Title

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Authors
Tancredi, S
Abdu, R
Abrahamson, D
et al.

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Proof of Concept: Applying Recurrence Quantification Analysis to Model Fluency in a Math Embodied Design

Sofia Tancredi, University of California Berkeley and San Francisco State University, sofiatancredi@berkeley.edu
Rotem Abdu, University of Haifa, rotem abdu rotem_abdu@yahoo.com
Dor Abrahamson, University of California Berkeley, dor@berkeley.edu
Ramesh Balasubramaniam, University of California Merced, ramesh@ucmerced.edu

Abstract: We report a subset of results from an exploratory study that modeled mathematics learning using a dynamical systems lens. This study applied Recurrence Quantification Analysis to model participants' interactions with a touchscreen-based embodied-design learning environment for proportionality, conducting both qualitative (case study) and quantitative (linear regression) analyses. Findings indicate an abrupt change in the RQA meanline metric associated with increased fluency, suggesting a phase transition into a new mode of interaction. These findings suggest theoretical and methodological traction for modeling embodied math learning as phase transitions in a human–technology dynamical system.

Dynamical systems theory (DST) approaches suggest that the complex interactions within and between body and environment shape cognition (e.g. Kelso, 1995; Richardson & Chemero, 2014; Thelen & Smith, 1994). How might a DST approach apply to higher-order cognition such as mathematics learning? We explored this question using a DST tool, *Recurrence Quantification Analysis (RQA),* on touchscreen data from a mathematics learning task. RQA begins with the construction of a recurrence plot, a way to map the alignment in states between two time series (Marwan et al., 2007) (Figure 1). RQA metrics quantify features of the recurrence plot. Our analysis explored five RQA metrics, but here, in the interest of space, we will present only the meanline metric. Meanline is the average length of diagonal lines on the plot. It reflects the level of predictability and stability of a system. Meanline trends are representative of those found across a panel of RQA metrics.

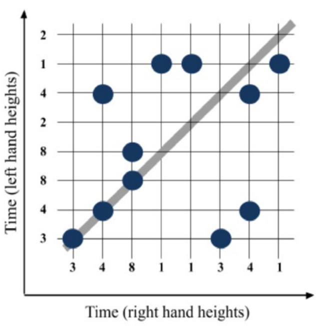

Figure 1. Example RQA plot for mock data of left and right hand heights (scale: 1-10 centimeters). Every position of each hand is compared to every position of the other hand. Aligned states are marked with a point.

We conducted a secondary analysis of touchscreen and video data of students learning in an embodied-design environment (Abrahamson et al., 2020): the Mathematics Imagery Trainer Parallel Bars problem (MIT-P) (Figure 2). In this task, participants manipulated red bars on a touchscreen with the goal of turning the bars green and maintaining them green while moving them. The bars were set to turn green only when the ratio of the left to right bar was 1:2; moving-in-green required participants to move the right bar at twice the rate of the left. This movement problem was designed to ground concepts from proportional reasoning.

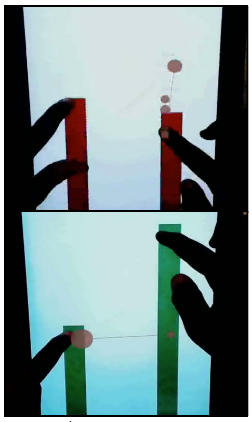

Figure 2. User interacting with the MIT-P. Upper image: bars are red. Lower image: The height of the left bar is half that of the right; bars turn green.

Our research question was: How does the predictability of hand coordination dynamics evolve as fluency increases in a math embodied-design learning task? To answer this, we conducted continuous cross-RQA of bimanual touchscreen data. We: (1) used linear regression to compare changes in meanline over time across all participants; and (2) analyzed the evolution of the meanline RQA metric for a pair of contrasting participants, one of whom reached fluency ("Nils") and the other of whom did not ("Liam"). For the regression analysis, we split each participants' time series into three phases—Exploration, Discovery, and Fluency—and regressed meanline on phase. We defined the onset of Discovery as when participants sustained green feedback above 50% of the time for 20 seconds, and of Fluency as when learners moved both hands at the same time in green at 80% of their personal best.

