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EJECTION OF LARGE FRAGMENTS IN HIGH-ENERGY NUCLEAR REACTIONS

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Vitor Pereira Crespo
(Ph.D. Thesis)

September 6, 1961

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ABSTRACT

Several features of the production of Na^{24} and Mg^{28} fragments produced in the interaction of protons and He ions with Cu, Ag, Au, and U have been investigated. Formation cross sections were determined for He ions of different energies between 320 and 880 Mev and for protons of 700 Mev. Thick-target recoil experiments were performed at bombarding energies of 0.7 and 3 Bev for protons, and 880 Mev for He ions. Also given are some recoil measurements of Na^{24} from Al.

Analysis of data obtained with target materials heavier than Al shows that for the bombarding energies used in this work Na^{24} and Mg^{28} are probably produced by the cleavage of the target nucleus into two heavy fragments. One of these fragments has a mass approximately equal to the mass of Na^{24} or Mg^{28} and the other contains most of the remaining mass of the target nucleus. However, Na^{24} and Mg^{28} are very probably not slowly evaporated particles nor products of a slow fission process.

The experimental information covering fragmentation from photographic emulsion studies and radiochemical studies is discussed. The various mechanisms proposed are considered and a new one suggested. According to this new mechanism, Na^{24}, Mg^{28}, and the more energetic fragments observed in nuclear emulsions are ejected promptly from the parent nucleus by very complex nucleon-nucleon cascades and by collective effects.

I. INTRODUCTION

This thesis presents the results of a radiochemical study of the mechanism of high-energy reactions. In particular we study the mechanism by which relatively complex aggregates of nucleons such as Na^{24} and Mg^{28} are produced during the bombardment of Al, Cu, Ag, Au, and U by charged particles of hundreds or thousands of Mev energy.

Before we outline the purposes, methods, and results of this study, it is necessary to review a number of general features of high-energy reactions, and to consider the findings of previous research made by radiochemical, nuclear emulsion, and other techniques.

The main features of nuclear reactions induced by high-energy particles (more than 100 Mev) have been described by a mechanism proposed by Serber,[1] in which the nucleus is considered to be a degenerate Fermi gas in which the high-energy incoming nucleon has a mean free path comparable to the nuclear radius. Because of its large mean free path, the bombarding nucleon may pass right through the nucleus, or it may collide with one or more nucleons. Except for the restrictions imposed by the exclusion principle, any collision is to be treated as a collision of two free nucleons. The particles involved in such a collision may escape the nucleus or may strike other nucleons. This prompt cascade will continue until all the involved particles either leave the nucleus or are slowed down to such a degree that they cannot directly escape.

Those nucleons which do not escape share their energy with the residual nucleus. This excited nucleus in turn loses its excess energy by evaporating a number of particles. Weisskopf's treatment of the statistical theory[2] leads us to expect evaporated particles with a modified Maxwellian distribution of energy, in most cases having a center-of-mass angular distribution symmetric about 90 deg.[3] The statistical theory predicts that most of the evaporated particles are nucleons, deuterons, and alpha particles, but does not exclude the possibility of emission of heavier aggregates. Indeed, calculations have been made[3-7] of the probability of emission of particles with $Z \geq 3$.

We designate as fragments those clusters of nucleons produced in high-energy nuclear reactions which are heavier than an α particle and lighter than the lightest species produced as a residue of a cascade-plus-evaporation process, and that cannot be identified beyond doubt as fission products. The process or processes by which they are produced are called, for the sake of brevity, fragmentation. In the framework of the cascade-evaporation model of nuclear reactions, fragments have to be produced as evaporated particles. Other possible mechanisms for the production of fragments are discussed later.

Fragment production has been studied by two different techniques: nuclear emulsions and radiochemistry. A brief review of the results obtained is given, in which nuclear emulsion studies and radiochemical studies are treated separately, since the masses of the fragments dealt with by the two methods have been different, and the mechanism responsible for the fragments observed in nuclear emulsions may not be the one that produces those identified in radiochemical studies.

In the nuclear emulsion studies, the target materials most frequently used have been the heavier constituents of nuclear emulsion, silver and bromine. In a few cases, however, other target materials have been used, either as external targets or in loaded emulsions. Using the latter technique, Denisenko et al.[8] studied uranium fission accompanied by production of fragments; by means of external target techniques, Katcoff[9] has studied the Li[8] fragment production from Cu, Ag, Au, and U; and Deutsch[10] has studied fragmentation in Ni, Ag, Au, and U. As projectiles, protons ranging from 100 Mev to 9 Bev[9-22] have been used as well as π^-[20,23-26] mesons, π^+ mesons,[27] fast neutrons,[28,29] deuterons,[10] α particles,[10] and cosmic rays.[20,30-36]

As a parenthetical comment on the last set of references, it should be noted that many investigators saw heavy fragments in the early work with cosmic rays, but owing to very poor statistics their observations do not give much information about the formative mechanism of the "heavy splinters". The works quoted give these early references.

Generally speaking, it is impossible to identify the masses of the fragments (Z>2), and difficult to identify their charges. Only Li^8, Be^8, and Li^9 are readily identifiable, through the two α particles that result from their decay. Since the cross section of formation of fragments decreases exponentially with Z of the fragment,[14,15,18,21,31] and because Li^8 is the fragment more easily identified, one finds that most of the data obtained with nuclear emulsions pertain to Z=3 fragments. Through nuclear emulsion techniques it is possible to study the energy spectra and angular distributions of the fragments. The observed spectra show a large number of forward-peaked high-energy particles. Analysis of the spectra in the framework of the evaporation theory requires temperatures greater than 10 Mev,[20,26,36] too high to be acceptable. Also, the forward peaking of the more energetic fragments is not clearly understood. Katcoff found that the spectrum of Li^8 produced in the interaction of 2.2-Bev protons with silver agrees with that calculated by using the cascade evaporation model, but not the Li^8 spectra from Cu and Au.[9] These results are contrary to the assertion made heretofore that fragments have to be produced as evaporated particles. This inability to explain the results fully by the cascade-evaporation model of nuclear reactions has led many workers to suggest that the more energetic fragments may arise directly from the prompt cascade process.[10,12,14,18,21,26,30,32,34] A behavior similar to that found for $Z \geq 3$ fragments has also been observed for some helium particles.[23,28,34,37-42]

Radiochemical techniques are best suited for studying particular radionuclides that show convenient radioactive properties. With the exception of part of this work, only the total production cross section of low-Z products has been measured radiochemically. The products that have been observed are He^6,(43) Be^7,(44-52) C^{11},(45-47,53,54,61,63) N^{13},(7) F^{18},(45-47,52,53,55-57,61,69) Na^{22},(46-49,54,56,58,59,69) Na^{24},(45-49,53,54,56-67,69) Mg^{28},(45-49,56,65,69) Si^{31},(47-48,59,63) P^{32},(46-49,56,58,59,62-64,67,68,61,69) and P^{33}.(46-49,56,61,69) Among the data available we should single out the determinations of the excitation functions for the production of F^{18}, Na^{24}, Mg^{28}, and P^{32} from lead[56], and the studies at

one or more energies of the cross-section dependence on the mass number of the target nucleus for He^6,[43] Be^7,[44,50] N^{13},[7] F^{18},[55,57] Na^{24},[57,67] and P^{32}.[67] These works represent a systematic study of the behavior of products of nuclear reactions lying in the mass region where fragmentation is known to occur. Another study of fragmentation is the determination by Wright of Li^8 cross-section dependence on the target mass number.[70] The excitation functions show a sharp rise from about 200 Mev up to about 2 Bev and then become almost constant.[43,50,56,57,67] This same behavior is observed as well with the fragments identified by nuclear emulsion techniques.[22] It has also been found that, for the same bombarding energy, the cross sections of F^{18}, Na^{24}, and P^{32}[55,57,67] decrease with increasing mass number of target material up to about mass 140, and then increase with increasing target mass (see Figs. 15,16,17). On the other hand, the cross section of He^6,[43] Li^8,[70] Be^7,[44,50] and N^{13},[7] do not clearly show the same trend. These always seem to increase or decrease with increasing target mass number. To explain the excitation functions they obtained, Wolfgang et al. proposed a mechanism for fragmentation.[56] According to these authors, fragmentation is a fast process involving creation and reabsorption of mesons. The absorbed meson creates hot spots and gives rise to fragment production. The dependence of F^{18} and Na^{24} cross sections on the mass number of the target nucleus was explained by Caretto and co-workers,[57] assuming that different mechanisms take place in different target mass regions: spallation in the Cu region, fission and spallation in the Ag region, and fragmentation by the Wolfgang mechanism in the Au-U region.

The cross sections for production of a group of fragments have been calculated by using the evaporation theory.[3-7,14] Agreement between the calculated cross sections and the measured ones is in general poor, although acceptable in a few cases.

The study of secondary reactions[71-76,54,58] can also give information about the behavior of low-Z products. Secondary reactions result from the interaction of the target nucleus with particles heavier than the bombarding particle. These heavy projectiles are themselves products

of the disintegration of the target nucleus. In order to react with a nucleus identical to the one from which they are formed, these heavy bombarding particles have to be emitted with energies exceeding the Coulomb barrier. The super-barrier particles have been considered to be evaporated particles by some workers[75] but other workers point out inconsistencies with the evaporation mechanism.[76]

Fragmentation has been recently reviewed by Perfilov et al.[77] Some features have also been reviewed by Camerini et al.,[78] Hudis and Miller,[79] and Lavrukhina.[80]

The work presented here was undertaken to obtain more information about the fragmentation mechanism. The recoil properties of Na^{24} and Mg^{28} originating from the interaction of 700-Mev protons, 3-Bev protons, and 880-Mev He ions with Cu, Ag, Au, and U have been measured by using thick-target techniques.[81] Thick-target recoil ranges of Na^{24} produced in aluminum have also been determined. Formation cross sections of Na^{24} and Mg^{28} have also been measured for Cu, Ag, Au, and U irradiated with 880-, 700-, 500-, and 320-Mev He ions and with 700-Mev protons. The results are analyzed with a simple model of the nuclear reactions.

Sodium and magnesium have been chosen as products mainly because of their convenient radioactive properties. Also, they are the lowest-mass products for which it is possible to get suitable recoil-catcher materials.

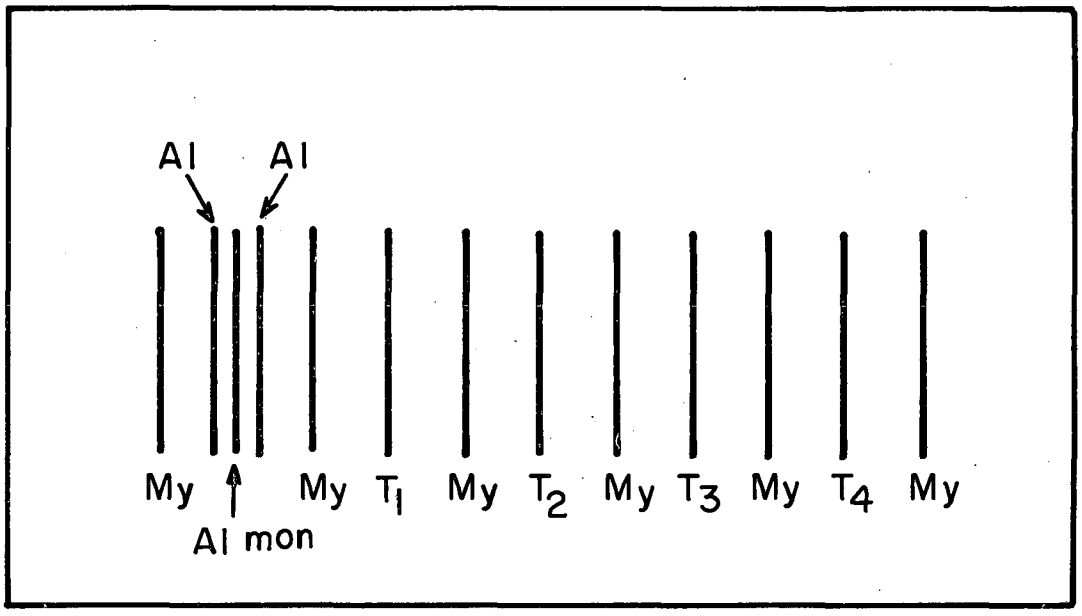
II. EXPERIMENTAL PROCEDURES

A. Target Assemblies

Two types of target assembly were used in these experiments. We shall refer to them for convenience as the cross-section assembly and the recoil assembly.

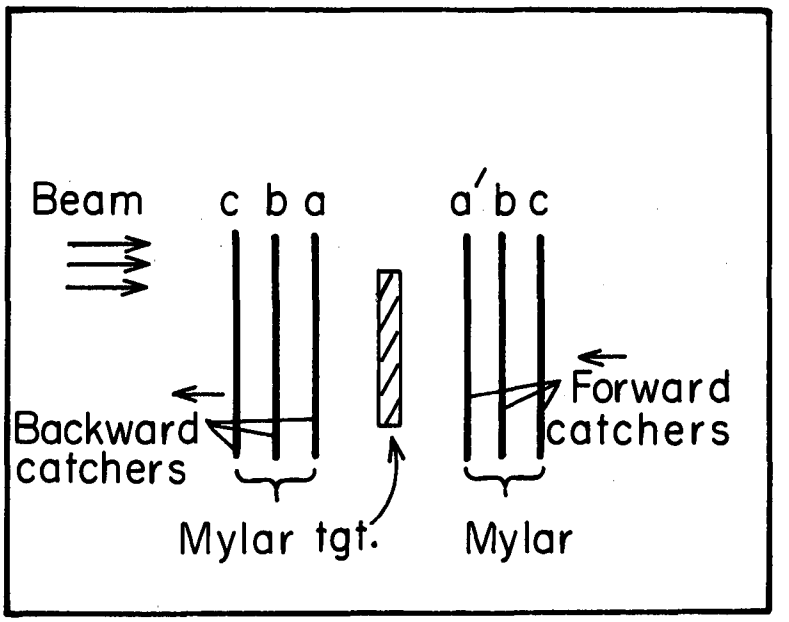
The cross-section assembly consisted of a stack of 1.5x2-cm foils mounted as shown in Fig. 1. The target foils T_1, T_2, T_3, and T_4 were separated from one another by 0.003-in. Mylar foils (My). All cross sections were measured relative to Na^{24} produced in the aluminum monitor foil (Al-mon). In an actual experiment the number of target foils used varied from one to four. The Copper target foils used were 0.002 in. thick; all others were 0.001 in. thick. Spectrochemical analyses of the target foils showed the following contaminations: copper – 0.03% Al; silver – 0.1% Cu and 0.02% Al; gold – 0.3% Cu; uranium – 100 ppm Si and 10 ppm Cu, Co, and Mg; aluminum – 0.05% Cu and Fe. The stack was mounted in target holders commonly used at Berkeley. Because most of the beam hits the leading edge of the target assembly, it is necessary and critical to obtain a good alignment of the different foils in the stack, in order to ensure that the same beam flux is received by the monitor and also the different target foils. To ensure proper alignment, the leading edge was machined after the stack was fastened to the target holder. The stack was then wrapped in 0.0001-in. Mylar.

The recoil assembly is shown in Fig. 2. The target foil (T) had dimensions 1.5x2 cm, and the catcher foils (a, a') as well as the blank foils (b) and guard foils (c) had dimensions 1.9x2.2 cm. The Mylar foils (My) were always 0.003 in. thick (much thicker than the recoil ranges of Na^{24} or Mg^{28}). The production of Na^{24} and Mg^{28} by activation of impurities in the Mylar was measured by analysis of the blank foils. Recoils in the forward and backward directions were studied by making the normal to the plane of the stack parallel to the beam. Recoils predominantly perpendicular to the beam were studied in experiments with an 80 deg angle between the beam and to the normal target. The standard Bevatron target holders were used for mounting



MU-24559

Fig. 1. The cross-section assembly. T_1, T_2, T_3, and T_4 are the target foils separated by 0.003-in.-thick Mylar spacers (My); Al_{mon} is the Al monitor foil. The other Al foils are guard foils.



MU-24554

Fig. 2. The recoil assembly. The recoils generated in the target foil T are caught in the catcher foils a and a'. Foils b are used as blank foils; foils c are guard foils. All the foils are 0.003-in.-thick Mylar.

targets either parallel or nearly perpendicular (80 deg) to the beam. In the perpendicular experiments at the 184-inch cyclotron a special target holder, shown in Fig. 3, was used.

All foils were cleaned before mounting for irradiation. Copper and gold foils were cleaned with dilute nitric acid, uranium foils with 2 to 6 M nitric acid, and silver foils with ammonia. All foils were then washed with acetone and rinsed with distilled water.

