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Essays on Monetary Policy, International Macroeconomics, and Expectations

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Essays on Monetary Policy, International Macroeconomics, and Expectations

DISSERTATION

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for the degree of

DOCTOR OF PHILOSOPHY

in Economics

by

Francisco Elias Ilabaca

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2020

DEDICATION

To Edmundo and Maria Gabriela.

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VITA

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ABSTRACT OF THE DISSERTATION

Essays on Monetary Policy, International Macroeconomics, and Expectations

By

Francisco Elias Ilabaca

Doctor of Philosophy in Economics

University of California, Irvine, 2020

Fabio Milani, Co-Chair
Eric Swanson, Co-Chair

The primary contribution of my dissertation is to understand more about the role of expectations in the transmission of monetary policy within and between countries. In the first chapter of my dissertation titled 'Measuring the Effects of US Unconventional Monetary Policy on International Financial Markets', I estimate and separately identify the effects of the US Federal Reserve's forward guidance and large-scale asset purchases on bond yields, stock indices, and exchange rates across countries. I identify these two monetary policy factors, namely forward guidance and large scale asset purchases, as in Swanson (2018), and show that they have substantial effects on a large panel of international bond yields, but these effects exhibit heterogeneity across countries and across maturities. I find small effects on exchange rates, and no effect on stock indices. I then employ a panel regression framework, with country and time fixed effects, to further explore the heterogeneity of this effect. I find for 5- and 10-year yields, bonds of emerging countries reacted more strongly to forward guidance, while industrial countries reacted more strongly to asset purchases. Across all the specifications, I find that industrial countries' 10-year yields had the largest estimated effect to expansionary monetary policy conducted via large scale asset purchases.

In the second chapter of my dissertation titled '*Anchoring Japanese Inflation Expectations*',

I use survey and financial data to look at the movement in long run inflation expectations in Japan following the introduction of Abenomics and inflation targeting. I wrote this chapter throughout the course of my internship at the International Monetary Fund in 2018 under the supervision of Paul Cashin. Using a Nelson-Siegel framework, we combine firm level and professional forecaster data along with inflation swaps from financial markets to generate inflation expectations curves. Since it's a parametric approach, I am able to examine the behavior of the Level parameter, as a proxy for long run inflation expectations, following several rounds of unconventional monetary policy. The findings show that long run inflation expectations rose to around 1% following Bank of Japan actions between 2012 and 2017. In addition, inflation expectations became more invariant to movements in the CPI, which signals they are stabilizing. I conclude that while the Bank of Japan missed their target, inflation expectations seem to becoming "anchored" at 1%.

Finally, in the third chapter of my dissertation titled '*Interest Rate Rules and Indeterminacy in an Estimated Open Economy Model*', I estimate a small open economy model for Canada and the United States while allowing for equilibrium indeterminacy, to examine the role of sunspot shocks and self-fulfilling expectations in the transmission of shocks across the economies. Open economy considerations alter the conditions for a determinate equilibrium, yet traditional estimation approaches usually assume the equilibrium is always unique. My empirical results show that in a benchmark model, the data prefer a unique equilibrium. I consider two alternative specifications to the benchmark model. First, I employ an inflation forecast based rule to match the inflation targeting period for Canada, and find that the data prefer indeterminacy in this case, along with structural shocks that exhibit different dynamics. For example, now monetary policy and preference shocks are inflationary rather than deflationary. In the second extension, I introduce heterogeneous expectations into the benchmark model, and find that the data prefer adaptive agents over fully rational expectations, along with a unique equilibrium.

Chapter 1

Measuring the Effects of U.S. Unconventional Monetary Policy on International Financial Markets

1.1 Background

Monetary policy announcements in the US can have effects on international financial markets, even when interest rates are constrained by the zero lower bound (henceforth, ZLB). From December 2008 to October 2015, the Federal Reserve kept the federal funds rate at essentially zero, and moved to using unconventional monetary policy tools to lower longer term rates in hopes of influencing domestic interest rates and stimulating the economy. These policies had effects that spilled over onto international asset prices (a fact well-documented in the literature[1]). Agents in international financial markets were reacting to the new information contained in the various Federal Open Market Committee (henceforth, FOMC)

[1] See, for example, Rogers et al. (2014), Bhattarai and Neely (2016), Gilchrist et al. (2019), Rogers et al. (2018)

announcements during this period, as international asset prices moved in response, even when the federal funds rate remained unchanged.

Shifts in US monetary policy can be a source of unintended spillovers onto international financial markets by shifting foreign yields and exchange rates (Engel (2014)). It is important for central bankers and policymakers in other countries to understand the spillovers of monetary policy, both conventional and unconventional. These effects can alter the financial conditions of a country, as floating exchange rates alone are not enough[2] to insulate financial conditions abroad from shocks coming from US monetary policy. During the ZLB, rather than shifting the federal funds rate target, the FOMC moved to pursue unconventional policy in the form of forward guidance and large scale asset purchases (LSAP). It is valuable for international policymakers and central bankers to know *which* components of FOMC announcements have strong spillover effects on their domestic asset prices during the ZLB. Are asset prices moving in response to the forward guidance component of FOMC announcements, the asset purchase component, or both? Is the response similar across countries and maturities? The present paper provides separate estimates for each of these effects in order to answer these questions.

This paper jointly estimates and separately identifies the effect of US forward guidance and large-scale asset purchases[3] on international financial markets, focusing on bond yields, exchange rates, and stock indices in Canada, Australia, Japan, Germany, and the UK. Specifically, I use the methodology of Swanson (2018) to obtain both a forward guidance factor and a LSAP factor, and then use high frequency regressions to estimate the effect of each component on different assets. The main result of the paper is that I find significant effects of

[2]Work by Rey (Rey (2015), Rey (2016)) argues that floating exchange rates without additional capital restrictions are not enough to insulate countries from foreign shocks, an idea that is further examined by Aizenman et al. (2016) and Han and Wei (2018) who indeed find that flexible exchange rate regimes cannot fully prevent monetary policy spillovers

[3]Specifically, "Forward Guidance" (FWG) refers to communications made by the FOMC about the likely future path of the federal funds rate, while "Large Scale Asset Purchases" (LSAP) are purchases by the Federal Reserve of US Treasury bonds and other securities.

both components of unanticipated US unconventional monetary policy on international asset prices, particularly on bond yields and exchange rates. These effects exhibit heterogeneity across countries and across maturities in the case of bonds, with some responding strongly to forward guidance, others responding strongly to the asset purchase component, and some to a mix of both. To facilitate comparison with the response of US yields to both components of unconventional policy, I compute a spillover rate for each country across the 2-year, 5-year, and 10-year. The results indicate that some yields move more than one-for-one in comparison with US yields. To assess whether this occurs across other countries, I extend the analysis to a panel of bond yields for 32 additional countries, which shows that there are several other countries that also have large spillover rates. Lastly, I employ a panel regression framework to see if these effects are different in emerging versus industrial economies, and during expansionary versus contractionary monetary policy.

Separately identifying each component is difficult for several reasons. For starters, FOMC announcements may contain both news about purchases and news about future policy. In addition, asset purchases and balance sheet operations themselves have signaling channels (Krishnamurthy and Vissing-Jorgensen (2011)), as the size and composition of these operations is influenced by the FOMC's beliefs about the future path of policy. When it comes to unconventional policy during the ZLB period, we have no good data on market expectations about the announcements *a priori*, making it difficult to measure the surprise component[4]. Additionally, FOMC inaction during announcements can have effects on asset prices, in particular if the FOMC foregoes an action that the markets were expecting, such as a tapering of asset purchases. To address these problems, Swanson (2018) extends the methods of Gürkaynak et al. (2005) (henceforth GSS) to separately identify the effects of forward guid-

[4] In this, "good data" refers to either asset prices or survey data. In the case of conventional policy, information about financial market expectations before the announcement can be obtained through federal funds futures and various other financial instruments (See Gürkaynak et al. (2005)). During the ZLB period, these contracts had rates that were also zero or very close to zero. In order to introduce a good measure for market expectations during the ZLB period, the New York Federal Reserve in 2012 made several adjustments to the Survey of Primary Dealers that introduced new questions on expectations of policy announcements.

ance and LSAPs on domestic assets during the ZLB period using a high frequency approach. I build on this analysis to estimate the effects on international assets. I use the data set from Swanson (2018), which looks at the 30-minute response of a number of asset prices to each FOMC announcement. The assumption in this approach is reasonable: movements in these asset prices during a narrow window bracketing the announcements are due to unanticipated changes in monetary policy. I compute the first three[5] principal components of those asset price responses. By using a set of identifying assumptions, I can separately identify the size of each of the three components of every FOMC announcement in the sample. I search over all possible rotations of these principal components so that the first factor corresponds to the change in the federal funds rates, the second factor to the change in forward guidance, and the third factor to the change in LSAPs. I use daily regressions to estimate the response of international bond yields and asset prices to the forward guidance and LSAP component during the zero lower bound period. As mentioned, I find statistically significant effects of US unconventional monetary policy on international bond yields, and exchange rates using this procedure.

The present paper fits into a growing empirical literature that seeks to estimate and explain the effects of unconventional policy measures on financial asset prices. A large part of this research focuses on the domestic effects of unconventional policy at the ZLB . Important contributions to this literature include Wright (2011), Gagnon et al. (2011), D'Amico and King (2013), Hamilton and Wu (2012), Gürkaynak et al. (2005), Swanson (2018), generally concluding that unconventional policy at the ZLB has effects on both domestic yields and the US real economy at large.

In the case of the response of international asset prices to unconventional monetary policy during the ZLB period, the literature has focused on a number of topics. Several papers focus on the announcements themselves, using a number of different techniques to find that

[5]See Swanson (2018) for details. Announcement effects can be well described by three dimensions over the 1991-2015 period.

international yields and asset prices indeed respond to FOMC announcements during this period (See Krishnamurthy and Vissing-Jorgensen (2011), Berge and Cao (2014), Bowman et al. (2015), Rogers et al. (2014), and the survey by Bhattarai and Neely (2016)). Other work has focused on asset purchases or the effects of purchase announcements themselves on prices or portfolio flows (Glick and Leduc (2012), Fratzscher et al. (2017), Neely (2015), Rosa (2012), Bernoth et al. (2015)). Two recent papers are directly related, with a different approach but comparable results to the present paper. In work by Gilchrist et al. (2019), the authors use high frequency data to analyze the effects of US monetary policy, before and during the ZLB, on foreign government bond markets. This paper employs changes in the two-year nominal US treasury yield as an instrument, following Hanson and Stein (2015) and Gertler and Karadi (2015). To separate the effects of asset purchases, the authors consider a sub-sample of the unconventional policy announcements that exclude the announcements most closely identified with these balance sheet policies. Their results indicate that the pass-through of unconventional monetary policy to foreign bond yields is comparable to that of conventional policy. Similarly, the paper by Rogers et al. (2018) assesses the relationship between unconventional policy during the ZLB period and term premia. They make use of high frequency jumps in the term structure of interest rates around announcements, employing the method of external instruments following Gertler and Karadi (2015) and Stock and Watson (2012). Using a structural VAR, they are able to trace out the effects of monetary policy shocks on interest rates and exchange rates. Their methodology allows them to split the effect of forward guidance and asset purchases, and they find that both monetary policy surprises have effects on foreign asset prices. While the present paper finds similar results to the aforementioned work, the main difference lies in the identification strategy for the two types of monetary policy surprises, as I employ the approach in Swanson (2018) rather than using the method of external instruments.

The rest of the paper is structured as follows. Section 2 describes the methodology and data used for the identification procedures. In Section 3, I estimate the effect of both policies on bond yields, foreign exchange rates, and stock indices for the UK, Canada, Australia, Japan, and Germany. Section 4 goes over the results, and computes spillover rates to facilitate the analysis. In Section 5, I extend the analysis to 32 additional countries and discuss the results. I employ a panel regression framework in Section 6 to analyze the overall response of unconventional monetary policy across countries. Finally, in Section 6 I discuss the significance of these effects on other economies, and the implications of these spillovers.

1.2 Methodology

1.2.1 Identification of Forward Guidance, LSAPs

I estimate and identify the forward guidance and LSAP component of each FOMC announcement replicating the analysis from Swanson (2018). This data set contains the dates of each FOMC announcement from July 1991 through October 2015, along with the change in a number of asset prices in a 30-minute window bracketing each announcement. Among asset prices included are first and third federal fund futures, second, third and fourth eurodollar futures contracts, 2-, 5-, and 10-year Treasury bond yields[6]. These asset price responses are then arranged into a $T \times n$ matrix X, with rows corresponding to FOMC announcements, and columns corresponding to n different assets. Therefore, each x_{ij} element corresponds to the 30-minute response of the jth asset to the ith FOMC announcement.

These data can be described with a factor model,

$$X = F\Lambda + \epsilon \tag{1.1}$$

[6]Swanson (2018) extends the original data set used in GSS (2005) using data obtained from staff at the Federal Reserve Board. I also chose to focus on these assets as they are the most closely related to monetary policy, and to reduce sampling variability. Federal funds futures, in particular, reflect revisions to the market's expectations for the fed funds rate in their daily changes. The change in a current month's contract rate on the day of an announcement can be used to measure the surprise component on the federal funds rate. Treasury yields provide information about market interest rate expectations and risk premia over longer horizons.

where F is a $T \times k$ matrix containing $k < n$ unobserved factors, Λ is a $k \times n$ matrix of loadings of the asset prices responses on the k factors, and ϵ is a matrix of white noise residuals. The approach here is to think of the columns of F as corresponding to the surprise component of: (1) the change in the federal funds rates, (2) the change in forward guidance, (3) any LSAP announcements, and (4) any additional dimensions of news about policy or the economy that are systematically revealed during FOMC announcements. The matrix F is unobserved, and so it needs to be estimated and identified. The approach I take in this paper is to replicate the full-sample identification strategy developed in Swanson (2018), using the entire sample from July 1991 to October 2015, and restricting X to only the first and third federal funds futures, along with the second, third, and fourth Eurodollar futures, and the 2-, 5-, 10-year Treasury bond yields. During this period there were 213 FOMC announcements, thus X has a dimension of 213×8. As mentioned earlier, the data can be well-explained by three dimensions underlying each announcement[7].

We can think of these three dimensions as corresponding to the aforementioned surprise components (changes in the federal funds rate, forward guidance, and large scale asset purchases). I estimate the matrix of factors F by principal components, following Gürkaynak et al. (2005), to get the three factors that explain a maximal fraction of the variation in X, ensuring that the scale of F is normalized so that each column has unit standard deviation. I then impose a structural interpretation to these three principal components, since these principal components are just a statistical decomposition, thus lacking structure. In order to do this, a $U_{3\times3}$ orthogonal matrix needs to be determined such that our matrix and loadings can be represented as $\tilde{F} \equiv FU$ and $\tilde{\Lambda} \equiv U'\Lambda$ (which fits the data exactly as well as F and Λ), and some structure can be then imposed on the columns of F to represent the surprise component of the three factors. Following Swanson (2018) and Gürkaynak et al. (2005), I impose that: (1) changes in LSAPs have no effect on the current federal funds rate, (2)

[7] This is done by investigating the rank of F following Cragg and Donald (1997)

changes in forward guidance have no effect on the current federal funds rate, [8] and (3) the LSAP factor is as small as possible in the pre-ZLB period. This is done by imposing three restrictions on the three parameters that completely determine U so as to uniquely identify it, and hence uniquely identify $\tilde{F}$ and $\tilde{\Lambda}$. Since each principal component was normalized to have a unit standard deviation, and this rotation matrix does not change the scaling of the factors, the rotated factors have a unit standard deviation as well.

1.2.2 Effects on Asset Prices

Following the estimation and identification of the forward guidance and LSAP components of the announcements, I then estimate their effects on asset prices using high-frequency OLS regressions. These two factors correspond to the 2$^{\text{nd}}$ and 3$^{\text{rd}}$ columns of $\tilde{F}$, since the 1$^{\text{st}}$ corresponds to changes in the federal funds rate. The coefficients in these regressions here are in units of basis points per standard deviation surprise in the announcement. For practical policy applications, it is better to relate these factors to a more observable scale. Following the results from Swanson (2018), we can think of a forward guidance surprise for the federal funds rate that was about 25bp lower one year ahead (a large surprise by historical standards) as a 4-6 standard deviation surprise. Similarly, for LSAPs, a standard deviation surprise can be thought of as a $250 billion surprise change in the size of the purchases. The sample period for all asset prices is from January 2009 to October 2015, which coincides with the zero-lower bound period.

These tables report estimates from a regression of the form:

$$\Delta y_t = \alpha + \beta \tilde{F}_t + \epsilon_t \tag{1.2}$$

[8] Recall that forward guidance is defined as the component of FOMC announcements that have information about the path of interest rates not contained in the changes to the federal funds rate itself. This definition is laid out in GSS, used in Swanson (2018), and likewise will be used here. For details on these restrictions, refer to these 2 papers

where y_t refers to the various yields or prices, t indexes each FOMC announcement during this period, $\tilde{F}$ denotes the factors as estimated and identified (the second and third columns of $\tilde{F}$), ϵ is a regression residual, and α is a parameter. β is the parameter of interest and this is the estimated effect of each factor. Δ denotes the change over a daily window bracketing each announcement, done in such a way as to ensure that the FOMC announcement occurred in between the yields or prices that were reported in the data. In regards to bond markets that were closed during the FOMC meetings (all except Canada), I ensured that the timing of the announcements coincided with the time index for the yields that were quoted. That is, I took the difference between the closing yield of the date of the announcement, and the opening yield for the following day. For example, if the announcement was at 2:15PM EST (7:15PM in London) on November 3rd, I'd take the difference between the November 3rd gilt closing yield and the November 4th closing yield. FOMC announcements throughout my sample occurred either at 12:30PM EST or 2:15PM EST, so overseas markets were closed during each announcement in my sample.

1.2.3 Data

British bond yield data comes from the Macro Financial Analysis division of the Bank of England, and they are the nominal zero coupon yields for government securities with 1-, 2-, 5-, 10-, and 25-year maturities. Bond yield data for Canada comes from the Bank of Canada, where daily rates are the mid-market closing yields. I examine the 6-month, and 1-, 2-, 5-, 10-year and Long Term rates. Data for Australian government bonds comes from the Reserve Bank of Australia, and reports the daily estimated yield at end of day on Australian Government Bonds of 2-, 5-, and 10-year maturities, as well as New South Wales Treasury bond yields for bonds of 3-, 5-, and 10-year maturities. Japanese government bond data comes from the Ministry of Finance of Japan, and they are the secondary market rates at market closing time. I examine the 1-, 2-, 5-, 10- and 30-year rates. German bunds yield data comes from the Deutsche Bundesbank, and I examine the 6-month, 2-, 5-, 10- and 30-year bond yields.

Data on foreign exchange rates comes from FRED, and I use the daily noon buying rates in New York City for cable transfers payable in foreign currencies [9]. The dependent variable in this case is 100 times the log change of daily exchange rates. Since none of the announcements happened before noon, I use the difference between the day of the announcement and the following day. The series are in terms of dollars per foreign currency for easier comparison. Stock index data comes from Yahoo Finance and is cross referenced, when possible, with data published on the websites of each respective index.

1.3 Results

I will discuss some results in comparison with what Swanson (2018) estimated for US variables. Results for changes in forward guidance and LSAPs during the ZLB period are in the following tables, with a discussion of each result for each country in my analysis. Standard errors and t-statistics are heteroskedasticity-consistent [10]:

1.3.1 Exchange Rates

Table 1.1 reports the estimated effects on 3 exchange rates to the US dollar: Australian dollars, Canadian dollars, and British pounds. Estimated effects for the Japanese yen, Euro to US dollar with the asset price change over a 30 minute window can be found in Swanson (2018).

[9] I omit analysis of the effect on USD/Yen, USD/Euro as this is done in Swanson (2018). Results can be found in Appendix A

[10] Stars around coefficients can be read as follows: * = significant at the 10% level, ** = significant at the 5% level, *** = significant at the 1% level

Table 1.1: Estimated Effects of Changes in US Forward Guidance, and LSAPs on Exchange Rates

	\$/AUS	\$/£	\$/CAN
Change in forward guidance	-0.55*	-0.16	-0.30
(Std. err)	(.296)	(.215)	(.227)
[t-stat]	[-1.88]	[-0.78]	[-1.32]
P>$\|t\|$	0.065	0.436	0.190
Change in LSAPs	0.43	0.43**	0.26
(Std. err)	(.289)	(.214)	(.198)
[t-stat]	[1.51]	[2.031]	[1.35]
P>$\|t\|$	0.136	0.047	0.182
Regression R^2	0.268	0.305	0.222
# of Observations	55	55	55

As mentioned above, the data for this analysis is over a 24-hour window bracketing FOMC announcements. I find, with a small level of significance, that the USD/AUD and USD/GBP exchange rates react to unconventional monetary policy. The effect of changes in forward guidance on the Australian dollar is intuitive, as increases in US interest rates make US investments more attractive relative to Australian investments, and tends to depreciate the Australian dollar against the US dollar. I also find that changes in LSAPs depreciate the USD/GBP exchange rate, with a one-standard-deviation increase causing a depreciation of the dollar against the pound of about 0.4 percent. The R^2 for these regressions is relatively small, and I found no significant results for the USD/CAN exchange rate. I find no effect of either factor on the Canadian exchange rate, which indicates that better data is needed for this analysis, as Canada is the US 2$^{\text{nd}}$ largest trading partner and should show *some* level of response to US monetary policy in its exchange rate. To get more precise estimates, a minute-by-minute data set would be needed in order to employ a 30-minute or 1-hour window around the announcements due to the high volatility of exchange rates. The effects I find for exchange rates are much less significant than those found in Swanson (2018), but with similar point estimates and signs. I find that the magnitude of the effect of changes in LSAPs on the USD/GBP rate is larger, but less significant, than what was estimated for USD/EUR and USD/YEN. The USD/AUD exchange rate seems to react to changes in forward guidance only, with an effect that is about double from what was estimated for the euro and yen. These two estimated effects were statistically significant but with relatively small t-stats, in stark contrast to what was found for the euro and yen.

1.3.2 Stock Markets

I collected data for a number of stock market indices and conducted my analysis using regressions of the same form as in equation (1.2). The dependent variable in this case was 100 times either the overnight or open-to-close change in the (log) price of the index. The results reported on Table 1.2 show the effect on Japanese, Canadian, German, and Australian stock indices[11]. Nevertheless, none of the results were significant as the t-statistics found were particularly small. Similarly, the magnitude of the estimated coefficients was small across the three regressions, with none of them larger than 0.31, and the regression R^2 for all three showed that the factors explained very little of the variation in stock index prices.

When estimating the effects of forward guidance and LSAPs on the S&P 500, Swanson (2018) finds that the effects of forward guidance led to a drop in stock prices of about 0.25 percent during the ZLB period. The estimated effect of LSAPs was a rise in stock prices of about 0.12 percent during the same period, with a relatively lower R^2 for the regressions due to the high volatility of stock prices after FOMC announcements. My estimates are close to those point estimates, but lack statistical significance. This is reasonable due to the following: First, I am using daily data as opposed to a 30 minute window, which would be much better for assets of high volatility such as equity prices. Second, the estimated effect on US stocks is small in comparison with some of the other results, and thus we would

[11]For the FTSE100, intra-daily data would be needed as the previous day closing and current day opening prices are almost always identical. This was the case across several different databases at the time of this writing. For this reason, the effect on the FTSE100 is omitted

Table 1.2: Estimated Effects of Changes in U.S. Forward Guidance, and LSAPs on Japanese, Canadian, German stock indices

	Nikkei	TSX	DAX	ASX
Change in forward guidance	0.10	0.29	0.31	-0.02
(Std. error)	(.164)	(.238)	(.230)	(.030)
[t-stat]	[0.64]	[1.23]	[1.35]	[-0.76]
P>$\|t\|$	0.527	0.221	0.183	0.451
Change in LSAP	0.08	-0.04	0.10	-0.01
(Std. error)	(.123)	(.165)	(.154)	(0.02)
[t-stat]	[0.72]	[-0.25]	[0.65]	[-0.64]
P>$\|t\|$	0.475	0.801	0.517	0.526
Regression R^2	0.011	0.074	0.067	0.017
# of Observations	55	55	55	55

expect the effect on international stocks to be smaller in comparison, as these indices are comprised of the asset prices of foreign companies, and thus are much more dependent on their corresponding domestic variables rather than US variables. If I extended this analysis using a minute-by-minute data set, I would expect the estimated effects to be small if they are found to be statistically significant.

1.4 Effect on Bond Markets

I will begin the analysis by looking at the response of sovereign bond yields of three developed countries: the United Kingdom, Canada, and Germany. These three countries all issue a number of short term and long term bonds with good data availability. We will see from the results that the response of each asset price to unconventional monetary policy by the US is heterogeneous, with some yields responding strongly to forward guidance but not to asset purchases, vice versa, or responding to both. An analysis of the full sample of countries follows. As mentioned earlier, the coefficients in these regressions here are in units of basis points per standard deviation surprise in the announcement, and we can think of a forward guidance surprise for the federal funds rate that was about 25bp lower one year ahead (a large surprise by historical standards) as a 4-6 standard deviation surprise. Similarly, for LSAPs, a standard deviation surprise can be thought of as a $250 billion surprise change in the size of the purchases.

1.4.1 United Kingdom

Table 1.3: Estimated Effects of Changes in U.S. Forward Guidance, and LSAPs on British Bond Yields

	1Y	2Y	5Y	10Y	20Y
Change in forward guidance	1.18**	2.62**	4.04***	3.34**	2.26*
(Std. err)	(.482)	(.984)	(1.43)	(1.47)	(1.20)
[t-stat]	[2.45]	[2.664]	[2.83]	[2.26]	[1.87]
P>$\|t\|$	0.018	0.010	0.007	0.028	0.066
Change in LSAPs	-0.515**	-0.96*	-1.88	-2.39*	-1.45
(Std. err)	(.222)	(.551)	(1.31)	(1.34)	(1.18)
[t-stat]	[-2.32]	[-1.74]	[-1.44]	[-1.78]	[-1.22]
P>$\|t\|$	0.024	0.086	0.156	0.081	0.227
Regression R^2	0.269	0.337	0.353	0.303	0.206
# of Observations	55	55	55	55	55

Table 1.3 reports results for the response of gilt yields to FOMC announcements. These results show that British bond yields respond systematically to changes in forward guidance and LSAPs. The effects of forward guidance all have some level of significance across every maturity I analyzed. These effects are largest and most significant for the maturities between 2-years and 10-years, peaking at the 5-year maturity. The estimated effects of LSAPs are all negative, even for the shortest maturities. The t-statistics for these estimated effects are relatively small, but there is some level of significance to these findings. Regression R^2 values indicate that these two factors explain about a third of the variation in British bond yields

around FOMC announcements.

When compared to the estimated effect on US yields[12], the effect of US forward guidance on British yields is, similarly, larger at the medium maturities. I find a larger but less significant estimated effect on the 10-year gilt yield, and smaller, less significant estimated effects on the 2-year and 5-year yields. In contrast with US response, I find that LSAPs had a relatively larger negative effect on the shorter end of the yield curve at the 1-year and 2-year maturities. I find no significant effect on either the 5-year or 20-year bond, which is in stark contrast to the effects found in the US yields of longer maturities (t-stats between 10 and 20), and only a slightly significant effect on the 10-year yield of about half the magnitude estimated for the US 10-year bond. British bonds seems to react more strongly to unanticipated changes in forward guidance, with the short end of the curve responding to LSAPs.

1.4.2 Canada

Table 1.4: Estimated Effects of Changes in U.S. Forward Guidance, and LSAPs on Canadian Bond Yields

	6mo	1Y	2Y	5Y	10Y	LT
Change in forward guidance	0.04	0.36	1.17**	1.49**	0.74	0.34
(Std. err)	(.198)	(.384)	(.558)	(.601)	(.624)	(.638)
[t-stat]	[0.24]	[0.95]	[2.09]	[2.48]	[1.19]	[0.53]
P>|t|	0.806	0.345	0.041	0.016	0.236	0.592
Change in LSAPs	-0.28	-0.66***	-1.18***	-3.36***	-4.03***	-2.56***
(std. err)	(.194)	(.220)	(.320)	(.354)	(.382)	(.548)
[t-stat]	[-1.47]	[-2.99]	[-3.71]	[-9.51]	[-10.57]	[-4.68]
P>|t|	0.147	0.004	0.000	0	0	0
Regression R^2	0.042	0.164	0.256	0.539	0.582	0.425
# of observations	55	55	55	55	55	55

Table 1.4 reports results for the estimated effects of changes in forward guidance and LSAPs

[12]See Apendix A for a table with results from Swanson (2018) of the effect on US yields.

on Canadian Bond yields. Again, the results show these bonds respond systematically to FOMC announcements, with the exception of the 6-month yield. In particular, unanticipated changes in LSAPs have extraordinarily statistically significant negative effects on bond yields, particularly for bonds of longer maturities, with t-statistics around 10 for the 5-year and 10-year bonds. These effects are hump shaped, starting out at around a half basis point negative effect, with a peak at the 10-year rate of around 4 basis points, and then a taper to a smaller effect for long term bonds. I find that Canadian bond yields respond strongly to changes in LSAPs for all yields except for the 6-month yield. The estimated effects are comparable to the US in magnitude for the 5-year and 10-year yield, with the effect on the 5-year yield being even larger than what was estimated for the US, but smaller for the long term yield. Also, in contrast to the US response to these factors, I find a highly significant negative effect on the 1-year and 2-year yields of -0.6bp and -1.2bp per standard deviation change, respectively. Forward guidance, on the other hand, only has significant effects on 2-year and 5-year bond yields, of about 1-1.5 basis point per standard deviation change. Canadian bond yields did not respond to forward guidance across all maturities, with only significant effects at the 2-year and 5-year yield. The estimated effects of forward guidance are about a fifth of what was estimated for the US.

Canadian bond yields seem to react more strongly to unanticipated changes in LSAPs, especially around maturities that the Federal Reserve targeted during their bond buying program. The estimated response to forward guidance reveals small effects on medium term maturities, but no significant effect on longer term maturities. The regression R^2 shows that these two factors explain very little of the variation in the short end of the yield curve, but about half of the variation of medium and long term assets.

Table 1.5: Estimated Effects of Changes in U.S. Forward Guidance, and LSAPs on German Bond Yields

	6 Mo.	2Y	5Y	10Y	30Y
Change in fwd guidance	-0.69	0.21	0.79	1.18	0.23
(Std. err)	(.612)	(.710)	(.764)	(.869)	(1.11)
[t-stat]	[-1.14]	[0.29]	[1.04]	[1.36]	[0.207]
P>$\|t\|$	0.261	0.768	0.303	0.180	0.837
Change in LSAPs	-0.64*	-1.25***	-2.37***	-3.19***	-3.10***
(Std. err)	(.345)	(.329)	(.625)	(.804)	(.979)
[t-stat]	[-1.851]	[-3.79]	[-3.79]	[-3.97]	[-3.17]
P>$\|t\|$	0.07	0.000	0.000	0.000	0.003
Regression R^2	0.062	0.159	0.297	0.354	0.217
# of Observations	55	55	55	55	55

1.4.3 Germany

Table 1.5 reports the estimated effects of changes in forward guidance and LSAPs on German bunds yields. Here, I find that these yields respond systematically to FOMC announcements, but only to the LSAP component of each announcement, as I find no significant effect of the forward guidance factor. The effect of LSAPs, on the other hand, all have some level of significance, with relatively large t-statistics for all except for the bond of 6-month maturity. The effect is slightly hump shaped, with a peak of -3.2 basis points at bonds of 10-year maturity. However, note that the estimated effect on the 30-year bond is -3.1, so the drop off is not large. The regression R^2 varies from 0.06 to 0.35, and thus our two factors only explain up to one third of the variation of these yields around FOMC announcement. Note that the largest R^2 is for the 10 year bond in this case.

