

Broken $SU(3)$ antidecuplet for Θ^+ and $\Xi_{\frac{3}{2}}$

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Abstract

If the narrow exotic baryon resonances $\Theta^+(1540)$ and $\Xi_{\frac{3}{2}}(1862)$ are members of the $J^P = \frac{1}{2}^+$ antidecuplet with $N^*(1710)$, the octet-antidecuplet mixing is required not only by the mass spectrum but also by the decay pattern of $N^*(1710)$. This casts doubt on validity of the Θ^+ mass prediction by the chiral soliton model. While all pieces of the existing experimental information point to a small octet-decuplet mixing, the magnitude of mixing required by the mass spectrum is not consistent with the value needed to account for the hadronic decay rates. The discrepancy is not resolved even after the large experimental uncertainty is taken into consideration. We fail to find an alternative $SU(3)$ assignment even with different spin-parity assignment. When we extend the analysis to mixing with a higher $SU(3)$ multiplet, we find one experimentally testable scenario in the case of mixing with a 27-plet.

PACS number(s):11.30.Hv,13.30.Eg,12.39.Dc,12.39.Jh

I. INTRODUCTION

Two baryon states with exotic quantum numbers were recently reported by experimenters [1,2]. Existence of the isosinglet resonance $\Theta^+(1540)$ of $Y = +1$ was corroborated by several subsequent experiment [3]. The discovery of Θ^+ led to search of the exotic $\Xi_{\frac{3}{2}}$ resonance. The NA49 Collaboration at CERN [2] surveyed over a wide mass range of $\Xi\pi$ and reported a narrow resonance, $\Xi_{\frac{3}{2}}$ at mass 1862 MeV. The H1 Collaboration [4] reported discovery of an exotic charmed baryon. Despite numerous positive reports, it is fair to say that some theorists and experimentalists are still skeptical about even their existence. There are hints of inconsistency among the reported values of the mass and the width. A very recent high-resolution mass plot by the HERA-B Collaboration [5] showed no evidence for Θ^+ nor $\Xi_{\frac{3}{2}}$. From the theoretical standpoint these exotic resonances do not contradict the basic principles of quantum chromodynamics. Nonetheless they do challenge our long-standing beliefs and practice in hadron spectroscopy.

The search of Θ^+ was motivated by the flavor $SU(3)$ extension of the Skyrme model or the chiral soliton model [6,7]. In this model the baryon number is identified with the topological quantum number associated the hedgehog configuration interlocking spin and flavors. After quantum chromodynamics was introduced, the large N_c picture was used to argue for the classical soliton solution [7]. The original Skyrme model was then extended to three flavors by embedding the Skyrme solution into the $SU(2)$ subgroups [8]. The exotic baryons Θ^+ and $\Xi_{\frac{3}{2}}$ fit in the antidecuplet $\overline{\mathbf{10}}$ of $SU(3)$ [8–10]. Their spin-parity is $\frac{1}{2}^+$ since $\overline{\mathbf{10}}$ appears as the rotationally excited levels of the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons. The soliton model describes the baryons as made of an infinite number of quarks in the large N_c limit. Consequently the infinite Skyrme tower emerges for the baryons.

After the announcement of the discovery of Θ^+ , numerous studies have been made in the constituent quark picture [11–14]. In the constituent quark model the antidecuplet baryons can be realized when five quarks $qqqq\bar{q}$ or more form baryons. When the five quarks are bound all in relative s -wave, the resulting baryons would be in the $J^P = \frac{1}{2}^-$ states. This is a general feature and persists in some attempts [15] with the lattice calculation. A strong spin-orbit force must play a role for the lowest pentaquark states to have positive parity in the constituent model. Dynamical arguments were put forth to lower the $J^P = \frac{1}{2}^+$ pentaquarks [11,12]. While both the constituent pentaquark model and the chiral soliton model can accommodate the $\overline{\mathbf{10}}$ representation of $SU(3)$, they are very different models in nature. Experimentally, it is possible in principle to determine the parity of Θ^+ through the angular distribution in the production process [16].

The $SU(3)$ extension of the chiral soliton model predicts the $\overline{\mathbf{10}}$ baryons below the second excited baryon multiplet $\mathbf{35}$ of the Skyrme tower [8]. The exotic quantum numbers of Θ^+ are a good signature of the $\overline{\mathbf{10}}$ multiplet. The prediction of the Θ^+ mass in the chiral soliton model is based on the assumption that the $N^*(1710)$ resonance is the nonstrange member of $\overline{\mathbf{10}}$ [9,10]. While both Θ^+ and $\Xi_{\frac{3}{2}}$ fit nicely to $\overline{\mathbf{10}}$, the nearby $\frac{1}{2}^+$ octet baryon resonances inevitably mix with the antidecuplet through the symmetry breaking. The existing data already require mixing and contradict the assumption that $N^*(1710)$ is the pure $\overline{\mathbf{10}}$ partner of Θ^+ and $\Xi_{\frac{3}{2}}$.

In this paper we make a flavor $SU(3)$ analysis of the baryon masses and decay branchings without using the dynamical details of the constituent pentaquark model nor of the chiral

soliton model. Our results are therefore model independent and based mostly on the group theory. While the group-theory analysis has been presented for masses in very recent papers [17], we focus on the consistency of broken $SU(3)$ symmetry not only among the masses but also between the masses and the decay rates.

Though purely group-theoretical analyses are less model dependent, they have an obvious disadvantage to the dynamical analyses [18] that incorporate the inputs of the chiral soliton model. Only when a sufficient number of experimental inputs exist, can we draw definite conclusions or make interesting predictions from a group-theory analysis. In carrying out our analysis, we must control the number of independent free parameters of group theory. As for the higher-order symmetry breaking in the mass spectrum, we incorporate the second-order effects only where they are enhanced by a small mass difference between nearby states through representation mixing. In the decay rates, the dominant symmetry breaking is by far in the phase space corrections. Following conventional wisdom, we do not include symmetry breaking in the coupling constants.

Going through this standard group-theory analysis, we find difficulty in accommodating Θ^+ and $\Xi_{\frac{3}{2}}$ in the decuplet even after the $\mathbf{8}\text{-}\overline{\mathbf{10}}$ mixing is taken into account. In our analysis we encounter a large uncertainty in the masses and widths of the $SU(3)$ partners of Θ^+ and $\Xi_{\frac{3}{2}}$. The source of the uncertainty is mainly in the phase-shift analysis that exposed the higher baryon resonances mostly in the 1960's to 70's. However, even this large uncertainty in the masses and widths of N^* resonances fails to resolve the discrepancy we report.