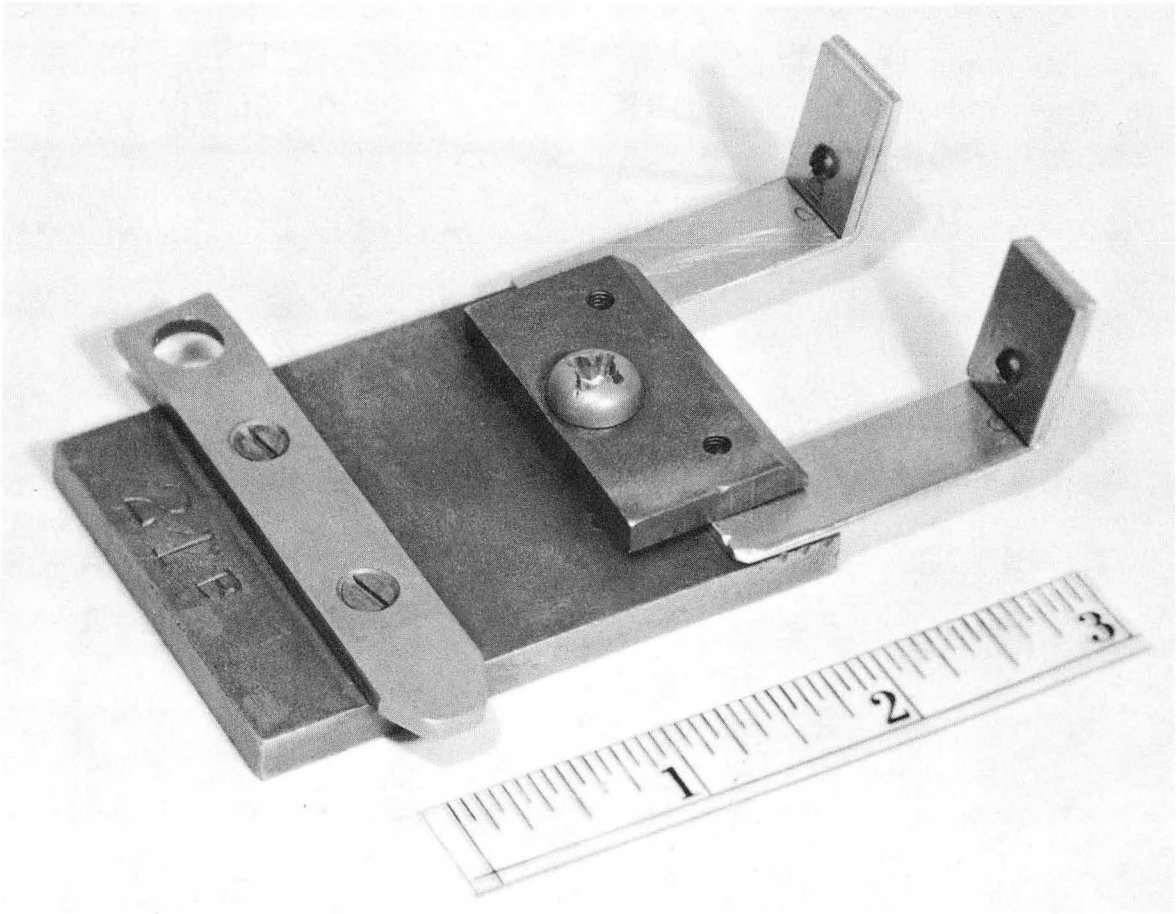
B. Irradiations and Beam Monitoring

Irradiations were performed at the 184-inch cyclotron and the Bevatron. Irradiation times varied between 1 and 1.5 hours.

The Na^{24} produced in the reactions $Al^{27}(p,3pn)Na^{24}$ and $Al^{27}(\alpha,\alpha 2pn)Na^{24}$ was used as a beam monitor. The monitor foil was cut and mounted so as to have very nearly identical counting conditions as the radiochemical samples. The monitor cross sections were taken as 10.7 mb for $Al^{27}(p,3pn)Na^{24}$, and 24.0 mb for $Al^{27}(\alpha,\alpha 2pn)Na^{24}$ reactions, for all energies. The value of 24 mb for the $Al^{27}(\alpha,\alpha 2pn)Na^{24}$ reaction is the value measured for 380-Mev He ions.[82,83] However, since this cross section has not been measured above 380 Mev, some error may have been introduced into the cross sections obtained with high-energy He ions.

C. Chemistry

The chemical procedures used were adaptations of standard radiochemical methods.[49,84] The metal foils were dissolved with appropriate acids and the plastic foils destroyed with a hot mixture of sulfuric and nitric acids in the presence of 10 mg of Na carrier and 5 mg of Mg carrier. The sulfuric-nitric acid mixture was evaporated to dryness and the residue dissolved in distilled water. The target materials were removed by precipitation of copper sulfide, silver chloride, or uranium tetroxide,[85] or by extraction of gold with ethyl acetate. The volume of all samples was made 10 milliliters. Iron hydroxide was precipitated once in the presence of ammonium chloride



ZN-2885

Fig. 3. Target holder used in the perpendicular experiments at the 184-inch cyclotron.

to scavenge out unwanted hydroxide-insoluble contaminants, and then copper, antimony, and nickel sulfides were precipitated. The excess of hydrogen sulfide was removed and another precipitation of iron hydroxide was performed. Magnesium was precipitated with 8-hydroxyquinoline and the solution kept for sodium analysis. The magnesium hydroxyquinolate was destroyed. The hydroxide and sulfide precipitations were repeated. The calcium, strontium, and barium oxalates were twice precipitated, and the solutions evaporated to dryness. The residue was taken up in water, and strontium sulphate was precipitated (hP=6). Iron hydroxide was precipitated and the solution filtered. Magnesium was precipitated with 8-hydroxyquinoline, filtered, dried at 110°C, weighed, and mounted.

The solution containing the sodium fraction was treated with a mixture of benzene and 1-butanol. The organic solution was discarded and the aqueous solution evaporated to dryness. The residue was dissolved in water. Sodium was precipitated as sodium uranyl magnesium acetate and converted to sodium chloride with hot 1-butanol saturated with dry HCl. This step was repeated. The sodium chloride precipitate was treated with concentrated perchloric acid, 5 mg of potassium chloride was added, and the solution was evaporated to dryness. Sodium perchlorate was extracted with 1-butanol and converted into sodium chloride with 1-butanol saturated with hydrogen chloride. The sodium chloride was filtered, dried at 110°C, weighed, and mounted.

Whenever only sodium was taken from a particular run, the sodium chemistry used was either the one just described, or the one given in Ref. 84. The filtration apparatus has been described by Blann.[86] In part of this work, glass filter pads were used to minimize weighing errors due to absorption of moisture.

D. Mounting and Counting of Samples

The samples were mounted in the centers of aluminum plates (350 mg/cm^2) and covered with 0.1-mil pliofilm. Double-sided Scotch tape was used to adhere the filter paper to the aluminum plate and to keep the pliofilm cover in place. The aluminum plates were visually inspected and the defective ones rejected.

The samples were counted, in several counters in rotation, on the third shelf (0.9 cm from the 2.54-cm-diam end-window) of β proportional counter. This procedure eliminated the need for decay and efficiency corrections, and provided a severe test of counter performance. The counters used were considered operative only if the following tests gave satisfactory results: (a) that the 0.001-in. uranium standard counted within ±0.75% of the average counting rate, and (b) that any background variation was equally observed in all counters. Use of this relative criterion for the background was found advisable, since all counters showed similar variations of as much as one count per minute (from the average of 7.3 counts/min). These large variations were correlated with the operation of the near-by Bevatron. A counter was considered inoperative whenever it showed small variations of the plateau characteristics. An acceptable plateau was about 400 v wide with a slope less than 0.5% per 100 volts.

In the sodium recoil experiments, the chemical yields of all samples were very similar, making counting-efficiency differences negligible. In the magnesium determinations that was not always the case. It was assumed that within the range of sample thickness (4 to 16 mg/cm^2) the magnesium counting efficiency was constant (see Ref. 86).

In the cross-section determinations, however, it was necessary to apply counting-efficiency corrections; Na^{24} counting efficiencies as a function of sample thickness were obtained by bombarding a stack of aluminum foils of different thicknesses, counting the foils in the standard way, and observing the counting rate per unit thickness as a function of foil thickness. The results are shown in Fig. 4. The relative efficiencies of Na^{24} and of Mg^{28} and its daughter (Al^{28}) were taken from Bayhurst and Prestwood.[87]

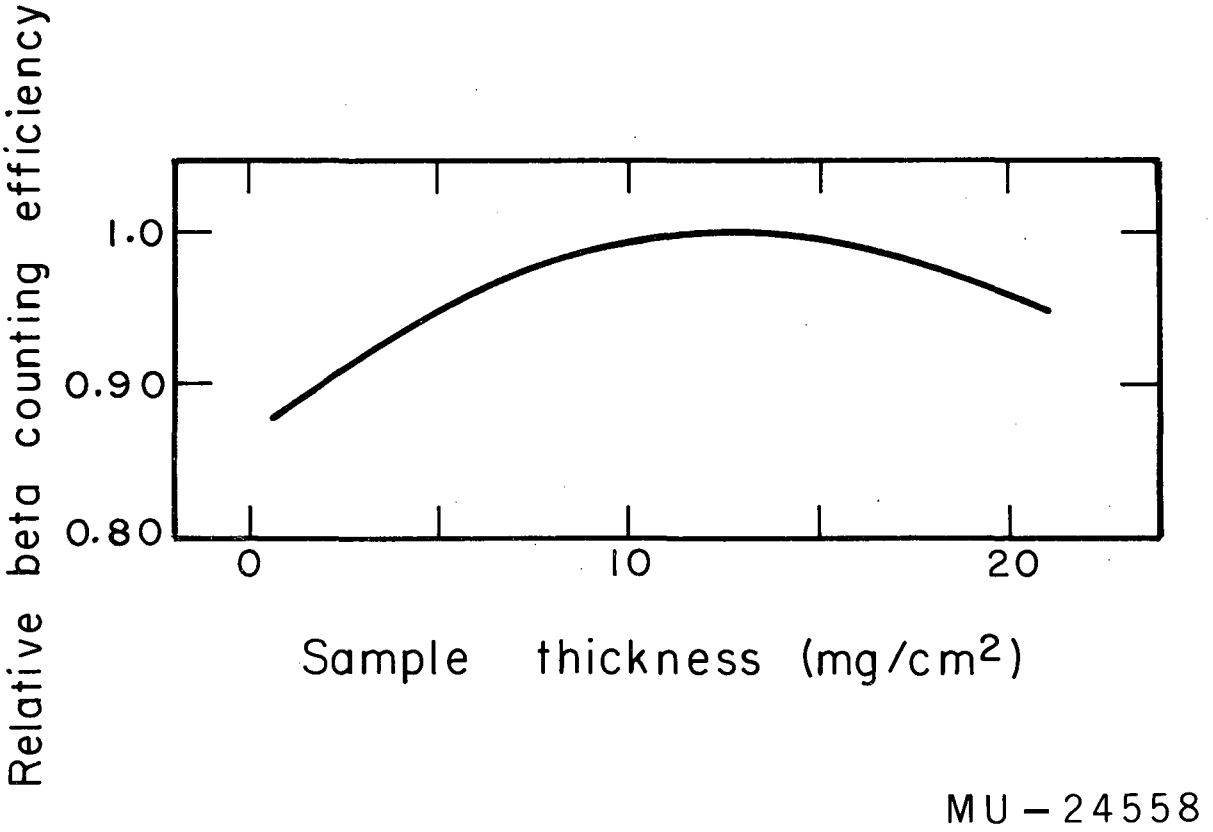


Fig. 4. Relative beta-counting efficiency vs sample thickness for Na^{24} on shelf 3.

E. Experimental results and analysis

In this section are presented the experimental measurements—the cross sections and thick-target recoil properties of Na^{24} and Mg^{28} formed in high-energy irradiations of Al, Cu, Ag, Au, and U. The cross-section data are given first, followed by a brief description of the thick-target recoil method. The data from the recoil experiments are presented and analyzed with certain simplifying assumptions.

The data are analyzed in terms of a fast-slow reaction as follows: A fast process imparts energy and velocity to the struck nucleus. A slow process de-excites the struck nucleus, and gives an additional velocity component to the final product. From the excitation functions it is possible to make estimates of the average deposition energy from the fast process. From the recoil data one may obtain the average values of the velocities imparted by the fast process and by the slow de-excitation. The average velocity imparted by the fast process may be related to the energy deposited and thus a comparison of the recoil data and cross-section data can be made.

F. Cross-Section Data

Cross sections for the production of Na^{24} and Mg^{28} from Cu, Ag, Au, and U are listed in Table I. Column 1 gives the target element; columns 3 to 7 give cross sections for the various bombarding energies and particles. Also shown are the cross-section ratios of Na^{24} to Mg^{28}, $\sigma\ Mg^{28}/Na^{24}$. The number of determinations made of any particular cross-section is given in parentheses after the cross-section value. The numbers given are the average values, and the errors quoted are the standard error

$$\sigma_x = \left(\frac{1}{n(n-1)} \sum_{1}^{n} (x_i - \bar{x})^2 \right)^{1/2},$$

where n is the number of determinations, x_i the measured values, and $\bar{x}$ the average of the measured values.

Table I

Formation cross sections (in mb) of Na^{24} and Mg^{28} from various target elements

		alphas				protons	
		320 Mev α	500 Mev α	700 Mev α	880 Mev α	700 Mev p	5.7 Bev p
Cu	σ Na^{24}	8.26×10^{-2}(1)	$2.80\pm0.13\times10^{-1}$(3)	$6.98\pm0.13\times10^{-1}$(3)	1.38 ± 0.57 (3)	3.68×10^{-1}(1)	
	σ Mg^{28}	1.15×10^{-2}(1)	0.37×10^{-1}(1)	0.91×10^{-1}(1)	0.19 (2)	0.49×10^{-1}(1)	
	$\sigma \frac{Mg^{28}}{Na^{24}}$	0.14	0.13	0.13	0.13	0.12	
	$\sigma \frac{Na^{24}}{Na^{22}}$					7.44[a]	1.65[b]
Ag	σ Na^{24}	3.25×10^{-2}(1)	$9.52\pm0.72\times10^{-2}$(3)	$2.27\pm0.005\times10^{-1}$(3)	$4.46\pm0.02\times10^{-1}$(3)	1.00×10^{-1}(1)	
	σ Mg^{28}	0.39×10^{-2}(1)	1.10×10^{-2}(1)	0.26×10^{-1}(1)	0.49×10^{-1}(2)	0.12×10^{-1}(1)	
	$\sigma \frac{Mg^{28}}{Na^{24}}$	0.12	0.12	0.12	0.11	0.12	
Au	σ Na^{24}	3.23×10^{-2}(1)	$10.68\pm0.28\times10^{-2}$(3)	$3.08\pm0.87\times10^{-1}$(3)	$5.94\pm0.16\times10^{-1}$(3)	1.35×10^{-1}(1)	
	σ Mg^{28}	2.07×10^{-2}(1)	6.24×10^{-2}(1)	1.02×10^{-1}(1)	1.98×10^{-1}(2)	0.54×10^{-1}(1)	
	$\sigma \frac{Mg^{28}}{Na^{24}}$	0.64	0.58	0.33	0.33	0.23	
	$\sigma \frac{Na^{24}}{Na^{22}}$						5.29

(Contined)

Table I (Continued)

		alphas				protons	
		320 Mev α	500 Mev α	700 Mev α	880 Mev α	700 Mev p	5.7 Bev p
U	$\sigma\ Na^{24}$	9.41×10^{-2}(1)	$21.02 \pm 1.38 \times 10^{-2}$(2)	$5.02 \pm 0.05 \times 10^{-1}$(2)	$8.75 \pm 0.25 \times 10^{-1}$(3)	2.30×10^{-1}(1)	
	$\sigma\ Mg^{28}$	6.42×10^{-2}(1)	12.29×10^{-2}(1)	2.38×10^{-1}(1)	4.50×10^{-1}(2)	1.15×10^{-1}(1)	
	$\sigma \frac{Mg^{28}}{Na^{24}}$	0.68	0.58	0.47	0.51	0.32	
	$\sigma \frac{Na^{24}}{Na^{22}}$						5.00[c]

a. Reference 54.

b. Reference 47

c. Reference 48

The Na^{24} data are considered more accurate than the Mg^{28} data because of higher counting rates, more reproducible chemical yields, and higher radiochemical purity for Na^{24}. In very few cases a small amount of radioactive impurity was observed in the Mg^{28} samples. In these cases, the Mg^{28} activity was obtained by analysis of the decay curve.

In Figs. 5 and 6, the Na^{24} and Mg^{28} cross sections, respectively, are plotted vs α-particle bombarding energy for the different targets. Figure 7 contains a similar plot for Na^{24} from proton bombardments (obtained from data found in the literature). As shown in Figs. 5 through 7, the probability of formation of Na^{24} and Mg^{28} increases very rapidly with increasing bombarding energy, for all targets, in the region of hundreds of Mev. At very high bombarding energies the cross sections of formation are relatively more constant. Similar results have been observed for other fragments.[22,43,50,56,57,67]

At 700 Mev the yields of Na^{24} and Mg^{28} obtained with He-ion bombardments are twice as large as those obtained with protons of the same energy. Since the monitor cross section is not known for He ions of this energy, little significance can be attached to this observation. The cross sections obtained with 700-Mev protons in this study are in agreement with data for those obtained with 660 to 680-Mev protons[54,67] by other workers. Values of $\sigma\ Na^{24}/Mg^{28}$ are larger for the heavier targets (Au,U) than for the lighter ones (Cu,Ag), and are, within experimental error, independent of the bombarding energy. This same behavior was found for the ratios $\sigma\ Na^{24}/F^{18}$,[57] $\sigma\ Na^{24}/Na^{22}$ (Table I), and $\sigma\ Na^{24}/P^{32}$.[67]

G. The Thick-Target Recoil Experiments

Sugarman and co-workers have initiated a very simple technique for studying recoil properties of nuclear reaction products.[81,88] In this method, a thick target is sandwiched between thick recoil-catcher foils. The stack is irradiated and a radiochemical analysis made to determine the relative activities of the product in question that are brought to rest in the target and catcher foils. Foil stacks are exposed both perpendicular and parallel to the beam direction. The average components

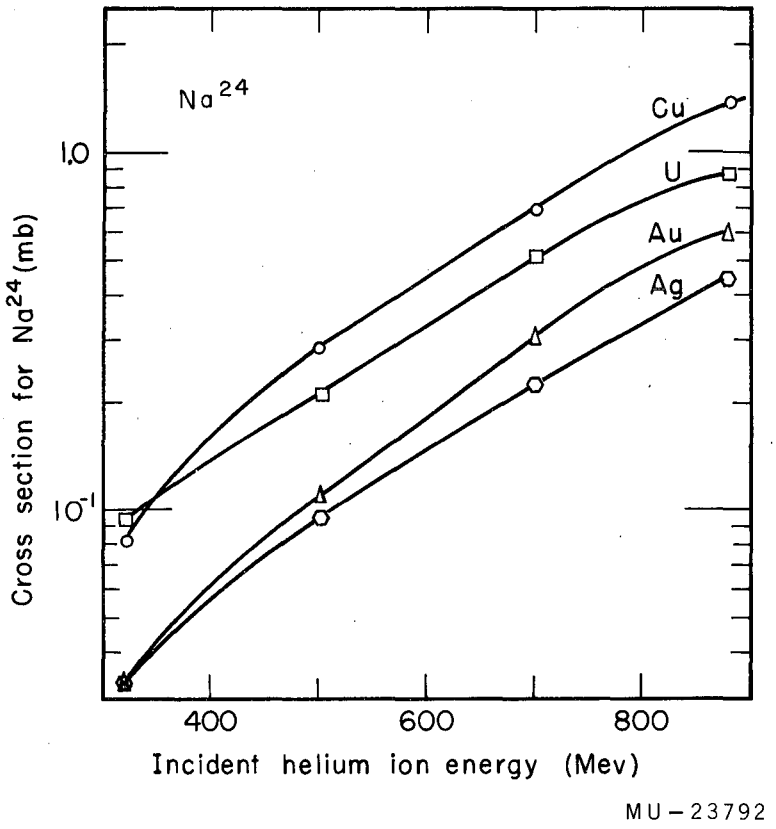


Fig. 5. Excitation functions for the production of Na^{24} from He ion bombardments of Cu, Ag, Au, and U.

