$  to a
surface point  $y$  at  $z=0$ . The extraction would simply amount to
evaluating  $T(x,z,\theta)$  at the level set of  $X(x,z,\theta) = y$ .

```

--Band_of_Gorillas_852_000--

Traveltime from a fixed source $\tau(\mathbf{x})$ in an isotropic medium ($\mathbf{x} \in \mathbb{R}^N$) is governed by the eikonal equation

$$\begin{aligned} &\text{\label{eq:eikonal}} \\ &\left| \nabla \tau \right|^2 v^2(\mathbf{x}) = 1; \end{aligned}$$

where $v(\mathbf{x})$ is the velocity distribution.

The rays [characteristics of equation~(\ref{eq:eikonal})] are defined by the system of Hamilton-Jacobi ordinary differential equations:

$$\begin{aligned} &\text{\label{eq:xray}} \\ &\frac{d\mathbf{x}}{dt} = v^2(\mathbf{x}) \nabla \tau; \quad \text{\label{eq:pray}} \\ &\frac{d\mathbf{p}}{dt} = - \nabla v(\mathbf{x}); \end{aligned}$$

where t has the meaning of the traveltime along the ray, and $\mathbf{p}$ corresponds to $\nabla \tau$ and is constrained by the Hamilton equation

$$\begin{aligned} &\text{\label{eq:hamilton}} \\ &\left| \mathbf{p} \right|^2 v^2(\mathbf{x}) = 1; \end{aligned}$$

equivalent to~(\ref{eq:eikonal}).

In the two-dimensional case, where $\mathbf{x} = \{x, z\}$, it is convenient to introduce the angle~ θ between the vertical and the slowness vector~ $\mathbf{p}$ such that $\mathbf{p} = \left(\frac{\sin \theta}{v(x, z)}, \frac{\cos \theta}{v(x, z)} \right)$.

Equation~(\ref{eq:hamilton}) is then automatically satisfied, and we can rewrite system~(\ref{eq:xray}-\ref{eq:pray}) in the form

$$\begin{aligned} &\text{\label{eq:xt}} \\ &\frac{dx}{dt} = v(x, z) \sin \theta; \quad \text{\label{eq:zt}} \\ &\frac{dz}{dt} = v(x, z) \cos \theta; \quad \text{\label{eq:thetat}} \\ &\frac{d\theta}{dt} = - \frac{\partial v}{\partial z} \sin \theta - \frac{\partial v}{\partial x} \cos \theta. \end{aligned}$$

The initial conditions for solving system~(\ref{eq:xt}-\ref{eq:thetat}) consist of the initial point~ $\{x_0, z_0\}$ and the take-off angle~ θ_0 .

The solution of system~(\ref{eq:xt}-\ref{eq:thetat}) as a function of time t and the initial conditions x_0 , z_0 , and θ_0 satisfies the Liouville partial differential equations:

$$\begin{aligned} &\text{\label{eq:xl}} \\ &\frac{\partial x}{\partial t} + v \sin \theta_0, \\ &\frac{\partial x}{\partial x_0} + v \cos \theta_0, \\ &\frac{\partial x}{\partial z_0} + \left(\frac{\partial v}{\partial z_0} \sin \theta_0 - \frac{\partial v}{\partial x_0} \cos \theta_0 \right), \\ &\frac{\partial x}{\partial \theta_0} = 0; \quad \text{\label{eq:zl}} \\ &\frac{\partial z}{\partial t} + v \sin \theta_0, \\ &\frac{\partial z}{\partial x_0} + v \cos \theta_0, \\ &\frac{\partial z}{\partial z_0} + \left(\frac{\partial v}{\partial z_0} \sin \theta_0 - \frac{\partial v}{\partial x_0} \cos \theta_0 \right), \end{aligned}$$

$$\begin{aligned} & \frac{\partial v}{\partial z_0} \sin \theta_0 - \frac{\partial v}{\partial x_0} \cos \theta_0 \Big|_{x_0, z_0} = 0, \\ & \frac{\partial \theta}{\partial t} + v \sin \theta_0, \\ & \frac{\partial \theta}{\partial x_0} + v \cos \theta_0, \\ & \frac{\partial \theta}{\partial z_0} + \left(\frac{\partial v}{\partial z_0} \sin \theta_0 - \frac{\partial v}{\partial x_0} \cos \theta_0 \Big|_{x_0, z_0} \right) = 0, \end{aligned}$$

where the velocity v is evaluated at (x_0, z_0) . The appropriate initial conditions for system~(\ref{eq:xl})-(\ref{eq:thetal}) are $(x, z, \theta) = (x_0, z_0, \theta_0)$ at $t=0$.

