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UNIVERSITY OF CALIFORNIA SAN DIEGO

Highly Integrated Hybrid DC-DC Converters for High Performance Power Delivery Systems: Design, Analysis and Implementation

A dissertation submitted in partial satisfaction of the requirements for the degree Doctor of Philosophy

in

Electrical Engineering (Electronic Circuits and Systems)

by

Tianshi Xie

Committee in charge:

Professor Hanh-Phuc Le, Chair Professor Shengqiang Cai Professor Patrick Mercier Professor Chris Mi Professor Tse Nga Ng

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University of California San Diego

2023

DEDICATION

To my beloved family: my wife Jingxiang Tian, my daughter Ada Xie, my father Jiadong Xie and mother Yanling Liu.

In memory of my grandfather Zhiyuan Xie, a pioneer in Nuclear Medicine in China

Dissertation Approval Page	iii
Dedication	iv
Table of Contents	v
List of Figures	vii
List of Tables	xi
Acknowledgements	xii
Vita	xiv
Abstract of the Dissertation	xv
Chapter 1 Demand for Miniaturized and Efficient Power Delivery Network	1
Chapter 2Overview of the Converters with Reduced Magnetic Devices2.1High frequency Switched-inductor Converter2.2Switched-capacitor Converter2.3Hybrid Converter2.4Resonant Converter	6 6 8 10 13
Chapter 3Multi-phase Dual-inductor Hybrid Converter3.1Operation of MP-DIH Converter3.2Experimental Results3.3Summary3.4Acknowledgements	15 15 21 24 25
Chapter 4 Soft-switching Assisted by Additional Circuitry 4.1 Topology and Operation of the ZVS-3LB Converter 4.2 Design Consideration of the ZVS-3LB Converter 4.2.1 Auxiliary Circuit Conduction Loss 4.2.2 Auxiliary Circuit Switching Loss 4.2.3 Auxiliary Inductor Loss 4.3 System Design 4.4 Hardware and Experimental Results	26 27 31 31 32 33 35 39
 4.5 Limitation of the Prototype 4.6 Summary 4.7 Acknowledgements 	41 43 43
Chapter 5 ITSAB Converter	45 45

TABLE OF CONTENTS

	5.1.1 Soft-switching	49
5.2	Loss Analysis of ITSAB Converter	51
	5.2.1 Conduction Loss	52
	5.2.2 Switching Loss	56
	5.2.3 Inductor Loss	57
5.3	Optimization Methodology	57
	5.3.1 Minimum RMS Current	59
	5.3.2 Optimization Process	61
5.4	Implementation Details	63
	5.4.1 Control Signal Generation	66
	5.4.2 Gate Drivers & Supply Voltage Generation	67
	5.4.3 Control Loop Design	71
5.5	Experimental Verification	72
5.6	Summary	83
5.7	Acknowledgements	84
Chapter 6.1 6.2 6.3 6.4 Chapter 7.1 7.2	 6 ITSAB Converter for Heavy-Load Applications System Design of High-Power ITSAB Converter Circuit Design of High-Power ITSAB Converter 6.2.1 Switch Partitioning 6.2.2 Fine-tuned Deadtime Generator Experiment Results Summary 7 Conclusion and Future Work Conclusion Future Work 7.2.1 Bi-directional Operation 7.2.2 Constant Current (CC) Mode for Battery charger 	85 85 90 91 93 97 98 98 98 99 99
Appendi	ix A FPGA Embedded System Design for ZVS-3LB Converter	102
Appendi	ix B Automatic Parameter Sweeping Source Code in Matlab for ZVS-3LB Con- verter	108
Appendi	ix C ITSAB Converter Optimization Source Code in Matlab	117
Bibliogr	aphy	123

LIST OF FIGURES

Figure 1.1.	Moore's Law [1]	2	
Figure 1.2.	Collected data showing trend in microprocessor technology [2]		
Figure 1.3.	Trend in power efficiency of top performing super-computers [3]		
Figure 1.4.	iPhone 13 Pro (2021) main board, the highlighted parts are power manage- ment circuits and obviously a lot of inductors occupy the area		
Figure 1.5.	A survey of inductance vs. volume for Coilcraft XEL series inductors	5	
Figure 2.1.	The concept of variable switching frequency that improves light load efficiency		
Figure 2.2.	BCM operation that enables soft-switching	8	
Figure 2.3.	Switched-capacitor converter model	9	
Figure 2.4.	Charge flow analysis of 2:1 switched-capacitor converter	9	
Figure 2.5.	Output resistor R_{OUT} of a SC converter	10	
Figure 2.6.	Various type of hybrid converter	11	
Figure 2.7.	3-level buck converter and its 1)inductive operation (top), 2)resonant oper- ation (bottom)	14	
Figure 3.1.	Topology of the Multi-Phase Dual-Inductor Hybrid (MP-DIC) converter	16	
Figure 3.2.	Operating states of the proposed MP-DIH converter	17	
Figure 3.3.	Operational waveforms of the proposed MP-DIH converter	18	
Figure 3.4.	Prototype Board	21	
Figure 3.5.	Operational waveforms of prototype at 48V-1.8V/4A	22	
Figure 3.6.	Flying capacitor voltage with no hard-charging.	22	
Figure 3.7.	Measured efficiency for different input voltages at $V_{out} = 1.8V$ regulated.	23	
Figure 3.8.	Measured efficiency for different output voltages at $V_{in} = 48V$ regulated	23	
Figure 3.9.	Loss breakdown at 1.8V/4A output	24	

Figure 4.1.	. The proposed Zero-Voltage-Switching 3 Level Buck (ZVS-3LB) converter schematic	
Figure 4.2.	. Operation principle of the proposed ZVS-3LB converter	
Figure 4.3.	Key operation states of the proposed ZVS-3LB converter during the time interval: (a) before t_2 , (b) $t_2 \sim t_3$, (c) $t_3 \sim t_4$, (d) $t_4 \sim t_6$, (e) $t_6 \sim t_7$, (f) $t_7 \sim t_9$, (g) $t_9 \sim t_{10}$	
Figure 4.4.	Normalized auxiliary circuit conduction loss vs. L_{aux} at different: (a) output inductor current I_L , (b) required charge Q_{req} for soft-switching	33
Figure 4.5.	Convert pulsed current into continuous sawtooth current for convenient core loss estimation	34
Figure 4.6.	An example of closed-loop design	36
Figure 4.7.	The testbench for ZVS-3LB converter demonstration	38
Figure 4.8.	Prototype of the proposed ZVS-3LB converter: (a) front side, (b) back side.	38
Figure 4.9.	Measured waveforms @15 V/4 V, 1 A, 1 MHz condition (a) without ZVS, (b) with ZVS because of auxiliary circuit.	40
Figure 4.10.	(a) loss breakdown at 20 V/5 V; and measured efficiencies of the proposed ZVS-3LB converter and their comparison with the conventional 3LB converter at (b) 20 V/5 V , (c) 15 V/4 V	42
Figure 4.11.	The efficiency comparison of ZVS-3LB converter with (solid lines) 1.5μ H main inductor and 100nH auxiliary inductor, and the one (dash lines) with 500nH main inductor and 10nH auxiliary inductor	43
Figure 5.1.	Schematic of the proposed integrated transformerless stacked active bridge (ITSAB) converter.	46
Figure 5.2.	Power stage operation of the ITSAB converter.	47
Figure 5.3.	Operating waveforms in the ITSAB converter.	48
Figure 5.4.	Zero-voltage-switching (ZVS) of Q3 and Q4 between (a) state 4 and 1, (b) state 2 and 3	50
Figure 5.5.	Definitions of parameters f_o , R_o and R_s	51
Figure 5.6.	Calculated I_{L1} , I_{L3} waveforms for 12 V-to-3 V conversion, 2 A load, $f_s = 3.8 \text{ MHz}$, with $R_s = 55 \text{ m}\Omega$.	52

Figure 5.7.	Loss model verification: (a) efficiency comparison; Loss breakdown at (b) 0.5 A load, (c) 2 A load	
Figure 5.8.	Illustration of how inductor RMS current is minimized using $k = f_s/f_o$ and $Q = R_o/R_s$ as optimization parameters	58
Figure 5.9.	Optimum k as a function of Q for $R_s = 50 \text{ m}\Omega$ and $I_o = 2 \text{ A}$	59
Figure 5.10.	Block diagram of the prototype ITSAB converter realized on a 130 nm BCD die flip-chipped on a package substrate together with power capacitors and two 10 nH inductors.	64
Figure 5.11.	Phase-shifted control signal generation, including block diagrams of (a) Ramp Generator (RG), and (b) Phase Shift Modulator (PSM); (c) timing diagram: generation of ϕ , ϕ_S	65
Figure 5.12.	Block diagrams of (a) subtraction LR to generate V_{SS8} and V_{SSL7} , (b) summation LR to generate $V_{DD5}(V_{DDH6})$, (c) gate driver for Q6, and (d) level shifter.	68
Figure 5.13.	Magnitude and phase responses of the loop gain obtained under the follow- ing conditions: $V_{in} = 9.6 \text{ V}$, $V_o = 2.38 \text{ V}$, with $\sim 74 \text{ nH}$ parasitic inductors of the wire loops used to measure inductor currents, as shown in Fig. 5.15.	70
Figure 5.14.	Top and side views of the ITSAB converter prototype, including (a) $1.7 \text{ mm} \times 1.9 \text{ mm}$ die in a 130 nm BCD process, and $6.5 \text{ mm} \times 6.5 \text{ mm}$ package substrate with flip-chipped die, power capacitors and $2 \times 10 \text{ nH}$ (b) IPD inductors or (c) discrete (Coilcraft 0807SQ-10N) inductors	73
Figure 5.15.	Current-measurement test setup with wire loops having (\sim 74 nH parasitic inductances serving as the power-stage inductors	74
Figure 5.16.	Measured steady-state converter waveforms, with 74 nH inductors due to the current measurement setup shown in Fig. 5.15. The converter operates at $V_{in} = 9.6$ V, $V_o = 2.38$ V, $I_o = 1$ A	75
Figure 5.17.	Step-load transient responses of the experimental prototype operating at $f_s = 3$ MHz with wire loops as inductors. The output voltage is closed-loop regulated at $V_o = 2.38$ V from $V_{in} = 9.6$ V.	75
Figure 5.18.	Line transient responses of the experimental prototype operating at $f_s = 3.8 \text{ MHz}$ with 10 nH discrete inductors. The output is closed-loop regulated at $V_o = 2.5 \text{ V}$.	76

Figure 5.19.	Step-load transient responses of the experimental prototype with 10 nH discrete inductors operating at $f_s = 3.8$ MHz. The output is closed-loop regulated at $V_o = 2.38$ V from $V_{in} = 9.6$ V.		
Figure 5.20.	V_{sw2} and I_{L1} of two converters (m and s) with 180° phase shift @8.2 V/2 V, 3.8 MHz, 1 A (0.5 A each), open-loop		
Figure 5.21.	Efficiency vs. load current I_o measured at (a) $V_{in} = 12$ V, $f_s = 3.8$ MHz, various V_o , (b) $V_{in} = 12$ V, $V_o = 2.93$ V, various f_s , and (c) $V_{in} = 9.6$ V, $V_o = 2.33$ V, various f_s .	79	
Figure 5.22.	Efficiency vs. V_o at 1 A load, 3.8 MHz f_s	80	
Figure 5.23.	Thermal image of the prototype with IPD inductors taken at full load (1.5 A) .	81	
Figure 6.1.	3D floorplan of high power ITSAB converter package	87	
Figure 6.2.	(a) efficiency with different inductor ESRs; (b) Loss breakdown at 8 A load	89	
Figure 6.3.	Efficiency improvement with switch partitioning	90	
Figure 6.4.	(redraw with signal names, 1x to x1) Block diagram of switch partitioning circuit	91	
Figure 6.5.	Block diagram of fine-tuned deadtime generator	92	
Figure 6.6.	PCB prototype of high-power ITSAB converter	93	
Figure 6.7.	Die photo of high-power ITSAB converter	94	
Figure 6.8.	Measured switching nodes voltages	95	
Figure 6.9.	(a) Efficiency with different output voltages at 1.8 MHz; (b) efficiency with $(x1)$ and without $(x4)$ switch partitioning at 2.38 V output voltage	96	
Figure 7.1.	Bi-directional operation of DAB converter	100	
Figure 7.2.	Modulation of inductor current in ITSAB converter	101	
Figure A.1.	Embedded system design of the ZVS-3LB controller based on NiosII system	103	

LIST OF TABLES

Table 2.1.	Comparison between synchronous buck converter and n-level dickson-star hybrid converter	12
Table 3.1.	Parts List	20
Table 4.1.	Parameters to define time delay	37
Table 4.2.	Parameters setup at various load	39
Table 4.3.	Key components	39
Table 5.1.	Optimization results.	63
Table 5.2.	Source and gate voltages of each switch during ON and OFF intervals, when the ITSAB prototype is generating $V_o = 2.4$ V from $V_{in} = 9.6$ V	69
Table 5.3.	Components for the PI (Type-II) compensator.	72
Table 5.4.	Components on the package substrate	72
Table 5.5.	Performance comparison: discrete vs. IPD inductors	81
Table 5.6.	Comparison with prior works	82
Table 5.7.	Comparison between converter topologies	83
Table 6.1.	Optimization results.	86
Table 6.2.	High-power ITSAB converter optimization results	88
Table 6.3.	Components on the demonstration PCB	95

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T. Xie, J. Zhu, D. Maksimovic and H. -P. Le, "A Highly Integrated Hybrid DC–DC Converter With nH-Scale IPD Inductors," in IEEE Journal of Solid-State Circuits, vol. 58, no. 3, pp. 705-719, March 2023, doi: 10.1109/JSSC.2022.3227163.

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T. Xie, M. R. Oltra, and H. Le, "A 5kV/15W Dual-Transformer Hybrid Converter with Extreme 2000X Conversion Ratios for Soft Mobile Robots," in 2020 IEEE Applied Power Electronics Conference and Exposition (APEC), Mar. 2020.

T. Xie, R. Das, G.-S. Seo, D. Maksimovic, and H.-P. Le, "Multiphase Control for Robust and Complete Soft-charging Operation of Dual Inductor Hybrid Converter," in 2019 IEEE Applied Power Electronics Conference and Exposition (APEC).

ABSTRACT OF THE DISSERTATION

Highly Integrated Hybrid DC-DC Converters for High Performance Power Delivery Systems: Design, Analysis and Implementation

by

Tianshi Xie

Doctor of Philosophy in Electrical Engineering (Electronic Circuits and Systems)

University of California San Diego, 2023

Professor Hanh-Phuc Le, Chair

As the development of technology, devices, particularly semiconductor, have scaled down dramatically enabling the high performance of computing systems. However, when examine the entire electronics system, the total dimension is not equally scaled. It is found that magnetic devices are the bottleneck for the dimensional scaling. In this dissertation, several commonly employed methods for reducing the usage of inductors are discussed and compared, including high switching frequency converter, switched-capacitor converter, hybrid converter and resonant converter. Each of these converters has been proved that partially meet the requirement of miniaturized system with fine regulation and high energy efficiency. In the following chapters,

three approaches that address the problem from circuit operation, circuit technique and circuit topology are proposed.