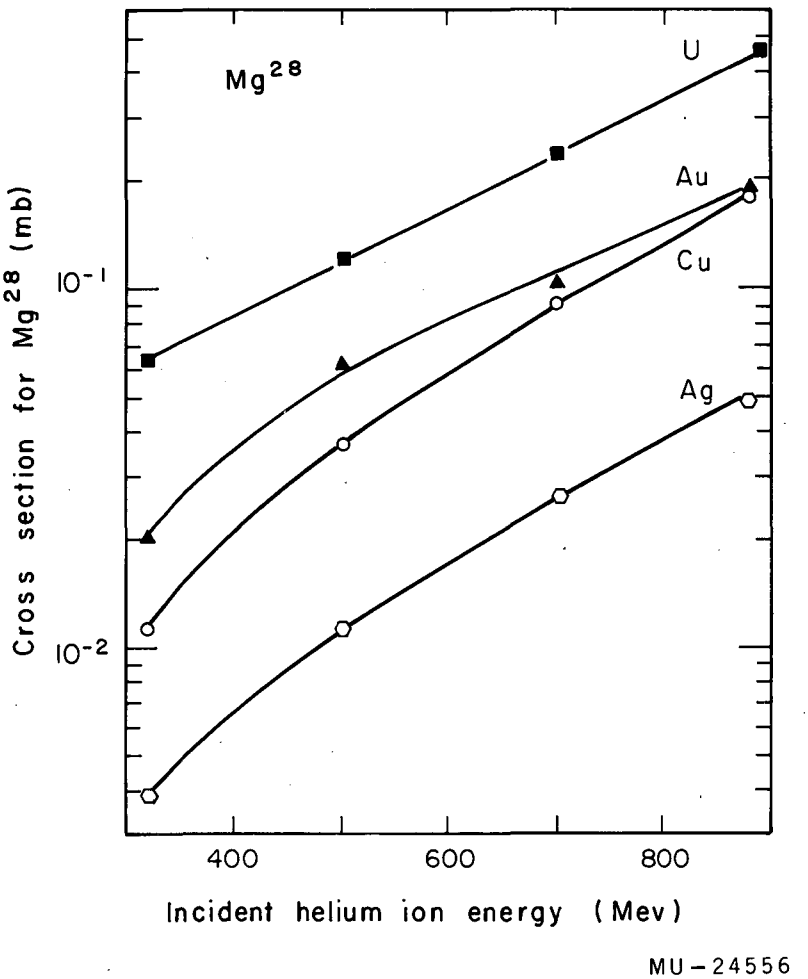


Fig. 6. Excitation functions for the production of Mg^{28} from He ion bombardments of Cu, Ag, Au, and U.

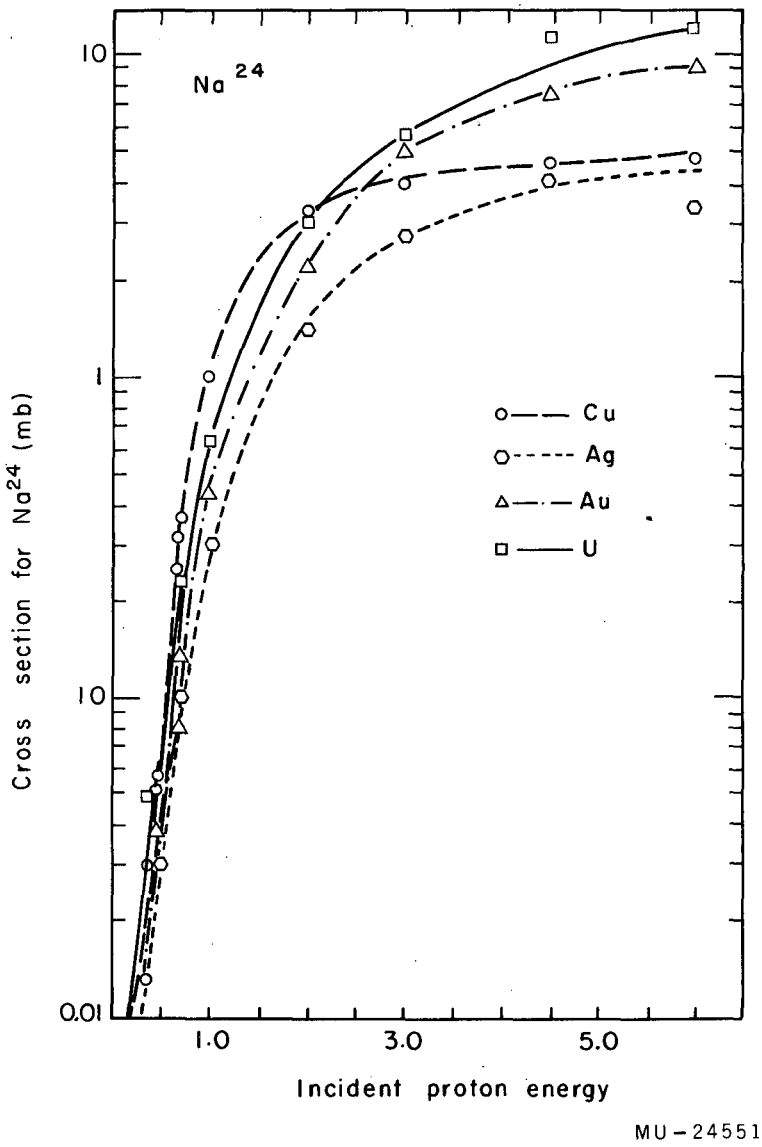


Fig. 7. Excitation functions for the production of Na^{24} from proton bombardments of Cu, Ag, Au, and U. Data from 1 to 5.9 Bev from Ref. 57. Data for Cu are from Refs. 53, 54, 58, 67; for Ag from Refs. 45 and 63; for Au from Ref. 67; and for U from Refs. 64 and 66. The 700-Mev points are from this work.

of the recoil range are measured (a) along the beam direction, (b) opposite to the beam direction, and (c) in a direction perpendicular to that of the beam. These values refer to the range in the material of the target foil. These average components of the recoil range give information about the velocities of the product if the range-energy relationship is known and if an assumption is made concerning the angular distribution of the products.

The experimentally measured quantities for targets exposed to a beam parallel to the normal to the target plane are (a) the target foil thickness W (in mg/cm^2), (b) the fraction F_T of the activity of a given radionuclide remaining in the target foil, and (c) the corresponding fractions F_F and F_B that leave the target foil and are collected in the forward and backward catchers. Similar quantities are measured for targets exposed at 80 deg to the beam, which are designated F_{TP}, F_{FP}, and F_{BP}. The recoil activities were obtained by measuring the activity found in the catchers, and subtracting the activity found in the blanks after correcting for chemical yield and possible thickness differences between blanks and catchers.

In a true perpendicular experiment, F_{PF} should be equal to F_{PB}. The differences found are due to the residual forward-backward effects. The perpendicular recoil fraction will be taken as $F_P = (F_{PF} + F_{PB})/2$.

The analysis of data is made with the following assumptions:

(a) that the bombarding particle imparts to the target nucleus a velocity $v_{\parallel}$ in the laboratory system along the beam direction, and a velocity $v_{\perp}$ in a direction perpendicular to the beam;

(b) that the fragment receives an additional velocity component, $\vec{V}$, from the process that leads to its formation (the velocity $\vec{V}$ reflecting the intrinsic kinetic energy of the fragment in the frame of reference of the moving nucleus);

(c) that the range of the fragments R is proportional to the vectorial sum of $\vec{V}$ and $\vec{v}$ ($\vec{v} = v_{\parallel}\hat{i} + v_{\perp}\hat{j}$, where $\hat{i}$ and $\hat{j}$ are two unit vectors parallel and perpendicular to the beam direction respectively), and the range is given by $R = k\,(|\vec{v} + \vec{V}|)^N$, k and N being constants;

(d) that the angular distribution $\Omega(\theta)$ of $\vec{V}$ in the moving frame is given by $\Omega(\theta) = a + b \cos^2\theta$;

(e) the magnitude of $\vec{v}$ and $\vec{V}$ are unique; and

(f) that the path of the fragment is a straight line.

N. Sugarman[89] and L. Winsberg[90] have derived the necessary equations for the analysis of these experiments with the above assumptions. Using the notation $R_o = k V^N$, $\eta_{\parallel} = v_{\parallel}/V$ and $\eta_{\perp} = v_{\perp}/V$, and neglecting terms of order larger than $\eta_{\parallel}^2$, $\eta_{\perp}^2$, and (b/a), one gets for $\eta \leq 1$ [89]

$$F_F = \frac{R_o}{4W}\left\{1 + \frac{1}{6}\frac{b}{a} + \eta_{\parallel}\left[2/3\ (N+2) + \frac{b}{a}\ 8\ \frac{(N-1)}{45}\right] + \eta_{\parallel}^2\left[\frac{(N+1)^2}{4} + \frac{b}{a}\ \frac{N^2-N-4}{12}\right] + \right.$$

$$\left. + \eta_{\perp}^2\left[\frac{N^2-1}{8} + \frac{b}{a}\ \frac{N-1}{12}\right]\right\}, \qquad (1)$$

$$F_B = \frac{R_o}{4W}\left\{1 + \frac{1}{6}\frac{b}{a} - \eta_{\parallel}\left[2/3\ (N+2) + \frac{b}{a}\ 8\ \frac{(N-1)}{45}\right] + \eta_{\parallel}^2\left[\frac{(N+1)^2}{4} + \frac{b}{a}\ \frac{N^2-N-4}{12}\right] + \right.$$

$$\left. + \eta_{\perp}^2\left[\frac{N^2-1}{8} + \frac{b}{a}\ \frac{N-1}{12}\right]\right\}, \qquad (2)$$

and

$$F_P = \frac{R_{oP}}{4W}\left\{1 - \frac{1}{12}\frac{b}{a} + \eta_{\perp}^2\ \frac{(N+1)}{16}\left((3N+1) + \frac{b}{a}\left(\frac{7-3N}{6}\right)\right) + \eta_{\parallel}^2\ \frac{(N-1)}{8}\left((N+1) + \frac{b}{a}\left(\frac{N-5}{6}\right)\right)\right\}. \qquad (3)$$

It is clear that there are four unknowns ($\eta_{\parallel}$, $\eta_{\perp}$, R_o and b/a) and only three equations. We will first proceed, assuming $\eta_{\perp} = 0$ and $b/a = 0$, and calling $\eta_{\parallel} = \eta$ and $v_{\parallel} = v$. These assumptions introduce only small errors into the calculated values of v and E (the kinetic energy corresponding to V) and do not alter the conclusions reached. Their effect, as well as violations of above assumptions (e- uniqueness of v and V) - and (f- straight line path)- are discussed in Appendix A.

It will be seen in Appendix B that N ($R_o = kV^N$) takes the values of 1 for Na^{24} in Al and 1.5 for both Na^{24} and Mg^{28} in all other targets. Therefore one gets as working equations:

$$F_F = \left(\frac{R_o}{4W}\right)\left(1 + 2.333\eta + 1.562\,\eta^2\right), \tag{4}$$

$$W(F_F - F_B) = 1.166\eta\, R_o, \tag{5}$$

$$WF_P = \frac{R_{oP}}{4}\,(1 + 0.156\,\eta^2), \tag{6}$$

for N = 1.5, and

$$F_F = \frac{R_o}{4W}\,(1+\eta)^2, \tag{7}$$

$$W(F_F - F_B) = R_o\eta = kv, \tag{8}$$

$$WF_P = \frac{R_{oP}}{4}, \tag{9}$$

for N = 1.

By means of these equations, the ranges R_o of the fragments have been calculated.

The results obtained are given in Tables II and III for Na^{24} and IV and V for Mg^{28}. The tables are arranged in the following way: column 1 gives the target materials, column 2 the number of determinations, column 3 the bombarding particles and their energies, columns 4 and 5 respectively W x F_F and W x F_B; columns 6 (Tables III, V) and 8 (Tables II and IV) the ranges (R_o) of the fragments. The errors quoted are the standard error.

From a knowledge of the range of the fragments it is possible to obtain their energy provided that the range-energy relationships are known. These relationships are discussed in Appendix B. By making use of the results there obtained, the energy E of the fragments was calculated and is given in column 9 of Tables II and IV. Note that the energy E in this analysis is the average kinetic energy in the moving frame of reference.

Table II

Summary of Na^{24} recoil results from the forward-backward experiments

1 Tgt.	2 No.	3 Beam	4 $W F_F$ (mg/cm^2)	5 $W F_B$ (mg/cm^2)	6 F_F/F_B	7 η	8 R_o (mg/cm^2)	9 E (Mev)	10 v (Mev/amu)$^{1/2}$	11 P_T^* (amu Mev)$^{1/2}$	12 P_T^*/P_{CN}
Al	5	880 Mev α	0.301±0.001	0.0319±0.0007	9.42±.21				0.22	5.81	0.06
	7	660 Mev α	0.365±0.002	0.0296±0.0008	12.40±0.32				0.27	7.27	0.09
	3	320 Mev α	0.572±0.004	0.0199±0.0009	28.72±1.39				0.44	11.95	0.23
	2	660 Mev p	0.220±0.002	0.0483±0.0000	4.56±0.05				0.14	3.71	0.10
Cu	2	880 Mev α	1.678±0.055	0.202 ±0.000	8.23±0.29	0.42	2.99	14.4	0.46	29.4	0.33
	1	700 Mev p	1.184	0.352	3.37	0.25	2.81	13.2	0.27	16.9	0.38
	1	3 Bev p	1.036	0.370	2.79	0.22	2.63	12.1	0.22	13.8	0.10
	1	320 Mev α	(2.440)	(0.338)	(7.19)						
Ag	2	880 Mev α	3.345±0.072	0.654±0.002	5.09±0.13	0.33	6.88	30.2	0.53	57.3	0.64
	1	700 Mev p	2.253	0.969	2.32	0.18	6.18	25.7	0.26	28.1	0.63
	1	3 Bev p	1.848	0.813	2.27	0.17	5.10	20.4	0.23	24.4	0.21
Au	3	880 Mev α	6.576±0.096	2.166±0.072	3.03±0.06	0.23	16.22	60.0	0.52	102.6	1.15
	1	700 Mev p	4.728	2.796	1.68	0.11	14.92	54.0	0.23	46.4	1.05
	1	3 Bev p	4.560	2.184	2.09	0.16	13.05	45.6	0.30	59.9	0.52
	1	6.2,4.5 Bev p	4.452	2.202	2.01						
	1	320 Mev α	(5.370)	(2.718)	(2.01)						
U	5	880 Mev α	6.200±0.035	2.230±0.024	2.77±0.03	0.21	15.82	58.1	0.47	112.6	1.26
	3	700 Mev p	4.165±0.081	2.520±0.043	1.64±0.006	0.106	13.30	46.1	0.21	49.3	1.12
	1	3 Bev p	5.450	2.635	2.06	0.15	15.66	56.6	0.33	79.5	0.68

Table III

Summary of Na^{24} recoil results from the perpendicular experiments

1 Target	2 Number of determ.	3 Bombarding particle and its energy	4 $W \cdot F_{PF}$ (mg/cm^2)	5 $W \cdot F_{PB}$ (mg/cm^2)	6 R_{oP} (mg/cm^2)	7 R_o/R_{oP}
Al	2	880 Mev α	0.162	0.114		
	3	660 Mev α	0.174	0.125		
	2	320 Mev α	0.223	0.127		
	2	660 Mev p	0.141	0.116		
Cu	1	880 Mev α	0.818	0.642	2.84	1.05
	1	700 Mev p	0.664	0.630	2.56	1.10
	1	3 Bev p	0.742	0.576	2.62	1.00
Ag	1	880 Mev α	1.866	1.494	6.60	1.04
	1	700 Mev p	1.506	1.443	5.86	1.05
	1	3 Bev p	1.335	1.155	4.95	1.03
Au	1	880 Mev α	4.218	3.498	15.30	1.06
	1	700 Mev p	3.816	3.720	15.04	0.99
	1	3 Bev p	3.702	3.198	13.73	0.95
U	2	880 Mev α	4.025 ±0.001	3.285 ±0.169	14.51	1.09
	1	700 Mev p	3.340	3.235	13.11	1.01
	1	3 Bev p	4.140	3.515	15.24	1.03

Table IV

Summary of Mg^{28} recoil results from the forward-backward experiments

1 Tgt.	2 No.	3 Beam	4 $W.F_F$ (mg/cm^2)	5 $W.F_B$ (mg/cm^2)	6 F_F/F_B	7 η	8 R_o (mg/cm^2)	9 E (Mev)	10 v $(Mev/amu)^{1/2}$	11 P^*_T $(amu\ Mev)^{1/2}$	12 P^*_T/P_{CN}
Cu	2	880 Mev α	1.546 ±0.013	0.176 ±0.011	8.80 ±0.66	0.43	2.70	14.5	0.44	28.2	0.31
	1	700 Mev p	1.122	0.272	4.13	0.29	2.47	13.1	0.29	18.2	0.41
	1	3 Bev p	0.990	0.328	3.02	0.23	2.44	12.9	0.22	14.2	0.12
Ag	2	880 Mev α	3.042 ±0.050	0.669 ±0.014	4.54 ±0.03	0.31	6.50	32.7	0.48	51.6	0.58
	1	700 Mev p	1.995	0.897	2.22	0.17	5.53	26.6	0.23	25.3	0.57
	1	3 Bev p	1.749	0.801	2.18	0.16	4.92	22.9	0.21	22.8	0.19
Au	3	880 Mev α	6.744 ±0.221	2.520 ±0.041	2.67 ±0.07	0.21	17.49	77.0	0.48	95.6	1.07
	1	700 Mev p	5.484	3.570	1.53	0.09	18.02	80.3	0.22	42.9	0.97
	1	3 Bev p	5.160	2.514	2.05	0.15	14.92	63.0	0.32	63.5	0.54
U	2	880 Mev α	7.420	3.085	2.46	0.19	19.55	89.6	0.48	114.3	1.27
	1	700 Mev p	5.320	3.330	1.52	0.09	18.94	85.4	0.22	52.9	1.20
	1	3 Bev p	5.870	3.145	1.86	0.13	17.69	78.4	0.31	74.3	0.64

Table V

Summary of Mg^{28} recoil results from the perpendicular experiments

1 Target	2 Number of Determ.	3 Bombarding particle and its energy	4 $W F_{PF}$ (mg/cm^2)	5 $W F_{PB}$ (mg/cm^2)	6 R_{oP} (mg/cm^2)	7 R_o/R_{oP}
Cu	1	3 Bev p	0.688	0.538	2.43	1.00
Au	1	880 Mev α	4.932	3.684	17.12	1.02
	1	700 Mev p	4.842	4.644	18.95	0.95
	1	3 Bev p	4.236	3.816	16.04	0.93
U	1	880 Mev α	4.080	3.710	15.5	1.26
	1	3 Bev p	4.800	4.290	18.1	0.98

The average velocity v imparted to the progenitor of the fragment is given by $v = \eta(2E/A)^{1/2}$. Tables II and IV, and Figs. 8(a) and 9(a) give the results in units of $(\text{Mev/amu})^{1/2}$. The results for aluminum have been calculated by using Equation (8). Values of η, R_o, and E have not been calculated for Na^{24} for α bombardments in aluminum because the analysis in terms of η and R_o is not correct for such large values of F_F/F_B.

