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Tariff Reductions, Entry, and Welfare: 
Theory and Evidence for the Last Two Decades*

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Abstract
We use a multi-sector, heterogeneous-firm trade model to study the trade and welfare effects of commercial policy. We show that the effect of tariffs on entry, especially in the presence of production linkages, can reverse the traditional positive optimal-tariff argument. We then use a new tariff dataset, and apply it to a 189-country, 15-sector version of our model, to quantify the trade, entry, and welfare effects of trade liberalization over the period 1990–2010. We find that the impact on firm entry was larger in Advanced relative to Emerging and Developing countries; that more than 90% of the gains from trade are a consequence of the reductions in MFN tariffs (the Uruguay Round); and that for some countries, particularly some Emerging and Developing countries, there are additional gains from a further move to complete free trade. The countries gaining from the elimination of tariffs have a strong rank correlation with those that gain from a negative optimal tariff, which comprise one-quarter of the countries in the world.

Keywords: trade policy, monopolistic competition, gains from trade, input-output linkages, multilateralism, bilateralism.

JEL Codes: F10, F11, F12, F13, F15, F17, F60, F62.

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1 Introduction

Tariffs have fallen significantly around the globe over the last two decades. Yet, very little is known about the trade, entry, and welfare effects generated by this unprecedented shift in trade policy. To study this, we build upon the most up-to-date model in international trade—with heterogeneous firms in the tradition of Melitz (2003) and Chaney (2008)—and extend this model to incorporate tariffs and the kind of input-output structure that is realistic for modern economies, following Caliendo and Parro (2015, henceforth CP). With these more general model foundations, we find that sectoral linkages and firm entry decisions can have meaningful impacts on trade and welfare, in ways not captured hitherto in many current-generation trade models.

After presenting our general model, we use a two-sector, two-country version of the Melitz-Chaney model to theoretically characterize the effects of tariffs on firm entry and welfare. We obtain clean and intuitive conditions by specializing to the case where in one sector the “manufacturing” firms produce differentiated varieties under monopolistic competition, and in the other sector the “services” firms produce a non-tradable good under perfect competition. We first show that tariffs reduce entry relative to a free trade equilibrium. We then show that this reduction in firm entry, contracts the output of the differentiated sector, raises its price index, and therefore lowers welfare, with tariff revenue only offsetting a part of this effect.

We further show that the usual (positive) optimal-tariff argument can be reversed by the impact of tariffs on firm entry. To understand this result, recall that the equilibrium of a one-sector Melitz-Chaney model is socially optimal, as shown by Dhingra and Morrow (2014). In our two-sector model, by contrast, given the presence of an outside competitive “service” sector, too few resources are devoted to the monopolistic differentiated sector and entry there is sub-optimal. This creates a domestic distortion that is exacerbated by any reduction in entry or output in that sector. We characterize the conditions under which import tariffs can be used to reduce this distortion and show that, in the absence of any other policy instrument, a negative tariff is the optimal policy. We also show how the presence of production linkages can magnify this unusual result.

We then use a 189-country/15-sector quantitative version of our model and go well beyond recent quantitative exercises in expanding the data universe to build a tariff dataset that includes not just the usual sample of Advanced (e.g., OECD) economies, but also a large subsample of Emerging and Developing economies, using newly collected data going back to the 1980s. Our work therefore permits a broader and more realistic computation of the retrospective, and prospective, gains from trade liberalization in both rich and poor nations, a step we think is crucial since it is in the poorer

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1The importance of the input-output structure has been made clear in CP and in recent work by Costinot and Rodríguez-Clare (CR, 2014). CR used stylized, uniform tariff cuts to show how the gains from trade are systematically larger when the input-output structure is taken into account. Here are echoes of an earlier trade literature on distortions due to high effective rates of protection, and more recent empirical trade and growth papers highlighting the damaging effects of tariffs on inputs (Goldberg et al. 2010; Estevadeordal and Taylor 2013).

2We unify tariff schedules from five different sources. With more than 1 million observations per year in the 1980s, rising to 2 million by the 2000s, with our tariff data we can perform tariff policy experiments which could not be explored before now.
countries that trade liberalization has proceeded most rapidly since 1990, and in which there may be still significant scope for further tariff reductions in the future.

To sum up, our paper develops new theoretical results about optimal tariffs, entry, and welfare; it builds a new tariff dataset and compiles other data from high- and low-income countries in order to calibrate the model; and it uses the model to perform policy experiments to evaluate the gains from actual past trade liberalization and possible future gains yet to be realized.

**Major findings** We implement four policy experiments. First, we quantify the effects of arguably the most successful GATT/WTO process, the Uruguay Round.\(^3\) We do so by using the model to evaluate the economic effects of the observed change in Most Favored Nations (MFN) tariffs for countries at the product level from 1990 to 2010, focusing on the trade, entry, and welfare impacts. We then go beyond this Uruguay Round experiment and evaluate the impact of all observed changes in tariffs, namely MFN and preferential tariffs, over the same period; we refer to this model experiment as Uruguay Round + Preference. After that, we ask if there are any further potential gains in the world today from zeroing all tariffs, a counterfactual experiment we refer to as Free Trade. Finally, we also investigate whether, starting from a Free Trade position, the imposition of negative tariffs would be optimal for each country acting individually.

We find that the Uruguay Round had a profound impact. Almost all the gains from tariff elimination in the last two decades result from the MFN tariff cuts in the Uruguay Round. The effects from other tariff reductions, namely PTAs, contributed virtually nothing to total world trade and welfare. In fact, we find that PTAs generated only a tiny average increase in the world trade share (measured as imports/GDP), whereas on its own the Uruguay Round doubled the trade share. In terms of welfare, the Uruguay Round generated an average increase in welfare of 1.43%, while the additional effect from PTAs was only 0.13%, an order of magnitude smaller.

When looking at countries by income group, we find that both the Advanced and the Emerging and Developing economies gained most from Uruguay Round tariff elimination relative to PTAs. We also find that the distribution of gains across these two groups are quite different. For the Advanced economies, most countries gain and the gains do not vary widely. However, for Emerging and Developing economies, not all countries win, but the ones that do gain substantially.

We also evaluate how commercial policy has affected the entry and exit of firms across markets. We find that tariffs affect firm entry in very different ways across countries. For instance, the reductions in tariffs as a consequence of the Uruguay Round generated considerable changes in entry and exit of firms across industries in Advanced economies, while there was a much smaller effect on Emerging economies. This is despite the fact that the Emerging economies have greater dispersion in the welfare impact of the Uruguay Round tariff cuts.

The results are striking when we consider the counterfactual of moving to a Free Trade world with zero tariffs. Our results show that there are extra gains for some Emerging and Developing economies, in particular. Furthermore, there is a strong rank correlation between the countries gaining from complete free trade and those which are found to have negative optimal tariffs. One-quarter of the countries in the world have negative optimal tariffs, with the majority of these being small and remote, and a minority being more developed countries that appear to have strong production linkages.

The remainder of the paper is structured as follows. After briefly comparing our results to the existing literature below, in Section 2 we present the quantitative model. In Section 3, to develop intuition, we present some key results with the aid of a simplified two-sector two-country model. Section 4 describes the new tariff dataset and the rest of the data sources that we use in order to calibrate the 189-country/15-sector version of the model. Section 5 explains how we take the model to the data, and section 6 presents the empirical results which quantify the gains from tariff liberalization in the last 20 years, and the potential remaining gains from tariff liberalization going forward. Section 7 concludes. All proofs are relegated to the Appendix.

Comparison to the literature  Our study is related to Spearot (2016) who analyzes the tariff cuts of 1994–2000 over a large group of countries, one that is only slightly smaller than the set of countries and the time period that we shall analyze. In his model, he finds that while the majority of countries benefited from those tariff cuts, those benefits were skewed towards developing countries. In contrast, the benefits from zeroing all tariffs from their 2000 levels would be skewed towards advanced countries. Significantly, in his model, only about one-half of countries benefit from both tariff cuts (i.e., going from 1994 levels to zero), and few countries benefit from unilateral tariff cuts starting from 2000 levels (though the countries that do gain include India, Japan, Korea, and the U.S.). These results from Spearot (2016), emphasizing the disparate gains across groups of countries and the losses from unilateral tariff cuts in most cases, are very much in line with the conventional optimal tariff argument.

Our results are quite different. We shall find that the Advanced and the Emerging and Developing countries both gain roughly the same amount on average from the actual tariff cuts seen over the period 1990–2010 (and likewise, from going all the way to zero tariffs), though there is greater variation in the benefits for the latter group. Most important, we find that mutual gains would have occurred even if either one of these groups had cuts its tariffs, with no tariff cuts by the other. In other words, we find quantitative evidence of a negative optimal tariff, despite the fact that we share the heterogeneous firm, monopolistic competition framework in the tradition of Melitz (2003) and Chaney (2008).\footnote{Spearot (2016) actually relies on the quadratic utility function in the spirit of Melitz and Ottaviano (2008). Because he does not assume an outside good, however, he argues that the results are much the same when using a CES utility function.}

\footnote{For a recent quantitative study on optimal tariffs, see Ossa (2014).}
Optimal tariffs have been examined in a heterogeneous firm monopolistic competition model by Costinot, Rodríguez-Clare, and Werning (CRW, 2016). They find that the selection of heterogeneous firms into exporting leads to an aggregate nonconvexity in the foreign production possibilities set between domestic goods and exports, which damps the incentive for the home country to apply a tariff to improve its terms of trade. Nevertheless, if there is a Pareto distribution for firm productivities then the optimal tariff is still positive, but lower than it would otherwise be. It follows that individual countries will lose from removing small tariffs, so that mutual gains require multilateral tariff cuts.

Three important features of our model are responsible for some very profound differences between our results and those of Spearot and CRW. First, we allow for production linkages with the kind of input-output structure that is realistic for modern economies. Specifically, we have traded intermediate inputs making use of the non-traded finished goods as material inputs in their production. Second, we analyze only a simple import tariff, and not the full range of policy instruments as used by CRW. As they stress, having the full range of instruments available means that tariffs are never used to offset domestic distortions. Third, there is indeed a domestic distortion present in our model because we allow for the free entry of firms, and we find that entry is impacted by the use of tariffs. So, while a reduction in tariffs generates a terms-of-trade loss it generates a welfare gain by adjusting entry to its optimal level. As a result, the impact of tariffs on entry, especially in the presence of production linkages, can reverse the traditional positive optimal tariff argument.

This study also relates to recent work by Melitz and Redding (2015) who show, in a Melitz (2003) model, that after relaxing the assumption of a Pareto distribution of firm productivities assumed in Chaney (2008), changes in iceberg trade cost impact entry and welfare. A contribution of this paper is to clearly explain how tariffs affect entry, and ultimately welfare, in a Melitz (2003) model, even without relaxing the maintained assumption of a Pareto distribution of firm productivities.

The potential for tariffs to impact entry has not received sufficient attention in the literature. We believe that one reason for this is that iceberg transport costs do not affect entry in a one-sector Melitz-Chaney model, as shown most clearly by Arkolakis, Costinot, and Rodríguez-Clare (2012, henceforth ACR). One of ACR’s “macro” assumptions—which they label R2—is that aggregate profits in any country $i$ ($\Pi_i$, measured gross of the entry fee) are a constant share of aggregate revenue ($R_i$), and that assumption is indeed satisfied in the special case of a Pareto distribution on productivity draws. In the further special case of a symmetric, one-sector, one-factor model, revenue equals the factor supply ($L_i$), since without loss of generality we can normalize wages $w_i = 1$. In turn, revenue is fixed, aggregate profits are also fixed, and since these equal the number of entrants $N$ times the fixed costs of entry $f_i^E$, it follows that $N_i = \Pi_i/f_i^E \propto R_i/f_i^E = L_i/f_i^E$, which in turn is also then fixed. Therefore, changes in iceberg transport costs have no impact on tariffs.

entry in this very special case. In a multi-sector model, however, the factor supply to each sector is not fixed so it is quite possible that changes in iceberg transport costs will affect entry, as ACR (section IV.A) note.

Balistreri, Hillberry, and Rutherford (2011) were the first to introduce ad valorem tariffs into a Melitz-Chaney model. They obtain substantial changes in entry in their quantitative model, which is based on GTAP and models the heterogeneous-firm sector as a single, aggregate manufacturing sector, with additional constant-returns sectors in the economy. As we show here, the presence of the additional sectors guarantees that changes in tariffs applied to the manufacturing sector will affect entry. Our approach makes further advances in several respects. We analytically solve for the impact of ad valorem tariffs on entry in a two-country version of our model with a single manufacturing sector, while in our more general quantitative model we use multiple heterogeneous-firm sectors. In addition, our tariff data are much more detailed than Balistreri et al. (2011), who consider a 50% tariff cut rather than the actual impact of the Uruguay Round.\footnote{Another difference is that Balistreri et al. (2011) estimate all the fixed costs in their model from GTAP data. In contrast, we use the “hat” algebra (Dekle, Eaton, and Kortum 2008) to solve for changes in the key variables, which avoids the need to estimate fixed costs.}

Two more recent contributions have also sought to consider ad valorem tariffs as opposed to iceberg transport costs in a Melitz-Chaney model: these are the works by Felbermayr, Jung, and Larch (2015) and Costinot and Rodríguez-Clare (2014, henceforth CR). The latter include tariffs in their analysis, but apply them to the variable production cost of imports; they allow for changes in entry in their theoretical model but do not focus on this margin in reporting their quantitative work. The former use tariffs applied to either the revenue or production cost of imports; but they hold entry fixed in their one-sector model. In our working paper, we carefully compare the difference between applying tariffs to the revenue cost of imports versus applying tariffs to the variable production cost of imports, and that analysis is briefly summarized in Appendix A. There are some notable theoretical differences between these two cases—in particular, regarding whether changes in tariffs affect entry in a one-sector model. As explained more fully in Appendix A, we assert that modeling tariffs as applying to the revenue cost of imports is clearly the realistic choice that matches customs practices, and is also a theoretically parsimonious benchmark case, so we will focus only on that case here.

Finally, we note that strong evidence of the impact of trade policy on entry is provided for the case of apparel exporters from Bangladesh by Cherkashin, Demidova, Kee, and Krishna (2015). They analyze how European Union (EU) preferences provided to these exporters led to an increase in entry and an increase in exports to both the EU and to the United States. We confirm in our quantitative exercise that changes in foreign tariffs impact entry in the exporting countries, and we find the greatest changes in entry for Advanced countries, which face the largest tariff reductions in Emerging and Developing markets.
2 Model

Consider a world with $M$ countries, indexed by $i$ and $j$. There is a mass $L_i$ of identical agents in economy $i$. There are $S$ sectors, making final or intermediate goods, indexed by $s$ and $s'$. Agents consume nontradable finished goods from all sectors. The finished goods in turn are produced with intermediate goods from different sources, either traded or nontraded. Finished goods are also used as materials, i.e., inputs, for the production of intermediate goods, along with raw labor. Intermediate goods producers in sector $s$ have heterogeneous productivities $\varphi$ (which, following convention, we will also use as an index for each producer, or firm). Specifically, upon entry, for which it pays a fixed cost, a firm’s $\varphi$ is drawn from the known distribution of productivities $G_s(\varphi)$, where we assume that $G_s(\varphi) = 1 - \varphi^{-\theta_s}$ follows a Pareto distribution with coefficient $\theta_s > 0$. We further impose the standard condition that $\theta_s + 1 > \sigma_s$, where $\sigma_s$ is the elasticity of substitution of intermediate varieties defined later, so as to ensure that average aggregate productivity under constant elasticity of substitution (CES) aggregation is well defined. The schematic production structure of the model is shown in Figure 1, where the significant inclusion of inter-sectoral production linkages is shown by the crossed highlighted arrows.

In addition to fixed entry costs, the intermediate goods producers face fixed operating costs, and costs of trading, in all markets. As regards trading costs, traded intermediate goods are subject to two types of bilateral trade frictions. First, as is conventional there is an iceberg trade cost in the \textit{ad valorem} form $\tau_{ji,s} - 1 > 0$ of shipping goods from $j$ to $i$, where we assume $\tau_{ii,s} = 1$ for all $i, s$. Second, we introduce the \textit{ad valorem} tariff $t_{ji,s}$ which is applied to the \textit{revenue cost} of imports from
j to i, where we assume that \( t_{ii,s} = 0 \). Intermediate goods producers decide how much to supply to the domestic market and how much to supply abroad. Intermediate producers in sector \( s \) and country \( j \) pay a fixed operating cost \( f_{ji,s} \) in order to produce goods for market \( i \), and we make the standard assumption that home operation is less costly than export operation, so that \( f_{ii,s} < f_{ji,s} \) for all \( j \neq i \). As a result of these fixed costs, less efficient producers of intermediate goods do not find it profitable to supply certain markets, and some do not operate even in the home market. We denote by \( \varphi_{ji,s}^* \) the cutoff or threshold productivity level such that all firms in each sector \( s \) and country \( j \) with \( \varphi < \varphi_{ji,s}^* \) are not active in exporting to country \( i \), or not active in the home market, in the case where \( \varphi < \varphi_{ii,s}^* \).

Denote by \( N_{j,s} \) the mass of entering firms in equilibrium in each sector \( s \) and country \( j \). By virtue of the Pareto distribution, the number of firms/products actually sold in sector \( s \), from country \( j \), into market \( i \) is the total number of entering firms times the mass of firms above the relevant threshold, which is given by \( N_{j,s} \left[ 1 - G_s \left( \varphi_{ji,s}^* \right) \right] = N_{j,s} \varphi_{ji,s}^* \).

### 2.1 Households

Assume that agents consume only domestically produced nontraded finished goods with preferences given by

\[
U_i(C_i) = \prod_{s=1}^{S} (C_{i,s})^{\alpha_{i,s}},
\]

where \( C_{i,s} \) is the consumption of a finished good with sector index \( s \) and produced in country \( i \), and the \( \alpha_{i,s} \) are standard expenditure shares.\(^8\)

Demand is then given by

\[
C_{i,s} = \frac{\alpha_{i,s} R_i}{P_{i,s}},
\]

where \( R_i \) represents the income of the agents in country \( i \), and \( P_{i,s} \) is the price of finished good \( s \) in country \( i \). As explained below, agents derive income from two sources, labor income and rebated tariff revenue, and firm profits will be equal to zero by an assumption of free entry.

### 2.2 Finished goods producers

Assume finished goods are assembled from tradable intermediates using no labor. Specifically, finished goods are produced with a nested CES production function: the upper-level distinguishes home and foreign inputs, with an elasticity of substitution of \( \omega_s > 1 \) between these two groups; and the lower-level is defined over varieties of home and varieties of foreign intermediate inputs, with an elasticity of substitution \( \sigma_s > \omega_s \) between varieties within each group.\(^9\)

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\(^8\)The final goods are inherently nontraded by assumption, e.g., due to prohibitive iceberg costs.

\(^9\)This nested structure is also used by Feenstra, Luck, Obstfeld, and Russ (2014). We use this nested structure here (in contrast to our working paper) because Kucheryavyy, Lyn, and Rodriguez-Clare (2016) have recently shown the potential for corner solutions in multi-sector monopolistic competition models. That potential is offset by adding the extra upper-level curvature in the nested CES structure.
The cost minimization problem of finished good firms in sector \( s \) and country \( i \) is:

\[
\min_{\{q_{ji,s}(\varphi)\}_j} \sum_{j=1}^{M} N_{j,s} \int_{\varphi_{ji,s}}^{\infty} p_{ji,s}(\varphi) q_{ji,s}(\varphi) g_s(\varphi) d\varphi,
\]

subject to

\[
Q_{i,s} = \left( (Q_{i,s})^{\omega_s^{-1}} + (Q_{i,s}^{F})^{\omega_s^{-1}} \right)^{-\omega_s},
\]

where

\[
Q_{i,s} = \left( N_{i,s} \int_{\varphi_{i,s}}^{\infty} q_{i,s}(\varphi)^{\frac{\omega_s}{\sigma_s-1}} dG_s(\varphi) \right)^{\frac{\sigma_s}{\sigma_s-1}},
\]

\[
Q_{i,s}^{F} = \left( \sum_{j\neq i}^M N_{j,s} \int_{\varphi_{j,s}}^{\infty} q_{j,i,s}(\varphi)^{\frac{\omega_s}{\sigma_s-1}} dG_s(\varphi) \right)^{\frac{\sigma_s}{\sigma_s-1}},
\]

and \( q_{ji,s}(\varphi) \) is the demand by country \( i \) and sector \( s \) of an intermediate variety \( \varphi \) from country \( j \) with the tariff-inclusive price \( p_{ji,s}(\varphi) \). \( Q_{i,s} \) is the total quantity of finished goods produced, and \( N_{j,s} \) is the number of entering firms in country \( j \) and sector \( s \). As noted above, the number of firms/products actually sold to market \( i \) is \( N_{j,s} \left[ 1 - G_s(\varphi_{ji,s}^*) \right] = N_{j,s} \varphi_{ji,s}^{*,-\varphi} \). Note that \( q_{ji,s}(\varphi) > 0 \), and the good is produced by \( j \) for \( i \), if and only if \( \varphi \geq \varphi_{ji,s}^* \). Otherwise \( q_{ji,s}(\varphi) = 0 \), which accounts for the lower limit of the integral.