Although there is no experimental evidence for the $\mathbf{27}$ -plet baryon resonances at present, we look for a possible resolution with the $\mathbf{27}\text{-}\overline{\mathbf{10}}$ mixing too. We assign the existing resonances, normally assigned to $\mathbf{8}$, to the nonexotic members of $\mathbf{27}$ and see if the difficulty in the $\mathbf{8}\text{-}\overline{\mathbf{10}}$ mixing can be resolved or not and what predictions can be made for future experiment. This attempt turns up with one possible resolution.

II. MASS SPECTRUM

A. The octet-antidecuplet mixing

The smallest multiplet that can accommodate both $\Theta^+(1540)$ and $\Xi_{\frac{3}{2}}$ is the antidecuplet $\overline{\mathbf{10}}$. In the simple-minded constituent pentaquark model, the states of $J^P = \frac{1}{2}^-$, namely all in s -wave, are more likely lower in mass. However, it is the chiral soliton model that triggered our interest in these exotic resonances. Furthermore, the original idea of Skyrme incorporated in the chiral soliton model, if it should apply to the real world, is revolutionary in understanding of the baryon number. Therefore we follow the prediction of the chiral soliton model for the quantum numbers of Θ^+ in this paper and consider the case of spin-parity equal to $\frac{1}{2}^+$ for Θ^+ and $\Xi_{\frac{3}{2}}$. The representation mixing is a second-order symmetry breaking effect. Only when two masses are close, can it be enhanced by a small mass difference to compete with or even dominate over the first-order breaking. In the chiral soliton model the mixing of $\overline{\mathbf{10}}$ with the ground-state octet was studied because of the mass spectrum of the model [18]. The mixing effect with the ground-state octet is not large enough to affect the antidecuplet masses substantially. Among the established $\frac{1}{2}^+$ resonances, there is the so-called Roper resonance at 1440 MeV, which is potentially more important because

of its proximity to $N^*(1710)$ in mass. We focus here on the Roper resonance $N^*(1440)$ as the mixing partner of $N^*(1710)$. Then the excited Σ resonances at 1660 MeV and at 1880 MeV are the candidates of the $Y = 0$ partners. Reviews of Particle Physics (RPP) [19] lists another N^* at 2100 MeV and Σ^* at 1770 MeV, but both are rated with “one star” (evidence weak). Furthermore $N^*(2100)$ is too far off from the mass region of our interest. Table I lists the $\frac{1}{2}^+$ resonances relevant to our analysis. These baryon resonances were discovered many years ago through the partial-wave analysis of meson-baryon scattering. Unfortunately, the values of mass and width spread substantially from one experiment to another because they are sensitive to details of the angular distributions and the methods of partial-wave analysis. Some recent analyses [20] in the baryon channels after the discovery of Θ^+ have revealed substantial uncertainties. The RPP makes its own estimate of the masses and widths from the reported results for each resonance and lists their best fits or likely ranges of values as “our estimates”. Without a better alternative, we shall adopt the values of “our estimate” as the experimental values keeping in mind that uncertainties are fairly large in many cases.

The assignment of Θ^+ and $\Xi_{\frac{3}{2}}$ to $\overline{\mathbf{10}}$ raises an immediate problem. The Gell-Mann-Okubo (GMO) mass formula,

$$M(\mathbf{n}, I, Y) = M_n + m_n Y + m'_n [I(I+1) - \frac{1}{4}Y^2] \quad (1)$$

requires the equal spacing rule between the adjacent Y members of $\overline{\mathbf{10}}$. Therefore, the mass of the pure antidecuplet $N^*(\overline{\mathbf{10}})$ should be at

$$\begin{aligned} M_{N^*(\overline{\mathbf{10}})} &= \frac{1}{3}(M_{\Xi_{\frac{3}{2}}} + 2M_{\Theta^+}), \\ &= 1647 \text{ MeV}. \end{aligned} \quad (2)$$

The GMO formula holds very well for the $\mathbf{8}$ and $\mathbf{10}$ baryons almost up to the intra-isospin splittings (*i.e.*, up to the electromagnetic and the u - d mass splitting). Therefore the prediction of Eq. (2) should hold with the accuracy of a few MeV. Since the masses of Θ^+ and $\Xi_{\frac{3}{2}}$ are sharply determined in experiment and their widths are within the experimental resolution, the number on the right-hand side of Eq. (2) should have little uncertainty. The $N^*(1710)$ resonance therefore does not fit well to the partner of Θ^+ and $\Xi_{\frac{3}{2}}$. Putting it in another way, if $N^*(1710)$ were a pure $\overline{\mathbf{10}}$, $\Xi_{\frac{3}{2}}(1862)$ would be at 2080 MeV. A simple and natural way to reconcile with this difficulty is to postulate that $N^*(1710)$ is not a pure $\overline{\mathbf{10}}$, but contains an octet component.

If the amount of $\mathbf{8}$ - $\overline{\mathbf{10}}$ mixing present in $N^*(1710)$ is too large, the chiral soliton model prediction on the Θ^+ mass made by Diakonov *et al.* [10] and by their predecessors [9] would lose its basis since it assumes that $N^*(1710)$ is the pure $\overline{\mathbf{10}}$ and does not include the $O(m_s^2)$ mass correction. On the other hand, one may take an optimistic viewpoint that the mass 1647 MeV is not so far off the lower end of the likely range (1680~1740 MeV) suggested for $N^*(1710)$ by RPP. It may be premature to jump onto the negative conclusion in view of the large experimental uncertainty. With this optimism or skepticism, let us proceed to make a more quantitative analysis of the mass spectrum incorporating the $\mathbf{8}$ - $\overline{\mathbf{10}}$ mixing.

In the presence of the $\mathbf{8}$ - $\overline{\mathbf{10}}$ mixing, the mass eigenstates of N^* and Σ^* are defined by

$$\begin{aligned} |N^*(1440)\rangle &= |\mathbf{8}\rangle \cos \theta_N - |\overline{\mathbf{10}}\rangle \sin \theta_N, \\ |N^*(1710)\rangle &= |\overline{\mathbf{10}}\rangle \cos \theta_N + |\mathbf{8}\rangle \sin \theta_N, \end{aligned} \quad (3)$$

and by the corresponding relations

$$\begin{aligned} |\Sigma^*(1660)\rangle &= |\mathbf{8}\rangle \cos \theta_\Sigma - |\overline{\mathbf{10}}\rangle \sin \theta_\Sigma, \\ |\Sigma^*(1880)\rangle &= |\overline{\mathbf{10}}\rangle \cos \theta_\Sigma + |\mathbf{8}\rangle \sin \theta_\Sigma. \end{aligned} \quad (4)$$