The first one is multi-phase operation of a hybrid converter, as discussed in Chapter. 3. With multi-phase operation instead of dual-phase which is more commonly used, the effective frequency over the output inductors is 3 times higher leading to smaller inductors. While retaining the benefits of using a Dickson-Star hybrid converter, such as high conversion ratio, soft-charging of flying capacitors, the multi-phase operation has also been proved to gain the robustness on the flying capacitor mismatch. A demonstration PCB has been made to show the experimental results. The peak efficiency was captured at 40 V-1.8 V/4 A with a number of 92.4 %.

Secondly, an auxiliary circuit assisted Zero-Voltage-Switching 3-Level Buck (ZVS-3LB) converter is introduced in Chapter. 4 with its fundamental operations, design analysis and key experimental results. By inserting an auxiliary ZVS circuit with around 20% area overhead, the converter achieves maximum 30% loss reduction compared with a conventional 3-level buck converter when the output current is below 1.5 A. A peak efficiency of 92.4% is reached for 20 V/5 V conversion at 1 A load. As another significant benefit, ZVS operation allows the switching frequency to reach a multiple Mega-Hertz range as a result of low switching loss, making the topology a good candidate for minimizing inductor usage.

Furthermore, an Integrated Transformer-less Stacked Active Bridge (ITSAB) converter that uses only nano-Henry scale inductors at 2-5 MHz switching frequency is proposed in Chapter. 5. Since it is derived from a Dickson-Star Switched Capacitor converter, the proposed converter inherits the benefit of low voltage stress on switches while enjoying an efficient fine regulation by phase shift, similar to a Dual Active Bridge (DAB) converter. The converter is optimized, designed and fabricated in 1.7 mm \times 1.9 mm area of a 130 nm BCD process. The active die is flip-chipped on a 6.5 mm \times 6.5 mm package substrate together with power capacitors and two 10 nH IPD inductors for demonstration, illustrating the feasibility of passive components integration, resulting in a peak efficiency of 91.2% and a peak power density of 1.36 W/mm³ from 9.6-12 V input to 2.15-3.3 V output. Another demonstration is constructed on the same package substrate but with discrete air-core inductors. It achieves a peak efficiency of 92.4% and a peak power density of 0.62 W/mm³, while delivering a max power of 7.5 W. To achieve the performance, a detailed loss analysis and a unique optimization methodology for the converter, together with the design of key sub-blocks, including gate drivers (GDs), phase shift modulator (PSM), and ramp generator (RG), are provided in this chapter.

Additionally, a revised prototype of ITSAB converter that is capable of higher output current is presented in Chapter. 6. It has been demonstrated 5.2 A output current with 10 nH inductors. The chip is fabricated with the same technology. The necessary techniques to achieve such high output current, including the 3D integration and switch partitioning, are addressed.

In last, a conclusion and future works, such as CC-CV mode, bidirectional conversion are presented.

Chapter 1

Demand for Miniaturized and Efficient Power Delivery Network

Pushed by the famous Moore's Law (Fig. 1.1), technology has been continuously evolving and brought unprecedented change to modern human life. Albeit the physics limitation, the performance trend predicted by the Moore's Law is well maintained with transistor structure innovation, multi-threading operation, 3D integration and many more technology. However, the trend of another vital metrics does slowdown (Fig. 1.2), which is the microprocessor typical power. It hits the limitation that is usually rephrased as power wall. In other words, with this power limitation, the High Performance Computing (HPC) unit can't really work as designed despite the growth of number of transistors.

What's more, as indicated in [3], the power consumption of a major supercomputer in the year of 2035 would reach the power level of a nuclear power plant, if the same energy efficiency of computing is maintained, as shown in Fig. 1.3. Apparently, this is insane. Therefore, many improvements that prioritize energy efficiency must be done. From the power delivery point of view, the improvements could include new converter topology, innovation in integration and packaging.

On the other hand, when reviewing the history of electronic devices, there is a noticeable trend of dimension shrinking. A computer, for example, changes from a gigantic machine when it was first invented, to a portable device that is used in everyday life. That being said, to satisfy



Figure 1.1. Moore's Law [1]



Figure 1.2. Collected data showing trend in microprocessor technology [2]



Green500 Supercomputer GFLOPs/Watt

Figure 1.3. Trend in power efficiency of top performing super-computers [3]

the aforementioned computation capability, the required power density within these electronic devices has been continuously increasing [4, 5], making the power delivery design increasingly challenging. Active devices, who are benefited from semiconductor technology scaling, decline in dimension without losing in performance, whereas passive devices, particularly the magnetic components, do not keep up with the rate of scaling and miniaturization [6]. As a consequence, as implied in the main board of a major brand smart phone (Fig. 1.4), inductors dominate the size of modern power converter implementations. In other words, to diminish the board dimension and increase the power density, it is important to reduce the number of inductors used on board and the individual inductor size.

From the survey of inductance vs. volume for Coilcraft XEL series inductors depicted in Fig. 1.5, the dimension of an inductor linearly changes with its inductance. Therefore, the desired high power density converter design prioritizes a small inductance for power delivery.

In addition to improving the quality of inductors, significant efforts have been invested in coming up with new converter topologies and different operating principles to reduce the required inductance [7–11]. The most straightforward approach to minimize inductance is operating



Figure 1.4. iPhone 13 Pro (2021) main board, the highlighted parts are power management circuits and obviously a lot of inductors occupy the area

a power converter at very high frequency (VHF) [12, 13]. However, the benefit of reduced inductance comes with increased switching losses leading to reduced efficiency. Alternatively, Switched capacitor (SC) converters, which require no inductors, are good candidates for realization of fully integrated power converters [14–16]. However, drawbacks of an SC converter include significantly degraded efficiency when operating away from the nominal conversion ratio, and large output noise with variable frequency that could cause EMI issues in the system. Therefore, new topologies that efficiently utilize inductors are emerging, such as hybrid converters [7, 9–11, 17, 18]. With the help of capacitors that blocks high DC voltage, the inductors are magnetized with much lower voltage such that smaller inductors can be employed. Nonetheless, most of the previously proposed hybrid converters are based on discrete-circuit realizations, still requiring inductances in the range of 100 nH or larger. In an another effort to use less inductance, resonant or quasi-resonant converters are employed. Unfortunately, output regulation is often challenging for converters with a resonant tank while satisfying high efficiency and small size requirements. The quasi-resonant converter demonstrated in [8] was a successful demonstration of this type of converter, capable of regulation at relatively low switching frequency. However, its efficiency is relatively low, and more importantly, the 36 nH inductor used is still large, which makes it difficult for on-chip realization. Further increasing the switching frequency to achieve a



Figure 1.5. A survey of inductance vs. volume for Coilcraft XEL series inductors. smaller inductor would face significant challenge in maintaining high efficiency [6]. In the next chapter, detailed analysis and comparison between these topologies would be addressed.

Chapter 2

Overview of the Converters with Reduced Magnetic Devices

As illustrated in Chapter 1, magnetic components are the bottleneck to reduce the overall dimension and to improve the power density. To reduce the usage of magnetic parts, several applicable methods have been analyzed and implemented in prior arts. In this chapter, they are evaluated and commented.

2.1 High frequency Switched-inductor Converter

As for a buck converter, the inductor current ripple is inversely proportional to the switching frequency f and the inductor value L:

$$\Delta i_L \propto \frac{V_{in}}{fL} \tag{2.1}$$

To achieve the same current ripple, if a smaller inductance is selected, the switching frequency must be increased. In most cases, a smaller inductance leads to a lower profile inductor and hence a miniaturized power module. However, as analyzed in [13], if considering maintaining a consistent quality factor Q and an acceptable temperature rise ΔT , a cored inductor will cease scaling down once hit the temperature rise limit, while a coreless inductors can keep declining in size but with a lower rate when approaching the temperature rise limit. In other words, if using an coreless inductor, it is a viable method to shrink the overall converter size by increasing switching frequency.

On the other hand, even though the analysis above implies nearly the same the loss over inductors after scaling as a result of consistent Q, increasing switching frequency results in poor efficiency at light load due to excessive switching loss. To deal with this design challenge, several approaches have been published.

Reported in [19], the solution of actively tuning switching frequency at light load has been effectively proved for USB-C application. The idea can be summarized as Fig. 2.1. However, the reduced switching loss comes with the penalty of the circuit complexity and dynamic performance.



Figure 2.1. The concept of variable switching frequency that improves light load efficiency

Boundary Conduction Mode (BCM) operation, as shown in Fig. 2.2 is another way to achieve soft-switching at light load as demonstrated in[20–22]. However, the limited ZVS range and high conduction loss resulted from large current ripple remain problems.



Figure 2.2. BCM operation that enables soft-switching

In addition to these methods, using auxiliary passive components to generate required charge for ZVS is another viable method [23, 24]. Through an inductor, the parasitic capacitors associated with the switching node will be softly charged/discharged once the auxiliary current is higher than the main inductor current and hence the converter achieves soft-switching. Despite the switching loss is effectively reduced as reported in these works, a lack of precise timing control causes more charge delivered from auxiliary circuit than required for ZVS and therefore hurts efficiency.

2.2 Switched-capacitor Converter

Switched-capacitor (SC) topology is a potential candidate for Point-of-Load (PoL) converter [25] due to the fact of zero inductor and hence possible fully integration of the converter. As discussed in numerous publications, a SC converter can be modeled as a DC transformer (DCX) with an output resistance as shown in Fig. 2.3.

The regulation of the converter is to modulate the output resistance R_{out} by tuning one or combination of the listing parameters: switching frequency $f_s[15]$, size of the power switches and the effective flying capacitance $C_{fly}[26]$.

The calculation of the output resistance is evaluated by charge flow analysis described in [27]. Take one-capacitor 2:1 switched capacitor (SC) converter as an example. The charge flow



Figure 2.3. Switched-capacitor converter model



Figure 2.4. Charge flow analysis of 2:1 switched-capacitor converter

can be depicted as in Fig. 2.4. The charge transferred through the capacitor is the same during state A and state B, which is q, while the total charge flows to the output is 2q. The conversion ratio M, slow switching limit (SSL) resistance R_{SSL} and fast switching limit (FSL) resistance R_{FSL} are as follows.

$$M = \frac{q_{IN}}{q_O} = \frac{q}{2q} = \frac{1}{2},$$

$$R_{SSL} = \frac{1}{4f_sC},$$

$$R_{FSL} = 2R_{ON}$$
(2.2)

where q_{IN} , q_O , f_s , C and R_{ON} are input and output charge, switching frequency, capacitance and ON resistance of each switch, respectively. The total output resistance R_{out} is therefore expressed as,

$$R_{OUT} = \sqrt{R_{SSL}^2 + R_{FSL}^2} \tag{2.3}$$

The value of (2.3) is sketched in Fig. 2.5. It indicates the output resistance of a SC



Figure 2.5. Output resistor R_{OUT} of a SC converter

converter is bounded by R_{SSL} at low switching frequency while limited to R_{FSL} at high switching frequency. This feature implies a narrow regulated output voltage range. Moreover, the efficiency drops quickly when the output voltage is away from the nominal value which is slightly lower than $M * V_{IN}$. Because a larger R_{OUT} and hence higher conduction loss is inevitable when the converter is regulated to lower output voltage, whereas a smaller R_{OUT} and hence worse switching loss is required for higher output voltage. A more completed loss analysis including the influence of bottom plate capacitors is given in [16]. The output voltage range problem can be alleviated by reconfigurable topology [16]. But it inevitably increases the complexity of the gate drivers and control logic.

2.3 Hybrid Converter

Intrinsically, R_{SSL} of an SC converter represents the total charge sharing loss of capacitors. To alleviate this type of loss, the most straightforward method is to add inductors in series with these capacitors during charging and discharging. Indeed, if the inductor is selected properly,



Figure 2.6. Various type of hybrid converter

the total loss would be reduced. No charge sharing loss would be observed and the current spikes during charging and discharging are limited. The term soft-charging is used to describe this circumstance. The converter with both inductor and capacitor is called hybrid converter, which has been demonstrated a good candidate for high-conversion-ratio and high-power-density applications in many publications [17, 28–30].

A hybrid converter can have three types of combinations as shown in Fig. 2.6. The first one is to place the inductor at the output, which is called inductor-last hybrid hybrid converter. The advantage of this type of arrangement is continuous output current and hence smaller output capacitor is required. The next one is to connect the inductor to the input, which is called inductor-first topology [31]. The benefit of this kind of connection is light inductor current. It's particularly suitable for step-down converter as the input current of it is less than the output one, so that less conduction loss on the inductor compared with putting the inductor at the output node. Emerging more recently, the last type is called inductor-first topology, the

conduction loss is reduced. Meanwhile, the circuit can be reconfigured to different structures that provides wide output range without compromising performance.

In order to have a fair comparison between a hybrid converter and a conventional switchedinductor converter, let's only consider the inductor-last topology and assume they have the same switching frequency at the switching node and the same current ripple over the inductors, as well as the identical input and output voltages. Thus, for a n-level dickson-star hybrid converter [17] and a synchronous buck converter, their duty ratios and inductor voltages as well as the required inductance are listed in Table 2.1.

 Table 2.1. Comparison between synchronous buck converter and n-level dickson-star hybrid converter

Topology	Duty ratio	Voltage over inductor(s)	Inductance
Buck	D	V_L	L
Hybrid (n-level)	n * D	$\frac{V_L}{n}$	L

From the well-know Steinmetz equation for calculating magnetic core loss,

$$P_{core} = V_{core} C_M f^{\alpha} \Delta B^{\beta} \tag{2.4}$$

the inductor loss for these two converters are the same, given the fact that ΔB , determined by volt*sec value, and switching frequency *f* are identical. In other words, using a hybrid topology won't reduce the volume of inductors. Nevertheless, the capacitors have three orders of magnitude higher energy per unit volume than a ferrite magnetic material [5], which means for a practical application, e.g. power supplies for servers, one hybrid converter can substitute multiple stages of conventional switched-inductor converters [17], in other words, it greatly reduces the usage of inductors.

To further decrease the inductor ripple and hence inductance value required, a multi-phase operation can be applied [18]. The detailed operation and its analysis as well as the experimental

results of this converter will be addressed in Chapter. 3.

2.4 Resonant Converter

Resonant converter can be viewed as a special case of a hybrid converter. The sinusoidal current and voltage help achieve Zero-Voltage-Switching (ZVS) and Zero-Current-Switching (ZCS). As a matter of fact, a resonant converter can use very switching frequency without worrying about switching loss and hence passive components, especially magnetic devices, are miniaturized.

Traditionally, the magnetized or leakage inductance of a transformer resonates with an added capacitor. Even though the transformer provides the additional conversion ratio and isolation between input and output, which is crucial for high power applications such as data center power supplies and EVs, the dimension of the magnetic parts makes it difficult to be implemented onto size-limited applications such as mobile phones, USB-C chargers, *etc*.

Alternatively, inductors are inserted into conventional switched-capacitor converters to resonate with existing capacitors. One example is shown in Fig. 2.7 which combines 2:1 switched-capacitor converter with an resonant inductor. Despite the topology is identical to a 3-level buck converter, the resonant operation determines a different sinusoidal shape of the current and the voltage which helps achieving zero-voltage-switching (ZVS) and zero-current-switching (ZCS) and results in higher efficiency.