The magnitude of the estimated response to LSAPs is similar to the US response for the 5-year yield, but about half of what was estimated for the US 10-year and 30-year yields. German 6-month and 2-year yields had statistically significant negative responses, of -0.6 and -1.25 basis points respectively, in contrast with the small positive effect found for the

US short end of the curve. The lack of response to forward guidance would stand against what previous literature has found for the effect of "surprises" in monetary policy on German yields. For example, Berge and Cao (2014) find that the 10-year German yield had an average movement of -0.2 basis point on announcement days. However, the sample that they used for their analysis included announcements that had both forward guidance and asset purchases in them. The analysis in this paper disentangles these two, and I can attribute all of the response to unanticipated changes in LSAPs.

1.4.4 Effect on Bonds: Extended Analysis

I extend the analysis to a larger panel of countries to see if there is heterogeneity in response to US unconventional monetary policy across the world. I will look at the response of 2-, 5-, and 10-year bond yields as these were the most widely available for the sample chosen. The data in this case is the close-to-close or close-to-open change in yields for sovereign government bonds around FOMC announcements, and it was obtained from Global Financial Data, with the exception of Switzerland which came directly from the the Swiss National Bank website. This panel includes all countries for which the data was available at the time of this writing, and I also include the Euro benchmark bond rates for the 3 maturities analyzed.

Table 1.6: Estimated Effect of Forward Guidance, LSAP: Extended Analysis

	Forward Guidance			**LSAP**		
Country	2 Year	5 Year	10 Year	2 Year	5 Year	10 Year
Austria	1.64	2.63**	2.17*	-0.23	-1.93*	-2.80**
	(0.907)	(1.154)	(1.099)	(0.725)	(0.970)	(1.165)
Australia	1.44	1.85**	2.35**	-3.62***	-4.64***	-5.48***
	(0.876)	(0.862)	(0.904)	(0.868)	(1.03)	(0.997)

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Table 1.6 – *Continued from previous page*

	Forward Guidance			**LSAP**		
Country	2 Year	5 Year	10 Year	2 Year	5 Year	10 Year
Belgium	0.72	1.76*	1.81*	-1.061	-3.07**	-3.56***
	(0.845)	(0.916)	(1.007)	(0.685)	(1.264)	(0.855)
Canada	1.17**	1.49**	0.74	-1.18***	-3.36***	-4.03***
	(0.558)	(0.601)	(0.624)	(0.320)	(0.354)	(0.382)
Switzerland	1.24*	0.69	0.74	0.81	-0.87**	-1.07
	(0.669)	(0.522)	(0.571)	(0.242)	(0.365)	(0.696)
China	-0.39	0.43	0.82	-1.61***	-0.62	1.13
	(0.818)	(1.026)	(0.995)	(0.456)	(0.412)	(0.652)
Czech Rep.		0.46	2.50***		-0.58	-0.90
		(0.508)	(0.755)		(1.287)	(1.231)
Denmark	1.48*	2.26*	1.89*	-0.89	-2.09*	-2.76***
	(0.869)	(1.253)	(1.117)	(1.227)	(1.247)	(1.020)
Spain	0.25	2.17*	3.43**	-2.60	-3.50*	-2.95*
	(1.168)	(1.129)	(1.339)	(1.929)	(2.063)	(1.491)
Euro	1.91**	2.57**	0.95	-0.77	-3.50	-2.64***
	(0.936)	(1.160)	(1.102)	(0.779)	(1.030)	(0.694)
Finland	1.25	2.12*	2.24*	-0.51	-1.93	-2.67**
	(0.839)	(1.116)	(1.132)	(0.641)	(1.062)	(1.082)
France			2.12*		-2.74***	
			(1.123)		(0.840)	
Greece			6.75			-5.30
			(15.42)			(11.02)
Hong Kong			1.89**			-4.11***

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Table 1.6 – *Continued from previous page*

	Forward Guidance			**LSAP**		
Country	2 Year	5 Year	10 Year	2 Year	5 Year	10 Year
			(0.796)			(0.978)
Hungary		3.34**	5.22***		5.01	4.31
		(1.605)	(1.711)		(3.110)	(4.400)
India		0.67	1.47**		1.55	-1.71**
		(2.639)	(0.553)		(4.285)	(0.771)
Ireland	4.25	7.10	1.63	-0.47	-0.82	-2.44**
	(5.033)	(4.477)	(1.009)	(2.093)	(2.076)	(1.065)
Iceland		1.04	0.72		3.67	2.44
		(1.593)	(1.569)		(1.661)	(1.462)
Italy	1.04	2.08*	3.41***	-2.06	-2.93*	-3.14**
	(1.038)	(1.218)	(1.142)	(1.506)	(1.576)	(1.280)
Japan	0.02	0.54***	0.75***	-0.06	-0.58***	-0.62***
	(0.111)	(0.139)	(0.200)	(0.066)	(0.083)	(0.202)
Korea		1.70*	2.73***		-2.89***	-2.62***
		(0.875)	(0.642)		(0.574)	(0.513)
Luxembourg			1.84			-2.87***
			(1.165)			(0.839)
Mexico		5.46			-1.06	
		(3.842)			(2.917)	
Malta			1.74**		-0.28	
			(0.725)		(0.625)	
Netherlands	0.88	2.30*	2.15*	-0.35	-1.99*	-3.08***
	(0.790)	(1.163)	(1.133)	(0.627)	(0.697)	(1.032)

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Table 1.6 – *Continued from previous page*

	Forward Guidance			**LSAP**		
Country	2 Year	5 Year	10 Year	2 Year	5 Year	10 Year
Norway	-1.18	-0.60		-1.39**	-2.47***	
	(1.457)	(1.389)		(0.548)	(0.697)	
New Zealand		0.94	0.40		-1.40**	-2.81***
		(1.176)	(1.077)		(0.630)	(0.629)
Poland	1.97**	4.11***	5.48***	-2.78***	0.17	-0.98
	(0.818)	(0.849)	(1.007)	(1.027)	(0.686)	(1.505)
Portugal	6.90	6.10	2.61	2.95	0.31	-2.18
	(5.478)	(4.693)	(1.857)	(2.445)	(2.114)	(1.560)
Romania		-0.33			0.76	
		(1.851)			(1.194)	
Singapore	0.62	1.16	2.57**	-0.55*	-2.08**	-3.25*
	(0.553)	(1.014)	(1.244)	(0.307)	(0.831)	(1.674)
Slovenia			2.27			-1.32
			(1.412)			(0.929)
Sweden	1.36	2.24*	1.87	-0.97	-1.93	-2.77**
	(0.924)	(1.211)	(1.142)	(1.031)	(1.438)	(1.103)
Turkey	10.26***	7.08**		-3.14	-0.12	
	(3.447)	(3.059)		(2.432)	(4.073)	
United Kingdom	2.62**	4.04***	3.34**	-0.96*	-1.88	-2.39*
	(0.984)	(1.43)	(1.47)	(0.551)	(1.31)	(1.34)
South Africa			6.45***			-1.46
			(1.827)			(2.108)

1.5 Discussion

I find significant results for bond yields for most of the countries in my analysis as shown in Table 1.6, with a good deal of variation in magnitude of the estimated effect, and in the maturity of bond yields affected. In addition, note that some countries, such as Turkey and Hungary, had a stronger estimated response to forward guidance, with no response to unanticipated asset purchases. Similarly, some countries only had an estimated response to asset purchases, such as Germany. For the most part, the effects I find are less significant, have smaller t-statistics, and have smaller regression R^2 in comparison to the tables found in Swanson (2018)[13] and the estimated effects of forward guidance and LSAPs on US assets found therein. This is to be expected due to the fact that US assets respond most strongly to US monetary policy, and likewise international bond yields will respond most strongly to their respective countries' economic circumstances.

An important difference to note with the estimated effects on this paper is that, in contrast with what was found for the US, unanticipated changes in LSAPs had small, negative effects on the short end of the yield curve. One explanation for this could be that international market participants didn't sell off short-term bonds to finance the purchase of longer maturity bonds when re-balancing their portfolios. It could also be that demand for international short term bonds responds differently than demand for US short term bonds due to differences in safety premiums. Another difference worth noting is the estimated effects of forward guidance on some of the long term bonds, such as British and Japanese yields. For these two, not only is the factor affecting the yield different than what we see for the US, but also the magnitude of the estimated effects.

[13] Table included in the Appendix

These estimated results show that unconventional policy does have spillover effects on international bond yields; the main difference between other literature and this paper is that I am able to disentangle the forward guidance and the asset purchase component of each announcement, whereas most of the literature looks at either surprise components as a whole (by doing event studies), or just asset purchases (by analyzing the size and composition of each purchase). I will present a simple, intuitive way to examine these cross country effects in the following section.

1.5.1 Spillover Rate

In order to better compare this effect across my analysis, I compute estimated Spillover Rates of each factor for the 2-year, 5-year, and 10-year yields for each country. I compute these rates by taking the ratio of estimated effect for the US and each corresponding country, and report them on Table 1.7. Spillover rates that are in italics refer to those estimated effect on international bond yields that were found to not be statistically significant at any level. I omit analysis of a spillover rate of the effect of LSAPs on 2-year yields as the estimated coefficient for US bond yields was not found to be statistically significant).

Spillover rates greater than 1 mean that the estimated effect was larger in the foreign country than in the US for that particular maturity. Negative spillover rates means yields of that particular maturity responded in an opposite manner than what was estimated for the US (if US yield rose, foreign yield fell).

Table 1.7: Spillover Rates

	Forward Guidance			**LSAP**	
Country	2 Year	5 Year	10 Year	5 Year	10 Year
Austria	0.318	0.423	0.709	0.661	0.432

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Table 1.7 – *Continued from previous page*

	Forward Guidance			LSAP	
Country	2 Year	5 Year	10 Year	5 Year	10 Year
Australia	*0.280*	0.297	0.767	1.589	0.844
Belgium	*0.14*	0.283	0.592	1.049	0.549
Canada	0.227	0.239	*0.241*	1.151	0.621
Switzerland	0.242	*0.111*	*0.243*	0.297	*0.164*
China	*-0.077*	*0.069*	*0.266*	*0.212*	-0.174
Czech Rep.		*0.074*	0.815	*0.197*	*0.138*
Denmark	0.288	0.364	0.618	0.717	0.426
Spain	*0.047*	0.348	1.122	1.198	0.455
Euro	0.371	0.413	*0.311*	*0.573*	0.406
Finland	*0.244*	0.340	0.734	*0.444*	0.412
France			0.693		0.422
Germany	*0.040*	*0.127*	*0.385*	0.811	0.491
Greece			*2.208*		*0.817*
Hong Kong			0.619		0.633
Hungary		0.5377	1.707	*-1.714*	*-0.665*
India		*0.107*	0.481	*-0.531*	0.262
Ireland	*0.827*	*1.140*	*0.533*	*0.279*	0.376
Iceland		*0.166*	*0.236*	-1.258	*-0.377*
Italy	*0.203*	0.333	1.117	1.003	0.485
Japan	*0.003*	0.086	0.245	0.198	0.095
Korea		0.272	0.891	0.991	0.403
Luxembourg			*0.602*		0.441
Mexico		*0.878*		*0.363*	

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Table 1.7 – *Continued from previous page*

	Forward Guidance			LSAP	
Country	2 Year	5 Year	10 Year	5 Year	10 Year
Malta			0.567		*0.042*
Netherlands	*0.171*	0.369	0.703	0.680	0.474
Norway	*-0.229*	*-0.096*		0.846	
New Zealand		*0.150*	*0.130*	0.480	0.433
Poland	0.383	0.660	1.79	*-0.059*	*0.151*
Portugal	*1.341*	*0.981*	*0.853*	*-0.108*	*0.336*
Romania		*-0.053*		*-0.2615*	
Singapore	*0.121*	*0.186*	0.839	0.714	0.501
Slovenia			*0.742*		*0.202*
Sweden	*0.264*	0.360	*0.609*	*0.662*	0.426
Turkey	1.997	1.138		*0.041*	
United Kingdom	0.509	0.649	1.091	*0.643*	0.368
South Africa			2.108		*0.225*

For the most part, spillover rates lie between 0 and 1 across all maturities, with a fair degree of cross country variation. I compute the average spillover rates for all countries in my analysis where the estimated effects of forward guidance and LSAPs was significant at any level ($p < 0.10$). For the 2-, 5-, and 10-year yields the average spillover rates for forward guidance are 0.542, 0.418, and 0.910 respectively[14]. For the 5- and 10-year yields, the average spillover rates for the LSAP component are 0.695 and 0.425, respectively. This signals a strong pass through effect of the Fed's unconventional policy to international sovereign bond yields during the zero lower bound period, especially for forward guidance on the 10-year

[14] It is worth nothing that for the 2-year yield, a number of the spillover rates were either not significant, or not available. If Turkey is excluded, the average spillover rate is 0.334

yield which is nearly one to one. Note, however, that the estimated effect of US forward guidance on the US 10-year yield is small, which could be driving these results.

A few interesting points stand out from this analysis. Several countries had spillover rates that were both significant and larger than 1. This is the case across the 3 maturities, and for both components. A couple of rates were near or larger than 2, as is the case for Turkey, Poland, and South Africa. To reiterate, a spillover rate larger than 1 indicates that their bond yields had a stronger reaction than what was estimated for the US. Surprisingly, for Iceland I find a significant, negative spillover rate that's also larger than 1 in absolute value, signaling that yields of that maturity moved in the opposite direction than for the US at a larger magnitude in response to the LSAP component of Fed announcements.

Table 1.8: S&P Sovereign Rating

Country	S&P April 2010 Rating	S&P January 2013 Rating
Australia	AAA	AAA
Belgium	AAA	AA+
Canada	AAA	AAA
Spain	AA	BBB-
Greece	BB+	BB-
Hungary	BBB-	BB
Ireland	BBB+	BBB+
Italy	A+	BBB+
Poland	A-	A-
Portugal	A-	BB
Turkey	BB	BB
South Africa	BB+	BBB

In an effort to find a preliminary explanation for these large effects, I collected the Standard & Poor's Sovereign Ratings for April 2010 and January 2013 for each country that reported a spillover rate larger than 1, and report them on Table 1.8. Eight out of the twelve countries in this short sample had a rating of BBB or BB by 2013. The majority of these countries saw their sovereign ratings downgraded during this period, with only South Africa receiving an improvement in their rating, while Canada and Australia stayed put. On the other hand, a few countries had large spillover rates even though they were not subject to a credit downgrade. While this is not a complete analysis, it suggests that a good avenue for further research would be to study the role of sovereign credit ratings in the magnitude of pass through of unconventional monetary policy in large economies to international financial markets. It would be interesting to study whether countries with poor ratings or who've been recently subjected to a downgrade are more susceptible to the effects of monetary policy in large economies during the ZLB period, but that is left for future research.

1.6 Panel Regression

To examine whether there are differences across groups of countries, I employ a panel regression framework to exploit the cross country variation in response. I will estimate the effect of unanticipated changes in forward guidance and asset purchases on foreign bonds using country and time fixed effects and accounting for two different specifications: emerging and industrial countries[15], and "Expansionary" versus "Contractionary" monetary policy to study whether there is asymmetry of the effect of unconventional policy. I define "Expansionary" policy as the FWG factor being negative (an easing of the expected path of future policy) and the LSAP factor being positive (signaling an increase in asset purchases). The results are reported below.

Some interesting results arise from this procedure. Results for 10-year yields can be found on Table 1.9, where I find that forward guidance has a stronger and significant easing effect on emerging countries using country and time fixed effects. I employ a test of parameter restrictions and find that the null hypothesis that these coefficients are the same is rejected at the 10% level. On the other hand, unanticipated changes in asset purchases have a significant effect on industrial countries, even while accounting for country and time fixed effects, but no

[15]I will be using the International Monetary Funds' classification of "Emerging" and "Industrial".

Table 1.9: Ten Year Yield

	(1) Standard	(2) Standard	(3) Ind / Eme	(4) Ind / Eme	(5) Exp / Con	(6) Exp / Con
FWG	2.279***	1.718***				
	(0.272)	(0.366)				
LSAP	-2.031***	-2.258***				
	(0.353)	(0.478)				
FWGEme			3.187***	2.626***		
			(0.639)	(0.743)		
FWGInd			1.952***	1.391***		
			(0.259)	(0.297)		
LSAPEme			-0.920	-1.147		
			(0.785)	(0.773)		
LSAPInd			-2.431***	-2.658***		
			(0.354)	(0.545)		
FWGexp					2.240***	1.211***
					(0.518)	(0.426)
FWGcon					2.060***	2.351***
					(0.512)	(0.601)
LSAPexp					-6.279***	-8.404***
					(0.621)	(0.959)
LSAPcon					-0.940**	-0.821*
					(0.391)	(0.453)
Country FEs	Yes	Yes	Yes	Yes	Yes	Yes
Time FEs	No	Yes	No	Yes	No	Yes
Adjusted R-squared	0.028	0.064	0.029	0.065	0.035	0.072
N	1870	1870	1870	1870	1870	1870

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

significant effect on emerging economies. In the case of expansionary versus contractionary monetary policy, contractionary forward guidance seems to have a stronger effect than expansionary when accounting for country and time fixed effects, but the coefficients are not statistically different from zero[16]. Meanwhile, expansionary asset purchase announcements had much stronger effects than contractionary announcements, which is mostly being driven by the effect on industrial economies.

Table 1.10: Five Year Yield

	(1) Standard	(2) Standard	(3) Ind / Eme	(4) Ind / Eme	(5) Exp / Con	(6) Exp / Con
FWG	2.092***	2.142***				
	(0.361)	(0.613)				
LSAP	-1.085***	-0.643				
	(0.369)	(0.585)				
FWGEme			2.493***	2.543**		
			(0.834)	(1.006)		
FWGInd			1.928***	1.977***		
			(0.371)	(0.606)		
LSAPEme			0.343	0.785		
			(0.653)	(0.691)		
LSAPInd			-1.669***	-1.227*		
			(0.379)	(0.710)		
FWGexp					2.084***	2.296***
					(0.463)	(0.610)
FWGcon					1.746***	1.539
					(0.407)	(1.075)
LSAPexp					-6.555***	-6.796***
					(1.012)	(0.846)
LSAPcon					0.326	0.927
					(0.478)	(0.680)
Country FEs	Yes	Yes	Yes	Yes	Yes	Yes
Time FEs	No	Yes	No	Yes	No	Yes
Adjusted R-squared	0.049	0.098	0.056	0.104	0.087	0.122
N	1705	1705	1705	1705	1705	1705

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Results for the response of 5-year yields to unconventional policy are reported on Table 1.10,

[16]The parameter test almost rejects the null, with a p value of 0.107.

where I find that the forward guidance factor remains statistically significant when using country and time fixed effects, but not the asset purchase factor. In addition, the forward guidance factor has an estimated effect on emerging economies that seems stronger, yet it is not statistically different than the one found for industrial countries. As in the case for 10-year yields, the asset purchase factor has a strong, statistically significant effect during an easing of policy but not during contractionary policy when country and time fixed effects are included.

Table 1.11: Two Year Yield

	(1) Standard	(2) Standard	(3) Ind / Eme	(4) Ind / Eme	(5) Exp / Con	(6) Exp / Con
FWG	1.579***	1.806**				
	(0.553)	(0.749)				
LSAP	-0.987***	-0.084				
	(0.294)	(0.610)				
FWGEme			3.115	3.342		
			(2.159)	(2.041)		
FWGInd			1.237***	1.464*		
			(0.436)	(0.833)		
LSAPEme			-2.019***	-1.116*		
			(0.523)	(0.609)		
LSAPInd			-0.758**	0.145		
			(0.314)	(0.644)		
FWGexp					1.720**	1.938**
					(0.682)	(0.892)
FWGcon					1.155*	1.381
					(0.573)	(1.466)
LSAPexp					-4.206***	-3.570***
					(0.558)	(0.805)
LSAPcon					-0.132	0.814
					(0.352)	(0.597)
Country FEs	Yes	Yes	Yes	Yes	Yes	Yes
Time FEs	No	Yes	No	Yes	No	Yes
Adjusted R-squared	0.032	0.079	0.040	0.087	0.045	0.087
N	1210	1210	1210	1210	1210	1210

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Results for the estimated response of 2-year yields to unconventional monetary policy can be found on Table 1.11. The estimated response to forward guidance remains statistically significant when using time and country fixed effects, but that isn't the case for asset purchases, as we saw for the 5-year yield. In addition, the estimated effect of forward guidance seems to be the strongest for industrial economies, while asset purchases have stronger estimated effects on the 2-year yields of emerging markets. Lastly, as we've found before, expansionary monetary policy has the strongest estimated effect when using country and time fixed effects.

Results from this procedure seem to highlight two main results. First, there is heterogeneity in the response of bond yields to unconventional monetary policy between emerging and industrial economies, with forward guidance seemingly having a stronger effect on emerging economies for 10- and 5-year yields. Unanticipated changes in asset purchases on the other hand have a stronger, statistically significant effect on industrial economies for the 10- and 5-year yields. Second, across the board I estimated a strong, statistically significant effect of asset purchases on bond yields when the policy was expansionary, but not in the contractionary case. This hints at an asymmetric response of sovereign bond yields to US monetary policy during the ZLB period.

1.7 Concluding Remarks

I build on the analysis of Swanson (2018) and use separately identified forward guidance and LSAP announcements factors to estimate the effects of both types of unconventional policies on international bond yields, stock indices, and exchange rates. Both types of policies had significant effects on bond yields, but weak effects on exchange rates, and no significant effects on stock market indices. The magnitudes of the effects found on bond yields were smaller than the US response with a few exceptions, as would be expected. The effectiveness of each type of unconventional policy varied across countries. British bond yields reacted more strongly to changes in forward guidance. Canadian, Australian, and German bond yields were estimated to react more strongly to changes in LSAPs. Japanese bond yields reacted to both factors, but only bonds of long maturities showed significant responses. Asset purchases during the ZLB seemed to have a negative effect on the short end of international yield curves, in contrast to the positive effect on the short end reported in Swanson (2018). Yields of maturities that the Federal Reserve targeted during their asset buying program responded to unconventional policy across all countries analyzed in this paper.

I find that there are significant spillover effects on international bond yields from US unconventional monetary policy. I also find that countries react differently to these two estimated factors. I compute a spillover rate for each factor to better compare the effects across countries and maturities, and extend this analysis to a larger panel of countries. I find that the spillover rates are similar across the countries studied, with some interesting results also coming from this extended analysis, providing avenues for further research. Lastly, I find that there is heterogeneity in the estimated response between emerging and industrial economies, and between contractionary and expansionary monetary policy using a panel regression framework. In particular, I find a strong estimated effect of asset purchases during expansionary monetary policy.

Central bankers in other countries can use these results to better anticipate the effects of Federal Reserve unconventional monetary policy on their own economies, financial markets, and their peers. There is still work to be done in this framework, namely the issue of persistence and the effect of these factors on international private-sector rates. An analysis of the spillover effects of unconventional policy on emerging economies vs. developed economies could also prove fruitful. The heterogeneity in spillover rates also give some avenues for future research, as it would be interesting to see what drives the difference between the levels of spillover for each factor. We are also some years away from the end of the ZLB period, so an analysis of monetary policy now that we are back in the conventional world could prove fruitful, in particular to assess whether spillovers rates to the forward guidance factor are different when outside of the ZLB.

Chapter 2

Anchoring Japanese Inflation Expectations

Over the past two decades, Japan found itself in several economic difficulties, such as prolonged deflation, declining potential growth, several financial crises, and structural impediments arising from a rapidly aging population and dwindling labor force. Given such experiences, I stress one observation in thinking about the resilience of monetary policy framework; that is, in the long run, low inflation and low nominal interest rates are most likely to coexist. Under such a circumstance, central banks have very little latitude for cutting policy rates due to the existence of the zero lower bound of nominal interest rates. In other words, the resiliency of monetary policy – more precisely, conventional or standard monetary policy – is considerably eroded.

- Bank of Japan Governor Haruhiko Kuroda

Remarks at the 2016 Economic Policy Symposium Held by the FRB of Kansas City

2.1 Background

In the current Japanese economic environment, as described above by Governor Kuroda, the central goal of long-term price stability at reasonable levels has proven difficult to achieve. One of the key components of long term inflation is expected inflation, and efforts to achieve reasonable price stability can be strengthened by a better understanding of the dynamics of expectations. This paper examines inflation expectations in Japan and their evolution following developments in Japanese monetary policy during recent years. Using data on inflation expectations and financial market measures, we use a Nelson Siegel model to analyze the dynamics of long-run expectations following the introduction of new monetary policies by the Bank of Japan. The main finding of this paper is that while recent Bank of Japan measures have increased long-run inflation expectations and their degree of anchoring[1], they have not been able to effectively anchor expectations at the 2-percent target.

Control of inflation is central to good monetary policy (Bernanke (2007)), and inflation is undoubtedly affected by expected inflation, influencing the central bank's ability to achieve stability in the aggregate economy [2]. The Japanese economy has experienced weak inflation for the past two decades, with continued efforts to raise inflation and inflation expectations that have so far fallen short (IMF 2016a; 2018a; 2018b). Moreover, inflation expectations seem to be strongly backward looking, and largely influenced by observed current and past inflation (Bank of Japan, 2016). Among G7 countries, Japan has the lowest contribution to CPI core[3] and CPI headline inflation dynamics coming from expected inflation (Figure

[1] Specifically, increasing their degree of anchoring refers to inflation expectations increasing towards the target and becoming less dispersed.

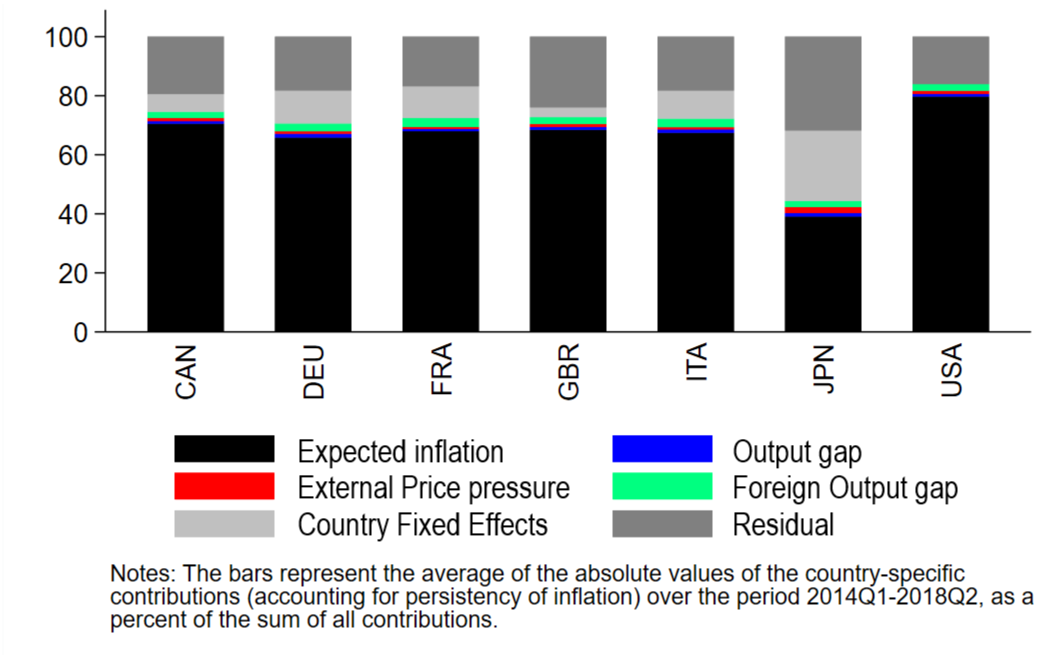
[2] Inflation expectations, in a New Keynesian model, will shift up the trade-off between unemployment (or some measure of lost output) and actual inflation, leading to higher actual inflation at a given rate of employment, and vice-versa. In an economy operating under a Taylor Rule, the central bank can offset the rise in inflation by adjusting the nominal interest rate. On the other hand, expectations that are *too low* can lead an economy to find itself constrained by the zero lower bound on interest rates (as cash carries a nominal rate of zero), which leads to the depression of real aggregate demand as a result of an expected decline in the rate of inflation. This is currently the case in Japan: both inflation, and inflation expectations, are remarkably low.

[3] CPI core excludes food and fuel commodities.

(2.1)). Furthermore, inflation expectations have a strong negative contribution to deviations of core and headline[4] inflation from the headline CPI target (See Figure (2.2))[5].

Figure 2.1: Contributions to G7 Core Inflation Dynamics

(Percent)



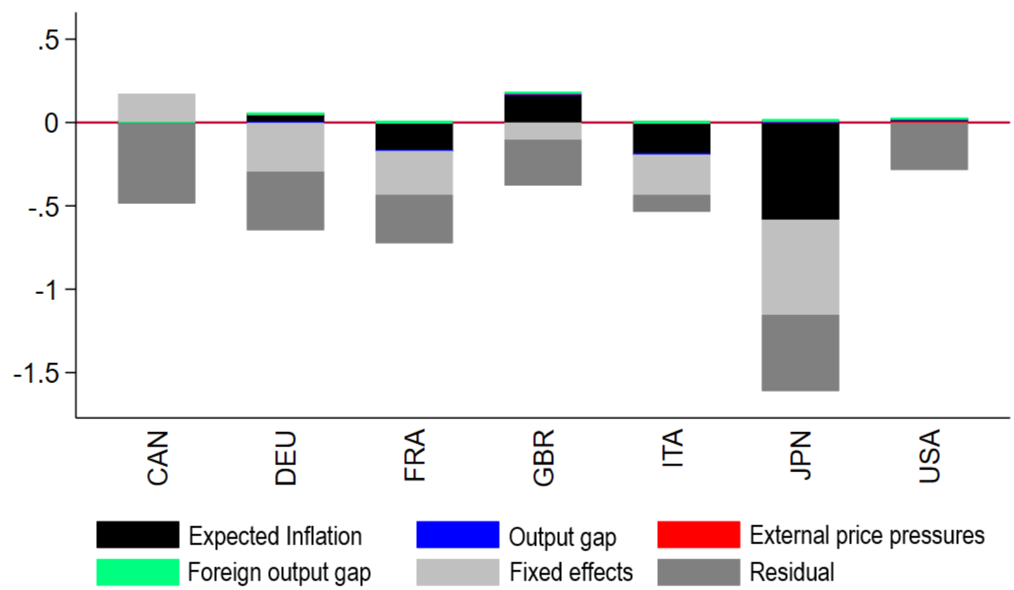
In this environment, beginning in 2013 the Bank of Japan introduced a number of monetary policy actions with the objective of raising inflation, partly by raising inflation expectations. Success in the fight against deflation is important for the future economic development of Japan. Strongly anchored inflation expectations can significantly improve economic resilience to adverse external shocks. Strongly anchored expectations at the target reduces inflationary or deflationary persistence, and provides a buffer against the zero lower bound that allows the Bank of Japan flexibility in monetary policy. Sustained deflation has acted as a headwind for the Japanese economy in recent years, inducing firms to become reluctant in their investment and pricing decisions. Additionally, sustained deflation can also cause consumers to be cautious in their purchases. Deflation also increases the real debt burden of borrowers,

[4]See Appendix B.

[5]These figures were generated using a methodology found in Chapter 3 of the *World Economic Outlook* (International Monetary Fund, 2018). Figures 2.1 and 2.2 are derived from the authors' own calculations using current IMF data.

Figure 2.2: Contributions to Deviations of G7 Core Inflation from Target

(Percentage Points)



Notes: The bars represent country-specific averages over the period 2014Q1-2018Q2.

raising default risk while reducing asset prices and collateral valuations. In many advanced economies, declining prices and stagnant wages all reinforce the weakening of demand in a continuous cycle (International Monetary Fund, 2016b).

Section 2.2 surveys the literature on inflation expectations, focusing on its measurement and formation. Section 3 reviews the related literature on measuring Japanese inflation expectations, while Section 4 describes the derivation of the Nelson-Siegel model and data used in the paper. Section 5 estimates the changes in long term inflation expectations following a number of monetary policy episodes in Japan, derived from Nelson-Siegel curves as used by Lewis and McDermott (2016). Section 6 discusses the robustness of results and Section 7 compares them with the current literature, and with results for New Zealand. Section 2.8 concludes.