Let us denote the coefficients of the GMO formula for $\mathbf{8}$ and $\overline{\mathbf{10}}$ as

$$\begin{aligned} M(\overline{\mathbf{10}}; Y) &= M_{\overline{\mathbf{10}}} - aY, \\ M(\mathbf{8}; I, Y) &= M_8 - bY + c[I(I+1) - \frac{1}{4}Y^2] \end{aligned} \quad (5)$$

and introduce the $\mathbf{8}$ - $\overline{\mathbf{10}}$ mixing parameter by

$$\delta \equiv \langle N^*(\mathbf{8}) | N^*(\overline{\mathbf{10}}) \rangle = \langle \Sigma^*(\mathbf{8}) | \Sigma^*(\overline{\mathbf{10}}) \rangle. \quad (6)$$

Then the mixing angles θ_N and θ_Σ are related to these parameters by

$$\begin{aligned} \tan 2\theta_N &= \frac{2\delta}{M_{\overline{\mathbf{10}}} - M_8 - a + b - \frac{1}{2}c} \\ \tan 2\theta_\Sigma &= \frac{2\delta}{M_{\overline{\mathbf{10}}} - M_8 - 2c}, \end{aligned} \quad (7)$$

and the baryon masses are expressed as

$$\begin{aligned} \Xi_{\frac{3}{2}}(1862) &= M_{\overline{\mathbf{10}}} + a, \\ \Xi_{\frac{1}{2}}^* &= M_8 + b + \frac{1}{2}c, \\ \Sigma^*(1880) + \Sigma^*(1660) &= M_{\overline{\mathbf{10}}} + M_8 + 2c, \\ (\Sigma^*(1880) - \Sigma^*(1660)) \cos 2\theta_\Sigma &= M_{\overline{\mathbf{10}}} - M_8 - 2c, \\ \Lambda^*(1600) &= M_8, \\ N^*(1710) + N^*(1440) &= M_{\overline{\mathbf{10}}} + M_8 - a - b + \frac{1}{2}c, \\ ((N^*(1710) - N^*(1440)) \cos 2\theta_N) &= M_{\overline{\mathbf{10}}} - M_8 - a + b - \frac{1}{2}c, \\ \Theta^+(1540) &= M_{\overline{\mathbf{10}}} - 2a, \end{aligned} \quad (8)$$

where the baryons denote their masses. The masses of Θ^+ and $\Xi_{\frac{3}{2}}$ immediately determine $M_{\overline{\mathbf{10}}}$ and a accurately. M_8 is fixed by the mass of $\Lambda^*(1600)$ alone, albeit with a fairly large uncertainty (1600_{-50}^{+100} MeV). The remaining three parameters, b , c and δ , are determined by other four masses. There is one redundancy. When we fit b and c to $N^*(1710)$ and $N^*(1440)$, the central values of our fit are as follows:

$$\begin{aligned} M_{\overline{\mathbf{10}}} &\simeq 1755 \text{ MeV}, & M_8 &\simeq 1600 \text{ MeV} \\ a &\simeq 107 \text{ MeV}, & b &\simeq 144 \text{ MeV}, & c &\simeq 93 \text{ MeV}, \\ \delta &\simeq \pm 123 \text{ MeV}. \end{aligned} \quad (9)$$

Alternatively, we can determine δ using $\Sigma^*(1660)$ and $\Sigma^*(1880)$. The value thus determined ($\delta \simeq \pm 109$ MeV) is consistent with ± 123 MeV in Eq. (9) after the large uncertainty of the masses are taken into account. The value of $\delta \simeq \pm 123$ MeV leads to

$$\tan \theta_N \simeq \pm 0.59 \quad \tan \theta_\Sigma \simeq \pm 1.12. \quad (10)$$

We have chosen $|\tan \theta_N| < 1$ and $|\tan \theta_\Sigma| > 1$ since $N^*(1710)$ is dominantly $\overline{\mathbf{10}}$ and $\Sigma(\mathbf{8})$ (1785 MeV) is heavier than $\Sigma(\overline{\mathbf{10}})$ (1755 MeV) (cf. Eq. (4)). Within the uncertainties due to the spread of the mass values reported by different phase-shift analyses [19], the fit shows not only that $\tan \theta_N$ is nonzero, but also that the $\mathbf{8}\text{-}\overline{\mathbf{10}}$ mixing is substantial. We quote from Eq. (10)

$$\tan^2 \theta_N^{(m)} \simeq 0.35, \quad (11)$$

where the superscript of $\theta_N^{(m)}$ indicates the value determined by the mass spectrum. Let us compare this mixing with that of the ‘‘correlated-diquark’’ model by Jaffe and Wilczek [11]. Their $N^*(1440)$ is a linear combination of

$$|N^*(1440)\rangle = \sqrt{\frac{2}{3}}|\mathbf{8}\rangle + \sqrt{\frac{1}{3}}|\overline{\mathbf{10}}\rangle, \quad (12)$$

namely $\tan^2 \theta_N = 0.5$. This mixing is considerably larger than the value obtained from our analysis of the mass spectrum.

The set of parameter values in Eq. (9) leads us to something unusual. By substituting the values of b and c in the octet mass formula, we obtain

$$\begin{aligned} M_{\Xi_{\frac{1}{2}}^*} &= 1790 \text{ MeV} \\ M_{\Sigma^*(\mathbf{8})} &= 1785 \text{ MeV} \\ M_{\Lambda^*} &= 1600 \text{ MeV} \\ M_{N^*(\mathbf{8})} &= 1520 \text{ MeV}, \end{aligned} \quad (13)$$

and $M_{\Sigma^*(\overline{\mathbf{10}})}$ at 1755 MeV which is 30 MeV below $M_{\Sigma^*(\mathbf{8})}$. The large Λ^* - $\Sigma^*(\mathbf{8})$ mass splitting and the near degeneracy of $\Sigma^*(\mathbf{8})$ and $\Xi_{\frac{1}{2}}^*$, though allowed by group theory, would not look natural in the constituent quark model.