However, a successful soft-switching operation requires precise timing control. Moreover, similar to traditional transformer-included resonant converter, because inductance *L* and capacitance *C* are fixed so as the resonant frequency $f_0 = \frac{1}{2\pi\sqrt{LC}}$, the converter in Fig. 2.7 can't be regulated by varying frequency while achieving ZVS and ZCS similar to the traditional transformer-included resonant converters. But it's still a good candidate for fixed conversion ratio application [33].

Meanwhile, to gain the regulation capability of a resonant converter, many demonstrations



Figure 2.7. 3-level buck converter and its 1)inductive operation (top), 2)resonant operation (bottom)

have been published. The work in [8] has presented precisely on-time control which means modulating the time of resonant. However, the efficiency is therefore sacrificed. Another approach is adding a regulated converter stage to a fixed conversion ratio resonant converter stage. One good example is [34]. A multi-phase buck converter is plugged between input and the resonant stage. It has been proved an outstanding efficiency but as additional stage is added, more inductors are presented which offsetting the motivation of using a resonant converter.

Chapter 3

Multi-phase Dual-inductor Hybrid Converter

As discussed in Chapter. 2, the required inductor can't be reduced solely by replacing the conventional switched-inductor converter, e.g. buck converter, with a hybrid converter unless the effective switching frequency over the inductors is increased. Modified from the Dual-Inductor Hybrid (DIH) converter [35], rather than dual-phase operation, the Multi-Phase Dual-Inductor Hybrid (MP-DIH) converter [18] achieves 3x higher effective frequency on the inductors while reserves other benefits such as soft-charging, high conversion ratio, *etc.* In this chapter, the detailed operation and experimental results of it would be given.

3.1 Operation of MP-DIH Converter

The proposed MP-DIH converter operation can be applied to any DIH converter topology that has an even number of levels. The number of non-overlapped phases equals the number of the converter's levels (ignoring the zero level). In this work, a six-level version of the topology shown in Fig. 3.1 is demonstrated as a proof of concept. This 6-to-1 MP-DIH converter has two inductors L_{1-2} , five capacitors C_{1-5} , and eight switches S_{1-8} . Its operation can be explained using the illustrations of states and operational waveforms in Fig. 3.3. In State 1, a charge drawn from the input by C_1 and L_1 is stored in C_1 . It is then sequentially transferred to C_2 , C_3 , C_4 , and C_5 in other odd states with the operation of L_1 and L_2 . In each odd state other than State 1 and



Figure 3.1. Topology of the Multi-Phase Dual-Inductor Hybrid (MP-DIC) converter

11, two capacitors and one inductor form a conducting branch where the inductor enables a soft charge transfer from the higher-level capacitor to the lower-level one, e.g. from C_1 to C_2 using L_1 in State 3. In State 1 and 11, C_1 receives the charge from the source and C_5 delivers the charge to the load, respectively. These odd states are also labeled as the six identical non-overlapped phases A-F evenly distributed over an operational cycle T. For even states, all the capacitors are



Figure 3.2. Operating states of the proposed MP-DIH converter



(a) Operational waveforms

Figure 3.3. Operational waveforms of the proposed MP-DIH converter
inactive and open-circuited. Inductor L_1 (L_2) free-wheels the stored charge to the output when S_8 (S_7) is activated during even states and during odd states when L_2 (L_1) is getting charged. The operational PWM signals, related capacitor voltage and inductor current ripple waveforms are shown in Fig. 3.3a.

To simplify the analysis on the steady-state voltages V_{C1} to V_{C5} of the capacitors C_{1-5} , we first assume small voltage ripples across the capacitors and inductor volt-second balance. Similar to a regular 6-to-1 Dickson SC converter, C_{1-5} have their steady-state voltages gradually decreased by $\frac{V_{in}}{6}$ from $V_{C1} = \frac{5V_{in}}{6}$ to $V_{C5} = \frac{V_{in}}{6}$. Switching nodes V_{X1} and V_{X2} swing from 0 to $\frac{V_{in}}{6}$ in states 1,5,9 and 3,7,11, respectively. Therefore, each inductor blocks only $\frac{V_{in}}{6}$ and is effectively switched at a frequency three times higher than that of S_{1-6} . This creates an opportunity to select an inductor of lower inductance, and thus lower equivalent series resistance (ESR) for the same size, while sacrificing switching loss for only $S_{7,8}$, i.e. $S_{7,8}$ is operated at 3X the fundamental switching frequency.

As can be seen in Fig. 3.5, on top of the high-frequency ripple, associated with switching nodes $V_{X1,2}$, $I_{L1,2}$ and V_{out} also have a low-frequency ripple at the operational frequency of the high-side switches, i.e. $\frac{1}{T}$. This phenomenon is caused by the different impedance and as a result, different voltage swing seen by the inductors in States 3, 5, 7, and 9 compared with States 1 and 11 [36]. Particularly, L_1 (L_2) only sees one capacitor C_1 (C_5) when it gets charged in State 1 (11), while in all other odd phases the inductors see two capacitors in series at switching nodes $V_{X1,2}$. Since the voltage swing difference created by equivalent capacitor mismatch only causes a small predictable perturbation in the fundamental operations of the converter, it does not alter either the charge balance of C_{out} and C_{1-5} or the volt-second balance of the inductors achieved in the entire period.

General steady-state values of the output and flying capacitor voltages for an MP-DIH converter are given as:

$$V_{out} = \frac{DV_{in}}{2} \quad \text{and} \quad V_{C_k} = \frac{(N-k)V_{in}}{N}$$
(3.1)

where, k=1, 2,...., N-1

One can calculate from this analysis that for an N-level N-phase operation, the maximum duty cycle D is limited to $\frac{1}{N}$ due to non-overlapping phase distribution. In other words, the conversion ratio $\frac{V_{in}}{V_{out}}$ is limited to 2N. However, the converter is still capable of supporting a wide range of conversion ratios from a 40V-54V input supply to an output voltage up to 3.33V-4.5V, covering the 1V-2V range typically required in data center and telecommunication systems. It is also noted that, for the same voltage conversion ratio as a three-level Buck converter the voltage stress across each high-side switch of the MP-DIH converter is $\frac{2V_{in}}{N}$, significantly lower than the three level Buck converter's switch stress of $\frac{V_{in}}{2}$ [37–39]. This difference brings advantages in both switching loss and conduction loss, i.e. allowing switches with lower breakdown voltage and on resistance, promising a higher efficiency for a larger conversion ratio.

Item	Design Selection
Flying Capacitors C_{1-5}	2.2, 1.5, 1.5, 1, 1µF, X7R, 1812/1210, TDK
Output Capacitor C ₀	6.8μ <i>F</i> , X5R, 0603, 10V, TDK
Inductors L_1, L_2	1.5μ <i>H</i> , IHLP-5050CE-01
High Side Switches, S_{1-6}	EPC2014C
Low Side Switches, S _{7,8}	EPC2023
$S_{7,8}$ Paralleled Diodes, $D_{7,8}$	CRS08, 30 V, 1.5 A
Gate Drivers	LM5113, LM5114
Signal Isolators	Si8422, Silicon Labs
Microcontroller	TMS320F28377 Delfino, TI

Table 3.1. Parts List



Figure 3.4. Prototype Board

3.2 Experimental Results

To verify the feasibility of this novel topology and operation, a 48V 20W prototype has been implemented using the components listed in Table 3.1. With a switching frequency of 150 KHz applied to S_{1-6} , $S_{7,8}$ and $L_{1,2}$ are switched effectively at 450 KHz, as described in Section 3.1. The operational waveforms of I_{L1} , I_{L2} , V_{X1} , V_{X2} , and V_{out} are captured in Fig. 3.5, while capacitor voltages V_{C1-5} for one operational cycle with 12 states shown in Fig. 3.6. These measured waveforms verify intended converter operations and analysis of Section 3.1.

As shown in Fig. 3.7 and Fig. 3.8, the converter is also demonstrated to maintain efficiency higher than 90% over most of the 1A-8A load range for wide ranges of input voltages, 40V-54V, and output voltages, 1.4V-2V, with a peak efficiency of 92.4% at 40V-1.8V/4A and 92.1% at 48V-1.8V/4A.

At a mid load of 4A for 48V-to-1.8V conversion, the converter is analyzed to have switch-



Figure 3.5. Operational waveforms of prototype at 48V-1.8V/4A.



Figure 3.6. Flying capacitor voltage with no hard-charging.

ing loss, magnetic component loss, conduction loss and non-ideal parasitic loss of 40%, 37%, 20%, and 3%, respectively, as given in the Fig. 3.9. It can be concluded that to further increase the



Figure 3.7. Measured efficiency for different input voltages at $V_{out} = 1.8$ V regulated



Figure 3.8. Measured efficiency for different output voltages at $V_{in} = 48$ V regulated



Figure 3.9. Loss breakdown at 1.8V/4A output

efficiency of this Multi-Phase Dual Inductor Hybrid (MP-DIH) converter, a customized and more optimized inductor design is required. Moreover, a soft-switching approach and discontinuous conduction mode (DCM) operation are necessary for better efficiency at light load operations.

3.3 Summary

In this chapter, a multi-phase hybrid converter, namely Multi-Phase Dual-Inductor Hybrid (MP-DIH) converter is proposed. With multi-phase operation, the effective frequency over the output inductors is 3 times higher leading to smaller inductors. While retaining the benefits of using a Dickson-Star hybrid converter, such as high conversion ratio, soft-charging of flying capacitors, the multi-phase operation has also been proved to gain the robustness on the flying capacitor mismatch. A demonstration PCB has been made to show the experimental results. The peak efficiency was captured at 40 V-1.8 V/4 A with a number of 92.4 %.

3.4 Acknowledgements

This chapter, in full, is a reprint of the material as it appears in 2019 IEEE Applied Power Electronics Conference and Exposition (APEC). Tianshi Xie; R. Das, G.-S. Seo, D. Maksimovic, and H.-P. Le, "Multiphase Control for Robust and Complete Soft-charging Operation of Dual Inductor Hybrid Converter," APEC 2019. The dissertation author was the primary investigator and author of this paper.

Chapter 4

Soft-switching Assisted by Additional Circuitry

Introduced in Chapter. 2, the volume of inductors and hence the overall system size can be shrinked if operating the converter at a higher switching frequency. However, this comes with the penalty of higher switching loss. To alleviate it, several methods including variable frequency, BCM operation and auxiliary circuit assisted ZVS have been published. Among them, the last one is the most applicable approach as it requires no complexity circuit for tuning frequency and maintains the acceptable current ripple. But the current solutions don't have sophisticated timing control and therefore lots of charge have been wasted.

Alternatively, a new topology named Zero-Voltage-Switching 3-Level Buck (ZVS-3LB) converter, shown in Fig. 4.1, is proposed. A small auxiliary circuit is added to a 3LB converter to achieves full ZVS for both high-side and low-side switches. Because the significant reduction of switching loss, the converter can operate at Mega-Hertz switching frequency, while maintaining > 92.2% efficiency at light load. In Section. 4.1, the detailed operation principle is introduced. The design considerations are discussed in Section. 4.2. In Section. 4.4, the experimental results are given. And the paper is concluded in Section. 4.6.



Figure 4.1. The proposed Zero-Voltage-Switching 3 Level Buck (ZVS-3LB) converter schematic

4.1 Topology and Operation of the ZVS-3LB Converter

The proposed ZVS-3LB converter is constructed by adding an auxiliary circuit to the conventional 3LB converter. As shown in Fig. 4.1, the auxiliary circuit contains two N-type transistors $Aux_a \& Aux_b$, two diodes $D_a \& D_b$, and one small inductor L_{aux} . The presence of the auxiliary circuit is to provide additional charge for soft-switching when High Side (HS) switches, namely Q_1 and Q_2 , turn on.

The complete operation of the proposed converter is depicted in Fig. 4.2(a) where the ZVS parts are highlighted for Φ_1 and Φ_2 . Because the auxiliary circuit operates the same in Φ_1 and Φ_2 to softly turn-on Q_1 and Q_2 . The operation details during Φ_2 with all time intervals of the ZVS operation are presented in Fig. 4.2(b) and explained below, while Fig. 4.3 shows the status of main power switches Q_1 - Q_4 corresponding to the time intervals during both Φ_1 (in red) and Φ_2 (in blue).

• <u>Before t_1 </u>: Only Low Side (LS) switches $Q_3 \& Q_4$ are on. The auxiliary circuit is inactive.



(b) Zoomed-in ZVS operation during Φ_2

Figure 4.2. Operation principle of the proposed ZVS-3LB converter.

















Figure 4.3. Key operation states of the proposed ZVS-3LB converter during the time interval: (a) before t_2 , (b) $t_2 \sim t_3$, (c) $t_3 \sim t_4$, (d) $t_4 \sim t_6$, (e) $t_6 \sim t_7$, (f) $t_7 \sim t_9$, (g) $t_9 \sim t_{10}$

- $t_1 \sim t_2$ (Fig. 4.3(a)): one of the LS switches (Q_3 or Q_4) is turning off. Since it has not been fully off yet, the auxiliary circuit remains hibernated to avoid flowing extra charge to the ground.
- $t_2 \sim t_3$ (Fig. 4.3(b)): Despite the LS switch is not completely off yet, L_{aux} must be magnetized in advance so as to ramp its current I_{aux} to I_L at the rate of $\frac{V_g}{L_{aux}}$. When $I_{aux} > I_L$, the extra charge starts flowing into the switching node once the LS switch is off at t_3 .
- $t_3 \sim t_4$ (Fig. 4.3(c)): As the LS switches are off, the additional charge resulted from $(I_{aux} I_L)$ is transferred to the switching node parasitic capacitor C_x . Aux_b is turned off at the end of this time interval since L_{aux} is sufficiently magnetized for completely soft-switching of the HS switches.
- $t_4 \sim t_6$ (Fig. 4.3(d)): The free-wheeling diode D_a conducts, leading to ramping down of I_{aux} . Since $I_{aux} > I_L$, C_x is continuously soft-charged until the HS switch is completely on at t_6 . The extra charge from the auxiliary circuit for soft-switching one HS switch is given by

$$Q_{aux} = \int_{t_3}^{t_6} (i_{aux} - i_L) dt$$
(4.1)

And the total charge required for ZVS is

$$Q_{req} = Q_{oss,HS}(HS\,switch) + Q_{oss,LS}(LS\,switch).$$
(4.2)

To exhaust Q_{aux} for ZVS HS switches and to minimize power loss, $Q_{aux} = Q_{req}$ is desired.

- $\underline{t_6 \sim t_7}$ (Fig. 4.3(e)): The I_{aux} continues ramping down to zero at the rate of $\frac{V_g}{2L_{aux}}$. Note that $V_{Cfly} = \frac{V_g}{2}$.
- $t_7 \sim t_9$ (Fig. 4.3(f)): Aux_a turns off at zero current putting L_{aux} into inactive status again. The rest of the circuit operates the same as a conventional 3LB converter. The presence of D_b helps reduce current ringings during auxiliary circuit inactive states.

• $t_9 \sim t_{10}$ (Fig. 4.3(g)): The HS switches are completely off letting I_L fully discharge switching node parasitic capacitor. As a result, the LS switch will be zero-voltage switched on.