Regression results showed no statistically significant change from Exploration to Discovery. From Discovery to Fluency, meanline increased by an estimate mean of 14.8 deciseconds (t=3.01, d.f.=85, p=0.003), reaching about double the estimated meanline length of 15.63 deciseconds during the initial Exploration phase.

Examining the contrasting case studies (Figure 3), Nils' (reached fluency) meanline dynamics mirrored that of the overall group with longer meanline in the Fluency phase. His meanline increased abruptly synchronously with the onset of fluent moving-together-in-green (between the second dotted line and the second filled line in Figure 3, left) just after he articulated aloud an arithmetic rule to "make half" with his hands, identifying a multiplicative relation. In contrast, Liam's (did not reach fluency) meanline actually decreased over the course of the task. He, too, articulated a rule aloud, to "keep the right hand higher." Liam's qualitative rule did not yet appear to offer him as strong of a grip on the problem as with Nils: Liam did not manifest increased coordination fluency, nor an increase in meanline.

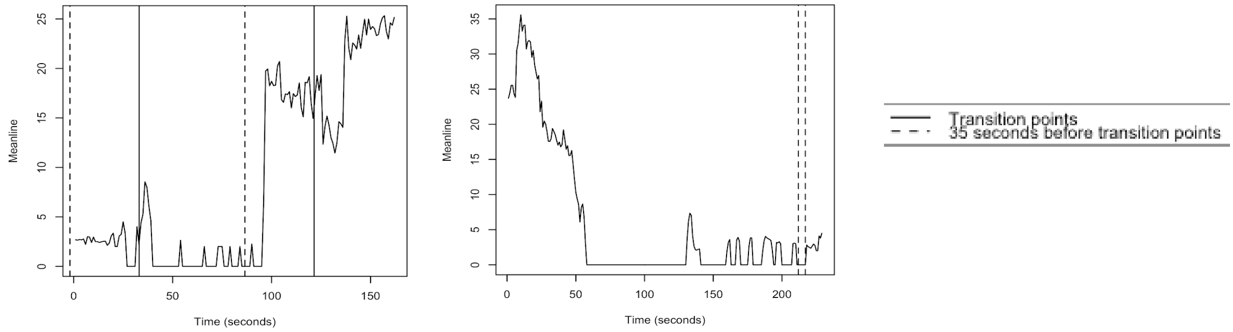

Figure 3. Windowed recurrence plots of meanline over time for Nils and Liam.

We found Fluency but not Discovery to be associated with increases in the RQA meanline metric, such that stability and predictability dynamics transformed when enacting moving-together-in-green. In the case study of a learner who attained fluency, meanline analysis showed an abrupt qualitative change. This finding evokes the dynamical-systems-theory phenomenon of a *phase transition*, whereby a system abruptly shifts from one state to another, such as from liquid to gaseous state of water. Our results suggest that the onset of fluent movement in the embodied learning of math content might constitute a phase transition in the learner–technology system. To the extent that we view higher-order cognitive activity, such as mathematical reasoning, as emerging from recurrent patterns of perceptually guided motor action (Varela et al., 1991), the Mathematics Imagery Trainer appears to constitute a *field of promoted action* (Reed & Bril, 1996) geared to foster transition into a new conceptually-salient ways of moving.

This study illustrates the traction of RQA on embodied learning data. RQA is apt for dynamical systems analyses, because it does not carry the assumption of linearity inherent in traditional quantitative methods, nor does it treat variability as noise. This proof-of-concept suggests that RQA can serve to characterize and potentially predict key moments of transition in learning. More broadly, RQA shows promise for studying the evolution of embodied-interaction learning dynamics as they unfold in time.

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