It is appropriate here to make some comments about the experimental data. In the recoil experiments the counting rates were low, especially in the backward recoil catcher. Nevertheless, the method of counting samples and blanks in rotation canceled out most of the sources for counting errors in the determinations of Na ranges. In the determinations of Mg ranges there may be appreciable counting errors because of the extremely low activities obtained. The Na^{24} produced by activation of impurities in the blank foils was always measurable, and corresponded to a small percentage of the total Na^{24} activity found in the catchers. On the other hand, the errors in the Mg^{28} determination because of activation were not measurable. However, since the counting rates of Mg samples were very low, especially in the backward recoil catcher, activations of the order of 1 count/min could introduce errors up to 10%. Also owing to the low counting rates, the possible radioactive impurities that could not be detected may have introduced errors into the determinations of the Mg ranges. The Na^{24} samples always showed a pure 15-hour half period. Errors due to counting-efficiency corrections are considered small in Na^{24} experiments because the chemical yields of the five samples of a given bombardment differed by only a few percent. In magnesium determinations the counting-efficiency errors may be as large as 10%. In most of the experiments, radiochemical samples were weighed twice as a check on weighing errors. The selection of mounting plates and careful centering of the samples should have reduced geometric errors to less than 2%. Chemical impurities in the target foils were so low that no appreciable errors are expected from this source. Cu, Au, and U foils were quite uniform in average thickness. The Ag foil, however, was not uniform; its thickness decreased from one edge to the other. Since the beam principally hits the leading edge of the target foil it is expected that the Ag foil inhomogeneity may introduce errors up to 5%. Uranium foils

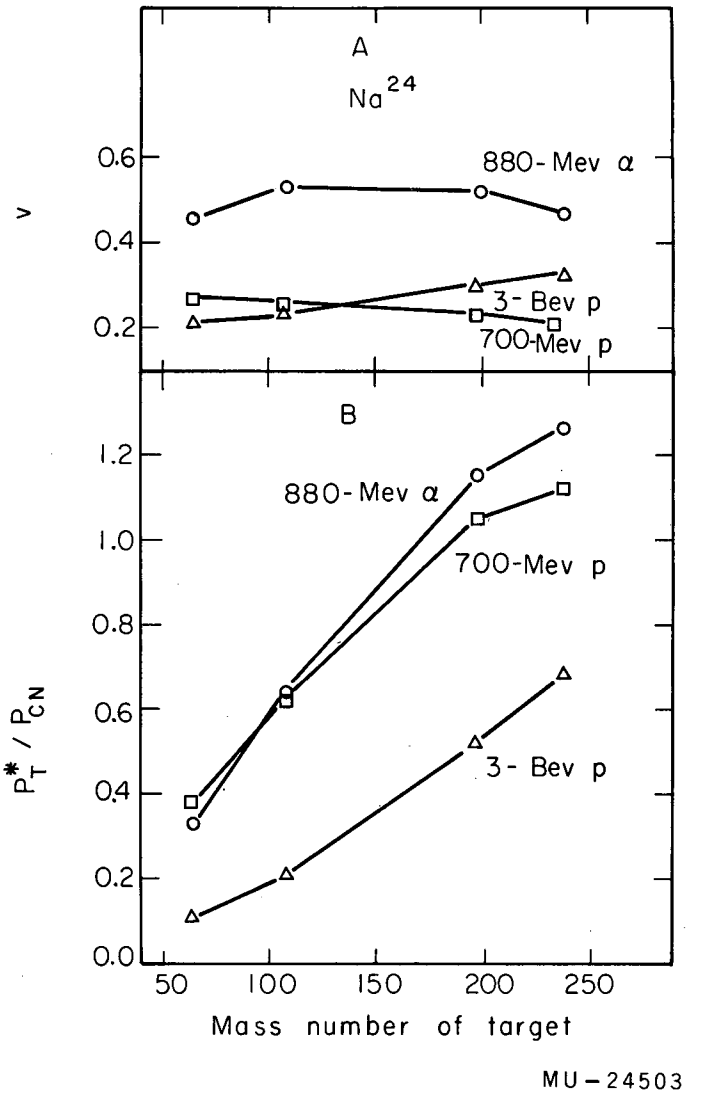


Fig. 8. Imparted velocity v (A) and fractional imparted momentum P^*_T/P_{CN} (B) for Na^{24} as a function of the target mass.

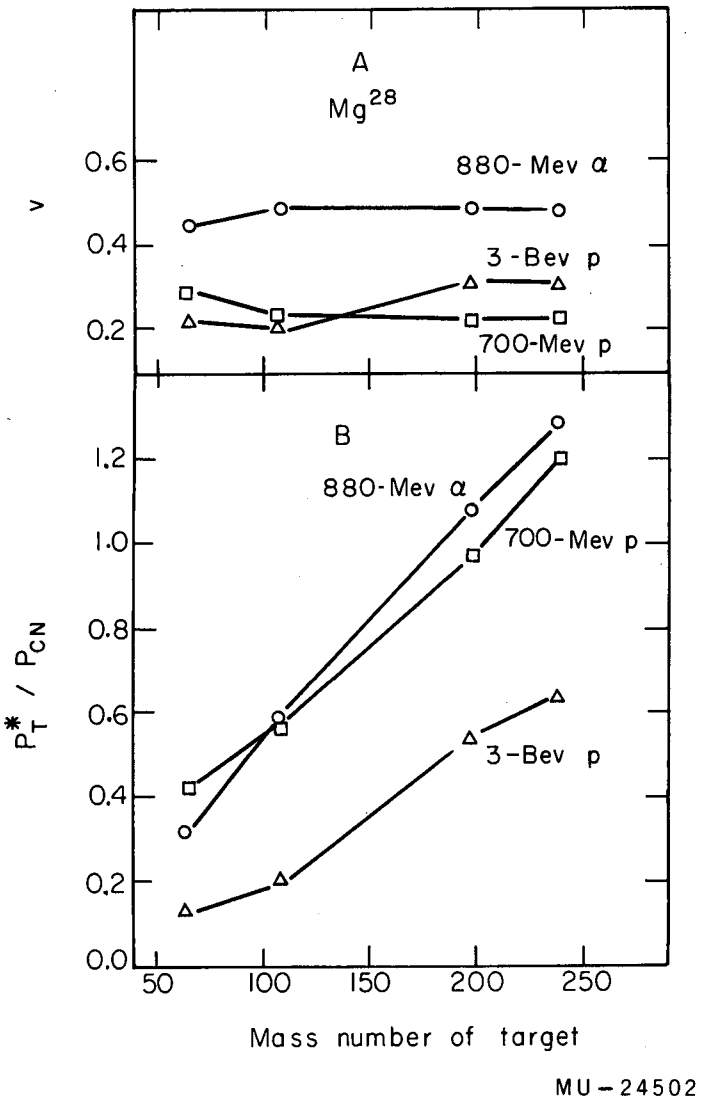


Fig. 9. Imparted velocity v (A) and fractional imparted momentum P^*_T/P_{CN} (B) for Mg^{28} as a function of target mass.

were cleaned with HNO_3. A thin layer of uranium oxide remained. The effect of this layer is not known. However, the reproducibility of these results, and Niday's range measurements of fission products from polished and acid-cleaned uranium foils[91] lead us to conclude that the errors due to the oxide layer are small. The over-all error of the measurements of W x F values for Na is estimated to be of the order of 5% from Cu, Au, and U, and 10% from Ag.

The Mg errors are probably twice as large. The values of $W \times F_{FP}$ and $W \times F_{BP}$ are not very reproducible because of fluctuations of the angle between target and beam. However, the sum of these two quantities was more reproducible. Since it is the sum of WF_{FP} and WF_{BP} that enters in the calculations of R_{oP}, the effect of the variations of target position are not expected to introduce appreciable errors in R_{oP}.

Forward-backward ratios of Na^{24} recoils from thick U targets have been measured by Lavrukinina et al.[92] Their results are apparently very different from the ones reported here. This difference is probably due to the large catcher activation in their experiments. Volkova and Denisov have also measured Na^{24} recoils produced in the interaction of 660-Mev protons with Al.[93] Some disagreement exists (not exceeding 25%) between their results and the results reported here. Our values of WF_F and WF_B are, however, in better agreement with the values obtained by interpolation of the measurements made at Brookhaven.[94]

III. DISCUSSION OF EXPERIMENTAL RESULTS

First we discuss the cross-section results, and then the results of the recoil experiments. Some comments about the mechanism of fragmentation follow. Since the Mg^{28} results are very similar to the Na^{24} results, only the latter are discussed, with the understanding that the conclusions pertain to both Na^{24} and Mg^{28}.

A. Cross Sections

The energies required for the formation of a given product A of a nuclear reaction can be obtained from the analysis of the excitation function data by a method described by Porile and Sugarman.[95] In this analysis the probability of formation of the product A is given by the sum over all possible cases of the product of the probability of forming a nucleus with excitation energy E^*, times the probability that this excited nucleus decays to the product A. In the interaction of a projectile of energy E_B with a target nucleus, the probability of formation of the residual nuclei of the nuclear cascade with excitation energies E^* is given by $\sigma_g N(E^*,E_B)$, where σ_g is the geometric cross section of the target nucleus and $N(E^*,E_B)$ is the probability that the projectile will deposit the excitation energy E^*. The probability that the residual nuclei with excitation energies E^* decay to the product A is written $f_A(E^*)$. The formation cross section of A at a bombarding energy E_B for a given target nucleus is therefore given by

$$\sigma_A(E_B) = \int_0^{E^*_{max}} \sigma_g N(E^*,E_B)\, f_A(E^*)\, dE^*,$$

where E^*_{max} is the maximum excitation energy that can be deposited by the bombarding particle. The values of $f_A(E^*)$ can be obtained by trial and error from this analysis of the cross-section data. Once $f_A(E^*)$ and $N(E^*,E_B)$ are known, it is possible to calculate the average value

of excitation energies $\overline{E}^*$ required to form the product A. The value of $\overline{E}^*$ is given by[95]

$$\overline{E}^* = \int_0^{E^*_{max}} \sigma_g E^* N(E^*,E_B)\, f_A(E^*)\, dE^* \Big/ \int_0^{E^*_{max}} \sigma_g N(E^*,E_B)\, f_A(E^*)\, dE^* .$$

Since the fragmentation cross sections have an onset of about 250 Mev, it is necessary only to know $N(E^*,E_B)$ for deposition energies larger than 250 Mev. The deposition-energy spectra necessary for the analysis are not yet available in the literature. However, the published spectra by Metropolis et al.[96] show that as a first approximation the spectra of deposition energies can be considered as having, for the different target nuclei, the same form although not necessarily the same area. If that is the case, the tails of the deposition energy spectra for targets x and y are related by the equation

$$\left(N(E^*,E_B)\right)_x = k\left(N(E^*,E_B)\right)_y,$$

where k is a constant.

As shown in Fig. 5, the excitation functions for production of Na^{24} in the interaction of He ions with Cu, Ag, Au, and U are essentially proportional one to another. This is true also of the excitation functions for the production of Na^{24}, with protons, from Ag, Au, and U (see Fig. 7); the excitation function for production of Na^{24} in the interaction of protons with Cu increases faster with the bombarding energy up to 3 Bev, and at higher energies stays more constant than the corresponding excitation functions for Ag, Au, and U. If the assumption

$$\left[N(E^*,E_B)\right]_x = k\left[N(E^*,E_B)\right]_y$$

for large E^* is correct, one expects the functions $f_A(E^*)$ for Na^{24} production from Ag, Au, and U to differ only by a constant factor,

and the function $f_A(E^*)$ for Na^{24} production from Cu to be displaced towards lower energies in relation to the $f_A(E^*)$ functions for the other targets. This conclusion can be proved in the following way. For very high bombarding energies, $E^*_{max} \approx E_B$. For $[N(E^*,E_B)]_x = k[N(E^*,E_B)]_y$, where k is a constant and subscripts x and y refer to target nuclei x and y, then

$$1-\left(\sigma_{Ax}(E_B)/\sigma_{Ay}(E_B)\right)=\frac{\int_0^{E_B} k'\, N(E^*,E_B)\left(f_{Ax}(E^*)-k''f_{Ay}(E^*)\right) dE^*}{\int_0^{E_B} \sigma_{gx}\, N(E^*,E_B)\, f_{Ax}(E^*)\, dE^*}$$

If $f_{Ax}(E^*) = k'''\, f_{Ay}(E^*)$, then $1-\left(\sigma_{Ax}(E_B)/\sigma_{Ay}(E_B)\right)$ is a constant, and the excitation functions are proportional one to another. If $f_{Ax}(E^*)/f_{Ay}(E^*)$ is a function of E^*, then the excitation functions cannot be proportional. The relative behavior of $f_A(E^*)$ for Cu and Au can be seen from Figs. 10 and 11. The assumption of any reasonable form of $N(E^*,E_B)$ nonvanishing up to the bombarding energy gives an average excitation energy for production of Na^{24} with 700-Mev protons lying between 500 and 700 Mev, for all target nuclei. The shapes of the excitation functions indicate that at 3 Bev the average excitation energy required to produce Na^{24} fragments is the same for Ag, Au, and U, and smaller for Cu than for the other target materials, but not by more than a factor of 2. Since $N(E^*,E_B)$ very probably has nonzero values for all possible E^* for E_B up to 1 Bev,[95] it can be concluded from the excitation-function data that $\bar{E}^*$ is larger than 500 Mev for bombarding energies of 700 Mev and 3 Bev, and that $\bar{E}^*$ for E_B=3 Bev is larger than $\bar{E}^*$ for E_B= 700 Mev. This question is discussed in more detail in Appendix C.

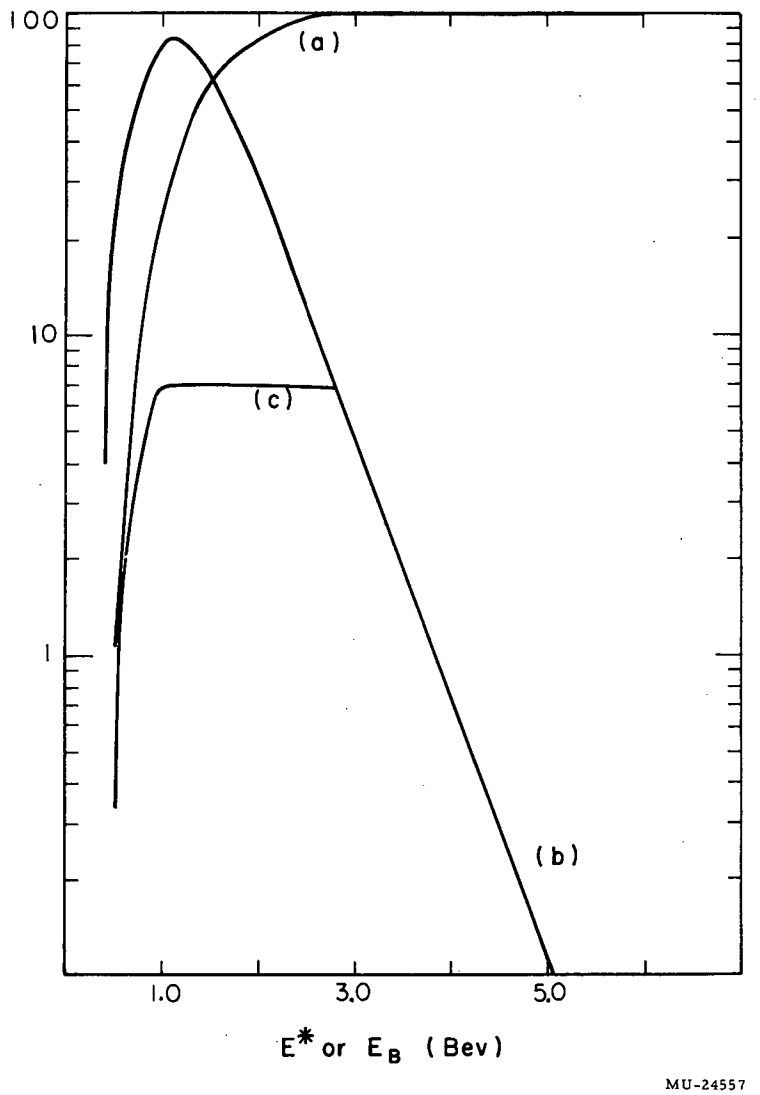


Fig. 10. Variation of $f_A(E^*)$ with E^* for Na^{24} from proton bombardments of Cu. Curve (a) gives the shape of the excitation functions. Curve (b) gives the shape of $f_A(E^*)$ for $N(E^*, E_B)$ = constant. Curve (c) gives the shape of $f_A(E^*)$ for $N(E^*, E_B) = 2(E_B - E^*)/ E_B$.