B. Adding a singlet-octet mixing

A comment is in order on the Λ^* states. There is another ‘‘three-star’’ Λ^* state of $\frac{1}{2}^+$ at 1810 MeV. If the antidecuplet is absent, $\Lambda^*(1810)$ would be assigned to the second excited $\frac{1}{2}^+$ octet with $N^*(1710)$, $\Sigma^*(1880)$, and $\Xi_{\frac{1}{2}}^*$ that is yet to be found. In the case that $\overline{\mathbf{10}}$ is present, the natural assignment of $\Lambda(1810)$ is an $SU(3)$ singlet by itself. Then a singlet-octet mixing occurs between two Λ^* states with a mixing angle,

$$\tan 2\theta_\Lambda = \frac{2\delta_\Lambda}{M_1 - M_8}. \quad (14)$$

The mass eigenvalues after the mixing are expressed as

$$\begin{aligned} \Lambda^*(1810) + \Lambda^*(1600) &= M_1 + M_8, \\ (\Lambda^*(1810) - \Lambda^*(1600)) \cos 2\theta_\Lambda &= M_1 - M_8. \end{aligned} \quad (15)$$

Irrespective of the value of θ_Λ , we have

$$1600 \text{ MeV} \leq M_8 \leq 1810 \text{ MeV}, \quad (16)$$

since two levels repel each other by mixing. This creates a room only to raise the value of M_8 keeping the $\Sigma^*(\mathbf{8})$ mass unchanged so that the octet mass spectrum is more in line with the constituent quark picture. However, the value of θ_N can be determined without knowledge of the Λ^* sector since

$$\begin{aligned} \tan 2\theta_N &= \frac{2\delta}{M_{N^*(1710)} + M_{N^*(1440)} - 2(M_{\overline{10}} - a)}, \\ \delta^2 &= (M_{N^*(1710)} + M_{N^*(1440)})(M_{\overline{10}} - a) + (M_{\overline{10}} - a)^2 - M_{N^*(1710)}M_{N^*(1440)}, \end{aligned} \quad (17)$$

where $M_{\overline{10}}$ and a are determined by the decuplet alone. For the same reason, θ_Σ is also not affected by the $\mathbf{1-8}$ mixing.

C. Experimental uncertainties

A difficult question is what uncertainties should be attached to the values that we have obtained by use of the central values or the most likely values for the masses. In contrast to the masses of the lower baryon resonances that are determined directly with the peaks in cross sections, the higher resonance masses are subject to the systematic uncertainties involved in the methods of partial-wave analysis. We expect little chance of improvement in the experimental uncertainties in the foreseeable future. We now explore how much our results would be affected by the dominant uncertainties.

In the 2×2 mass matrix of $N^*(\mathbf{8})$ - $N^*(\overline{\mathbf{10}})$, the diagonal entry of $N^*(\overline{\mathbf{10}})$ is accurately known from the Θ^+ and Ξ^* masses. Then we have two unknowns left in the mass matrix. The mixing angle is determined according to Eqs. (9) and (10) by

$$\cos 2\theta_N = \frac{2 \times 1647 \text{ MeV} - M_{N^*(1710)} - M_{N^*(1440)}}{M_{N^*(1710)} - M_{N^*(1440)}}, \quad (18)$$

where 1647 MeV is the $N^*(\overline{\mathbf{10}})$ mass. It is easy to see that the minimum mixing (the largest $\cos 2\theta_N$) occurs when $N^*(1710)$ is at its minimum value, while the maximum mixing occurs when $N^*(1710)$ is at its maximum value. Sweeping the value of $N^*(1440)$ over the likely range of 1430~1470 MeV [19], we find the allowed range of values for the mixing angle $|\theta_N|$,

$$0.15 \leq \tan^2 \theta_N^{(m)} \leq 0.53. \quad (19)$$

In terms of the mixing angle, Eq. (19) corresponds to $21^\circ < |\theta_N^{(m)}| < 36^\circ$. The minimum mixing $\theta_N^{(m)} = 21^\circ$ would shift the pure antidecuplet mass value 1647 MeV upward by 33 MeV to the lowest edge of the likely mass range of $N^*(1710)$. The large mixing angle of the Jaffe-Wilczek model ($\tan^2 \theta_N = 0.5$) is only marginally consistent with the upper edge of the large experimental uncertainties. The conclusion drawn from our mass spectrum analysis is summarized as follows: If the baryon resonance masses are within the ‘‘likely ranges’’ suggested by the RPP [19], we would have to question validity of the high-precision prediction of the Θ^+ mass by the chiral soliton model. It should be added before concluding this section that within the chiral soliton model a critical review [21] was presented for the precision and the robustness of the mass prediction. By now many chiral soliton advocates seem to admit an uncertainty larger than claimed in some of the earlier papers.

III. HADRONIC DECAY MODES

The two-body hadronic decay branching fractions of $N^*(1710)$ are listed by RRP [19] as

$$N^*(1710) \rightarrow \begin{cases} N\pi, & 10 \sim 20\% \\ N\eta, & 6.0 \pm 1.0\% \\ \Lambda K, & 5 \sim 25\% \\ \Delta\pi, & 15 \sim 40\% \\ N\rho, & 5 \sim 25\%. \end{cases} \quad (20)$$

We should notice here among others that the decay $N^*(1710) \rightarrow \Delta\pi$ occurs as strongly as $N^*(1710) \rightarrow N\pi$ in spite of the smaller phase space. This is a clear evidence for the $\mathbf{8}\text{-}\overline{\mathbf{10}}$ mixing. If $N^*(1710)$ were a pure $\overline{\mathbf{10}}$, the decay $N^*(1710) \rightarrow \Delta\pi$ would be $SU(3)$ -forbidden¹

$$\overline{\mathbf{10}} \not\rightarrow \mathbf{10} + \mathbf{8}. \quad (21)$$

An educated guess is that $B(N^*(\overline{\mathbf{10}}) \rightarrow \Delta\pi)$ would be less than a tenth of $B(N^*(\overline{\mathbf{10}}) \rightarrow N\pi)$. Only if there exists a nonnegligible mixing of $\mathbf{8}$, can $N^*(1710)$ decay substantially into $\Delta\pi$ through the octet component. When we compare the decay $N^*(1710) \rightarrow \Delta\pi$ with the decay $N^*(1440) \rightarrow \Delta\pi$ by separating the phase space factors, the reduced decay rate of $N^*(1710) \rightarrow \Delta\pi$ is considerably smaller than that of $N^*(1440) \rightarrow \Delta\pi$. This indicates that $N^*(1710)$ is dominantly $\overline{\mathbf{10}}$ while the $\mathbf{8}$ component definitely exists. We can determine the $\mathbf{8}\text{-}\overline{\mathbf{10}}$ mixing from the partial decay widths into $\Delta\pi$. Then we can compare it with the value required by the mass spectrum. We proceed as follows.