From the standard solutions to this nested CES problem we find that home demand for home intermediates of variety \( \varphi \) sold in sector \( s \) in country \( i \) is given by

\[
q_{i,i,s}(\varphi) = \left( \frac{p_{i,i,s}(\varphi)}{P_{i,i,s}} \right)^{-\sigma_s} \left( \frac{P_{i,i,s}}{P_{i,s}} \right)^{-\omega_s} Y_{i,s} P_{i,s}^{1-\sigma_s}, \tag{3}
\]

where \( Y_{i,s} = P_{i,s} Q_{i,s} \) is the value of output of the finished good \( s \) in \( i \), and \( P_{i,i,s} \) is the CES price index for home intermediate inputs in sector \( s \), which is given by

\[
P_{i,i,s} = \left( N_{i,s} \int_{\varphi_{i,s}}^{\infty} p_{i,i,s}(\varphi)^{1-\sigma_s} dG_s(\varphi) \right)^{\frac{1}{1-\sigma_s}}.
\]

Likewise, home demand for imported intermediates sold from country \( j \neq i \) in country \( i \) is

\[
q_{j,i,s}(\varphi) = \left( \frac{p_{j,i,s}(\varphi)}{P_{j,i,s}} \right)^{-\sigma_s} \left( \frac{P_{j,i,s}}{P_{i,s}} \right)^{-\omega_s} Y_{j,s} P_{i,s}^{1-\sigma_s}, \tag{4}
\]

\(^{10}\)Intermediate good producers are heterogeneous in their productivity levels and since a particular variety is related to a particular productivity throughout the paper we will abuse notation and denote by \( \varphi \) both the productivity level and variety of the firm.
where \( P_{i,s}^F \) is the CES price index of foreign intermediate inputs, inclusive of tariffs, is given by

\[
P_{i,s}^F = \left( \sum_{j \neq i}^{M} N_{j,s} \int_{\varphi_{j_{i,s}}}^{\infty} p_{j_{i,s}}(\varphi)^{1-\sigma_s} dG_s(\varphi) \right)^{\frac{1}{1-\sigma_s}}.
\]

Finally, with these results, we can derive the aggregate CES prices index \( P_{i,s} \) over all varieties,

\[
P_{i,s} = \left[ (P_{i,s}^F)^{1-\omega_s} + (P_{i,s}^F)^{1-\omega_s} \right]^{\frac{1}{1-\omega_s}}.
\]

(5)

### 2.3 Intermediate goods producers

Denote the output of a tradable intermediate goods firm in sector \( s \) in country \( i \) with variety \( \varphi \) by \( q_{i,s}(\varphi) \). In order to produce, the intermediate goods producer must incur fixed costs, which are discussed below. In addition, the producer employs labor and uses materials from all sectors (the production linkages) and combines them using the following production function

\[
q_{i,s}(\varphi) = \varphi \ l_{i,s}(\varphi) \prod_{s' = 1}^{S} m_{i,s'}(\varphi)^{\gamma_{i,s'}},
\]

(6)

where \( \varphi \) is the productivity draw of the firm, \( l_{i,s}(\varphi) \) is labor demand, \( m_{i,s'}(\varphi) \) is the quantity of materials used from sector \( s' \), \( \gamma_{i,s} \geq 0 \) is the share in output of value added (here, labor costs), and \( \gamma_{i,s'} \geq 0 \) is the share in output of the cost of inputs from sector \( s' \) used by sector \( s \) (input-output coefficients). The technology will be assumed to be constant returns, and this requires that the production cost shares sum to unity, so that \( \gamma_{i,s} + \sum_{s' = 1}^{S} \gamma_{i,s'} = 1 \).

**Cost minimization**  
We solve the problem of the tradable intermediate variety producer in two stages. First, we determine the minimum cost of producing a given quantity. The solution to this problem is the variable cost function of the firm. Second, we solve the profit maximization problem of the firm using the cost function derived in the first stage and allowing for the fixed costs.

The cost minimization problem of tradable intermediate firms of variety \( \varphi \) in country \( i \) is

\[
C \left( q_{i,s}(\varphi); w_i, \{P_{i,s'}\}_{s' = 1}^{S} \right) = \min_{(l_{i}(\varphi), \{m_{i,s'}(\varphi)\}_{s' = 1}^{S}, \gamma_{i,s} \geq 0)} w_i l_{i,s}(\varphi) + \sum_{s' = 1}^{S} P_{i,s'} m_{i,s'}(\varphi),
\]

subject to (6), where \( w_i \) denotes the wage in country \( i \).

From the first order conditions of this problem, the demand for labor in the production of variety \( \varphi \) in each sector \( s \) is given by

\[
l_{i,s}(\varphi) = \gamma_{i,s} \ \frac{x_{i,s} q_{i,s}(\varphi)}{w_i \ \varphi},
\]
and the demand for intermediate inputs is given by

\[ m_{i,s'}(\varphi) = \gamma_{i,s'} \frac{x_{i,s} q_{i,s}(\varphi)}{P_{i,s} \varphi}, \]

where in the last expression we introduce a newly-defined term

\[ x_{i,s} \equiv (w_i / \gamma_{i,s}) \prod_{s'=1}^{S} \left( \frac{P_{i,s'} / \gamma_{i,s'}}{\gamma_{i,s'}} \right)^{\gamma_{i,s'}}, \tag{7} \]

and we refer to this price index \( x_{i,s} \) as the cost of the input bundle or more simply as the input cost index. The input cost index contains information on prices from all sectors in the economy and, clearly, the input cost directly affects production decisions in all sectors. This feature is a key distinction of our model, as compared to a one-sector model or a multi-sector model without input-output linkages.

The solution to the cost minimization problem yields the following variable cost function for each producer of variety \( \varphi \) in country \( i \) and sector \( s \):

\[ C(q_{i,s}(\varphi); x_{i,s}) = \frac{x_{i,s}}{\varphi} q_{i,s}(\varphi). \tag{8} \]

The marginal cost of each producer is then given by

\[ MC_{i,s}(q_{i,s}(\varphi); x_{i,s}) = \frac{x_{i,s}}{\varphi}. \tag{9} \]

**Profit maximization** We now solve for the profit maximizing quantity of output of the intermediate variety producer assuming monopolistic competition. Producers in country \( i \) pay a sector-specific fixed operating cost to sell into each market \( j \), denoted by \( f_{ij,s} \) and paid in units of labor. Note that since the production technology is linear we can solve the profit maximization problem for each individual market separately. Consider the profit maximization problem of supplying goods to market \( j \). Profits are given by

\[ \pi_{ij,s}(\varphi) = \max_{p_{ij,s}(\varphi) \geq 0} \left\{ \frac{p_{ij,s}(\varphi)}{1 + t_{ij,s}} q_{ij,s}(\varphi) - \frac{x_{i,s}}{\varphi} \tau_{ij,s} q_{ij,s}(\varphi) - w_i f_{ij,s} \right\}, \tag{10} \]

subject to (\ref{4}). The control variable in this problem is \( \frac{p_{ij,s}(\varphi)}{1 + t_{ij,s}} \), the net-of-tariff price received by the exporting firm.

As we can see, this price differs from the tariff-inclusive price \( p_{ij,s}(\varphi) \) paid by the importer, and means that the sales revenue \( p_{ij,s} q_{ij,s} \) is divided by the tariff factor \( 1 + t_{ij,s} \) in order to obtain producer revenue in (\ref{10}). Note that the quantity sold by the firm is \( \tau_{ij,s} q_{ij,s}(\varphi) \) because of the iceberg trade costs. So the costs of production \( (x_{i,s}/\varphi) q_{ij,s} \) are multiplied by the iceberg trade costs \( \tau_{ij,s} \) to obtain the costs in (\ref{10}).
These are subtle but very important details. This discussion shows how the tariffs and iceberg trade costs enter the profit equation in slightly different ways, and follows from our reality-based assumption that the ad valorem tariff is applied to the sales revenue. In contrast, if the tariff was applied to only the costs of the imported product then the costs \((x_{i,s}/\varphi)q_{ij,s}\) would be multiplied by the product of the iceberg trade costs and the tariff factor, \(\tau_{ij,s}(1 + t_{ij,s})\) in (11), so that the tariffs and iceberg costs would enter the firm’s problem symmetrically.\footnote{We will see that this distinction between how tariffs and iceberg costs are modeled makes an important difference to the zero-profit-cutoff productivity that we solve for below.}

The first order conditions of this CES producer problem can be solved for the quantity sold and price charged, as follows, making use of the CES demand functions at (3) and (4). As in the standard solution, price charged is the usual markup over unit cost pre-tariff (input cost index, adjusted for productivity, and also scaled by the iceberg factor since it is a destination pre-tariff price). The quantity demanded for imported inputs is then a function of this price plus the tariff, relative to the import price index of all intermediates in sector \(s\) in destination market \(i\). Thus,

\[
p_{ij,s}(\varphi) = \frac{\sigma_s}{\sigma_s - 1} \frac{x_{i,s} \tau_{ij,s}}{\varphi}, \tag{11}
\]

\[
q_{ij,s}(\varphi) = \left( \frac{\sigma_s}{\sigma_s - 1} \frac{x_{i,s} \tau_{ij,s}}{\varphi} \right)^{-\sigma_s} \frac{P_{j,s}^{F(\sigma_s-\omega_s)} P_{j,s}^{\omega_s-1} Y_{j,s}}{(1 + t_{ij,s})^{\sigma_s}}. \tag{12}
\]

The profits for sector \(s\) in country \(i\) from selling to market \(j\neq i\) are given by the markup minus one, times unit cost pre-tariff, times output, less fixed costs:

\[
\pi_{ij,s}(\varphi) = \frac{x_{i,s} \tau_{ij,s} q_{ij,s}(\varphi)}{(\sigma_s - 1)\varphi} - w_i f_{ij,s}. \tag{13}
\]

The price \(p_{ii,s}(\varphi)\) and quantity \(q_{ii,s}(\varphi)\) for selling to the home market are obtained by using \(t_{ij,s} = 0\), \(\tau_{ij,s} = 1\), and replacing the import price index \(P_{i,s}^{F}\) with the home price index \(P_{ii,s}\) in the above expressions.

\footnote{For clarity, the profit maximization equation in the case where the tariff was applied to firm revenue for the imported product, profits would be as in (11) and we can scale that up by a factor \((1 + t_{ij,s})\) to get,

\[
(1 + t_{ij,s}) \pi_{ij,s}(\varphi) = \max_{p_{ij,s}(\varphi) \geq 0} \left\{ p_{ij,s}(\varphi) q_{ij,s}(\varphi) - \frac{x_{i,s}}{\varphi} \tau_{ij,s}(1 + t_{ij,s}) q_{ij,s}(\varphi) - w_i f_{ij,s}(1 + t_{ij,s}) \right\}.
\]

In contrast, when the tariff is applied to only the firm cost for the imported product profits would be,

\[
\pi_{ij,s}(\varphi) = \max_{p_{ij,s}(\varphi) \geq 0} \left\{ p_{ij,s}(\varphi) q_{ij,s}(\varphi) - \frac{x_{i,s}}{\varphi} \tau_{ij,s}(1 + t_{ij,s}) q_{ij,s}(\varphi) - w_i f_{ij,s} \right\}.
\]

In both expressions we use the firm’s destination price \(p_{ij,s}\) and quantity sold \(q_{ij,s}\) to make for comparability. From these two equations, viewed side-by-side, it is obvious that in the latter case the effect of cost tariffs and icebergs are totally symmetric, entering as \(\tau_{ij,s}(1 + t_{ij,s})\), and setting aside the income effects arising for the cost tariff rebate which are absent in the case of icebergs.}
2.4 Selection and Entry

Zero cutoff profit condition As usual, following Melitz (2003), the first-stage fixed costs of entry $f_{i,s}^E$ in each sector $s$ and country $i$ are assumed to be covered by a lump-sum mutual-fund arrangement which pays out to all firms that enter, whether they are nonoperators, domestic operators, or export operators. This scheme operates in the background, and ensures ex ante expected profits are zero at the first-stage decision, which governs the entry of firms. This leaves only the second-stage fixed costs of operation $f_{ij,s}$ for each sector $s$ and exporter-importer pair $ij$ to be considered, which govern the selection of firms into nonoperators, domestic operators, or export operators according to another set of zero expected profit conditions.

Given the presence of fixed operating costs, there exits a threshold level of productivity such that a firm in a given sector makes zero profit. We can characterize the threshold or cut-off level of productivity of operating firms by looking at the profits of the marginal firm producer. In particular, the zero cutoff profit (ZCP) level of productivity is determined by

$$\pi_{ij,s} (\varphi_{ij,s}^*) = 0.$$  

Using the equilibrium conditions for prices and quantities derived before, the ZCP level of productivity in sector $s$ for export sales is given for $i \neq j$ by

$$\varphi_{ij,s}^* = \left(\frac{\sigma_s}{\sigma_s - 1}\right) \left(\frac{\sigma_s w_i f_{i,j,s} (1 + t_{ij,s})}{Y_{j,s} P_{j,s}^F (\sigma_s - \omega_s) P_{j,s}^{\omega_s - 1}}\right)^{\frac{1}{\sigma_s - 1}} x_{i,s} \tau_{ij,s} (1 + t_{ij,s}).$$  

(14)

Note that a reduction in the tariff level affects the ZCP condition in a way that is different from a reduction in iceberg trade costs. This follows from our assumption that tariffs are applied to the sales value of the import, as discussed above. In practice, this means that a reduction in actual tariffs acts in the ZCP condition very similarly to a joint reduction in iceberg trade costs and in fixed costs. In contrast, if tariffs are applied only to the costs of imported products, then they would have exactly the same effect on the zero-cutoff-profit condition as do iceberg trade costs $\tau_{ij,s}$, and would appear only as multiplying those trade costs above (i.e., as in the final terms in (14)). Under our maintained assumption that tariffs are applied to the sales revenue, they have the “extra” impact of effectively reduced fixed costs, too. The gains from tariff reduction will take into account this implicit reduction in fixed costs, which will act so as to encourage the entry of exporters and increase export variety, as we show below.

Another feature of (14) that deserves attention is that the output $Y_{j,s}$ of sector $s$ in country $j$ appears in the denominator on the right. With country $i$ exporting to country $j$ in that sector, a higher output means that exporters can spread their fixed costs over greater sales, which therefore allows more firms to self-select into exporting. We therefore refer to the presence of $Y_{j,s}$ in (14) as a selection effect, and we will find that it enters our later equations, too.
Free entry  As noted earlier, firms pay a fixed cost of entry \( f_{i,s}^E \) in each sector, in units of labor, in order to allow them to take a draw from the known distribution of productivities \( G_s(\varphi) \). Free entry implies that expected profits of firms have to equal entry costs in sector \( s \) and country \( i \),

\[
\sum_{j=1}^{M} \int_{\varphi_{ij,s}}^{\infty} \pi_{ij,s}(\varphi) \, dG_s(\varphi) = w_i \, f_{i,s}^E.
\]

Using the equilibrium conditions (13) and (14), and the analogous conditions for the home market, and given the assumption of a Pareto distribution of productivities, we end up with the following equilibrium condition

\[
\sum_{j=1}^{M} f_{ij,s} \varphi_{ij,s}^{\vartheta_s} = \frac{\theta_s - \sigma_s + 1}{\sigma_s - 1} \, f_{i,s}^E,
\]

which relates the ZCP levels of productivities to the fixed operating and entry costs \( f_{ij,s} \) and \( f_{i,s}^E \).

2.5 Price index

We define the average productivity level of the firms making intermediate goods in sector \( s \) sold in \( i \) and sourced from \( j \) as

\[
\hat{\varphi}_{ji,s} = \left( \int_{\varphi_{ji,s}}^{\infty} \varphi^{\sigma_s-1} \mu_{ji,s}(\varphi) \, d\varphi \right)^{\frac{1}{\sigma_s-1}},
\]

where \( \mu_{ji,s}(\varphi) = g_s(\varphi)/\left[1 - G_s(\varphi_{ji,s})\right] \) is the conditional distribution of productivities (that is, conditional on the variety \( \varphi \) being actively produced for this \( \{i, j, s\} \) combination). Then using (3) and (4) we obtain

\[
P_{i,s} = \left\{ \left( \varphi_{ii,s}^{\vartheta_s} N_{i,s} \right)^{1-\omega_s} \left( \frac{\sigma_s}{\sigma_s - 1} \frac{x_{i,s}}{\varphi_{ii,s}} \right)^{1-\omega_s} + \sum_{j \neq i}^{M} \left( \varphi_{ji,s}^{\vartheta_s} N_{ji,s} \right) \left( \frac{\sigma_s}{\sigma_s - 1} \frac{x_{j,s} \gamma_{ji,s} (1 + t_{ji,s})}{\varphi_{ji,s}} \right)^{1-\sigma_s} \right\}^{\frac{1}{1-\omega_s}},
\]

where \( \varphi_{ji,s}^{\vartheta_s} = \left[1 - G_s(\varphi_{ji,s}^{\vartheta_s})\right] \) is the probability that an entering firm in country \( j \) actually exports to market \( i \), so that the number of products actually sold are \( N_{ji,s} \equiv \varphi_{ji,s}^{\vartheta_s} N_{ji,s} \).

2.6 Trade balance and market clearing

Two steps remain to close the model, the first being to ensure that all entities obey their budget constraints, markets clear, and trade is balanced.
Expenditure shares Recall that \( Y_{i,s} = P_{i,s} Q_{i,s} \) is the value of the output of the finished good \( s \) in country \( i \), which is produced entirely from intermediate goods, these being either imported or produced domestically. Hence, this value of output equals the total expenditure on those intermediate goods.

Let \( \lambda_{ji,s} \) denote the share of country’s \( i \) total expenditure in sector \( s \) on intermediate goods from market \( j \). In this share, integrating over sales of all varieties of \( s \) from \( j \) to \( i \) yields the numerator, and summing over all markets \( j \) gives the denominator:

\[
\lambda_{ji,s} = \frac{N_{j,s} \int_{\varphi_{ji,s}}^{\infty} p_{ji,s}(\varphi) q_{ji,s}(\varphi) dG_s(\varphi)}{\sum_{k=1}^{M} N_{k,s} \int_{\varphi_{ki,s}}^{\infty} p_{ki,s}(\varphi) q_{ki,s}(\varphi) dG_s(\varphi)}. \tag{18}
\]

Using the conditions (11), (12), (16), and (17) we can obtain the following expression for the expenditure share on domestic inputs

\[
\lambda_{ii,s} = \varphi_{ii,s}^{*} - \theta_s N_{i,s} \left( \frac{\sigma_s}{\sigma_s - 1} \frac{x_{i,s}}{\varphi_{ii,s} P_{ii,s}} \right)^{1-\sigma_s} \left( \frac{P_{ii,s}}{P_{i,s}} \right)^{1-\omega_s}, \tag{19}
\]

and on imported inputs

\[
\lambda_{ji,s} = \varphi_{ji,s}^{*} - \theta_s N_{j,s} \left( \frac{\sigma_s}{\sigma_s - 1} \frac{\tau_{ji,s} x_{j,s} (1 + t_{ji,s})}{\varphi_{ji,s} P_{i,s}^{F}} \right)^{1-\sigma_s} \left( \frac{P_{i,s}^{F}}{P_{i,s}} \right)^{1-\omega_s}. \tag{20}
\]

Sectoral trade flows We now solve for sectoral exports and imports and impose balanced trade.

Consider sector \( s \) imports first. The total expenditure by country \( i \) on country \( j \) intermediate goods is given by \( \lambda_{ji,s} Y_{j,s} \). Due to the presence of tariffs not all of this expenditure reaches producers in country \( j \). The tariff-adjusted expenditure in country \( j \) on goods produced in country \( i \), or exports from \( i \) to \( j \), is \( E_{ij,s} = \lambda_{ji,s} Y_{j,s} \).

Of course, that term is identical to imports arriving in \( j \) from \( i \). Therefore, total exports from country \( i \), not including goods that are sold domestically, are given by

\[
E_{i,s} = \sum_{j \neq i} E_{ij,s} = \sum_{j \neq i} \frac{\lambda_{ij,s}}{1 + t_{ij,s}} Y_{j,s}, \tag{21}
\]

and total imports are given by

\[
\sum_{j \neq i} E_{ji,s} = \sum_{j \neq i} \frac{\lambda_{ji,s}}{1 + t_{ji,s}} Y_{i,s}. \tag{22}
\]
Now we have derived the sectoral trade flows, we define the trade balance condition,

$$\sum_{s=1}^{S} \sum_{j \neq i} \frac{\lambda_{ji,s}}{1 + t_{ji,s}} Y_{i,s} = \sum_{s=1}^{S} \sum_{j \neq i} \frac{\lambda_{ij,s}}{1 + t_{ij,s}} Y_{j,s}. \tag{23}$$

**Goods Market Equilibrium** We can also define sectoral, $T_{i,s}$, and total, $T_{i}$, tariff revenue as

$$T_{i} = \sum_{s=1}^{S} T_{is} = \sum_{s=1}^{S} \sum_{j \neq i} t_{ji,s} F_{ji,s}. \tag{24}$$

With that, the expenditure on finished goods from sector $s$ by households in country $i$ is given by $\alpha_{i,s} R_{i}$, where $R_{i}$ is total expenditure consisting of labor income plus this redistributed tariff revenue, $R_{i} = w_{i} L_{i} + T_{i}$.

The total value of production of all intermediate goods in sector $s$ in country $i$ is given by $\frac{\sigma_{s} - 1}{\sigma_{s}} \sum_{j=1}^{M} \frac{\lambda_{ij,s}}{1 + t_{ij,s}} Y_{j,s}$; namely, the net-of-tariff value of sector $s$ goods that are sold locally and abroad adjusted by markups. Given the input-output coefficients, a share $\gamma_{i,s's}$ of this gross production is then spent on intermediate inputs from sector $s'$. Therefore, the materials from sector $s'$ demanded in sector $s$ for the production of intermediate goods is then given by $\gamma_{i,s's} \frac{\sigma_{s} - 1}{\sigma_{s}} \sum_{j=1}^{M} \frac{\lambda_{ij,s}}{1 + t_{ij,s}} Y_{j,s}$.