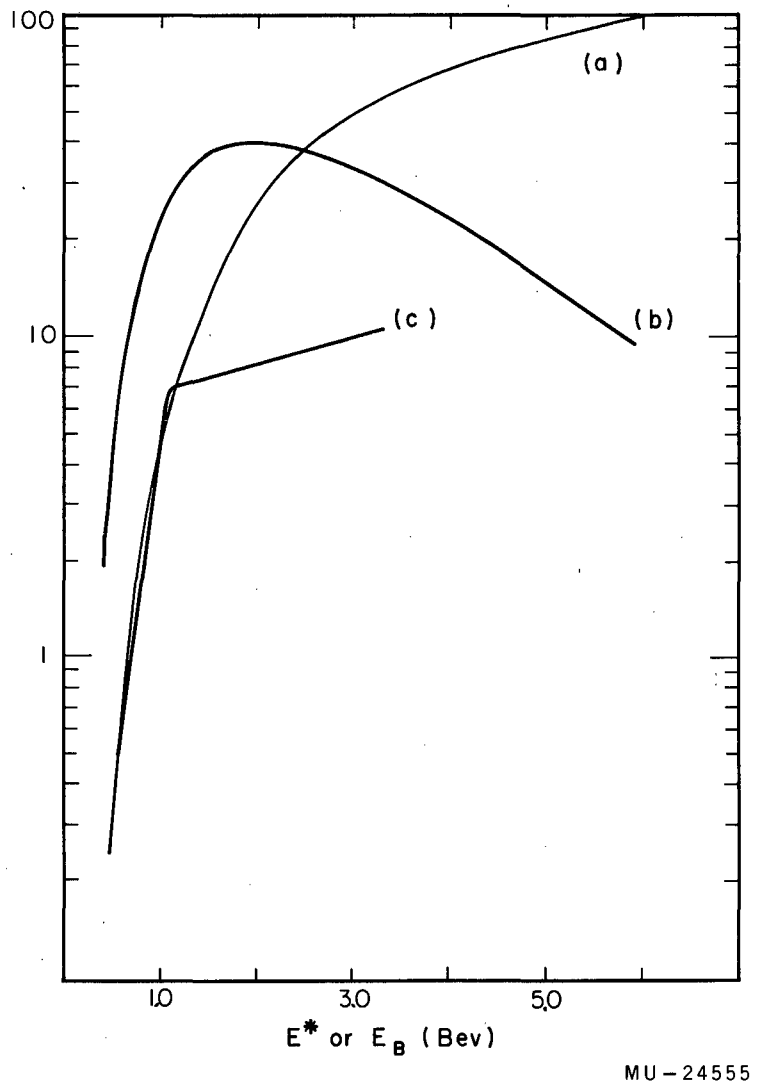


Fig. 11. Variation of $f_A(E^*)$ with E^* for Na^{24} from proton bombardments of Au. Curve (a) gives the shape of the excitation function. Curve (b) gives the shape of $f_A(E^*)$ for $N(E^*, E_B)$ = constant. Curve (c) gives the shape of $f_A(E^*)$ for $N(E^*,E_B) = 2\ (E_B-E^*)/E_B^{\ 2}$.

B. Recoil Determinations

The thick-target recoil experiments give information about the average intrinsic velocity V given to the final nucleus, in the process of its formation, by decay of an excited progenitor and about the velocity component v of the progenitor in the beam direction. The values obtained are given in Tables II and IV. We discuss first the significance of the intrinsic velocity V.

If target nucleus A_Z is divided into two spheres, Na^{24}_{11} and its complementary fragment $^{A-24}_{Z-11}$, it is possible to calculate the energy of Na^{24} due to the Coulomb repulsion E_{coul} of these two fragments:

$$E_{coul} = \left[11\ (Z-11)\ \times\ e^2/r_o\ (24^{1/3} + (A-24)^{1/3})\right] \cdot \left[(A-24)/A\right]$$

with r_o equal to 1.45 f. Some information about the mechanism by which Na^{24} fragments are produced can be obtained by comparing the intrinsic recoil energy E (given in Tables II and IV) with the Coulomb energy E_{coul}. If $E/E_{coul} < 0.5$ it is concluded that the excited nuclei that remain after the prompt cascade produce Na^{24} by emission of many small particles (p,α, etc.); on the other hand, if $0.5 < E/E_{coul} < 1$, the Coulomb repulsion of the fragments dominates the recoil velocity. If $E/E_{coul} > 1$, then deposition energy from the beam is contributing to the intrinsic recoil velocities or the Coulomb energy is underestimated. The limits of 0.5 and 1 for E/E_{coul} for a fission-type process are derived from the fact that E_{coul} as calculated is an upper limit of the Coulomb energy, because no corrections for thermal expansion and surface vibrations have been applied. Also, the average parent of Na^{24} is not expected to be the target nucleus, but a lighter nucleus resulting from the nuclear cascade. It should be kept in mind that these effects are more important for Cu than for Ag, Au, or U. All these effects are not expected to reduce the value of the Coulomb energy by a factor of more than 2. The values of E/E_{coul} are given in Fig. 12. They are approximately equal to 0.7. These large values show that even at bombarding energies of 3 Bev, Na^{24} is not produced in copper by a

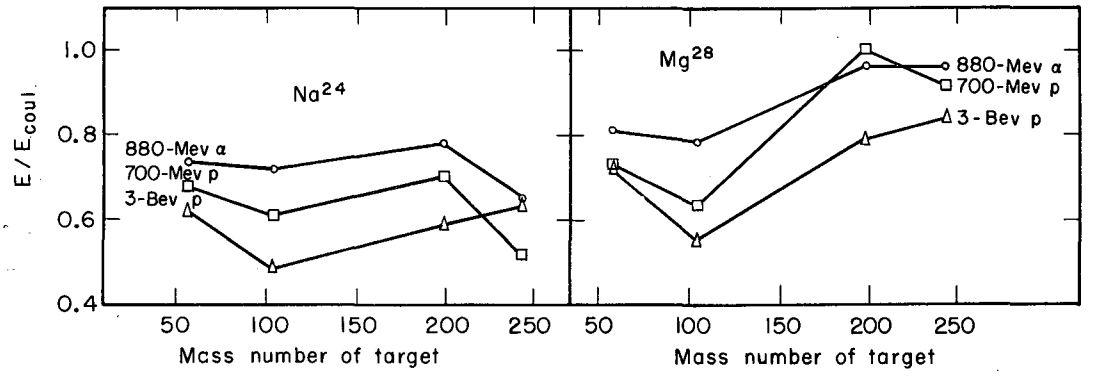


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Fig. 12. Ratio for Na^{24} and Mg^{28} of the intrinsic recoil energy to the Coulomb energy (see text) as a function of the target mass.

cascade process followed by evaporation of small particles, contrary to what is generally assumed. They also show that very probably the same mechanism is responsible for fragment production in all target nuclei. This is a surprising result, since, as indicated earlier, the excitation-function data show that at the bombarding energies of this work, more than 500 Mev of excitation is required in order to have "fragmentation". Such deposition energies are large enough to allow production of Na from Cu as the residue of a cascade-evaporation process. If any reasonable amount of this latter process took place, E/E_{coul} should be smaller for copper than for the other targets. This is emphasized by the fact that the total binding energy of copper is approximately 550 Mev,[98] only slightly larger than the average deposited energy, if at all. These large excitation energies should reduce the Coulomb barrier enormously (by more than a factor of 2),[99] making the ratio of the recoil energy to the Coulomb energy much larger for Cu (and also for Ag) than for Au and U. The combined effects of the cascade and the reduction of the Coulomb barrier make the true E/E'_{coul} for Cu larger than 1 (E'_{coul} is the corrected Coulomb energy).

It has been pointed out earlier in this paper that the values of E/E_{coul} of about 0.7 imply that Na^{24} is produced by rupture of the target nucleus A_T into Na^{24} and a heavy fragment of mass about A_T-24. Such a process may be similar to evaporation (as described by Dostrovsky et al.[5]), or to a fission process. Both these processes are usually considered as slow processes. However, Ericson estimated, for a nucleus of mass 100 and with neutron binding energy of 8 Mev, a lifetime of the order of 3×10^{-22} sec at 200 Mev of excitation energy.[100] Thus Na^{24} cannot be, in the usual sense, either a fission product or an evaporated particle.

Under the assumption that the progenitors of the fragments have mass values very nearly equal to the target mass, the momenta $P_T^* (\sigma \times A_T)$ imparted to the struck nuclei in the beam direction have

been calculated. The values obtained are given in column 10 of Tables II and IV. In Figs. 8(b) and 9(b) the values of P_T^*/P_{CN} are plotted for Na^{24} and Mg^{28} respectively, where P_{CN} is the momentum of the bombarding particle. Because the progenitors of the fragments are not expected to have the mass of the target nuclei, but a rather smaller mass, owing to the cascade, P_T^* is a slight overestimate of the actual momentum transferred. The values of P_T^*/P_{CN} are larger than unity for 700-Mev protons and 880-Mev α particles on Au and U. One possible reason for this large ratio is the aforementioned fact that P_T^*/P_{CN} is overestimated; another possibility is that particularly complex cascades (cascades with a negative forward momentum) may deposit momenta larger than the momentum corresponding to compound-nucleus formation.[101] Another reason, and probably the true one, is given below; it is a consequence of the proposed mechanism for fragmentation.

C. Comparison of Results of Cross-Section and Recoil Measurements

A number of simple models have been used to relate momentum and excitation energy deposited by the bombarding particle in the cascade process. Recently Porile[101] was able to obtain a more realistic correlation by means of the results of the Monte Carlo cascade calculations by Metropolis et al.[96,97] Porile's results are shown in Fig. 13, which plots the average deposited momentum versus the deposited energy, both in units of the corresponding values for compound-nucleus formation. The calculations have been made for U, Bi, and Ru. It is seen that the momentum deposited increases almost linearly with increasing deposition energy, and as a first approximation the relationship between $\overline{P}_F/P_{CN}$ and E^*/E_{CN}^* can be considered to be independent of the bombarding energy and target material. It will be assumed that this is also true for targets used in this study. The only published results of the Monte Carlo calculation are for intranuclear cascades initiated by nucleon bombardments. Owing to the similarity of the results obtained with 700-Mev protons and 880-Mev alpha particles, it is assumed that the Porile results are in this particular case valid for cascades induced by α-particles. This may very well be incorrect; however, the assumption is not essential to the conclusions.

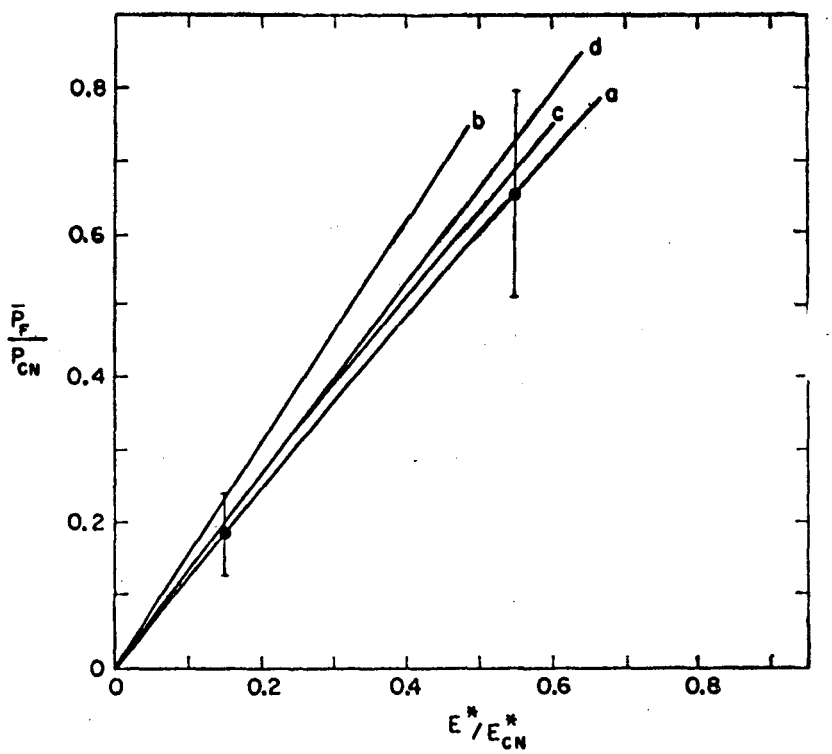

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Fig. 13. Variation of the average forward momentum with excitation energy. a: U or Bi, 0.46 Bev; b: Bi, 0.94 Bev; c: Bi, 1.84 Bev; d: Ru, 0.46 Bev (after Porile, Ref. 101).

From the results shown in Fig. 13, it is seen that if fragmentation is the result of a typical process described by the cascade calculations, then the measured value of the average deposited momenta in the forward direction (P^*_T/P_{CN}) give a measure of the average deposition energy. However, the measured values of P^*_T/P_{CN}, and thus the values of E^*, increase with target mass number in spite of the fact that the similar excitation functions imply that E^* is independent of the target mass. This inconsistency between E^* values from recoil data and from excitation-function data is both interesting and puzzling. No real explanation of this inconsistency can be made until the theoretical calculations are improved by more statistical significance and the inclusion of angular momentum. Several possibilities may be cited that may make fragmentation reactions atypical, and thus invalidate the comparison with average prompt cascade calculations. With 700-Mev protons (and 880-Mev alpha particles) it may be that the requirement that deposition energies be close to the bombarding energy makes very particular selections of the nuclear cascades. It is not clear, however, why this same selection should be made at 3 Bev. It is possible that special selections of angular momentum may explain the results obtained.

Unless the improbable assumption is made that the determinations of the excitation energies are in error by a factor of more than 3, the inconsistency cannot be removed. It can be concluded that the treatment of fragmentation as an average process in the nucleon-cascade model of nuclear reactions gives rise to inconsistent results. Therefore, either the cascades leading to high excitation energies or angular momenta are not average cascades, or else the fragmentation mechanism is alien to the cascade-evaporation model.

It is noteworthy that the measurements imply a strong similarity in the average processes leading to Na^{24} production from all targets. Excitation functions are proportional; the values of v and E/E_{coul} are only slightly dependent on target mass for each incident energy. The prompt cascade calculations lead one to expect that these quantities should not be as regular. These facts suggest that some other mechanism may be useful to correlate all the data.

D. Other Studies of Fragmentation

In other studies of production of fragments the behaviors of fragments observed in nuclear emulsion (usually Z=3), He^6, Be^7, and N^{13} have been observed. Generally speaking, the cross sections, energy spectra, and forward-backward asymmetry are known for the fragments observed with nuclear emulsions. The behavior of He^6, Be^7, N^{13}, and the fragments observed with nuclear emulsion have been compared with the expectations from the evaporation theory. The agreement between calculated and measured cross sections varies from fair to bad[3-7,14] (see Figs. 14 and 15). Katcoff studied Li^8 from 2.2-Bev protons on Cu, Ag, Au, and U.[9] He calculated the Li^8 energy spectra for a particular set of parameters, taking account of the effects of the nuclear cascade. With the parameters used he found that the agreement between calculated and measured spectra was approached only for Ag. In Cu, Ag, and possibly U there were too many high-energy particles. Skjeggestad and Sörensen, analyzing the spectra of Li^8 produced in Ag-Br by cosmic rays, obtained for the average temperature of the evaporating nuclei the value of 11.5 Mev, too high to be acceptable.[36,102] The existence of broad energy spectra with too many high-energy particles for the nuclear emulsion fragments is a generally accepted fact among emulsion workers, as indicated in the introduction. A large probability of multiple fragment emission has also been observed,[14,18,21,22,31] contrary to what could be reasonably expected. All these observations indicate that the evaporation theory cannot easily account for the behavior of some of the emitted fragments. The similarity of the excitation functions for nuclear emulsion fragments and heavier fragments[22,56] makes it possible that the same mechanism may be responsible for production of fragments detected by both radiochemical and nuclear emulsion methods, or at least the more energetic fragments seen in nuclear emulsion. Also, the similar behavior of Na^{24} production from Cu to U targets suggests a mechanism common to all targets.