Four relevant $SU(3)$ -couplings are involved in the decays of Eq. (20). We denote them by

$$\begin{aligned} g_{8S}, g_{8A} & \text{ for } \mathbf{8} \rightarrow \mathbf{8} + \mathbf{8}, \\ g_{10} & \text{ for } \mathbf{8} \rightarrow \mathbf{10} + \mathbf{8}, \\ g_{\overline{10}} & \text{ for } \overline{\mathbf{10}} \rightarrow \mathbf{8} + \mathbf{8} \end{aligned} \quad (22)$$

in the obvious notation. According to a simple, commonly accepted practice, we separate the phase space factor and assume the $SU(3)$ symmetry holds for the dimensionless coupling constants. Specifically, we express the decay rates in the form of $\Gamma = g^2(p^{2l+1}/M_i^{2l})$ where g is the $SU(3)$ -symmetric coupling, M_i is the initial baryon mass, and $l = 1$ in the present case. We shall not include the $SU(3)$ breaking in these coupling constants since the phase space corrections are by far the dominant $SU(3)$ breaking effect. Once the $SU(3)$ breaking were included for the coupling constants in a general form, we would lose simple group-theory predictions among the decay rates. We have tabulated our $SU(3)$ -parametrization of the decay amplitudes in Table II and the relevant experimental numbers in Table III by defining the reduced width $\overline{\Gamma}$ with $\overline{\Gamma} = (M_i^2/p^3)\Gamma$.

¹Relevance of this selection rule was pointed out in [22] and its consequence particularly in the decay $\Xi_{\frac{3}{2}} \rightarrow \Xi(\mathbf{10})^*\pi$ was studied in [23].

While the experiment has so far given only the upper bounds on the Θ^+ decay width, Cahn and Trilling [24] were able to deduce it with a minimum of theoretical input by combining the Θ^+ production in K^+Xe with the charge-exchange KN scattering data at the continuum from the 1960's-70's. Since their argument is simple, model-independent and robust, we adopt the Cahn-Trilling value for Γ_{Θ^+} ;

$$\Gamma(\Theta^+ \rightarrow KN) = 0.9 \pm 0.3 \text{ MeV}. \quad (23)$$

This constrains the value of $g_{\overline{10}}$ tightly, reflecting the abnormal narrowness of the Θ^+ decay width:

$$g_{\overline{10}}^2 = 0.11 \pm 0.04. \quad (24)$$

The value of Eq. (23) predicts the $\Xi_{\frac{3}{2}} \rightarrow \Xi\pi$ decay width by $SU(3)$ symmetry: $\Gamma(\Xi_{\frac{3}{2}} \rightarrow \Xi\pi) < 1.9 \pm 0.6 \text{ MeV}$. The experiment [2] has set an upper bound on the $\Xi_{\frac{3}{2}}$ total width at less than 18 MeV.

Next we compare the $N\pi$ decay modes of $N^*(1710)$ and $N^*(1440)$. By summing two decay rates, we obtain from Tables II and III

$$\left(\frac{3\sqrt{5}}{10}g_{8S} + \frac{1}{2}g_{8A}\right)^2 + \frac{1}{4}(g_{\overline{10}})^2 = 7.8. \quad (25)$$

Comparing this relation with Eq. (24), we find that the octet couplings completely dominate over the abnormally small antidecuplet coupling. In the approximation of dropping the coupling $g_{\overline{10}}$ in Eq. (25), the mixing angle of N^* is simply determined according to Eq. (3) by

$$\tan^2 \theta_N \simeq \frac{\overline{\Gamma}(N^*(1710) \rightarrow N\pi)}{\overline{\Gamma}(N^*(1440) \rightarrow N\pi)}. \quad (26)$$

Substituting the experimental numbers in Eq. (26), we obtain

$$\tan^2 \theta_N^{(d)} \simeq 0.030, \quad (27)$$

where the superscript of $\theta_N^{(d)}$ indicates the value determined by the decays. The value of $\tan^2 \theta_N^{(d)}$ turns out small despite the sizable $N^*(1710) \rightarrow \Delta\pi$ branching fraction. The reason is that $N^*(1710)$ is much narrower in width than $N^*(1440)$ and that the phase space factor p^3 is much larger for $N^*(1710)$. The value of $\tan^2 \theta_N^{(d)}$ may look qualitatively similar to the value or the range of values for $\tan^2 \theta_N^{(m)}$ in Eqs. (11) or (19). However, when we compute the decay rate of $N^*(1710) \rightarrow N\pi$ from Eq. (26) with $\tan^2 \theta_N = 0.15$ (cf Eq. (19)), the branching fraction $B(N^*(1710) \rightarrow N\pi)$ would come out close to 100%. That is, even the minimum value $\tan^2 \theta_N = 0.15$ allowed by the mass spectrum is much too large for the branching fraction $B(N^*(1710) \rightarrow N\pi)$. From the experimental uncertainties quoted in RPP [19], we can put $\tan^2 \theta_N^{(d)}$ in the range of

$$0.008 < \tan^2 \theta^{(d)} < 0.078. \quad (28)$$

The resonance $N^*(1440)$, known as the Roper resonance, has been best studied since the 1960's among the higher baryon resonances. It is difficult to reconcile the mixing angle

$\tan^2 \theta_N^{(d)} < 0.08$ required by the decay with the value from the mass spectrum, $\tan^2 \theta_N^{(m)} > 0.15$.

Our conclusion from the decay widths is that the $\Delta\pi$ decay mode requires the presence of $\mathbf{8}-\overline{\mathbf{10}}$ mixing in $N^*(1440)$ and $N^*(1710)$. However, the range of the values for the mixing angle $\theta_N^{(d)}$ required by the decays is outside of the corresponding range required by the mass spectrum.

It should be noted here that very recently Praszalowicz proposed the $\overline{\mathbf{10}} - \mathbf{27}$ mixing [25] in order to improve the chiral soliton model predictions on the decay rates. Such a mixing would also generate the decay $N^*(1710) \rightarrow \Delta\pi$ through the mixing. We discuss this possibility below in section VI.

IV. RADIATIVE DECAYS

The $\overline{\mathbf{10}}$ representation has a unique feature in the electromagnetic property, as was already pointed out in literature [26]. The electromagnetic current is a U -spin singlet while the positively charged members of $\overline{\mathbf{10}}$ form a $U = \frac{3}{2}$ multiplet unlike the positively charged octet members that form a U -spin doublet. Consequently, the radiative transitions obey the $SU(3)$ selection rule,

$$\overline{\mathbf{10}}(Q = +1) \not\rightarrow \gamma + \mathbf{8}(Q = +1). \quad (29)$$

We have some pieces of experimental information on the helicity amplitudes of $N^*(1440)$ and $N^*(1710)$ into γN , which are listed in Table IV [19]. Since the $SU(3)$ relations hold for individual amplitudes, we use the information of helicity amplitudes for our $SU(3)$ analysis. From the $N^{*+} \rightarrow \gamma p$ decay amplitudes, we obtain the mixing angle by the relation,

$$\begin{aligned} \tan^2 \theta_N^{(r)} &= \left| \frac{A_{1/2}(N^*(1710) \rightarrow \gamma p)}{A_{1/2}(N^*(1440) \rightarrow \gamma p)} \right|^2, \\ &= 0.02_{-0.02}^{+0.21}. \end{aligned} \quad (30)$$

The mixing angle $\theta_N^{(r)}$ in Eq. (30) again favors the $\overline{\mathbf{10}}$ dominance for $N^*(1710)$. It does not conflict either $\theta_N^{(m)}$ or $\theta_N^{(d)}$. While the uncertainty is too large to say anything more quantitative, it does not accommodate the Jaffe-Wilczek model. No useful result can be extracted from the ratio of $A(N^* \rightarrow \gamma n)/A(N^* \rightarrow \gamma p)$ since the $N^* \rightarrow \gamma n$ amplitudes involve two more $SU(3)$ constants.