We can then obtain the total demand for the output of sector $s$ of country $i$, which goes to both consumers as finished goods and to firms for intermediate use (the term here in braces), and which must equal total supply of that output:

$$Y_{i,s} = \alpha_{i,s} (w_{i} L_{i} + T_{i}) + \left\{ \sum_{s'=1}^{S} \gamma_{i,s's} \sum_{j=1}^{M} \frac{\lambda_{ij,s'}}{1 + t_{ij,s'}} Y_{j,s'} \right\}. \tag{25}$$

To explain this specification, recall that fixed costs are paid in units of labor. Then the value of output net of markups in each sector, $\left( \frac{\sigma_{s} - 1}{\sigma_{s'}} \right) Y_{j,s'}$, equals the value of intermediate inputs used in their production, and these generate demand for the output $Y_{i,s}$ used as materials to produce those intermediate inputs. We define the combined parameters $\bar{\gamma}_{i,s's} \equiv \gamma_{i,s's} \left( \frac{\sigma_{s} - 1}{\sigma_{s'}} \right)$ to reflect the demand generated in sector $s'$ for the output in sector $s$.

### 2.7 Firm Entry and Product Variety

To close the model we need to tackle selection and entry, solving for the mass of firms $N_{i,s}$ entering in country $i$ and sector $s$, and the productivity cutoffs $\varphi_{i,s}^{j}$ for the varieties produced for market $j$.

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12 If fixed costs are instead paid with the input bundle that costs $x_{i,s'}$, the same bundle used in variable costs, then the value of those fixed costs are measured by the markups earned in sector $s'$. So rather than deducting the markup from the value of final goods, we use the full value $Y_{j,s'}$ in sector $s'$ to generate demand for the final goods in sector $s$, according to the input-output coefficient $\bar{\gamma}_{i,s's} \equiv \gamma_{i,s's'}$. 

To solve for product variety, we first rewrite (14) for \( i \neq j \) as

\[
\left( \frac{\sigma_s}{\sigma_s - 1} \right)^{1-\sigma_s} \left( \frac{\phi_{ij,s}^*}{P_{j,s}^F} \right) = \sigma_s w_i f_{ij,s} (1 + t_{ij,s}) \frac{1 - \omega_s}{Y_{j,s}}.
\]

We then note that the average value \( \bar{\phi}_{ij,s} \) is related to the cutoff \( \phi_{ij,s}^* \) by

\[
\bar{\phi}_{ij,s} = \left( \int_{\phi_{ij,s}^*}^{\infty} \phi^{\sigma_s - 1} \mu_{ij,s}(\phi) d\phi \right)^{1/\sigma_s - 1} = \phi_{ij,s}^* \left( \frac{\theta_s}{\theta_s + 1 - \sigma_s} \right)^{1/\sigma_s - 1},
\]

by the properties of the Pareto distribution, where the integral runs over the varieties produced.

Substituting these last two equations into (20) we obtain an equation governing the cutoffs \( \phi_{ij,s}^* \),

\[
\lambda_{ij,s} = \phi_{ij,s}^* - \theta_s N_{i,s} \left( \frac{\sigma_s w_i f_{ij,s} (1 + t_{ij,s})}{Y_{j,s}} \right) \left( \frac{\theta_s}{\theta_s + 1 - \sigma_s} \right).
\]

Next, multiplying this equation by \( Y_{j,s}/(1 + t_{ij,s}) \), summing over \( j \) and making use of (21) and (13), we obtain an expression for total domestic plus international sales of intermediate inputs in sector \( s \) by country \( i \),

\[
E_{ii,s} + E_{i,s} = \sum_{j=1}^{M} \phi_{ij,s}^* - \theta_s N_{i,s} \left( \frac{\sigma_s w_i f_{ij,s}}{\theta_s + 1 - \sigma_s} \right) = N_{i,s} w_i f_{i,s}^E \left( \frac{\theta_s \sigma_s}{\sigma_s - 1} \right),
\]

from which we can obtain an equation governing the mass of entrants \( N_{i,s} \), namely

\[
N_{i,s} = \left( E_{ii,s} + E_{i,s} \right) / \left[ w_i f_{i,s}^E \left( \frac{\theta_s \sigma_s}{\sigma_s - 1} \right) \right].
\]

It may appear surprising that the total domestic plus international sales of intermediate inputs \( E_{ii,s} + E_{i,s} \) is so tightly linked to the mass of entrants \( N_{i,s} \). But recall from the introduction that the condition from ACR that aggregate profits in an economy, which equal entry times the fixed costs of entry, are proportional to the labor force: therefore, entry is fully determined by the labor force in each country. Equation (28) is the analogous result here: entry times fixed costs of entry is proportional to domestic sales plus exports in each sector. But here, exports will depend on ad valorem tariffs, as is clear from (21) and the share equations in (13).

### 2.8 Changes in Welfare

Our final step is to solve for changes in welfare in country \( i \) due to any changes in ad valorem tariffs or trade costs. For this purpose, we substitute the solution for the ZCP level of productivity from (13) into the expression for the home share \( \lambda_{ij,s} \) in (21). For convenience when comparing to the existing literature, let us focus on the case where \( \omega_s = \sigma_s \), so that the domestic and foreign
varieties all substitute with the elasticity $\sigma_s$. In addition, let us choose the wage of country $i$ as the numeraire, $w_i \equiv 1$. Then substituting (13) into (27) and differentiating, we readily obtain

$$\frac{d\lambda_{i,s}}{\lambda_{i,s}} = \frac{dN_{i,s}}{N_{i,s}} + \theta_s \left( \frac{dP_{i,s}}{P_{i,s}} - \sum_{s' = 1}^{S} \gamma_{i,s,s'} \frac{dP_{i,s'}}{P_{i,s'}} \right) + \left( \frac{\theta_s}{\sigma_s - 1} - 1 \right) \frac{dY_{i,s}}{Y_{i,s}}.$$ 

We can invert this equation to solve for price index changes $d\ln P_{i,s}$ in all sectors $s$, but it requires matrix notation to deal with the input-output coefficients $i;ss'$. Once again, for convenience in comparing to existing literature, let us simplify and suppose that $i;ss' = 0$ for $s \neq s'$, so the input-output matrix is diagonal. Then we can readily solve for the price index changes, with

$$\frac{dP_{i,s}}{P_{i,s}} = \frac{1}{\theta_s(1 - \gamma_{i,s,s})} \left[ \frac{d\lambda_{i,s}}{\lambda_{i,s}} - \frac{dN_{i,s}}{N_{i,s}} - \left( \frac{\theta_s}{\sigma_s - 1} - 1 \right) \frac{dY_{i,s}}{Y_{i,s}} \right].$$

The first term on the right of this equation is precisely the rise in prices, and hence loss in welfare, due to the change in trade volume, which here, is in the form of the change in home share, $d\ln \lambda_{i,s}$, just as in ACR. What is new are the next two terms on the right. The second term is due to the entry of firms into sector $s$ in country $i$, which ceteris paribus serves to lower the price index and raise welfare. This term does not appear in a one-sector version of the Melitz-Chaney model, because entry is fixed in that case. The third term on the right reflects the change in output of the finished good in sector $i$, $d\ln Y_{i,s}$. From our prior discussion, just after equation (13), we can regard this term as capturing the selection of firms into this sector. Thus, in general, we see that both entry and selection are needed, in addition to the change in the home share, to obtain the true change in the price index, and in welfare. These additional terms could also arise in principle in multi-sector versions of ACR and CR, though they are not stressed by those authors.

We can determine the overall change in welfare by differentiating the utility function from (2) and (3), and substituting from (30) to obtain

$$\frac{dU_i}{U_i} = -\sum_{s = 1}^{S} \alpha_{i,s} \frac{dP_{i,s}}{P_{i,s}} + \frac{dT_i}{L_i + T_i} = -\sum_{s = 1}^{S} \theta_s(1 - \gamma_{i,s,s}) \left[ \frac{d\lambda_{i,s}}{\lambda_{i,s}} - \frac{dN_{i,s}}{N_{i,s}} - \left( \frac{\theta_s}{\sigma_s - 1} - 1 \right) \frac{dY_{i,s}}{Y_{i,s}} \right] + \frac{dT_i}{L_i + T_i}.$$ 

We note that it is immediately obvious that the first term $d\ln \lambda_{i,i}$, even when naively adjusted for the income effect of the tariff rebate, is certainly not a sufficient statistic for welfare changes when entry ($N_{i,s}$) and the value of output ($Y_{i,s}$) are changing, an important point throughout this paper.
Thus, calculating the overall change in welfare will involve summing all of these endogenous effects, and in the next section we solve for them in a simplified version of our model. To motivate that analysis, we note that the sectoral outputs are determined by the goods market equilibrium conditions in (25). The entry of firms is determined by the conditions shown in (28) and (29). By summing (28) over all sectors, we obtain the payments to labor obtained from all exports and domestic sales of intermediate inputs, which equals total factor earnings, so that with the normalization we have made, \( w_i \equiv 1 \),

\[
L_i = \sum_{s=1}^{S} E_{i,i,s} + E_{i,s} = \sum_{s=1}^{S} N_{i,s} E_{i,s} \frac{\theta_s \sigma_s}{\sigma_s - 1}.
\]

(32)

Totally differentiating this condition and using (28), we readily obtain

\[
\sum_{s=1}^{S} \frac{dN_{i,s}}{N_{i,s}} \beta_{i,s} = 0, \quad \text{with} \quad \beta_{i,s} \equiv \left( \frac{E_{i,i,s} + E_{i,s}}{L_i} \right).
\]

(33)

We interpret the endogenous coefficients \( \beta_{i,s} \) as the production shares of each intermediate-goods sector in the overall economy. This equation shows that a weighted average of the proportional changes in entry, \( d\ln N_{i,s} \), sum to zero, as also obtained by Spearot (2016). In particular, a one-sector economy will have no changes in entry due to changes in ad valorem tariffs, or in iceberg costs; but a multi-sector model will generally experience entry in some sectors and exit in others.\(^{13}\)

As a final step, we consider solving from the change in entry in, say, sector 1, using (33). Substituting the result into the change in utility from (31), we obtain

\[
\frac{dU_i}{U_i} = \frac{dT_i}{L_i + T_i} - \sum_{s=1}^{S} \frac{\alpha_{i,s}}{\theta_s (1 - \gamma_{i,ss})} \frac{d\lambda_{i,i,s}}{\lambda_{i,i,s}}
\]

\[
+ \sum_{s=2}^{S} \frac{\alpha_{i,s}}{\beta_{i,s} \theta_s (1 - \gamma_{i,ss})} - \frac{\alpha_{i,1}}{\beta_{i,1} \theta_1 (1 - \gamma_{i,11})} \frac{dN_{i,s}}{N_{i,s}}
\]

\[
+ \sum_{s=1}^{S} \frac{\alpha_{i,s}}{\theta_s (1 - \gamma_{i,ss})} \left( \frac{\theta_s}{\sigma_s - 1} - 1 \right) \frac{dY_{i,s}}{Y_{i,s}}.
\]

(34)

The terms on the first line of (34) are the changes in tariff revenue rebated minus the trade share loss (the ACR term). On the second line we have the changes in entry, for sectors \( s = 2, ..., S \), multiplied by a term reflecting the combined parameters \( \alpha_{i,s}/[\beta_{i,s} \theta_s (1 - \gamma_{i,ss})] \) in each sector relative to those in sector 1, while the third line is the impact of output (i.e., selection) in each sector.

From this last calculation, it would appear that in order to raise welfare on the second line, the social planner should inhibit entry into the sector with the smallest value of the combined

\(^{13}\)As we show in Appendix A, obtaining this result for a one-sector economy with a change to the ad valorem tariff requires that the tariff revenue is redistributed to consumers. If instead the revenue is wasted on a zero-utility good, then that will withdraw labor from the economy and therefore lead to some net exit.
parameters, such as a sector with $\alpha_{i,1} = 0$ so that it has no consumer demand, and thereby encourage entry into the other sectors (which are multiplied by a positive coefficient on the second line provided that $\alpha_{s,s} > 0$). This reasoning is too simplistic, however, because the changes in the home shares $d \ln \lambda_{ii,s}$ and in outputs $d \ln Y_{i,s}$ will depend on what happens to entry. To make further progress on determining the overall change in welfare, we must solve for all these endogenous changes.

3 Illustrative Two-Country, Two-Sector Model

To illustrate some key insights from our model, we now consider a simplified case where there are two initially identical countries and two sectors, with only the home country $i = H$ then applying a tariff $t_H \equiv t$ on intermediate inputs imported from the foreign country, $j = F$. As we shall see, this case allows us to obtain a closed-form solution for the comparative statics with respect to small changes in the home tariff $dt$.

Having just two sectors allows us to be more specific about the input-output structure. The first sector (“manufactures”) will be as we have assumed above, with traded intermediate inputs and a nontraded output good that is consumed and is also used as a material in the production of intermediate inputs domestically. So this sector has both backward and forward linkages. For convenience we ignore the nested CES structure and treat the upper- and lower-level elasticities as both equal to $\sigma_s = \sigma_s = \gamma$, while the Pareto parameter is denoted by $\theta_s = \theta$.

The second sector is much simpler and will consist of purely nontraded consumer services (“haircuts”) which are produced with labor and which neither use nor are used as intermediate inputs. In other words, this residual sector has no backward or forward linkages. This second sector plays a role mainly on the demand side where it has a consumption expenditure share of $1 - \alpha$, while the first sector has an expenditure share of $\alpha$. For convenience, we assume that this second service sector is perfectly competitive and that, without loss of generality, its productivity level is unity so that the price of a unit of the service equals the wage $w_i$.

The condition (25) applies to the first sector only, and for clarity we drop the summation over sectors $s$ in (25); in fact, we can drop the sector subscript altogether. We let $\tilde{\gamma} = \tilde{\gamma}_{i,11} = \gamma \left( \frac{\sigma - 1}{\sigma} \right)$ denote the single nonzero term in the input-output matrix for the first sector in both countries, with $0 < \gamma < 1$. Finally, we normalize the wage in the home country $H$ as unity, $w_H \equiv 1$. The labor force in both countries is of the same size $L$, and the foreign wage $w_F$ will be determined endogenously. We assume that there are iceberg costs $\tau > 1$.

For simplicity, we start with a zero tariff on the traded intermediate imports in both countries, $t = 0$, which we refer to the symmetric free trade equilibrium (SFTE). In this situation, the iceberg costs $\tau > 1$ ensure that $\lambda_{HH} = \lambda_{FF} > 0.5$. We then allow that the home country applies a small
change of tariff $dt$. In this setting, the change in home welfare is simplified from (31) as,

$$\frac{dU_H}{U_H} = -\alpha \frac{dP_H}{P_H} + \frac{dT_H}{L + T_H},$$  \hspace{1cm} (35)

where $P_H$ is the price index for the differentiated good that uses the traded inputs. The change in this price index can be rewritten using (30) as

$$\frac{dP_H}{P_H} = \frac{1}{(1 - \gamma) \theta} \left[ \frac{d\lambda_{HH}}{\lambda_{HH}} - \frac{dN_H}{N_H} - (\kappa - 1) \frac{dY_H}{Y_H} \right], \text{ with } \kappa \equiv \frac{\theta}{\sigma - 1} > 1. \hspace{1cm} (36)$$

Here we see that any reduction in entry or in the output of the differentiated sector raises its price index, and therefore lowers welfare in (35) unless there is some offsetting change in tariff revenue.

To explain this result, recall from Dhingra and Morrow (2014) that the equilibrium of a one-sector Melitz-Chaney model is socially optimal. In a two sector model, by contrast, we expect that the competitive, service sector means that too few resources are devoted to the differentiated sector and that entry there is sub-optimal. This creates a domestic distortion that is exacerbated by any reduction in entry or output of the differentiated sector. The question we need to address is whether protecting the sector with an import tariff will lead to such a reduction in entry or output.

While that outcome may sound counter-intuitive, recall that by Lerner symmetry, an import tariff will be equivalent to an export tax. We would not be so surprised if an export tax reduces entry and/or output in the differentiated sector, and that is what we explore next.

**Output and Entry** Consider next the goods market clearing conditions from (25). With both countries having the same labor force of size $L$, and the home country $H$ imposing an ad valorem tariff of $t$ on its imports of the differentiated intermediate inputs, we obtain

$$Y_H = \tilde{\gamma} (\lambda_{HH} Y_H + \lambda_{HF} Y_F) + \alpha (L + T_H)$$  \hspace{1cm} (37)

and

$$Y_F = \tilde{\gamma} \left( \lambda_{FF} Y_F + \frac{\lambda_{FH}}{1 + t} Y_H \right) + \alpha w_F L,$$  \hspace{1cm} (38)

with home tariff revenue

$$T_H = \frac{t}{1 + t} \lambda_{FH} Y_H.$$  \hspace{1cm} (39)

The trade balance condition from (23) becomes

$$\frac{\lambda_{FH}}{1 + t} Y_H = \lambda_{HF} Y_F.$$  \hspace{1cm} (40)
Finally, the free entry conditions are from (29) with \( E_{ij,s} \equiv \frac{\lambda_{ij,s}}{1+t_{ij,s}} Y_{j,s} \), leading to

\[
N_H = \frac{\lambda_{HH} Y_H + \lambda_{HF} Y_F}{f_F \theta \sigma / (\sigma - 1)},
\]

\[
N_F = \frac{\lambda_{FF} Y_F + \lambda_{FH} Y_H}{w_F f_F \theta \sigma / (\sigma - 1)},
\]

where recall that we have normalized the home wage at unity, \( w_H \equiv 1 \), and the tariff only applies to the home country imports of foreign intermediate inputs.

We now differentiate these conditions and evaluate them at the initial SFTE. In the symmetric equilibrium we have that \( \lambda_{HH} = \lambda_{FF} \equiv \lambda \), and that \( \lambda_{HF} = \lambda_{FH} \equiv 1 - \lambda \). While the shares are changing with the tariff in the above equations, when evaluated at \( t = 0 \) these changes all conveniently cancel out, because \( \lambda_{ii} + \lambda_{ji} = 1 \Leftrightarrow d\lambda_{ii} + d\lambda_{ji} = 0 \), for \( j \neq i \). Then, for a small change in the home tariff \( dt \), from (37) using (39) and (40), we obtain, with \( t = 0 \) in the SFTE,

\[
\frac{dY_H}{Y_H} = \Delta (1 - \lambda) dt, \quad \text{with} \quad \Delta \equiv \frac{\alpha - \tilde{\gamma}}{1 - \tilde{\gamma}} \leq 1.
\]

Similarly, using (48) and (40), we obtain

\[
\frac{dY_F}{Y_F} = \frac{dw_F}{w_F}.
\]

Changes in entry are obtained in much the same way. From (41) and (42), using (40), we have

\[
\frac{dN_H}{N_H} = (\Delta - 1) (1 - \lambda) dt.
\]

\[
\frac{dN_F}{N_F} = 0.
\]

From expression (43), we see that the value of output in the differentiated sector falls if and only if \( \Delta < 0 \), meaning that \( \alpha < \tilde{\gamma} \). We see from (45) that entry falls due to a rise in the tariff if and only if \( \alpha < 1 \) so that \( \Delta < 1 \), meaning that the second sector exists to absorb some of the redistributed tariff revenue. Without the second sector, however, the tariff applied to the revenue cost of imports does not affect entry (though it still affects sectoral output).

We have shown that entry falls for a slight increase in the tariff from the SFTE. What about for a large increase in the tariff? As \( t \to \infty \), then trade is eliminated and we are in the autarky equilibrium for both countries, which is again symmetric, so that we can treat \( w_F = w_H \) as unity. The conditions (41) or (42) will give the same level of entry as in the SFTE, because output in the differentiated sector becomes once again \( Y_H = Y_F = \alpha L/(1 - \tilde{\gamma}) \), from (23). So entry is back at the same level as in the SFTE.

We can summarize these results for the two-country, two-sector model in the following theorem, where part (c) is proved in Appendix B.
Figure 2: Entry effects of tariff changes in the two-sector model

Note: This figure shows how the level of firm entry $N_H$ and the domestic share $\lambda = \lambda_{HH}$ vary as the tariff $t$ changes, for different values of the traded sector share $\alpha \in \{1, 0.75, 0.5, 0.25\}$. For each of these four cases, tariffs vary in the range $t \in (0, 1)$, i.e., from the symmetric free trade equilibrium (SFTE) to autarky (AUT). Iceberg costs are set at $\tau = 1.1$, which creates a small home bias even under free trade, with $\lambda \approx 0.66$. The other model parameters are $\gamma = 0, \sigma = 2, \theta = 4, f_D = f_H = 1, f_X = f_{ij} = 1.1$, and $f_{iD}^H = 1$. See text.

Theorem 1 The mass of entering firms $N_i$ is the same under free trade and prohibitive tariffs. If and only if $\alpha < 1$, then: (a) near the free trade equilibrium reducing the tariff will increase entry; (b) near the prohibitive tariff, reducing the tariff will decrease entry; (c) entry is lower at all intermediate tariff levels than under free trade or prohibitive tariffs.

To go a little further, we can turn to numerical simulations of the model to see more clearly how entry is affected by tariffs in different configurations of the model. Figure 2 shows how the level of firm entry $N_H$ and the domestic share $\lambda_{HH}$ varies as the tariff level $t$ changes over the range from free trade to autarky, for different values of the traded sector share $\alpha$.