4.2 Design Consideration of the ZVS-3LB Converter

A careful analysis and selection of auxiliary circuit devices are necessary to achieve the efficiency benefit from ZVS operation. The auxiliary circuit will reduce the main circuit switching loss by

$$P_{sw,redu} = 2\left(\frac{C_{oss}}{2}\left(\frac{V_g}{2}\right)^2 + \frac{V_g}{2}I_L t_{on} + Q_{rr(LS)}\frac{V_g}{2}\right)f_s$$
(4.3)

where C_{oss} , t_{on} and $Q_{rr(LS)}$ are HS switches output capacitance, turn-on time, and LS switches body diode reverse recovery charge, respectively. Meanwhile, the auxiliary circuit incurs additional power loss, namely $P_{tot,aux}$, which can be divided into conduction loss $P_{con,aux}$, switching loss $P_{sw,aux}$ and inductor loss P_{Laux} .

4.2.1 Auxiliary Circuit Conduction Loss

When computing conduction losses, it can be assumed that I_{aux} rises from t_2 to t_4 and falls from t_4 to t_7 at constant rates of $k_r = \frac{V_g}{L_{aux}}$ and $k_f = \frac{V_g}{2L_{aux}}$, respectively, for simplicity. Thus, (4.1) becomes

$$Q_{aux} = 0.5 \left(\left(t_r - \frac{I_L}{k_r} \right) + \left(t_f - \frac{I_L}{k_f} \right) \right) \left(k_r t_r - I_L \right)$$
(4.4)

where t_r is the time duration from t_2 to t_4 and t_f represents the time duration from t_4 to t_7 . Apparently, $t_f = 2t_r$. On the other hand, $Q_{aux} = Q_{req}$ is required. Therefore, the relation between t_r and Q_{req} can be derived as

$$t_r = \sqrt{\frac{2Q_{req}}{3k_r}} + \frac{I_L}{k_r} \tag{4.5}$$

Then, the conduction loss of the auxiliary circuit can be computed by

$$P_{con,aux} = \frac{I_{pk}^2}{3} (R_{on,a} + R_{on,b}) t_r f_s + \frac{I_{pk}^2}{3} R_{on,a} t_f f_s$$

$$= \frac{k_r^2 t_r^3}{3} f_s (3R_{on,a} + R_{on,b})$$
(4.6)

where I_{pk} is the peak value of I_{aux} , $R_{on,a}$ and $R_{on,b}$ are the on-resistance of the switches Aux_a and Aux_b , respectively. Interestingly, the conduction loss of the auxiliary circuit has strong relation with the switching frequency f_s which is not very common in PWM controlled switched-inductor converters. This is simply because Q_{req} , which is only determined by the devices, doesn't change along with f_s . And to achieve fully soft-switching in a faster switching converter, the auxiliary circuit has to be activated more frequently which resulting in higher average conduction loss. Moreover, plugging (4.5) into (4.6) would help better understand the relation between $P_{con,aux}$ and L_{aux} . To simplify, let's assume one converter, which means fixed on-resistance of the switches, operating at a constant f_s .

$$P_{con,aux} \propto \sqrt{k_r} \left(\sqrt{\frac{2}{3}Q_{req}} + \frac{I_L}{\sqrt{k_r}} \right)^3 \tag{4.7}$$

By analyzing (4.7), it can be observed that for each set of I_L and Q_{req} , a minimum conduction loss can be approached by varying L_{aux} , which is inversely proportional to k_r . From Fig. 4.4(a), it can be summarized that a higher I_L or output current would require a smaller L_{aux} to achieve the corresponding minimum auxiliary circuit conduction loss. Meanwhile, a greater Q_{req} , which means larger main switches of 3LB converter, requires a higher value of auxiliary inductor, as implied in Fig. 4.4(b).

4.2.2 Auxiliary Circuit Switching Loss

As the auxiliary switches are all small, the switching time is short. Besides, the auxiliary switches are switched at zero current. As a result, the auxiliary circuit switching loss is dominated



Figure 4.4. Normalized auxiliary circuit conduction loss vs. L_{aux} at different: (a) output inductor current I_L , (b) required charge Q_{req} for soft-switching.

by output capacitance and reverse recovery charge $(Q_{rr,aux})$ of D_a and D_b , given by

$$P_{sw,aux} \approx 2\left(C_{oss,aux}\left(\frac{V_g}{2}\right)^2 + Q_{rr,aux}\frac{V_g}{2}\right)(2f_s)$$

$$\tag{4.8}$$

The expression indicates trade-off between auxiliary circuit switching loss and conduction loss just like other switch mode power converters. To achieve the most optimized efficiency, the total loss of the auxiliary switches must be considered.

4.2.3 Auxiliary Inductor Loss

In general, inductor loss can be estimated by the summation of copper loss, which is determined mostly by the winding resistance, and core loss, which is related to switching frequency of the converter.

$$P_{Laux} = P_{cu}(Copperloss) + P_{core}(Core \, loss) \tag{4.9}$$

The copper loss is computed by

$$P_{cu} = I_{aux,rms}^2 (R_{ac} + R_{dc})$$
(4.10)

Where the ac resistance R_{ac} and dc resistance R_{dc} can usually be found in the data sheet of a commercial inductor or be calculated by the winding parameters if it is customized designed.



Figure 4.5. Convert pulsed current into continuous sawtooth current for convenient core loss estimation

The modeling of the core loss is however tricky as the current going through is not continuous. One easy approach to calculate its core loss is to convert it to a continuous triangle-shape current with switching frequency of $f_{aux} = \frac{1}{t_r + t_f}$ as sketched in Fig. 4.5. Its core loss can be modeled by using the provided data from the inductor vendor and the iGSE method described in [40] after determining the peak flux density ΔB which can be written as *Volt* · *Sec* expression $V_g t_r$. The rising time t_r is approximated as one third of the auxiliary current period $\frac{1}{3f_{aux}}$. Then, the actual core loss is the calculation value from the continuous current divided by the number of repeating cycles $N = \frac{f_{aux}}{f_s}$. Thus, the actual core loss is given by

$$P_{core,aux} = \frac{V_{core} C_M f_{aux}{}^{\alpha} \Delta B^{\beta}}{N}$$

$$= V_{core} C_M f_{aux}{}^{\alpha-1-\beta} (\frac{V_g}{3})^{\beta} * f_s$$
(4.11)

where V_{core} is the core volume and C_M , α and β is from fitting data. For a typical ferrite material, α is in a range of 1.4-2.0, while β is within 2.4-3.0 [13]. That being said, the term $f_{aux}^{\alpha-1-\beta}$ is always reversely proportional to f_{aux} . Therefore, when optimizing the auxiliary circuit for a given 3LB converter, a higher f_{aux} would result in less core loss if assuming the same core is used.

An optimization flow is created using the loss calculations above. To achieve the goal of improving efficiency, the total loss of auxiliary circuit $P_{tot,aux}$ must be smaller than the reduced switching loss $P_{sw,redu}$ of main circuit. As shown in the loss breakdown graph in Fig. 4.10(a), this can be achieved with load currents smaller than 1.7 A.

4.3 System Design

The system design of the proposed ZVS-3LB converter is critical as the auxiliary circuit is sensitive to tiny timing change which fails the fulfillment of (4.2) and thus leads to unexpected power loss.

To control the timing in a closed-loop manner, one example of the system configuration can be set as Fig. 4.6 shown. First of all, the switching node voltage must be sensed. The soft-switching sequence is initialized at t_2 when both Aux_a and Aux_b are turned on. Reset of Q_3 is upon sensing of the switching node voltage V_x and comparing it with zero voltage. Ideally, Q_3 is turned off at t_3 when I_L equals to I_{aux} , which means $V_x = 0$. Here, the turn-on/off time of the switches is ignored for simplification of the description. In reality, to guarantee Q_3 is off at the time t_3 , the control signal of it has to be pulled down before t_3 . The set of Q_3 is based on zero voltage sensing at t_{10} and the circuit design is not shown. The set of Q_2 happens when V_x reaches zero slope at t_6 as it means $I_L = I_{aux}$ and no charge current flows into the switching node. Once the zero-slope point is found, a comparision will be conducted between V_x and half V_g . When $V_x > \frac{V_x}{2}$, abundant Q_{aux} is generated and therefore a Delay- signal is triggered. And vice versa. After going through an integrator and a voltage-controlled delay block, a reset signal R_b will determine the falling edge of Aux_b . While the set of both Aux_a and Aux_b is generated from zero-current detection (ZCD) of I_{aux} .



Figure 4.6. An example of closed-loop design

From the control loop diagram shown above, it's obviously that the success of the closedloop control, and thus the efficiency of the converter, is a result of precise sensing. From (4.5), the rising time of L_{aux} can be derived as 3.25 ns if assuming 15 V input voltage, 1 A load current, 10 nH L_{aux} and 15 nC Q_{req} . However, sub-ns window of sensing on PCB is apparently very difficult. As a consequence, the soft-switching timing including the status of the auxiliary circuit is controlled in an open-loop manner.

In addition, as for a 3-level buck converter, there is always a balancing issue along with the flying capacitor [41]. Therefore, while the output voltage is modulated by varying duty cycles of Q_1 and Q_2 , the flying capacitor voltage is balanced by changing their difference [41]. Due to the aforementioned voltage and current sensing difficulty, the timing for soft-switching function together with output voltage regulation as well as flying capacitor voltage balancing is determined by parameter sweeping. The timing for all the signals displayed in Fig. 4.2 can be broken down into the time delay parameters listed in Table. 4.1. To exhaustively sweep these parameters and to capture the optimized soft-switching operation, an FPGA board ADC-SOC from Terasic with designed embedded system based on NiosII is employed. This complete automation system, as sketched in Fig. 4.7, formed by the FPGA board, ZVS-3LB demonstration board, testing equipments, remote desktop and a control program in Matlab, all connected to a PC, is designed to accomplish over 3000 groups of data searching within one hour. Each data group contains a set of control parameters and measured performance of the converter. Thus, the optimized points are found and the results are presented in Section. 4.4. The resulted parameter values are listed in Table. 4.2. The parameter sweeping is based on different load currents. Since it's open-loop, the output voltage must be kept consistently 4 V in order to have a fair comparison between ZVS-3LB converter and conventional 3LB converter. Because the minimal time duration of 1 bit is translated into 5 ns, at some load current, V_o is hard to well-regulated to desired voltage, e.g 4 V. But the same output voltage is achieved for performance comparison between topologies. Please also be noted, the duty ratios of Q_1 and Q_2 are different due to flying capacitor balancing requirement as discussed above. This duty ratio difference is also programmed. As a result of the 5 ns minimum time step set by the maximum counter clock frequency on FPGA, the minimum load current where the timing of auxiliary circuit is optimized is 0.7 A. It can be seen that the duty ratio of Aux_b is 1 bit which is 5 ns. As the load current increases, the number of bits accumulates but with 5 ns step. Consequently, the load current set for measurement has to follow this step. It's impossible to capture the optimized timing between the load currents shown in the table.

Parameters	Delay from	То
QbarF_AuxR	Falling edge of Qbar	Rising edge of Aux _{a,b}
duty_Aux	Rising edge of Aux _b	Falling edge of Aux _b
AuxF_QR	Falling edge of Aux _b	Rising edge of Q
Auxa_ext	Falling edge of Aux _b	Falling edge of Aux _a
QF_QbarR	Falling edge of Q	Rising edge of Qbar

Table 4.1. Parameters to define time delay.



Figure 4.7. The testbench for ZVS-3LB converter demonstration



Figure 4.8. Prototype of the proposed ZVS-3LB converter: (a) front side, (b) back side.

	Numbers					Comments
ار	0.7A	1A	1.6A	2A	2.5A	Where integer number of Duty_Aux provides required Q
Vg	15V	15V	15V	15V	15V	
V _o	4.38V	3.99V	3.98	4.07	4.09	Output voltage points where Vc is balanced
Eff. of ZVS-3LB	92.33%	92.2%	92.33%	91.99%	90.84%	Efficiency of ZVS 3LB
Eff. of 3LB only	89.4%	91.66%	93.31%	93.84%	94.39%	Efficiency of normal 3LB with the same Vo
duty ratio	0.525	0.47	0.475	0.505	0.505	
∆d	0.035	0.02	0.015	0.005	0.005	Q1 duty = duty+ Δ d; Q2 duty = duty- Δ d;
Duty_Aux3	1	2	3	4	5	For the edge ZVS Q2. t=n*5ns, 5ns is the resolution.
Duty_Aux1	1	2	3	4	6	For the edge ZVS Q1
QbarF_AuxR3	3	2	1	0	-1	Q3 falling edge to Aux rising edge. ZVS Q2
QbarF_AuxR1	3	2	1	0	-1	Q4 falling edge to Aux rising edge. ZVS Q1
AuxF_QR3	4	4	4	3	3	Aux falling to Q2 rising
AuxF_QR1	6	6	6	6	5	Aux falling to Q1 rising
Auxa_ext1	6	12	15	20	25	On-time of Auxa minus on-time of Auxb at ZVS Q1 edge
Auxa_ext3	6	12	15	20	25	On-time of Auxa minus on-time of Auxb at ZVS Q2 edge

 Table 4.2. Parameters setup at various load.

 Table 4.3. Key components

Components	Details
L	IHLP-5050 (1.5 μH)
L _{aux}	IHLP1616BZ (100 nH)
C _{fly}	$4\mu\mathrm{F}$
Q_1 - Q_4	IRF8714 $\times 2$ (4 m Ω)
Aux _a , Aux _b	PMF250XNE $\times 2 (127 \text{ m}\Omega)$
D_a, D_b	CRS01
Main switches gate driver	UCC27201
Auxiliary switches gate driver	LMG1210
Controller	Terasic ADC-SOC FPGA

4.4 Hardware and Experimental Results

As a result of the optimization described in Section. 4.2, the selection of each device is listed in Table. 4.3. Particularly, auxiliary inductor L_{aux} is only 100nH, which is ~15X smaller than the main inductor *L*. Note that, to achieve the peak efficiency at heavy load, two paralleled devices are used for each power switch. To minimize the timing mismatch, two sets of devices are mirrored on each side of the demonstration PCB as shown in Fig. 4.8. The total footprint

area of the auxiliary circuit is around 20% of the conventional 3LB converter.

Using the test automation system, the maximum efficiency points for each operating condition were found and the feature of full ZVS was verified as shown in Fig. 4.9. The results clearly reflected that at 15 V/4V conversion, and 1 MHz switching frequency, the switching node voltage V_x ringings due to hard switching are significantly alleviated by the auxiliary circuit.



Figure 4.9. Measured waveforms @15 V/4 V, 1 A, 1 MHz condition (a) without ZVS, (b) with ZVS because of auxiliary circuit.

To compare the conventional 3LB converter and the proposed ZVS-3LB converter, their measured efficiencies (Exp.) are plotted in Fig. 4.10 together with analytical calculation results (Cal.) at 20V/5V and 15V/4V conversions and various loads. At light load conditions below 1.5A, the ZVS-3LB converter significantly reduces power loss by up to 30% and improves efficiency by up to 3%. The improvement is proved to come from the auxiliary circuit for ZVS, which occupies only 20% additional area. The peak light load efficiencies are 92.4% and 92.2% for 20V/5V and 15V/4V conversion at 1A and 0.7A, respectively.