2.2 Inflation Expectations

2.2.1 Formation

In Japan, the formation process for inflation expectations differs from that of the rest of the world. For one, it has become more difficult to make the case that Japan's inflation expectations are mainly determined by the actions of the Bank of Japan. Due to the asymmetry in the effects of monetary policy at the zero lower bound[6], the Bank is unable to counter excessively low inflation by cutting rates. This results in a crisis of credibility, where the public is aware that the central bank's hands are tied, and that its ability to influence inflation is constrained. Another interesting episode highlighting this difference occurred in 2014 as a decline in crude oil prices led to a decline in long term inflation forecasts in Japan, while for the United States long term forecasts remained unchanged (where the United States has strongly anchored inflation expectations at around 2 percent). Indeed, Governor Kuroda of the Bank of Japan pointed to the 2014 decline in energy prices as one of the key causes for the weakening of long-term inflation expectations in Japan (Kuroda, 2016). This exogenous development, along with weakness in demand following the consumption tax rate increase of April 2014, and the slowdown in emerging economies and volatile global financial markets, lowered the observed Japanese inflation rate. This then led to a weakening of inflation expectations, reflecting that expectations formation in Japan is more dependent on lagged components rather than the future path of inflation (Bank of Japan, 2016).

Inflation expectations in Japan seem to be strongly backward looking and influenced by current and past inflation (Bank of Japan, 2016). Nishino et al. (2016) show that the adaptive component (the contribution of observed current and past inflation) dominates in the formation of inflation expectations in Japan. Compared with the United States, United

[6]Current research has shown there is now a difference between the "zero lower bound" and the "effective lower bound". See for example Agarwal and Kimball (2019). In this context, we are referring to the zero lower bound on interest rates.

Kingdom, and the Euro area, inflation expectations in Japan are more sensitive to changes in the one quarter lagged observed inflation rate. The adaptive formation mechanism can lead exogenous factors to exert pressure on inflation expectations, as was the case with the 2014 decline in crude oil prices. These findings are in agreement with the view of the Bank of Japan as set out in their "Comprehensive Assessment" of QQE1 (2016)[7]. Furthermore, least squares estimation shows that the observed inflation rate accounts for 70 percent of short-term inflation expectations and 40 percent of medium- and long-term expectations, suggesting that the adaptive component plays a considerably large role in Japan. When running the same regression on data for the United States, United Kingdom, and the Euro area, Japan has the largest adaptive component in inflation expectations. A possible but incomplete explanation for this, as noted by the Bank of Japan, is that wage negotiations in Japan – for example, the annual *blackshunto* – are more sensitive to developments in the observed year-to-year inflation rate.

2.2.2 Measurement

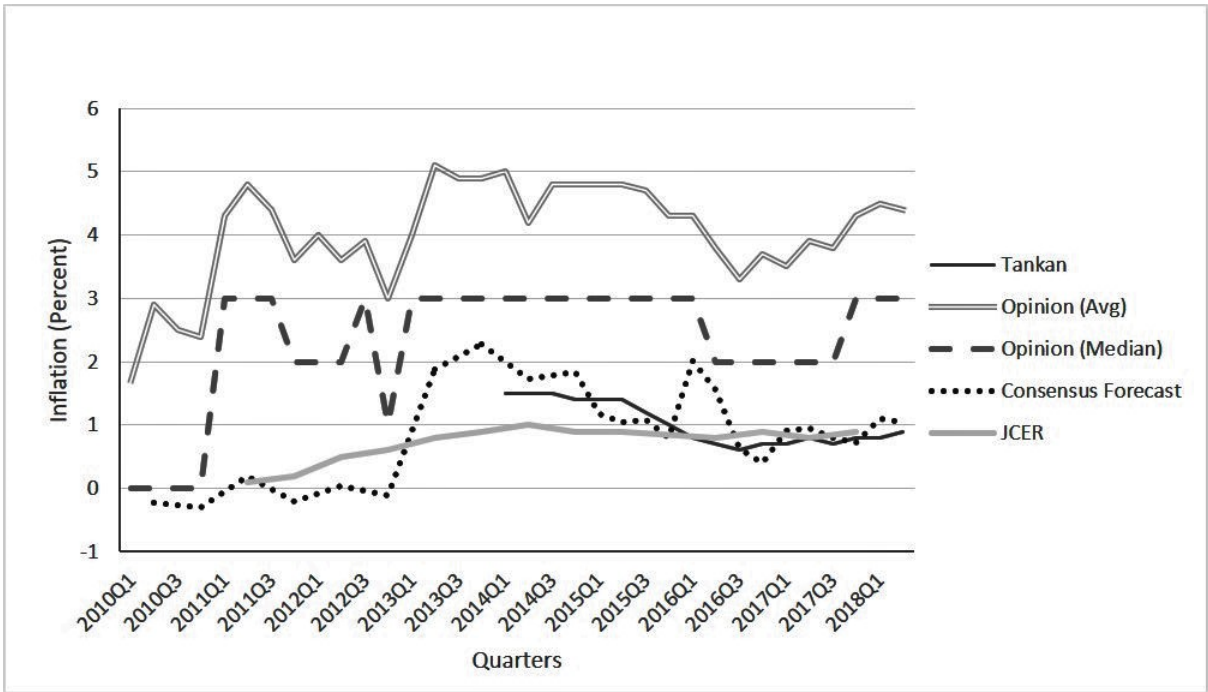
Using survey data is the most direct way to measure inflation expectations, as surveys often ask respondents their views regarding price dynamics both at present and in future horizons. Financial markets also contain information about inflation expectations and inflation risk. These include over-the-counter inflation swaps and the break-even inflation rate, which is the difference between far-ahead forward rates on nominal and inflation-indexed bonds. No single measure is a perfect measure, with advantages and drawbacks to each of them. In theory, inflation expectations of agents in the economy should be both backward and forward looking, paying attention to past inflation as well as central bank forward guidance. As a

[7] "QQE1" stands for Quantitative and Qualitative Monetary Easing, and it refers to the first round of unconventional monetary policy introduced by the Bank of Japan in April 2013. Section 2.7 has more details. See Appendix B for the full timeline of monetary policy frameworks introduced by the Bank of Japan since the launch of Abenomics. Details on the new framework after a comprehensive assessment in 2016 can be found here: `https://www.boj.or.jp/en/announcements/release_2016/k160921a.pdf`

result, one would expect the inflation expectations of different agents to closely match one another. In practice, inflation expectations (and beliefs about past inflation) of households, firms, and forecasters do not often match up. There is widely documented disagreement in the forecast of inflation expectations by different agents in the European Union and New Zealand (Coibion et al., 2018,c), and this is also the case for Japan. Figure 2.3 plots the time series of mean inflation one year ahead expectations for firms (Tankan), households (Bank of Japan's Opinion Survey, both average and median), and professional forecasters (Consensus Economics, Japanese Center for Economic Research (JCER)) confirming that agents in Japan have differing inflation expectations. While surveys for firms and professional forecasters begin to match closely after 2016Q3, households report expected inflation with an average four times larger than those of firms. Furthermore, in the case of Japan, there is wide cross sectional disagreement on inflation expectations not only among households but also forecasters. Work by Nakazono (2016) finds that information rigidities are the main determinants of this disagreement in inflation expectations, with revisions to the forecast of inflation occurring infrequently.

In addition, financial market data can be used to obtain information about long run inflation expectations. The difference between yields on nominal government bonds and inflation-protected government bonds (or inflation swap rates) can be a proxy for the inflation expectations of financial market participants. Two papers, Gürkaynak et al. (2007a) and Gürkaynak et al. (2010a) use forward inflation compensation to investigate the effects of inflation targeting on future expectations. The advantage of using bond data is the availability of series with thousands of daily observations for yields of different maturities in highly liquid markets, such as the United States and United Kingdom bond markets. In the case of Japan, however, an issue may arise as the market for inflation-indexed bonds (JGBi) is very thinly traded, with the Ministry of Finance having purchased a majority of the issuance in the last decade. In addition, the market for nominal bonds has exhibited low volume in recent years, exacerbating liquidity concerns. Illiquid bond markets make implied forward

Figure 2.3: Japan One-Year-Ahead Inflation Expectations, Different Surveys



Source: Authors' calculations using data from Consensus Forecast, and Haver Analytics.

rates difficult to estimate, and the resulting series could feature a lot of noise. In addition to liquidity, there are also other risk premia that might influence the movement in yields.

2.3 Related Literature

A number of papers have taken different approaches to evaluate the effects of the Bank of Japan's actions on inflation expectations, with all reporting similar results to ours. A paper by Kamada et al. (2015) investigates household inflation expectations in Japan and, using micro-data from household surveys on inflation expectations, examines to what extent household inflation expectations have been affected by the Bank of Japan's policies. Their methodology relies on fitting a normal inverse Gaussian distribution to remove distortions found in survey data, such as downward rigidity of price expectations. Their approach also allows one to take a term-structure point of view in characterizing inflation expectations, which is similar to our approach. The authors show that the introduction of an inflation target, along with unconventional monetary easing introduced by the Bank of Japan, contributed to strengthening the anchor of inflation expectations as measured by household surveys.

A paper by Hattori and Yetman (2017) assesses the evolution of inflation expectations for Japan, using previous work by Yetman and fitting a model to inflation forecasts. They find that an estimated inflation anchor has tended to rise in recent years, and the degree to which these anchors pin down inflation expectations at longer horizons has increased. Work by De Michelis and Iacoviello (2016) employs a VAR model to examine how macroeconomic variables respond to an identified inflation target shock. They then calibrate the effect of inflation targeting in a new Keynesian DSGE model of the Japanese economy. They find that Japan has made some progress toward overcoming deflation following the introduction of the target, but further measures are needed to raise inflation in a durable manner. Nakazono (2016) uses survey data to examine two types of disagreements regarding Japanese inflation expectations and their implications for monetary policy: disagreements among forecasters, and disagreements between the central bank and private agents. His methodology involves estimating noisy and sticky information models of inflation to determine the level of information asymmetry and how agents process information. The key findings in this paper are as follows: (i) information rigidities are the determinants of cross-sectional disagreement among both households and market forecasters; (ii) there is disagreement in the long-run forecasts of inflation between central bank and private agents in the economy, despite both the adoption of an inflation target and use of unconventional monetary policy; and (iii) the revised policy regime of the Bank of Japan does not abruptly change following the introduction of inflation targeting, mainly because Japanese monetary policy was already perceived to be accommodative.

Modeling term structures using Nelson Siegel curves has long been done for interest rates, but recent work has brought this approach to modeling inflation expectations. The original paper by Nelson and Siegel (1987) introduced a model for a parametrically parsimonious approach to modeling yield curves. Their work was motivated by the expectations theory of the term structure of yields. This representation effectively works as a dynamic three factor model, where the framework of the model imposes a structure such that each of the factors can be thought of as the level, slope, and curvature of expectations curves (Diebold and Li, 2005). Drawbacks of this model are that the Nelson-Siegel factors are unobserved, which allows for measurement error, and that there are plausible economic restrictions on the associated loadings (strictly positive, etc...). A generalization of this model (Svensson, 1995) has been in use by the Federal Reserve for quite some time (see Gürkaynak et al. 2007b), with Christensen et al. (2011) providing an arbitrage free version of the model in a dynamic form. Use of these models has focused on forecasting (Diebold and Li, 2005); modeling in a macro-finance setting (Diebold et al., 2005; Christensen et al., 2011); and more recently measuring inflation expectations (Christensen et al., 2010; Gürkaynak et al., 2010b).

A number of recent papers have used Nelson-Siegel curves to model inflation expectations, namely Aruoba (2014, 2019) and Lewis and McDermott (2016); Lewis (2015). Aruoba uses this statistical model to combine surveys, and produce a term structure of inflation expectations, which is the approach we take in this paper. He shows that this methodology works quite well as inflation expectations track realized inflation, display good forecast accuracy when compared with other popular alternatives, and show that long-run inflation expectations in the United States remained anchored in the 2008-2015 period[8]. Lewis and McDermott (2016) examine the experience of the Reserve Bank of New Zealand in changing its inflation target, particularly its effect on inflation expectations. They use surveys of inflation expectations and a Nelson-Siegel model to combine them into a continuous expectation

[8]The work by Aruoba is now featured by the Federal Reserve Bank of Philadelphia as one of the series made available by the Real-Time Data Research Center. For a more in-depth treatment of this subject, Diebold and Rudebusch (2012) provide an excellent text, addressing a number of extensions to the dynamic Nelson-Siegel model framework.

curve where expectations can be plotted as a function of the forecast horizon. Analyzing their estimate of long-run inflation expectations, they find that changes in the inflation target by the Reserve Bank of New Zealand result in an immediate change in inflation expectations (see also Section 7).

2.4 Inflation Expectations Curves

Following the methodology used in Lewis (2015) and Lewis and McDermott (2016), we combine a number of Japanese surveys along with other data using a Nelson-Siegel model. This approach produces information on the level of long-run inflation expectations by using a procedure that, when applied to various expectation measures, is akin to model averaging. Furthermore, we test the impact of monetary policy events in Japan on the perceived long-run inflation level, which can be used as a proxy for the perceived inflation target. Due to data limitations, this paper examines inflation expectation curves before and after recent specific monetary policy events: the beginning of inflation targeting in January 2013; introduction of Quantitative and Qualitative Monetary Easing (QQE1) in April 2013; the second round of Quantitative and Qualitative Monetary Easing (QQE2) in October 2014; the introduction of Negative Interest Rates (NIRP) in January of 2016; and Quantitative and Qualitative Monetary Easing with Yield Curve Control (YCC) which began in September 2016.

2.4.1 The Model: Nelson-Siegel and Inflation Expectations

Using a Nelson-Siegel model to form an inflation expectations curve is useful for a number of reasons. First, inflation expectations survey data are known to be a more accurate measure of agents' expectations (Faust and Wright, 2013), and their combination is akin to model averaging under this framework, which helps to deal with forecast error, measurement error, or survey-specific bias. Also, the underlying components from the Nelson-Siegel model can be given economic interpretations, since we rely on a parametric approach (the aforementioned level, slope, curvature). This is different from competing empirical techniques to model term structures, such as non-parametric splines, which cannot assign economic meaning to components of the model. Furthermore, the Nelson-Siegel class of models provides a stable fit over time, and is a popular choice of models both at central banks and in the literature. By combining surveys, it provides an inflation expectations curve at any point in time, which is useful given that available surveys tend to have sparse forecast horizons.

Since survey data relate directly to agents' inflation expectations at a point in the future, we use this representation rather than the arbitrage-free adjustment for market price of risk that is typically applied to bond yield data. However, one key assumption that needs to be made is that respondents are consistent in their survey responses, providing their best assessment of future inflation at each given horizon. The forward rate representation of the Nelson-Siegel model is the following:

$$\pi^e(\tau)_t = H\beta_t + e_t \tag{2.1}$$

where $\pi^e(\tau)_t$ is the vector of expectations observed at time t for K horizons collected in the $K \times 1$ vector τ, where τ is the horizon of the survey measured in years, β_t is the 3×1 vector of expectation components (Level, Slope, Curvature) at t, and the error term is $e_t \sim N(0, R)$. The sensitivity matrix H is given by:

$$H = \begin{bmatrix} 1 & \exp(-\lambda\tau_1) & \lambda\tau_{t1}\exp(-\lambda\tau_1) \\ 1 & \exp(-\lambda\tau_2) & \lambda\tau_{t2}\exp(-\lambda\tau_2) \\ \vdots & \vdots & \vdots \\ 1 & \exp(-\lambda\tau_K) & \lambda\tau_{tK}\exp(-\lambda\tau_K) \end{bmatrix} \tag{2.2}$$

However, we need to derive the analogous Nelson-Siegel functional form that is applicable to annual inflation expectations by adjusting the instantaneous forward rate. This is due to the original framework representing the term structure of forward rates at a given point in time using the latent factor model. We can derive this representation to create an annual rate $\pi^e(\tau, \tau-1)_t = H\beta_t + e_t$ where:

$$H = \begin{bmatrix} 1 & [\text{Slope Loading}]_1 & [\text{Curvature Loading}]_1 \\ 1 & [\text{Slope Loading}]_2 & [\text{Curvature Loading}]_2 \\ \vdots & \vdots & \vdots \\ 1 & [\text{Slope Loading}]_K & [\text{Curvature Loading}]_K \end{bmatrix}$$

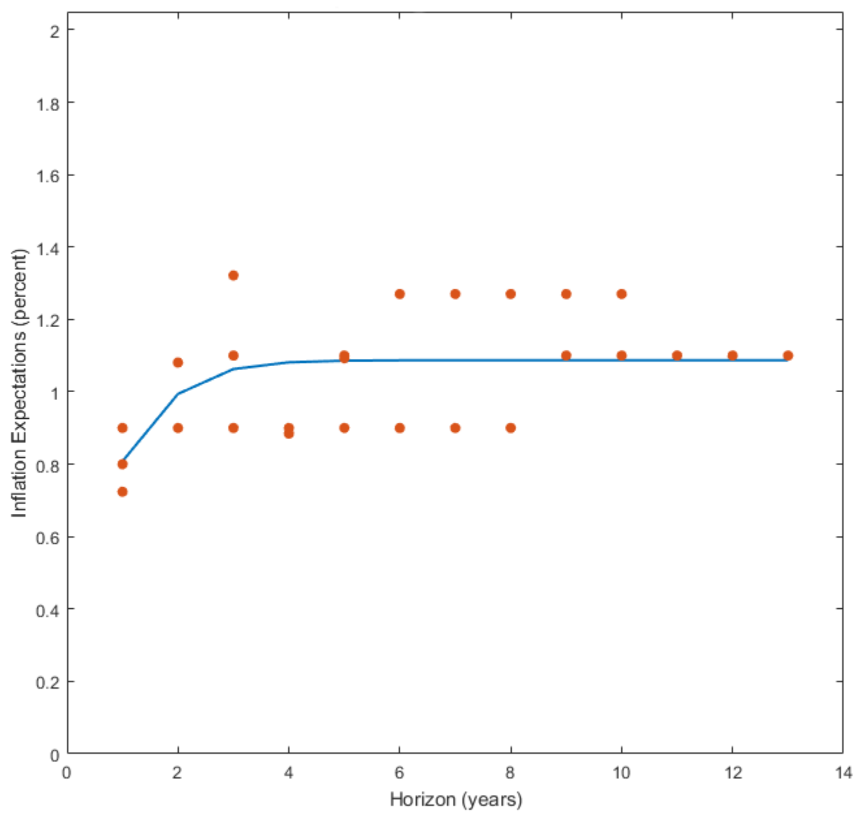
$$[\text{Slope Loading}] = \frac{1}{\lambda}\{\exp(-\lambda[\tau_i - 1]) - \exp(-\lambda\tau_i)\}$$

$$[\text{Curvature Loading}] = \frac{1}{\lambda}\{\exp(-\lambda[\tau_i - 1]) - \exp(-\lambda\tau_i)\} + \{[\tau_i - 1]\exp(-\lambda[\tau_i - 1]) - \lambda\exp(-\lambda\tau_i)\}$$

The parameter λ determines the speed of time decay for the Slope and Curvature components towards zero as the horizon τ increases. For this exercise, λ will be kept constant across time though it could have a time subscript. The optimal λ is globally estimated by minimizing the squared residuals of the fitted and actual inflation expectations over the entire sample of expectations data. Globally optimizing λ ensures that the expectations curves are consistent across time and cross-sectionally. Values for the Level, Slope, and Curvature are estimated for each expectations curve observation in the sample. The Level component can be interpreted as a measure of long-run inflation expectations, which in turn can be used as a proxy for the perceived inflation target or survey respondents and market participants. The loading on the Slope factor starts near one and decays towards zero monotonically as $\tau \to \infty$ and can be thought of as a short-term factor. The Curvature loading starts near zero, rises near the medium-term horizon, then decays towards zero as $\tau \to \infty$. Similarly, this loading can be thought of as a medium-term factor. The combination of these two components can be used to indicate the "expected time to target" implied by market and survey participants.

Figure 2.4 shows one such curve, for October 2017.

Figure 2.4: Japan: Nelson-Siegel Inflation Expectations Curve on Oct. 2017



Japanese surveys used in this paper include the Consensus Forecast for 1-year to 10-years ahead from 2010Q1 to 2018Q2; Japanese Center for Economic Research inflation forecasts, for 1-year to 14-years ahead from 2011Q1 to 2017Q4; QUICK Monthly Market Survey forecasts for 10-year ahead inflation from 2010Q1 to 2013Q4; and the Tankan survey of firms' inflation forecasts for 1-year, 3-year and 5-year ahead for 2014Q2 to 2018Q2. Japanese financial market data includes Inflation Swaps for 1-year, 5-year and 10-year, as well as the 5-year 5-year forward swap rates. Household surveys were omitted from this analysis due to varying distortions and bias, including the ones documented in Kamada et al. (2015). Furthermore, we are also particularly interested in the inflation expectations of firms, due to their role in setting prices across the economy.

Table 2.1: Japan Survey Measure Summary Statistics

	2010Q2 - 2018Q2		2010Q2 - 2013Q4		2014Q1 - 2018Q2	
	$\bar{\pi}$	$\hat{\sigma}$	$\bar{\pi}$	$\hat{\sigma}$	$\bar{\pi}$	$\hat{\sigma}$
Consensus 1yr	0.92	0.74	0.44	0.96	1.15	0.45
JCER 1yr	0.72	0.27	0.52	0.29	0.89	0.06
Tankan 1yr	0.99	0.32	-	-	0.99	0.32
Consensus 2yr	1.27	0.57	0.74	0.49	1.52	0.42
JCER 2yr	0.74	0.34	0.52	0.29	0.93	0.24
Consensus 3yr	1.18	0.30	0.92	0.31	1.30	0.21
JCER 3yr	0.75	0.39	0.55	0.42	0.93	0.24
Tankan 3yr	1.25	0.23	-	-	1.25	0.23
Consensus 4yr	1.13	0.29	0.96	0.28	1.20	0.27
JCER 4yr	0.72	0.43	0.45	0.45	0.94	0.24
Consensus 5yr	1.13	0.30	0.89	0.30	1.25	0.21
JCER 5yr	0.82	0.38	0.63	0.41	0.97	0.27
Tankan 5yr	1.31	0.26	-	-	1.31	0.26
Consensus 6yr	1.30	0.20	1.12	0.18	1.39	0.15
JCER 6yr	0.78	0.39	0.63	0.41	0.91	0.31
Consensus 7yr	1.30	0.20	1.12	0.18	1.39	0.15
JCER 7yr	0.80	0.42	0.63	0.41	0.94	0.37
Consensus 8yr	1.30	0.20	1.12	0.18	1.39	0.15
JCER 8yr	0.85	0.43	0.75	0.47	0.94	0.37
Consensus 9yr	1.30	0.20	1.12	0.18	1.39	0.15
JCER 9yr	0.99	0.36	0.96	0.33	1.01	0.38
Consensus 10yr	1.30	0.20	1.12	0.18	1.39	0.15
JCER 10yr	1.05	0.36	1.13	0.29	1.01	0.38
Quick 10y	0.92	0.20	0.92	0.20	-	-
JCER 11yr	1.11	0.32	1.13	0.29	1.10	0.34
JCER 12yr	1.09	0.25	1.13	0.29	1.05	0.21
JCER 13yr	1.00	0.21	-	-	1.05	0.21
JCER 14yr	0.95	0.25	-	-	0.95	0.25

Source: Authors' calculations

The model fits relatively well, in particular when one plots the actual data versus the fitted curve as in Figure 2.4. However, average error statistics are larger than those found by Lewis (2015) for New Zealand. This is to be expected, as the literature examined earlier pointed out that inflation expectations disagreement in Japan has been larger than that observed in other economies (Nakazono, 2016).

2.5 Results

Below, we look at the movement in the long-term inflation expectations of Japan following various Bank of Japan monetary policy framework announcements. In Figures 5 through 8, the left-hand-side panel is based on a combination of financial market data and survey data, while the right-hand-side panel is based only on survey data[9]. One important caveat is the possibility that other exogenous events exerted a considerable influence on inflation expectations during these episodes. Due to the quarterly (and at times, biannual) availability of the surveys used for this exercise, it is difficult to rule this out. In particular, the oil price shock that Japan experienced during some of these events might have exerted a strong deflationary effect. Nevertheless, it is likely that the actions of the Bank of Japan during this period had a large effect on inflation expectations.

2.5.1 QQE1

The first monetary policy announcement relevant to this paper is the initial "Quantitative and Qualitative Monetary Easing" (QQE1) announcement of April 2013 that set the framework for a price stability target[10] of 2 percent for the year-on-year rate of change in the headline CPI[11].

The change in inflation expectations following this announcement can be seen in the following figures. Figure 2.5 shows that following this announcement, inflation expectations shifted upwards. The shift occurred for all horizons, but it did not manage to anchor expectations at the 2 percent target. For precise estimates, refer to Table 2.2, in particular the QQE1

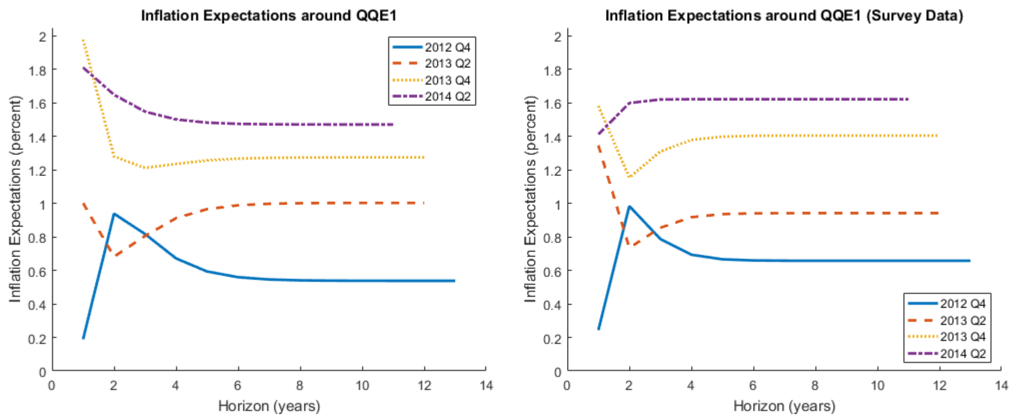
[9]Using financial market data in combination with survey data gave inflation expectations results that were on average lower than our results for the survey-only dataset. This downward bias could be due to liquidity issues in Japanese inflation protected bond markets. We include these combination financial market-survey results here for completeness.

[10]For more on Japanese inflation targeting, the reasoning behind its 2 percent inflation target, and targeting "from below", see Appendix A.

[11]Note that for this period, data constraints limit the sample to only biannual observations.

section. The level component, as noted earlier, can be interpreted as a measure of long-run inflation expectations held by the survey respondents. By 2014Q2, the level component had reached 1.622 percent, which was strikingly close to target.

Figure 2.5: Japan: Nelson-Siegel Curves QQE1

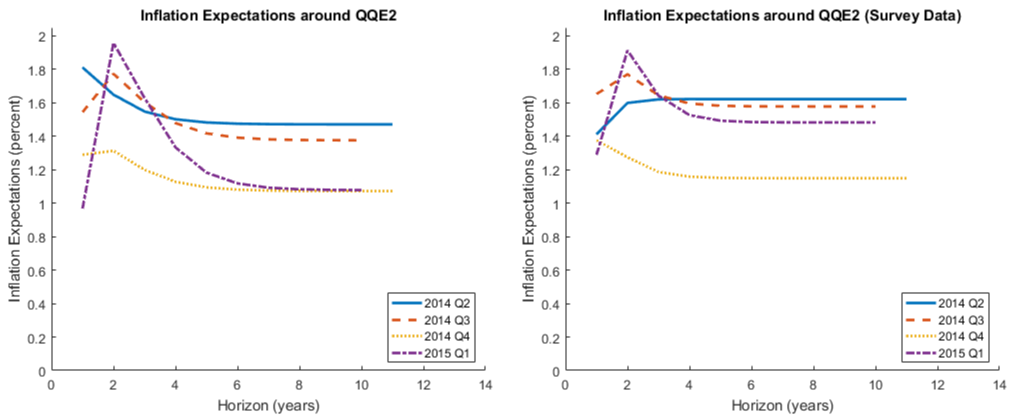


2.5.2 QQE2

The Bank of Japan introduced additional measures to expand the Quantitative and Qualitative Monetary Easing program (QQE2) on October 2014. In a pair of narrow votes, the Bank of Japan moved to accelerate the pace of increases in the monetary base, increase asset purchases, and extend the average remaining maturity of bond purchases. This was in response to downward pressures on prices due to weak demand after the 2014 consumption tax rate increase, along with the decline in crude oil prices. The Bank of Japan again committed itself in this announcement to achieving its price stability target of 2 percent.

Inflation expectations decreased in the quarters that followed, falling to close to 1 percent for the medium to long term horizon, but shifting up for the shorter horizons in 2015Q1, as seen in Figure 2.6. Expectations initially increased for short horizons, then dropped for all horizons in 2014Q4, with long run expectations settling around 1.5 percent by 2015Q1.

Figure 2.6: Japan: Nelson-Siegel Curves QQE2

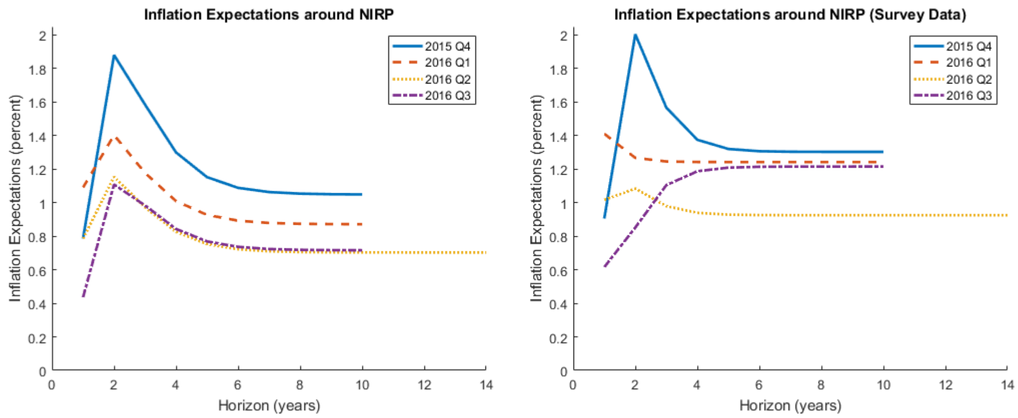


2.5.3 Negative Interest Rate

Following its January 2016 Monetary Policy meeting, the Bank of Japan announced that they would be imposing an interest rate of minus 0.1 percent to a portion of banks' accounts held at the Bank. In addition, the Bank of Japan maintained that rates would go further negative if deemed necessary, potentially to minus 0.5 percent. This was in response to financial turmoil abroad, and a reinforcement of the modest gains in inflation which had yet to achieve the 2 percent inflation target, to which the Bank of Japan remains strongly committed.

As shown in Figure 2.7, inflation expectations for all horizons slightly decreased in the quarters following the negative interest rate announcement, with a noticeable drop in the quarter following the announcement but a bounceback in Q3 of 2016. However, it is worth noting the difference between long run and short run expectations: while the curves themselves moved together, there was a lot of movement at short horizons while long run expectations moved relatively little.