Entry is the same under free trade and autarky. Entry is also constant when the nontraded sector is absent and $\alpha = 1$. Otherwise, starting from free trade, entry falls as tariffs increase, before then rising again in a $\cup$-shape after some point as tariffs approach infinity. The $\cup$-shape is more pronounced as the nontraded sector grows in size (i.e., as $\alpha$ falls further below 1). Theorem 1 shows that this $\cup$-shape holds in general for changes in the home tariff, i.e., that the graph of entry has a single local minimum. The fact that entry is reduced for all tariffs short of the prohibitive level shows a key contrast between this two-sector model with a non-traded, competitive sector, compared to a multi-sector model with all differentiated-goods sectors. When all sectors have heterogeneous firms, changes in entry are constrained to have a weighted sum of zero, as shown in (33).
The reduction in entry that we have found for a small increase in tariffs, assuming that \( \alpha < 1 \), necessarily reduces welfare in (43), but this effect is potentially offset by any increase in output \( Y_H \) and also by any increase in tariff revenue rebated. Evaluated at the SFTE, the increase in tariff revenue can be calculated from (39) as \( dT_H = dt (1 - \gamma) Y_H = dt (1 - \gamma) \alpha L (1 - \tilde{\gamma}) \).

Substituting this expression along with (43) and (45) into (36) and (35), we obtain,

\[
\left. \frac{dU_H}{U_H} \right|_{t=0} = - \frac{\alpha}{(1 - \gamma) \theta} \frac{d\lambda_{HH}}{\lambda_{HH}} + \frac{\alpha (1 - \gamma)}{(1 - \gamma) \theta} (\kappa \Delta - 1) dt + \frac{\alpha (1 - \gamma)}{(1 - \gamma)} \frac{dt}{(1 - \tilde{\gamma})}.
\]

The first term on the right of (47) is the effect of changes in trade volume (which is valid in ACR for the case of changes in iceberg costs). Beyond this, the second and third terms reflect changes in the entry and selection margins of the differentiated sector and in the tariff revenue rebate, respectively.

Looking at (47), if we are in the case where \( \frac{d\ln U_H}{\ln \lambda_{HH}} > - \frac{\alpha}{(1 - \gamma) \theta} \frac{d\ln \lambda_{HH}}{\lambda_{HH}} \), so that the positive impact of the tariff rebate term overwhelms any negative impact of reduced entry, then we get the seemingly normal result that welfare with tariffs exceeds that with iceberg transport costs, for a given change in trade volume (i.e., the first term in (47)). But this isn’t guaranteed: if, on the other hand, we are in the case where \( \frac{d\ln U_H}{\ln \lambda_{HH}} < - \frac{\alpha}{(1 - \gamma) \theta} \frac{d\ln \lambda_{HH}}{\lambda_{HH}} \), then we get a seemingly paradoxical outcome that, for a given change of trade volume, the welfare effect of a tariff—\textit{with rebate}—ends up being worse than iceberg costs.

To see whether these different cases can arise, recall that \( \Delta < 1 \) and that \( \kappa \equiv \theta / (\sigma - 1) > 1 \). It follows that the coefficient \( (\kappa \Delta - 1) \) on the entry+selection term in (47) can take on either a positive or negative sign. If the term is negative, then any increase in entry+selection that accompanies a tariff reduction will \textit{further reduce} the price index in (51) and will \textit{further increase} the resulting welfare gains. The magnitude of this welfare gain is sensitive to the value of \( \gamma \), which indicates the extent to which the differentiated products are used as intermediate inputs: as \( \gamma \) is larger, just as the gains via trade volumes \( - \frac{\alpha}{(1 - \gamma) \theta} \frac{d\ln \lambda_{HH}}{\lambda_{HH}} \) get larger, so too do the gains from entry correspondingly increase.

The welfare impact of a change in the tariff also depends on the change in tariff revenue, the final term on the right of (47). Notice that a reduction in the tariff directly lowers tariff revenue in the final term, and this effect is stronger as \( \tilde{\gamma} \) is larger. In other words, just as increased linkages magnify the the welfare gain from increased trade volume and entry in (47), so too the increased linkages would lead to an offsetting fall in tariff revenue.

If the magnitude of the second term in (47) is large enough so that it overwhelms the third term (so the two combined terms change sign) then we will obtain the aforementioned odd outcome where \( \frac{d\ln U_H}{\ln \lambda_{HH}} < - \frac{\alpha}{(1 - \gamma) \theta} \frac{d\ln \lambda_{HH}}{\lambda_{HH}} \), so that an increase in the tariff is worse than an increase in iceberg costs. This condition holds if and only if \( \left[ (\kappa \Delta - 1) / (\gamma - 1) \theta \right] + 1 / (1 - \tilde{\gamma}) < 0 \). Simplifying this condition, we obtain the following result that holds in a neighborhood of the SFTE.
Theorem 2  For a small increase in the tariff $dt$ in country $H$ starting from free trade, a necessary and sufficient condition for a small increase in the tariff to be worse than a small increase in iceberg transport costs, where both lead to the same change in $\lambda_{HH}$, namely, for $d \ln U_H < -\frac{\alpha}{1-\gamma}\theta d \ln \lambda_{HH}$, is that:

$$\tilde{\gamma} \equiv \gamma \left(\frac{\sigma - 1}{\sigma}\right) > 1 - \left(\frac{2 - \alpha}{\sigma + 1 - (\sigma - 1)\theta}\right).$$

(48)

This condition can hold only if $\gamma > 0$ (production linkages are present) and $\alpha < 1$ (the service sector is present).

This result gives us the necessary and sufficient condition for a small increase in the tariff to be worse than an increase in iceberg transport costs. For example, with $\alpha = 0.5$ and $\theta = \sigma = 3$, then condition (48) is equivalent to the condition $\tilde{\gamma} > 0.55$, or $\gamma > 0.825$. Experimenting with other parameter values, we get a very clear sense that the role of intermediate inputs must be substantial in order for the increased entry due to a tariff cut to result in welfare gains larger than $-\frac{\alpha}{1-\gamma}\theta d \ln \lambda_{HH}$, so that the tariff increase is worse than iceberg costs. An inspection of condition (48) shows that it can hold only if we have both production linkages ($\gamma > 0$) and the service sector is present ($\alpha > 1$), as stated.

In another somewhat counterintuitive finding, when condition (48) holds it is not necessarily the case that home utility falls as we lower tariffs so that the optimal tariff is negative. Since $d \ln U_H < -\frac{\alpha}{1-\gamma}\theta d \ln \lambda_{HH}$ when (48) holds, home utility falls due to a tariff $dt > 0$ if only if the home shares rises, $d \ln \lambda_{HH} > 0$. That is certainly the outcome that we are familiar with from the one-sector version of ACR with increases in iceberg transport costs. But now with a two-sector model allowing for changes in entry and the output of the differentiated sector, we cannot be sure that $d \ln \lambda_{HH} > 0$ when $dt > 0$.

To see this, we go back to the change in the home price index in (36), and now substitute in the changes in entry (45) and sectoral output (43) and rearrange to obtain,

$$\frac{d \lambda_{HH}}{\lambda_{HH}} = (\kappa \Delta - 1) dt + (1 - \gamma) \theta \frac{d P_H}{P_H}.$$  

(49)

Entry falls with the tariff provided that $\alpha < 1$, since then $\Delta < 1$ in (45), while sectoral output can rise (if $\Delta > 0$) or fall (if $\Delta < 0$) in (43). But the combined effect of these two terms is to reduce the home share whenever $\kappa \Delta < 1$, as shown by the first term on the right of (49). In order to offset this tendency for the home share to fall due to reduced entry (and possibly reduced output), it must be the case that the rise in the home price index $d \ln P_H$, the second term on the right, is positive and sufficiently large to lead to an overall positive value for $d \ln \lambda_{HH}$. This outcome is certainly not guaranteed, because any terms of trade gain due to a reduction in foreign wages tends to reduce the increase in $P_H$ caused by the home tariff.

In the remainder of this section we identify several cases where $d \ln \lambda_{HH} > 0$ holds, however, so that home welfare falls with the increase in the tariff. The first case was discussed in our working paper (Caliendo, Feenstra, Romalis and Taylor, CFRT, 2015), and assumes that both countries
apply the same tariff $t$. Because of full symmetry in that case, we can normalize both the home and foreign wages at unity. It turns out that the changes in home entry and sectoral output shown in (43) and (44) around the SFTE are not affected by the presence of a small, foreign tariff. To understand this, notice that the impact of the home tariff on foreign entry and sectoral output are as shown in (46) and (45), but with foreign wages normalized at unity, then both of these terms are zero. It follows that the impact of the foreign tariff on home entry and output are likewise zero, and so the changes that we have solved for (43) and (44) apply equally well to the joint increase in the (equal) home and foreign tariffs.

The general expression for the change in home welfare in (35) continues to hold in home and foreign tariffs, and so the decomposition of the change in home welfare in (47) continues to hold as well. Then condition (48) of Theorem 2 still gives the necessary and sufficient condition to have $d\ln U_H < -\frac{\alpha}{(1-\gamma)\theta} d\ln \lambda_{HH}$. The difference with our earlier analysis is now that an equal increase in the ad valorem tariff $t$ or in the iceberg trade costs $\tau$ in both countries, $\lambda_{HH} = \lambda_{FF} = \lambda$, will rise. Indeed, in our working paper (CFRT, 2015, Appendix B.1) we show that with an increase in the ad valorem tariff $t$ or in the iceberg trade costs $\tau$ in both countries, then the domestic share changes by,

$$\frac{d\lambda}{\lambda} = \frac{(1-\lambda)\theta}{\tau} \frac{d\tau}{\tau} + (1-\lambda)(\theta + \kappa - 1) \frac{dt}{1 + t}. \quad (50)$$

Equation (50) shows the change in home share. The first term on the right shows the effect due to a change in iceberg trade costs, with the familiar elasticity of $\theta$, while the second term shows how the effect is magnified for the same size of change in the ad valorem tariff. As we have emphasized, an increase in the ad valorem tariff also acts as an effective increase in fixed costs, which accounts for the additional magnifying term $(\kappa - 1) > 0$. Clearly, an increase in either type of trade cost raises the domestic share, and since $d\ln U_H < -\frac{\alpha}{(1-\gamma)\theta} d\ln \lambda_{HH}$ when condition (48) holds, we have a fall in home and foreign welfare that exceeds the fall which would occur from an increase in iceberg costs. In this case, it would be optimal for both countries to subsidize their imports so as to encourage entry into the differentiated sector.

We summarize these results with the following Theorem:14

**Theorem 3** When both the home and foreign country apply a small equal tariff $dt$, then a necessary and sufficient condition to have $d\ln U_i < -\frac{\alpha}{(1-\gamma)\theta} d\ln \lambda_{ii}$, for $i = H, F$, is still condition (48). Because the domestic shares in both countries rise with the tariff, then welfare in both countries falls. The countries would gain when both apply a small import subsidy from the symmetric free trade equilibrium.

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14In our working paper, CFRT, (48) ensures that the welfare change in both countries is less than the ACR loss, $d\ln U_i < -\frac{\alpha}{(1-\gamma)\theta} d\ln \lambda_{ii}$, $i = H, F$, even for tariffs away from the free trade equilibrium. Likewise, (44) applies away from the free trade equilibrium, too.
Returning to the case where only the home country applies a tariff, we still want to determine whether the change in home welfare for an increase in the tariff around the SFTE is negative, which would imply the optimal tariff for that country alone is also negative. Condition \((48)\) in Theorem 2 is actually stronger than we need to reverse the conventional (positive) optimal-tariff argument. Even when \((48)\) does not hold, so that the second and third terms on the right of \((47)\) are positive for an increase in the tariff, welfare can still fall if they are overwhelmed by the first term, that is, if the home share indeed rises enough with the tariff.

To evaluate whether this outcome occurs, we now need to solve for the change in home welfare while incorporating the change the home share \(d\ln \lambda_{HH}\), as we do in Appendix C. That results in two expressions for the change in the price indexes in each country, involving the changes in wages, entry, and output of the differentiated sector in both countries. As before, we normalize the home wage at unity, \(w_H = 1\). Now the decomposition of home welfare analogous to that in \((47)\) now includes an additional term indicating a terms-of-trade gain for the home country based on the fall in foreign wages \(w_F\). In Appendix C, we solve for this terms of trade effect, and for the overall change in home welfare. These results are summarized as follows.

**Theorem 4** Consider an economy near an SFTE with zero tariffs initially, but with \(\tau > 1\). Restrict attention to the cases (a) \(\gamma = 0\) (no linkages); and (b) \(\gamma \to 1\) (strong linkages).

Then for small increases in the home tariff \(dt > 0\), the welfare change is:

(a) \(\gamma = 0\) (no linkages):

\[
\frac{d\ln U_H}{dt} \bigg|_{t=0, \gamma=0} = -\alpha (1 - \lambda) - \alpha (1 - \lambda)(1 - \alpha) \left[ \frac{1}{\sigma - 1} - \frac{(1 - \lambda)}{\theta} \right] \\
+ \alpha (1 - \lambda) + \frac{2\alpha \lambda (1 - \lambda)^2 (\theta \sigma + 1)}{(2\lambda - 1) \theta (\sigma - 1)},
\]

which is negative when \(\alpha < 1\) and \(\tau\) is sufficiently large so that \(\lambda\) is close to unity. It follows that the optimal home tariff is negative.

(b) \(\gamma \to 1\) (strong linkages):

\[
\lim_{\gamma \to 1} (1 - \gamma) \frac{d\ln U_H}{dt} \bigg|_{t=0} = -\frac{\alpha (1 - \lambda)}{2} - \frac{\alpha (1 - \lambda)(1 - \lambda) \sigma}{2(\sigma - 1)} \\
+ \underbrace{0}_{\text{vanishing revenue and terms of trade effects}}
\]

which is negative for all \(0.5 < \lambda < 1\) and \(\alpha \leq 1\). It follows again that the optimal home tariff is negative.
To interpret part (a), the first term on the right, \(-\alpha (1 - \lambda)\), reflects the efficiency loss in welfare due to the tariff, and is much the same as the conventional ACR term.\(^{15}\) The second term reflects the welfare loss due to the reduction in entry, and is still negative whenever \(\alpha < 1\). The third term is the revenue gain from the small tariff, which now exactly offsets the efficiency loss. So with the first and third terms canceling when there are no production linkages, we are left with the welfare loss due to reduced entry and the fourth term, which is positive and reflect the terms of trade gain due to reduced foreign wages.

A condition on parameters to guarantee that the loss from entry exceeds the terms of trade gain when \(\gamma = 0\) is that \(\alpha < 1\) and \(\tau\) is large so that \(\lambda\) is sufficiently close to unity. Notice that the terms of trade gain depends on \((1 - \lambda)^2\) because the drop in foreign wages is proportional to \((1 - \lambda)\), and then to obtain the gain in welfare we must multiply by \((1 - \lambda)\) again to reflect the magnitude of imports. In contrast, it can be seen that the loss due to reduced entry includes the term \(\alpha (1 - \lambda) (1 - \alpha) / (\sigma - 1)\), which it proportional to \((1 - \lambda)\). It follows that for iceberg costs \(\tau\) sufficiently large so that \(\lambda\) is sufficiently close to unity, then the terms of trade gain is necessarily smaller than the entry loss, provided that \(\alpha < 1\). In that case, welfare declines with a slight increase in the tariff, and hence the optimal tariff is negative.

In part (b) we consider the alternative case of very strong production linkages, so that \(\gamma \to 1\). In that case the model is not well-behaved, with the price elasticity \(d \ln P_H / dt\) approaching infinity.\(^{16}\) To obtain a bounded expression for welfare we consider the limit of \((1 - \gamma) d \ln U_H / dt\). The first two terms on the right of (60) are the efficiency loss and the entry loss, which both reduce welfare. It turns out that the revenue and the terms-of-trade gains are both bounded as \(\gamma \to 1\), so when multiplying by \((1 - \gamma)\) these terms become vanishingly small. It follows that in this case we obtain a welfare loss due to a small tariff when either \(\alpha < 1\), so both the efficiency and entry losses are present, or when \(\alpha = 1\), so there is just the efficiency loss with no entry loss (because entry is fixed). Here then, once again, the optimal tariff is negative.

It is evident that the conditions to obtain a negative optimal tariff in Theorem 4 are weaker than condition (48) in Theorem 2: that condition requires that both \(\gamma > 0\) and \(\alpha < 1\), whereas parts (a) and (b) of Theorem 4 can result in welfare falling due to a tariff increase when \(\gamma = 0\) or \(\alpha = 1\), respectively. The reason that we do not require (48) is that under the conditions stated in parts (a) and (b), the home share \(\lambda_{HH}\) is in fact rising due to an increase in the home tariff, or is likely to rise. We state this result formally as:

**Theorem 5** Under the conditions of part (a) in Theorem 4, the home share rises with the tariff, \(d \ln \lambda_{HH} / dt > 0\). Under the conditions of part (b), the home share rises provided that \(\kappa \equiv \theta / (\sigma - 1) > 2 / (1 + \alpha \sigma)\).

\(^{15}\)The ACR loss due to an increase in the iceberg transport cost is, \(d \ln U_H = -(\alpha / \theta) d \ln \lambda_{HH} = -\alpha (1 - \lambda) d \ln \tau\), as obtained by substituting from (60). This is the same as the first term on the right of (61).

\(^{16}\)Melitz and Redding (2014) likewise find that the gains from trade approach infinity as the production linkages become very strong.
The condition $\theta/(\sigma - 1) > 2/(1 + \alpha \sigma)$ to have the home share rise with the tariff in part (b) is empirically plausible, and will hold for the average parameters in our quantitative model below.

It is especially surprising that we obtain a welfare decline when $\gamma \to 1$ and $\alpha = 1$ in part (b), since in that case the tariff has no impact on entry. Evidently, the presence of strong production linkages in our model plays an independent role in leading to a negative optimal tariff. In our decomposition of welfare, the negative optimal tariff arises because there is always an efficiency loss due to the tariff (even without the added loss due to declining entry), and that loss becomes infinitely larger than the revenue gain as $\gamma \to 1$. That result can be seen mathematically in our first decomposition (14), where the coefficient $\alpha / \theta (1 - \gamma)$ on the trade volume term approaches infinity as $\gamma \to 1$, but the coefficient $\alpha (1 - \lambda)(1 - \gamma)$ on the tariff rebate term remains bounded since $\gamma = \gamma (\sigma - 1)/\sigma < 1$. We have found that similar results continue to hold even in other specifications for the fixed costs in our model, leading to tariff revenue terms that are unbounded as $\gamma \to 1$.

The significance of our findings is that when welfare falls for small tariffs, then both countries can be expected to gain from a reduction in tariffs in either country. In such a setting, trade agreements are not necessary for mutual gains. To explain this result, recall that from Lerner symmetry, the import tariff in each country is acting like an export tax which can be expected to inhibit entry. To offset this distortion, either country would prefer to reduce tariffs and encourage entry into the differentiated sector. The gain for the country reducing its tariff is indicated by the sufficient conditions given in Theorem 3, and the gain for the other country follows because it will experience a rise in its relative wage. In this scenario, the mutual gains from tariff reductions are quite different from what is predicted from competitive models, or even from monopolistically competitive models in the absence of production linkages.

Is this new result empirically relevant? To find out, we evaluate whether these results will hold in a more realistic setting than our two-country, two-sector model, using our multi-country, multi-sector quantitative model. There are several important differences between the quantitative model and our simple illustrative model in this section. Most important, in the quantitative model we do not treat the services sector as having no production linkages with the rest of the economy, or as being perfectly competitive: those assumptions were made here just for convenience. Rather, we will allow the services sector to be composed of heterogeneous firms operating under monopolistic competition, as explained before—which, all else equal, by placing a domestic distortion in the nontraded sector as well as in the traded sector, might be expected to bias against a finding of negative optimal tariffs.

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17We have assumed that fixed costs are paid entirely in terms of labor and do not use any materials, which indirectly use imported inputs. In the alternative case where the fixed costs are paid in terms of the same input bundle as variable costs, then the coefficient $\bar{\gamma}$ in the goods equilibrium condition would instead be $\gamma$, as discussed in footnote 12. In that case, the coefficient on the tariff revenue term in (14) would be $\alpha (1 - \lambda)/(1 - \bar{\gamma})$, which approaches infinity as $\gamma \to 1$. With this alternative specification of fixed costs, however, we find that the efficiency loss of the tariff is also higher, because the tariff has a direct impact on raising fixed costs and reducing entry. It turns out that the efficiency cost of the tariff approach infinity as $\gamma \to \theta/(\theta + \kappa) < 1$, in which case the tariff revenue gain is still bounded. So once again, under this alternative specification, we find that the efficiency cost of the tariff dominates its revenue gain as production linkages become strong enough.
4 Data Description

In order to quantify the effects of actual, and counterfactual, tariff changes we need detailed information on tariffs, as well as on production and trade flows for a large set of countries. Moreover, we are interested in understanding how both high- and low-income countries have been impacted by changes in trade policy, and this can only be done if the data have good coverage of both sets of countries. We start this section by first describing the sources and the way we obtain tariff data, and we then on move to explain the sources for production and trade flow data.