4.5 Limitation of the Prototype

As already introduced in Section. 4.3, the parasitics on a PCB would add difficulties on rapid and precise sensing. Consequently, an open-loop design with parameter sweeping is necessary. To facilitate this system design requirement, an FPGA with NiosII embedded system is employed. However, the minimum time step on FPGA becomes one bottleneck of achieving optimized inductor dimension while maintaining high efficiency.

For a smaller L_{aux} , it requires shorter time period for ramping the auxiliary inductor current up to acquire adequate Q_{req} for complete soft-switching. Numerically, by using 10 nH auxiliary inductor, to achieve the desired soft-switching, the pulse width of Aux_b is 3.25 ns if assuming 15 V input voltage, 1 A load current, 10 nH L_{aux} and 15 nC Q_{req} , way lower than the 5 ns minimal time step on the FPGA. Therefore, a larger auxiliary inductor has to be selected, e.g. 100 nH. From the analysis in Section. 4.2, a longer Aux_b would cause higher conduction loss on the auxiliary circuit limiting the range where efficiency is benefited from ZVS. A comparison of the efficiency with different auxiliary inductor is conducted as shown in Fig. 4.11. The result suggests that when the pulse width boundary of PWM signals can decrease to a few nano-second or even sub-ns, the L_{aux} would drop to a lower value while the system efficiency as well as the power density would be greatly enhanced.



Figure 4.10. (a) loss breakdown at 20 V/5 V; and measured efficiencies of the proposed ZVS-3LB converter and their comparison with the conventional 3LB converter at (b) 20 V/5 V, (c) 15 V/4 V



Figure 4.11. The efficiency comparison of ZVS-3LB converter with (solid lines) 1.5μ H main inductor and 100nH auxiliary inductor, and the one (dash lines) with 500nH main inductor and 10nH auxiliary inductor

4.6 Summary

The proposed ZVS-3LB converter has been proved a good candidate for reducing the switching loss related to high side switches. The same idea of adding auxiliary circuit to assist soft-switching is also applicable for other topologies when there is a need for alleviating switching loss. As a consequence, it results in a higher switching frequency and thus smaller inductors. Despite the limitation of the prototype testing, analysis shown in Fig. 4.11 has indicated the possible improvement of the results when the circuit is built on a more compact package, e.g. an integrated circuit, and controlled more precisely. The potential value of inductance would rest in 100s of nH, which is a great improvement compared with the conventional buck converter, but is still outside the acceptable range for fully integration.

4.7 Acknowledgements

This chapter, in full, is a reprint of the material as it appears in 2023 IEEE Applied Power Electronics Conference and Exposition (APEC). Tianshi Xie; H. Le, "A Zero-Voltage-Switching 3-Level Buck Converter Achieving 30% Loss Reduction at Light Load for USB-C Charger Applications," APEC 2023. The dissertation author was the primary investigator and author of this paper.

Chapter 5 ITSAB Converter

5.1 **Operation Principle**

Figure 5.1 shows the power stage of the proposed ITSAB converter. It consists of 2 P-type power switches Q7 and Q8, 6 N-type power switches Q1-Q6, 3 flying capacitors C1-C3, and two nH-scale inductors L1 and L3. The proposed converter can be viewed as adding inductors to the flying capacitors of a Dickson-star Switched Capacitor (SC) converter. The presence of these inductors helps with soft-charging the flying capacitors to achieve high efficiency. Although the circuit topology resembles the prior works that rely on resonant operation [8, 34, 42, 43], the ITSAB converter has completely different operation that enables the use of nH-scale inductors at moderate switching frequency. A detailed comparison between these topologies has been presented in [44].

Operation of the ITSAB converter can be divided into 4 different states as shown in Fig. 5.2. Two phases of gate control signals, $\phi - \phi_b$ and $\phi_S - \phi_{Sb}$, are illustrated in Fig. 5.3 together with the waveforms of the inductor currents and flying capacitor voltages. Phase $\phi_S - \phi_{Sb}$ is shifted by t_{ϕ} from $\phi - \phi_b$ forming the two phase-shift states 2 and 4. The operation of capacitor charge transfer in the ITSAB converter resembles that of a Dickson-Star SC converter. The key difference is that the charge transfer between flying capacitors and to the output in the Dickson-star SC converter is hard-charging, while it is soft-charging in the ITSAB using the two inductors L1 and L3. As a result, the steady-state voltages across C1, C2 and C3 are $\frac{3Vin}{4}$,



Figure 5.1. Schematic of the proposed integrated transformerless stacked active bridge (ITSAB) converter.

 $\frac{V_{in}}{2}$ and $\frac{V_{in}}{4}$, respectively. Unlike the resonant switched-capacitor (ReSC) converters in [45][8], the passive components values are selected to ensure the *L*1 (*L*3) and *C*1&*C*2 (*C*3&*C*2) tank resonant frequency is well below the switching frequency f_s .

State 1: Unique in this converter, both inductors carry near-constant $I_o/2$ currents to the output as a result of near-zero voltage across each of the inductors during this time interval. Subsequently, both flying capacitor C1 and C3 are softly charged, while C2 is softly discharged, all by the same amount of current.

State 2: This phase-shift time interval starts when all the switches toggle except for Q3 and Q4. As a consequence, both inductors have $V_L = -V_{in}/4$ across them, which is the voltage of C3 for L3 and the voltage difference across C1 and C2 for L1. As the result, both inductor



Figure 5.2. Power stage operation of the ITSAB converter.



Figure 5.3. Operating waveforms in the ITSAB converter.

currents ramp down to zero then continue ramping up in the opposite direction. All flying capacitors shift their charging/discharging status during this time interval when the inductor currents are zero. This state ends when Q3, Q4 change their ON/OFF status as the inductor current reaches approximately $-I_o/2$.

State 3: This state is the same as State 1, except the inductor currents are flipped. The capacitors' charging status is also altered.

State 4: Inductor currents are flipped compared with State 2. The voltages across the inductors are $V_L = V_{in}/4$.

During the two phase-shift states 2 and 4, the inductor currents ramp up and down at a same rate of

$$\frac{di_L}{dt} = \frac{V_{in}}{4L} \tag{5.1}$$

From charge balance, the output current I_o can be found as [44]

$$I_o = \frac{V_{in}}{8Lf_s}\phi(1-\phi) \tag{5.2}$$

where $\phi = \frac{2t_{\phi}}{T_s}$ is the phase shift ratio. Therefore, by modulating t_{ϕ} , I_o and thus the output voltage V_o can be regulated.

The length of t_{ϕ} depends on the inductance, the load current, as well as the input voltage. Practically, it is in nano-second range when using nano-Henry inductors, considering 1 A output current and 12 V input voltage. The trapezoidal shape with ripple-free maximum values of currents I_{L1} and I_{L3} not only leads to small RMS value and thus small conduction loss but also results in small output voltage ripple. This eventually reduces the required output capacitance, which further contributes to high power density.

5.1.1 Soft-switching

As an additional benefit of flipping the inductor current flowing direction, at the end of states 2 and 4, the switching node V_{sw1} is fully soft-discharged and soft-charged by the inductor



Figure 5.4. Zero-voltage-switching (ZVS) of Q3 and Q4 between (a) state 4 and 1, (b) state 2 and 3.

currents during the deadtime of ϕ_S and ϕ_{Sb} , as illustrated in Fig. 5.4. Consequently, Q3 and Q4 are fully soft-switched. On the contrary, I_{L3} and I_{L1} keep forward biasing the body diodes of Q2 and Q1 during the deadtime between state 1 and 2, and the one between state 3 and 4, respectively. In other words, Q1 and Q2 are hard-switched. Meanwhile, the switches Q5-8 are partially soft-switched, because the switching nodes V_{x1-3} are partially charged/discharged by I_{L1} and I_{L3} while being impacted by hard-switching node V_{sw2} . As an example, V_{x1} , also the Source of Q7, is soft-charged to V_{in} by I_{L1} once Q7 is turned off at the end of state 3. When Q1 and Q2 hard-switch in state 4, V_{x2} jumps from $3V_{in}/4$ to $V_{in}/2$ causing drain-to-source capacitor C_{ds} of Q7 being hard-charged. Even though Q1 and Q2 are hard-switched, they are half the size of the switches Q3 and Q4 because they carry half the current and thus cause lower additional switching loss. Because of the ZVS operation, the ITSAB power stage can operate efficiently at switching frequency in the Mega-Hertz range.

5.2 Loss Analysis of ITSAB Converter



Figure 5.5. Definitions of parameters f_o , R_o and R_s .

The power loss of the ITSAB converter can be categorized into two major parts: conduction loss and switching loss. To accurately calculate the conduction loss, the RMS value of the inductor currents is needed. In this section, the detailed loss analysis with RMS current value calculation is given.



Figure 5.6. Calculated I_{L1} , I_{L3} waveforms for 12 V-to-3 V conversion, 2 A load, $f_s = 3.8$ MHz, with $R_s = 55$ m Ω .

5.2.1 Conduction Loss

The conduction loss can be written as

$$P_{cond} = \sum_{i=7}^{8} R_{on_i} I_{L1,RMS}^2 + \sum_{i=5}^{6} R_{on_i} I_{L3,RMS}^2 + \sum_{i=3}^{4} R_{on_i} I_{L1+L3,RMS}^2 + R_{on_2} I_{L3,RMS}^2 + R_{on_1} I_{L1,RMS}^2$$
(5.3)

where R_{on_i} is the on resistance of each power switch and $I_{L1,RMS}$, $I_{L3,RMS}$ are the RMS current of L1 and L3, respectively. While the inductor currents are displayed in Fig. 5.3 as having an ideally trapezoidal shape with $\frac{I_o}{2}$ value at the top, they slightly curve in practice owing to the response of the LC tank network with parasitic series resistances. To improve the conduction loss model accuracy, it is necessary to include these effects in calculation of the current RMS values.

The approach described in [46] can be extended to the 4-to-1 ITSAB circuit. In each switch state shown in Fig. 5.2, the state-space equation can be written as

$$\dot{x} = A_i x + b_i V_{in}, i = 1, 2, 3, 4 \tag{5.4}$$

where $x = [i_{L1}, i_{L3}, v_{C1}, v_{C3}, v_{C2}, v_O]^T$ and

$$A_{1} = \begin{vmatrix} -\frac{Rs}{L1} & 0 & -\frac{1}{L1} & 0 & 0 & -\frac{1}{L1} \\ 0 & -\frac{Rs}{L3} & 0 & -\frac{1}{L3} & \frac{1}{L3} & -\frac{1}{L3} \\ \frac{1}{C1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{C3} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{C} & \frac{1}{C} & 0 & 0 & 0 & 0 & -\frac{1}{R_{LCG}} \end{vmatrix}, b_{1} = \begin{vmatrix} \frac{1}{0} \\ 0 \\ 0 \\ \frac{1}{C} & \frac{1}{C} & 0 & 0 & 0 & 0 & -\frac{1}{R_{LCG}} \end{vmatrix}$$

(5.5)

where R_s is the total loop resistance as drawn in Fig. 5.5.

To numerically compute the inductor currents at various time, the augmented state-space approach [47] is utilized in this work. It provides a viable way to accurately calculate the exact solution of the aforementioned state-space equations, especially when the system matrix A_i is singular. Examples of the computed I_{L1} and I_{L3} are plotted in Fig. 5.6, assuming $R_s = 55 \text{ m}\Omega$


Figure 5.7. Loss model verification: (a) efficiency comparison; Loss breakdown at (b) 0.5 A load, (c) 2 A load

. When the load current is 2 A, and each inductor delivers 1 A on average, with ~ 1 A ripple during states 1 and 3. One may note how the exact RMS values of these currents are higher than ideal $I_o/2$, which yields a more accurate conduction loss evaluated from (5.3).

5.2.2 Switching Loss

Switching loss is primarily related to power switches gate capacitances C_{gg} and the parasitic capacitors of the switching nodes, namely, C_{X1} , C_{X2} , C_{X3} , C_{SW1} , C_{SW2} . The gate capacitors switching loss can be written as

$$P_{gate} = \sum_{i=1}^{8} C_{ggi} V_{DD}^2 f_s$$
(5.6)

where V_{DD} is the V_{GS} driving voltage for the power switches. As mentioned in Section 5.1, Q5-Q8 are partially soft-switched, Q3 and Q4 are fully soft-switched, while Q1 and Q2 are completely hard-switched. Therefore, the total switching loss of the switching nodes parasitic capacitors can be approximated by

$$P_{coss} = \left(\sum_{i=1}^{3} \frac{1}{2} C_{Xi} + C_{SW2}\right) V_{ds}^2 f_s$$
(5.7)

where $V_{ds} = V_{in}/4$. The last part of switching loss is caused by V-I overlap [48] during the hard-switching transitions of Q1,2 and partial hard-switching transitions of Q5-8, which can be expressed by

$$P_{tr} = P_{tr,on} + P_{tr,off} = 8 \times \frac{1}{2} V_{ds} I_{L1,3} t_{tr} f_s$$
(5.8)

Where the transition time t_{tr} indicates V-I overlap time. Practically in this design, the P_{tr} is small compared to other losses and therefore can be ignored. This is because: 1) the nature of Dickson-Star SC that blocks most of V_{in} so that Q1,2 are only stressed by $\frac{V_{in}}{4}$; and 2) t_{tr} is short as for small power switches. However, this type of loss can't be ignored when the ITSAB converter is designed for heavier loads that the power switches are large enough to have significant reverse

recovery charge and long t_{tr} .

5.2.3 Inductor Loss

Considering an air-core inductor realization, no core loss needs to be accounted for. As a consequence, only DCR and ACR loss of the inductor are considered. The DCR loss is given by

$$P_{ind,DCR} = (I_{L1,RMS}^2 + I_{L3,RMS}^2)R_{DC}$$
(5.9)

where the RMS value of the inductor currents can be computed by the same approach as described in Section 5.3. The AC loss of the specific inductor can be calculated using the model provided by the manufacturer. For simplicity, the inductor loss is approximated as the DCR loss, $P_{ind} \approx P_{ind,DCR}$.

The loss model is verified by comparing the calculated efficiency to the CAD simulated results, as shown in Fig. 5.7, which shows a good match. Two pie charts of the loss breakdown computed at 12 V/3 V, $f_s = 3.8$ MHz, 0.5 A and 2 A loads are given in Fig. 5.7(b) and Fig. 5.7(c), respectively. In the calculation, switching loss P_{sw} includes P_{coss} and P_{tr} . It can be seen that the frequency related losses, mostly P_{sw} and P_{gate} , dominate the total loss at light load as a result of less conduction loss and insufficient I_o for completely soft-switching Q3 and Q4 [44]. At heavy load, however, the conduction loss dominates and it is therefore crucial to properly design and operate the converter so that the minimum RMS value of the inductor currents is achieved. In the next section, the optimization methodology is discussed in more detail.

5.3 Optimization Methodology

As discussed in Section 5.2, two major loss mechanisms in the ITSAB converter are conduction loss P_{cond} and switching loss $P_{sw} + P_{gate}$. To minimize the power loss at a desired operating point, one can sweep the converter parameters, including power switch area, switching frequency, etc. However, as shown in Fig. 5.6 and the analysis in Section 5.2, the shape of the



(a) Inductor current waveshape for Q = 2.6 and three different values of $k = f_s/f_o$. The minimum $I_{L1,RMS}$ is obtained for k = 2.4391.