Figure 2.7: Japan: Nelson-Siegel Curves NIRP

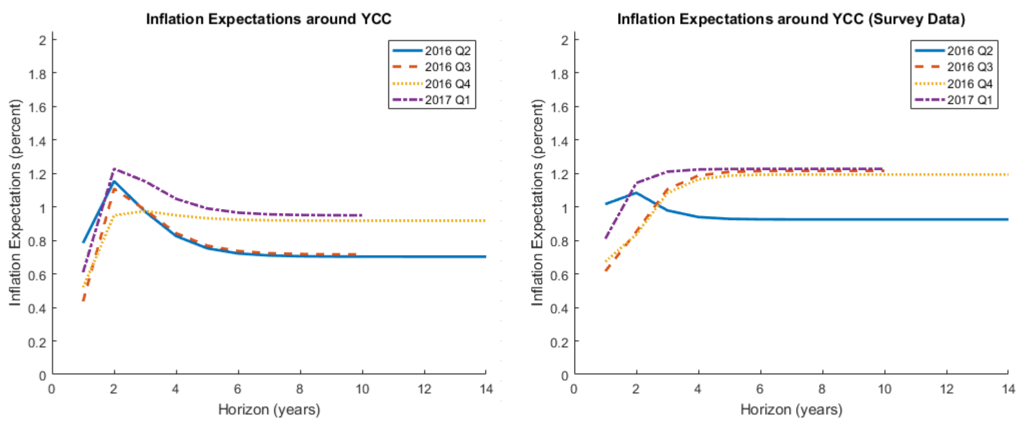


2.5.4 Yield Curve Control

The Bank of Japan Monetary Policy meeting of September 2016 introduced yet another framework for the conduct of monetary policy, in light of the Japanese economy's inability to reach the price stability target of 2 percent despite two rounds of QQE and the recent introduction of negative interest rates. The Bank of Japan decided to introduce two additional tools: "QQE with Yield Curve Control" (YCC), and a commitment to inflation overshooting where the Bank will expand the monetary base until inflation exceeds the price stability target. The BOJ decided to conduct market operations and asset purchases in a more flexible manner in order to enhance the sustainability of QQE with Yield Curve Control (see International Monetary Fund (2018a)). The overshooting commitment serves two purposes: 1) this policy is consistent with theoretical papers that conclude central banks should commit to keep rates lower following an effective lower bound episode than it otherwise would; and 2) the overshooting commitment signals to agents in the economy that the Bank of Japan has not abandoned the fight against deflation. Yield Curve Control, on the other hand, changes the targeting framework of the central bank when it comes to monetary easing through asset purchases. It can interpreted as a change from a quantity target (80 trillion in purchases per year) to a price target (the Japanese Government Bond yield itself). Following the announcement, inflation expectations in the short run moved as seen in Figure

2.8, increasing at short horizons by Q1 2017. Long run expectations, on the other hand, showed little movement and seem to be fixed at around 1.2 percent.

Figure 2.8: Japan: Nelson-Siegel Curves YCC



2.5.5 Parameter Estimates

Parameter estimates for each of the episodes are provided in Table 2.2. The first column denotes a dataset using surveys as well as financial market data, while the second column displays our results using only survey data. Note the shift in long run inflation expectations for each episode by examining the changes in the Level parameter. The Slope parameter indicates an expectation of generally increasing inflation when positive. For example, by 2013Q4 the Level component had increased to 1.405 percent, and the Slope component indicated expectations for generally increasing inflation. Curvature parameters are estimated and reported, but they are mostly not statistically significant.

2.5.6 Level Parameter

Figure 2.9 plots the evolution of the Level estimate against the percent change in the headline CPI. Additional graphs showing this estimate versus CPI Core and CPI Core-Core are provided as well (see Figure 2.10).

Table 2.2: Parameter Estimates

	Date	Financial + Survey			Survey		
		Level	Slope	Curvature	Level	Slope	Curvature
QQE1	2012 Q4	0.538	-1.227	1.654	0.658	-1.663	2.240
		(0.062)	(0.409)	(0.634)	(0.060)	(0.561)	(1.051)
	2013 Q2	1.003	0.399	-1.045	0.942	1.404	-1.563
		(0.068)	(0.442)	(0.688)	(0.077)	(0.693)	(1.303)
	2013 Q4	1.275	1.462	-0.686	1.405	0.932	-1.578
		(0.076)	(0.488)	(0.760)	(0.059)	(0.537)	(1.010)
	2014 Q2	1.471	0.489	0.238	1.622	-0.471	0.084
		(0.081)	(0.446)	(0.753)	(0.039)	(0.302)	(0.641)
QQE2	2014 Q2	1.471	0.489	0.238	1.622	-0.471	0.084
		(0.081)	(0.446)	(0.753)	(0.039)	(0.302)	(0.641)
	2014 Q3	1.376	-0.145	1.127	1.578	-0.200	1.002
		(0.066)	(0.357)	(0.659)	(0.055)	(0.396)	(0.905)
	2014 Q4	1.072	0.153	0.567	1.149	0.308	0.475
		(0.092)	(0.507)	(0.857)	(0.119)	(0.916)	(1.942)
	2015 Q1	1.078	-1.327	2.976	1.483	-1.327	2.596
		(0.112)	(0.606)	(1.119)	(0.032)	(0.233)	(0.533)
NIRP	2015 Q4	1.048	-1.581	2.969	1.303	-2.362	4.313
		(0.075)	(0.404)	(0.746)	(0.048)	(0.345)	(0.788)
	2016 Q1	0.871	-0.205	1.511	1.242	0.364	-0.029
		(0.122)	(0.658)	(1.217)	(0.102)	(0.725)	(1.657)
	2016 Q2	0.703	-0.390	1.386	0.925	-0.091	0.796
		(0.084)	(0.495)	(0.825)	(0.065)	(0.545)	(1.139)
	2016 Q3	0.716	-1.072	1.556	1.215	-0.749	-1.430
		(0.133)	(0.719)	(1.329)	(0.037)	(0.263)	(0.601)
YCC	2016 Q2	0.703	-0.390	1.386	0.925	-0.091	0.796
		(0.084)	(0.495)	(0.825)	(0.065)	(0.545)	(1.139)
	2016 Q3	0.716	-1.072	1.556	1.215	-0.749	-1.430
		(0.133)	(0.719)	(1.329)	(0.037)	(0.263)	(0.601)
	2016 Q4	0.918	-0.872	0.502	1.194	-0.568	-1.478
		(0.090)	(0.530)	(0.884)	(0.033)	(0.279)	(0.585)
	2017 Q1	0.950	-1.056	1.246	1.227	-0.856	-0.051
		(0.089)	(0.482)	(0.891)	(0.077)	(0.552)	(1.261)

Source: Authors' calculations. Standard errors in parentheses.

The entire expectation yield curve has shifted slightly higher in the past few years, following the implementation of QQE1. More importantly, toward the end of the sample, notice that there is very little movement in the Level factor despite relatively large movements in the CPI, which would suggest long run expectations are becoming less sensitive to current

Figure 2.9: Evolution of Level Parameter

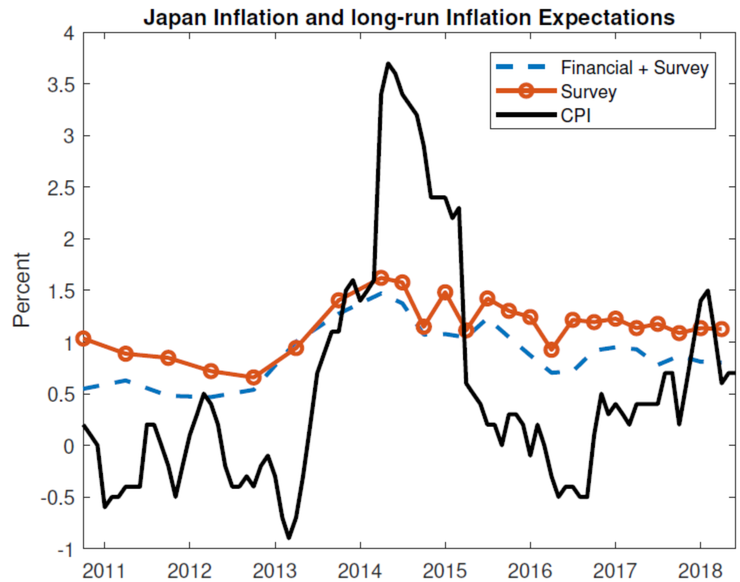
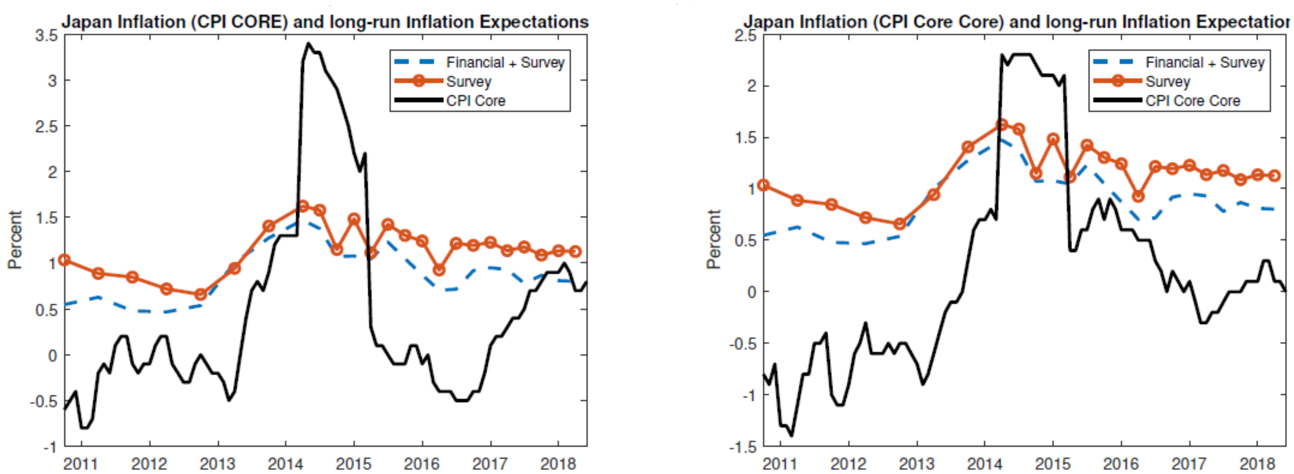
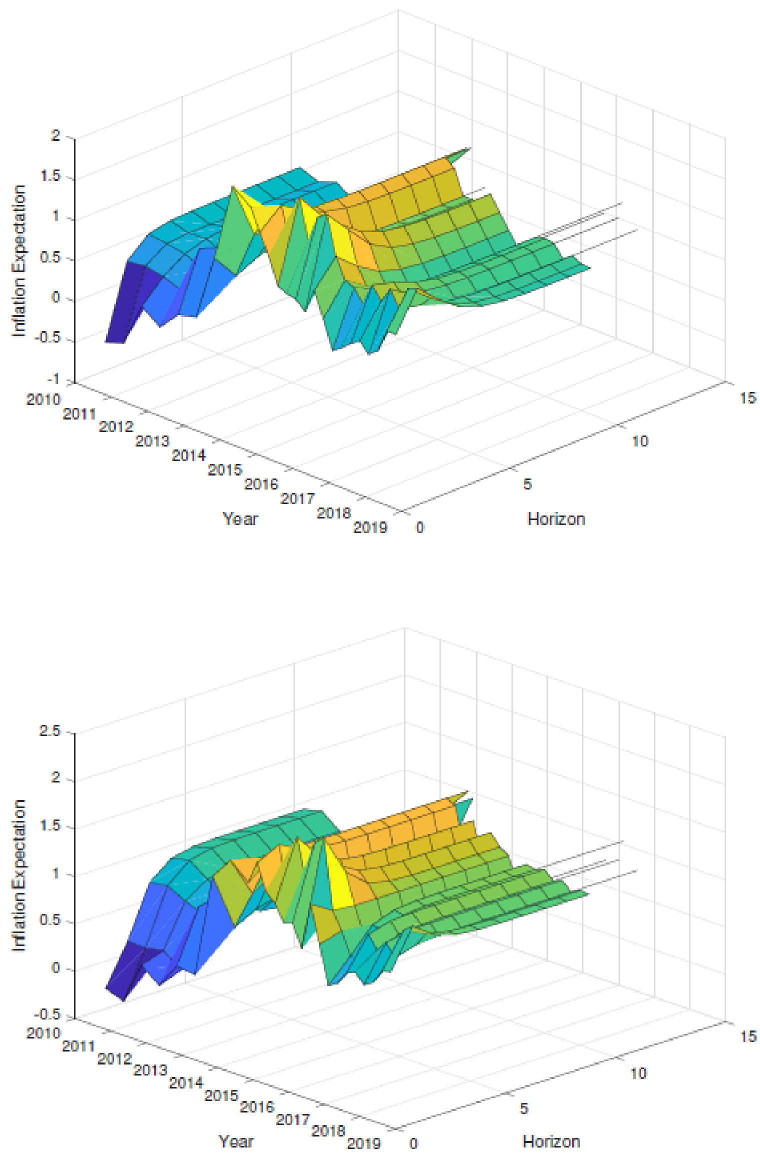


Figure 2.10: Evolution of Level Parameter, Core Inflation



price movements. This suggests that inflation expectations may be beginning to anchor, but anchoring themselves at around 1 percent rather than the Bank of Japan's 2 percent target. The same result shows up in the three dimensional plots of Nelson-Siegel curves for all horizons across all t (see Figure 2.11). In particular, the shape and level of the curves for the survey data set not only show a shift in the level, but also more stable curves at longer horizons both in shape (as represented by the slope of the curves across horizons and time) and level (as represented by the more uniform color).

Figure 2.11: Three-Dimensional Nelson-Siegel Curves



Note: Colors represent a heat map, warmer colors are higher inflation expectations.

2.6 Robustness

2.6.1 Regression

The above results suggest that the monetary policy actions of the Bank of Japan have had an effect on inflation expectations. These effects can be seen both in financial markets, as well as by firms and forecasters through their survey responses. However, these results could be capturing the fact that changes in inflation expectations may be driven by changes in past and current inflation. To address this, we follow the procedure from Lewis and McDermott (2016) and regress the Level estimate from the Nelson-Siegel curves on a constant, lagged Level, dummy variables for each episode, and lagged measures of inflation to control for the impact of past inflation. The least-squares regression is of the following form:

$$\hat{\beta}_{1t} = \alpha_0 + \hat{\beta}_{1t-1} + \sum_{k=0}^{2} \gamma_k \pi_{t-k*4} + \sum_{k=1}^{4} \delta_k D_k + \epsilon_t \tag{2.3}$$

where we obtain $\hat{\beta}_{1t}$ from the estimate of the Level from the Nelson-Siegel curves at each time t, π_t annual inflation at t, and the parameters δ_k show the marginal impact of monetary policy events relative to the preceding period. For the dummy variables, D_1 =1 if $t \geq 2013Q2$ and 0 otherwise (QQE1), D_2 =1 if $t \geq 2014Q3$ and 0 otherwise (QQE2), D_3 =1 if $t \geq 2016Q1$ and 0 otherwise (QQE1), D_4 =1 if $t \geq 2016Q3$ and 0 otherwise (QQE1), with ϵ as the regression residual. The results from this regression, for both data sets, can be found in Tables 2.3 and 2.4.[12]

Table 2.3: Impact of Lag Level and Past Inflation on Inflation Expectations

	Financial + Survey		Survey	
	Estimate	SE	Estimate	SE
(Intercept)	0.363**	0.142	1.099***	0.272
Level (lagged)	0.533**	0.217	0.062	0.255
Inflation	0.062	0.047	0.173***	0.034
Inflation (lagged 1 year)	0.023	0.032	0.058**	0.024
Inflation (lagged 2 years)	0.013	0.025	0.069***	0.018
R^2	0.667		0.65	

HAC standard errors & covariance, Bartlett kernel with bandwidth 4.

These results show that the effect of lagged Level is statistically significant for the data

[12] Appendix B contains tables for alternative dummy specifications, and different measures of inflation.

Table 2.4: Impact of Monetary Policy on Inflation Expectations

	Financial + Survey		Survey	
	Estimate	SE	Estimate	SE
(Intercept)	0.334***	0.089	0.831***	0.180
Level (lagged)	0.441***	0.113	0.062	0.156
Inflation	0.035	0.023	0.111**	0.050
Inflation (lagged 1 year)	0.037	0.027	0.034	0.067
Inflation (lagged 2 years)	0.030***	0.005	0.054***	0.016
QQE1	0.469***	0.015	0.302***	0.091
QQE2	-0.331***	0.054	-0.130	0.103
NIRP	-0.228***	0.049	-0.156**	0.056
YCC	0.184***	0.026	0.136	0.085
R^2	0.925		0.757	

HAC standard errors & covariance, Bartlett kernel with bandwidth 4.

set which includes financial data, but not for the data set with only survey data. In contrast, the effect of current inflation is statistically significant for the survey-only data set, but not the including-financial data set, while the effect of inflation lagged 2 years is statistically significant for both, albeit with small effects. In agreement with the literature, this seems to suggest that inflation expectations have an adaptive adjustment to past inflation, in particular if we look at only survey data. Importantly for both data sets, the estimated coefficient for the dummy variable for the 2013 inflation target announcement is highly statistically significant, with a large coefficient for both data sets, and a very large t-statistic for the including-financial data set. The results suggest that long run inflation expectations increased by 0.302 percentage points when the Bank of Japan introduced the QQE1 framework in April 2013. Using the including-financial data, the regression results suggest that long run inflation expectations increased by 0.469 percent when the Bank of Japan introduced QQE1. For the including-financial data set, all episodes of monetary policy had statistically significant effects on long run inflation expectations, while only QQE1 and NIRP had significant effects when using the set of survey-only data. Surprisingly, the coefficient on the dummy variable for NIRP is negative and statistically significant for both data sets. This would imply that the introduction of negative interest rates lowered inflation expectations at long horizons, with a stronger effect on inflation expectations if the views of financial markets are included. However, note that this is a dummy variable from 2015Q4 and onwards, and thus the identification is not strong. Lower energy prices and an appreciating yen led to lower inflation through 2016 (International Monetary Fund, 2017), which is most likely being picked up by this regression (if one considers the adaptive nature of Japanese inflation expectations) rather than a causal effect of NIRP.

2.6.2 Additional Measures of Inflation

In order to further complement these results, we construct additional metrics aimed at determining the degree of anchoring of inflation expectations in Japan. This is similar to work found in Chapter 3 of the *World Economic Outlook* (International Monetary Fund, 2018) on inflation expectations.

Data

For this section, we use only data from Consensus Economics, particularly for horizons between three and seven years ahead. As before, these surveys are available quarterly for part of the sample, and biannually prior to 2013. The choice of three to seven years ahead is to capture beliefs about inflation in the medium to long term, while omitting transitory shocks that could show up in short term forecast. The inflation target for Japan is fixed at 2 percent beginning February 2013, while the one-year moving average of the ten-year ahead inflation forecasts is used as a proxy prior to the target announcement. Three different periods will be analyzed, using four different measures of dispersion and anchoring of inflation expectations.

Root Mean Square Deviation from Target

The root mean square deviation of the mean inflation forecast with horizon h from the announced inflation target of 2 percent over period τ is:

$$\sqrt{\frac{1}{T}\sum_{t=1}^{T}(\pi_t^{e,h} - \pi^*)^2}; t \in \tau \tag{2.4}$$

Since we are looking at horizons between 3 and 7 years, $h = 3, ..., 7$. The length of the episodes being analyzed is represented by τ. The inflation target is denoted by π^*or the aforementioned one year moving average of ten year ahead forecasts ($\pi_t^{e,10}$). The lower the root mean square deviation (RMSD) from target is, the higher the degree of anchoring. If expectations are well anchored around the target, then beliefs about future inflation from survey data should be close to the target itself (Demertzis et al., 2012).

Standard Deviation of Mean Inflation Forecasts

If agents have large revisions of their long term forecasts, then it is clear that their expectations are not well anchored to the target. Inflation expectations that are anchored to the target would result in stable forecasts over time (Kumar et al., 2015). A good way to measure this would be to look at the standard deviation of the mean inflation forecast at different horizons h over period τ. It is given by:

$$\sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(\pi_t^{e,h} - \bar{\pi}^{e,h})^2}; t \in \tau \tag{2.5}$$

As before, $h = 3, ..., 7$ and τ represents the length of each period. The average of mean inflation forecast over τ is given by $\bar{\pi}^{e,h}$. A lower standard deviation of mean (SDM) inflation forecast denotes better-anchored expectations.

Dispersion of Inflation Forecasts

Forecasters should not have many disagreements about long run inflation if expectations are well anchored, and as such their forecasts should have little dispersion (Ehrmann, 2015; Kumar et al., 2015). We can capture this dispersion by the standard deviation of h-years ahead inflation forecasts for individual respondents of the survey, at each t and averaged over τ. This can be given by:

$$\frac{1}{T}\sum_{t=1}^{T}\left(\sqrt{\frac{1}{J-1}\sum_{j=1}^{J}(\pi_{j,t}^{e,h} - \pi_{t}^{e,h})^2}\right); t \in \tau \tag{2.6}$$

In this case, $\pi_{j,t}^{e,h}$ refers to the inflation forecast of agent j at time t for horizon h, with $\pi_{t}^{e,h}$ as the average across forecasters. A lower value for this metric denotes lower dispersion, signaling better anchoring of inflation expectations.

Sensitivity to Unexpected Inflation

Inflations that are well anchored should show little co-movement between long-term and short-term expectations. Inflation shocks would be then treated as transitory, without having much of an effect on the anchor. This is another way of thinking about expectations that are backward looking versus forward looking. We can estimate the sensitivity of h-years ahead inflation forecasts to short term forecasts by running the following least-squares regression over τ:

$$\Delta\pi_t^{e,h} = \alpha + \beta^h \Delta\pi_t^{e,1} + \varepsilon_t; t \in \tau \tag{2.7}$$

The parameter β^h denotes the sensitivity of h year ahead inflation forecasts to short term forecasts, with $\Delta\pi$ denoting the change in mean inflation forecasts between $t-1$ and t. Lower sensitivity points to better anchored expectations, as this signals that short-term price fluctuations are not affecting the beliefs about long-run price dynamics.

2.6.3 Results

As mentioned above, lower values for these metrics denotes better-anchored inflation expectations. These measures are computed using forecasts between three and seven years ahead, with the highest value being reported across horizons (the lowest degree of anchoring)[13] . In Table 2.5, we report the values found for each of these measures, as well as a simple average of all four measures which we refer to as an Anchoring Indicator:

Table 2.5: Additional Measures of Anchoring - Pre-crisis and Abenomics

Period	RMSD	SDM	Dispersion	Sensitivity	Anchoring Indicator
2000Q1 - 2008Q3	0.762	0.704	0.834	0.885	0.796
2012Q4 - 2018Q2	0.856	0.422	0.594	0.255	0.532

Table 2.6: Additional Measures of Anchoring - QQE and NIRP/YCC

Period	RMSD	SDM	Dispersion	Sensitivity	Anchoring Indicator
2000Q1 - 2008Q3	0.762	0.704	0.834	0.885	0.796
2012Q4 - 2015Q4	0.685	0.426	0.631	0.356	0.524
2016Q1 - 2018Q2	0.986	0.267	0.561	0.233	0.512

[13]The IMF (2018) *World Economic Outlook* reports that these results are consistent across metrics, undertaking a cross country analysis of relative ranking and finding high correlations between anchoring measures.

The Anchoring Indicator shows that inflation expectations have become better anchored during the Abenomics era relative to the pre-Global Financial Crisis era, with three out of the four measures showing improvement in the form of lower values. When comparing the two QQE episodes with the NIRP/YCC episode up until Q2 of 2018, we see large improvements across three out of the four metrics, with only RMSD showing a higher value for the metric. Recall that for this metric, we are minimizing the root mean squared deviation of the mean inflation forecast from the announced inflation target of 2 percent. Given that Japanese inflation expectations are strongly backward looking, it is no surprise that the RMSD was lower during the 2012Q4 -2015Q4 period since inflation (headline CPI) was above 3.5 percent in 2014. Thus, inflation expectations were closer to the 2 percent target during this period than in the 2016Q1 - 2018Q2 period. However, if we look at the other three measures, we see a clear improvement in the anchoring of inflation expectations[14]. This result concurs with the finding from the previous section, where Japanese inflation expectations seem to be becoming better anchored.

2.7 Discussion

These results suggest that while inflation expectations have failed to converge to the Bank of Japan's 2 percent inflation target, they may be showing signs of better anchoring. This finding agrees with those of Nakazono (2016), and the Bank of Japan's Comprehensive Assessment (2016), in that the introduction of the target failed to anchor inflation expectations at 2 percent. Similarly, these results mirror the findings of Kamada et al. (2015) that, much like households, the introduction of the inflation target *improved* the anchoring of long run expectations for firms. This is also what was found by Hattori and Yetman (2017), in which they model the behavior of inflation forecasts using a decay function. They report that the weight on the estimated anchor has been increasing over time, yet remains lower than that

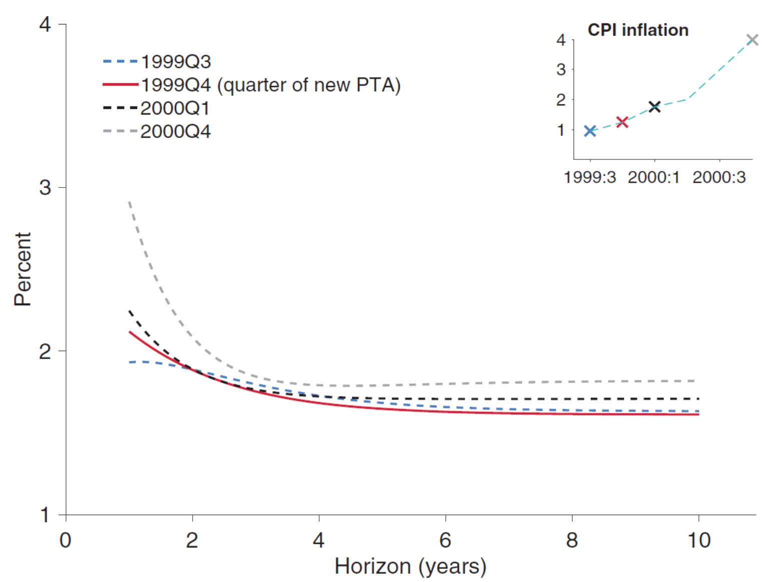
[14]If we exclude RMSD from the Anchoring Indicator, the value of the latter becomes 0.353.

found for Canada and the United States. Work by Dell'Ariccia et al. (2018) surveys recent literature on the effects of unconventional monetary policy in Japan. They find that there are positive effects on inflation from QQE1 and QQE2, with long term inflation expectations increasing to about 1 percent (a result also found by De Michelis and Iacoviello (2016)).

Another paper that finds evidence for better anchoring is a Bank of Japan paper by Hogen and Okuma (2018), which uses a learning model to jointly estimate long-term inflation expectations and their degree of anchoring over the last half century in Japan. They find estimates of inflation expectations that are largely consistent with Japanese survey data as well as the Synthesized Inflation Expectations Indicators from the Bank of Japan's Comprehensive Assessment (2016). Hogen and Okuma find that movement in inflation expectations in Japan following QQE1 was characterized by a rise up until summer 2014, then flat expectations until summer 2015, and a weakening of expectations from then on. Additionally, they find that inflation expectations rose following the introduction of the target, but have failed to anchor at the 2 percent level. They suggest that the decline in crude oil prices, weak global economic conditions, and markups in domestic goods markets have all exerted downward pressure on inflation expectations. A recent paper by Christensen and Spiegel (2019) takes a thorough approach to exploiting information contained in prices for inflation indexed Japanese bonds, accounting for the value of the deflation protection option embedded in Japanese inflation-indexed bonds issues since 2013. They find that Japanese option-adjusted break even inflation is lower than expected inflation as reported in survey data, which agrees with our result that long-run inflation expectations are lower when using a data set that incorporates financial data.

The introduction of QQE1 had the most significant and strongest effect on inflation expectations in Japan in recent years, according to the regression results. This is evident from the change in the numerical Level component, the shift of inflation expectations curves in the graphs, and the strong results from the dummy variable included in the regressions. The results for New Zealand from Lewis and McDermott (2016) do not include the introduction of the inflation target, and unlike New Zealand, Japan has had only a fixed inflation target at 2 percent without any movements or a band, so it becomes difficult to compare these two particular cases. However, when comparing the rest of these results with those found for New Zealand, a couple of items do stand out. First, the Reserve Bank of New Zealand has been successful in anchoring long run expectations at 2 percent, as the Level component from the Lewis and McDermott estimation is around this value for the different dates they investigate. Figure 2.12 shows the evolution of Nelson-Siegel curves for inflation expectations in New Zealand after the introduction of the 1999 Policy Target Agreement, as estimated by Lewis and McDermott (2016). This announcement by the Reserve Bank of New Zealand widened the inflation target band to 0%-3%, up from 0%-2%. The estimated inflation expectations curve around this announcement shows that the long-term anchor for inflation expectations shifts upwards as inflation increases through the period.

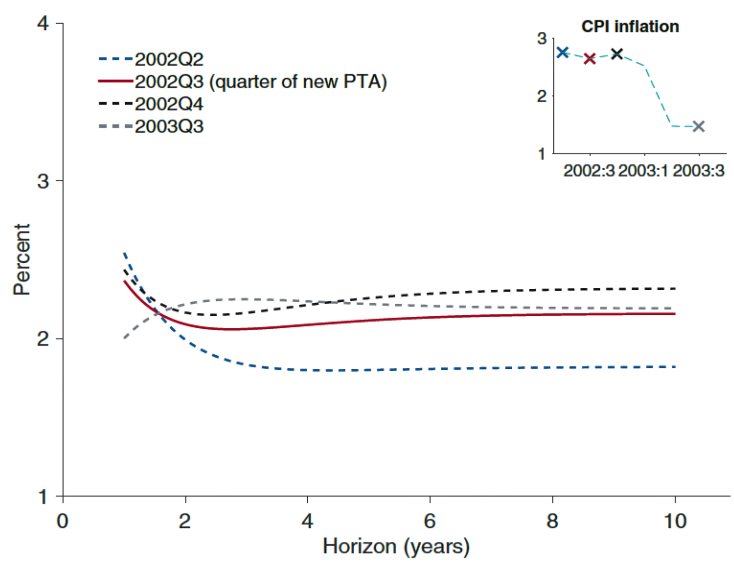
Figure 2.12: Nelson-Siegel Curves in New Zealand after 1999 Policy Target Agreement



Source: Lewis and McDermott (2016)

The next policy announcement from the Reserve Bank of New Zealand came in September 2002, where the Bank narrowed the target band to 1%-3% while also allowing more flexibility over the target horizon. This numerical change to the band both narrowed the tolerance interval for what was deemed satisfactory inflation, and also raised the midpoint of the target band. Figure 2.13 shows the evolution of Nelson-Siegel curves for inflation expectations in New Zealand after the introduction of the 2002 Policy Target Agreement, as estimated by Lewis and McDermott (2016). Inflation expectations in New Zealand increased, particularly at long-horizons, following the announcement. The estimates show that the Level component jumped 0.34 percentage points in the quarter the new Policy Target Agreement was introduced.

Figure 2.13: Nelson-Siegel Curves in New Zealand after 2002 Policy Target Agreement



Source: Lewis and McDermott (2016)

Using Nelson Siegel curves to model inflation expectations has proved quite useful in the case of New Zealand. One can infer that monetary policy announcements by the Reserve Bank of New Zealand have been successful at both changing long run inflation expectations (as shown by the movement in long term expectations after the 2002 PTA) and anchoring them at their target (as evidenced in the invariance of this Level component even when short run expectations are affected by developments in the economy). In contrast, Japan seems to have successfully anchored inflation expectations at around 1 percent rather the inflation target of 2 percent. It also seems to be the case for Japan that the anchoring of expectations is improving, as while the inflation expectations curves exhibit movement at short horizons following the last two policy announcements (NIRP and YCC), the curves seem to be invariant at long horizons, which is similar to the case of New Zealand. Second, when comparing the regression results from this paper and Lewis and McDermott (2016), there are differences depending on which data set is used for Japan. When using the mixed data set of financials and survey data, the coefficient on the lagged level is statistically significant at the 1 percent level, which is similar to the result found for New Zealand. However, current and lagged inflation were not found to be statistically significant. In contrast, the survey-only data set has no statistically significant coefficient on the lagged level, while current inflation and inflation lagged 2 years are found to be significant, signaling that inflation expectations adjust adaptively in Japan, unlike New Zealand.