4.1 New Tariff Data

We build a new comprehensive, disaggregated, annual tariff dataset from the early 1980s onwards. We obtain tariff schedules from five primary sources: (i) raw tariff schedules from the TRAINS and IDB databases accessed via the World Bank’s WITS website as far back as 1988 for some countries; (ii) manually collected tariff schedules published by the International Customs Tariffs Bureau (BITD), some dating back as far as the 1950s; (iii) U.S. tariff schedules from the U.S. International Trade Commission from 1989 onwards (Feenstra, Romalis, and Schott 2002); (iv) U.S. tariff schedules derived from detailed U.S. tariff revenue and trade data from 1974 to 1988 maintained by the Center for International Data at UC Davis; and (v) the texts of preferential trade agreements primarily sourced from the WTO’s website, the World Bank’s Global Preferential Trade Agreements Database, or the Tuck Center for International Business Trade Agreements Database. For the U.S., specific tariffs have been converted into ad valorem tariffs by dividing by the average unit value of matching imported products. Due to the difficulties of extracting specific tariff information for other countries and matching it to appropriate unit values, only the ad valorem component of their tariffs are used. The vast majority of tariffs are ad valorem. Switzerland is a key exception here, with tariffs being specific. We proxy Swiss tariffs with tariffs of another EFTA member (Norway). We aggregate MFN and each non-MFN tariff program to the 4-digit SITC Revision 2 level by taking the simple average of tariff lines within each SITC code.

Tariff schedules are often not available in each year, especially for smaller countries. Updated schedules are more likely to be available after significant tariff changes. Rather than replacing “missing” MFN tariffs by linearly interpolating observations, missing observations are set equal to the nearest preceding observation. If there is no preceding observation, missing MFN tariffs are set equal to the nearest observation. Missing non-MFN tariff data (other than punitive tariffs applied in a handful of bilateral relationships) are more difficult to construct for two reasons: (i) they are often not published in a given tariff schedule; and (ii) preferential trade agreements have often been phased in. To address this we researched the text of over 100 regional trade agreements and Generalized System of Preferences (GSP) programs to ascertain the start date of each agreement or

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18 Most tariff schedules can be fairly readily matched to the SITC classification.
19 Multiple preferential tariffs may be applicable for trade in a particular product between two countries. Since the most favorable one may change over time, we keep track of each potentially applicable tariff program.
program and how the typical tariff preference was phased in. To simplify our construction of missing preferential tariffs we express observed preferential tariffs as a fraction of the applicable MFN tariff. We fill in missing values of this fraction based on information on how the tariff preferences were phased in. Preferential tariffs are then constructed as the product of this fraction and the MFN tariff. We keep the most favorable potentially applicable preferential tariff. Punitive non-MFN tariff levels tend not to change over time (though the countries they apply to do change). We replace missing observations in the same way we replace missing MFN tariff observations.

An overview of our new tariff data is shown in Figures 3 to 7. These data show, with country coverage and disaggregated detail of a kind never seen before, the remarkable impacts of the Uruguay Round on the levels and dispersion of tariffs around the world from the 1980s to the 2010s.

To start, Figure 3 plots the average (mean) ad valorem tariff rates, both MFN and Preferential, across all countries and all goods at the SITC 4-digit goods level, in each year from 1984 to 2011, for the full sample, the Advanced economies, and the Emerging and Developing economies. At the start of the period shown, in the 1980s, the typical sample size for the calculations of these statistics is about 1 million distinct tariff lines. By the late 2000s, at the end of the period shown, the sample size in a given year is well over 2 million distinct tariff lines. It is clear that both types of tariffs fell over the period, by about 9 percentage points, with essentially all of the reductions concentrated after 1990.

Given the similar trends, we focus henceforth on MFN tariffs in this section. Figure 4 plots the median MFN ad valorem tariff rate across all goods at the SITC 4-digit level, in each year, for the full sample, the Advanced economies and the Emerging and Developing economies. It also plots a fan showing ten percentiles from 5th, 15th, 25th, . . . to 95th in each year to give an idea of the dispersion of tariff rates. This figure shows very clearly that the Uruguay Round was followed by a dramatic reduction in both the levels and dispersion of tariff rates, with these trends being particularly concentrated in the subsample of Emerging and Developing economies. In part this reflects the fact that these countries started with higher levels and dispersion to begin with, and so had more scope for these kinds of policy adjustments. In contrast, the Advanced countries had made much greater progress in this direction during earlier GATT rounds going back to the 1940s.

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Figure 5 uses histograms and kernel density plots to show the distributions of ad valorem tariff rates across countries and goods, for two snapshot years that we will use for our policy experiments: a pre-Uruguay 1990 sample year and a post-Uruguay 2010 sample year. The histograms are truncated at the 50% tariff level; a small number of tariffs over this level (some well over 100%) appear in both sample years for a few unusual goods and countries, but this right tail is not very representative. Within the range shown, tariff peaks at certain round numbers are clearly visible (0, 5, 10, 15, etc.), as one would expect. However, looking past those peaks, we can clearly see again the impacts of changes in tariff policy over this period. The spike at zero rises, as more zero-tariff rates appear across goods and countries, and in the strictly positive region mass is shifted from the above-20% region and into the below-20% region.
Figure 3: Average MFN and Preferential *ad valorem* tariff rates

![Graph showing average MFN and Preferential ad valorem tariff rates from 1985 to 2010.](image)

Note: Averages are taken over all 4-digit SITC level good, all countries, by year 1984–2011.

Figure 4: Distributions of MFN *ad valorem* tariff rates

![Graph showing distributions of MFN ad valorem tariff rates from 1985 to 2010 for All countries, Advanced, and Emerging/Developing.](image)

Note: 4-digit SITC level data, in 3 samples, by year 1984–2011 (percentiles 5/10/25/50/75/90/95).
Figure 5: Distributions of MFN *ad valorem* tariff (4-digit SITC, all countries, 1990 and 2010)


Figure 6: MFN *ad valorem* tariff rates, 10 sectors, all countries, in 1990 and 2010
Figure 7: MFN *ad valorem* tariff rates, 10 sectors, all countries, in 1990 and 2010
Finally, Figures 6 and 7 provides sectoral detail for tariffs aggregated up to the level of 10 tradable sectors which we use in our calibrated model. This figure shows clearly that the Uruguay Round did not have a peculiar compositional impact across sectors. It lowered average tariffs pretty much across the board in all sectors, and was not just confined to some limited areas of the tradable economy. And again, the figure clearly shows the much larger scope for tariff reductions in the Emerging and Developing sample, given the relatively high tariff rates they had at the start of the period in all sectors as compared to the Advanced economies.

4.2 Production and Trade Data

To obtain production and trade data, we relied on Eora MRIO, a global multi-region IO database. This dataset, to our knowledge, is the most comprehensive dataset available that contains information on production, trade flows and input-output (IO) tables for 189 countries. Six sources are used to construct these multi-region IO tables. The sources are are: (1) input-output tables and production data from national statistical offices; (2) IO from Eurostat, IDE-JETRO, and OECD; (3) the UN National Accounts Main Aggregates Database; (4) the UN National Accounts Official Data; (5) the UN Comtrade international trade database; and (6) the UN Service Trade Statistics Database. For further information, refer to Lenzen et al. (2012; 2013). We use Eora MRIO to obtain data on value added shares, share of intermediate inputs in production, gross output, and total exports. These are mapped into our model concepts as explained in Appendix E.

A key advantage of this database, compared to others, is the fact that it contains information for a large set of countries (high- and low-income countries) and for the early years in the sample period we wish to study. In particular we can make use of the 1990 multi-region table with 25-sector harmonized classifications. As a reference point, in comparison with the World Input-Output Database (WIOD), we have more than three times the number of countries and account for a number of developing countries, some of them quite small. Moreover, there is no WIOD for the year 1990, the period immediately before the Uruguay Round tariff cuts. Having data as far back as circa 1990 allows us to take the model to the data and evaluate the effects of every single tariff reform after that period.

5 Taking the Quantitative Model to the Data

Several issues need to be dealt with in order to take the model to the data. First, we need to find a way to infer a large set of unobservable parameters. Second, we need to deal with the fact that trade is unbalanced and that our static model cannot accommodate this feature of the data. Third, we need estimates for parameters such as the trade and the home versus foreign input elasticities.

20 Tariffs are aggregated using trade weights as discussed in Appendix E.
21 Please refer to http://worldmrio.com/ for more information.
22 Several parameters from our model are directly observable, like value added shares and input-output coefficients. However, there are a large number of parameters, like fixed entry, production, and exports costs, that are not observed.
The way we solve the first issue is by using the equilibrium conditions of the model in relative changes, where we use the “hat” notation for the ratio of after-versus-before levels of any variable for a given perturbation; that is, we define $\hat{z} = z'/z$ for any variable $z$. It was first shown by Dekle, Eaton, and Kortum (2007, 2008) that their model could be expressed conveniently using this type notation. As we show in Appendix D, this approach allows us to condition on an observed allocation in a given baseline year and solve the model without needing estimates of fixed costs and other parameters which are not directly observable. The way we solve the second issue is by first calibrating the model with trade deficits as a residual and then use the model to net out the deficits.

Finally, solving our quantitative model requires estimates, by sector, of the elasticity of substitution across varieties $\sigma_s$, the home versus foreign input elasticity $\omega_s$, and the Pareto shape parameter $\theta_s$. In order to obtain estimates for the elasticity of substitution and the Pareto parameter we use the estimates from CP. CP show that by triple differencing the gravity equation one can identify the elasticities using tariff policy variation. In the context of our model the elasticity that is estimated is given by $1 - \sigma_s \theta_s / (\sigma_s - 1)$.

In order to separately identify $\theta_s$ and $\sigma_s$ we rely on estimates from the literature to obtain $\theta_s / (\sigma_s - 1)$. The two most cited studies to deal with this issue are Chaney (2008) and Eaton, Kortum, and Kramarz (2011). Chaney (2008) obtains the coefficient by regressing the log of the rank of US firms according to their sales in the United States on the log of sales using Compustat data on US listed firms. Eaton, Kortum, and Kramarz (2011) use a different procedure and data on the propensity of French firms to export to multiple markets. Chaney (2008) finds that $\theta_s / (\sigma_s - 1) \approx 2$, while Eaton, Kortum, and Kramarz (2011) find $\theta_s / (\sigma_s - 1) \approx 1.5$. We take this latter estimate and apply it to our sectoral elasticities estimated using CP.

The values for the elasticities that we obtain are shown in Table 1. Note that these values imply that $\sigma_s$ for the tradable sectors are 6.7, 9.7, and 4.4 respectively. These numbers are clearly within the range of values estimated by Broda and Weinstein (2006), where they find that a simple average of the elasticities of substitution are 17 at a seven-digit level of goods disaggregation (TSUSA), 7 at the three-digit level (TSUSA), 12 at a ten-digit level (HTS) and 4 at a three-digit level (HTS).

We also need an elasticity for the service sector. Gervais and Jensen (2013) find that services have an elasticity of substitution that is smaller than for manufacturing: about three-quarters the size of the elasticity in manufacturing (though they obtain rather high values for both elasticities using accounting data). Given this, we likewise adopt an elasticity of substitution in services that is below what we use for the manufacturing sector. In particular, for services we use a value of $\sigma_s = 2.8$ and, given $\theta_s / (\sigma_s - 1) = 1.5$, this implies $\theta_s = 2.7$.

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23This idea was first advanced by Dekle, Eaton, and Kortum (2008) in the context of a Ricardian trade model. CP and Ossa (2014) show that one can use this method to analyze the effects of tariff policy. CR refer to this as the “exact hat algebra” and show how it works for a variety of trade models, including a multi-county, multi-industry Melitz model similar to the one we use here. We show how it works with a nested CES structure and for the case of revenue tariffs.
Table 1: Elasticities

<table>
<thead>
<tr>
<th>Sector(s)</th>
<th>$\frac{\sigma_s \omega_s}{\sigma_s \omega_s - 1}$</th>
<th>$\theta_s$</th>
<th>$\sigma_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture and Fishing (1 sector)</td>
<td>9.11</td>
<td>8.6</td>
<td>6.7</td>
</tr>
<tr>
<td>Mining and Quarrying (1 sector)</td>
<td>13.53</td>
<td>13.0</td>
<td>9.7</td>
</tr>
<tr>
<td>Manufacturing Sectors (all 8 sectors)</td>
<td>5.55</td>
<td>5.1</td>
<td>4.4</td>
</tr>
<tr>
<td>Nontraded services (all 5 sectors)</td>
<td>—</td>
<td>2.7</td>
<td>2.8</td>
</tr>
</tbody>
</table>

Table 2: Comparing across models

<table>
<thead>
<tr>
<th>Welfare effects</th>
<th>$\sigma_s/\omega_s = 2$</th>
<th>$\sigma_s/\omega_s = 1.25$</th>
<th>$\sigma_s/\omega_s = 1.1$</th>
<th>$\sigma_s/\omega_s = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>0.64%</td>
<td>1.56%</td>
<td>3.14%</td>
<td>5.93%</td>
</tr>
<tr>
<td>Median</td>
<td>0.27%</td>
<td>0.54%</td>
<td>1.74%</td>
<td>3.66%</td>
</tr>
<tr>
<td>Maximum</td>
<td>9.89%</td>
<td>26.87%</td>
<td>35.01%</td>
<td>41.11%</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.96%</td>
<td>-1.93%</td>
<td>-4.95%</td>
<td>-5.39%</td>
</tr>
</tbody>
</table>

Trade effects (growth in imports/GDP)

| Median | 8% | 31% | 73% | 143% |

Finally, we also need an elasticity for home versus foreign input substitution, $\omega_s$. We obtain this elasticity by calibrating the model to the baseline year of 1990 and then solving the model after adding the actual, observed, tariff changes. We do this for different values of $\omega_s$, while fixing all other elasticities, and choose the value of $\sigma_s/\omega_s$ such that we match the actual growth rate of imports/GDP which was 35%. Table 2 shows how sensitive the trade effects are to the value of $\omega_s$. We find that the best fit is achieved with a value of $\omega_s = \sigma_s/1.25$ and in all our quantitative results below we use this elasticity.

6 A Quantitative Assessment

In this section we evaluate the trade, entry, and welfare effects of the observed change in trade policy over the years 1990 to 2010. We take as our initial baseline the levels of tariffs in the year 1990, the year before tariffs started falling as a consequence of the Uruguay Round. We quantify the economic effects of tariff changes by performing four different exercises, as follows.

- We first impose on the model the actual changes in MFN tariffs from the year 1990 to the year 2010, holding fixed the preferential tariffs (PTA) in place in the year 1990. This exercise we think of as informative on the effects of changes principally due to multilateral negotiations, i.e., the Uruguay Round, so we label this case the “Uruguay Round” experiment.  

- We then go beyond the Uruguay Round effects on MFN tariffs, and aim to quantify the effects from all tariff changes, MFN together with any preferential PTA tariffs in place in the year 2010. We refer to this last exercise as the “Uruguay Round + Preference” experiment.

24Specifically, we set the 2010 tariff equal to minimum of the 1990 preference tariff and the 2010 MFN tariff.
In addition, we explore whether there are any extra gains from tariff changes by moving to a world with zero tariffs, what we refer as the “Free Trade” experiment.

Finally, starting from a free trade equilibrium of the world, we solve for the unilateral uniform optimal tariff across countries.

**Trade Effects**  We start by showing the trade effects from the change in tariffs in our experiments. We calculate the share of total expenditure in each country on foreign goods, a model counterpart of the trade share of GDP. Figure 8 uses smooth histograms, or kernel density plots, to show the effects on the trade share of GDP in all countries in the world in the baseline and the three experiments. The results are stark, and Uruguay Round tariff reductions generate considerable trade effects. The distribution of trade shares in 1990 had its mass concentrated in the 0%–10% region. After the Uruguay Round experiment this mass is more spread out in the 0%–20% region. There is little difference between the three experiments, suggesting that most of the impact that could have been achieved by a move towards free trade was achieved by the Uruguay Round experiment; still, the Free Trade case shows some extra trade might be generated by the removal of all tariffs.

Figure 9 shows the effects on the trade share of GDP for the case of Advanced and Emerging countries. The world, on average, became more open with a roughly twofold increase in the median trade share in both subsamples. Interestingly, the median level of openness increases slightly more for Advanced economies relative to Emerging and Developing economies. The trade effects for the latter are very dispersed. Some countries, like Hong Kong and Singapore display a substantial increase in trade share, even from an initial high level, while for other countries the trade share remain almost constant.

The second takeaway from both of figures is that Uruguay Round + Preference does not generate a large increase in world trade relative to Uruguay Round only. This is clearly seen by comparing the median change in openness for Advanced and Emerging and Developing countries as we move from the Uruguay Round case to the Uruguay Round + Preference. The line is flat, as it is at almost all marked deciles. The histogram makes the same point. Finally, note that moving to zero tariffs generates considerable trade share effects for Emerging economies, but little in the way of extra trade share effects for Advanced economies. This result unmasks the asymmetrical impact of further reducing tariffs for Emerging and Developing countries.

**Entry Effects**  We now discuss our findings on firm entry. Figure 10 presents the distribution of changes in entry across all countries and sectors by trade policy relative to the 1990 baseline (normalized to 1). Concretely, we are showing the change in entry, in hat notation \( \hat{N}_{i,s} = N'_{i,s}/N_{i,s} \). The histogram in Figure 11 shows that the entry margin is very active and heavily impacted by the changes in tariffs. As we can see, there is mass in both tails reflecting that in some country-sector cases entry goes up, while in others it falls. As we compare experiments it is evident that both Uruguay Round and Uruguay Round + Preference generate very similar entry effects, while moving
Figure 8: Trade effects from tariff changes, world, histograms, 1990–2010

![Histogram of Trade/GDP across countries by trade policy](image)

1990 Baseline
Uruguay Round
Uruguay Round + Preference
Free Trade

Figure 9: Trade effects from tariff changes, subsamples, detail, 1990–2010

![Distribution of Trade/GDP](image)

Trade - Advanced
Trade - Emerging/Developing

Trade/GDP: all countries, median, and 10 deciles (5-95), by trade policy

1990 Baseline
Uruguay Round
Uruguay Round + Preference
Free Trade

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to Free trade affects entry a little bit more. In particular, it tends to reduce entry sharply in many cases, in part, as a consequence of increased import competition.

Figure 11 separates the distribution of entry effects in the Advanced versus Emerging and Developing countries. The left hand side panel shows the distribution of the change in entry for Advanced economies while the right hand side panel presents the distribution of the change in entry for Emerging markets. As we can see, trade policy impacts firm entry across these types of countries in very different ways. In particular, we find that firm entry reacts more in Advanced economies (where tariff changes were smaller) relative to Emerging economies (where tariff changes were bigger): for all three experiments the results on firm entry are very concentrated for Emerging markets while this is not the case for Advanced economies. These results show clearly that entry is impacted by tariffs not only theoretically, as we discussed several times in the paper, but that it is also affected in a quantitatively significant way by a realistic change in trade policy.

Welfare Effects Figures 12 and 13 present the welfare effects for the world, namely the change in welfare relative to the base year 1990 (normalized to 1) for each of our first three experiments. Here, the Uruguay Round accounts for most of the welfare effects from tariff changes, with little further difference made by the other two experiments. In fact, the average gains across countries in our sample are +1.43% for the Uruguay Round experiment, +1.56% for Uruguay Round + Preference, and +2.04% for Free trade. Yet there is substantial heterogeneity in terms of winners and losers, as the histogram makes very clear, even if most countries are winners. Importantly, we find gains for some countries, notably some Emerging and Developing countries, from the move to complete free trade. That will help to motivate the next exercise, which is the investigation of negative optimal tariffs. These findings are reinforced when we split the sample according to Advanced and Emerging economies, as we can see in Figure 14.

Optimal Tariffs Finally, starting from a free trade equilibrium, we evaluate whether a negative uniform unilateral tariff is optimal or not. We do so by unilaterally changing tariffs uniformly across sectors for each of the countries in the sample one by one. We find that a negative tariff is optimal for one-quarter of the countries in the world, or 47 out of 189 countries. A minority of these are economies that appear to enjoy strong production linkages, as suggested by the sufficient conditions for a negative optimal tariff in Theorem 4(b): these are Belgium, France, Italy, Luxembourg, Portugal, Sweden, and also Malaysia and the Philippines (with Hungary on the borderline with a zero optimal tariff). The vast majority of these countries are not suggestive of strong production linkages, however, and are Emerging or Developing economies that are mostly remote from other other countries, including a number of islands. These economies are suggestive

\footnote{Concretely what we do is to evaluate the welfare effects from imposing unilateral uniform tariffs across sectors in the range between -20% to +20%. We do this in an interval of 2.5% one country at a time.}

\footnote{These Emerging and Developing economies with negative optimal tariffs are: Andorra, Angola, Aruba, Bahamas, Bahrain, Belarus, Belize, Brazil, British Virgin Islands, Brunei, Cameroon, Cayman Islands, Croatia, Cyprus, Czechia, Republic of the Congo, Greenland, Haiti, Jamaica, Latvia, Liberia, Libya, Macao, Maldives, Mauritius, Moldova,}
Figure 10: Entry effects from tariff changes, world, histograms, 1990–2010

Figure 11: Entry effects from tariff changes, subsamples, histograms, 1990–2010

Morocco, Namibia, Netherlands Antilles, New Caledonia, Palestine, Rwanda, San Marino, Sao Tome and Principe, Saudi Arabia, Slovenia, Syria, and Yemen.
Figure 12: Welfare effects from tariff changes, world, histograms, 1990–2010

Figure 13: Welfare effects from tariff changes, world, detail, 1990–2010
of the sufficient conditions for a negative optimal tariff in Theorem 4(a), which requires remoteness from other countries and therefore a high share of expenditure on home production.

If we rank countries by the negative of their optimal tariff, and also by the magnitude of gains from the complete removal of tariffs starting from their 2010 values, then we obtain a rank correlation of 0.56. Thus, it appears that the countries gaining from the complete removal of tariffs benefit more from the removal of their own tariffs than from the removal of tariffs in the rest of the world. In comparison, if we rank countries by the negative of their optimal tariff and also by the magnitude of gains from the actual removal of tariffs over 1990–2010, then we obtain the lower rank correlation of 0.39. Sensibly, the removal of tariffs in the rest of the world plays a greater role when evaluating the welfare gains over the entire two decades.