We can predict the radiative decay ratio for $\Sigma^{*+} \rightarrow \gamma \Sigma^+$ by the selection rule of Eq. (29) if we use the mixing angle θ_Σ obtained in Eq. (9). After the phase space corrections. we obtain with the value of θ_Σ from Eq. (10)

$$\begin{aligned} \frac{\overline{\Gamma}(\Sigma^{*+}(1880) \rightarrow \gamma \Sigma^+)}{\overline{\Gamma}(\Sigma^{*+}(1660) \rightarrow \gamma \Sigma^+)} &= \tan^2 \theta_\Sigma \times \text{phase space} \\ &\simeq 2.5. \end{aligned} \quad (31)$$

Unfortunately the prospects for experimental test of this prediction are not very promising.

V. PROSPECTS OF OCTET-ANTIDECUPLET MIXING

It is not very likely that a phase-shift analysis will uncover another $\frac{1}{2}^+$ resonance of $Y = +1$ in the mass region of 1650 to 1700 MeV in the near future. Given the current data on the mass spectrum and the decay pattern, we have ruled out zero $\mathbf{8}\text{-}\overline{\mathbf{10}}$ mixing solutions. Since the presence of the Roper resonance $N^*(1440)$ has been well established, the $\mathbf{8}\text{-}\overline{\mathbf{10}}$ mixing between $N^*(1710)$ and the $N^*(1440)$ is the most natural solution to the problem. Experiment is less certain for the Σ^* resonances. But the Σ^* masses at 1660 MeV and 1880 MeV also show clear $\mathbf{8}\text{-}\overline{\mathbf{10}}$ mixing. The difficulty is that the $\mathbf{8}\text{-}\overline{\mathbf{10}}$ mixing determined from the N^* masses is not consistent with the value of the mixing determined from the N^* decay modes even after the large experimental uncertainties are taken into account.

Is it possible that $\Xi_{\frac{3}{2}}$ and Θ^+ belong to different antidecuplets? Let us examine it on a purely phenomenological basis even though it does not fit in the chiral soliton model nor expected in any other theoretical model. When we inspect the mass plots of the NA49 experiment [2], a less prominent peak can be seen just below 2 GeV in the $\Xi\pi$ and $\overline{\Xi}\pi$ plots. If Θ^+ and $N^*(1710)$ belong to a single unmixed antidecuplet, their $Y = -1$ partner should be found above 2 GeV ($\simeq 2080$ MeV). Therefore a $\Xi_{\frac{3}{2}}$ below 2 MeV cannot be their partner. By deducing the antidecuplet mass splitting in Y from Θ^+ and $N^*(1710)$, we would expect the $Y = +2$ partner of $\Xi_{\frac{3}{2}}$ at mass below 2 MeV should be below the KN threshold, that is, a stable baryon of positive strangeness. The two-decuplet scenario is therefore ruled out provided that the mass splitting in Y of the second $\overline{\mathbf{10}}$ is roughly of the same magnitude as $m_{N^*(1710)} - m_{\Theta^+}$. If this estimate should be grossly wrong, one should look for another Θ^+ .

The argument presented above in fact holds valid even in the case that Θ^+ and $\Xi_{\frac{3}{2}}(1862)$ have different spin-parities, say $J^P = \frac{1}{2}^+$ and $\frac{1}{2}^-$. The same line of argument rules out most other possibilities. How about the case that Θ^+ and $\Xi_{\frac{3}{2}}$ both carry $\frac{1}{2}^-$? The spin-parity of $\frac{1}{2}^-$ is what one would naively expect from the constituent quark model when the ground-state pentaquarks are all in relative s -wave. There is a well-established $\frac{1}{2}^-$ resonance $N^*(1650)$. As far as the GMO mass formula is concerned, $N^*(1650)$ fits nicely to a pure $\overline{\mathbf{10}}$. Recall that the pure $N(\overline{\mathbf{10}})$ mass should be at 1647 MeV according to Eq. (2). However, the decay pattern is in variance with the antidecuplet assignment of $N^*(1650)$: The decay branching into $N\eta$ is too small ($0.09_{-0.06}^{+0.05}$) as compared with the $SU(3)$ prediction (0.6). The $SU(3)$ -forbidden decay mode $N^*(1650) \rightarrow \Delta\pi$ has been observed with a branching fraction of $1 \sim 7\%$. Another potentially serious problem of the $\frac{1}{2}^-$ baryons is that they decay into $\frac{1}{2}^+0^-$ in s -wave without the partial-wave phase space suppression of $l = 1$. The narrow decay widths of Θ^+ and $\Xi_{\frac{3}{2}}$ would become even more difficult to understand.

Within the assignment of $\frac{1}{2}^+$, the only way out of the difficulty is to postulate that for some unknown reason the second order $SU(3)$ violations of $O(m_s^2)$ are unexpectedly larger than what we have seen in the lower hadron states. If this should be the case, corrections to the GMO formulae would be large and the flavor $SU(3)$ prediction on the decay branching fractions would be subject to not only the mixing effect but also the symmetry breaking correction to the coupling constants themselves. The phase space correction incorporates a large $SU(3)$ violation effect in a specific manner. However, altering the phase space correction does not change our qualitative conclusion.

VI. MIXING WITH HIGHER REPRESENTATIONS

There is no experimental evidence that calls for a 27-plet or higher representation of SU(3) for baryons. Nonetheless, we should explore for such possibilities as well since the higher representations naturally appear in the chiral soliton models.