2.8 Concluding Remarks

Using Nelson-Siegel curves adapted for use in measuring inflation expectations, this paper examines the behavior of long run inflation expectations in Japan following several recent episodes of unconventional monetary policy actions taken by the Bank of Japan. The first episode of Quantitative and Qualitative Easing (which began in early 2013) was effective in raising inflation expectations, but the results for later policy initiatives have been more mixed. Our results are robust to different data specifications, and different methods for measuring the anchoring of inflation expectations. The results also indicate that inflation expectations in Japan may be experiencing a greater degree of anchoring at around the 1 percent level.

Chapter 3

Interest Rate Rules and Indeterminacy in an Estimated Open Economy Model

3.1 Background

In an increasingly interconnected world, macroeconomic stability has become a primary concern for central banks in emerging and developed open economies. Policy makers in small countries pay close attention to the international transmission of structural shocks. In an open economy setting, the way these shocks propagate depends not only on monetary policy, but also on openness to the rest of the world. During the past decade, open economy DSGE models building off of Gali and Monacelli (2005) have become a benchmark for this type of analysis in international macroeconomics. These studies mainly employ New Keynesian models and typically assume that the equilibrium of the model is unique. However, open economy considerations interact with the conditions that ensure a unique equilibrium, such that what resembles "good" monetary policy in a closed economy setting may lead to indeterminacy of equilibria in an open one. In addition, a small open economy can become susceptible to indeterminacy due to monetary policy in the foreign economy, as the joint actions of policymakers impinge on the determinacy of equilibrium in a two country setting.

Indeterminate equilibria pose a problem to policy makers for a number of reasons. First, the economy becomes subject to non-fundamental shocks (so-called 'sunspot' shocks), which generate an additional source of extrinsic uncertainty and can bring large welfare losses. Second, the propagation of structural shocks changes when the equilibrium is not unique, as these now depend on self-fulfilling expectations that generate additional persistence. But traditional estimation approaches assume, *a priori*, that these type of equilibria can't occur and, as a result, the economy is never subject to self-fulfilling beliefs or extrinsic uncertainty. In this paper, I use an estimated small open economy model with potential indeterminacy to consider important questions, such as: How can monetary policy in a small open economy create self-fulfilling expectations? Can self-fulfilling expectations be a driving force of business cycles in open economies? Do sunspot shocks play a role in the propagation of shocks across borders?

To answer these questions, this paper examines empirically whether the two-country equilibrium is determinate or not using a small open economy model for the U.S. and Canada, employing Bayesian methods and a methodology that allows for the solution and estimation of the model under indeterminacy. In open economy models, a number of papers have established that the conditions for equilibrium determinacy are different than in the closed economy case[1]. Bullard and Schaling (2009) examine the determinacy conditions in a two country framework representing the U.S. and Europe, and how monetary policy choices in-

[1] In a classic sticky-price New Keynesian model of a closed economy, indeterminacy can emerge from both passive and aggressive central bank responses to inflation. If central bank policy is passive, interest rates respond less than one for one to inflation, causing real rates to fall which increases demand and turns any exogenous expected inflation into a self-fulfilling belief. On the other hand, indeterminacy is also a possibility if the policy rule responds too aggressively and produces cycles of positive and negative inflation (Giannoni and Woodford (2002)).

teract across borders to affect these conditions. They find that open economy considerations alter the conditions for a unique rational expectations equilibrium. Llosa and Tuesta (2008) evaluate under which conditions different Taylor rules lead to determinacy in the small open economy of Gali and Monacelli (2005), showing that the degree of openness, elasticity of substitution between home and foreign goods, and the form of Taylor rule all interact to affect the region of determinacy.

Benchmark Model

I pursue this analysis using a benchmark small open economy framework proposed by Justiniano and Preston (2010b), which extends Gali and Monacelli (2005) by introducing habits in consumption, price indexation, incomplete asset markets, and a large set of structural shocks. The model is closed with a contemporaneous Taylor (1999) rule that responds to inflation, output, output growth, and nominal exchange rates. In this model, the region of determinacy can shrink if the central bank targets consumer CPI and responds strongly to exchange rates.

Estimations of open economy models typically do not allow for the possibility of a non unique equilibrium, as the algorithm will discard the parameter draws that lead to indeterminacy[2]. However, this assumption disregards feasible parameter values for the monetary policy response. In order to consider these feasible values for the Taylor rule when the equilibrium is indeterminate, I use the methodology developed in Bianchi and Nicolò (2017)[3].

My empirical analysis shows that, in this benchmark case, the equilibrium is determinate, suggesting that self-fulfilling expectations and extrinsic uncertainty play no role in driving

[2]For closed economy estimations, see Smets and Wouters (2007), Arias et al. (2014). For open economy estimations, see Elekdag et al. (2006), Adolfson et al. (2007), Justiniano and Preston (2010b)

[3]This approach is equivalent to previous papers in the literature that generate a model solution under indeterminacy, such as Lubik and Schorfheide (2004) and Farmer et al. (2015), and has recently been used to study models with long run stagnation (Cuba-Borda and Singh (2018)), New Keynesian models with asset bubbles (Bianchi and Nicolò (2017)), and a New Keynesian model with bounded rationality (Ilabaca et al. (2019)).

the open economy. I find that the data prefer a unique equilibrium with a posterior model probability of 96%. To dig deeper into these findings, I also consider two extensions. First, I focus on the inflation targeting period for Canada and I explore whether the result holds under an inflation forecast based rule for monetary policy. Second, I introduce agents with heterogeneous expectations into the benchmark model.

Inflation Forecast Based Rules

The motivation for considering the case of inflation forecast based rules is twofold. First, Canada introduced inflation targeting in 1991, along with an explicit inflation forecast as the feedback variable for their policy rule. Second, as discussed in Svensson (1997) and Batini and Pearlman (2002), monetary policy typically affects variables with a lag, so it is reasonable for central banks to react in a pre-emptive way to price movements by responding to changes in their inflation forecasts. Simple rules as in Taylor (1993) can be modified to allow the policy instrument to respond to deviations of expected inflation from target, in what is known as "inflation-forecast-based" rules.

Inflation targeting banks, such as Canada's, have explicitly referred to these types of simple rules in their projection models (see for example Coletti et al. (1996)). Previous literature has found that these rules can lead to equilibrium indeterminacy, and a longer forecast horizon aggravates this problem, as shown by Batini et al. (2004). I estimate the model with an inflation forecast based rule on a sub-sample that coincides with the introduction of inflation targeting in Canada, allowing for indeterminacy.

In this setting, my benchmark result for determinacy reverses, as the data prefer a model that is consistent with an inflation forecast based rule, self-fulfilling expectations, and sunspot shocks. I find that the data prefer indeterminacy with a forecast horizon of 1 quarter ahead, but the central bank is responding more than one for one to changes in their inflation forecast. This has a number of implications on the propagation mechanism of the structural shocks, as now shocks to technology and monetary policy in the domestic economy become persistently inflationary. On the other hand, preference shocks become persistently deflationary. Furthermore, the volatility of inflation is driven by sunspot fluctuations in addition to the structural disturbances.

Heterogeneous Expectations

So far, the results from the previous sections were obtained under the assumption that agents have rational expectations. However, there is substantial empirical (Branch (2004), Branch and McGough (2009), Branch and McGough (2018)) and experimental evidence (Hommes (2011)) that individuals and firms have heterogeneous expectations when engaging in intertemporal decision-making. Additionally, the literature shows that even a small share of agents with non-rational expectations can change the dynamics of the model, including the properties that lead to a determinate equilibrium. To explore these results further, I allow for heterogeneous expectations in the model.

Introducing imperfections in the way expectations are formed has solved some traditional macro puzzles in recent studies. Heterogeneous expectations can generate models that are consistent with inflation inertia, output persistence, and the forward guidance puzzle. Beqiraj et al. (2018) find that a behavioral New Keynesian model derived with heterogeneous expectations fits the data better than one with rational expectations. I relax the rational expectations assumption and allow for expectations to feature a share of rational agents as well as a share of adaptive agents, as in Branch and McGough (2018). Rather than impose the distribution of heterogeneous agents, I let the data pick the share of agents with adaptive expectations. Since the region of equilibrium determinacy now depends on both the open economy parameters and the parameters governing heterogeneous expectations, I estimate this model allowing for both determinacy and indeterminacy, using the same methodology as in the benchmark case. I find that a small open economy model with heterogeneous expectations and a determinate equilibrium fits the data better. In addition, I find that this specification can better account for the influence of foreign shocks than the benchmark model when compared to the results from a Bayesian VAR. Under heterogeneous expectations, a shock to foreign output is persistently expansionary in the domestic economy, and doesn't generate a rise in inflation as in the benchmark case.

Related Literature

This paper contributes to different strands of the literature. First, it contributes to the literature that studies the empirical implications of dynamic indeterminacy. Work by Benhabib and Farmer (1999), Lubik and Schorfheide (2004), Farmer et al. (2015), and Bianchi and Nicolò (2017) provide the methodology to solve and estimate models with an indeterminate equilibrium. Closed economy applications of these methods include Bhattarai et al. (2016), Cuba-Borda and Singh (2018), Hirose et al. (2019), and Ilabaca et al. (2019). A recent survey by Farmer (2019) surveys a subset of the macroeconomics literature that adopts the existence of multiple equilibria. The present paper extends this type of analysis to open economy models where the equilibrium indeterminacy can be generated by interactions between the domestic and foreign economy.

Second, this paper contributes to the literature that analyzes these conditions for equilibrium indeterminacy for small open economy models, such as the work by Llosa and Tuesta (2008), Bullard and Schaling (2009), and Bullard and Singh (2008). It is also related to work that analyzes the conditions of determinacy for open economies with inflation forecast based rules, such as Batini et al. (2004), De Fiore and Liu (2002), Zanna (2003), and Buffie et al. (2018). The present paper finds the numerical region of equilibrium determinacy in a medium-scale open economy model, as well as one featuring heterogeneous expectations.

Third, this paper contributes to the literature on empirical small open economy models, and analyzes the role of sunspot shocks and self-fulfilling expectations in the propagation of structural shocks in an open economy model estimated using Bayesian methods. Previous studies in this literature, such as Justiniano and Preston (2010a), Elekdag et al. (2006), and Adolfson et al. (2007) don't allow for the possibility of equilibrium indeterminacy, and as such self-fulfilling expectations don't play a role. The closest papers in this regard are Hirose (2013) and Zheng and Guo (2013). Hirose (2013) estimates a two-country open economy version for the U.S. and the Euro area (following Gali and Monacelli (2005)) to examine empirically whether the worldwide equilibrium is determinate or not. Zheng and Guo (2013) estimate a simplified version of Gali and Monacelli (2005) on data for China and the rest of the world while allowing for indeterminacy. The present paper differs from these two in that the model features a larger set of disturbances, along with habits in consumption, incomplete asset markets, and price indexation.

Fourth, this paper contributes to the literature on heterogeneous expectations. Following Branch and McGough (2018), a share agents have rational expectations while the rest form adaptive expectations. Other papers in this literature have employed this approach to expectations and their role in optimal monetary policy, such as Gasteiger (2014), and Di Bartolomeo et al. (2016). I estimate the model using Bayesian methods, as in Beqiraj et al. (2018), in an open economy setting with heterogeneous expectations. I find that, as in the closed economy case, heterogeneous expectations provide a better fit to the data. This is also consistent with evidence found in Cole and Milani (2017) in a DSGE-VAR approach.

3.2 Model

This section outlines the key structural equations of the medium-scale small open-economy model. This framework follows Justiniano and Preston (2010b), which is an extension to the model proposed by Gali and Monacelli (2005), in which a small and a large country both specialize in the production of a continuum of goods subject to imperfect competition and price rigidities. Imports are subject to local currency pricing through a retail sector providing distribution services, which gives rise to deviations from the law of one price. The model allows for incomplete asset markets, a large set of structural disturbances, and rigidities such as habit formation, and indexation of prices to past inflation in the domestic economy. The disturbances in the model have been found crucial in taking closed-economy models to the data as documented in Smets and Wouters (2007), and Christiano et al. (2005).

3.2.1 Households

Assume households maximize their utility, given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \tilde{\varepsilon}_{g,t} \left[\frac{(C_t - H_t)^{(1-\sigma)}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right]$$

where N_t is labor input, $H_t \equiv hC_{t-1}$ are external habits assumed to be exogenous, $\sigma, \varphi > 0$ refer to inverse elasticities of intertemporal substitution and labor supply, respectively, and $\tilde{\varepsilon}_{g,t}$ is a preference shock. C_t represents the composite consumption index:

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

where $C_{H,t}$ and $C_{F,t}$ are Dixit-Stiglitz aggregates of the available domestic and foreign produced goods; α refers to the share of foreign goods in the domestic consumption bundle; $\eta > 0$ is the elasticity of substitution between domestic and foreign goods, with $\varepsilon > 1$ the elasticity of substitution between types of differentiated domestic or foreign goods in the Dixit-Stiglitz aggregates. The only available assets for the households are one-period domestic and foreign bonds. Households maximize their utility subject to the flow budget constraint:

$$P_t C_t + D_t + e_t B_t = D_{t-1}(1 + \tilde{\imath}_{t-1}) + e_{t-1} B_{t-1}(1 + \tilde{\imath}^*_{t-1}) \phi_t(A_t) + W_t N_t + \Pi_{H,t} + \Pi_{F,t} + T_t$$

for all $t > 0$, where D_t denotes the household's holdings of one-period domestic bonds, and B_t denotes holdings of one-period foreign bonds with corresponding interest rates $\tilde{\imath}_t$ and $\tilde{\imath}^*_t$. The nominal exchange rate is $\tilde{e}_t$. P_t, $P_{H,t}$, $P_{F,t}$ and P^*_t correspond to the domestic CPI, the domestic goods prices, the domestic currency price of imported goods, and the foreign price respectively. Wages W_t are earned on labor supplied and $\Pi_{H,t}$ and $\Pi_{F,t}$ denote profits from holding shares in domestic and imported goods firms. T_t denotes lump-sum taxes and transfers by the government. Real quantity of outstanding foreign debt

$$A_t \equiv \frac{\tilde{e}_{t-1} B_{t-1}}{\bar{T} P_{t-1}}$$

is expressed in terms of domestic currency as a fraction of steady state output. Along with a risk premium shock $\tilde{\phi}_t$, the function for debt elastic interest rate premium is given by

$$\phi_t = exp[-\chi(A_t + \tilde{\phi}_t)]$$

This functional form ensures stationarity of the foreign debt level in a log-linear approximation to the model as in Schmitt-Grohé and Uribe (2003).

In order to impose complete markets within the domestic economy, I assume that all households in the domestic economy receive an equal fraction of both domestic and retail firm profits. Thus, nominal income in each period is $W_t N_t + \Pi_{H,t} + \Pi_{F,t}$, equaling $P_{H,t} Y_{H,t} + (P_{F,t} - \tilde{e}_t P^*_t) C_{F,t}$ for all households in equilibrium. This assumption also guarantees that households choose identical state-contingent plans for consumption, as they face identical decision problems. The optimization problem for the household requires allocation of expenditures across all types of goods, both foreign and domestic, as well as intertemporally and intratemporally. This yields the following set of optimality conditions:

$$C_{H,t}(i) = \big(\frac{P_{H,t}(i)}{P_{H,t}}\big)^{-\theta} C_{H,t}, \forall i$$
$$C_{F,t}(i) = \big(\frac{P_{F,t}(i)}{P_{F,t}}\big)^{-\theta} C_{F,t}, \forall i$$

with associated price indexes for domestic and foreign consumption bundles given by $P_{H,t}$ and $P_{F,t}$. Optimal allocation of expenditure across domestic and foreign goods implies the demand functions:

$$C_{j,t} = (1-\alpha)\big(\frac{P_{j,t}}{P_t}\big)^{-\eta} C_t, j \in [H, F] \tag{3.1}$$

where $P_t = [(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}]^{\frac{1}{1-\eta}}$ is the consumer price index. Allocation of expenditures on the aggregate consumption bundle, and optimal labor supply satisfy

$$\lambda_t = \tilde{\varepsilon}_{g,t}(C_t - H_t)^{-\frac{1}{\sigma}} \tag{3.2}$$
$$\lambda_t = \tilde{\varepsilon}_{g,t} P_t N_t^{\varphi} / W_t \tag{3.3}$$

with portfolio allocation determined by optimality conditions

$$\lambda_t \tilde{e}_t P_t = E_t[(1+\tilde{\imath}^*)\beta\phi_{t+1}\lambda_{t+1}\tilde{e}_{t+1}P_{t+1}] \tag{3.4}$$
$$\lambda_t P_t = E_t[(1+\tilde{\imath})\beta\lambda_{t+1}P_{t+1}] \tag{3.5}$$

for Lagrange multiplier λ_t. Combining (3.4) with (3.2) gives the usual Euler equation in consumption.

3.2.2 Domestic Producers

There is a continuum of monopolistically competitive domestic firms producing differentiated goods. Calvo-style price setting is assumed, allowing for indexation to past domestic goods' price inflation. In any period t, a fraction $1-\theta_H$ of firms set price optimally, while a fraction $\theta_H \in (0,1)$ of goods' prices follow the indexation rule

$$logP_{H,t}(i) = logP_{H,t-1}(i) + \delta_H \pi_{H,t-1} \tag{3.6}$$

where $\delta_H \in (0,1)$ measures the degree of indexation, with $\pi_{H,t} = log(P_{H,t}/P_{H,t-1})$. Firms that reset their price do so while facing the same decision problem, and thus setting a common price $P'_{H,t}$. The Dixit-Siglitz aggregate price index evolves according to

$$P_{H,t} = \left[(1-\theta_H)P_{H,t}^{\prime(1-\varepsilon)} + \theta_H \Big(P_{H,t-1}\big(\frac{P_{H,t-1}}{P_{H,t-2}}\big)^{\delta}_{H}\Big)^{1-\varepsilon}\right]^{1/(1-\varepsilon)} \tag{3.7}$$

with firms setting prices in period t facing demand curve

$$y_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,T}}\Big(\frac{P_{H,T-1}}{P_{H,t-1}}\Big)^{\delta_H}\right)^{1-\varepsilon}(C_{H,T}+C^*_{H,T}), \forall t \tag{3.8}$$

and taking aggregate prices and consumption bundles as parametric. Good i is produced using a single labor input $N_t(i)$ with production function $y_{H,t}(i) = \tilde{\varepsilon}_{a,t}N_t(i)$, where $\tilde{\varepsilon}_{a,t}$ is an exogenous technology shock.

The firm's price-setting problem in period t is to maximize the expected present discounted value of profits subject to the demand curve (3.8). The firm's optimization problem implies the F.O.C.

$$E_t \sum_{T=t}^{\infty} \theta_H^{T-t} Q_{t,T} y_{H,T}(i)\left[P_{H,t}(i)\Big(\frac{P_{H,T-1}}{P_{H,t-1}}\Big)^{\delta_H} - \frac{\theta_H}{(\theta_H-1)}P_{H,T}MC_T\right] \tag{3.9}$$

where $MC_T = W_T/(P_{H,T}\tilde{\varepsilon}_{a,T})$ is the real marginal cost function for each firm assuming homogenous factor markets.

3.2.3 Retail Firms

Retail firms import foreign differentiated goods for which the law of one price holds at the border. In determining the domestic currency price of the imported goods, firms are assumed to be monopolistically competitive. This leads to a violation of the law of one price in the short run, due to the firms' small degree of pricing power. Retail firms face Calvo-style pricing allowing for indexation to past inflation. In any period t, a fraction $1-\theta_F$ of firms set prices optimally, while a fraction $\theta_F \in (0,1)$ of goods prices follow an indexation rule analogous to (3.6). The Dixit-Stiglitz aggregate price index consequently evolves according to

$$P_{F,t} = \left[(1-\theta_F)P_{F,t}^{'(1-\varepsilon)} + \theta_F\Big(P_{F,t-1}\big(\frac{P_{F,t-1}}{P_{F,t-2}}\big)^{\delta}_{F}\Big)^{1-\varepsilon}\right]^{1/(1-\varepsilon)} \tag{3.10}$$

with firms setting prices in period t facing demand curve

$$C_{F,T}(i) = \left(\frac{P_{F,t}(i)}{P_{F,T}}\Big(\frac{P_{F,T-1}}{P_{F,t-1}}\Big)^{\delta_F}\right)^{-\varepsilon}, \forall t \tag{3.11}$$

and taking aggregate prices and consumption bundles as parametric. The firm's price-setting problem is to maximize the expected present discounted value of profits subject to demand curve (3.11).

3.2.4 International Risk Sharing

From the asset-pricing conditions that determine domestic and foreign bond holdings, the uncovered interest rate parity condition

$$E_t \lambda_{t+1} P_{t+1}[(1+\tilde{\imath}_t) - (1+\tilde{\imath}_t^*)(\tilde{e}_{t+1}/\tilde{e}_t)\phi_{t+1}] = 0 \tag{3.12}$$

follows, placing restrictions on the relative movements of the domestic and foreign interest rate, and changes in the nominal exchange rate.

The real exchange rate is defined as $\tilde{q}_t \equiv \tilde{e}P_t^*/P_t$. Since $P_T^* = P_{F,T}^*$, when the law of one price fails to hold, we have $\tilde{\Psi}_{F,t} \equiv \tilde{e}_t P_t^*/P_{F,t} \neq 1$, called the law of one price gap in Monacelli (2005).

3.2.5 General Equilibrium

Goods market clearing requires

$$Y_{H,t} = C_{H,t} + C_{F,t}^* \tag{3.13}$$

to hold in the domestic economy. The model can be closed by assuming foreign demand for the domestically produced good is specified as

$$C_{H,t}^* = \left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\lambda} Y_t^*$$

where[4] $\lambda > 0$. Domestic debt is assumed to be in zero net supply, $D_t = 0, \forall t$. The analysis looks at the symmetric equilibrium where domestic producers and retailers set a common

[4]This demand function nests the specification in Monacelli (2005) by allowing λ to be different from η to give additional flexibility in the transmission mechanism of foreign disturbances to the domestic economy. This function for foreign demand is standard in small open-economy models.

price. Households are assumed to have identical initial wealth, ensuring symmetric consumption and portfolio decisions. Monetary policy follows a Taylor-type rule, with fiscal policy specified as a zero debt policy, with taxes equaling the subsidy required to eliminate the steady-state distortion induced by imperfect competition in the domestic and imported goods markets.

3.2.6 Log-Linearization

I employ a log-linear approximation of the optimality conditions of the model around a non-stochastic steady state. All variables in the following section are interpreted as log deviations from their steady state values. For simplicity, assume $\bar{s} = 1$.

The domestic household's Euler equation (3.5) becomes

$$c_t - hc_{t-1} = E_t(C_{t+1} - hc_t) - \frac{1}{\sigma}(1-h)(i_t - E_t\pi_{t+1}) + \frac{1}{\sigma}(1-h)(\varepsilon_{g,t} - E_t\varepsilon_{g,t+1}) \quad (3.14)$$

A log-linear approximation to the goods market clearing condition implies

$$(1-\alpha)c_t = y_t - \alpha\eta(2-\alpha)s_t - \alpha\eta\psi_{F,t} - \alpha y_t^* \quad (3.15)$$

where

$$\psi_{F,t} \equiv (e_t + p_t^*) - p_{F,t}$$

denotes the law of one price gap, i.e. the difference between world currency price and the domestic price of imports. Terms of trade are defined as $s_t = p_{F,t} - p_{H,t}$. Taking a difference of this definition implies

$$\Delta s_t = \pi_{F,t} - \pi_{H,t} \quad (3.16)$$

Equilibrium domestic consumption depends on domestic output and three sources of variation from the foreign block: terms of trade, deviations from the law of one price, and foreign output. Terms of trade and the real exchange rate are related according to

$$q_t = e_t + p_t^* - p_t = \psi_{F,t} + (1 - \alpha)s_t \tag{3.17}$$

such that the real exchange rate varies with deviations from the law of one price, and differences in consumption bundles across the domestic and foreign economies.

Taking a log-linear approximation to the optimality conditions for price setting and the price index of domestic firms implies the relation

$$\pi_{H,t} - \delta_H \pi_{H,t-1} = \frac{1 - \theta_H}{\theta_H}(1 - \theta_H \beta)mc_t + \beta E_t(\pi_{H,t+1} - \delta \pi_{H,t}) \tag{3.18}$$

where the real marginal cost function of each firm is denoted by

$$mc_t = \varphi y_t - (1 + \varphi)\varepsilon_{a,t} + \alpha s_t + \frac{\sigma}{1 - h}(c_t - hc_{t-1})$$

Domestic price inflation is thus determined by current marginal costs, expectations about inflation in the next period, and recently observed inflation[5]. Three sources of foreign variation affect domestic price inflation: direct and indirect effects of terms of trade on firms' marginal costs, foreign output, and deviations from the law of one price.

A log-linear approximation of the optimality conditions for retailer's pricing problem yields

$$\pi_{F,t} - \delta_F \pi_{F,t-1} = \frac{1 - \theta_F}{\theta_F}(1 - \theta_F \beta)\psi_{F,t} + \beta E_T(\pi_{F,t+1} - \delta \pi_{F,t}) + \varepsilon_{cp,t} \tag{3.19}$$

Inflation in the domestic currency price of imports is determined by current marginal cost conditions and expectations about next period's rate of inflation. A cost-push shock has

[5]Zero indexation implies $\delta = 0$, in which case the usual forward-looking Phillips curve appears

been added to capture the inefficient variations in mark-ups. Prices are again indexed to past inflation. Domestic CPI and home good prices follow the relationship:

$$\pi_t = \pi_{H,t} + \alpha \Delta s_t \tag{3.20}$$

CPI inflation and domestic goods' price inflation differ in that imported goods' prices deviate from domestic goods' prices, with the difference weighted by the importance of those goods in the CPI. Taking a log linear approximation of the uncovered interest rate parity condition gives

$$(i_t - E_t\pi_{t+1}) - (i_t^* - E_t\pi_{t+1}^*) = E_t\Delta q_{t+1} - \chi a_t - \phi_t \tag{3.21}$$

with the flow budget constraint

$$c_t + a_t = \beta^{-1}a_{t-1} - \alpha(s_t + \psi_{F,t}) + y_t \tag{3.22}$$

where a_t is the log real net foreign asset position as a fraction of steady state output, $a_t = log(e_t B_t/(P_t\bar{Y}))$, while the foreign economy is assumed to have a zero debt-to-GDP ratio in steady state.

A Taylor rule for monetary policy closes the model in the benchmark case:

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\{\psi_\pi \pi_t + \psi_y y_t + \psi_{\Delta y}\Delta y_t + \psi_e \Delta e_t\} + \varepsilon_{M,t} \tag{3.23}$$

The nominal interest rate is determined by lagged interest rates, and responds to inflation, output, output growth, and the change in the nominal exchange rate, with a monetary policy shock or implementation error $\varepsilon_{M,t}$.

3.2.7 Foreign Economy

The foreign economy is exogenous to the domestic economy, and it is represented by a simple 3 equation New Keynesian model with AR(1) processes for the shocks in the IS and NKPC:

$$
\begin{aligned}
y_y^* &= E_t y_{t+1}^* - \sigma^{-1}(i_t^* - E_t \pi_{t+1}^*) + g_t^* \\
\pi_t^* &= E_t \pi_{t+1}^* + \kappa^* y_t^* + u_t^* \\
i_t^* &= \rho_{i^*} i_{t-1}^* + (1-\rho_{i^*})(\psi_{\pi^*}^* \pi_t^* + \psi_{y^*}^* y_t^*) + \varepsilon_{m,t}
\end{aligned}
$$

The domestic block of the economy is given by equations (14)-(23) in the unknowns $\{c_t, y_t, i_t, q_t, s_t, \pi_t, \pi_{H,t}, \pi_{F,t}, \psi_{F,t}, a_t\}$, and the foreign economy block $\{\pi_t^*, y_t^*, i_t^*\}$. Combining this with the AR(1) processes for the domestic disturbances $\{\varepsilon_t^a, \varepsilon_t^g, \varepsilon_t^{cp}, \varepsilon_t^{rp}\}$ and exogenous disturbances $\{g_t^*, u_t^*\}$, and the definitions $\Delta s_t = s_t - s_{t-1}$ and $\Delta q_t = q_t - q_{t-1}$, we obtain a linear rational expectations model which can be solved using standard methods. In the estimation, I employ observables for the series $\{y_t, i_t, \pi_t, q_t, s_t, \pi_t^*, y_t^*, i_t^*\}$.

3.3 Estimation

Using Bayesian methods following Herbst and Schorfheide (2015), I aim to characterize the posterior distribution of the model parameters θ. Given a prior, $\pi(\theta)$, the posterior density is proportional to the product of the likelihood and the prior. Draws from the posterior are generated using a random-walk Metropolis Hastings algorithm along with the state space representation of the model and a diffuse Kalman filter. The posterior distributions come from two chains of 500,000 draws with a 50% burn-in. The scale for the proposal distribution in the MH algorithm has been calibrated through a Monte Carlo exercise to achieve an acceptance rate around 30%. I use the first four observations to initialize the Kalman filter, which excludes them from the computation of the likelihood and from the estimates.

3.3.1 Data and Priors

The discount factor β is set to 0.99, and the share of openness α has a tight prior centered around the average share of exports and imports to GDP in the sample, which equals 0.28. Following Benigno and Benigno (2001), the parameter governing the interest rate elasticity of debt χ is fixed at 0.01. The priors for the rest of the parameters are described in Table 3.2. Priors for the domestic block are set following the rationale in Justiniano and Preston (2010b), while I use priors from Herbst and Schorfheide (2015) for the 3 equation new Keynesian model. I adopt fairly loose Gamma priors with large tails for the inverse Frisch elasticity of labor supply ϕ and the elasticity of substitution between domestic and foreign goods η, considering the diverse estimates emerging from macro and micro studies in the literature. The prior for the intertemporal elasticity of substitution σ allows for values below 1 as well as substantially large estimates. Priors for the Calvo price parameters assume the presence of nominal rigidities while remaining flexible enough to accomodate values as large as those obtained in macro studies. The priors for the Taylor Rule parameters in the domestic and foreign economy are chosen following Lubik and Schorfheide (2007). Priors for the habit and indexation parameters are flat so as to allow both large and small values of persistence, since these dynamics could be coming from the foreign block. The autoregressive parameter on the exogenous stochastic disturbances (risk premium, technology, preferences, and import cost-push) has a beta prior with a mean of 0.8, which assumes a fairly persistent process for these shocks. The priors for the standard deviation of the shocks are all inverse gamma, with mean and standard deviation set following Justiniano and Preston (2010a).

The estimation uses quarterly data on output, inflation, interest rates, the real exchange rate and the terms of trade for the United States and Canada in a sample[6] covering 1982Q1

[6]The sample was chosen so as to avoid the Great Financial Crisis and the subsequent period where

to 2007Q1. The dataset matches the series used by Justiniano and Preston (2010b). For Canada, output is GDP per capita in log deviations from a linear trend. The inflation measure excludes food and energy, given the references to this series by the Bank of Canada in the conduct of their monetary policy. An average of 3-month bank rates in annualized percentages is used for interest rates. Justiniano and Preston (2010b) construct a model consistent real exchange rate data series using prices, CPI, and the bilateral nominal exchange rate, and I use it here expressed as log-differences for the estimations. Terms of trade measure the price of imports to exports using price deflators from the national accounts in each country, and it is also used as log-difference.

3.3.2 Estimation under Indeterminacy

In practice, standard Bayesian estimation methods for medium-scale DSGE models rule out indeterminacy altogether, as parameter combinations that do not satisfy the Blanchard and Kahn (1980) conditions once they are fed into the model will be discarded during the algorithm. In other words, researchers assume *a priori* that the solution of the model is unique.

As shown in work by Bullard and Schaling (2009) and Llosa and Tuesta (2008), the region of equilibrium determinacy depends on both open economy parameters (η, α), the response of the central banks to inflation and output, and the type of Taylor rule assumed by the central bank in the domestic economy. At times, indeterminate equilibria can arise from feasible and sensible parameter combinations for the central bank response. This is also the case in the present paper, as shown in Figure 3.1 below.