(b) Summary of inductor current waveshapes in the *k* versus *Q* plane. The minimum-RMS cases are highlighted (red dots).

Figure 5.8. Illustration of how inductor RMS current is minimized using $k = f_s/f_o$ and $Q = R_o/R_s$ as optimization parameters.



Figure 5.9. Optimum *k* as a function of *Q* for $R_s = 50 \text{ m}\Omega$ and $I_o = 2 \text{ A}$.

inductor currents is sensitive to the system parameters. In other words, even though a minimum power loss can be found by sweeping the parameters, the inductor currents could potentially be distorted and peaking above the saturation current of the inductors, potentially resulting in malfunction of the circuit. In this section, a novel optimization methodology is proposed so that the inductor current distortion can be prevented while the minimum power loss is found.

5.3.1 Minimum RMS Current

As marked in Fig. 5.5, the resonant frequency and the characteristic impedance of each resonant tank are

$$f_o = \frac{1}{2\pi\sqrt{LC}}, \text{ and } R_o = \sqrt{\frac{L}{C}}$$
(5.10)

where *L* is the inductance of L1 and L3, and *C* is the capacitance of C1 and C3. Two variables are used in this optimization process, k defined as

$$k = \frac{f_s}{f_o} \tag{5.11}$$

and the quality factor Q,

$$Q = \frac{R_o}{R_s} \tag{5.12}$$

where R_s is the total loop resistance as indicated in Fig. 5.5, which comprise of power switches on-resistance and ESR of L and C. Given the above equations, the inductance and the capacitance of each resonant tank can be expressed in terms of k and Q

$$L = \frac{kQR_s}{2\pi f_s}, \ C = \frac{k}{2QR_s\pi f_s}$$
(5.13)

where f_s is the switching frequency. In order to find the parameters that make the inductor current achieve minimum RMS value, a relation between k, Q and R_s must be carried out. The augmented state-space approach [47] is utilized to obtain the inductor current waveforms for various sets of parameters and operating conditions. For instance, when Q is picked as 2.6 and R_s equals 50 m Ω , current I_{L1} can be depicted in Fig. 5.8(a) for different k at 12 V-to-3 V, 2 A conversion at $f_s = 1$ MHz. The highlighted curve, which is for k = 2.4391, has minimum RMS value. When k is smaller, indicating a smaller L and the resonant frequency becomes closer to the switching frequency, the curve has higher ripple. On the other hand, an increased k means larger L and slower current ramping speed during states 2 and 4, leading to shorter times for states 1 and 3, and thus higher peak current to deliver the same load current. Neither of the above two cases can achieve smaller RMS value than the optimal point. A brief summary of how k and Q value change along with the circuit parameters L, C and R_s , and the resulted inductor current shapes is listed in Fig. 5.8(b). The dots on each column indicate the corresponding minimum RMS value point. Once a range of Q is evaluated in the same manner, the targeted parameters relation can be depicted by using a curve fitting method as shown in Fig. 5.9. It sets the boundary of distortion region which is the area below the curve. This curve-fit relation is:

$$k = 3.64Q^{-1.14}R_s^{0.02} + 1 \tag{5.14}$$

From this equation, it can be seen that the optimum k approaches 1 when Q approaches infinity, which represents operation at resonance. For a practical, finite Q, the optimum is for above-resonance k > 1 operation.

5.3.2 Optimization Process

Semiconductor parameters are scaled with the device area A_s

$$R_{on} = \frac{R_{on,sp}}{A_s} \tag{5.15}$$

$$Q_{tot} = (Q_{gg,sp} + Q_{dd,sp} + Q_{ss,sp})A_s$$
(5.16)

where $R_{on,sp}$, $Q_{gg,sp}$, $Q_{dd,sp}$ and $Q_{ss,sp}$ are the density of on-resistance, gate charge, drain charge and source charge per unit area, respectively. After plugging these scaling equations (5.15) and (5.7), the loss modeling expressions (5.3), (5.6) and (5.7) can then be formulated as

$$P_{cond} = \frac{P_{cond,sp}}{A_s} \tag{5.17}$$

$$P_{gate} = P_{gate,sp}A_s \tag{5.18}$$

$$P_{sw} = P_{sw,sp} A_s \tag{5.19}$$

where $P_{cond,sp}$, $P_{gate,sp}$ and $P_{sw,sp}$ represent the specific power loss per unit area. As a result, the total power loss can be turned into the following form

$$P_{tot} = \sum_{i=1}^{8} \left(\frac{P_{cond, sp_i}}{A_{s_i}} + (P_{gate, sp_i} + P_{sw, sp_i})A_{s_i} \right) + P_{ind}$$
(5.20)

The optimization process of the proposed circuit can be expressed in the following form

minimize
$$P_{tot}(X)$$

subject to $\sum_{i=1}^{8} A_{s_i} \le A_{tot,max}$
 $A_c \le A_{c,max}$
 $L \le L_{max}$
(5.21)

where the vector $X = [f_s, A_s, R_o]^T$ denotes the variables in this converter design, and $A_{tot,max}$, $A_{c,max}$ and L_{max} set the limit of chip area, capacitor footprint area and inductor value, respectively. Mathematically, minimizing P_{tot} in (5.20) is a geometric programming (GP) problem [49], which can be efficiently solved using tools such as CVX[50]. The optimization results, including selected passive component, switching frequency and areas of the power switches, are summarized in Table 5.1.

Parameters	Details
L	$10\mathrm{nH}$
C1, C3	$1.05\mu\mathrm{F}$
C2	$4\mu{ m F}$
$\mathbf{f}_{\mathbf{s}}$	$3.35\mathrm{MHz}$
Q1, Q2 area	$0.1139\mathrm{mm}^2$
Q3, Q4 area	$0.2007\mathrm{mm}^2$
Q5, Q6 area	$0.1194\mathrm{mm}^2$
Q7, Q8 area	$0.1373\mathrm{mm^2}$

 Table 5.1. Optimization results.

5.4 Implementation Details

The system block diagram including all key blocks and the power stage is shown in Fig. 5.10. The blocks within the solid rectangle are implemented on-chip, while the passive components between the solid line and dash line are on the package substrate. The key on-chip blocks include Gate Drivers (GD1-GD8) and their voltage-supply Linear Regulators (LR); Ramp Generator (RG) for synchronization and generating ramp signal at the required frequency; Phase Shift Modulator (PSM) for closed-loop regulation; Non-overlapped Signal Generator for dead-time control; serial Programming Register for setting up frequency and dead time; Biasing circuit for creating bias currents for other sub-blocks throughout the chip; OTA-based Error Amplifier (EA) to form the closed-loop controller. In the following parts of this section, the control signal generation, gate drivers and LRs, control loop design and synchronization are addressed separately.



Figure 5.10. Block diagram of the prototype ITSAB converter realized on a 130 nm BCD die flip-chipped on a package substrate together with power capacitors and two 10 nH inductors.



Figure 5.11. Phase-shifted control signal generation, including block diagrams of (a) Ramp Generator (RG), and (b) Phase Shift Modulator (PSM); (c) timing diagram: generation of ϕ , ϕ_S .

5.4.1 Control Signal Generation

As derived in (5.2), the output voltage of the implemented ITSAB converter is fully regulated by the phase shift between ϕ and ϕ_S . These control signals are generated from the Phase Shift Modulator (PSM) and the Ramp Generator (RG), illustrated in Fig. 5.11 along with the timing diagram of the corresponding signals. Particularly, ϕ_S is periodically toggled by the rising edge of cmp_out (Fig. 5.11(b)) which is the output of comparing the error voltage m with the ramp signal from RG (Fig. 5.11(a)). Meanwhile, ϕ is triggered by the rising edge of the *clk* signal after a short delay as indicated in Fig. 5.11(c). Practical delay mismatches on signal paths from this control circuit to the final power gates would potentially result in delay mismatch on ϕ and ϕ_S at the converter power stage, which could lead to a minimum phase shift that is larger than zero even if the control circuit sets the phase shift to zero. A non-zero minimum phase shift would limit the minimum load current that the converter can support, following (5.2). To ensure that the converter can support an output current at light load current down to open-circuited load, it is desirable to generate a negative phase shift value of t_{ϕ} at this control circuit to compensate for any possible timing mismatch in the signal paths. Therefore, a short delay of ~ 15 ns is added after *clk* to give ϕ the phase shift offset for this purpose, as shown in Fig. 5.11(c).

The sawtooth waveform is generated by charging a MiM capacitor with I_{chrg} between ground and V_H . In this design $V_H = 1.5 - 2.5V$ depending on the target frequency, while V_{DD} can vary from 3V to 5V to optimize switching loss and conduction loss of power switches. From (5.2), it can be seen that the maximum output current is achieved when the phase shift time $t_{\phi} = \frac{T_s}{4}$. $t_{\phi} > \frac{T_s}{4}$ will lead to smaller average output current, because there is not sufficient time left after the phase shift to allow inductor currents to flow to the output. Therefore, the comparison to generate ϕ_S for phase-shift control need only be in the lower half of the sawtooth waveform. Exploiting this characteristic to get better noise immunity in generating the phaseshift control signal, *ramp_pre* is generated by a second branch with two times larger charge current. Finally, *ramp_pre* is slightly shifted up using a source-follower stage to make the *ramp* signal that fits in the input common-mode range of the comparator inside PSM and the output range of Error Amplifier, m. To prevent the regulation loop from falling into a positive feedback when $t_{\phi} > \frac{T_s}{4}$, the phase shift must be limited using the signal max_ ϕ_S , which is a byproduct of ramp generation, by comparing the intermediate signal *ramp_pre* with the peak voltage V_H of the sawtooth signal.

Another feature of this design is to allow multiple converter chips to be synchronized, possibly in an interleaved manner, to improve the output current capability. This synchronization mode, therefore, is added and enabled by turning *SYNC_EN* of the RG to 1. With this setting, the current chip frequency can be triggered and synchronized by an off-chip clock signal to *SYNC_IN*. The synchronization is proved to work well when two chips are synchronized to the same frequency with 180° out of phase operations, as shown in the experiments in Section 5.5.

5.4.2 Gate Drivers & Supply Voltage Generation

The gate driver design is more challenging for a multi-level converter, because of flying source voltages and a larger number of power switches. The source voltages and the required gate voltages during ON and OFF state of all the power switches are listed in Table 5.2. There are four additional voltage levels defined as follows:

$$V_{SS8} = V_{in} - V_{DD}$$

$$V_{SSL7} = V_{SS8} - V_o$$

$$V_{DD5} = V_{DD} + V_o$$

$$V_{DDH6} = V_{DD5} + V_o$$
(5.22)

A bootstrap driver containing a large capacitor and a diode is the most common way of driving high-side switches. However, the large bootstrap capacitor and diode would require to be offchip and take significant area and volume. Furthermore, the diode voltage drop may reduce the available gate-drive swing resulting in weak driving, especially in the context of a multi-level



Figure 5.12. Block diagrams of (a) subtraction LR to generate V_{SS8} and V_{SSL7} , (b) summation LR to generate $V_{DD5}(V_{DDH6})$, (c) gate driver for Q6, and (d) level shifter.

Switch	Switch Source Voltage		Gate Voltage		
Switch	ON	OFF	Low Swing	High Swing	
Q8	V _{in}	V _{in}	V _{SS8}	V _{in}	
	(9.6V)	(9.6V)	(6V)	(9.6V)	
Q7	3V _{in} /4	V _{in}	V _{SSL7}	V _{in}	
	(7.2V)	(9.6V)	(3.6V)	(9.6V)	
Q6	V _{in} /2	V _{in} /4	V _o	V _{DDH6}	
	(4.8V)	(2.4V)	(2.4V)	(8.4V)	
Q5	V _{in} /4	V _{in} /4	V _o	V _{DD5}	
	(2.4V)	(2.4V)	(2.4V)	(6V)	
Q1,4	V _{in} /4	gnd	gnd	V _{DD5}	
	(2.4V)	(0V)	(0V)	(6V)	
Q2,3	gnd	gnd	gnd	V _{DD}	
	(0V)	(0V)	(0V)	(3.6V)	

Table 5.2. Source and gate voltages of each switch during ON and OFF intervals, when the ITSAB prototype is generating $V_o = 2.4$ V from $V_{in} = 9.6$ V.

power converter with multiple high-side switches. To overcome these obstacles, and to guarantee the same voltage (V_{DD}) over Vgs during ON time, regardless of the value of V_{in} and V_{DD} , fixed-level gate driver structures with the driver supply voltages generated by Linear Regulators (LR) are designed and presented in Fig. 5.12.

The subtraction LR, which is implemented using two high-voltage op-amps as shown in Fig. 5.12(a), is used to generate V_{SS8} and V_{SSL7} . The stacked structure is used to reduce power loss as the efficiency of a linear regulator depends on the voltage difference between its input and output voltage. Another type of LR is for summation function as shown in Fig. 5.12(b). Two circuits of this type are used for supplying V_{DD5} and V_{DDH6} , respectively. Two high voltage op-amps are connected so that the reference voltages are summed at the output. Both of the



Figure 5.13. Magnitude and phase responses of the loop gain obtained under the following conditions: $V_{in} = 9.6$ V, $V_o = 2.38$ V, with ~ 74 nH parasitic inductors of the wire loops used to measure inductor currents, as shown in Fig. 5.15.

Subtraction LR and Summation LR recycle the residual charge to Vo to further save power.

As an example, the gate driver for Q6 is given in Fig. 5.12(c). To minimize the shootthrough current at the last stage of the gate driver, the PMOS M1 and NMOS M4 are driven separately with dead time created by the Non-overlapped Signal Generator at the front of this block. Stacking of M2 and M3 reduces the voltage stress on M1 and M4 so that when driving the switches with flying source voltages, such as Q1, Q4, Q6, Q7, it is still safe to use only thin-oxide devices. M2 and M3 are not present in the GD2, GD3, GD5 and GD8 gate drivers.

Level shifters are also required for non-gnd referenced signals. The level shifter circuit is shown in Fig. 5.12(d), with MiM capacitors used for coupling signals between different voltage domains with minimal latency while keeping small area. The cross-coupled inverters at the high voltage domain employ weak NMOS transistors to ensure rail-to-rail operation[9].

5.4.3 Control Loop Design

To achieve a large DC gain and adequate phase margin as well as fast closed-loop transient response, a proprtional-integral (PI, or Type-II) compensator is used in this implementation, as indicated in Fig. 5.10. An on-chip OTA together with PSM enables the design of a fast analog control loop. The control-to-output transfer function can be expressed as [46]:

$$\hat{I}_{o} = \frac{V_{in}}{8Lf_{s}}\hat{\phi} = K_{\phi}\hat{\phi}$$

$$G(s) = \frac{\hat{V}_{o}}{\hat{\phi}} = K_{\phi}\frac{1}{1+s/\omega_{p}}$$
(5.23)

where $\omega_p = 1/R_L Co$. The transfer function of the compensator is

$$G_c(s) = G_0 \frac{1 + \omega_z/s}{1 + s/\omega_p}$$
(5.24)

where $G_0 = \frac{R2}{R1+R2} \frac{g_m R3C5}{C4+C5}$, $\omega_p = \frac{C4+C5}{R3C4C5}$ and $\omega_z = \frac{1}{R3C5}$. The position and connection of the passive components R1-R3, C4 and C5 are shown in Fig. 5.10. As part of the control loop, the ramp signal has the gain of $\frac{1}{2V_H}$, because its peak voltage is about $2V_H$. Consequently, the loop gain can be written as

$$T(s) = \frac{R_2}{R_1 + R_2} G_c(s) G(s) \frac{1}{2V_H}$$
(5.25)

The system is designed to work at a switching frequency f_s of around 3 MHz. The crossover frequency f_c is selected to be around $f_s/10$. As (5.23) suggests, the system dynamic performance is highly related to the circuit parameters L, f_s , output capacitor *Co* and load current. Based on the values of passive components in the converter, the Type-II compensator is designed for 3 MHz switching frequency and 0.2 A load is listed in Table 5.3. The Bode plot of the computed loop gain is shown in Fig. 5.13. The designed 150 kHz crossover frequency and 60° phase margin ensure the fast response and stable operation.