Let us summarize the difficulty that we have encountered above. The $N^*(1710) \rightarrow \Delta\pi$ decay definitely requires representation mixing for $\overline{\mathbf{10}}$. Since the $\overline{\mathbf{10}}$ widths are very narrow, however, only a tiny amount of mixing is needed to account for the $\Delta\pi$ decay. On the other hand the mass spectrum deviates substantially from the equal-spacing rule of GMO for $\overline{\mathbf{10}}$. The magnitude of the $\mathbf{8}\text{-}\overline{\mathbf{10}}$ mixing needed to fix this deviation is much larger than what the decay pattern calls for. Can we resolve this difficulty by mixing $\overline{\mathbf{10}}$ with $\mathbf{27}$? In order to make our argument concrete and quantitative, we assign the existing higher $\frac{1}{2}^+$ resonances of $Y = 1$ and $Y = 0$ to the nonexotic members of a 27-plet and see whether such assignment can resolve the inconsistency between the mass spectrum and the decay pattern. With the abnormal narrowness of the $\overline{\mathbf{10}}$ decay width leading to the small mixing, we ask whether the mass spectrum can be consistent with an equally small mixing in the case of $\mathbf{27}\text{-}\overline{\mathbf{10}}$ mixing.

The $\mathbf{27}$ -plet masses are parametrized with the same GMO formula as Eq. (1) but with different values for the parameter M_{27} , m_{27} , and m'_{27} . The $\mathbf{27}\text{-}\overline{\mathbf{10}}$ mixing strength depends on (Y, I) unlike the $\mathbf{8}\text{-}\overline{\mathbf{10}}$ mixing. We find instead of Eq. (7)

$$\delta_{27} \equiv \langle N^*(\mathbf{27}) | N^*(\overline{\mathbf{10}}) \rangle = \sqrt{\frac{3}{8}} \langle \Sigma^*(\mathbf{27}, 1) | \Sigma^*(\overline{\mathbf{10}}) \rangle = \sqrt{\frac{1}{5}} \langle \Xi^*(\mathbf{27}, \frac{3}{2}) | \Xi^*(\overline{\mathbf{10}}) \rangle. \quad (32)$$

For a mixing as tiny as $\tan^2 \theta^{(d)} \simeq 0.03$ as suggested by the decay widths, however, we may ignore the mass shifts due to mixing in a good approximation. So we proceed with this approximation. Then Θ^+ , $N^*(1710)$, and $\Sigma^*(1880)$ fit nicely to the equal spacing rule with $\Delta m = 170$ MeV. The mass of $\Xi_{\overline{\mathbf{10}}, \frac{3}{2}}^*$ is predicted as

$$M_{\Xi^*(\overline{\mathbf{10}})} = 2050 \text{ MeV}. \quad (33)$$

This state is certainly not the $\Xi^*(1862)$ state discovered by NA49 [2]. Then, does $\Xi^*(1862)$ fit in a $\mathbf{27}$? Since three states, $N^*(1440)$, $\Lambda(1600)$, and $\Sigma^*(1660)$ are assigned to $\mathbf{27}$, the values of all three parameters in the GMO formula are fixed. The mass of $\Xi^*(1862)$ is given with the GMO formula by

$$\begin{aligned} M(\Xi_{27, \frac{3}{2}}^*) &= 2M_{\Sigma^*} - M_{N^*} \\ &= 1880 \text{ MeV}. \end{aligned} \quad (34)$$

The predicted value of 1880 MeV is close enough to $\Xi^*(1862)$, considering the uncertainty of M_{N^*} and M_{Σ^*} . We now have an interesting alternative of assigning $\Xi^*(1862)$ to the $I = \frac{3}{2}$ member of $\mathbf{27}$ instead of $\overline{\mathbf{10}}$. The mass of $\Xi_{27, \frac{1}{2}}^*$ is the same as in Eq. (13),

$$M_{\Xi_{27, \frac{1}{2}}^*} = 1790 \text{ MeV}, \quad (35)$$

provided that it does not mix much with $\mathbf{8}$ or $\mathbf{10}$. The state $\Sigma_{27, 2}$ of $I = 2$ is exotic and does not mix even with $\overline{\mathbf{10}}$ so that its mass prediction

$$M_{\Sigma_{27,2}} = 1720 \text{ MeV} \quad (36)$$

is robust. The $(I = \frac{3}{2}, Y = 1)$ member of **27** should be at

$$M_{\Delta(27)} = 1530 \text{ MeV}. \quad (37)$$

Experimentally, the only established Δ resonance with $\frac{1}{2}^+$ is at 1910 MeV, which is too far away. Although a Δ resonance of $\frac{1}{2}^+$ is listed at 1750 MeV in RPP [19], this mass is still too far away and evidence is very weak for its existence (“a one-star” resonance). The viability of the 27-plet scenario depends on whether $\Xi(1862)$ decays into $\Xi^*(1530)\pi$ without suppression and on whether a Δ resonance of $J^P = \frac{1}{2}^+$ will be uncovered in the neighborhood of 1530 MeV in future. In addition, a $\Sigma^\pm\pi^\pm$ resonance should be searched for around 1720 MeV. The ratio of the reduced decay rates $\bar{\Gamma}(\Xi^*(1862) \rightarrow \Xi\pi)$ and $\bar{\Gamma}(\Xi^*(1862) \rightarrow \Sigma\bar{K})$ is unity for $\Xi^*(1862)$ of **27**, which happens to be the same as for the pure $\bar{\mathbf{10}}$. However, interference between **27** and $\bar{\mathbf{10}}$ can cause departure from unity. This ratio will therefore provide an independent test of **27**- $\bar{\mathbf{10}}$ mixing for $\Xi^*(1862)$.

Finally a remark is in order on the mixing with $\bar{\mathbf{35}}$. A state of $\bar{\mathbf{35}}$ cannot decay into $\mathbf{8} + \mathbf{8}$ nor $\mathbf{10} + \mathbf{8}$. Therefore this mixing would not explain the decay mode $N^*(1710) \rightarrow \Delta\pi$ for $N^*(1710)$ of $\bar{\mathbf{10}}$. If the known $\frac{1}{2}^+$ resonances are assigned to the nonexotic members of $\bar{\mathbf{35}}$, then the $Y = 3$ (strangeness 2) member of $\bar{\mathbf{35}}$ would have to be approximately at 1000 MeV, which is totally unacceptable. In fact, when any one of those higher $\frac{1}{2}^+$ resonances is an nonexotic member of $\bar{\mathbf{35}}$, the $Y = 3$ member would be most likely too light and stable against weak and radiative decays since its mass would be far below the decay threshold 1940 MeV (KKN). Making both the $Y = 3$ and the $Y = -2$ members heavier than the $|Y| \leq 1$ members in $\bar{\mathbf{35}}$ is against any dynamical picture of quarks. However, one farfetched but interesting possibility exists about $\bar{\mathbf{35}}$: If we assign Θ^+ and $\Xi^*(1862)$ to $\bar{\mathbf{35}}$, leaving $N^*(1710)$ and $\Sigma^*(1880)$ out, the abnormal narrow widths of Θ^+ and $\Xi^*(1862)$ could be explained by the $SU(3)$ selection rule of $\bar{\mathbf{35}} \not\rightarrow \mathbf{8} + \mathbf{8}$ and $\not\rightarrow \mathbf{10} + \mathbf{8}$. The nonresonant three-body decay into $\Xi\pi\pi$ would be the leading $SU(3)$ -allowed decay for $\Xi^*(1862)$. Their nonexotic partners N^* and Σ^* are predicted at mass 1647 MeV and 1755 MeV, respectively, by the equal-spacing rule. Despite the interesting features, we would have to resolve the problem of the “too light $Y = 3$ state” before we consider $\bar{\mathbf{35}}$ seriously. Without more experimental input, it is not fruitful to explore any further along this direction.