In addition, introducing an inflation forecast Taylor rule or heterogeneous expectations into the model also modifies the region of equilibrium determinacy. In order to properly account

interest rates approached the zero lower bound in the US, and to coincide with the abandonment of monetary targeting in Canada. Four observations before the start of the sample are used to initialize the Kalman Filter.

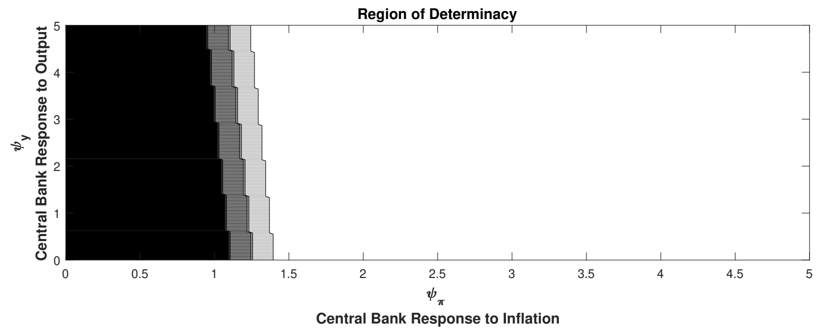


Figure 3.1: Region of Determinacy and Response to Exchange Rates- Benchmark

(a) This figure was generated by numerically finding the region of determinacy (white) for the benchmark model, parametrized to the posterior means in Justiniano and Preston (2010b). The parameter $\psi_{\Delta e}$, the central bank response to changes in the nominal exchange rate, varies from 0.1 (Black) to 0.4 (light gray).

for these feasible regions of the parameter space rather than discarding them *a priori*, I utilize the methodology developed in Bianchi and Nicolò (2017). This method is a technical approach of existing solution methods for medium scale models, such as those of Lubik and Schorfheide (2004) and Farmer et al. (2015), that can be employed to solve and estimate LRE models allowing for indeterminacy of the model solution. The methodology proposes to augment the original model with an additional set of equations that provide the adequate number of explosive roots in the presence of indeterminacy. The representation introduces a non-fundamental sunspot shock to construct the solution under indeterminacy, and the characterization of the full set of equilibria under indeterminacy is parametrized by the additional parameters related to the standard deviation of the sunspot shock and its correlation with the structural shocks in the model.

The methodology in Bianchi and Nicolò (2017) accommodates both the case of determinacy and indeterminacy while considering the same set of equations in the model. The solution in the augmented state space is always determinate if it exists, and is identical to the indeterminate solution of the original model. This methodology will allow the exploration of the full parameter space, by estimating the model under determinacy or indeterminacy. Afterwards, model fit can be assessed by comparing the marginal likelihoods obtained during the estimation procedure, to ascertain whether the data are better explained by a model that features a unique solution, or by multiplicity of equilibrium paths. The augmented representation in Bianchi and Nicolò (2017) appends an auxiliary process that does not affect the law of motion of the model variables. When the model is indeterminate, the additional process is explosive so as to satisfy the Blanchard and Kahn (1980) conditions. Take the original medium-scale, open economy model from Section 3.2 and write it as an Linear Rational Expectations (LRE) model:

$$\Gamma_0(\theta)X_t = \Gamma_1(\theta)X_{t-1} + \Psi(\theta)\varepsilon_t + \Pi(\theta)\eta_t \tag{3.24}$$

where $X_t \in R^k$ is a vector of endogenous variables, $\varepsilon_t \in R^l$ is a vector of structural shocks, $\eta_t \in R^p$ represents a vector of one-step ahead forecast errors for the deviations of the endogenous variables from their steady states, and θ is a vector of parameters. The matrices Γ_0, Γ_1 are of dimension $k \times k$, while Φ, Π are of dimension $k \times l$ and $k \times p$, respectively. The equilibrium is determinate when the number of unstable roots of the system equals the number of forward looking variables. When the model is indeterminate, the additional process should however be explosive so that the Blanchard-Kahn condition is satisfied for the augmented system. The representation of the model in (3.24) is augmented by appending the AR process:

$$\omega_t = (\frac{1}{\xi})\omega_{t-1} + \nu_t - \eta_{2,t}$$

where ω_t is an independent auto-regressive process, $\xi \in [0, 2]$ and ν_t is a newly defined mean zero sunspot shock with standard deviation σ_ν. Define a new vector of endogenous variables $\hat{X}_t \equiv (X_t, \omega_t)^T$ and a newly defined vector of exogenous shocks, and now the system can be rewritten as:

$$\hat{\Gamma}_0\hat{X}_t = \hat{\Gamma}_1\hat{X}_{t-1} + \hat{\Psi}\hat{\varepsilon}_t + \hat{\Pi}\eta_t$$

and along with the vector of observables, the model can now be solved and estimated using standard methods. When the model is determinate, $1/\xi < 1$ and the appended autoregressive process is stationary but its evolution does not affect the law of motion of the model variables. Under indeterminacy of the original model, $1/\xi > 1$,the appended process is explosive, and ω_t is always equal to zero. The process only serve the purpose of providing the necessary explosive roots under indeterminacy, and of creating the mapping between the sunspot shocks and the expectations errors. As shown in Bianchi and Nicolò (2017), this methodology is equivalent to that developed by Lubik and Schorfheide (2004) and Farmer et al. (2015). Once put in this form, the model can then be solved and estimated using standard Bayesian procedures.

3.4 Results

The results from the posterior estimation of the benchmark model under determinacy and indeterminacy are reported in Table 3.2. The model with determinacy provides a better fit to the data, with a posterior model probability of 96%, as shown in Table 3.1. Some differences emerge between parameter estimates across the two cases. Under indeterminacy, the data prefer a model with higher inverse intertemporal elasticity of substitution, inverse elasticity of labor, and elasticity between home and foreign. The Taylor Rule parameters also differ across the two estimations, with the indeterminate case displaying higher indexation to past interest rates, a more muted response to inflation, and more weight on output and changes in the nominal exchange rate. Calvo and indexation parameters are similar across the two estimations, implying that firms reoptimize prices around every 3 quarters[7]. Prices in the imported goods sector are adjusted more frequently in both estimations. The elasticity of substitution between home and foreign goods are low, but higher in the indeterminate case, with median estimates of 0.77 and 0.92.

[7]In line with values reported in Woodford and Walsh (2005)

	Determinacy	Indeterminacy
Log Marginal Likelihood	-1373.36	-1376.56
Posterior Model Probabilities	96.06%	3.94%

Table 3.1: Benchmark Model: Posterior Model Probability

The cost-push, preference, technology, and risk premium disturbances are highly persistent, with autoregressive coefficients between 0.85 and 0.96 across both estimations. The standard deviations of the shocks are plausible for both of the cases, with preference shocks and cost push shocks emerging as the the most volatile. In the indeterminate case, the sunspot shock seems to be correlated with the tecnhnology, cost push, and foreign output shocks. The estimates of the foreign block are similar across both estimations, as expected. The model provides a similarly reasonable characterization of the date, as it matches the volatility and autocorrelation of most of the data series. The model in the indeterminate case performs poorly in this regard, generating quite large standard deviations for inflation and interest rates. Overall, the model performs reasonably well, and much better in the determinate case.

Coefficients			Determinacy			Indeterminacy		
Structural Parameters		Prior	Posterior	90% HPD		Posterior	90% HPD	
Openness	α	B(0.28,0.01)	0.2755	0.2592	0.2922	0.2776	0.2610	0.2939
Inverse intertemporal elasticity of substitution	σ	G(1.2,0.4)	0.3552	0.1973	0.5091	0.5592	0.2833	0.8295
Inverse Frisch	ψ	G(1.5,0.75)	1.1858	0.4385	1.8823	1.6217	0.5912	2.6358
Calvo domestic prices	θ_H	B(0.5,0.1)	0.5316	0.3948	0.6686	0.5955	0.4516	0.7365
Calvo import prices	θ_F	B(0.5,0.1)	0.4093	0.2807	0.5100	0.4720	0.3790	0.5660
Indexation domestic	δ_H	B(0.5,0.25)	0.1510	0.0021	0.3057	0.0936	0.0002	0.1954
Indexation foreign	δ_F	B(0.5,0.25)	0.1195	0.0006	0.2634	0.0846	0.0004	0.1761
Elasticity H-F goods	η	G(1.5,0.75)	0.7790	0.6288	0.9263	0.9212	0.6909	1.1407
Habit	h	B(0.5,0.25)	0.4305	0.3076	0.5490	0.3920	0.2231	0.5584
Taylor rule, smoothing	ψ_r	B(0.5,0.25)	0.6326	0.5204	0.7422	0.9224	0.8779	0.9693
Taylor rule, inflation	ψ_π	G(1.5,0.3)	2.1230	1.8512	2.3764	0.9613	0.7809	1.1232
Taylor rule, output	ψ_y	G(0.25,0.13)	0.0325	0.0077	0.0563	0.2868	0.1136	0.4574
Taylor rule, output growth	$\psi_{\Delta y}$	G(0.25,0.13)	0.6711	0.4272	0.9045	0.4007	0.1025	0.6796
Taylor rule, exchange rate	$\psi_{\Delta e}$	G(0.25,0.13)	0.0242	0.0049	0.0428	0.0981	0.0155	0.1776
Technology	ρ_a	B(0.8,0.1)	0.8988	0.8460	0.9536	0.9049	0.8491	0.9647
Preferences	ρ_g	B(0.8,0.1)	0.9463	0.9225	0.9735	0.9190	0.8780	0.9631
Risk premium	ρ_{rp}	B(0.8,0.1)	0.8764	0.8265	0.9283	0.8548	0.7873	0.9278
Import cost-push shock	ρ_{cp}	B(0.5,0.25)	0.9699	0.9471	0.9938	0.9612	0.9320	0.9915
Foreign Output	ρ_{y*}	N(0.9,0.1)	0.8369	0.7977	0.8756	0.8460	0.8067	0.8874
Foreign Inflation	$\rho_{\pi*}$	N(0.6,0.1)	0.5045	0.3936	0.6134	0.5199	0.4168	0.6238
Foreign intertemporal elasticity of substitution	$\sigma*$	G(1,0.75)	4.0257	2.2597	5.7850	3.6673	2.0999	5.2841
Foreign NKPC slope	$\kappa*$	G(0.1,0.05)	0.0305	0.0117	0.0488	0.0308	0.0117	0.0490
Foreign Taylor rule, smoothing	ψ^*_{r*}	B(0.5,0.2)	0.8301	0.7964	0.8616	0.8361	0.8021	0.8700
Foreign Taylor rule, inflation	$\psi^*_{\pi*}$	G(1.5,0.3)	1.2807	1.0040	1.5442	1.2740	0.9690	1.5558
Foreign Taylor rule, output	ψ^*_{y*}	G(0.25,0.15)	0.2126	0.1166	0.3030	0.2333	0.1255	0.3325
Shocks		Prior	Posterior	90% HPD		Posterior	90% HPD	
sd Technology Shock	ε_a	IG(0.5,0.5)	0.8545	0.5774	1.1195	0.9980	0.6469	1.3729
sd MP Shock	ε_m	IG(0.15,0.5)	0.3891	0.3002	0.4746	0.2259	0.1999	0.2518
sd Preference Shock	ε_g	IG(1,0.5)	2.9536	1.6598	4.3022	3.9773	2.4822	5.3896
sd Risk Premium Shock	ε_{rp}	IG(1,0.5)	0.3618	0.2858	0.4362	0.4337	0.3254	0.5418
sd Cost Push Shock	ε_{cp}	IG(0.15,0.5)	2.4547	1.2981	4.1924	1.7676	1.0135	2.4791
sd Foreign Output	ε_{y*}	IG(0.5,0.5)	0.1628	0.1239	0.2009	0.1627	0.1246	0.2007
sd Foreign Inflation	$\varepsilon_{\pi*}$	IG(0.5,0.5)	0.2132	0.1654	0.2546	0.2083	0.1658	0.2488
sd Foreign MP	ε_{m*}	IG(0.5,0.5)	0.1924	0.1682	0.2167	0.1916	0.1678	0.2146
sd Sunspot	ν	IG(1,0.5)	-	-	-	0.4387	0.3633	0.5116
Correlations		Prior	Posterior	90% HPD		Posterior	90% HPD	
Sunspot, Technology Shock	ν, ε_a	U[-1,1]				-0.5058	-0.7062	-0.3077
Sunspot, MP Shock	ν, ε_m	U[-1,1]				0.0972	-0.1171	0.3048
Sunspot, Preference Shock	ν, ε_g	U[-1,1]				0.1001	-0.3000	0.4830
Sunspot, Risk Premium Shock	ν, ε_{rp}	U[-1,1]				0.1502	-0.2067	0.5575
Sunspot, Cost Push Shock	ν, ε_{cp}	U[-1,1]				0.4112	0.2578	0.5639
Sunspot, Foreign Output	ν, ε_{y*}	U[-1,1]				0.2156	0.0115	0.4202
Sunspot, Foreign Inflation	$\nu, \varepsilon_{\pi*}$	U[-1,1]				0.0708	-0.0993	0.2399
Sunspot, Foreign MP	ν, ε_{m*}	U[-1,1]				-0.0791	-0.2815	0.1169
Log Marginal Likelihood			**-1373.36**			**-1376.56**		

Table 3.2: Benchmark Model - Posterior Estimates

3.5 Inflation Targeting and Inflation Forecast Based Rules

Previous literature has shown that adopting an inflation forecast based rule can alter the conditions that ensure a unique equilibrium. Under inflation targeting, the task of the central bank is to respond with the policy instrument in such a way so as to maintain inflation close to a pre-announced level. Inflation, however, responds with a lag to monetary policy. Simple rules as in Taylor (1993) can be modified to allow the policy instrument to respond to deviations of expected inflation from target, in what is known as "inflation-forecast-based" (IFB) rules (Batini and Haldane (1999)). These rules are not just interesting due to the effects they have on the equilibrium determinacy conditions of the model, but also because inflation targeting banks have explicitly referred to these types of rules in their projection models (see for example Coletti et al. (1996)).

Previous literature has found that these rules can lead to equilibrium indeterminacy, and a longer forecast horizon aggravates this problem, as shown by Batini et al. (2004). To explore this in the context of the model, I produce the analytical region of equilibrium determinacy for a number of simple inflation forecast based rules. The results are shown in Figures 3.3, 3.4, and 3.5. Figure 3.3 plots the region of equilibrium determinacy (in white) for the following IFB Taylor rule:

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\{\psi_\pi E_t \pi_{t+j} + \psi_y E_t y_{t+j} + \psi_{\Delta y} \Delta y_t + \psi_e \Delta e_t\} + \varepsilon_{M,t}$$

for two different forecast horizons, $j = 1, 3$.

In this case, we see that the region of determinacy shrinks as the forecast horizon goes from 1 quarter ahead to 3 quarters ahead. Therefore, in order to estimate an model with this type of Taylor rule, one must properly account for the feasible regions of the parameter space

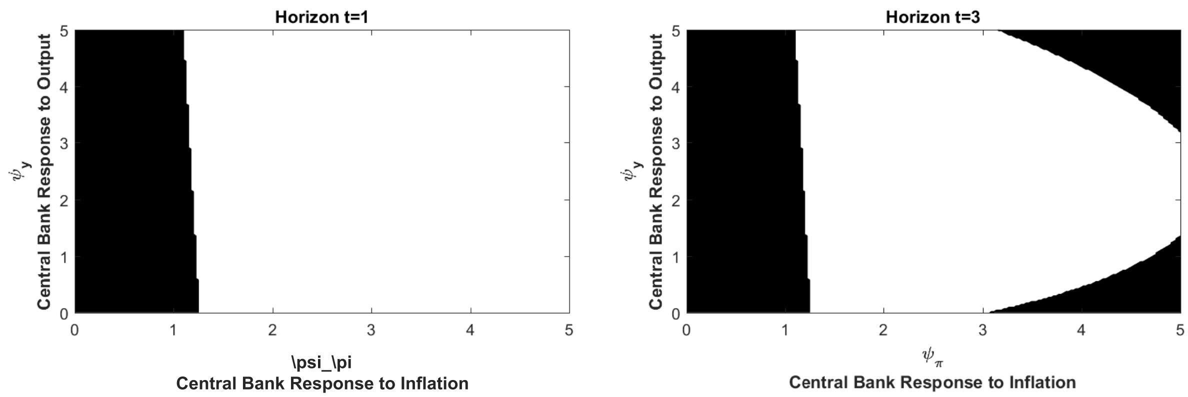


Figure 3.3: Region of Determinacy - Inflation Forecast Based Rule

that may lead to indeterminacy. In addition to forecast horizon, the region of determinacy also interacts with the open economy parameters in the model. Figures 3.4 and 3.5 show the different regions of determinacy as the parameters α, trade openness, and η, elasticity of substitution between home and foreign, are iterated. In the case of α, the parameter takes on values from 0.2 to 0.8, while η ranges from 0.5 to 2. As we can see, both of these parameters cause the region of determinacy to shrink when the forecast horizon is $j = 3$. In this case, feasible regions of the parameter space for the policy response of the central bank can now lead to an equilibrium that is not unique and susceptible to sunspot fluctuations. This is exacerbated once the forecast horizon goes out to $j = 4$, as the region shrinks even further and the central bank is constrained as to which policy choices will lead to a unique equilibrium.

In order to ascertain whether this is the case in the data, I estimate the model using an IFB rule on a sub-sample for Canadian data after the introduction of inflation targeting from 1991Q1 to 2007Q1, across a two forecast horizons and under both determinacy and indeterminacy of order 1 as done for the benchmark model. In addition, in the case of a horizon of 4 quarters, I will conduct the estimation under the assumption that the degree of indeterminacy is equal to 2 for a region of the parameter space since for a region of the parameter space, there are two fewer explosive roots than forward looking variables.

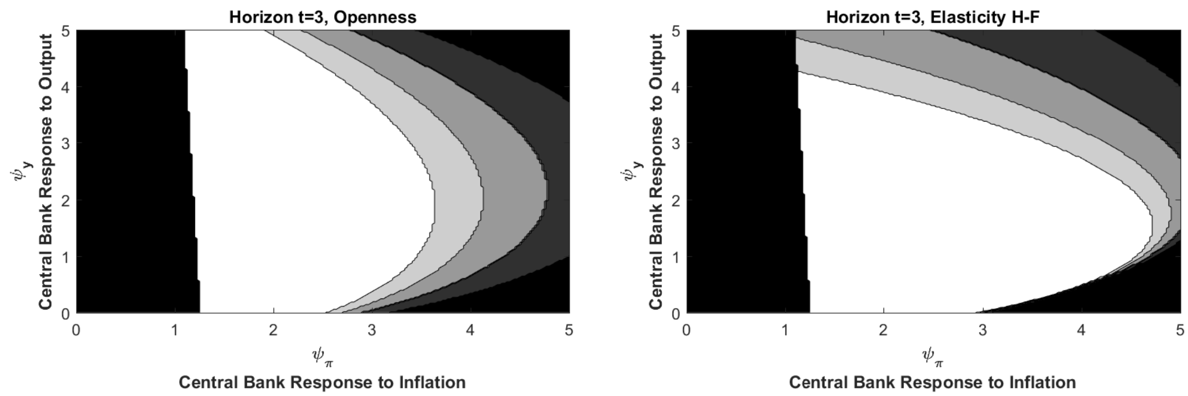


Figure 3.4: Region of Determinacy and Open Economy parameters - IFB Rule 3 quarters ahead

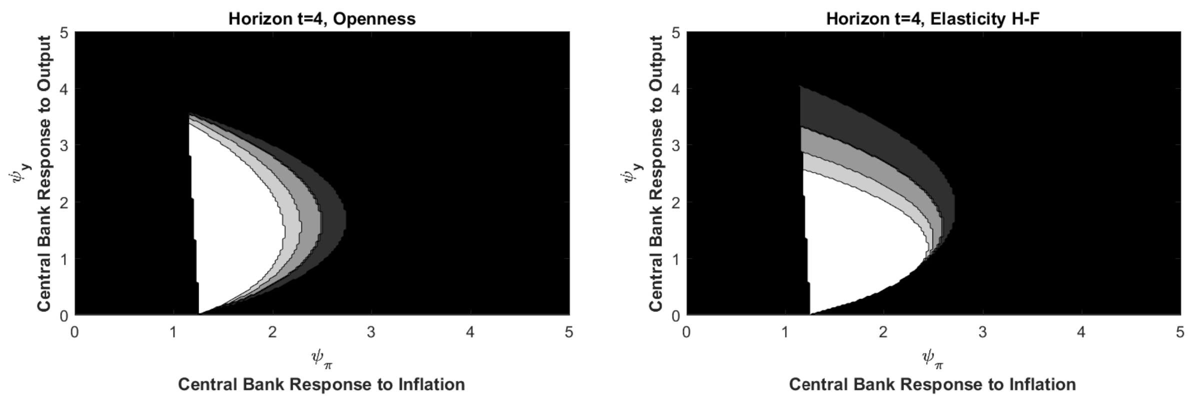


Figure 3.5: Region of Determinacy and Open Economy parameters - IFB Rule 4 quarters ahead

(a) Figures 3.4 and 3.5 were generated by numerically finding the region of equilibrium determinacy (white) for the model with an inflation forecast based rule with horizon at $t = 3$ and $t = 4$. "Openness" refers to the parameter α, which varied from less open at 0.2 (black) to more open 0.8 (light gray). For the inverse elasticity of substitution between Home and Foreign, η, this parameter ranged from 0.5 (black) to 2 (light grey) in increments of 0.5.

3.5.1 Results

First, I adopt an IFB Taylor rule with a forecast horizon of 1 quarter ahead and under the assumption that there is one degree of indeterminacy. Results for this estimation are found in Table 3.4, with marginal likelihoods reported on the bottom rows. The data favor the indeterminate case, with a log marginal likelihood of -752.61 versus -760.74 for the determinate case. These marginal likelihoods imply a Bayes factor of 8.13 which indicates substantial evidence in favor of indeterminacy. Next, I increase the forecast horizon to 4 quarters ahead so now the central bank is responding the their forecast of inflation a year ahead. Results for this estimation are found in Table C.1 in Appendix C. Again, the data favor the indeterminate case, with a log marginal likelihood of -753.02 versus -814.01 for the determinate case. The Bayes factor in this case is 61, which is very strong evidence in favor of the indeterminate case. Lastly, I estimate the model with a forecast horizon of 4 quarters allowing for 2 degrees of indeterminacy, and find that the data favor indeterminacy of order 1 for the sample. Nevertheless, the log marginal likelihoods indicate that the data favor indeterminacy of degree 2 over the determinate case. In addition, the data prefer the IFB Taylor over the contemporaneous benchmark policy rule for this sample, with posterior model probabilities of around 70% in favor of the forecast rule. These results are summarized below in Table 3.3.

Forecast Horizon	Determinacy	Indeterminacy (Order = 1)	Indeterminacy (Order = 2)
1 quarter	-760.74	-752.61	-
4 quarters	-814.01	-753.02	-800.50

Table 3.3: IFB: Log Marginal Likelihoods

Since the marginal likelihoods imply posterior model probabilities of 60% in favor of the model with a forecast horizon of 1 quarter, this will be the focus of my analysis. The estimated structural parameters display some differences across the two cases, particularly in the case of the monetary policy parameters. The results show that data prefers a model with an indeterminate equilibrium, higher degrees of habits and elasticity of substitution, that's subject to a sunspot shock correlated with the domestic monetary policy and cost-push shock. The estimates for the Taylor rule show higher persistence, a lower response to inflation (but still consistent with the Taylor principle), and a stronger response to output and nominal exchange rates. The autoregressive coefficients for the disturbances are similar across the two cases, and the parameters for the foreign block are nearly identical. Shocks to preferences and the cost-push shock emerge as the most volatile in the estimation, with similar estimated for the standard deviations of the shocks across the two cases. The sunspot shock is estimated to be correlated with technology, monetary policy, and cost push shocks.

Coefficients			Determinacy			Indeterminacy		
Structural Parameters		Prior	Posterior	90% HPD		Posterior	90% HPD	
Openness	α	B(0.28,0.01)	0.2735	0.2570	0.2896	0.2748	0.2584	0.2910
Inverse intertemporal elasticity of substitution	σ	G(1.2,0.4)	0.5443	0.3094	0.7633	0.6369	0.3357	0.9484
Inverse Frisch	ψ	G(1.5,0.75)	1.8242	0.7715	2.7728	2.1015	0.8242	3.3407
Calvo domestic prices	θ_H	B(0.5,0.1)	0.4767	0.3772	0.5896	0.4567	0.3206	0.6020
Calvo import prices	θ_F	B(0.5,0.1)	0.4320	0.3357	0.5332	0.4491	0.2825	0.5961
Indexation domestic	δ_H	B(0.5,0.25)	0.1447	0.0031	0.2819	0.2241	0.0025	0.4548
Indexation foreign	δ_F	B(0.5,0.25)	0.1595	0.0049	0.3004	0.1498	0.0008	0.3069
Elasticity H-F goods	η	G(1.5,0.75)	0.7219	0.5839	0.8546	0.7185	0.5447	0.8893
Habit	h	B(0.5,0.25)	0.1715	0.0513	0.2882	0.2855	0.0859	0.4787
Taylor rule, smoothing	ψ_r	B(0.5,0.25)	0.4159	0.2154	0.5966	0.9739	0.9499	0.9993
Taylor rule, inflation	ψ_π	G(1.5,0.3)	2.8503	2.3745	3.3526	1.0791	0.8228	1.3218
Taylor rule, output	ψ_y	G(0.25,0.13)	0.0470	0.0097	0.0822	0.2268	0.0491	0.3982
Taylor rule, output growth	$\psi_{\Delta y}$	G(0.25,0.13)	0.3293	0.1518	0.4998	0.2732	0.0572	0.4694
Taylor rule, exchange rate	$\psi_{\Delta e}$	G(0.25,0.13)	0.0172	0.0038	0.0304	0.2426	0.0410	0.4379
Technology	ρ_a	B(0.8,0.1)	0.9548	0.9244	0.9862	0.9203	0.8707	0.9702
Preferences	ρ_g	B(0.8,0.1)	0.9329	0.8952	0.9723	0.8527	0.7495	0.9614
Risk premium	ρ_{rp}	B(0.8,0.1)	0.8705	0.8168	0.9263	0.8406	0.7434	0.9327
Import cost-push shock	ρ_{cp}	B(0.5,0.25)	0.9549	0.9216	0.9890	0.9341	0.8844	0.9960
Foreign Output	ρ_{y*}	N(0.9,0.1)	0.8289	0.7820	0.8760	0.8367	0.7887	0.8852
Foreign Inflation	$\rho_{\pi*}$	N(0.6,0.1)	0.4131	0.2812	0.5472	0.4231	0.2932	0.5533
Foreign intertemporal elasticity of substitution	$\sigma*$	G(1,0.75)	2.2977	1.4378	3.2332	2.5400	1.3348	3.6765
Foreign NKPC slope	$\kappa*$	G(0.1,0.05)	0.0364	0.0093	0.0620	0.0345	0.0092	0.0592
Foreign Taylor rule, smoothing	ψ^*_{r*}	B(0.5,0.2)	0.8720	0.8326	0.9103	0.8725	0.8315	0.9151
Foreign Taylor rule, inflation	$\psi^*_{\pi*}$	G(1.5,0.3)	1.4650	1.0323	1.8932	1.4901	1.0115	1.9511
Foreign Taylor rule, output	ψ^*_{y*}	G(0.25,0.15)	0.3041	0.1565	0.4475	0.3099	0.1563	0.4551
Shocks		Prior	Posterior	90% HPD		Posterior	90% HPD	
sd Technology Shock	ε_a	IG(0.5,0.5)	0.5278	0.4080	0.6423	0.5699	0.4046	0.7358
sd MP Shock	ε_m	IG(0.15,0.5)	0.2355	0.1694	0.2963	0.1697	0.1447	0.1943
sd Preference Shock	ε_g	IG(1,0.5)	2.1834	1.0827	3.3016	2.5692	1.0547	4.3254
sd Risk Premium Shock	ε_{rp}	IG(1,0.5)	0.4226	0.3228	0.5189	0.4858	0.3431	0.6244
sd Cost Push Shock	ε_{cp}	IG(0.15,0.5)	2.4560	1.2522	3.4810	2.4760	0.9162	4.2151
sd Foreign Output	ε_{y*}	IG(0.5,0.5)	0.1627	0.1203	0.2024	0.1551	0.1162	0.1924
sd Foreign Inflation	$\varepsilon_{\pi*}$	IG(0.5,0.5)	0.2182	0.1737	0.2616	0.2134	0.1695	0.2554
sd Foreign MP	ε_{m*}	IG(0.5,0.5)	0.1396	0.1167	0.1620	0.1413	0.1171	0.1649
sd Sunspot	ν	IG(1,0.5)	-	-	-	0.4453	0.3576	0.5307
Correlations		Prior	Posterior	90% HPD		Posterior	90% HPD	
Sunspot, Technology Shock	ν, ε_a	U[-1,1]				-0.3131	-0.5832	-0.0544
Sunspot, MP Shock	ν, ε_m	U[-1,1]				0.2586	0.0650	0.4561
Sunspot, Preference Shock	ν, ε_g	U[-1,1]				-0.1350	-0.6358	0.3297
Sunspot, Risk Premium Shock	ν, ε_{rp}	U[-1,1]				0.1431	-0.2887	0.5642
Sunspot, Cost Push Shock	ν, ε_{cp}	U[-1,1]				0.3750	0.1686	0.5923
Sunspot, Foreign Output	ν, ε_{y*}	U[-1,1]				0.0181	-0.2466	0.2800
Sunspot, Foreign Inflation	$\nu, \varepsilon_{\pi*}$	U[-1,1]				0.1444	-0.1133	0.4045
Sunspot, Foreign MP	ν, ε_{m*}	U[-1,1]				-0.0331	-0.4249	0.3423
Log Marginal Likelihood			**-760.74**			**-752.61**		

Table 3.4: Inflation Forecast Rule Horizon 1 quarter ahead - Posterior Estimates

Some differences arise once we look at the historical shock decomposition for output and inflation for these results, which can be found in Appendix C. In the case of output, technology and cost-push shocks explain most of the variation in the sample, both under determinacy and indeterminacy. Monetary policy shocks seem to drive some of the variation in the determinate case, but they have a much smaller role in the indeterminate case. In addition, sunspot shocks seem to have very little effect in driving the variation in output. In the case of CPI inflation, however, the assumption of a unique equilibrium substantially affects the interpretation of the data. Under determinacy, shocks to technology, preferences, risk premium, and monetary policy seem to all play a role in driving volatility in inflation. The indeterminate case, however, shows a different story: monetary policy shocks and preference shocks emerge as the main drivers of volatility, with a smaller role for technology shocks. In addition, sunspot shocks and the initial observations also play a role in driving volatility in CPI inflation.

3.5.2 Propagation

In this section, I focus on the results for the inflation forecast based rule with a horizon of 1 period ahead and examine the implications that an indeterminate equilibrium subject to sunspot shocks has on the transmission of the structural shocks. I pay close attention to the shocks that explain most of the fluctuations in the endogenous variables highlighted in the previous section: technology, cost-push, preference, and monetary. In addition, I also look at the effect of foreign output. Figures 3.7, 3.8, 3.9 along with C.6, C.5 in Appendix C, show the Bayesian impulse responses of output, inflation, and interest rates to these shocks. The figures for technology, preference, and monetary policy shocks will be shown here while the rest are contained in Appendix C.