Component	Details
C4	$2.7\mathrm{pF}$
C5	$56\mathrm{pF}$
R3	$43.2\mathrm{k}\Omega$
R1,2	$80\mathrm{k}\Omega$

Table 5.3. Components for the PI (Type-II) compensator.

5.5 Experimental Verification

Component	Details
C1	2 x 2.2 µF, 16 V 0402 (0.92 µF)
C2	2 x 10 µF, 10 V 0402 (4 µF)
C3	$1 \ge 2.2 \mu\text{F}, \ 6.3 \ge 0.201 \ (0.87 \mu\text{F})$
Cin(V _{in})	2 x 10 µF, 25 V 0603 (3.2 µF)
Cin(V _{DD})	1 x 10 μF, 10 V 0402 (2 μF)
Со	3 x 2.2 µF, 6.3 V 0201 (2.6 µF)
Decap(V _{DDH6})	2 x 1 µF, 16 V 0201 (304 nF)
Decap(V _{DD5})	1 x 1 µF, 16 V 0201 (244 nF)
Decap(V _{SS8})	2 x 1 µF, 16 V 0201 (408 nF)
Decap(V _{SSL7})	1 x 1 µF, 16 V 0201 (353 nF)
L1,3	10 nH, Ferric IPD or 0807SQ-Coilcraft

Table 5.4. Components on the package substrate

The demonstration chip was fabricated in TSMC 130 nm BCD technology with a dimension of $1.9 \text{ mm} \times 1.7 \text{ mm}$ as indicated in Fig. 5.14(a). Aiming for a high power density performance, the chip die was flip-chipped on a $6.5 \times 6.5 \text{ mm}^2$ 6-layer organic package substrate



6.5mm 6.5mm Cin L1 Decaps Decaps 6.5mm 6.5mm Die Die Decaps Decaps R Cin 0.85mm without thinning (only 50µm metal+ Package Package substrate/ magnetic core) substrate 1.524mm **(b)** (c)

Figure 5.14. Top and side views of the ITSAB converter prototype, including (a) $1.7 \text{ mm} \times 1.9 \text{ mm}$ die in a 130 nm BCD process, and $6.5 \text{ mm} \times 6.5 \text{ mm}$ package substrate with flip-chipped die, power capacitors and $2 \times 10 \text{ nH}$ (b) IPD inductors or (c) discrete (Coilcraft 0807SQ-10N) inductors.



Figure 5.15. Current-measurement test setup with wire loops having (\sim 74 nH parasitic inductances serving as the power-stage inductors.

together with flying capacitors, decoupling capacitors, input/output capacitors and two 10 nH IPD inductors supplied by Ferric Semiconductor. Note that the IPD inductors promise much higher power density because they achieve a thickness of $50 \,\mu$ m (plus $800 \,\mu$ m unused silicon carrier as shown in Fig. 5.14(b), which could be thinned substantially down to $10s \,\mu$ m), while discrete inductors are 1.524 mm thick. While the flip-chip die has $180 \,\mu$ m bump pitch, the organic substrate has a ball grid array (BGA) with 0.8 mm ball pitch to reduce the cost of the PCB. The details of each passive component are listed in the Table 5.4. The selection of the flying capacitors guarantees the actual capacitance of C1 and C3 are equal and much less than the one of C2 so that R_o and f_o are relatively the same for both LC tanks.

Since the design uses small inductors of 10 nH, any additional wires with reasonable length added in series with the inductors to allow measuring their currents with current probes would add significant inductance beyond the inductors themselves. Therefore, to measure



Figure 5.16. Measured steady-state converter waveforms, with 74 nH inductors due to the current measurement setup shown in Fig. 5.15. The converter operates at $V_{in} = 9.6$ V, $V_o = 2.38$ V, $I_o = 1$ A.

operational waveforms, the inductors are removed and replaced by two short wire loops, as shown in Fig. 5.15. Note that even when the two wires are sized just long enough to clip the small current probes, their effective parasitic inductances are approximately 74 nH, which is about $7 \times$ more than the inductors used on the package. The significantly larger wire-loop inductances limit the maximum load current according to (5.2), but the measurement setup still provides verification of operating waveforms at light to medium loads.

Fig. 5.16 shows the measured steady-state waveforms of switching node voltages V_{sw1}



Figure 5.17. Step-load transient responses of the experimental prototype operating at $f_s = 3$ MHz with wire loops as inductors. The output voltage is closed-loop regulated at $V_o = 2.38$ V from $V_{in} = 9.6$ V.



Figure 5.18. Line transient responses of the experimental prototype operating at $f_s = 3.8 \text{ MHz}$ with 10 nH discrete inductors. The output is closed-loop regulated at $V_o = 2.5 \text{ V}$.

and V_{sw2} , flying capacitor voltages, and inductor currents. The noticeable time difference between V_{sw1} and V_{sw2} is the phase shift time t_{ϕ} , during which the inductors are magnetized or demagnetized with a slope of ~ 30 mA/ns, implying ~ 74 nH parasitic inductance of the wires. The top of I_{L1} and I_{L3} is not as flat as in the theoretical waveforms in Fig. 5.3 due to the additional path resistances. The flying capacitor voltages are measured by a differential probe, verifying the soft charging behavior as discussed in Section 5.1.

The synchronization feature is verified by running two ITSAB converters at the same time. The phases are set up by two 180°-phase-shifted synchronization control signals generated by an FPGA, which also allows any other amount of phase shift if more converters are connected in parallel for a larger load. The measured switching node V_{sw2} voltages and the inductor current I_{L1} for both converters are shown in Fig. 5.20.

Fast closed-loop load-step transient performance is measured at a 9.6 V-to-2.38 V conversion and $f_s = 3$ MHz, with 0.5 A and 0.4 A load steps. The waveforms of AC-coupled and DC-coupled output voltage V_o , output current I_o and inductor current I_{L1} are shown in Fig. 5.17. Corresponding to the two load steps, the output voltage takes 4.4 μ s and 3.6 μ s to settle within 1%, and the undershoot and overshoot during the step-load transients are within 2% of its DC value. In addition, the full-load step response is performed on the prototype with 10 nH discrete inductors as in Fig. 5.14(c). V_o settles in 14 μ s and 10 μ s with undershoot and overshoot of 30 mV when responding to 2.3 A step load as shown in Fig. 5.19.



Figure 5.19. Step-load transient responses of the experimental prototype with 10 nH discrete inductors operating at $f_s = 3.8$ MHz. The output is closed-loop regulated at $V_o = 2.38$ V from $V_{in} = 9.6$ V.

A line transient of 1.4V step at V_{in} is performed on the experimental prototype that has two 10 nH discrete inductors and operates at 1 A load. As shown in Fig. 5.18, V_o is well regulated to 2.5 V with no significant fluctuation.

On Fig. 5.21, the efficiency is measured with two types of inductors, discrete and IPD, at various input/output voltages and different switching frequencies. The peak efficiency measured at 9.6 V/2.33 V, 3.1 MHz, 1 A load is 92.4% with 0807SQ discrete inductors, compared to 91.2% with Ferric IPD inductors at 0.6 A. The efficiency difference between the prototypes with two types of inductors is largely caused by the large difference in the inductor series resistance, which is consistent with the differences in size. Fig. 5.21(a) also illustrates the matched efficiency between analytical calculation and simulation in addition to the drop in measurement results. This is largely due to the paths and connection resistances and capacitances in both the silicon layout and the PCB prototype. The additional resistances not only directly affect the conduction loss, but also result in larger inductor current ripple, which further increases the total conduction



Figure 5.20. V_{sw2} and I_{L1} of two converters (m and s) with 180° phase shift @8.2 V/2 V, 3.8 MHz, 1 A (0.5 A each), open-loop.

loss across all the load conditions. An additional efficiency calculation considering 50% more on-resistance due to paths and vias on silicon layout, 30% more parasitic resistance from PCB routing and connection, and 40% more parasitic capacitance from dense layout routing is plotted in Fig. 5.21(a), which shows an excellent match with the measurement results.

The converter prototype is also tested for its output regulation range. As shown in Fig. 5.22, the output voltage ranges from 2.15 V to 2.65 V and 2.7 V to 3.3 V with an input voltage of 9.6 V and 12 V, respectively. The lower boundary of V_o is set once there is zero phase shift, meaning the inductors couple and resonate directly with C1,3 and deliver charge to the output during states 1 and 3, without the charging states 2 and 4. The peak values of I_{L1} and I_{L3} are determined by f_o and f_s as well as the desired load current. On the other hand, the upper boundary is limited by maximum phase shift ratio which is one quarter of T_s as calculated from (5.2) and Section 5.4.1. Hence, the upper boundary of V_o can be larger than the measured values. However, as the inductors are charged longer time during states 2 and 4, the peak current can be



Figure 5.21. Efficiency vs. load current I_o measured at (a) $V_{in} = 12$ V, $f_s = 3.8$ MHz, various V_o , (b) $V_{in} = 12$ V, $V_o = 2.93$ V, various f_s , and (c) $V_{in} = 9.6$ V, $V_o = 2.33$ V, various f_s .



Figure 5.22. Efficiency vs. V_o at 1 A load, 3.8 MHz f_s .

significantly larger than the saturation current of small and integrated inductors. The large RMS currents in steady state can also be detrimental for other parts of the circuits, including the power switches and capacitors. To keep the prototype in a safe and reliable operating region, a range of $\pm 10\%$ of nominal V_o is chosen for the demonstration. In this output voltage range, the efficiency remains greater than 75%.

The power density of the prototype with IPD inductors is about two times larger compared to the power density of the prototype with the discrete inductors, counting the volume of the active die, the flying capacitors and the effective inductors volume. The comparison is summarized in Table 5.5. One may note that the package substrate is not fully optimized for power density. As indicated by the thermal image in Fig. 5.23, which is measured at 12 V/2.93 V, 3.8 MHz, 1.5 A load, the temperature rise is only $10^{\circ}C$ from the room temperature. This suggests that a finer pitch package design can be utilized to further increase the overall power density.

The connection of inductors and capacitors in ITSAB converter form resonant tanks similar to well-known resonant converters. In contrast, however, the ITSAB converter requires



Figure 5.23. Thermal image of the prototype with IPD inductors taken at full load (1.5 A). **Table 5.5.** Performance comparison: discrete vs. IPD inductors

Ind	luctor	V _{in} /V _o	I _{o,max} [A]	Peak Eff. @I _o	Pout,max [W]	Peak power density [W/mm ³]*
10 nH	Discrete	9.6 V/2.33 V	2.5	92.4% @1 A	7.5	0.62
10 111	IPD	9.6 V/2.33 V	1.5	91.2% @0.6 A	4.4	1.36

*Space counts active die, flying capacitors and effective inductors volume only.

less inductance and much lower switching frequency because of phase shift modulation, although both these types of converters achieve best efficiency at nominal conversion ratio, i.e. 4. Moreover, as (5.23) suggests, ITSAB converter is a first-order system, which allows for faster closed-loop design and transient responses.

The proposed ITSAB converter employs only two 10 nH inductors, at least $1.8 \times$ smaller values compared to the state-of-the-art designs with otherwise similar specifications, as shown in the comparison Table 5.6. Because of the small inductors, the prototype converter has an outstanding power density of 1.36 W/mm^3 . Furthermore, it is capable of operating over a wide input and output range with similar peak efficiency of 92.4%. As summarized in Table 5.7, the ITSAB converter in this paper exhibits an effort to overcome challenges in other types of converter that either require much larger inductors (conventional hybrid converter), have no

	This Work	[4]	[5]	[6]	[7]	[8]
Topology	ITSAB	Tri-State DSD	3:1 ReSC	Hybrid SC	Hybrid Dickson	Cascade Hybrid
Technology	130 nm	180 nm	180 nm	130 nm	65 nm	180 nm
Inductor	2x10 nH	2x560 nH	36 nH	$1 \mu H$	$180\mathrm{nH}$	$240\mathrm{nH}$
Inductor switching frequency	2-5 MHz	200 k-2 MHz	1.7 MHz	2.3 MHz	400 k-10 MHz	1.5 MHz*
[V] Inuit voltage [V]		$V_{\rm in} \ge 9V$			$V_{\rm in}$	<9V
Input votage [v]	9.6-12	12/24	12	6	3-4.5	4-6
Output voltage [V]	2.15-3.3	1	3.5-3.8	3-4.2	0.3-1	0.4-1.2
Peak output current [A]	2.5	3	1.24*	3.4	1.53	1
Peak power density [W/mm ³] ***	1.36 @ 4.1**	0.06 @ 12*	1.12 @ 3.4*	0.41 @ 2.7*	0.14 @ 5.2*	0.18 @ 4.5*
Efficiency [%] @ CR	92.4 @ 4.12	91.2 @ 12	82 @ 3.4	94.3 @ 2.5	88.3 @ 5.2	96.9 @ 4.2
Regulation Method	Phase shift modulation	Duty cycle	Deadtime control	Duty cycle	Duty cycle	Ripple injection
Package	Flip-chip	Wire-bonding	Flip-chip	Flip-chip	Flip-chip	Wire-bonding
*Estimation from reported	measurement results. ** Ass	uming Ferric IPD	inductor being used			

Table 5.6. Comparison with prior works

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***Area counts active die, flying capacitors and inductors only. Calculated at given conversion ratio (CR)=V_{in}/V₀

Converter	Inductor	Regulation	Efficiency
Resonant	tens to hundreds of nH	Limited in most cases	Good
Switched-capacitor (SC)	N/A	Limited	Poor trade-off with regulation
Conventional hybrid	hundreds of nH to μH	Yes	Good
ITSAB converter (this work)	up to tens of nH	Yes	Good

 Table 5.7. Comparison between converter topologies

regulation or difficulty in achieving regulation together with high efficiency and compact size (resonant converter), or suffer from lower efficiency for fine regulation (SC converter).

5.6 Summary

In this chapter, a new hybrid converter, named integrated transformerless stacked active bridge (ITSAB) converter, is presented. It requires only 10 nH inductors, while operating at a modest switching frequency in the MHz range. A voltage regulation approach is provided as well as a detailed loss analysis for the ITSAB converter. Additionally, an augmented state space method is employed to accurately calculate the inductor current waveshapes and RMS values. A novel optimization methodology is described, which is used to determine passive components, switching frequency and areas of the power devices so as to minimize the total loss. Moreover, circuit design details are provided for the key sub-blocks, including an adaptive gate driver design and phase-shifted control signal generation. The article also includes an analysis of open-loop control-to-output frequency responses, based on which a voltage control loop with a PI (Type-II) compensator is designed.