Our analysis uses as input the existing higher baryon resonances that have been suggested by the phase shift analysis. When we remove this constraint, our analysis quickly loses its effectiveness. We need more experimental input before extending a scope of analysis.

VII. SUMMARY

Not only do the masses of Θ^+ , $N^*(1710)$, and $\Xi^*(1862)$ fail to fit to a pure antidecuplet, but also is the decay mode $N^*(1710) \rightarrow \Delta\pi$ in conflict with the pure antidecuplet assignment of $N^*(1710)$. Although the $\mathbf{8}$ - $\bar{\mathbf{10}}$ mixing can explain the failure of the equal spacing of the masses, one loses the basis of the prediction [10] of the Θ^+ mass from the $N^*(1710)$ mass. Furthermore, the mixing required for the decay $N^*(1710) \rightarrow \Delta\pi$ is not compatible with the mixing deduced from the masses. The only viable alternative is to assign Θ^+ to $\bar{\mathbf{10}}$ and

$\Xi^*(1862)$ to **27**. In addition to the antidecuplet $\Xi_{\frac{3}{2}}^*$ predicted at 2050 MeV, existence of $\Delta(1530)$ of $\frac{1}{2}^+$ and $\Sigma(1720)$ of $I = 2$, and presence of $\Xi^*(1862) \rightarrow \Xi^*(1530)\pi$ will test this scenario.

ACKNOWLEDGMENTS

This work was supported in part by the Director, Office of Science, Office of High Energy and Nuclear Physics, Division of High Energy Physics, of the U.S. Department of Energy under contract DE-FG03-94ER40833 and DE-AC03-76SF00098, and in part by the National Science Foundation under grant PHY-0098840.

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TABLES

TABLE I. The spin $\frac{1}{2}^+$ resonances. All resonance states carry the rating of four or three stars by the Review of Particle Physics except for $\Sigma^*(1880)$ with two stars.

Baryons	I	Y	Mass (MeV)
Ξ^*	$\frac{3}{2}$	-1	1862
Σ^*	1	0	1880
Σ^*			1660
Λ^*	0	0	1600
N^*	$\frac{1}{2}$	+1	1440
N^*			1710
Θ^+	0	+2	1540

TABLE II. The isoscalar factors for $SU(3)$ parametrization of the decay amplitudes. The commonly defined F/D ratio is equal to $\sqrt{5}g_{8A}/3g_{8S}$.

Decay amplitude	$SU(3)$ isoscalar factor
$\Xi^* \rightarrow \Xi\pi$	$-\frac{1}{\sqrt{2}}g_{10}$
$\Sigma^*(\mathbf{8}) \rightarrow N\bar{K}$	$\frac{\sqrt{30}}{10}g_{8S} + \frac{\sqrt{6}}{6}g_{8A}$
$\Lambda^* \rightarrow \Sigma\pi$	$-\frac{\sqrt{15}}{5}g_{8S}$
$\Lambda^* \rightarrow N\bar{K}$	$\frac{\sqrt{10}}{10}g_{8S} + \frac{\sqrt{2}}{2}g_{8A}$
$N^*(\mathbf{10}) \rightarrow N\pi$	$-\frac{1}{2}g_{10}$
$N^*(\mathbf{8}) \rightarrow \Delta\pi$	$-\frac{2\sqrt{5}}{5}g_{10}$
$N^*(\mathbf{8}) \rightarrow N\pi$	$\frac{3\sqrt{5}}{10}g_{8S} + \frac{1}{2}g_{8A}$
$N^*(\mathbf{8}) \rightarrow N\eta$	$-\frac{\sqrt{5}}{10}g_{8S} + \frac{1}{2}g_{8F}$
$\Theta^+ \rightarrow KN$	$-g_{10}$

TABLE III. The decay modes and the reduced decay width $\bar{\Gamma} = \Gamma M_{B^*}^2 / p^3$ for $B^* \rightarrow MB$. The range of the $\Sigma^*(1880)$ decay width is our estimate since RPP does not give one.

Baryons	Γ_{tot} (MeV)	Modes	Branching	$p^3/M_{B^*}^2$ (MeV)	$\bar{\Gamma}$
$\Xi^*(1862)$	< 18	$\rightarrow \Xi\pi$?	25.9	< 0.7
$\Sigma^*(1880)$	(80 ~ 200)				
$\Sigma^*(1660)$	100 (40~200)	$\rightarrow N\bar{K}$	0.2 ± 0.1	24.1	8.3 ± 4.1
$\Lambda^*(1600)$	150 (50~250)	$\rightarrow N\bar{K}$	0.23 ± 0.08	15.8	2.2 ± 0.8
		$\rightarrow \Sigma\pi$	0.35 ± 0.25	14.8	3.6 ± 2.5
$N^*(1710)$	100 (50~250)	$\rightarrow N\pi$	0.15 ± 0.05	69.2	0.22 ± 0.07
		$\rightarrow N\eta$	0.06 ± 0.01	23.6	2.5 ± 0.4
		$\rightarrow \Lambda K$	0.15 ± 0.10	6.3	2.4 ± 1.6
		$\rightarrow \Delta\pi$	0.28 ± 0.013	20.8	1.4 ± 0.7
$N^*(1440)$	350 (250~350)	$\rightarrow N\pi$	0.65 ± 0.05	30.2	7.5 ± 0.6
		$\rightarrow \Delta\pi$	0.25 ± 0.05	1.4	63 ± 13
Θ^+	0.9 ± 0.3 (< 7)	$\rightarrow NK$	1	8.2	0.11 ± 0.04

TABLE IV. The radiative decay amplitudes of $N^* \rightarrow N\gamma$.

Baryon	Decay mode	Amplitude ($\text{GeV}^{-1/2}$)
$N^*(1710)$	$\rightarrow p\gamma$	$+0.009\pm 0.022$
	$\rightarrow n\gamma$	-0.002 ± 0.014
$N^*(1440)$	$\rightarrow p\gamma$	-0.065 ± 0.004
	$\rightarrow n\gamma$	$+0.040\pm 0.010$