The impact of a technology shock on output is similar across the two cases, but its effects on inflation and interest rates differ under indeterminacy. Technology shocks are now per-

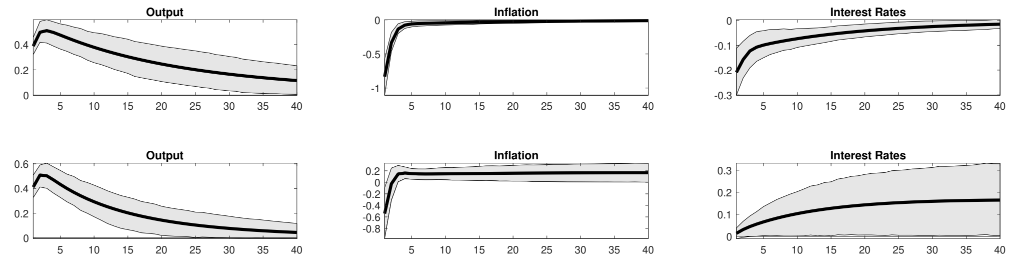


Figure 3.7: Bayesian IRF to technology shock - determinacy (top) and indeterminacy (bottom)

sistently inflationary, and interest rates respond in turn with a slow but steady rise in the interest rate. Inflation is now subject to the sunspot shock and to past inflation, which results in the steady rise in inflation following a technology shock. Cost-push shocks both lower output on impact, and increase inflation on impact. In the determinate case, interest rates rise to counter the rise in inflation but fall back to zero. The response of interest rates on the indeterminate case seems to be a rise on impact and a slight decrease over time, but it isn't clear if the response is much different from zero.

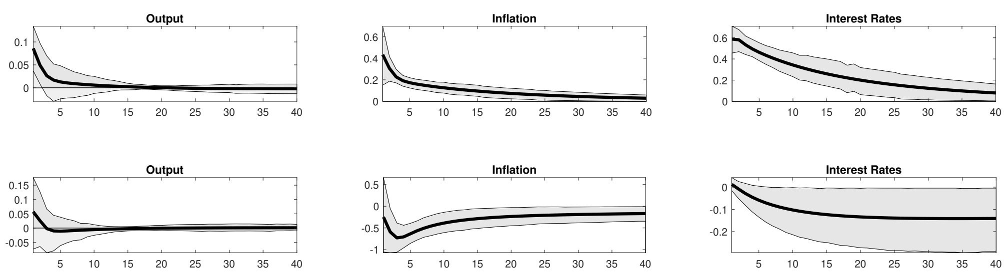


Figure 3.8: Bayesian IRF to a preference shock - determinacy (top) and indeterminacy (bottom)

Preference shocks affect inflation in the opposite directions, producing a deflationary response in the indeterminate case that's somewhat persistent. As a result, interest rates respond negative as well, with the opposite response for the determinate case. Monetary policy shocks produce a drop in output and inflation in the determinate case, with both variables

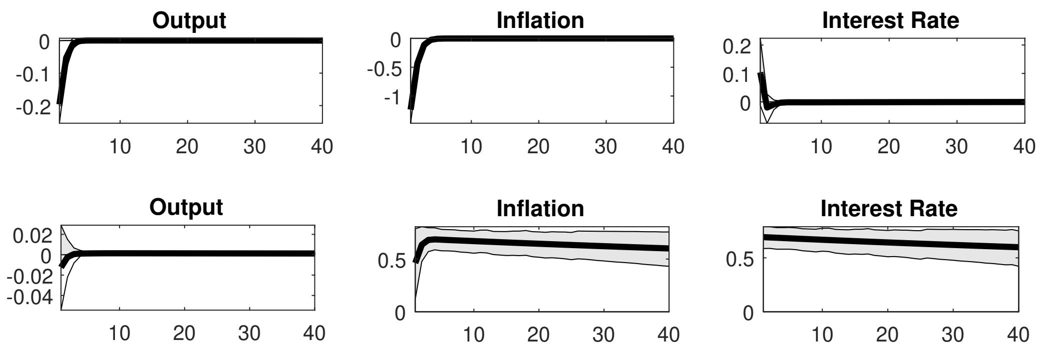


Figure 3.9: Bayesian IRF to a MP shock - determinacy (top) and indeterminacy (bottom)

quickly reverting to their steady states. When the equilibrium is indeterminate, however, the response to a monetary policy shock is a persistent inflationary effect, in line with agents forming inflationary expectations that become self-fulfilling. In the case of foreign output, under indeterminacy the shock produces a larger effect on impact but a couple with a deflationary response. As a result, interest rates don't move much, which isn't the case under determinacy. These figures show to highlight the difference in the impact and propagation of the structural disturbances when the equilibrium is not unique.

3.6 Heterogeneous Expectations

Most studies in international macroeconomics employ estimated, structural, small open economy models derived under the assumption that agents form rational expectations over both domestic and foreign variables. However, there is substantial empirical and experimental evidence that individuals and firms have heterogeneous expectations when engaging in intertemporal decision-making. Imperfections in expectations formations have solved some traditional macro puzzles in recent studies, as previous research has used heterogeneous expectations to generate models that are consistent with inflation inertia, output persistence, and the forward guidance puzzle. In addition, Beqiraj et al. (2018) find that a behavioral

New Keynesian model derived with heterogeneous expectations fits the data better than one with rational expectations. In the present paper, I relax the rational expectations assumption and allow for expectations to feature a share of rational agents as well as a share of adaptive agents. Since the region of equilibrium determinacy depends on both the open economy parameters and the parameters governing heterogeneous expectations, I estimate this model allowing for both determinacy and indeterminacy as in the previous sections. I find that the data are best explained by a model with heterogeneous expectations and a unique equilibrium.

I introduce heterogeneous expectations in a reduced form way, by assuming the expectations operator takes on a modified form. The model can be rewritten so that in the domestic economy, agents form expectations on $\{c_t, y_t^*, q_t, \pi_{H,t}, \pi_{F,t}, s_t\}$. Agents in the foreign economy will form expectations on $\{y_t^*, \pi_t^*\}$. In order to introduce heterogeneous expectations into this model, I will make some specific assumptions on the expectations operator, E_t, following Branch and McGough (2009). There are be two types of agents: rational[8] agents of type "1", and adaptive agents of type "2". Agents of the rational type have one step ahead perfect foresight on the variables of the model. Adaptive agents, on the other hand, have expectations:

$$E_t^2(x_{t+1}) = \phi x_{t-1}$$

and, following the learning literature, $E_t^2 x_{t+k}$ is constructed iteratively. With this in mind, the expectations operator for the variables in the model can be replaced by

$$\hat{E}_t(x_{t+1}) = nE_t x_{t+1} + (1-n)\phi^2 x_{t-1}$$

[8]Subtly, rational agents are "really good" forecasters, but not fully rational in the conventional sense. As noted in Branch and McGough (2009), full rationality would imply that $E_t^1 E_{t+1}^2 x_{t+1} = \phi x_t$. The assumptions they introduce on admissible forms of heterogeneous expectations that facilitate decoupling the aggregate dynamics from the dynamics of the wealth distribution require that $E_t^1 E_{t+1}^2 x_{t+1} = E_t x_{t+1}$. Essentially, rational agents of type "1" know the conditional distributions of the variables, but not the expectations of non-rational agents.

where n is the proportion of agents having rational expectations. The adaptive expectations operator can take on the form of adaptive ($\phi < 1$) or extrapolative ($\phi > 1$) expectations[9], and these expectations can be thought of as coming from a simple linear perceived law of motion for the variables. Note that as $n \to 1$, we obtain the canonical homogeneous rational expectations case. Despite the simple form of this expectations operator, there is evidence coming from studies using suvery data that suggest agents are distributed across rational expectations and adaptive expectations of this form (Branch (2004)).

As shown in Appendix C, these types of expectations can provide a better fit over homogeneous rational expectations in a closed economy, simple 3 equation New Keynesian model estimated on US data. Table 3.5 summarizes the results from this simple estimation, suggesting that the data strongly favors the model with heterogeneous expectations. We also know that under heterogeneous expectations, the effect of structural shocks on the variables of the model can exhibit more persistence than in the rational expectations case, or display different signs.

	Homogeneous Expectations	Heterogeneous Expectations
Log Marginal Likelihood	-440.39	-437.24
Posterior Model Probabilities	4.11%	95.89%

Table 3.5: Simple New Keynesian Model: Posterior Model Probability

3.6.1 Results

The estimation procedure is the same as in the benchmark model, using the full sample from 1982Q1 to 2007Q1. Table 3.6 shows the results of the estimation of the structural parameters of the benchmark model against the model with heterogeneous expectations. The table reports priors for the parameters, along with the posteriors (and their 90% highest posterior

[9]For simplicity, I will refer to agents of type "2" as adaptive, even though their expectations can be adaptive or extrapolative.

density interval) and the log marginal likelihood for each model. The best-fitting model for this sample is the one that features heterogeneous expectations exhibiting a log marginal likelihood of -1359.94 versus -1373.36 for the homogeneous expectations models. The difference in terms of marginal likelihood is 13.42 in favor of the model featuring heterogeneous expectations, indicating strong evidence towards this result.

In addition, Table 3.6 reports the share of agents that form rational expectations for each variable. I estimate a share of agents that form rational expectations over consumption (n_c) of 91.03%, implying that about 8.97% are boundedly rational, with an adaptation parameter of 1.17 which implies these are extrapolative expectations. I find extrapolative expectations ($\phi > 1$) on consumption and foreign inflation, and adaptive expectations for the rest. In the foreign block, the shares of agents with boundedly rational expectations over output and inflation are 25% and 48% respectively, albeit with relatively large 90% HPD intervals. I find that the expectations parameters estimated on the foreign block are both adaptive. The estimation under heterogeneous expectations produces a number of differences in the deep parameters for the domestic block in relation to the rational expectations case, while the structural parameters for the exogenous foreign block remain largely the same across both cases. I find a more muted central bank response to inflation in the heterogeneous expectations case, while the rest of the Taylor rule parameters are close to the rational expectations case.

3.6.2 Variance Decomposition

An earlier study by Justiniano and Preston (2010a) established that a benchmark New Keynesian open economy model cannot account for the substantial influence of foreign shocks. In order to analyze whether heterogeneous expectations can solve this issue, I look at the variance decomposition of the model versus the ones coming from the data. Table 3.7 shows the

Coefficients			Heterogeneous Expectations			Homogeneous Expectations		
Structural Parameters		Prior	Posterior Mean	90% HPD Interval		Posterior Mean	90% HPD Interval	
Openness	α	B(0.28,0.01)	0.2776	0.2610	0.2939	0.2755	0.2592	0.2922
Inverse intertemporal elasticity of substitution	σ	G(1.2,0.4)	0.5383	0.3567	0.7154	0.3552	0.1973	0.5091
Inverse Frisch	ψ	G(1.5,0.75)	0.5422	0.1261	0.9303	1.1858	0.4385	1.8823
Calvo domestic prices	θ_H	B(0.5,0.1)	0.4902	0.3699	0.6174	0.5316	0.3948	0.6686
Calvo import prices	θ_F	B(0.5,0.1)	0.8535	0.8067	0.8989	0.4093	0.2807	0.5100
Indexation domestic	δ_H	B(0.5,0.25)	0.0674	0.0003	0.1424	0.1510	0.0021	0.3057
Indexation foreign	δ_F	B(0.5,0.25)	0.5875	0.2201	0.9687	0.1195	0.0006	0.2634
Elasticity H-F goods	η	G(1.5,0.75)	0.4788	0.3576	0.5937	0.7790	0.6288	0.9263
Habit	h	B(0.5,0.1)	0.3865	0.2689	0.5119	0.4305	0.3076	0.5490
Taylor rule, smoothing	ψ_r	B(0.5,0.25)	0.7288	0.6419	0.8197	0.6326	0.5204	0.7422
Taylor rule, inflation	ψ_π	G(1.5,0.3)	1.7827	1.4719	2.1055	2.1230	1.8512	2.3764
Taylor rule, output	ψ_y	G(0.25,0.13)	0.0747	0.0190	0.1304	0.0325	0.0077	0.0563
Taylor rule, output growth	$\psi_{\Delta y}$	G(0.25,0.13)	0.7215	0.3621	1.0811	0.6711	0.4272	0.9045
Taylor rule, exchange rate	$\psi_{\Delta e}$	G(0.25,0.13)	0.0292	0.0059	0.0511	0.0242	0.0049	0.0428
Technology	ρ_a	B(0.8,0.1)	0.9021	0.8193	0.9824	0.8988	0.8460	0.9536
Preferences	ρ_g	B(0.8,0.1)	0.9185	0.8884	0.9510	0.9463	0.9225	0.9735
Risk premium	ρ_{rp}	B(0.8,0.1)	0.7929	0.6581	0.9386	0.8764	0.8265	0.9283
Import cost-push shock	ρ_{cp}	B(0.5,0.25)	0.4552	0.0489	0.8671	0.9699	0.9471	0.9938
Foreign Output	ρ_{y*}	N(0.9,0.1)	0.8965	0.8821	0.9118	0.8369	0.7977	0.8756
Foreign Inflation	$\rho_{\pi*}$	N(0.6,0.1)	0.5708	0.5109	0.6295	0.5045	0.3936	0.6134
Foreign intertemporal elasticity of substitution	$\sigma*$	G(1,0.75)	3.5132	1.7540	5.1785	4.0257	2.2597	5.7850
Foreign NKPC slope	$\kappa*$	G(0.1,0.05)	0.0459	0.0156	0.0739	0.0305	0.0117	0.0488
Foreign Taylor rule, smoothing	ψ^*_{r*}	B(0.75,0.1)	0.8895	0.8542	0.9254	0.8301	0.7964	0.8616
Foreign Taylor rule, inflation	$\psi^*_{\pi*}$	G(1.5,0.3)	1.3784	1.0488	1.6830	1.2807	1.0040	1.5442
Foreign Taylor rule, output	ψ^*_{y*}	G(0.25,0.13)	0.2189	0.0688	0.3467	0.2126	0.1166	0.3030
Expectations								
Consumption	n_c	B(0.5,0.2)	0.9109	0.8499	0.9742	-	-	-
Terms of trade	n_s	B(0.5,0.2)	0.7591	0.5845	0.9566	-	-	-
Real exchange rate	n_q	B(0.5,0.2)	0.1047	0.0202	0.1866	-	-	-
Domestic prices	n_{π_H}	B(0.5,0.2)	0.4795	0.2770	0.6882	-	-	-
Import prices	n_{π_F}	B(0.5,0.2)	0.3666	0.0821	0.6383	-	-	-
Inflation, foreign	$n_{\pi*}$	B(0.5,0.2)	0.3630	0.0831	0.6332	-	-	-
Foreign output	n^*_{y*}	B(0.5,0.2)	0.2550	0.0861	0.4212	-	-	-
Foreign inflation	$n^*_{\pi*}$	B(0.5,0.2)	0.4821	0.2962	0.6857	-	-	-
Consumption	ϕ_c	N(1,0.3)	1.1705	0.9730	1.3688	-	-	-
Terms of trade	ϕ_s	N(1,0.3)	0.8688	0.5745	1.1373	-	-	-
Real exchange rate	ϕ_q	N(1,0.3)	0.9703	0.9545	0.9857	-	-	-
Domestic prices	ϕ_{π_H}	N(1,0.3)	0.1707	0.0000	0.3017	-	-	-
Import prices	ϕ_{π_F}	N(1,0.3)	0.5893	0.3005	0.8623	-	-	-
Inflation, foreign	ϕ_{π^*}	N(1,0.3)	1.1277	0.7741	1.4890	-	-	-
Foreign output	ϕ^*_{y*}	N(1,0.3)	0.7182	0.6023	0.8333	-	-	-
Foreign inflation	$\phi^*_{\pi*}$	N(1,0.3)	0.2637	0.0303	0.4633	-	-	-
Shocks								
sd Technology Shock	ε_a	IG(0.5,0.5)	1.0203	0.6467	1.4127	0.8545	0.5774	1.1195
sd MP Shock	ε_m	IG(0.15,0.5)	0.3015	0.2392	0.3620	0.3891	0.3002	0.4746
sd Preference Shock	ε_g	IG(1,0.5)	2.4908	1.6658	3.2902	2.9536	1.6598	4.3022
sd Risk Premium Shock	ε_{rp}	IG(1,0.5)	2.0994	1.7835	2.4250	0.3618	0.2858	0.4362
sd Cost Push Shock	ε_{cp}	IG(0.15,0.5)	1.6122	1.1375	2.0632	2.4547	1.2981	4.1924
sd Foreign Output	ε_{y*}	IG(0.5,0.5)	0.4046	0.2677	0.5350	0.1628	0.1239	0.2009
sd Foreign Inflation	$\varepsilon_{\pi*}$	IG(0.5,0.5)	0.2892	0.2317	0.3468	0.2132	0.1654	0.2546
sd Foreign MP	ε_{m*}	IG(0.5,0.5)	0.1794	0.1592	0.1992	0.1924	0.1682	0.2167
Marginal Likelihood			**-1359.94**			**-1373.36**		

Table 3.6: Heterogeneous Expectations - Posterior Estimates

variance decomposition implied by the two models, a seemingly-unrelated-regression model reported in Justiniano and Preston (2010a) (labeled "JP2010"), and a structural VAR using the same sample as in the estimation (1982Q1 - 2007Q1). The SVAR is estimated on the 8 series used in the model, with zero restrictions that imply no feedback from Canada to the U.S., and I report the long horizon variance decomposition of Canadian Series attributed to all U.S. shocks in the SVAR. The SUR also has a zero restriction of no feedback from Canada to the U.S., but utilizes twelve series (two additional series per country: nominal compensation per hour and total hours) for inference using an efficient Gibbs sampling algorithm as in Zha (1999). I report the stationary posterior variance share of Canadian series attributed to US shocks as in Table 1 of Justiniano and Preston (2010a). As for the models, I report the sum of the variance decomposition of Canadian series attributed to U.S. shocks coming from the estimation procedure. For all series except inflation, the model with heterogeneous expectations outperforms the baseline model. In particular, the model significantly outperforms the baseline case when capturing the variance share of output attributed to foreign shocks. Under rational expectations, the model variance decomposition attributes less than 1% to U.S. shocks, which is significantly less than the SUR share of 76% or the SVAR share of 56%. Heterogeneous expectations in the model improve this significantly up to 13%, but it still falls short of the variance decomposition implied by the reduced form models.

Series	JP2010	BVAR	SVAR	RE	HE
Output	0.76	0.55	0.56	0.002	0.132
Inflation	0.65	0.22	0.22	0.021	0.016
Interest Rate	0.71	0.65	0.54	0.034	0.085
Real Exchange Rate	0.62	0.09	0.21	0.041	0.080
Terms of Trade	0.57	0.08	0.17	0.055	0.178

Note: RE - Baseline model. HE - Heterogeneous Expectations

Table 3.7: Variance Decomposition of Canadian Series Attributed to U.S. Shocks

3.6.3 Estimation under Indeterminacy

When agents have heterogeneous expectations, the conditions for equilibrium determinacy not only depend on the parameters governing openness or the policy rule, but also on the parameters of the expectations operator. As shown in Branch and McGough (2009), monetary policy rules can be stabilizing or destabilizing depending on the proportion of adaptive agents. Policy rules that are shown to be stabilizing under homogeneous rational expectations may actually destabilize under heterogeneous expectations. This section focuses on the region of determinacy of the model under the assumption of heterogeneous expectations. I calibrate the model to the parameters from the estimation procedure in section the previous section, and numerically find the region of equilibrium determinacy by testing whether the Blanchard and Kahn (1980) conditions hold for different combinations of monetary policy parameters in the domestic economy. To keep things simple, I assume that the share of rational agents n and the adaptive expectations parameter ϕ are the same for each expectational variable, but different across countries (denoted by a * in the case of foreign variables).

Figure 3.10 shows the region of determinacy when agents are "adaptive" ($\phi < 1$) as the share of rational agents goes from $n = 0.9$ (black) to $n = 0.7$ (light gray) in increments of 0.05 (implying that the share of agents with adaptive expectations increases). As the share increases to 0.3 ($n = 0.7$), the region of determinacy shrinks significantly, requiring a very strong response by the central bank to inflation while keeping its response to output sufficiently low. In the case of "extrapolative" expectations ($\phi > 1$) show in figure 3.11, the region of determinacy severely shrinks once the share of boundedly rational agents goes above 30%, as indicated by the gray area. Now, a strong response to inflation by the central bank needs to be coupled with a high enough response to output gap to ensure a unique equilibrium. Thus, reasonably feasible Taylor Rule parameter combinations (for example, $\psi_\pi = 1.5, \psi_y = 0.25$) lead to equilibrium indeterminacy (or in some cases, an explosive equilibrium), if the share of adaptive agents is large enough, depending on the nature of their adaptive expectations (whether $\phi > 1$ or $\phi < 1$). This is in line with previous research, such as Branch and McGough (2009) who establish this result for the closed economy case.

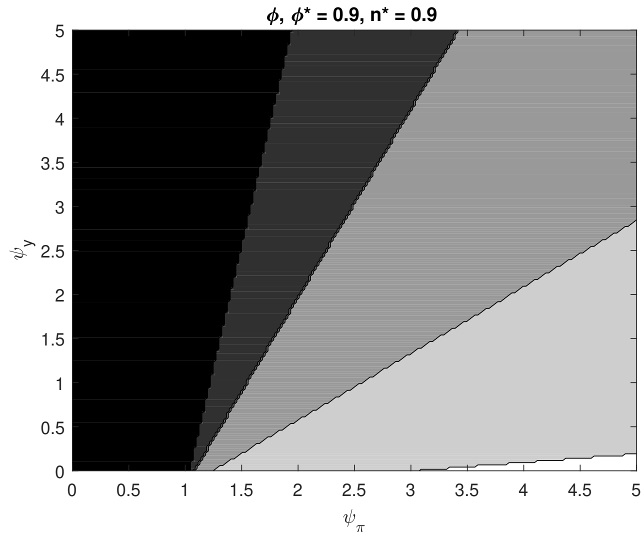


Figure 3.10: Determinacy Region and Share of Rational Agents

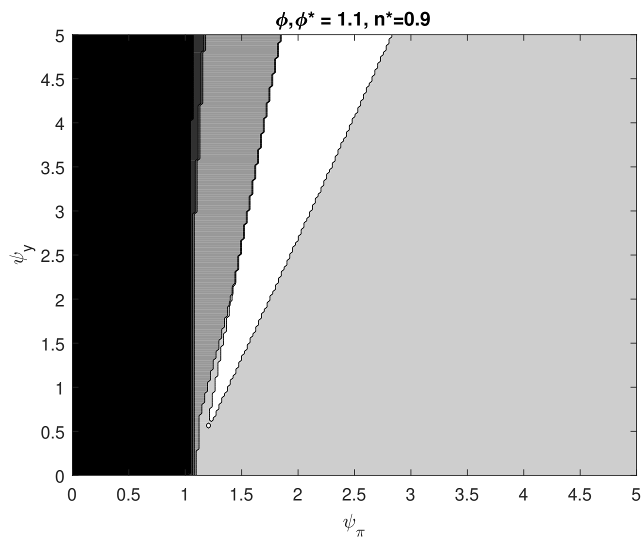


Figure 3.11: Determinacy Region and Share of Rational Agents

As in the previous sections, I estimate the model under determinacy and indeterminacy for the full sample. Results for model fit for both estimations can be seen in Table 3.8, where the data strongly prefer determinacy in both cases.

	Rational Expectations	Heterogeneous Expectations
Determinacy	-1373.36	**-1359.94**
Indeterminacy	-1376.56	-1444.24

Table 3.8: Model Comparison

3.6.4 Impulse Responses

Introducing heterogeneous agents changes the dynamics of structural shocks in the model. I will focus on the effect of a shock to foreign output. Figure 3.12 displays the impulse response from the Bayesian VAR to a shock to foreign output. Figures 3.13 and 3.14 display the Bayesian impulse response from the estimation procedure to a shock to foreign output. The model with heterogeneous expectations displays dynamics for domestic output that provide a better match to the one obtained under a reduced form Bayesian VAR. Heterogeneous expectations is able to better match the hump shape of the response, as well as the persistence, than the rational expectations case.

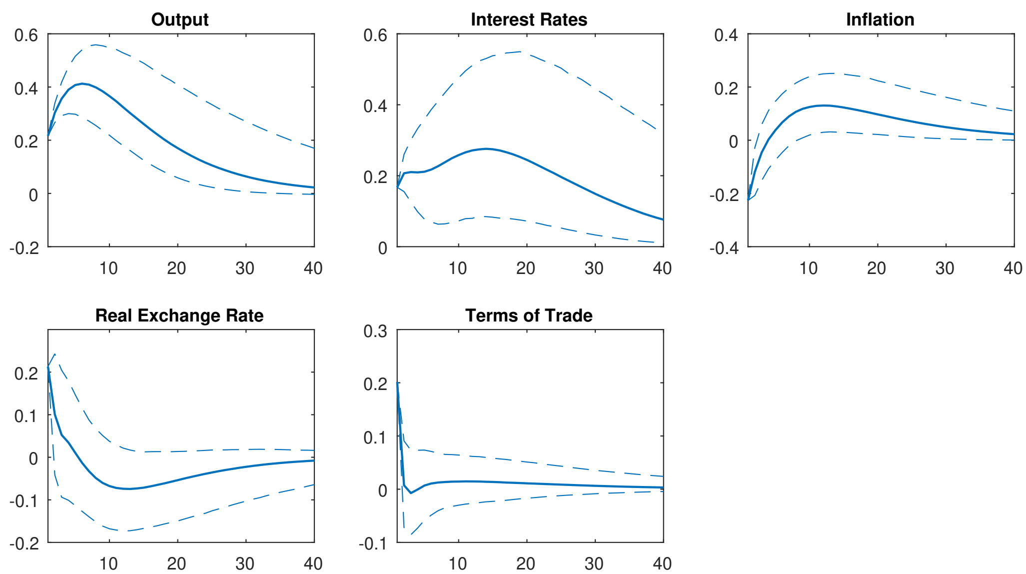


Figure 3.12: BVAR - Shock to Foreign Output

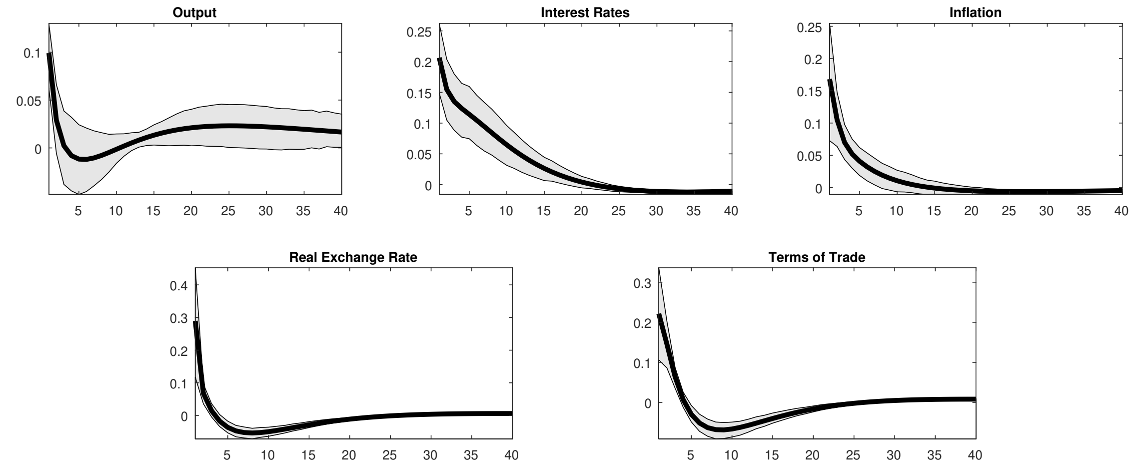


Figure 3.13: RE - Shock to Foreign Output

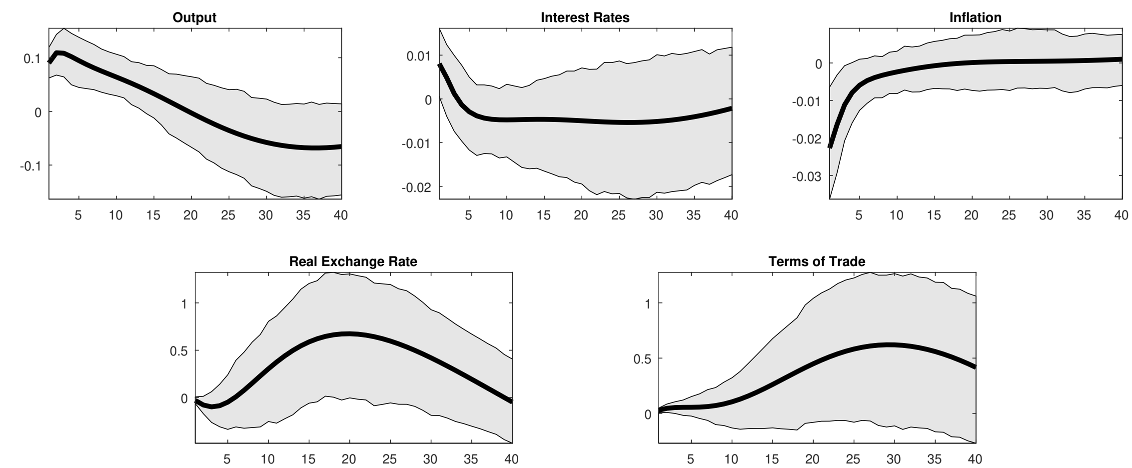


Figure 3.14: HE - Shock to Foreign Output

3.7 Concluding Remarks

This paper analyzes equilibrium determinacy in an estimated medium-scale small open economy model for Canada and the U.S. using Bayesian methods and a technique that allows for the solution and estimation of the model under indeterminacy. In addition, I consider two extensions: Forecast Based Taylor rules, and Heterogeneous Expectations.

I find three main results from these estimations. First, the benchmark medium scale open economy model estimated for Canada and the U.S. provides a better fit to the data under determinacy. Second, estimating the model with an inflation forecast based rule on a subsample matching the inflation targeting period in Canada reveals that the data are best explained by a model with indeterminacy of order 1 along with a forecast horizon of 1 quarter ahead. Even though the equilibrium is not unique, the central bank is reacting to inflation more than one for one. The dynamics of the structural shocks change under indeterminacy, as now shocks to technology and monetary policy become persistently inflationary. In addition, monetary policy, preference, and sunspot shocks emerge as the main drivers of volatility of inflation in the domestic economy. Lastly, I find that a small open economy model with heterogeneous expectations and a determinate equilibrium provides the best fit to the data and can account slightly better for the influence of foreign shocks than the benchmark model.

These results indicate that solutions that lead to an indeterminate equilibrium should be considered when taking open economy models to the data. Furthermore, deviations from the rational expectations framework such as heterogeneous expectations can provide a better fit in open economy models. Future avenues of research in this area includes the case of emerging economies, economies with fixed exchange rates or crawling pegs, and open economies with financial and labor market frictions.

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Appendix A

Corresponds to Table 4 in Swanson (2018).

Table A.1: *Estimated Effects of Changes in Forward Guidance, LSAPs on U.S. Treasury Yields, Jan. 2009 - Oct. 2015*

	6-month	2-year	5-year	10-year	30-year
Change in forward guidance	1.19***	5.14***	6.22***	3.06***	0.14
(Std. error)	(.322)	(.323)	(.363)	(.299)	(.886)
[t-stat]	[3.69]	[15.91]	[17.13]	[10.24]	[0.16]
Change in LSAP	0.19**	0.20	-2.92***	-6.49***	-5.77***
(Std. error)	(.094)	(.118)	(.514)	(.343)	(.554)
[t-stat]	[2.07]	[1.66]	[-5.69]	[-18.91]	[-10.42]
Regression R^2	0.40	0.93	0.95	0.98	0.81
# of Observations	55	55	55	55	55

Table A.2: *Estimated Effects of Changes in U.S. Forward Guidance, and LSAPs on Stock Prices and Exchange Rates, Jan. 2009-Oct. 2015*

	S&P500	$/euro	$/yen
Change in forward guidance	-0.26**	-0.37**	-0.24***
(Std. error)	(.100)	(.057)	(.050)
[t-stat]	[-2.61]	[-6.46]	[-4.86]
Change in LSAP	0.12*	0.21***	0.29***
(Std. error)	(.059)	(.053)	(.048)
[t-stat]	[1.99]	[4.00]	[6.08]
Regression R^2	0.28	0.68	0.79
# of Observations	55	55	55

Appendix B

More on Inflation Targeting in Japan

> *...Japan faces a different type of challenge. In the United States and Europe, people's inflation expectations have been anchored around the central banks' targeted inflation rates. In Japan, amid some 15 years of deflation, deflationary expectations have become entrenched among people – in other words, people's inflation expectations have been anchored at a substantially low level of around 0 percent. We need to de-anchor such expectations, increase them to the price stability target of 2 percent, and anchor the expectations again at this level.*
>
> \- Bank of Japan Governor Haruhiko Kuroda
>
> *Remarks at the Speech at the Kisaragai-kai Meeting in Tokyo, September 20, 2013.*

The rationale for the Bank of Japan's 2 percent inflation target has been outlined by Governor Kuroda in several speeches. The first and foremost reasoning is the Bank's commitment to achieving price stability for the sound development of the national economy, which is the basis for the 2 percent year-on-year rate of change in headline CPI the Bank of Japan has decided to target. In two speeches (Kuroda 2013b; 2014), he cites the upward bias in inflation rate and the risk of hitting the zero lower bound as additional reasons for a 2 percent inflation target. The tendency of the CPI growth rate to exhibit upward bias in its growth rate has been well documented, with a year-on-year rate of change in the CPI about 1 percent higher than that of the GDP deflator. In regards to the zero lower bound, he notes that in response to the Global Financial Crisis, the Bank of Japan was only able to reduce the nominal short-term rate by about 40 basis points, due to short-rates having been in a tight band between 0 and 50 basis points for about 20 years. In addition to upward bias and the need for an interest rate buffer, Governor Kuroda points to the emergence of 2 percent targeting as a global standard among major central banks as another reason for this setting target.