Let us denote by $T(x, z, \theta)$ the time at which the ray that starts at point (x, z) with the take-off angle θ first reaches the surface $z=0$. Correspondingly, the emergence point and the emergence angle of this ray at the surface will be defined by functions $X(x, z, \theta)$ and $\Theta(x, z, \theta)$. Differentiating the condition

$$\begin{aligned} & \text{\label{eq:zsurface}} \\ & z(T(x_0, z_0, \theta_0), x_0, z_0, \theta_0) = 0, \end{aligned}$$

where $z(t, x_0, z_0, \theta_0)$ is the solution of equation~(\ref{eq:zl}), we find that, in the region where $\frac{\partial z}{\partial t} \neq 0$, the function T has to satisfy the partial differential equation

$$\begin{aligned} & \text{\label{eq:tmarch}} \\ & v(x, z) \sin \theta, \\ & \frac{\partial T}{\partial x} + v(x, z) \cos \theta, \\ & \frac{\partial T}{\partial z} + \left(\frac{\partial v}{\partial z} \sin \theta - \frac{\partial v}{\partial x} \cos \theta \right), \\ & \frac{\partial T}{\partial \theta} = 1. \end{aligned}$$

with the boundary condition $T|_{z=0} = 0$. As follows from equations~(\ref{eq:tmarch}), (\ref{eq:xl}), and~(\ref{eq:thetal}), and the conditions

$$\begin{aligned} & \text{\label{eq:xsurface}} \\ & x(T(x_0, z_0, \theta_0), x_0, z_0, \theta_0) = X(x_0, z_0, \theta_0), \\ & \text{\label{eq:thetasurface}} \\ & \theta(T(x_0, z_0, \theta_0), x_0, z_0, \theta_0) = \Theta(x_0, z_0, \theta_0), \end{aligned}$$

the functions $X(x, z, \theta)$ and $\Theta(x, z, \theta)$ additionally satisfy the orthogonal equations

$$\begin{aligned} & \text{\label{eq:xmarch}} \\ & v(x, z) \sin \theta, \\ & \frac{\partial X}{\partial x} + v(x, z) \cos \theta, \\ & \frac{\partial X}{\partial z} + \left(\frac{\partial v}{\partial z} \sin \theta - \frac{\partial v}{\partial x} \cos \theta \right), \\ & \frac{\partial X}{\partial \theta} = 0, \\ & \text{\label{eq:thetamarch}} \\ & v(x, z) \sin \theta, \\ & \frac{\partial \Theta}{\partial x} + v(x, z) \cos \theta, \\ & \frac{\partial \Theta}{\partial z} + \left(\frac{\partial v}{\partial z} \sin \theta - \frac{\partial v}{\partial x} \cos \theta \right), \\ & \frac{\partial \Theta}{\partial \theta} = 0. \end{aligned}$$

\end{eqnarray}

with the boundary conditions $\left. X \right|_{z=0} = x$ and $\left. \Theta \right|_{z=0} = \theta$.

We propose to apply equations~(\ref{eq:tmarch}), (\ref{eq:xmarch}) and (\ref{eq:thetamarch}) for a numerical computations of traveltimes on a fixed $\{x, z\}$ grid. Although both $T(x, z, \theta)$ and $X(x, z, \theta)$ functions are strictly single-valued, we can extract from them the possibly multi-valued traveltimes from every grid point $\{x, z\}$ to a surface point y at $z=0$. The extraction would simply amount to evaluating $T(x, z, \theta)$ at the level set of $X(x, z, \theta) = y$.

Regen J. Cochrane
Operation Support MGR
8/1/2001

ERNEST ORLANDO LAWRENCE BERKELEY NATIONAL LABORATORY
ONE CYCLOTRON ROAD | BERKELEY, CALIFORNIA 94720