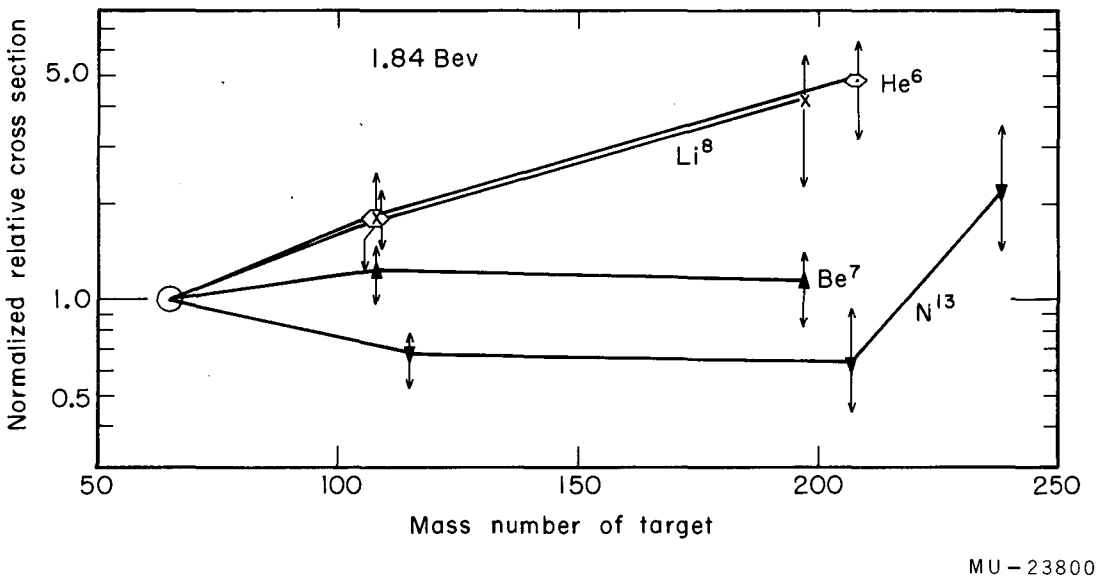

Fig. 14. Calculated cross section for production of He^6, Li^8, Be^7, and N^{13} with 1.84-Bev protons vs target mass number. The data are from Refs. 4, 5, and 7. The points are the averages of the calculated values and the tips of the arrows give the extreme calculated values.

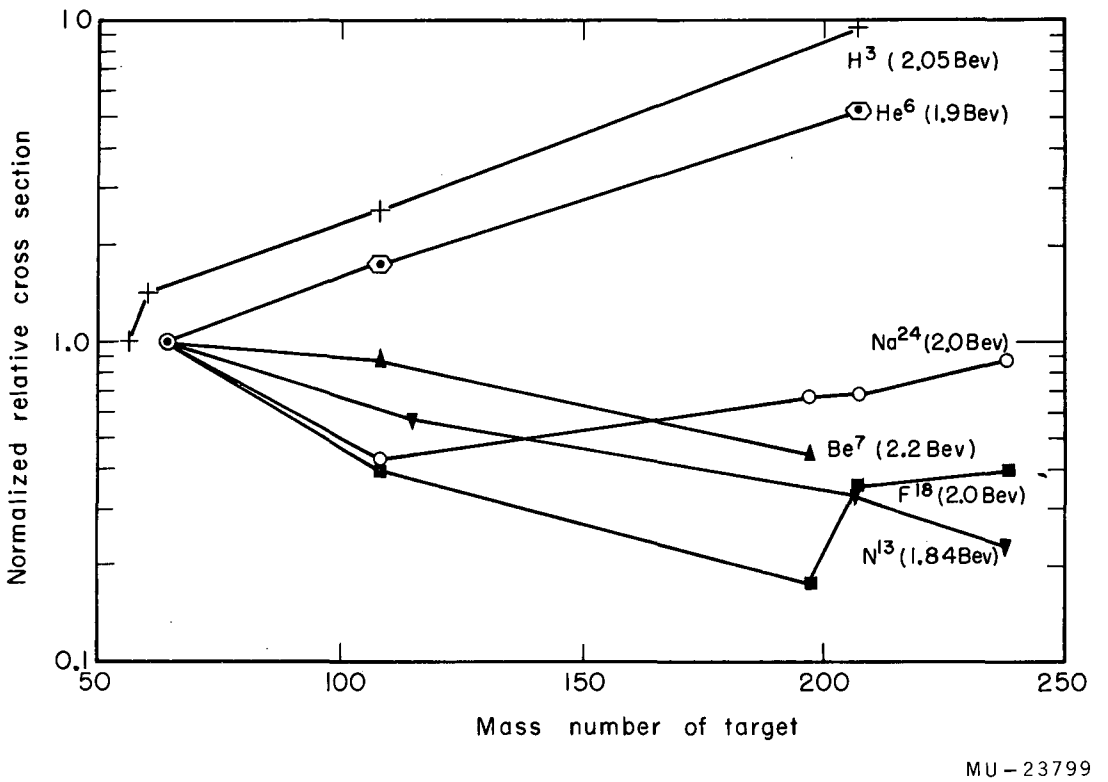

Fig. 15. Normalized relative corss sections of light nuclei produced by 2-Bev protons vs target mass number. The data are from Refs. 128 (H^3), 43 (He^6), 50 (Be^7), 7 (N^{13}), and 57 (F^{18} and Na^{24}).

E. The Mechanism of Fragmentation

A mechanism for fragmentation was proposed solely on the basis of cross-section data by Wolfgang et al.[56] In this mechanism, mesons are the means of depositing the large energies required for fragmentation. The mesons created by the bombarding particle, when reabsorbed, originate hot spots, with large disturbances breaking many bonds, and unbinding the fragments from the remaining nucleus. The fragments are separated by Coulomb repulsion and surface tension forces as well as by the momentum imparted by the knock-on cascade. It was assumed by Caretto et al. that this mechanism is responsible for the formation of fragments from heavy targets.[57] This assumption came as a result of the dependence of F^{18} and Na^{24} cross sections on the target mass number. The cross sections of these two products (and also Mg^{28} and P^{32}— see Figs. 15, 16, 17, 18, 19) decrease with mass number of the target nucleus up to about mass 140, and then increase with increasing target mass. This was explained by Caretto, Hudis, and Friedlander by assuming that Na^{24} and F^{18} were produced from Cu by a spallation mechanism,[57] from Ag by a fission process at low energies and by spallation at high energies, and from Au and U by the fragmentation mechanism proposed by Wolfgang et al.

The assumption that mesons are intrinsic to fragmentation does not seem to be supported by a number of experimental results. First there is no firm evidence from pion reactions (pion energy less than 250 Mev) that fragments accompany pion-nucleus interaction. Most of the products obtained from the bombardment of 122-Mev or lower-energy negative pions with Zinc,[103] Arsenic,[104] Bromine,[105] Silver,[106] Iodine,[107] and Mercury,[108] result from reactions of the type (π^-,xn) and (π^-,pxn), with x larger than 1. These products are not results of a fragmentation process. Only Pd^{109}, formed with 0.0066% yield as a result of the interaction of slow π^- with I^{127}, could be formed in a fragmentation process. But as Winsberg points out, Pd^{109} could also have been formed with pions in flight.[107] Stars produced in nuclear emulsions by 222-Mev π^+,[109] 95-Mev π^+,[110] and lower-energy pions[20,111,112] have less than

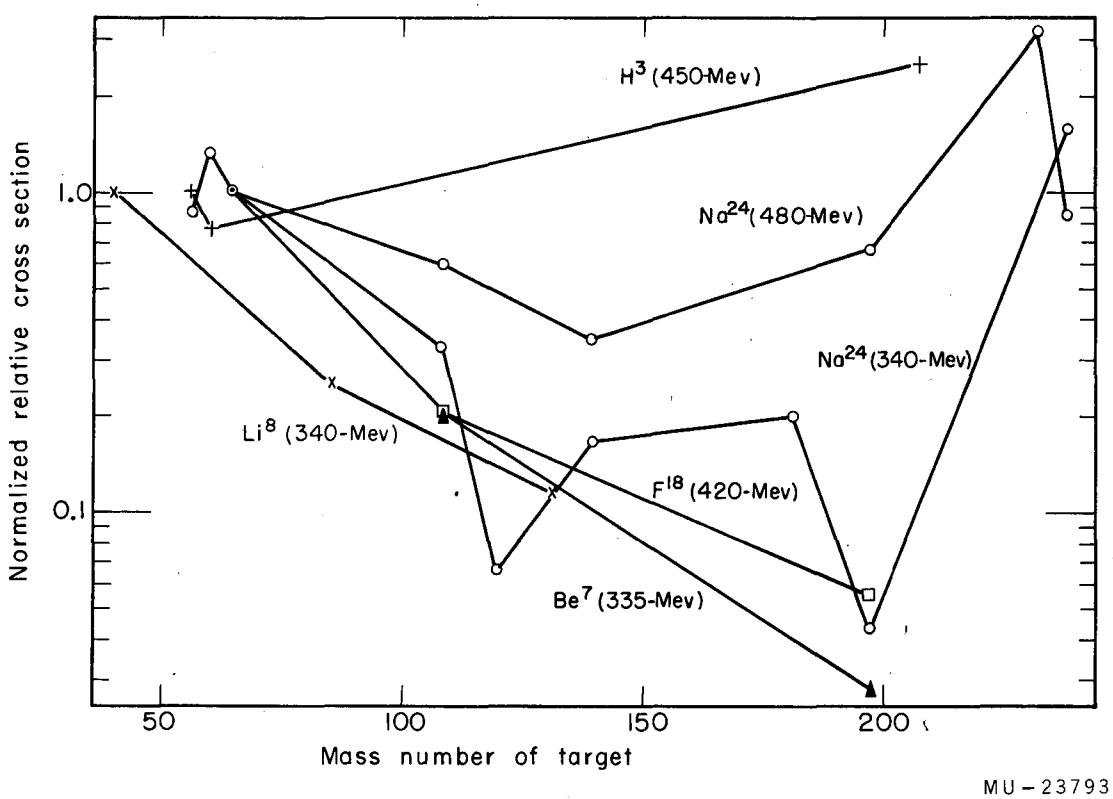

Fig. 16. Normalized relative cross sections of light nuclei produced by 450-Mev protons vs target mass number. The data are from Refs. 128 (H^3), 70 (Li^8), 44 (Be^7), 55 (F^{18}), and 63, 64, and 67 (Na^{24} at 480 Mev). The 340-Mev curve for Na^{24} was obtained with data from Refs. 45, 58-61, and 65-67.

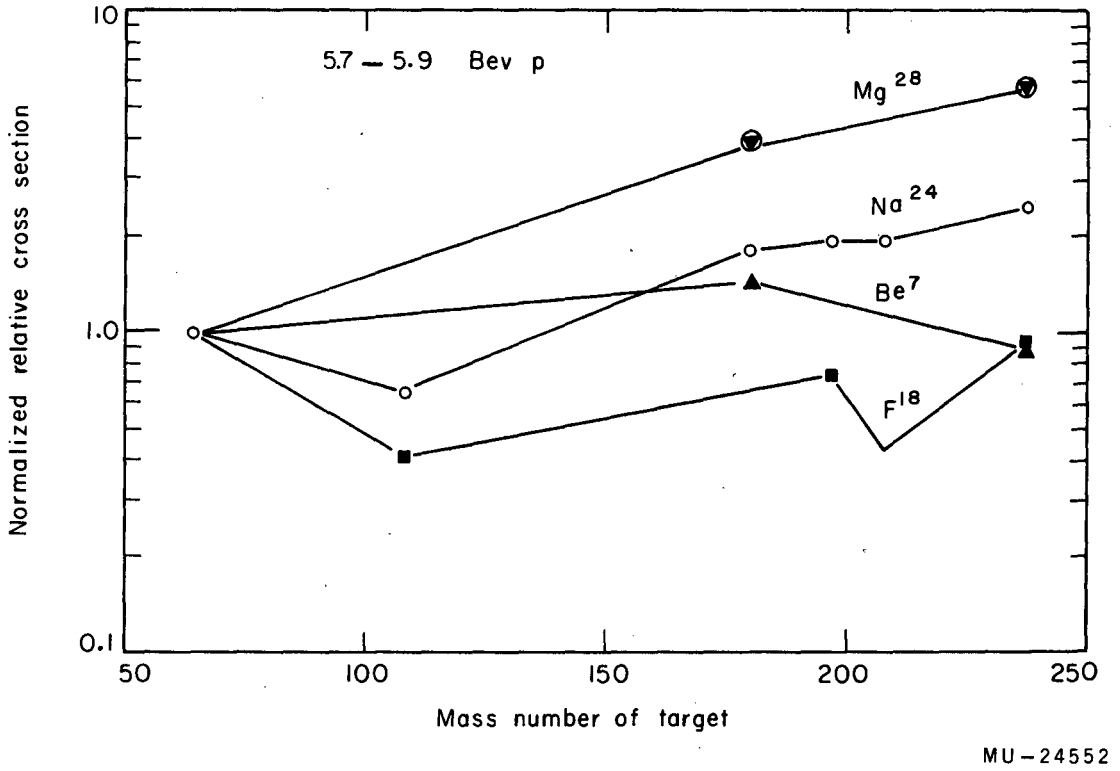

Fig. 17. Normalized relative cross sections of light nuclei produced by 5.9-Bev protons. The data are from Refs. 57 and 47-49.

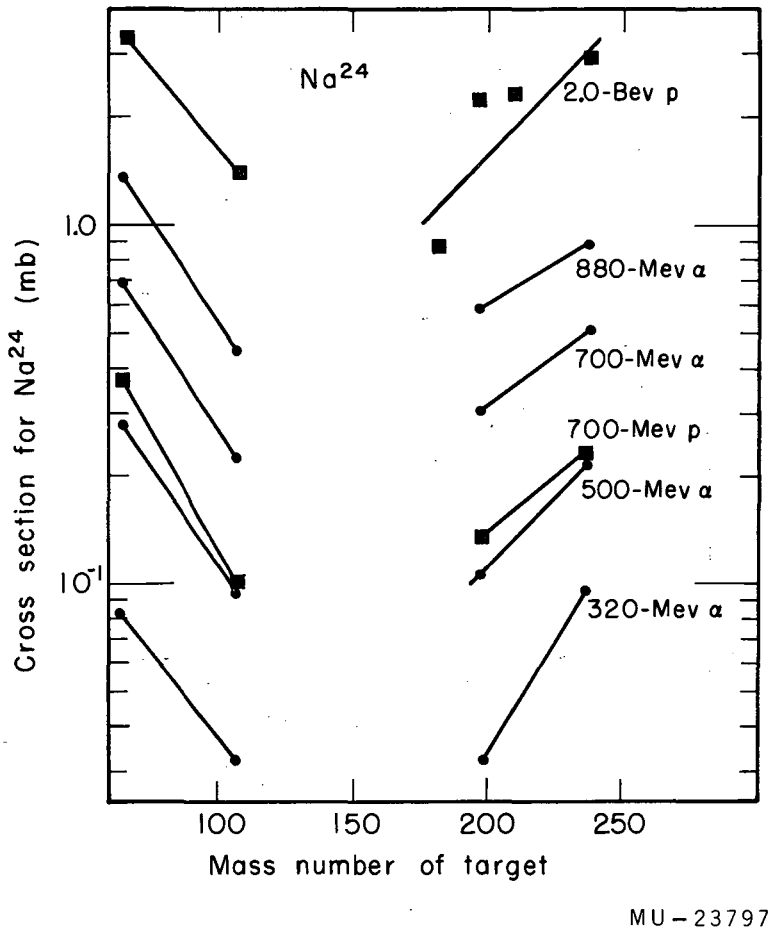

Fig. 18. Formation cross section of Na^{24} vs target mass number.

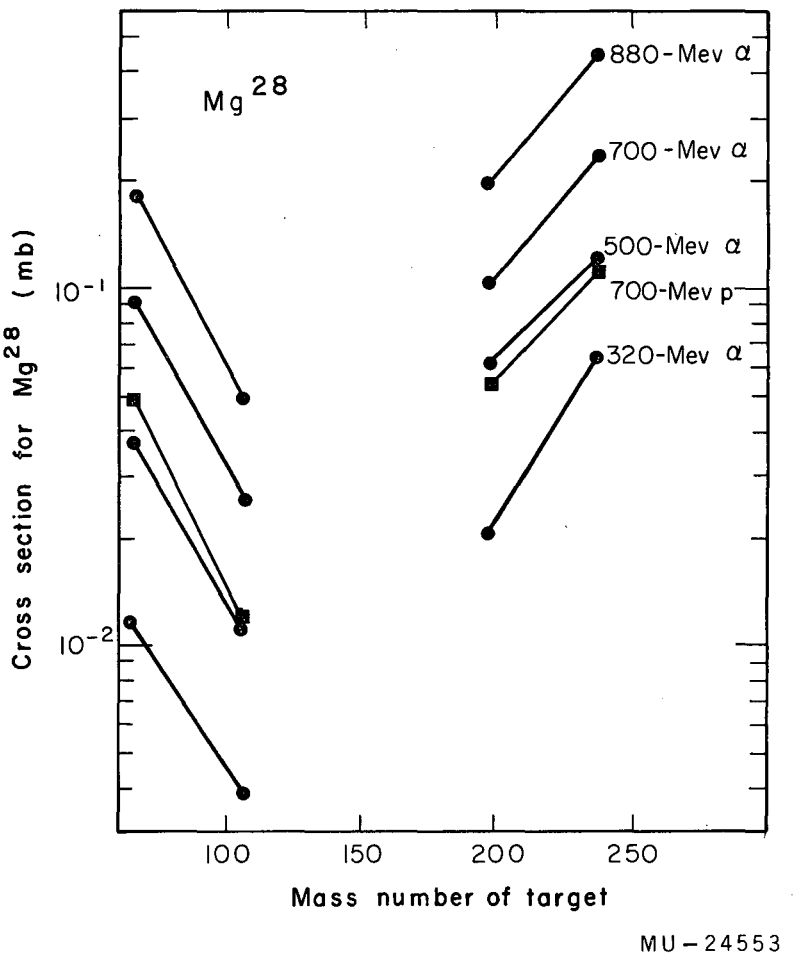

Fig. 19. Formation cross section of Mg^{28} vs target mass number.

seven prongs, in contrast with the very complex stars usually observed with fragment production (see Figs. 20 a) and b). With nuclear emulsions only Alumkal and co-workers[20] saw "fragments" produced by low-energy (stopped) π^-. It is not clear, however, what criterion was used to separate stars of light elements from stars of heavy elements.

Secondly fragments have been produced by bombarding particles with less than 200 Mev,[60,67,19,10,58,70] therefore in reactions in which mesons are very probably not involved. Also, at 320 Mev, the cross section for production of fragments is higher when α-particles are substituted for protons as bombarding projectiles.

High-energy mesons can produce fragments.[23] However, the yields of fragments detected with nuclear emulsions are independent of the number of shower particles.[18,21,22,31,34] This implies a lack of correlation of mesons and fragment production.