It might be thought that with a lower elasticity in services as we have used in our quantitative model, and therefore a higher markup than in manufacturing, the social planner would want to expand the services sector and contract manufacturing. But that is not what we find for this one-quarter of countries. Rather, we find that small tariffs in the manufacturing sector tend to worsen welfare for the country applying them, so that the optimal tariffs on these industries are negative. The reason for this result depends not only on the markups charged in different industries, but also on the extent of their openness and production linkages. The manufacturing industries in our quantitative model are more highly linked to the rest of the economy than are the services industries. The distortions that are present in the economy must take into account both monopolistic pricing and these production linkages and the extent of openness, as we have shown in Theorem 4. By not
treated the services sector in our quantitative model as perfectly competitive, as we did in the illustrative two-sector model, we are in effect “stacking the deck” against finding negative optimal tariffs in manufacturing. Despite that, we still find that tariffs in manufacturing create a domestic distortion that, in one-quarter of the countries, reverses the (positive) optimal tariff.

7 Conclusion

In this paper we study the trade, firm entry, and welfare effects arising from actual changes in trade policy in the last two decades. We do so with a multi-sector heterogeneous firm model that incorporates tariffs, traded intermediate goods, and an input-output structure that is realistic for modern economies.

First, we show that trade policy impacts firm entry and exit, a channel that has not been fully explored before. We provide a theoretical characterization of the conditions under which tariffs affect firm entry and, ultimately, welfare. We show that in a range of existing models the forces driving firm entry are inoperative only under restrictive, unrealistic assumptions about the tradability of goods, the production structure, or the way tariffs are modeled.

Next, we present a new comprehensive annual tariff dataset starting in the 1980s that allows us to measure how MFN and preferential tariffs have changed over time at a very disaggregated level. With these new data we can perform trade policy experiments which could not be explored before now, with many more interesting experiments left for future research.

Finally, with our model and data, we go beyond gains-from-trade estimates based largely on advanced economies, and use an 189-country/15-sector version of our model to quantify the effects of trade liberalization over the period 1990–2010, including the greatest round of global tariff elimination, the Uruguay Round. We find that the actual reductions in MFN tariffs in this period generated large trade, entry, and welfare effects. We also find that the effects from preferential tariff reductions have not contributed much to total world trade and welfare, and that meaningful gains from future liberalization may remain on the table only for a few developing countries.

Indeed, while some developing countries gain from the complete removal of tariffs, a number of remote Emerging and Developing economies would gain from going even further, to negative (optimal) tariffs. This policy would serve to facilitate trade into these countries and would raise welfare by more than the cost of providing the import subsidy—that is, even if consumers in these countries paid for the import subsidy, the policy would be welfare improving. One component of ongoing trade negotiations is to enable trade facilitation through other, non-monetary means, such as decreasing the time spent at the border, etc. We expect that such policies would be strongly welfare-improving in our model, through the usual channel of expanding trade and lowering prices on the intensive margin, and also through encouraging the entry of exporters on the extensive margin. In sum, our focus here on entry in a multi-sector, heterogeneous-firm model gives new insights into the potential welfare gains from trade liberalization.
References


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APPENDIX

A Tariffs, Icebergs, Entry, and Welfare

We draw from our working paper, Caliendo, Feenstra, Romalis, and Taylor (2015), to derive the impact of a change in *ad valorem* tariffs on entry in a one-sector Melitz-Chaney model. Consider first the case of tariffs applied to the variable cost of imports—or "cost" tariffs—with no rebate of the tariff revenue to consumers. The government instead wastes the revenue on a good with zero utility. Assume labor in country *i* is the numeraire, with *w*$_i$ = 1. The firm in country *i* selling to country *j* solves the profit-maximization problem

$$
\pi_{ij}(\varphi) = \max_{p_{ij}(\varphi) \geq 0} \left\{ p_{ij}(\varphi) q_{ij}(\varphi) - \frac{\tau_{ij}(1 + t_{ij})}{\varphi} q_{ij}(\varphi) - f_{ij} \right\},
$$

where $q_{ij}(\varphi)$ is the quantity chosen by consumers at the price $p_{ij}(\varphi)$, the firm’s marginal costs inclusive of iceberg costs $\tau_{ij}$ and the ad valorem cost tariff $t_{ij}$ are $\tau_{ij}(1 + t_{ij})/\varphi$, and $f_{ij}$ are the fixed operating costs. We assume CES demand with elasticity $\sigma$ and a Pareto distribution, $G(\varphi) = 1 - \varphi^{-\theta}$, for the firm productivities, with $\varphi \geq 1$. Then it can be shown by evaluating the integrals below that assumption R2 of Arkolakis, Costinot, and Rodriguez-Clare (ACR, 2012) holds, namely:

$$
\int_{\varphi_{ij}^*}^{\varphi_{ij}} \pi_{ij}(\varphi) dG(\varphi) = \frac{\sigma - 1}{\sigma \theta} \int_{\varphi_{ij}^*}^{\varphi_{ij}} p_{ij}(\varphi) q_{ij}(\varphi) dG(\varphi),
$$

where $\varphi_{ij}^*$ is the zero cutoff profit level of productivity at which $\pi_{ij}(\varphi_{ij}^*) = 0$. Now, summing over all destination markets *j*, denoting the mass of entrants by $N_i$ and the sunk costs of entry by $f_i^E$, and using the free-entry condition and equation (55), we can compute the integrals to obtain $N_i f_i^E = \Pi_i = \frac{\sigma - 1}{\sigma \theta} R_i = \left(\frac{\sigma - 1}{\sigma \theta}\right) L_i$, where $L_i$ is the labor earnings in this one-sector economy coming from the aggregate revenue of firms. It immediately follows that entry

$$
N_i = \frac{\sigma - 1}{\sigma \theta} \frac{1}{f_i^E} L_i
$$

is fixed and does not vary with iceberg trade costs or with un-rebated cost tariffs.

In comparison, now consider the realistic case of *ad valorem* tariffs applied to the import revenue gross of price markups—or "revenue" tariffs—with full rebate of the tariff revenue to consumers. In this case, the tariff-inclusive price $p_{ij}(\varphi)$ must be divided by $(1 + t_{ij})$ to obtain the net price $p_{ij}(\varphi)/(1 + t_{ij})$ earned by the firm, which is used to compute net revenue of the firm. Profits of the firm are then

$$
\pi_{ij}(\varphi) = \max_{p_{ij}(\varphi) \geq 0} \left\{ \frac{p_{ij}(\varphi)}{1 + t_{ij}} q_{ij}(\varphi) - \frac{x_i}{\varphi} \tau_{ij} q_{ij}(\varphi) - f_{ij} \right\}.
$$

Direct calculation of the integrals below shows that the analogous expression for R2, but now in the presence of revenue tariffs, becomes

$$
\int_{\varphi_{ij}^*}^{\varphi_{ij}} \pi_{ij}(\varphi) dG(\varphi) = \frac{\sigma - 1}{\sigma \theta} \int_{\varphi_{ij}^*}^{\varphi_{ij}} \frac{p_{ij}(\varphi)}{1 + t_{ij}} q_{ij}(\varphi) dG(\varphi).
$$

A clear difference between (55) and (57) is that the former uses revenue $R_{ij}$ paid by consumers, whereas the latter uses revenue $R_{ij}$ earned by firms, and these differ when *ad valorem* revenue tariffs are used. This difference is immaterial, however, when the tariff revenue is fully rebated. In that case the labor earnings

---

27In contrast, with iceberg trade costs, c.i.f. revenue paid by consumers (at c.i.f. prices but with quantity net of iceberg costs) equals f.o.b. revenue earned by firms (at lower f.o.b. prices but with quantity gross of iceberg costs).
Table 3: Operation of the entry margin under different forms of trade costs

<table>
<thead>
<tr>
<th></th>
<th>No rebate</th>
<th>Rebate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Icebergs</td>
<td>$N_i = \frac{\sigma - 1}{\sigma \theta} \frac{1}{\tau_i^L} L_i$</td>
<td>Not applicable</td>
</tr>
<tr>
<td>Cost tariffs</td>
<td>$N_i = \frac{\sigma - 1}{\sigma \theta} \frac{1}{\tau_i^L} L_i$</td>
<td>Yes: $N_i = \frac{\sigma - 1}{\sigma \theta} \frac{1}{w_i \tau_i^E} (w_i L_i + T_i)$</td>
</tr>
<tr>
<td>Revenue tariffs</td>
<td>Yes: $N_i = \frac{\sigma - 1}{\sigma \theta} \frac{1}{w_i \tau_i^E} (w_i L_i - T_i)$</td>
<td>No: $N_i = \frac{\sigma - 1}{\sigma \theta} \frac{1}{\tau_i^L} L_i$</td>
</tr>
</tbody>
</table>

paid by the firm are still $L_i$, equal to the labor endowment. Then summing over destination markets $j$, firm revenue net of tariffs is $R_i = \sum_j \frac{\sigma - 1}{\sigma \theta} \frac{1}{w_i \tau_i^E} L_i$. It follows that entry is determined by $N_i = \frac{\sigma - 1}{\sigma \theta} \frac{1}{w_i \tau_i^E} R_i$. The consumer expenditure $R_i$ in country $i$ is at tariff-inclusive prices is given by $R_i = w_i L_i + T_i$, which depends on the collected tariff revenue $T_i$. Therefore entry depends on the tariff, and is given by

$$N_i = \frac{\sigma - 1}{\sigma \theta} \frac{1}{w_i \tau_i^E} (w_i L_i + T_i).$$

Alternatively, with no rebate under revenue tariffs, then country $i$ tariff revenue $T_i$ is wasted. It follows that in this case we have that $N_i = \frac{\sigma - 1}{\sigma \theta} \frac{1}{w_i \tau_i^E} R_i$, where $R_i = w_i L_i - T_i$. Therefore entry again depends on the tariff, and is given by

$$N_i = \frac{\sigma - 1}{\sigma \theta} \frac{1}{w_i \tau_i^E} (w_i L_i - T_i).$$

Table 3 fully summarizes all of the above new results, which apply to the benchmark case of the one-sector model with Pareto productivity draws. It is worth emphasizing an important and novel insight from these results, which is that the existence of a revenue effect coming from a tariff rebate is neither necessary nor sufficient to generate changes in entry.

As the table shows, given the variety of possible trade cost formulations any analysis of the impact of tariffs on entry and welfare could in principle consider all four hypothetical tariff/rebate configurations. But in this paper we focus exclusively on ad valorem tariffs applied to the revenue of imports. This choice is made for two reasons. First, we note that these tariffs are the realistic choice, since the alternative cost-based tariffs in Costinot and Rodríguez-Clare (2014) and Felbermayr, Jung, and Larch (2015) are applied only to the variable costs of an import, and not to their full market value (variable costs plus fixed costs plus profits). But no such a distinction between variable and fixed costs and profits is made when the customs value of an import shipment is evaluated at the border, as the well known customs rules make abundantly clear.²⁸ But the second, more important reason, comes from our finding above that entry is fixed in the one-sector model when using revenue tariffs with rebate. This is a very convenient, and parsimonious, starting point for our broader analysis of tariffs, that now builds from the earlier literature.

### B Proofs of Theorems 1 to 5

**Theorem 1** The mass of entering firms $N_i$ is the same under free trade and prohibitive tariffs. If and only if $\alpha < 1$, then: (a) near the free trade equilibrium reducing the tariff will increase entry; (b) near the prohibitive tariff, reducing the tariff will decrease entry; (c) entry is lower at all intermediate tariff levels than under free trade or prohibitive tariffs.

²⁸Under the rules of the World Trade Organization, ad valorem tariffs are applied to the "customs value" of an import product, which is intended to reflect the price paid between unrelated parties. Such a price should, obviously, not exclude fixed costs or markups/profits. See: http://www.wto.org/english/tratop_e/cusval_e/cusval_info_e.htm.
Proof. Parts (a) and (b) have been shown already, but not part (c). The latter can be seen to follow from \((11), \,(14), \,(15), \) and \((18), \) as follows. From \((11)\) using \((14)\) and \((15)\) we see that

\[
Y_H = \alpha L \left(1 - \gamma \frac{1 + t\lambda_{HH}}{1 + t} - \alpha \frac{t(1 - \lambda_{HH})}{1 + t}\right).
\]

It follows from \((11)\) that \(N_H = (\sigma - 1)\alpha L/[(1 - \gamma) \sigma f^E]\) when \(t = 0\) or \(\lambda_{HH} = 1.\) It then follows that \(N_H \leq (\sigma - 1)\alpha L/[(1 - \gamma) \sigma f^E]\) at all other tariff equilibria provided that for \(t > 0\) and \(\lambda_{HH} < 1,
\[
\left(1 + \lambda_{HH} t \right) / \left[1 - \gamma \left(1 + \lambda_{HH} t \right) - \alpha \frac{t(1 - \lambda_{HH})}{1 + t}\right] < \frac{1}{1 - \gamma}.
\]

Straightforward but tedious algebra shows this condition is satisfied for \(\alpha < 1\) regardless of the value of \(\gamma.\)

\[\blacksquare\]

Theorem 2 For a small increase in the tariff \(dt\) in country \(H\) starting from free trade, a necessary and sufficient condition for a small increase in the tariff to be worse than a small increase in iceberg transport costs, where both lead to the same change in \(\lambda_{HH},\) namely, for \(d\ln U_H < -\frac{\alpha}{(1 - \gamma)\theta} d\ln \lambda_{HH},\) is that:

\[
\hat{\gamma} \equiv \gamma \left(\frac{\sigma - 1}{\sigma}\right) > 1 - \left(\frac{2 - \alpha}{\sigma + 1 - [(\sigma - 1)/\theta]}\right).
\]

This condition can hold only if \(\gamma > 0\) (production linkages are present) and \(\alpha < 1\) (the service sector is present).

Proof. Note that magnitude of the second term in \((17)\) is large enough to overwhems the third term and reverses its sign, if and only if \((\kappa\Delta - 1)/(1 - \gamma)\theta + 1/(1 - \gamma) < 0.\) Simplifying this condition, we obtain that \(\hat{\gamma} > 1 - \left(\frac{2 - \alpha}{\sigma + 1 - [(\sigma - 1)/\theta]}\right).\)

\[\blacksquare\]

Theorem 3 When both the home and foreign country apply a small equal tariff \(dt,\) then a necessary and sufficient condition to have \(d\ln U_i < -\frac{\alpha}{(1 - \gamma)\theta} d\ln \lambda_{ii},\) for \(i = H,F,\) is still condition \((18).\) Because the domestic shares in both countries rise with the tariff, then welfare in both countries falls. The countries would gain when both apply a small import subsidy from the symmetric free trade equilibrium.

Proof. We start by obtaining a closed-form expression for welfare in the symmetric two-sector, two-county model. Welfare for a change in tariffs in the model is given by the change in real income

\[
\frac{dU_i}{U_i} = -\alpha \frac{dP_i}{P_i} + \frac{dT_i}{w_i L_i + T_i}.
\]

We know \(T_i = t(1 - \lambda_{ii}) Y_i,\) and hence

\[
\frac{dT_i}{Y_i} = \frac{td\lambda_{ii}}{1 + t} + \frac{1 - \lambda_{ii} dt}{1 + t} + \frac{t(1 - \lambda_{ii}) dY_i}{1 + t}.
\]

We can then use the fact that \(\frac{Y_i}{w_i L_i + T_i} = \frac{1}{\frac{1}{\sigma - 1} \gamma (1 + \lambda_{ii}) / (1 + t)},\) whereby

\[
\frac{dU_i}{U_i} = -\alpha \frac{dP_i}{P_i} + \frac{\alpha}{1 - \frac{1}{\sigma - 1} \gamma (1 + \lambda_{ii}) / (1 + t)} \left(1 - \lambda_{ii} dt \frac{1 - t}{1 + t} + \frac{t(1 - \lambda_{ii}) dY_i}{1 + t} \right).
\]

Now, to develop this change in welfare expression, from the main text we have

\[
\frac{dN_i}{N_i} = (\Delta - 1) \left(1 - \lambda_{ii} \frac{dt}{1 + \lambda_{ii} t} - \frac{t}{1 + \lambda_{ii} t} d\lambda_{ii}\right),
\]

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and also
\[
\frac{dY_i}{Y_i} = \Delta \left( \frac{1 - \lambda_{ii}}{1 + \lambda_{ii} t} \right) dt - \frac{t}{1 + \lambda_{ii} t} d\lambda_{ii},
\]
so that
\[
\frac{1}{\Delta} \frac{dY_i}{Y_i} = \frac{1}{\Delta - 1} \frac{dN_i}{N_i},
\]
and thus the change in welfare expression can be written
\[
\frac{dU_i}{U_i} = \frac{1}{1 - \gamma} \frac{dP_i}{P_i} - \frac{1}{1 - \gamma} \left( \frac{1 - \gamma \left( \frac{\sigma - 1}{\sigma} \right)}{\sigma - 1 - \Delta} \right) \frac{1 + \lambda_{ii} t}{1 + t} d\lambda_{ii}.
\]
Then using
\[
\frac{dP_i}{P_i} = \frac{1}{1 - \gamma} \frac{d\lambda_{ii}}{\theta \lambda_{ii}} - \frac{1}{1 - \gamma} \left[ \frac{1}{\theta} + \frac{1}{\sigma - 1} - \frac{1}{\sigma - 1} \Delta \right] \frac{dN_i}{N_i},
\]
it follows that,
\[
\frac{dU_i}{U_i} = -\frac{\alpha}{1 - \gamma} \frac{d\lambda_{ii}}{\theta \lambda_{ii}} + \left[ \frac{\alpha}{1 - \gamma} \right] \frac{1}{\theta} \left[ \frac{\sigma - 1 - \Delta \theta}{\sigma - 1 - \Delta (\sigma - 1)} \right] \frac{dN_i}{N_i} - \frac{\alpha}{1 - \gamma} \left( \frac{1 - \gamma \left( \frac{\sigma - 1}{\sigma} \right)}{\sigma - 1 - \Delta (\sigma - 1)} \right) \frac{1 + \lambda_{ii} t}{1 + t} \frac{dN_i}{N_i}.
\]
In the case where we have \( dN_i > 0 \) by assumption, we will have \( \frac{dU_i}{U_i} > -\frac{\alpha}{1 - \gamma} \frac{d\lambda_{ii}}{\theta \lambda_{ii}} \) if and only if,
\[
\left( \frac{\alpha}{1 - \gamma} \right) \frac{1}{\theta} \left[ \frac{\sigma - 1 - \Delta \theta}{\sigma - 1 - \Delta (\sigma - 1)} \right] > \frac{\alpha}{1 - \gamma} \left( \frac{1 - \gamma \left( \frac{\sigma - 1}{\sigma} \right)}{\sigma - 1 - \Delta (\sigma - 1)} \right) \frac{1 + \lambda_{ii} t}{1 + t}.
\]
Now recall that
\[
\Delta = \left[ \frac{\alpha - \gamma}{1 + \lambda_{ii} t} \right] \Rightarrow \Delta \leq \left( \frac{\alpha - \gamma}{1 - \gamma} \right).
\]
In order for (58) to hold for all values of \( t \geq 0 \) and \( 0 \leq \lambda_{ii} \leq 1 \), we replace the right-hand side by its maximum value of \( \alpha/(1 - \gamma) \), and we replace its left-hand side by its minimum value when \( \Delta = \left( \frac{\alpha - \gamma}{1 - \gamma} \right) \), both obtained when \( t = 0 \) or \( \lambda_{ii} = 1 \), so the condition becomes
\[
\left( \frac{\alpha}{1 - \gamma} \right) \frac{1}{\theta} \left[ \frac{\sigma - 1 - \left( \frac{\alpha - \gamma}{1 - \gamma} \right) \theta}{(\sigma - 1) \left( \frac{1 - \alpha}{1 - \gamma} \right)} \right] > \frac{\alpha}{1 - \gamma}.
\]
Cross-multiplying terms and simplifying, we can rewrite (58) as
\[
\left( \frac{1 - \alpha}{1 - \gamma} \right) \left[ ((1 - \alpha) - (1 - \gamma)(\sigma - 1)) > (1 - \alpha) \frac{(\theta - \sigma + 1)}{\theta} \right].
\]
Dividing by \( \left( \frac{1 - \alpha}{1 - \gamma} \right) > 0 \), the condition becomes
\[
(1 - \alpha) \left[ 1 - \left( \frac{1 - \gamma}{1 - \alpha} \right) \frac{\theta - \sigma + 1}{\theta} \right] > (1 - \gamma)(\sigma - 1).
\]
Simplifying this condition, we obtain condition (58). The result for the case where we have \( dN_i < 0 \) follows directly. ■
The proofs of Theorems 4 and 5 use results derived in Appendix C.

**Theorem 4**  Consider an economy near an SFTE with zero tariffs initially, but with \( \tau > 1 \). Restrict attention to the cases (a) \( \gamma = 0 \) (no linkages); and (b) \( \gamma \rightarrow 1 \) (strong linkages).

Then for small increases in the home tariff \( dt > 0 \), the welfare change is:

(a) \( \gamma = 0 \) (no linkages):

\[
\frac{d\ln U_H}{dt} \bigg|_{\gamma = 0} = -\alpha (1 - \lambda) - \alpha (1 - \lambda) (1 - \alpha) \left[ \frac{1}{\sigma - 1} - \frac{(1 - \lambda)}{\theta} \right] + \alpha (1 - \lambda) + \frac{2 \alpha \lambda (1 - \lambda)^2 (\theta \sigma + 1)}{(2 \lambda - 1) \theta (\sigma - 1)},
\]

which is negative when \( \alpha < 1 \) and \( \tau \) is sufficiently large so that \( \lambda \) is close to unity. It follows that the optimal home tariff is negative.