Inflation targeting has been quite successful across the globe (Corbo et al., 2001; Abo-Zaid and Tuzemen, 2012) but the situation of Japan is quite different from that of most inflation targeting countries. The adoption of inflation targeting in Japan came after a prolonged period of subdued inflation[1], with the aim of lifting inflation expectations up to target rather than bringing them down. In addition, as Shirai (2018) points out, two features of the Japanese deflationary experience contribute to the difficulty in both achieving and anchoring expectations to the target. One is the "prevalence of deflation-oriented mindsets", meaning firms generally maintained deflationary expectations and cautious price-setting behavior as a result. The other is household's upwards bias in expectations, which meant their tolerance for price rises dropped as their inflation expectations rose, further contributing to firms' cautious price-setting.

One of the papers that examines "targeting from below" more closely is Ehrmann (2015), in which the case of Japan is examined thoroughly. The paper tests the extent of anchoring of inflation expectations when central banks find themselves with rates of inflation below their inflation targets. It does this by comparing across periods when inflation is around target, persistently high or persistently low, using a forecast survey data set from Consensus Economics. The main finding is that under persistently low inflation, expectations are not as well anchored as when inflation is around the explicit target. Evidence for this comes from

[1]In January 2013, the Bank of Japan implemented a monetary policy regime with an explicit inflation target based on Headline CPI for the first time.

three tests: the extent to which inflation expectations depend on lagged and realized inflation, the extent of disagreement among forecasters, and how agents conduct their revisions of inflation expectations in response to news about realized inflation. Low inflation countries had inflation expectations that are more dependent on lagged inflation, more forecaster disagreement about inflation, and downward revisions in their expectations in response to lower-than-expected inflation while failing to respond to higher-than-expected inflation.

In the particular case of Japan, the Ehrmann (2015) findings show that forecaster disagreement in Japan has been larger than that observed in other countries under persistently low inflation. In addition, the findings show that, under persistently low inflation, Japan has inflation expectations that are relatively more backward looking than in other inflation targeting countries. Furthermore, the findings for significant response to CPI announcement surprises is more muted, as the author finds no response to either negative or positive surprises in the announcement. This, however, could be due to a lack of power as the announcements were typically well predicted.

Bank of Japan's Monetary Policy Frameworks during Abenomics

Table B.1

Timeline	Monetary Policy Framework
January 2013	Price Stability Target of 2 percent
April 2013	Qualitatitve and Quantitative Easing (QQE1) with annual JGB purchase of 40 trillion
October 2014	Qualitatitve and Quantitative Easing (QQE2) with annual JGB purchase of 80 trillion
January 2016	Introducing Negative Interest Rate Policy
September 2016	QQE with Yield Curve Control, Inflation-Overshooting Commitment

January 22, 2013: The "Price Stability Target" under the Framework for the Conduct of Monetary Policy

> "The newly-introduced 'price stability target' is the inflation rate that the Bank judges to be consistent with price stability on a sustainable basis . . . [T]he Bank sets the 'price stability target' at 2 percent in terms of the year on-year rate of change in the consumer price index (CPI) – a main price index."

April 4, 2013: Introduction of the Quantitative and Qualitative Monetary Easing

> "The Bank will achieve the price stability target of 2 percent in terms of the year-on-year rate of change in the consumer price index (CPI) at the earliest possible time, with a time horizon of about two years. In order to do so, it will enter a new phase of monetary easing both in terms of quantity and quality."

October 31, 2014: "Expansion of the Quantitative and Qualitative Monetary Easing"

> "The Bank will conduct money market operations so that the monetary base will increase at an annual pace of about 80 trillion yen (an addition of about 10-20 trillion yen compared with the past)."

January 29, 2016: Introduction of "Quantitative and Qualitative Monetary Easing with a Negative Interest Rate"

> "The Bank will apply a negative interest rate of minus 0.1 percent to current accounts that financial institutions hold at the Bank... [S]peficially, the Bank will adopt a three-tier system in which the outstanding balance of each financial institution's current account at the Bank will be divided into three tiers, to each of which a positive interest rate, a zero interest rate, or a negative interest rate will be applied, respectively."

September 21, 2016: New Framework for Strengthening Monetary Easing: "Quantitative and Qualitative Monetary Easing with Yield Curve Control"

> "Based on these, with a view to achieving the price stability target of 2 percent at the earliest possible time, the Bank decided to introduce QQE with Yield Curve Control by strengthening the two previous policy frameworks mentioned above. The new policy framework consists of two major components: the first is yield curve control in which the Bank will control short-term and long-term interest rates; and the second is an inflation-overshooting commitment in which the Bank commits itself to expanding the monetary base until the year-on-year rate of increase in the observed consumer price index (CPI) exceeds the price stability target of 2 percent and stays above the target in a stable manner."

Figure B.1

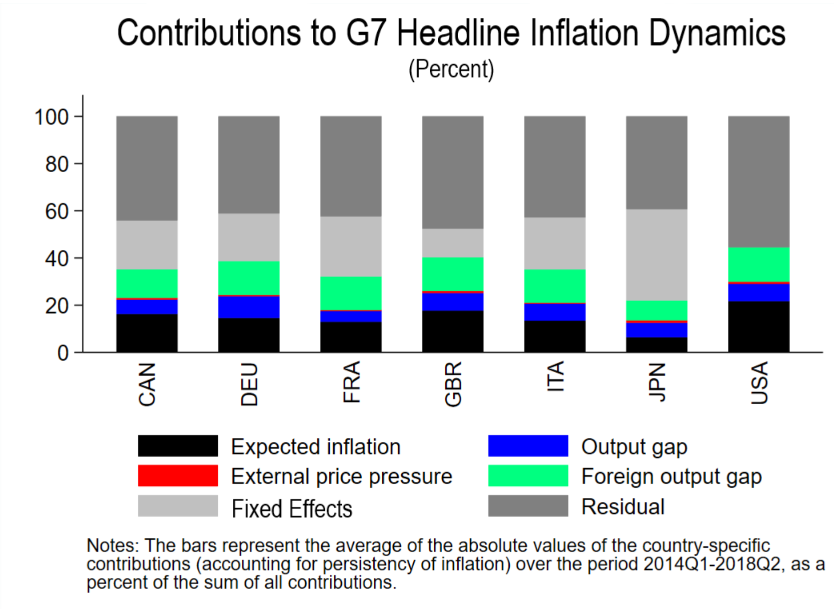
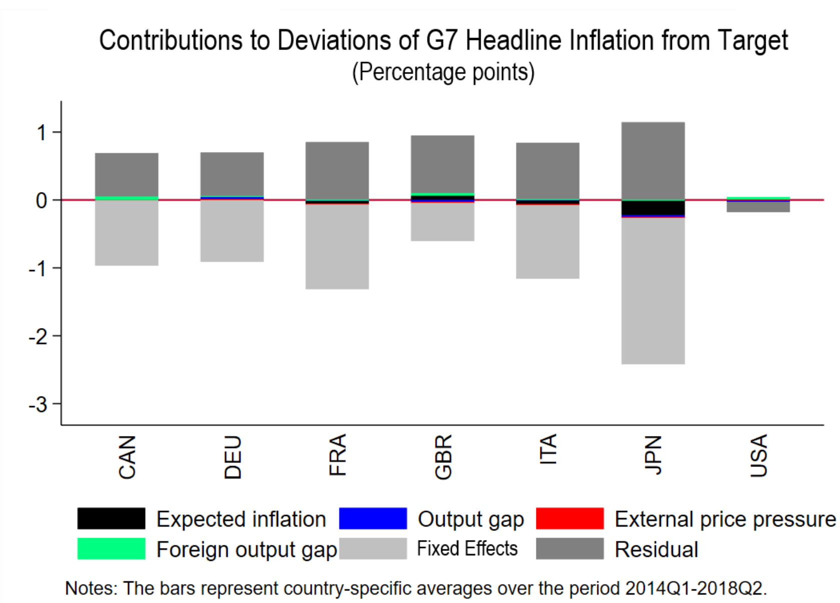


Figure B.2



These figures were generated using a methodology found in Chapter 3 of the *World Economic Outlook* (International Monetary Fund, 2018). Figures 14 and 15 are derived from the authors' own calculations using current IMF data.

Alternative Regression Specifications

The following tables contain alternative regression specifications for equation (2.3), using different dummies or different measures of inflation.

Table B.2: Testing Mon. Policy Changes: Different Measures of Inflation

	Financial + Survey		Survey	
	Estimate	SE	Estimate	SE
(Intercept)	0.371***	0.063	0.794***	0.165
Level (lagged)	0.410***	0.068	0.141	0.144
Inflation (Core)	0.045**	0.015	0.102*	0.050
Inflation (Core, lagged 1 year)	0.045**	0.020	0.046	0.064
Inflation (Core, lagged 2 years)	0.034***	0.002	0.069***	0.016
QQE1	0.443***	0.020	0.267***	0.066
QQE2	-0.329***	0.035	-0.165	0.098
NIRP	-0.211***	0.025	-0.179**	0.080
YCC	0.190***	0.015	0.191***	0.049
R^2	0.925		0.739	

	Financial + Survey		Survey	
	Estimate	SE	Estimate	SE
(Intercept)	0.352***	0.055	0.889*	0.461
Level (lagged)	0.443***	0.063	0.170	0.357
Inflation (Core-Core)	0.031**	0.014	0.145	0.084
Inflation (Core-Core, lagged 1 year)	0.025	0.027	0.070	0.058
Inflation (Core-Core, lagged 2 years)	0.018	0.019	0.036	0.030
QQE1	0.488***	0.023	0.228	0.167
QQE2	-0.302***	0.048	-0.290***	0.022
NIRP	-0.250***	0.040	-0.181*	0.088
YCC	0.159***	0.033	0.249***	0.045
R^2	0.918		0.7573	

Table B.3: Testing Mon. Policy Changes: Alternative Dummy Specification

	Financial + Survey		Survey	
	Estimate	SE	Estimate	SE
(Intercept)	0.380***	0.075	0.869*	0.432
Level (lagged)	0.226*	0.116	-0.017	0.414
Inflation	0.015	0.032	0.102**	0.042
Inflation (lagged 1 year)	-0.038*	0.021	0.000	0.049
Inflation (lagged 2 years)	0.007	0.016	0.042*	0.022
QQE	0.572***	0.042	0.336***	0.052
NIRP + YCC	0.260***	0.038	0.211***	0.026
R^2	0.886		0.743	
	Financial + Survey		Survey	
	Estimate	SE	Estimate	SE
(Intercept)	0.334***	0.089	0.831***	0.180
Level (lagged)	0.441***	0.113	0.062	0.156
Inflation	0.035	0.023	0.111**	0.050
Inflation (lagged 1 year)	0.037	0.027	0.034	0.067
Inflation (lagged 2 years)	0.030***	0.005	0.054***	0.016
QQE1	0.469***	0.015	0.302***	0.091
QQE2	0.138*	0.065	0.171	0.190
NIRP	-0.091***	0.028	0.015	0.147
YCC	0.093***	0.013	0.152	0.063
R^2	0.925		0.743	

(For this specification, dummy variables = 1 for the periods of the event, 0 otherwise)

Table B.4: Testing Mon. Policy Changes: Alternative Dummy Specification + CPI Core

	Financial + Survey		Survey	
	Estimate	SE	Estimate	SE
(Intercept)	0.386***	0.086	0.822**	0.369
Level (lagged)	0.218*	0.115	0.047	0.348
Inflation (Core)	0.021	0.036	0.086***	0.029
Inflation (Core, lagged 1 year)	-0.035	0.024	-0.002	0.044
Inflation (Core, lagged 2 years)	0.007	0.019	0.050***	0.013
QQE	0.565***	0.050	0.337***	0.042
NIRP + YCC	0.252***	0.039	0.191***	0.025
R^2	0.881		0.713	

	Financial + Survey		Survey	
	Estimate	SE	Estimate	SE
(Intercept)	0.371***	0.063	0.794***	0.165
Level (lagged)	0.410***	0.068	0.141	0.144
Inflation (Core)	0.045**	0.015	0.102*	0.050
Inflation (Core, lagged 1 year)	0.045**	0.020	0.046	0.064
Inflation (Core, lagged 2 years)	0.034***	0.002	0.069***	0.016
QQE1	0.443***	0.020	0.267***	0.066
QQE2	0.113**	0.048	0.102	0.163
NIRP	-0.098***	0.024	-0.077	0.086
YCC	0.092***	0.024	0.114**	0.039
R^2	0.925		0.739	

Table B.5: Testing Mon. Policy Changes: Alt. Dummy Specification + CPI Core Core

	Financial + Survey		Survey	
	Estimate	SE	Estimate	SE
(Intercept)	0.269***	0.080	0.844*	0.467
Level (lagged)	0.354***	0.100	0.068	0.406
Inflation (Core-Core)	-0.020	0.033	0.090	0.054
Inflation (Core-Core, lagged 1 year)	-0.058*	0.029	-0.016	0.051
Inflation (Core-Core, lagged 2 years)	0.017	0.039	0.037	0.029
QQE	0.584***	0.058	0.332***	0.095
NIRP + YCC	0.273***	0.053	0.171***	0.046
R^2	0.878		0.690	

	Financial + Survey		Survey	
	Estimate	SE	Estimate	SE
(Intercept)	0.352***	0.055	0.889*	0.461
Level (lagged)	0.443***	0.063	0.170	0.357
Inflation (Core-Core)	0.031**	0.014	0.145	0.084
Inflation (Core-Core, lagged 1 year)	0.025	0.027	0.070	0.058
Inflation (Core-Core, lagged 2 years)	0.018	0.019	0.036	0.030
QQE1	0.488***	0.023	0.228	0.167
QQE2	0.186**	0.070	-0.061	0.184
NIRP	-0.064	0.046	-0.242**	0.104
YCC	0.095***	0.017	0.007	0.063
R^2	0.918		0.753	

Appendix C

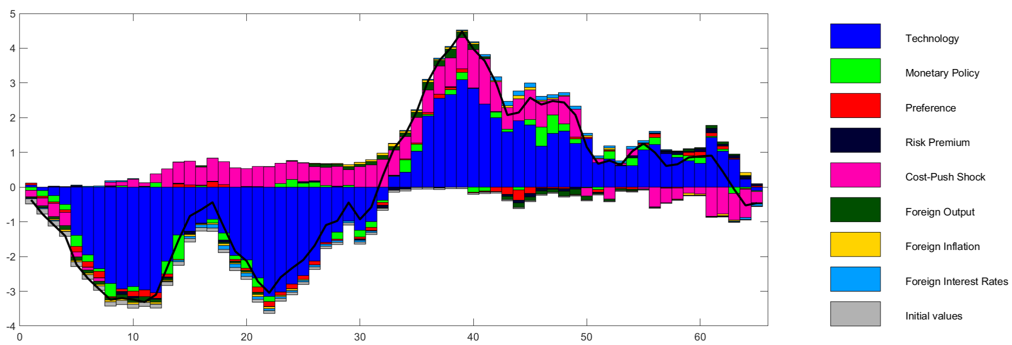


Figure C.1: Historical decomposition of output under determinacy at quarterly rates

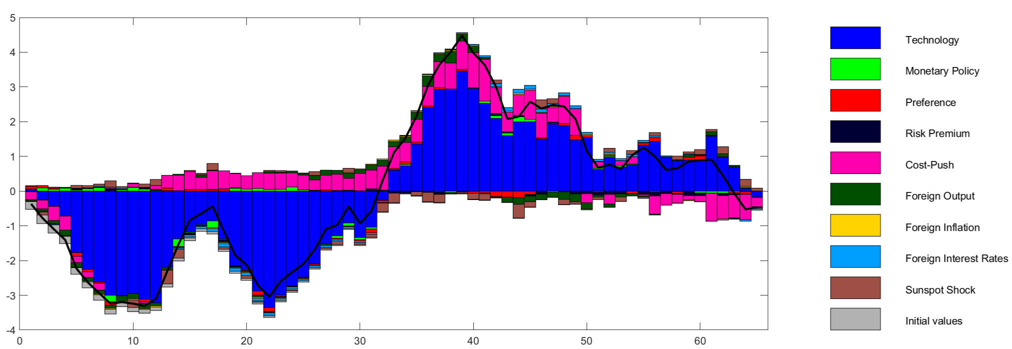


Figure C.2: Historical decomposition of output under indeterminacy at quarterly rates

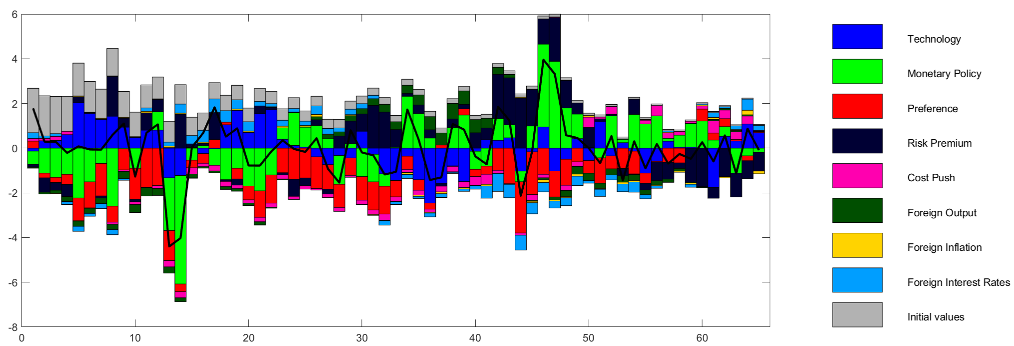


Figure C.3: Historical decomposition of CPI Inflation under determinacy at quarterly rates

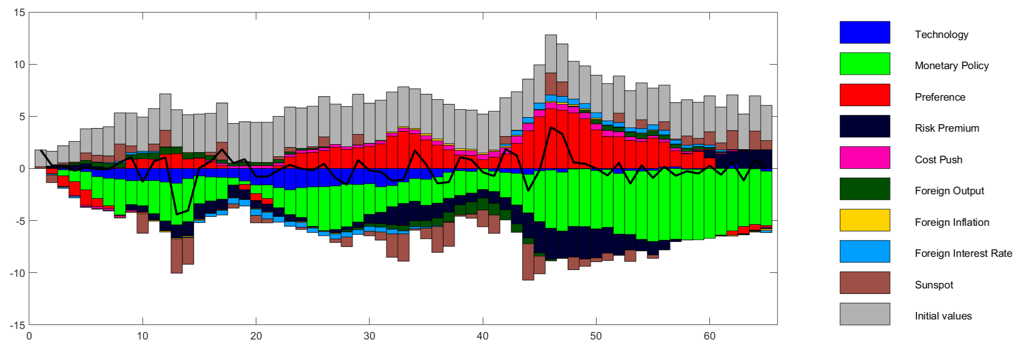


Figure C.4: Historical decomposition of CPI Inflation under indeterminacy at quarterly rates

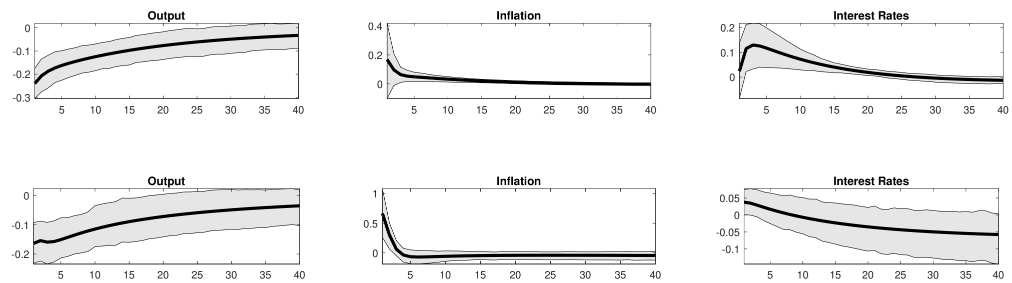


Figure C.5: Bayesian IRF to a cost-push shock under determinacy (top) and indeterminacy (bottom)

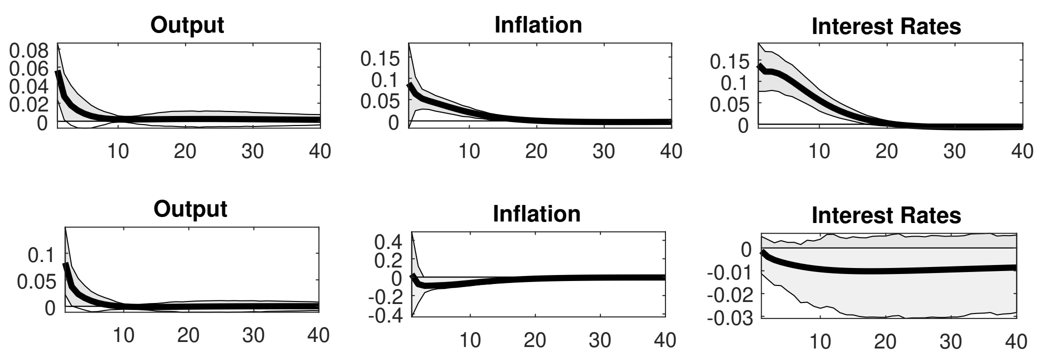


Figure C.6: Bayesian IRF to a shock to foreign output under determinacy (top) and indeterminacy (bottom)

Coefficients			Determinacy			Indeterminacy		
Structural Parameters		Prior	Posterior	90% HPD		Posterior	90% HPD	
Openness	α	B(0.28,0.01)	0.2610	0.2439	0.2778	0.2758	0.2599	0.2917
Inverse intertemporal elasticity of substitution	σ	G(1.2,0.4)	1.3869	0.8493	1.9746	0.6348	0.3324	0.9145
Inverse Frisch	ψ	G(1.5,0.75)	1.3528	0.4290	2.2455	2.2484	0.9154	3.5736
Calvo domestic prices	θ_H	B(0.5,0.1)	0.8311	0.7948	0.8661	0.4540	0.3182	0.5924
Calvo import prices	θ_F	B(0.5,0.1)	0.6780	0.5707	0.7799	0.4353	0.2614	0.6080
Indexation domestic	δ_H	B(0.5,0.25)	0.0398	0.0005	0.0792	0.2272	0.0024	0.4439
Indexation foreign	δ_F	B(0.5,0.25)	0.0450	0.0003	0.0929	0.1699	0.0003	0.3639
Elasticity H-F goods	η	G(1.5,0.75)	0.6784	0.5819	0.7682	0.7243	0.5424	0.8904
Habit	h	B(0.5,0.25)	0.3205	0.1950	0.4555	0.2819	0.0957	0.4688
Taylor rule, smoothing	ψ_r	B(0.5,0.25)	0.7497	0.7033	0.7932	0.9724	0.9474	0.9993
Taylor rule, inflation	ψ_π	G(1.5,0.3)	3.1391	2.5621	3.6653	1.0872	0.8531	1.3163
Taylor rule, output	ψ_y	G(0.25,0.13)	0.1634	0.0552	0.2659	0.2420	0.0542	0.4293
Taylor rule, output growth	$\psi_{\Delta y}$	G(0.25,0.13)	0.6237	0.3417	0.8906	0.2694	0.0578	0.4732
Taylor rule, exchange rate	$\psi_{\Delta e}$	G(0.25,0.13)	0.0210	0.0045	0.0364	0.2429	0.0554	0.4218
Technology	ρ_a	B(0.8,0.1)	0.9471	0.9110	0.9880	0.9238	0.8787	0.9716
Preferences	ρ_g	B(0.8,0.1)	0.9653	0.9421	0.9843	0.8462	0.7402	0.9560
Risk premium	ρ_{rp}	B(0.8,0.1)	0.8769	0.8247	0.9304	0.8464	0.7630	0.9343
Import cost-push shock	ρ_{cp}	B(0.5,0.25)	0.9097	0.8664	0.9543	0.9441	0.8848	0.9971
Foreign Output	ρ_{y*}	N(0.9,0.1)	0.8369	0.7892	0.8863	0.8343	0.7874	0.8816
Foreign Inflation	$\rho_{\pi*}$	N(0.6,0.1)	0.4156	0.2772	0.5512	0.4298	0.2889	0.5645
Foreign intertemporal elasticity of substitution	$\sigma*$	G(1,0.75)	2.3202	1.1595	3.4569	2.7269	1.3283	3.9541
Foreign NKPC slope	$\kappa*$	G(0.1,0.05)	0.0357	0.0079	0.0618	0.0344	0.0094	0.0594
Foreign Taylor rule, smoothing	ψ^*_{r*}	B(0.5,0.2)	0.8775	0.8416	0.9143	0.8739	0.8333	0.9140
Foreign Taylor rule, inflation	$\psi^*_{\pi*}$	G(1.5,0.3)	1.5226	1.0469	1.9823	1.4647	1.0244	1.9133
Foreign Taylor rule, output	ψ^*_{y*}	G(0.25,0.15)	0.3186	0.1630	0.4598	0.3016	0.1505	0.4484
Shocks		Prior	Posterior	90% HPD		Posterior	90% HPD	
sd Technology Shock	ε_a	IG(0.5,0.5)	1.9448	1.3725	2.5120	0.5531	0.4057	0.6947
sd MP Shock	ε_m	IG(0.15,0.5)	0.1849	0.1500	0.2168	0.1707	0.1449	0.1968
sd Preference Shock	ε_g	IG(1,0.5)	6.6399	2.8704	9.5902	2.4585	1.0810	3.7802
sd Risk Premium Shock	ε_{rp}	IG(1,0.5)	0.4060	0.3165	0.4944	0.4815	0.3463	0.6105
sd Cost Push Shock	ε_{cp}	IG(0.15,0.5)	0.8551	0.5194	1.2071	2.7002	0.7934	4.5228
sd Foreign Output	ε_{y*}	IG(0.5,0.5)	0.1625	0.1169	0.2053	0.1537	0.1151	0.1915
sd Foreign Inflation	$\varepsilon_{\pi*}$	IG(0.5,0.5)	0.2155	0.1711	0.2586	0.2120	0.1664	0.2558
sd Foreign MP	ε_{m*}	IG(0.5,0.5)	0.1387	0.1159	0.1600	0.1400	0.1162	0.1631
sd Sunspot	ν	IG(1,0.5)	-	-	-	0.4421	0.3505	0.5290
Correlations		Prior	Posterior	90% HPD		Posterior	90% HPD	
Sunspot, Technology Shock	ν, ε_a	U[-1,1]				-0.2996	-0.5870	-0.0350
Sunspot, MP Shock	ν, ε_m	U[-1,1]				0.2847	0.0829	0.4858
Sunspot, Preference Shock	ν, ε_g	U[-1,1]				-0.1183	-0.6169	0.3573
Sunspot, Risk Premium Shock	ν, ε_{rp}	U[-1,1]				0.1265	-0.3142	0.5668
Sunspot, Cost Push Shock	ν, ε_{cp}	U[-1,1]				0.3681	0.1555	0.5885
Sunspot, Foreign Output	ν, ε_{y*}	U[-1,1]				0.0113	-0.2564	0.2759
Sunspot, Foreign Inflation	$\nu, \varepsilon_{\pi*}$	U[-1,1]				0.1444	-0.0996	0.3972
Sunspot, Foreign MP	ν, ε_{m*}	U[-1,1]				-0.0491	-0.4335	0.3375
Log Marginal Likelihood			**-814.01**			**-753.02**		

Table C.1: Inflation Forecast Rule Horizon 4 quarter ahead - Posterior Estimates

Heterogeneous Expectations in a Simple NK Model

Consider the 3 equation New Keynesian model as in Lubik and Schorfheide (2004):

$$y_t = \tilde{E}_t^j y_{t+1} - \sigma^{-1}(i_t - \tilde{E}_t^j \pi_{t+1}) + g_t$$

$$\pi_t = \beta \tilde{E}_t^j \pi_{t+1} + \kappa y_t + u_t$$

$$r_t = \rho_r r_{t-1} + (1 - \rho_r)(\psi_\pi \pi_t + \psi_y y_t) + \varepsilon_t^m$$

where u_t, g_t are AR(1) processes, and the expectation operator $\tilde{E}_t^j$ for can take on the following forms, as outlined in section 3.6:

$$\tilde{E}_t^1 x_{t+1} = E_t x_{t+1}$$

$$\tilde{E}_t^2 x_{t+1} = n E_t x_{t+1} + (1 - n)\phi^2 x_{t-1}$$

I estimate this model using both types of expectations operators using US data for output, inflation, and interest rates for a sample covering 1980Q1 to 2007Q1. GDP is per capita in log deviations from a linear trend. The inflation series corresponds to the annualized quarterly log-difference in the consumer price index for all goods, and I use the annualized federal funds rate for interest rates. I estimate the model using Bayesian methods as outlined in Herbst and Schorfheide (2015), generating 100,000 draws using a Metropolis-Hastings algorithm, and discarding 50% as a burn-in. Results from this estimation are found in Table C.2. The log marginal likelihood is -437.24 for the model with heterogeneous expectations versus -440.39 for the model with homogeneous expectations. These values imply a Bayes Factor of 23.34 for the model with Heterogeneous expectations, which suggests the model with heterogeneous expectations is strongly supported by the data.

		Homogeneous Expectations			Heterogeneous Expectations		
Structural Parameters	Prior	Posterior Mean	90% HPD Interval		Posterior Mean	90% HPD Interval	
σ	G(2,0.5)	1.562	0.811	2.263	1.343	0.701	1.891
κ	G(0.5,0.2)	0.686	0.363	0.988	1.110	0.668	1.558
ρ_i	B(0.75,0.1)	0.790	0.740	0.841	0.761	0.703	0.8188
ψ_π	G(1.5,0.25)	2.013	1.639	2.394	2.034	1.682	2.382
ψ_y	G(0.25,0.13)	0.336	0.079	0.584	0.306	0.070	0.544
ρ_g	B(0.7,0.1)	0.843	0.793	0.892	0.867	0.817	0.917
ρ_u	B(0.7,0.1)	0.940	0.907	0.973	0.901	0.859	0.943
Heterogeneous Expectations							
n_y	B(0.5,0.2)	-	-	-	0.592	0.290	0.978
n_π	B(0.5,0.2)	-	-	-	0.493	0.187	0.821
ϕ_y	N(1,0.3)	-	-	-	1.222	0.988	1.726
ϕ_π	N(1,0.3)	-	-	-	0.819	0.405	1.215
Shocks							
ε_t^g	IG(0.5,0.5)	0.186	0.140	0.231	0.167	0.122	0.208
ε_t^u	IG(0.5,0.5)	0.650	0.570	0.733	0.637	0.560	0.709
ε_t^m	IG(0.5,0.5)	0.232	0.191	0.272	0.245	0.198	0.288
Log Marginal Likelihood		**-440.39**			**-437.24**		

Table C.2: Simple New Keynesian Model