Among other suggestions for the mechanism of production of fragments are those by Blokhintsev[113] and by Glassgold and co-workers[114]. Blokhintsev proposed that the fragments arise from the direct collision of an incoming bombarding nucleon with a cluster of nucleons. However, as pointed out first by Sörensen,[34] the angular distribution of the fragments shows that collisions between bombarding particles and the clusters are not allowed since too many fragments appear at large angle. In the mechanism proposed by Glassgold et al., the fragments are ejected from the nucleus by the action of shock waves originated by the "hole bored" by the passage of the bombarding particle through the nucleus.

Nakagawa, Tamai, and Nomoto measured the yield of fast Li^8 (more than 60 Mev) produced in nuclear emulsions by 6.2-Bev protons[18] and they found that it increases strongly with the number N_g of grey prongs (cascade particles), but is independent of the Number N_b of black prongs (evaporated particles), as shown in Figs. 20 a) and b) (see also Refs. 21 and 22). This is a surprising result,[78] showing that in events leading to the formation of energetic fragments the excitation energy of the nuclides resulting from the nuclear cascade is independent of the complexity of the cascade. This result contrasts markedly with the correlation of black and grey prongs in events without fragments.[78]

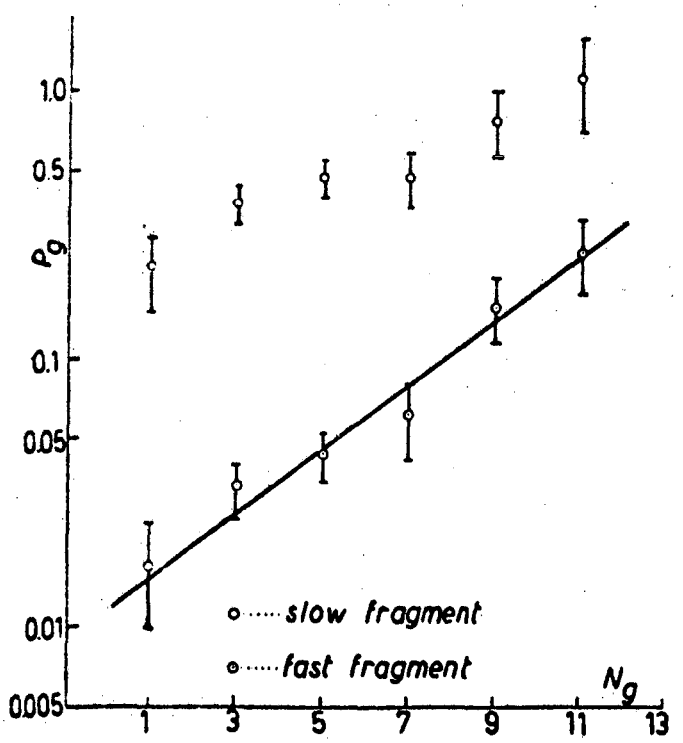

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Fig. 20-A. Probability of the ejection of a fast fragment as a function of the number of grey prongs N_g (after Nakagawa et al., Ref. 18).

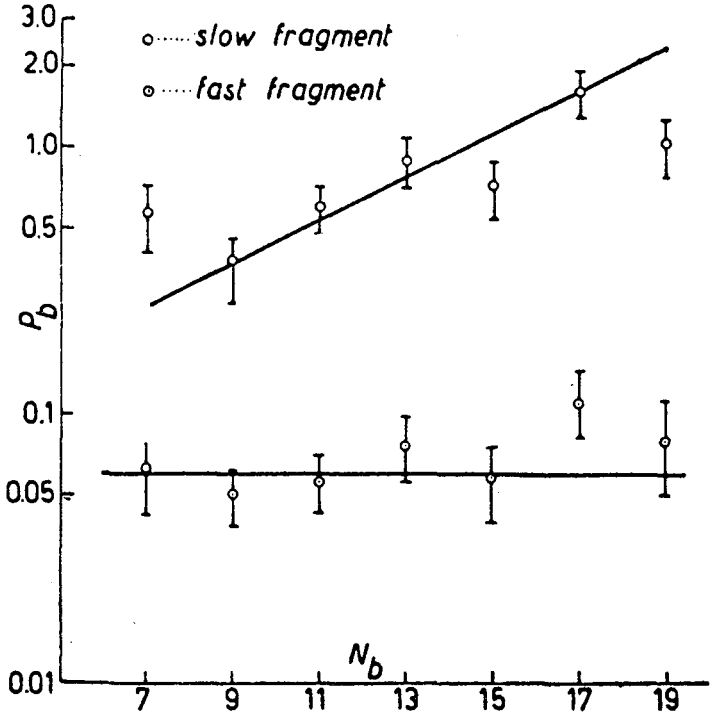

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Fig. 20-B. Probability of the ejection of a fast fragment as a function of the number of black prongs N_b (after Nakagawa et al., Ref. 18).

The various experimental observations of this work, and of Nakagawa and others, suggest the following mechanism of the fragmentation process. High-energy bombarding particles going through the nucleus may originate complex cascades in such a way as to cleave the nucleus into two or even more fragments. This can happen if the cascade particles have such a geometrical arrangement that they separate regions of relatively cool nuclear matter by a group of fast-moving nucleons. These cool regions, virtually unbound one to another, are subject to bombardment by the cascade nucleons. They can separate with resultant velocity due to Coulomb forces as well as to the momentum received from the cascade nucleons. From this mechanism the energy of Na^{24} may be due to Coulomb forces in the breakaway process, but fragments such as high-energy Li^{8} may receive most of their energy from the cascade nucleons. This process requires high-energy bombarding particles because only such particles can originate cascades with the complexity required. Such a mechanism can explain why v (speed of the progenitor) is practically independent of the target material and why P_T^* becomes larger than 1. The values of P_T^* become larger than 1 because they were calculated by assuming that the parent nucleus has a mass close to the mass of the target material, which is not true. It can also explain why the more energetic fragments observed in nuclear emulsions are more forward peaked (see, however, Refs. 9 and 36), and more abundant than the evaporation theory predicts.

With this mechanism the large probability for multiple fragment production becomes understandable. It can also explain the fast fission process observed by Faissner and Schneider.[115] Indeed, the complex cascade proposed here is not essentially different from a moving viscous fluid as suggested by Faissner et al.

Of course, other mechanisms may also take place. It is also possible that some amount of evaporated particles is always present. It is possible that the aforementioned mechanism can not be responsible for the production of fragments at bombarding energies of 100 to 200 Mev.

There is some experimental evidence for collisions between clusters of nucleons and the bombarding particle. Fragments have been seen with extremely high energies.[18,30] The emission of deuterons and α particles has been explained in a few cases as a result of the interaction of the bombarding particle with a deuteron[116,113] or alpha-particle[117,118,119] cluster. A collision of a cluster of nucleons with a nucleon inside the nucleus has been invoked to explain the high-energy tail of $(C^{12},2n)$ reactions.[120] It is unlikely that such direct collisions can take place at high bombarding energies without destroying the cluster.

IV. SUMMARY

We may summarize the experimental facts and the conclusions as follows: Excitation functions for many fragments of mass approximately 7 to 30 are similar for targets from Cu to U. The cross sections increase rapidly for energies of several hundred Mev and become almost energy-independent over approximately 3 Bev. The over-all energies of Na^{24} fragments from Cu, Ag, Au, and U targets are apparently related to Coulomb energy between Na^{24} and its heavy partner $({}^{A-24}_{24}\ {}_{Z-11})$. The average velocity imparted by the projectile to the Na^{24} progenitor is almost independent of the target mass for each bombarding energy. The energy and angular distribution of the fragments observed in nuclear emulsion (Z=3 to approximately 7) show the existence of many superbarrier fragments directed along the incident beam direction.

These facts require many separate explanations if they are to be correlated with the usual theoretical approach of the nucleon-nucleon cascade followed by slow processes of evaporation or fission. However, all these qualitative features may be explained by a fast reaction mechanism involving the cleavage of the nucleus by a complex nucleonic cascade followed by fast nucleon division, strongly influenced by Coulomb forces.

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APPENDICES

A. Discussion of the Effects of Approximations Made in Calculating Ranges of Fragments and Speeds of Their Progenitors.

By means of the recoil experiments one measures the ranges of the fragments and the speeds of their progenitors. Analysis of the data is made with the following assumptions:

(a) the bombarding particle imparts to the target nucleus a velocity $v_{\parallel}$ in the laboratory system along the beam direction and a velocity $v_{\perp}$ in a direction perpendicular to the beam;

(b) the fragment receives an additional velocity component, $\vec{V}$, from the process that leads to its formation, and this velocity $\vec{V}$ reflects the intrinsic kinetic energy of the fragment in the frame of reference of the moving nucleus;

(c) the range of the fragments R_o is defined as $R_o = k\,V^N$, where k and N are constants; and $V = |\vec{V}|$

(d) the angular distribution $\Omega(\theta)$ of $\vec{V}$ in the moving frame is given by $\Omega(\theta) = a + b\cos^2\theta$;

(e) the magnitude of $\vec{v}$ and $\vec{V}$ are unique; and

(f) the path of the fragment is a straight line.

Assumptions (e) and (f) are very probably not correct. Furthermore, the actual analysis is made by neglecting $v_{\perp}$ and b/a. The effects of these approximations are here discussed. As in the text, $\eta_{\parallel} = v_{\parallel}/V$ and $\eta_{\perp} = v_{\perp}/V$, W is the target thickness, and F_F, F_B, F_P are the measured activity fractions escaping from the target in the forward (F), backward (B), and perpendicular (P) directions.

1. The Effect of $\eta_{\perp}$

Equations (1), (2), and (3) of the text can be rewritten:

$$\frac{F_F}{F_B} = \frac{\left[1+\eta_{\parallel}\frac{2}{3}(N+2)+\eta_{\parallel}^2\frac{(N+1)^2}{4}\right]+\eta_{\perp}^2\frac{N^2-1}{8}+\frac{b}{a}\left[\frac{1}{6}+\eta_{\parallel}\frac{8(N-1)}{45}+\eta_{\parallel}^2\frac{N^2-N-4}{12}+\eta_{\perp}^2\frac{N-1}{12}\right]}{\left[1-\eta_{\parallel}\frac{2}{3}(N+2)+\eta_{\parallel}\frac{(N+1)^2}{4}\right]+\eta_{\perp}^2\frac{N^2-1}{8}+\frac{b}{a}\left[\frac{1}{6}+\eta_{\parallel}\frac{8(N-1)}{45}+\eta_{\parallel}^2\frac{N^2-N-4}{12}\quad\eta_{\perp}^2\frac{N-1}{12}\right]}, \tag{10}$$

$$W(F_F - F_B) = R_o \eta_{\parallel} \left[\frac{N+2}{3} + \frac{b}{a} \frac{4(N-1)}{45} \right] , \tag{11}$$

and

$$4WF_P = R_{oP}\ 1 + \eta_{\parallel}^2 \frac{(N-1)(N+1)}{8} + R_{oP} \left\{ \eta_{\perp}^2 \frac{(N+1)(3N+1)}{16} + \frac{1}{6} \frac{b}{a} \left[-\frac{1}{2} + \eta_{\perp}^2 \frac{(7-3N)(N+1)}{16} + \eta_{\parallel}^2 \frac{(N-1)(N-5)}{8} \right] \right\} . \tag{12}$$

It will be seen in Appendix B that N takes the values of 1 for Na^{24} in Al, and 1.5 for both Na^{24} and Mg^{28} in all other target materials. It is seen from Eqs. (10) and (11) that the coefficients of $\eta_{\perp}$ become 0 for N=1. Therefore the errors introduced by the neglect of $\eta_{\perp}$ have only to be discussed for N=1.5. In this case Eq. (10) becomes

$$\frac{F_F}{F_B} = \frac{1+2.33\ \eta_{\parallel} + 1.56\ \eta_{\parallel}^2 \left[1+0.1\ \eta_{\perp}^2/\eta_{\parallel}^2\right] + 0.04 \frac{b}{a}\ \eta_{\perp}^2 + b/a(\cdots)}{1-2.33\ \eta_{\parallel} + 1.56\ \eta_{\parallel}^2 \left[1+0.1\ \eta_{\perp}^2/\eta_{\parallel}^2\right] + 0.04 \frac{b}{a}\ \eta_{\perp}^2 + b/a(\cdots)} . \tag{10'}$$

In order to get an estimate of the relative magnitudes of $\eta_{\perp}$ and $\eta_{\parallel}$, we can refer to the Monte Carlo calculations of the nucleon-nucleon cascade. By doing so, Porile[101] has calculated the ratios of the transverse momentum to the forward momentum imparted by a projectile, to the products of the nuclear cascade as a function of the forward deposited momentum. His results are given in Fig. 21. They show that for large momentum transfers (the most frequent case of this work) $\eta_{\perp}/\eta_{\parallel} < 1$. Equation (10') shows that $\eta_{\perp}/\eta_{\parallel}$ must be much larger than 1 in order to introduce appreciable errors into the estimate of $\eta_{\parallel}$, if F_F/F_B is not extremely large. Thus the neglect of $\eta_{\perp}$ is justified (see Fig. 22). Since $R_o \eta_{\parallel}$ is independent of $\eta_{\perp}$ (Eq. 11), errors due to the neglect of $\eta_{\perp}$ are equally introduced in $\eta_{\parallel}$ and R_o. However, $W(F_F - F_B) \propto R_o \eta_{\parallel} \propto (V)^{1/2} v$ (for N=1.5), and therefore v is practically independent of $\eta_{\perp}$.

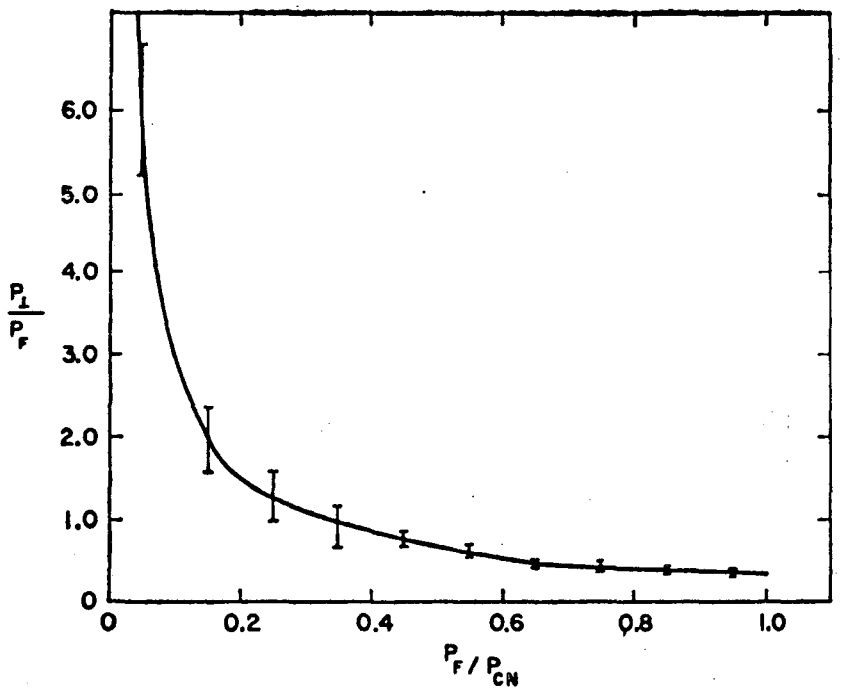

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Fig. 21. Variation of the ratio of transverse and forward components of momentum with the forward component of momentum. (after Porile, Ref. 101).

2. The Effect of b/a

In Eq. (11) the coefficient of b/a vanishes for N=1. For N=1.5 Eq. (11) becomes

$$W(F_F-F_B)=k^{2/3}(1.16 + 0.044\ b/a)\ (R_o)^{1/3}\ v. \qquad (11')$$

For values of b/a smaller than 0.5, 1.16 + 0.044 b/a $\approx$ 1.16. The experimental data indicate that b/a is always less than 0.5. Thus, the calculation of v is only affected by b/a through the influence of the latter on R_o. However, because R_o enters into Eq. (11') with a power of 1/3, v becomes practically independent of both b/a and $\eta_\perp$. The effects of b/a in the calculations of R_o can be obtained from Fig. 22. This figure shows (for N=1.5) the dependence of the values of $\eta_\parallel$ (calculated by means of Eq. (10)) on the assumptions made about b/a. As indicated earlier, Eq. (11) shows that the errors introduced in $\eta_\parallel$ are equally introduced into R_o, with an opposite sign. Equation (10) is correct to first order in b/a, and second order in $\eta_\parallel$ and $\eta_\perp$.

The assumption that v and V are unique is not expected to be strictly correct. This introduces certain errors in the analysis. The measured quantities are the average quantities F_F and F_B. For Na^{24} in Al N=1. For N=1,

$$\langle W(F_F-F_B)\rangle = \langle R_o\eta\rangle = k\langle v\rangle,$$

and $\langle v\rangle$ is exactly determined. For Na^{24} and Mg^{28} in all other targets N=1.5. For N=1.5 Eqs. (4) and (5) can be written

$$\langle W(F_F+F_B)\rangle \propto \langle V^N\ (1+ 1.562(v/V)^2)\ \rangle$$

and

$$\langle W(F_F-F_B)\rangle \propto \langle V^{N-1}\ v\ \rangle.$$

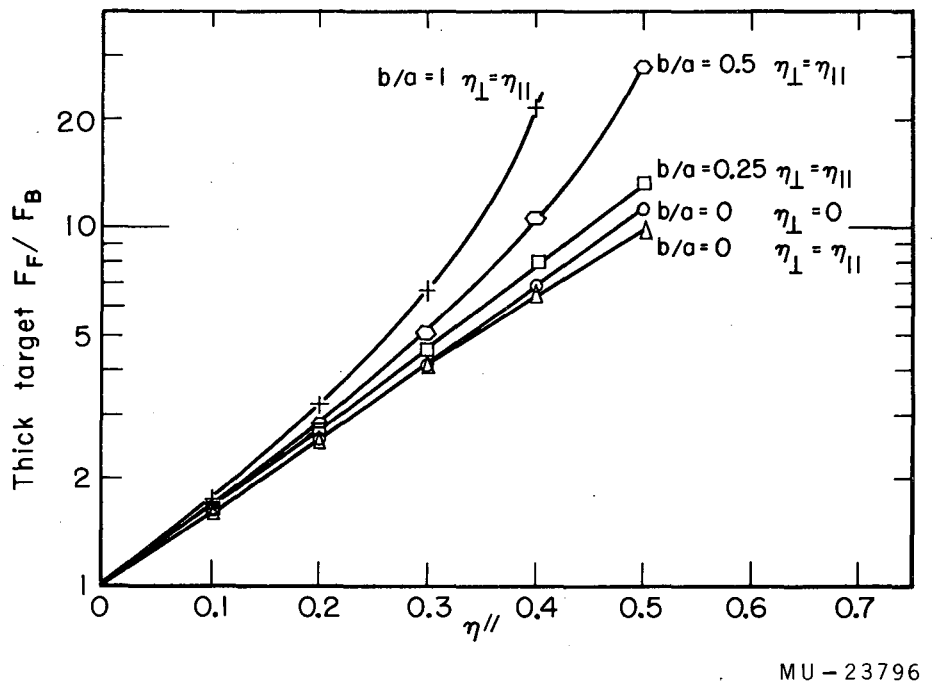

Fig. 22. Thick-target $\eta_{\parallel}$ as a function of forward-backward ratios for different $\eta_{\perp}$ and b/a.