(b) \( \gamma \rightarrow 1 \) (strong linkages):

\[
\lim_{\gamma \rightarrow 1} (1 - \gamma) \frac{d\ln U_H}{dt} \bigg|_{t=0} = -\frac{\alpha (1 - \lambda)}{2} - \frac{\alpha (1 - \lambda) (1 - \alpha) \sigma}{2(\sigma - 1)} + \frac{0}{\text{vanishing revenue and terms of trade effects}}
\]

which is negative for all \( 0.5 < \lambda < 1 \) and \( \alpha \leq 1 \). It follows again that the optimal home tariff is negative.

**Proof.** We substitute (53) into (51) and use \( dT_H = dt (1 - \lambda) Y_H = dt (1 - \lambda) \alpha L/(1 - \gamma) \) with \( T_H = 0 \) in the SFTE, to obtain the welfare change as,

\[
\frac{dU_H}{dU} = \frac{\alpha (1 - \lambda) dt}{(1 - \gamma)} \times \frac{1}{(1 + \Gamma) \theta} \left\{ -\theta - (1 - \Delta) \left[ \frac{\lambda + \Gamma (1 - \lambda) + \kappa - 1}{\kappa} \right] + \frac{(1 - \gamma)}{1 - \gamma} \right\},
\]

where we remind the reader that \( \kappa = \theta/(\sigma - 1) > 1 \) and \( \Gamma = [(1 - \lambda) \gamma]/(1 - \lambda) \leq 1 \). Notice that when \( \gamma = 0 \) then \( \Gamma = 0 \) and \( \Delta = \alpha \), so we simplify the above expression to obtain part (a). As \( \gamma \rightarrow 1 \) then \( \Gamma \rightarrow 1 \) and the first term on the right of (60) approaches infinity while the second term is bounded. So in that case we multiply both sides by \((1 - \gamma)\) and take the limit as \( \gamma \rightarrow 1 \), using \((1 - \Delta) = \sigma (1 - \alpha)\) to obtain part (b).

**Theorem 5** Under the conditions of part (a) in Theorem 4, the home share rises with the tariff, \( d\ln \lambda_H / dt > 0 \). Under the conditions of part (b), the home share rises provided that \( \kappa = \theta/(\sigma - 1) > 2/(1 + \alpha \sigma) \).

**Proof.** Substitute (53) along with (51) and (53) into the formula (55) for the change in the home share to obtain,

\[
d\ln \lambda_H = d\ln N_H + \theta (1 - \gamma) d\ln P_H - (1 - \kappa) d\ln Y_H = (\kappa \Delta - 1) (1 - \lambda) dt \left\{ -\theta - (1 - \Delta) \left[ \frac{\lambda + \Gamma (1 - \lambda) + \kappa - 1}{\kappa} \right] - \frac{(1 - \gamma)}{1 - \gamma} \right\},
\]
When $\gamma = 0$ so that $\Gamma = 0$ and $\Delta = \alpha$, under the conditions of part (a), we simplify the above expression as,

$$\frac{d\ln \lambda_{HH}}{dt} = \left[1 - \frac{2\lambda(1 - \lambda)}{2\lambda - 1}\right] (\theta + \kappa - 1) (1 - \lambda) - (1 - \alpha) (1 - \lambda)^2.$$  

For $\lambda$ sufficiently close to unity, this term is positive.

For part (b), we allow $\gamma \to 1$ and use $(1 - \Delta) = \sigma(1 - \alpha)$ to obtain,

$$\frac{d\ln \lambda_{HH}}{dt} = \frac{(1 - \lambda)}{2} \left[\frac{\theta(1 + \alpha\sigma)}{(\sigma - 1)} - 2\right].$$

This expression is positive provided that $\theta/(\sigma - 1) > 2/(1 + \alpha\sigma)$.  

C Two-Sector, Two-Country Model with a Change in the Home Tariff

We start at a symmetric free trade equilibrium, where the home country is unilaterally changing its tariffs, so that $t_{HF} = 0$ throughout while $t_{FH} = t$, and the latter is zero initially but then changes by $dt$. We further normalize $w_H = 1$ but determine $w_F$ endogenously. Unlike in the main text, we will not use the prime notation to denote the “new; equilibrium, but simply differentiate all terms starting at the symmetric free trade equilibrium (SFTE).

Define the terms,

$$A_{ji} = N_j \left( \frac{w_j (1 + t_{ji})}{Y_i} \right)^{\frac{\sigma-1-\theta}{\sigma-1}},$$

where

$$B_{ji} = \frac{\theta}{\theta + 1 - \sigma} \left( \sigma f_{ji} \right)^{\frac{\sigma+\sigma-1}{\sigma-1}} \left( \frac{\sigma}{\sigma - 1} \right)^{-\theta}.$$

The costs of production are,

$$x_i = \bar{\gamma} (w_i)^{-1-\gamma} (P_i)^{\gamma},$$

with $\bar{\gamma} = \gamma^\gamma (1 - \gamma)^{(1-\gamma)}$. Then using $A_{ji}$, $B_{ji}$, $x_i$, and (17) for the two-sector two-country case, we obtain

$$P_i = \left[ A_{ii} B_{ii} (x_i)^{-\theta} + A_{ji} B_{ji} (x_j t_{ji} (1 + t_{ji}))^{-\theta} \right]^{-\frac{1}{\theta}},$$

for $t_{FH} = t$ and $t_{HF} = 0$, where the trade shares are denoted by

$$\lambda_{ji} = A_{ji} B_{ji} \left( \frac{x_j (1 + t_{ji})}{P_i} \right)^{-\theta}.$$  

(62)

We need to develop equations for $dP_h / P_h$ and $dP_f / P_f$, which we can compute from (17). Differentiating this for home and foreign, and using $d\ln (1 + t) = dt$ since $t = 0$ initially,

$$\frac{dP_h}{P_h} = -\frac{1}{\theta} \left\{ \lambda_{HH} (d\ln A_{HH} - \theta d\ln x_{HH}) + \lambda_{FH} [d\ln A_{FH} - \theta d\ln x_{FH} - \theta dt] \right\},$$

$$\frac{dP_f}{P_f} = -\frac{1}{\theta} \left\{ \lambda_{FF} (d\ln A_{FF} - \theta d\ln x_{FF}) + \lambda_{HF} (d\ln A_{HF} - \theta d\ln x_{HF}) \right\},$$

Then using all the above equations with $\lambda_{FF} = \lambda_{HH} = \lambda$, we obtain
\[
\frac{dP_H}{P_H} = -\frac{1}{\theta} \left\{ \begin{array}{cl}
\lambda d\ln A_{HH} - \lambda \theta d\ln x_H \\
(1 - \lambda) d\ln A_{HF} - (1 - \lambda) \theta d\ln x_F - (1 - \lambda) \theta dt
\end{array} \right. \\
= -\frac{1}{\theta} \left\{ \begin{array}{cl}
\lambda d\ln N_H - \lambda \gamma d\ln P_H - (1 - \kappa) d\ln Y_H \\
(1 - \lambda) [1 - \kappa - \theta(1 - \gamma)] d\ln w_F + (1 - \lambda) \theta \gamma d\ln P_F - (1 - \lambda) \theta dt
\end{array} \right.
\]

where the second line uses \( dw_H = 0 \) and \( dN_F = 0 \), and with \( \kappa = \theta/\sigma - 1 > 1 \). Simplifying, we obtain,
\[
\frac{dP_H}{P_H} = -\frac{1}{(1 - \lambda \gamma)\theta} \left\{ \begin{array}{cl}
\lambda d\ln N_H - (1 - \kappa) d\ln Y_H - (1 - \lambda) \theta \gamma d\ln P_F \\
(1 - \lambda) [1 - \kappa - \theta(1 - \gamma)] d\ln w_F + (1 - \lambda) (1 - \kappa - \theta) dt
\end{array} \right. .
\] (63)

For the foreign country we have,
\[
\frac{dP_F}{P_F} = -\frac{1}{(1 - \lambda \gamma)\theta} \left\{ \begin{array}{cl}
\lambda d\ln A_{FF} - \lambda \theta d\ln x_F \\
(1 - \lambda) d\ln A_{HF} - (1 - \lambda) \theta d\ln x_H
\end{array} \right. \\
= -\frac{1}{\theta} \left\{ \begin{array}{cl}
-\lambda \theta (1 - \gamma) + (1 - \lambda) (1 - \kappa) d\ln w_F - \lambda \theta \gamma d\ln P_F \\
+ (1 - \lambda) d\ln N_H - (1 - \lambda) \theta \gamma d\ln P_H
\end{array} \right. .
\]

where the second line uses \( d\ln Y_F = d\ln w_F \) and \( dN_F = 0 \). Finally we obtain,
\[
\frac{dP_F}{P_F} = -\frac{1}{(1 - \lambda \gamma)\theta} \left\{ \begin{array}{cl}
(1 - \lambda) d\ln N_H - (1 - \lambda) \theta \gamma d\ln P_H \\
-[(1 - \lambda) (1 - \kappa) + \lambda \theta (1 - \gamma)] d\ln w_F
\end{array} \right. .
\] (64)

Substitute (64) into (63) and define \( \Gamma = [(1 - \lambda) \gamma]/(1 - \lambda \gamma) < 1 \) to obtain
\[
(1 - \lambda \gamma)\theta \frac{dP_H}{P_H} = -\frac{\lambda d\ln N_H - (1 - \kappa) d\ln Y_H}{\Gamma} \left\{ \begin{array}{cl}
[(1 - \lambda) - \Gamma (1 - \lambda)] d\ln N_H - (1 - \lambda) \theta \gamma d\ln P_H \\
+ (1 - \lambda) [1 - \kappa - \theta(1 - \gamma)] d\ln w_F + (1 - \lambda) (1 - \kappa - \theta) dt
\end{array} \right. .
\]

Note that \( (1 - \lambda \gamma) \theta - \Gamma (1 - \lambda) \gamma \theta = (1 - \lambda \gamma) \theta \left[ 1 - \Gamma \frac{(1 - \lambda) \gamma}{(1 - \lambda \gamma)} \right] = 0 \). Then we have
\[
(1 + \Gamma) \frac{dP_H}{P_H} = -\frac{1}{(1 - \gamma) \theta} \left\{ \begin{array}{cl}
\lambda \Gamma \frac{d\ln N_H - (1 - \kappa) d\ln Y_H}{\Gamma} \\
+ \{1 - (1 - \Gamma) (1 - \kappa) \theta(1 - \gamma) \theta \gamma (1 - \kappa) d\ln w_F \\
+ (1 - \lambda) (1 - \kappa - \theta) dt
\end{array} \right. .
\] (65)

We also solve for \( dP_F \) by substituting (63) into (63) and, using the same simplifications,
\[
(1 + \Gamma) \frac{dP_F}{P_F} = -\frac{1}{(1 - \gamma) \theta} \left\{ \begin{array}{cl}
\Gamma \lambda (1 - \lambda) d\ln N_H - \Gamma (1 - \lambda) d\ln Y_H \\
+ \{1 - (1 - \Gamma) (1 - \kappa) \theta(1 - \gamma) \theta \gamma (1 - \kappa) d\ln w_F \\
+ (1 - \lambda) (1 - \kappa - \theta) dt
\end{array} \right. .
\] (66)

Notice that one simplifying feature of (63) and (64) is that by summing them, the term \( (1 + \Gamma) \) appears on both sides and can be factored out, yielding
\[
\left( \frac{dP_H}{P_H} + \frac{dP_F}{P_F} \right) = -\frac{1}{(1 - \gamma) \theta} \left\{ \begin{array}{cl}
\lambda \theta (1 - \gamma) d\ln N_H - (1 - \kappa) d\ln Y_H - \theta(1 - \gamma) d\ln w_F \\
+ (1 - \lambda) (1 - \kappa - \theta) dt
\end{array} \right. .
\] (67)

Now we need to find an expression for \( d\ln w_F \). We use the equilibrium condition \( \lambda_{FF} + \lambda_{HH} = 1 \).
Totally differentiating at the SFTE, we obtain
\[ d \ln \lambda_{FH} = \frac{\lambda}{1 - \lambda} d \ln \lambda_{HH}. \]

Now use (64) for \( \lambda_{FH} \) to solve for \( d \ln \lambda_{FH} \), to get
\[ d \ln \lambda_{FH} = d \ln A_{FH} - \theta d \ln x_F - \theta dt + \theta d \ln P_H, \]
and, given that
\[ d \ln A_{FH} = (1 - \kappa) d \ln w_F + (1 - \kappa) dt - (1 - \kappa) d \ln Y_H, \]
we obtain
\[ d \ln \lambda_{FH} = [1 - \kappa - \theta(1 - \gamma)] d \ln w_F - \theta \gamma d \ln P_F + (1 - \kappa - \theta) dt \]
\[ - (1 - \kappa) d \ln Y_H + \theta d \ln P_H. \]

Likewise, we find that
\[ d \ln \lambda_{HH} = d \ln N_H + \theta(1 - \gamma) d \ln P_H - (1 - \kappa) d \ln Y_H, \]
so that \( d \ln \lambda_{FH} = \frac{\lambda}{1 - \lambda} d \ln \lambda_{HH} \) implies that
\[ d \ln N_H + (\frac{2 \lambda - \lambda \gamma - 1}{1 - \lambda}) \theta d \ln P_H + (1 - \kappa) d \ln Y_H \left( \frac{1 - 2 \lambda}{1 - \lambda} \right). \]

We can get an analogous expression using the other equilibrium condition \( \lambda_{HF} + \lambda_{FF} = 1 \). Totally differentiating again we obtain,
\[ d \ln \lambda_{HF} = \frac{\lambda}{1 - \lambda} d \ln \lambda_{HH}. \]

Now use the \( \lambda_{HF} \) to solve for \( d \ln \lambda_{HF} \), to get
\[ d \ln \lambda_{HF} = d \ln A_{HF} - \theta d \ln x_H + \theta d \ln P_F \]
\[ = d \ln N_H - (1 - \kappa) d \ln Y_F - \theta \gamma d \ln P_H + \theta d \ln P_F. \]

Here again, we find that
\[ d \ln \lambda_{FF} = d \ln A_{FF} - \theta d \ln x_F + \theta d \ln P_F \]
\[ = [1 - \kappa - \theta(1 - \gamma)] d \ln w_F - (1 - \kappa) d \ln Y_F + \theta(1 - \gamma) d \ln P_F, \]
so that \( d \ln \lambda_{FH} = \frac{\lambda}{1 - \lambda} d \ln \lambda_{HH} \) implies that
\[ d \ln N_H - \theta \gamma d \ln P_H \]
\[ = [1 - \kappa - \theta(1 - \gamma)] d \ln w_F + (1 - \kappa) \left( \frac{1 - 2 \lambda}{1 - \lambda} \right) d \ln Y_F + \theta \left( \frac{2 \lambda - \lambda \gamma - 1}{1 - \lambda} \right) d \ln P_F. \]
Summing (69) and (70) we get
\[
(1 - \kappa - \theta) \, dt = \left( \frac{2\lambda - 1}{1 - \lambda} \right) \ln N_H + (1 - \kappa) \, d\ln Y_H \left( \frac{1 - 2\lambda}{1 - \lambda} \right) + (1 - \kappa) \left( \frac{1 - 2\lambda}{1 - \lambda} \right) \ln Y_F \\
+ \theta \left( \gamma + \frac{2\lambda - \lambda_\gamma - 1}{1 - \lambda} \right) \left( d\ln P_H + d\ln P_F \right).
\]  

(71)

Now we can substitute \( d\ln Y_F = d\ln w_F \) in the above expression and use \( \left( \gamma + \frac{2\lambda - \lambda_\gamma - 1}{1 - \lambda} \right) = (1 - \gamma) \left( \frac{2\lambda - 1}{1 - \lambda} \right) \) and condition (71), along with (67) and (43) to get,
\[
(1 - \kappa - \theta) \, dt = \left[ (1 - \kappa) \left( \frac{1 - 2\lambda}{1 - \lambda} \right) - \theta(1 - \gamma) \left( \frac{1 - 2\lambda}{1 - \lambda} \right) \right] \, d\ln w_F
\]
\[
- \left( \frac{2\lambda - 1}{1 - \lambda} \right) (1 - \lambda) (1 - \kappa - \theta) \, dt,
\]
so that,
\[
d\ln w_F = - \frac{2\lambda(1 - \lambda)}{2\lambda - 1} \left( \frac{\theta + \kappa - 1}{\theta(1 - \gamma) + \kappa - 1} \right) \, dt.
\]  

(72)

Finally, substitute (72) into (13), and also use (13) and (15), to obtain
\[
dP_H \bigg/ P_H = - \frac{dt (1 - \lambda)}{(1 + \Gamma)(1 - \gamma)\theta} \left\{ \frac{(\Delta - 1) \left[ \lambda + \Gamma (1 - \lambda) + \kappa - 1 \right] - \theta}{(1 - \gamma)(1 - \lambda)(1 - \kappa - \theta) - \left( \frac{1 - \kappa - \theta}{1 - \gamma} \right)} \right\}.
\]  

(73)

With this, in Appendix B we can now take the final steps to prove Theorem 4 and 5 in the main text.

## D Equilibrium conditions of the model in relative changes

To gain traction with the model when taking it to the data, we express the equilibrium conditions in relative terms using “hat” notation for the ratio of after-versus-before levels for a given perturbation, that is, \( \hat{z} = z' / z \) for any variable \( z \). As shown below, the equilibrium conditions of our model can be expressed as follows, where the change in the price of the input bundle is given by

\[ \hat{x}_{i,s} = (\hat{w}_i)^{\gamma_{i,s}} \prod_{s' = 1}^{s} \left( \hat{P}_{i,s'} \right)^{\gamma_{i,s'}} \]  

(74)

the change in the price index

\[ \hat{P}_{i,s} = \left[ (\lambda_{ii,s})^{1 - \xi_s} \hat{A}_{ii}^{\xi_s} + (1 - \lambda_{ii,s})^{1 - \xi_s} \hat{A}_{ji}^{\xi_s} \right]^{-\frac{1}{\xi_s}} \]  

(75)

where \( \xi_s = \frac{(\sigma_s - 1)(1 - \omega_s)}{\sigma_s(\sigma_s - \omega_s) + (\sigma_s - 1)(1 - \omega_s)} \), and \( \hat{A}_{ji} = \sum_{s = 1}^{S} \sum_{j \neq i}^{M} \lambda_{ji,s} \left[ \hat{x}_{j,s} \hat{T}_{ji,s} (1 + t_{ji,s}) \right]^{-\theta_s} \hat{A}_{ji,s} \).

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The change in trade shares is given by

\[
\hat{\lambda}_{i;s} = \left( \frac{x_{i,s}}{P_{i,s}} \right)^{-\theta_s} \left( \frac{\hat{P}_{i,s}}{P_{i,s}} \right)^{-\frac{\omega_{i,s}}{\sigma_{i,s}} - \frac{1}{\sigma_{i,s}}} \hat{\lambda}_{i,s}, \tag{76}
\]

\[
\hat{\lambda}_{j;s} = \left[ \frac{x_{j,s}}{x_{j,s}} \right]^{-\theta_s} \left( \frac{\hat{P}_{j,s}}{P_{j,s}} \right)^{-\frac{\omega_{j,s}}{\sigma_{j,s}} - \frac{1}{\sigma_{j,s}}} \hat{\lambda}_{j,s},
\]

where \( \hat{P}_{i,s} = \hat{P}_{i,s}^{\sigma_{i,s}} \hat{\lambda}_{i,s}^{-\frac{\omega_{i,s}}{\sigma_{i,s}} - \frac{1}{\sigma_{i,s}}} \), \( \hat{P}_{j,s} = \hat{P}_{j,s}^{\sigma_{j,s}} \hat{\lambda}_{j,s}^{-\frac{\omega_{j,s}}{\sigma_{j,s}} - \frac{1}{\sigma_{j,s}}} \), and,

\[
\hat{A}_{j,s} = \hat{\lambda}_{j,s} (\frac{\hat{w}_{j}(1 + t_{j,i,s})}{Y_{i,s}})^{\frac{\sigma_{j,s}}{\sigma_{i,s}} - \frac{1}{\sigma_{i,s}}}. \tag{77}
\]

The remaining equilibrium conditions are,

\[
Y'_{i,s} = \sum_{s'=1}^{S} \hat{\zeta}_{i,s'} \sum_{j=1}^{M} \frac{\hat{\lambda}_{j,s}}{1 + t'_{j,i,s}} Y'_{j,s'} + \alpha_{i,s} \left( w'_i L'_i + T'_i \right), \tag{78}
\]

with tariff revenue given by

\[
T'_i = \sum_{s=1}^{S} \sum_{j=1 \neq i}^{M} \frac{t'_{j,i,s}}{1 + t'_{j,i,s}} \lambda_{j,i,s} Y'_{i,s}, \tag{79}
\]

trade balance

\[
\sum_{s=1}^{S} \sum_{j=1}^{M} \frac{\hat{\lambda}_{j,i,s}'}{1 + t'_{j,i,s}} Y'_{i,s} = \sum_{s=1}^{S} \sum_{j=1}^{M} \lambda_{j,i,s}' Y'_{j,s}, \tag{80}
\]

and the final condition for firm entry,

\[
\hat{N}_{i,s} = \frac{\sum_{j=1}^{M} \frac{\hat{\lambda}_{j,i,s}}{1 + t_{j,i,s}} Y_{j,s}}{\hat{w}_{i}}. \tag{81}
\]

As we can see, by expressing the model in this way we can analyze the effects of tariff changes without needing information of fixed entry and operating costs which are, in general, difficult to estimate in the data, especially at the necessary disaggregation. The only identification restriction we will impose is that these fixed have not changed over time. The above system of equations can then be used to study the impact of a change in tariffs \((1 + t_{j,i,s})\) (as well as the change in iceberg costs, \(\hat{\tau}_{j,i,s}\)).