The values of $1.562(v/V)^2$ are usually much smaller than 1. Thus

$$\langle V^N(1+1.562(v/V)^2)\rangle \cong \langle V^N\rangle\ (1+1.562\ \langle (v/V)\rangle^2\ ),$$

and since N is close to 1,

$$\langle V^N\rangle^{1/N} \cong \langle V\ \rangle .$$

Therefore $\langle W(F_F-F_B)\rangle \cong k'\ \langle V\rangle^N\ \langle v/V\rangle$, and $\langle v/V\rangle$ and $\langle V\rangle$ may be obtained. The estimate of $\langle v\rangle$ from the product $\langle V\rangle$ and $\langle v/V\rangle$ is more accurate than either factor separately. It is estimated that for the target materials other than Al, $\langle V\rangle$ can be obtained with an error smaller than 20% and $\langle v\rangle$ with an error smaller than 10%. Errors of this magnitude do not invalidate the arguments presented in the text. This is especially true because similar errors are probably introduced for all targets, and thus the relative errors are expected to be less than the absolute errors.

A charged particle going through matter loses its energy by electronic and atomic collisions. Electronic collisions are responsible for the losses of energy suffered by fast-moving recoils. On the other hand, slow recoils transfer their energy to the stopping medium by atomic collisions. When the masses of the atoms in the medium are comparable to or larger than the mass of the recoil, the recoiling atoms suffer large deflections in almost every atomic collision. As a result of these large deflections more fragments scatter out of the target material than back from the plastic catchers,[91] increasing the apparent recoil ranges. This nuclear scattering effect becomes important for speeds of the recoils lower than a critical speed V_c given by[121]

$$V_c = 2Z_1\ Z_2\ e^2\ (Z_1^{2/3} + Z_2^{2/3})^{1/2}/\mu\ a_o,$$

where Z_1 and Z_2 are the atomic numbers of the stopper and stopping nuclei, μ the reduced mass of the system, e the electronic charge, and a_o the first Bohr radius (0.528×10^{-8} cm). Niday has measured this effect for thick-target recoil studies of the fission products in U.[91] He found the apparent ranges of the fission products caught in Al to be larger by approximately 3% for heavy fission products and 5% for light fission products than the corresponding ranges of the fission products caught in lead. The V_c's for Na^{24} (and Mg^{28}) fragments are lower than the V_c's for the fission products. Furthermore, the ratios of the initial speed of the fragments to V_c are larger for Na^{24} (and Mg^{28}) than the corresponding ratios for the fission fragments. Therefore, the nuclear-scattering effects are expected to be relatively less important for the Na^{24} and Mg^{28} fragments observed in this study than for the fission products. Furthermore, the values of v are affected only by a difference in these nuclear scattering effects according to the expression

$$W((F_F - F_B) + (d' - d)) = k\ v\ (V)^{1/2},$$

where d' and d are the apparent increase due to scattering of the backward and forward fractions of recoiling nuclei, respectively. Therefore the corrections to the measured quantities due to scattering effects are of second order and have been neglected.

B. Range-Energy Relationships

The theories of stopping of charged particles in matter so far developed[121] are not adequate to give the needed range-energy relationships. The speeds of the Na^{24} and Mg^{28} fragments produced in Cu, Ag, Au, and U lie between 4 V_o and 10 V_o, where V_o is the Bohr velocity of the electron in the hydrogen atom ($e^2/\hbar$). This means that the stopping process is due to both electronic and atomic energy transfers and that the atomic charge of the recoil is changing markedly throughout the range. No experimental ranges have been published for the energies and stopping materials of interest to this work.

The ranges in nuclear emulsions of heavy ions with energies of 0.5 to 10 Mev/amu have been measured by Heckman and co-workers.[122] They analyzed the measured ranges $R(\beta)$ in terms of the range to be expected if there were no neutralization of charge $(M/Z^2)\ \lambda\ (\beta)$, and in terms of a range extension term $R_{ext}\ (\beta)$ defined by

$$R_{ext}\ (\beta) = R(\beta) - (M/Z^2)\ \lambda\ (\beta), \qquad (13)$$

where $\lambda\ (\beta)$ is the range of a proton of velocity $\beta = v/c$. Heckman et al. found that a plot of $R_{ext}/M\ Z^{2/3}$ vs the velocity of the ions (in terms of the velocity of their k electron) gives a universal curve. By means of this universal curve and Eq. (13), the ranges of Na^{24} and Mg^{28} in emulsions have been calculated. In the energy interval 0.4 to 3.0 Mev/amu, the ranges of Na^{24} and Mg^{28}, expressed in mg/cm^2, are given by $R=0.95\ (E/A)^{0.94}$. This range-energy relationship should be fairly good, since in this same energy interval the ranges of all heavy ions are, for a given speed, the same within 20%, as shown in Fig. 23. The relative stopping powers of the nuclear emulsions and the target materials used in this work, for ions of a given speed, were determined by the ratios of the ranges of α particles or protons of the same speed in nuclear emulsions and in the target material. This procedure is not absolutely correct, but it is a reasonable approximation. The ranges in emulsion of α particles and protons have been taken from Barkas et al.[123] and Heckman et al.[122] respectively. The ranges of α particles and protons in

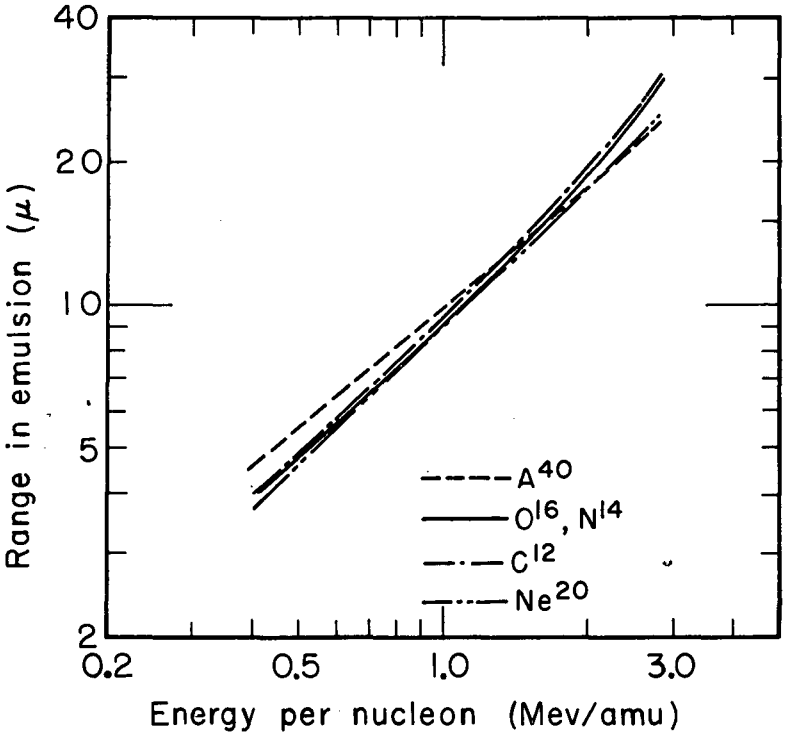

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Fig. 23. Range of heavy ions in nuclear emulsions vs the energy of the heavy ion (in Mev/amu).

Cu, Ag, and Au have been taken from Whaling.[124] The ranges of the fragments in U were assumed equal to the ranges of the fragments in Au. There is some evidence that this assumption is not strictly correct.[91,125,126] However, the errors introduced by this assumption in the calculated recoil energies of the fragments produced in U should not exceed 10%. The range-energy relationships obtained for both Na^{24} and Mg^{28} are of the form $R=k\ (E/A)^{0.76}$, where k is a constant. With R expressed in mg/cm^2 and E/A in Mev/amu, the constant k takes the values of 4.5 for Cu, 5.8 for Ag, and 8.1 for Au and U.

1. Range-Energy Relationships in Al

The range of a given nuclide produced in a nuclear reaction can be obtained by measurement of its recoil properties. If the energy of the recoiling radionuclide is known, one obtains a range-energy relationship. The energy of the recoils is known if they are formed by the compound-nucleus mechanism.[127]

Aluminum of 99.99% purity was bombarded at the Berkeley 60-inch cyclotron with deuterons and α particles, and the Na^{24} recoils were collected in 1-mil Be foils. (In one experiment, C foils were used.) Two other 1-mil Be foils were placed next to the recoil catchers in order to measure the amount of Na^{24} produced in the catcher foils by activation of impurities. These four foils constituted one target. In a particular run, a stack of identical targets was bombarded, the size of which was chosen in such a way as to cover the energy interval from the maximum bombarding energy to below the threshold energy.

In the α-particle bombardments a large activation due to the secondary reaction $Al^{27}\ (n,\alpha)\ Na^{24}$ was observed. This experimental difficulty prevented reliable measurements close to the threshold energy. However, the activation level at bombarding energies of 45 to 48 Mev (measured by extrapolation of the activity obtained in the part of the stack exposed to α particles whose energy had been reduced below threshold) proved negligible. At these bombarding energies it is not known a priori that a compound nucleus is formed. Furthermore,

whenever energetic particles are evaporated from an excited recoiling nucleus, a small fraction of the measured range of the recoiling nucleus is, in general, due to the evaporation process. The decay curves of the activities found in the target and catcher foils indicated the presence of Na^{24}, Mg^{28}, and Na^{22} nuclides. However, the average range of the mixture of these three products remained within experimental errors constant over long periods of observation, for any particular bombarding energy. This observation indicates that all three products have the same average recoil velocity and thus that the compound nucleus mechanism is probably operative in the production of all these products by 48-Mev α particles. The ranges of Na^{24} in Al in the energy interval under consideration were found to be proportional to the velocity of the recoiling nuclei, making it unnecessary to correct the measured ranges for the effects of the nuclear evaporation.[127]

Aluminum foil was also bombarded with deuterons and the recoiling fractions measured. It was observed in this particular case that the activation problem was much less important. In the bombarding energy interval ranging from 20.5- to 13-Mev deuterons the corrections due to activation amounted to less than 6%. The ranges of 12 determinations (two bombardments) in this energy interval plotted in log-log paper vs energy of the Na^{24} (calculated assuming compound nucleus formation) fell in a straight line. The extrapolation of this line fell within 5% of the range-energy points obtained with α-particle bombardments.

In the energy interval of 1/24 to 5/24 Mev/amu the range of Na^{24} in Al, in mg/cm^2, is given by $R=2.03\ (E/A)^{0.54}$. The Na^{24} activities were measured by means of beta proportional counters. The counting-efficiency corrections were obtained by counting a few stacks in a gross gamma counter.

C. Calculations of the Average Excitation Energy from the Cross-Section data.

Calculations from cross section data of the average excitation energy necessary for the production of a given product of a nuclear reaction require a knowledge of the spectrum of excitation energies of the products of the nuclear cascade, $N(E^*,E_B)$. The spectrum $N(E^*,E_B)$ represents the number of cascade product nuclei with excitation energy E^* produced by bombarding particles of energy E_B. The excitation functions of Cu and Au have been analyzed by assuming $N(E^*,E_B)$ to be a constant, and alternatively by assuming $N(E^*,E_B) = 2(E_B-E^*)/E_B^2$. In both assumptions the spectra are considered to be independent of the target mass. The assumed deposition-energy spectra are not necessarily realistic. However, these serve to illustrate the dependence of the calculated values of the average deposition energy necessary for fragmentation, on the assumptions made about $N(E^*,E_B)$.

As pointed out in the text, the cross sections for production of a given product A of a nuclear reaction at the bombarding energy E_B is given by

$$\sigma_A(E_B) = \int_0^{E^*_{max}} \sigma_g \, N(E^*,E_B) \, f_A(E^*) \, dE^*,$$

where $N(E^*,E_B)$ is normalized. The average excitation energy of the products of the nuclear cascade resulting in the formation of the product A is given by

$$\bar{E}^* = \int_0^{E^*_{max}} \sigma_g \, E^* \, N(E^*,E_B) \, f_A(E^*) \, dE^* \Big/ \int_0^{E^*_{max}} \sigma_g \, N(E^*,E_B) \, f_A(E^*) \, dE^*.$$

For $N(E^*,E_B)$ = const, $f_A(E^*)$ can be obtained by differentiation of the excitation function. For $N(E^*,E_B) = 2\,(E_B-E^*)/E_B^2$, the function $f_A(E^*)$ can be obtained by trial and error.

The excitation functions for production of Na^{24} from Ag, Au, and U differ only by a constant factor (approximately). Therefore, any form of the excitation spectra independent of the mass of the target nuclei will give the same results for the average excitation energy necessary to produce Na^{24} (at a given bombarding energy) from these targets. Thus, Au was taken as a typical case. A plot of the excitation functions on logarithmic probability paper revealed that the product of N f_A and (therefore f_A) (E^*) for $N(E^*, E_B)$ = constant, could be represented for both Cu and Au by a normal distribution in log E^*. For $N(E^*,E_B)$ =

$2(E_B-E^*)/E_B^2$ the values of $f_A(E^*)$ were obtained by trial and error. The shapes of $f_A(E^*)$ are shown in Figs. 10 and 11.

The values of $\bar{E}^*$ computed with the $f_A(E^*)$'s obtained were larger than 500 Mev for both Cu and Au at bombarding energies of 700 Mev. At 3 Bev the values of $\bar{E}^*$ obtained were, for the triangle-shape spectra, 1.47 and 1.61 for Cu and Au respectively, and for $N(E^*,E_B)$ = const. were 1.38 Bev (Cu) and 1.96 Bev (Au). These values indicate that $\bar{E}^*$ is not too sensitive to the assumptions made about $N(E^*,E_B)$.

The average energy deposited by the bombarding particle in the residues of the nuclear cascade is given by

$$\langle E\rangle = \int_0^{E^*_{max}} E^* N(E^*,E_B)\, dE^* \Big/ \int_0^{E^*_{max}} N(E^*,E_B)\, dE^*.$$

The values of $\langle E\rangle$ can give information about the character of $N(E^*,E_B)$. It can be seen from the Monte Carlo calculations by Metropolis et al.[96,97] that $\langle E\rangle$ increases with E_B faster for the heavy targets than for the lighter ones. That is to say that $N(E^*,E_B)$ for heavy targets is shifted towards higher energies in relation to the $N(E^*,E_B)$ for the lighter targets. Therefore the $f_A(E^*)$ for U is displaced towards the low-energy side in relation to the $f_A(E^*)$ for Ag, since Ag and U have excitation functions that differ only by a constant factor. This statement can be justified in the following way:

Let's assume that E = ne, where e is a unit of energy. Now any bombarding energy will be defined by the corresponding n. Let's call $N(E^*,E_B) = N$ and $f_A(E^*) = f$.

Then

$$\langle E\rangle_n = \sum_1^n E_i N_i \qquad \text{and} \qquad \sigma_n = \sum_1^n f_i N_i,$$

where $\langle E\rangle_n$ is the average deposition energy corresponding to the bombarding energy n and σ_n is the cross section at this bombarding energy for production of the product A. Divided by the geometrical cross section. Also

$$\langle E\rangle_{n+1} = \sum_1^n E_i N_i x_i + E_{n+1} \qquad \text{and}$$

$$\sigma_{n+1} = \sum_1^n f_i N_i x_i + f_{n+1} N_{n+1},$$

where x_i is the ratio of probabilities of obtaining excitation energies E_i for bombarding energies n and n + 1.

Then

$$\Delta\langle E\rangle = \langle E\rangle_{n+1} - \langle E\rangle_n = E_{n+1}N_{n+1} + \sum_1^n E_i N_i (x_i - 1),$$

$$\Delta\sigma = \sigma_{n+1} - \sigma_n = f_{n+1} N_{n+1} + \sum_1^n f_i N_i (x_i - 1).$$

Let's assume that the values of x_i are independent of i, then,

$$\Delta\langle E\rangle = E_{n+1}N_{n+1} + (x-1)\ \langle E\rangle_n \quad \text{and}$$

$$\Delta\sigma = f_{n+1}N_{n+1} + (x-1)\ \sigma_n.$$

For very small e we have $(1-x)=\delta$. The condition that N be normalized makes $N_{n+1} = \delta$.

Therefore

$$\Delta\langle E\rangle = \delta(E_{n+1} - \langle E\rangle_n) \quad \text{and}$$

$$\Delta\sigma = \delta(f_{n+1} - \sigma_n), \quad \text{or}$$

$$\Delta\sigma = \Delta\langle E\rangle \quad (f_{n+1} - \sigma_n)/(E_{n+1} - \langle E\rangle_n).$$

It is possible, therefore, to compare f_{n+1} for different targets if $\Delta\langle E\rangle$ and $E_{n+1} - \langle E\rangle_n$ are known.

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