To justify this set of equations, we return to the equilibrium conditions in the main text. The parameters of the model are \(\alpha_{i,s}, \sigma_{i,s}, \omega_{i,s}, f_{i,i,s}, \tau_{ij,s}, \theta_{i,s}, f_{ij,s}^{\delta}, \gamma_{i,s}, \) and \(\gamma_{i,s}, \) subject to the constraints \(\sum_{s=1}^{S} \alpha_{i,s} = 1\) and \(\sum_{s'=1}^{S} \gamma_{i,s} + \gamma_{i,s} = 1\). The equilibrium conditions to solve the model are then as follows: \(M \times M \times S\) ZCP conditions (13),

\[
\varphi_{i,s}^{*} = \left( \frac{\sigma_{i,s}}{\sigma_{i,s} - 1} \right) \left( \frac{\sigma_{i,s} w_{i} f_{i,s}}{Y_{i,s}} \right)^{\frac{\omega_{i,s}}{\sigma_{i,s} - 1}} \frac{x_{i,s}}{P_{i,s}} \left( \frac{P_{i,s}}{P_{i,s}} \right)^{\frac{\omega_{i,s}}{\sigma_{i,s} - 1}},
\]

\[
\varphi_{j,s}^{*} = \left( \frac{\sigma_{j,s}}{\sigma_{j,s} - 1} \right) \left( \frac{\sigma_{j,s} w_{j} f_{j,s}}{Y_{j,s}} \right)^{\frac{\omega_{j,s}}{\sigma_{j,s} - 1}} \frac{x_{i,s} \tau_{j,s}}{P_{j,s}} \left( \frac{P_{j,s}}{P_{j,s}} \right)^{\frac{\omega_{j,s}}{\sigma_{j,s} - 1}} \frac{(1 + t_{j,i,s})^{\frac{\omega_{j,s}}{\sigma_{j,s} - 1}}}{\frac{P_{j,s}}{P_{j,s}}}.
\]
\( M \times S \) goods market equilibria (23),

\[
Y_{i,s} = \frac{\sigma_s - 1}{\sigma_s} \sum_{s'=1}^{S} \gamma_{i,s'} \sum_{j=1}^{M} \frac{\lambda_{ij,s'}}{1 + \tau_{ij,s'}} Y_{j,s'} + \alpha_{i,s} (w_i L_i + T_i);
\]

\( M \times S \) sectoral prices (24),

\[
P_{i,s} = \left( (P_{i,s})^{1-\omega_s} + (P_{F,s})^{1-\omega_s} \right)^{1-\omega_s^{-1}}
\]

where

\[
P_{i,s} = \left( \frac{\phi_{i,s}^s - \theta_s N_{i,s} + \frac{x_{i,s}}{\sigma_s - 1 \phi_{i,s}^s}}{\sigma_s - 1 \phi_{i,s}^s} \right)^{1-\sigma_s} \left( \frac{P_{i,s}}{P_{i,s}} \right)^{1-\omega_s},
\]

\[
P_{F,s} = \left( \sum_{j \neq i} \phi_{j,s}^s - \theta_j N_{j,s} \left( \frac{\sigma_j x_{j,s} \tau_{ji,s} (1 + t_{ji,s})}{\sigma_j - 1 \phi_{j,s}^s} \right)^{1-\sigma_j} \right)^{1-\sigma_j} \left( \frac{P_{F,s}}{P_{i,s}} \right)^{1-\omega_s};
\]

\( M \times M \times S \) expenditure shares (21),

\[
\lambda_{i,s} = \phi_{i,s}^s - \theta_s N_{i,s} \left( \frac{\sigma_s x_{i,s}}{\sigma_s - 1 \phi_{i,s}^s} \right)^{1-\sigma_s} \left( \frac{P_{i,s}}{P_{i,s}} \right)^{1-\omega_s},
\]

\[
\lambda_{j,i,s} = \phi_{j,i,s}^s - \theta_j N_{j,s} \left( \frac{\sigma_j \tau_{ji,s} x_{j,s} (1 + t_{ji,s})}{\sigma_j - 1 \phi_{j,i,s}^s} \right)^{1-\sigma_j} \left( \frac{P_{F,s}}{P_{i,s}} \right)^{1-\omega_s};
\]

\( M \times S \) free entry conditions (17),

\[
\sum_{j=1}^{M} f_{ij,s} \phi_{j,i,s}^s - \theta_s = \frac{\theta_s - \sigma_s + 1}{\sigma_s - 1} f_{i,s};
\]

\( M \times S \) input bundle costs (11),

\[
x_{i,s} \equiv (w_i / \gamma_{i,s}) \gamma_{i,s} \prod_{s'=1}^{S} \left( \frac{P_{i,s'} / \gamma_{i,s'}}{\gamma_{i,s'}} \right)^{\gamma_{i,s'}};
\]

and \( M \) trade balances (23),

\[
\sum_{s=1}^{S} \sum_{j=1}^{M} \lambda_{j,i,s} Y_{i,s} = \sum_{s=1}^{S} \sum_{j=1}^{M} \frac{\lambda_{ij,s} Y_{j,s}}{1 + \tau_{ij,s}}
\]

We now show how we can express the model in relative changes. Consider the impact of a change in iceberg costs \( \tau_{ji,s} \) and/or tariffs \( t_{ji,s} \). Denote equilibrium prices and allocations under policy vector \((\pi, t)\), by the vector \( y \) and equilibrium prices and allocations under policy vector \((\pi', t')\), by the vector \( y' \). In the hat notation, we let \( \hat{y} = y' / y \) denote the relative change in equilibrium prices and allocations after a change in policy, for any element \( y \) of the vector \( y \).

Using input bundle costs (11) before and after a change in policy, we can easily obtain (21). We then proceed to solve for the change in sectoral prices. First we solve for prices after a change in policy using equation (24),

\[
(P'_{i,s})^{1-\omega_s} = \left( (P'_{i,s})^{1-\omega_s} + (P'_{F,s})^{1-\omega_s} \right)^{1-\omega_s^{-1}}
\]
\[ P'_{t,i,s} = \left( \varphi'_{t,i,s} - \theta_s \right) \tilde{N}'_{t,i,s} \left( \frac{x'_{t,i,s}}{\sigma_s - 1} \varphi'_{t,i,s} \right)^{1 - \sigma_s} \]

\[ P^{F'}_{t,i,s} = \sum_{j \neq i}^{M} \varphi'_{t,j,i,s} - \theta_s N'_{t,j,s} \left( \frac{\varphi'_{t,j,i,s}}{\sigma_s - 1} \right)^{1 - \sigma_s} \left( \frac{x'_{t,j,i,s}}{\varphi'_{t,j,i,s}} \right)^{1 - \sigma_s} \]

Next we use the definition of expenditure shares before the change in policy (81), and multiply and divide each expression in the summation by (84),

\[ \left( \tilde{P}_{t,i,s} \right)^{1 - \sigma_s} \left( \frac{P_{t,i,s}}{\tilde{P}_{t,i,s}} \right)^{1 - \omega_s} = \lambda_{t,i,s} \left( \varphi_{t,i,s} \right)^{\sigma_s - 1 - \theta_s} \tilde{N}_{t,i,s} \left( \tilde{x}_{t,i,s} \right)^{1 - \sigma_s} \quad (82) \]

\[ \left( \tilde{P}^{F}_{t,i,s} \right)^{1 - \sigma_s} \left( \frac{P^{F}_{t,i,s}}{\tilde{P}_{t,i,s}} \right)^{1 - \omega_s} = \sum_{j \neq i}^{M} \lambda_{t,j,i,s} \left( \varphi_{t,j,i,s} \right)^{\sigma_s - 1 - \theta_s} \tilde{N}_{t,j,s} \left( \tilde{x}_{t,j,i,s} \right)^{1 - \sigma_s} \quad (83) \]

where we use the fact that \( \varphi'_{t,j,i,s} = \varphi_{t,j,i,s} \). Now solve for the ZCP conditions (13) in relative changes,

\[ \varphi'_{t,i,s} = \left( \frac{\tilde{w}_{t,i}}{\tilde{Y}_{t,i,s}} \right)^{\frac{1}{\sigma_s - 1}} \frac{\tilde{x}_{t,i,s}}{\tilde{P}_{t,i,s}} \left( \frac{\tilde{P}_{t,i,s}}{\tilde{P}_{t,i,s}} \right)^{\frac{\sigma_s - \theta_s}{\sigma_s - 1}} \quad (84) \]

\[ \varphi'_{t,j,i,s} = \left( \frac{\tilde{w}_{t,j}}{\tilde{Y}_{t,j,s}} \right)^{\frac{1}{\sigma_s - 1}} \frac{\tilde{x}_{t,j,i,s} \tilde{x}_{t,j,i,s} \left( 1 + t_{t,j,i,s} \right)}{\tilde{P}^{F}_{t,i,s}} \left( \frac{\tilde{P}^{F}_{t,i,s}}{\tilde{P}_{t,i,s}} \right)^{\frac{\sigma_s - \theta_s}{\sigma_s - 1}} \quad (85) \]

and substitute it into (82) and (83) to obtain

\[ \tilde{P}_{t,i,s} = \tilde{P}_{t,i,s} \left( \frac{\lambda_{t,i,s} \left[ \tilde{x}_{t,i,s} \right] - \theta_s \tilde{A}_{t,i,s}}{\lambda_{t,i,s}} \right) \left( \frac{\xi_{t,i,s}}{1 - \omega_s} \right) \]

\[ \tilde{P}^{F}_{t,i,s} = \tilde{P}^{F}_{t,i,s} \left( \frac{\sum_{j \neq i}^{M} \lambda_{t,j,i,s} \left[ \tilde{x}_{t,j,i,s} \tilde{x}_{t,j,i,s} \left( 1 + t_{t,j,i,s} \right) \right] - \theta_s \tilde{A}_{t,j,i,s}}{1 - \lambda_{t,i,s}} \right) \left( \frac{\xi_{t,i,s}}{1 - \omega_s} \right) \]

where we define

\[ \xi_{s} = \frac{(\sigma_s - 1)(1 - \omega_s)}{-\theta_s (\sigma_s - \omega_s) + (\sigma_s - 1)(1 - \omega_s)} \]

\[ \tilde{A}_{t,i,s} = \tilde{N}_{t,i,s} \left( \frac{\tilde{w}_{t,i}}{\tilde{Y}_{t,i,s}} \right)^{\frac{\sigma_s - \theta_s}{\sigma_s - 1}} \]

\[ \tilde{A}_{t,j,i,s} = \tilde{N}_{t,j,s} \left( \frac{\tilde{w}_{t,j}}{\tilde{Y}_{t,j,s}} \right)^{\frac{\sigma_s - \theta_s}{\sigma_s - 1}} \]

and after combining terms using

\[ \left( \frac{\tilde{P}_{t,i,s}}{\tilde{P}_{t,i,s}} \right)^{1 - \omega_s} = \left( \frac{P_{t,i,s}}{\tilde{P}_{t,i,s}} \right)^{1 - \omega_s} \left( \frac{\tilde{P}_{t,i,s}}{\tilde{P}_{t,i,s}} \right)^{1 - \omega_s} + \left( \frac{P^{F}_{t,i,s}}{\tilde{P}_{t,i,s}} \right)^{1 - \omega_s} \left( \frac{\tilde{P}^{F}_{t,i,s}}{\tilde{P}_{t,i,s}} \right)^{1 - \omega_s} \]
we obtain (13)
\[ \hat{P}_{i,s} = \left( (\lambda_{ii,s})^{1-\xi_s} \hat{A}_{ii,s} + (1 - \lambda_{ii,s})^{1-\xi_s} \hat{A}_{ji,s} \right)^{-\theta_s} \]
and where \( \hat{A}_{ii} = \left( \lambda_{ii,s} \hat{x}_{i,s} \right)^{-\theta_s} \hat{A}_{ii,s} \), and \( \hat{A}_{ji} = \sum_{j \neq i} \lambda_{ji,s} \left[ \hat{x}_{j,s} \hat{r}_{j,s} (1 + t_{ji,s}) \right]^{-\theta_s} \hat{A}_{ji,s} \).

Expenditure shares in relative changes are solved in a similar way. Start from solving for the expenditure share after a change in policy using (20)
\[
\lambda'_{ji,s} = \varphi'_{ji,s}^{1-\theta_s} N_{ji,s} \left( \frac{\sigma_s}{\sigma_s - 1} \frac{r_{ji,s} x_{j,s}^t (1 + t_{ji,s})}{P'_{i,s} \varphi'_{ji,s}} \right)^{1-\sigma_s} \\
\lambda'_{ii,s} = \varphi'_{ii,s}^{1-\theta_s} N_{ii,s} \left( \frac{\sigma_s}{\sigma_s - 1} \frac{x_{i,s}^t}{\varphi'_{ji,s}} \right)^{1-\sigma_s} \left( \frac{P'_{i,s}}{P_{i,s}} \right)^{1-\omega_s} \\
\lambda'_{ji,s} = \varphi'_{ji,s}^{1-\theta_s} N_{ji,s} \left( \frac{\sigma_s}{\sigma_s - 1} \frac{x_{j,s}^t}{P'_{i,s} \varphi'_{ji,s}} \right)^{1-\sigma_s} \left( \frac{P'_{i,s}}{P_{i,s}} \right)^{1-\omega_s}
\]
take the ratio of this expression relative to the expenditure share before the change in policy,
\[
\frac{\lambda'_{ii,s}}{\lambda_{ii,s}} = \left( \varphi'_{ii,s} \right)^{1-\theta_s} \left( \hat{N}_{i,s} \right) \left( \frac{\hat{x}_{i,s}}{P_{i,s}} \right)^{1-\sigma_s} \left( \frac{\hat{P}_{i,s}}{P_{i,s}} \right)^{1-\omega_s} \\
\frac{\lambda'_{ji,s}}{\lambda_{ji,s}} = \left( \varphi'_{ji,s} \right)^{1-\theta_s} \left( \hat{N}_{j,s} \right) \left( \frac{\hat{x}_{j,s}}{P_{i,s}} \right)^{1-\sigma_s} \left( \frac{\hat{P}_{i,s}}{P_{i,s}} \right)^{1-\omega_s}
\]
Now use the ZCP condition in relative changes (33) and combine terms to obtain the expenditure shares in relative changes (33),
\[
\hat{\lambda}_{ii,s} = \left( \hat{x}_{i,s} \right)^{-\theta_s} \left( \frac{\hat{P}_{i,s}}{P_{i,s}} \right)^{-\theta_s \frac{1-\omega_s}{\sigma_s}} \hat{A}_{ii,s} \\
\hat{\lambda}_{ji,s} = \left( \hat{x}_{j,s} \hat{r}_{j,s} (1 + t_{ji,s}) \right)^{-\theta_s} \left( \frac{\hat{P}_{i,s}}{P_{i,s}} \right)^{-\theta_s \frac{1-\omega_s}{\sigma_s}} \hat{A}_{ji,s}
\]
The goods market equilibrium conditions (32) and the trade balance equilibrium conditions (30) are given by (24) and (26) at policy \( (\tau', t') \).
Finally, to solve for the change in entry, note that, from the free entry condition (13) and imposing trade balance (23), we obtain
\[
\hat{N}_{i,s} = \frac{E_{ii,s} + E_{i,s}}{w_t f_{i,s} \left( \frac{\alpha_s}{\sigma_s - 1} \right)}
\]
and expressing this in relative terms we end up with (31).

E  Domestic Sales, Expenditure Shares and Final Good Shares

In this Appendix we show how using information on tariffs, trade flows, production and with the estimated trade elasticities we can solve for the model domestic sales, expenditure shares and finished good shares. We then show how to aggregate tariffs in a model consistent way.
**Domestic Sales**

To calculate domestic sales ($E_{i,i,s}$) by country and sector, we need data on gross production ($GO_{i,s}$), and total exports ($E_{i,s} = \sum_{j \neq i} E_{ij,s} = \sum_{j \neq i} \frac{\lambda_{ij,s}}{1 + t_{ij,s}} Y_{j,s}$). Recall that gross production in sector $s$ is given by $\sum_{M_{jk}=1}^{M} \sum_{i} \frac{\lambda_{ijk,s}}{1 + t_{ijk,s}} Y_{j,s}$. We want to solve for $E_{i,i,s} = \lambda_{i,i,s} Y_{i,s}$. Therefore, domestic sales are given by

$$E_{i,i,s} = \frac{\sigma_{s}}{\sigma_{s} - 1} GO_{i,s} - E_{i,s}.$$  

**Expenditure Shares**

Denote by $Y_{ij,s}$ the total expenditure of country $j$ on sector $s$ goods from country $i$. Total expenditure includes tariffs, therefore in order to calculate $Y_{ij,s}$ we take imports and multiply by tariffs. We do this at the sectoral level, namely $Y_{ij,s} = E_{ij,s} (1 + t_{ij,s})$. Note that $Y_{ii,s} = E_{ii,s}$. We then calculate expenditure shares as

$$\lambda_{ij,s} = \frac{Y_{ij,s}}{\sum_{i} Y_{ij,s}}; \tag{86}$$

where $\sum_{i} Y_{ij,s} = Y_{j,s}$ is total expenditure.

**Finished goods consumption shares**

To calculate final consumption share, $\alpha_{i,s}$ we take the total expenditure of sector $s$ goods, subtract the intermediate goods expenditure and divide by total final absorption. Namely

$$\alpha_{i,s} = \frac{Y_{i,s} - \sum_{s'=1}^{S} \gamma_{i,ss'} GO_{i,s'}}{w_{i} L_{i} + T_{i}},$$

where $w_{i} L_{i}$ is total value added and $T_{i}$ tariff revenue which we calculate as $T_{i} = \sum_{s=1}^{S} \sum_{j \neq i} t_{ji,s} E_{ji,s}$.

**Tariff aggregation from the good level**

An important task is to find a model consistent procedure to aggregate goods-level tariffs at a fine level to the correct sectoral-level equivalent at a coarser level.

We make the assumption that in country $j$ and sector $s$ there are $G_{j,s}$ goods indexed by $g$. Our goal is to solve for a sectoral tariff $t_{ji,s}$ such that the change in this sectoral tariff $(1 + t_{ji,s})$ is equivalent to the effect of the observed changes in tariffs at a goods level $1 + t_{ji,s}(g)$, for $g = 1, ..., G_{j,s}$.

We calculate $\lambda_{ij,s}(g)$, namely the expenditure share on $g$ goods as

$$\lambda_{ij,s}(g) = E_{ij,s}(g) (1 + t_{ij,s}(g)) / \sum_{i} Y_{ij,s}. \tag{87}$$

Note that the expenditure share from country $i$ on all $G_{i,s}$ goods from country $j$ has to equal to the total expenditure on sector $s$ goods from country $j$, therefore

$$\sum_{g=1}^{G_{j,s}} \lambda_{ji,s}(g) = \lambda_{ji,s}. \tag{89}$$

Then the trade balance condition (23) can be re-written by adding the summation over goods $g$ as

$$\sum_{s=1}^{S} \sum_{j \neq i} \sum_{g=1}^{G_{j,s}} \lambda_{ij,s}(g) \frac{1 + t_{ji,s}(g)}{1 + t_{ij,s}(g)} Y_{i,s} = \sum_{s=1}^{S} \sum_{j \neq i} \sum_{g=1}^{G_{i,s}} \lambda_{ij,s}(g) \frac{1 + t_{ij,s}(g)}{1 + t_{ij,s}(g)} Y_{j,s}. \tag{88}$$

In order for (88) to be equivalent to (23), it is apparent that the tariffs must satisfy

$$\frac{\lambda_{ji,s}}{1 + t_{ji,s}} = \sum_{g=1}^{G_{j,s}} \lambda_{ji,s}(g) \frac{1 + t_{ji,s}(g)}{1 + t_{ji,s}(g)}. \tag{89}$$

Using (88), (89) and some manipulation we obtain a tariff aggregation formula:
\[
(1 + t_{ji,s}) = \frac{\sum_{g=1}^{G_{j,s}} E_{ji,s}(g) (1 + t_{ji,s}(g))}{\sum_{g=1}^{G_{j,s}} E_{ji,s}(g)} \iff t_{ji,s} = \frac{\sum_{g=1}^{G_{j,s}} E_{ji,s}(g) t_{ji,s}(g)}{\sum_{g=1}^{G_{j,s}} E_{ji,s}(g)}
\]  

(90)

In other words, when aggregating over a finer set of goods \(g\) to a coarse sector level, the sectoral aggregate tariff factor \(1 + t_{ji,s}\) should be computed as a trade-weighted mean of the tariff factors across the various goods \(g\). The analogous condition must hold for computing \(1 + t'_{ji,s}\) in the new equilibrium, evaluating the shares \(\lambda'_{ji,s}(g)/\lambda_{ji,s}(g)\) in this new equilibrium. Clearly, if there is a uniform change in the goods-level tariffs \(1 + t_{ji,s}(g)\) then the new shares would equal their initial values \(\lambda_{ji,s}(g)/\lambda_{ji,s}\), and in that case it is obvious from the above that the change in \(1 + t_{ji,s}(g)\) would equal the change in \(1 + t_{ji,s}\), i.e., the change in the sectoral tariff just equals the uniform change in the goods-level tariffs.