To validate operation and design of the proposed ITSAB converter, the power stage and the controller are implemented and fabricated on a $1.7 \text{ mm} \times 1.9 \text{ mm}$ die in a 130 nm BCD process. Two prototypes are constructed: one with Ferric IPD inductors and another one with discrete air-core inductors, both using an organic substrate and BGA package to house the flip-chip die, power capacitors and the inductors. Steady-state operation and load transients are verified by replacing the discrete inductors with two wire loops to facilitate capturing of inductor currents. Peak efficiencies of 91.2% and 92.4%, which are comparable to prior works, are measured for the two prototypes operating from 9.6-12 V input to 2.15-3.3 V output, respectively. Thanks to the low inductance requirements, and the usage of IPD inductors, the ITSAB converter prototype reaches a superior maximum power density of 1.36 W/mm^3 .

5.7 Acknowledgements

This chapter, in full, is a reprint of the materials as they appear in 2022 IEEE Custom Integrated Circuits Conference (CICC) and IEEE Journal of Solid-State Circuits. Tianshi Xie; J. Zhu, T. Byrd, D. Maksimovic, and H. Le, "A 0.66 W/mm2 Power Density, 92.4% Peak Efficiency Hybrid Converter with nH-Scale Inductors for 12 V System," in 2022 IEEE Custom Integrated Circuits Conference (CICC), Apr. 2022. Tianshi Xie; J. Zhu, D. Maksimovic and H. -P. Le, "A Highly Integrated Hybrid DC–DC Converter With nH-Scale IPD Inductors," in IEEE Journal of Solid-State Circuits, vol. 58, no. 3, pp. 705-719, March 2023, doi: 10.1109/JSSC.2022.3227163. The dissertation author was the primary investigator and author of these papers.

Chapter 6

ITSAB Converter for Heavy-Load Applications

As addressed in Chapter. 1, the demanding power for HPC continuously soars. Moreover, the miniaturization trend desires monolithic solutions. One chip or one package that supplies as much power as possible becomes a viable solution. As a consequence, co-design of chip die, which integrates power switches together with all the gate drivers and their supporting analog and digital blocks, with power inductors, which is proved to be the bottleneck of high power density design, is crucial. In this chapter, the design of a high-power version of ITSAB converter using the previously introduced methodology is presented together with an overview of a high power inductor.

6.1 System Design of High-Power ITSAB Converter

Targeting one compact package, the desired design would be sketched as Fig. 6.1, which shows top view, side view and bottom view of the package. The coppers with net names are labeled in different colors to show the preliminary floorplan. The chip die will be flipped on one side of the PCB, while two inductors are mounted on the other side of the PCB beneath the chip die so that the overall volume of the package is minimized. There are three sets of flying capacitors, C_1 , C_2 and C_3 . Because of de-rating effect due to voltage stress, each of them usually has nominal value two to three times of the actual capacitance needed. To maintain low ESR and to constrain the overall volume, four 0402 package capacitors are paralleled to form C_1 and C_2 , two 0201 package capacitors are paralleled to form C_3 .

Different from the package design in ITSAB converter introduced in Chapter. 5, the high-power ITSAB converter distributes components on both sides of substrate. Specifically, the big components, such as the chip die and the inductors, are placed on each side of the substrate to save the overall volume. The inductors are about 3.5 mm x 2.5 mm. To cover this area, the desired chip die area should be around 3.5 mm x 5 mm, which is the sum of the area of two inductors. From the 3D floorplan given in Fig. 6.1, this area is roughly the maximum allowable number of the chip die area, and this die area is desired, as the area is proportional to the maximum output power delivered by the converter.

Giving a little bit of headroom, the chip is designed in 3 mm x 5 mm. Based on the current density of each power switch, the chip die area can be divided into the following ratio. Basically, Q1,2 are the same area as Q5,6 and half of Q3,4. Because Q7,8 are PMOS, their area is roughly assigned as 1.5 X of Q1,2.

Area	Ratio
Q1, Q2 area	1 X
Q3, Q4 area	2 X
Q5, Q6 area	1 X
Q7, Q8 area	1.5 X

 Table 6.1. Optimization results.

After taking the area of gate drivers and peripheral circuits into consideration, using the optimization methodology given in Chapter. 5, the resulted components selection and the converter operating parameters as well as the size of power switches are listed in Table. 6.2.

The operating frequency is significantly lower than the low power ITSAB converter because the switch sizes for the high-power one are a lot larger and thus much higher parasitic capacitance. From the loss analysis presented in Section. 5.2, to reduce the switching loss, a



Figure 6.1. 3D floorplan of high power ITSAB converter package

Parameters	Details
L	10 nH
C1, C3	1.75μF
C2	$4\mu\mathrm{F}$
$\mathbf{f}_{\mathbf{s}}$	2.1 MHz
Q1, Q2 area	$0.882\mathrm{mm^2}$
Q3, Q4 area	$1.812\mathrm{mm}^2$
Q5, Q6 area	$0.866{ m mm^2}$
Q7, Q8 area	$1.343\mathrm{mm}^2$

 Table 6.2. High-power ITSAB converter optimization results.

lower switching frequency is required. Meanwhile, to keep $k = \frac{f_s}{f_o}$ much greater than 1, larger capacitance is therefore desired. The area of each power switch is measured directly from the chip layout and therefore does not strictly follow the ratio given in Table. 6.1.

The theoretical efficiency and loss breakdown at 8 A load are shown in Fig. 6.2. Apparently, the ESR of inductors affects the efficiency at heavy load. But at light load, unless shrinking the size of switches or reducing the switching frequency, which are not applicable for high-power applications, the efficiency can hardly be improved. To enhance the light load efficiency, a special switch partitioning approach is applied. Fundamentally, only one quarter of each switch is turned on at light load so that a total gate capacitance of $\frac{C_{gg}}{4}$ instead of C_{gg} is switched and thus P_{gate} is decreased by three quarters. The resulted efficiency is plotted in Fig. 6.3 on top of the efficiency curve with $20m\Omega$ inductor ESR and the entire power switch. It shows 15% loss reduction at light load. The detailed circuit design for this switch partitioning function will be introduced in Section. 6.2.1.



(b)

Figure 6.2. (a) efficiency with different inductor ESRs; (b) Loss breakdown at 8 A load



Figure 6.3. Efficiency improvement with switch partitioning

6.2 Circuit Design of High-Power ITSAB Converter

While the major analog blocks, such as Ramp Generator (RG), Phase Shift Modulator (PSM), OTA, gate drivers and Linear Regulator (LR), are inherited from the low-power ITSAB design and only scaled with switching frequency and switch size, two other blocks including switch partitioning logic cell and fine-tuned deadtime generator are newly added to support high-power circuit requirement.

6.2.1 Switch Partitioning

The power switch is divided into two pieces: one quarter (x1) and three quarters (x3) of the switch. The according gate driver must be also separated into one quarter and three quarters of the original size as designed in Fig. 5.12c. Specifically, the last stage of gate driver, which is an inverter or an inverter with stacked PMOS and NMOS, is divided into two cells to drive x1 and x3 power switches, respectively. For simplicity, the last stage inverter and its PMOS,


Figure 6.4. (redraw with signal names, 1x to x1) Block diagram of switch partitioning circuit

NMOS drivers are lumped in one gate driver symbol with "x1" or "x3" marked on it. The Non-overlapped Signal Generator (SG) is required for each VDD-domain driving signal S_x , as some deadtime is needed between the driving signals of the last stage inverter PMOS (S_x_PG) and NMOS (S_x_NG) to prevent shoot-through current. The following level shifter is required for every gate driver except GD2 and GD3. Again, the level shifter symbol in Fig. 6.4 represents one or two level shifters for S_x_PG and S_x_NG to generate $S_x_LS_PG$ and $S_x_LS_NG$, respectively, for PMOS and NMOS drivers. Every level shifter is the same as the design in Fig. 5.12d no matter x1 or x3 power switch is driven. There is an enable signal EN at front to control ON/OFF of the driving chain of x3 switch.

6.2.2 Fine-tuned Deadtime Generator

The deadtime generator is to produce proper deadtime to achieve soft-switching while avoiding shoot-through current. For ITSAB converter, since the inductors are only 10 nH, the inductor currents ramp up in a very steep slope, for example 3.3 ns of ramping time for 1 A load 12 V input. This creates a big challenge for deadtime tuning. Particularly at light load where the switching loss is dominant, it desires to achieve complete soft-switching for Q_3 and Q_4 while keeping body diode from conducting current. It's even more critical for high-power converter which has larger switches and thus higher switching loss if soft-switching fails. Moreover, the body diode conduction loss, which is proportional to the flowing current, can be significant at heavy load and becomes one major obstacle for achieving desired high output power, yet to mention the potential latch-up issue caused by reverse conduction within an integrated power switch. All in all, a fine-tuned deadtime generator is required for ITSAB converter, especially for a high-power design.



Figure 6.5. Block diagram of fine-tuned deadtime generator

The fine-tuned deadtime generator is sketched as Fig. 6.5, which is constructed from basic SR Latch. The rising edge of input signal *IN* sets the rising edge of *PH* and the trailing edge of *PHB*, while the trailing edge of *IN* resets *PH* and sets *PHB*. There are total 4 Programming Delay (PD) blocks. The PD#1 determines the deadtime from trailing edge of *PHB* to rising edge of *PH*, whereas PD#2 configures the deadtime between the trailing edge of *PH* and the rising edge of *PHB*. The presence of PD#3 pushes the trailing edge of *PH* to a later time. In other words, the total deadtime from the trailing edge of *PH* to the rising edge of *PD*#2 minus the one of PD#3. It's the same mechanism for generating the other deadtime by taking the difference between PD#1 and PD#4. The benefit of adding PD#3 and PD#4 is to compensate for any delay on the signal paths that causing none-zero deadtime even when PD#1 and PD#2 are set to zero. To gain the fine-tuned capability while maintaining wide range of tuning, each of PD#1 and PD#2 contains 18-bit Short Delay (SD) and 2-bit Long Delay (LD) blocks. Each of SD and LD block is made of several basic 2-bit programmable delay block as

indicated at the bottom of Fig. 6.5. The basic cell can be programmed from zero delay to 3 units of delay. For SD block, each of this unit delay is 80 ps/bit and this number is 1 ns/bit for each unit delay in LD block. That means the total deadtime ranges from 0 to 5.16 ns with 80 ps step. PD#3 and PD#4 are the same as 2-bit LD block.



6.3 Experiment Results

Figure 6.6. PCB prototype of high-power ITSAB converter

To evaluate the performance of this high-power ITSAB converter, a PCB prototype is designed and measured. Different from the demonstration PCB designed for low-power ITSAB converter in Chapter. 5, where all the power components are mounted on one fine-pitch substrate, the one for high-power ITSAB converter places components on both sides of PCB as enlightened by the 3D floorplan in Fig. 6.1. Specifically, the chip die, which is integrated with all the power switches and analog/mixed-signal blocks, is flipped-chip on the top side of the PCB together with the flying capacitor *C*2, input and output capacitors as well as resistors and capacitors for feedback loop. While on the bottom side of the PCB, inductors *L*1 and *L*3 are soldered beneath the position of the chip die surrounding by the other two flying capacitors *C*1 and *C*3. The part numbers and the details of each component are listed in Table. 6.3. Note that the flying capacitors



Figure 6.7. Die photo of high-power ITSAB converter

are selected based on their derated value after considering bias voltages so that C1 and C3 are matched in value, and C2 is much larger than the other two capacitors. The inductors are selected because of small ESR so that the conduction loss is minimized for heavy load capability.

The chip die is designed in TSMC 130 nm BCD technology with a dimension of $3.04 \text{ mm} \times 5.24 \text{ mm}$. The placement of power switches are labeled in the die photo (Fig. 6.7). Their positions are referred to the connections shown in the 3D floorplan in Fig. 6.1 for shortest path. The other function blocks are in the middle of the chip die, such as Linear Regulators (LR) and Ramp Generator (RG).

The major waveforms, such as the switching nodes V_{SW1} , V_{SW2} , V_{X2} and LR output V_{DD5} , are measured at $f_s = 1.8$ MHz, $V_{in} = 9.6$ V and $V_o = 2.3$ V with Coilcraft 1512SP as inductors. They are displayed in Fig. 6.8. It can be summarized that V_{DD5} is well regulated at $V_{DD} + V_o = 6.2$ V. What's more, from the rising edge of V_{SW1} , there's almost no overshoot or ringing can be noticed, meaning the deadtime is set precisely and thus complete soft-switching

Component	Part No.	Details
C1	GRM155C81C225ME15D	$4 \text{ x } 2.2 \mu\text{F}, 16 \text{ V } 0402 (1.83 \mu\text{F})$
C2	CL05A106MP8NUB8	4 x 10 µF, 10 V 0402 (8 µF)
C3	CL03A225MQ3CRNC	2 x 2.2 µF, 6.3 V 0201 (1.73 µF)
Cin(Vin)	GRM21BR61E106MA73L	6 x 10 μF, 25 V 0805 (10.5 μF)
Cin(V _{DD})	CL05A106MP8NUB8	1 x 10 µF, 10 V 0402 (2 µF)
Со	CL03A475MQ5C65	4 x 4.7 µF, 6.3 V 0201 (4.1 µF)
Decap(V _{DDH6})	GRM155C81C225ME15D	$3 \text{ x} 2.2 \mu\text{F}, 16 \text{ V} 0402 (1.24 \mu\text{F})$
Decap(V _{DD5})	GRM155C81C225ME15D	2 x 2.2 µF, 16 V 0201 (1.23 µF)
Decap(V _{SS8})	GRM155C81C225ME15D	$2 \text{ x} 2.2 \mu\text{F}, 16 \text{ V} 0201 (1.04 \mu\text{F})$
Decap(V _{SSL7})	GRM155C81C225ME15D	1 x 1 µF, 16 V 0201 (0.9 µF)
L1,3	Coilcraft 1512SP	$10 \mathrm{nH}, 2.5 \mathrm{m\Omega} \mathrm{ESR}$

Table 6.3. Components on the demonstration PCB



Figure 6.8. Measured switching nodes voltages

is achieved.

The efficiency curves are plotted in Fig. 6.9(a) at various regulated V_o . The peak efficiency



Figure 6.9. (a) Efficiency with different output voltages at 1.8 MHz; (b) efficiency with (x1) and without (x4) switch partitioning at 2.38 V output voltage

of 85.24% is captured at 9.6 V/2.31 V, 1.8MHz switching frequency, 2.6A load condition. The switch partitioning benefit can be verified in Fig. 6.9(b) at the same input voltage and switching frequency with 2.38V output voltage. By turning on switch partitioning, the efficiency at 2A load is increased from 75% to 83.16%. The recorded peak output current is 5.2A, twice of the value in the other ITSAB converter in Chapter. 5.

6.4 Summary

In this chapter, the application of ITSAB converter is extended to high output power. By co-design the chip die and the prototype PCB, the converter is now capable of 5.2A load in a compact 3D package. To achieve high efficiency at both heavy and light load, a switch partitioning mechanism is implemented such that only one quarter of the entire power switches are turned on at light load condition. Another fine-tuned deadtime generator is also added for minimized body diode conduction and fine-tuned timing of complete soft-switching. The chip die is fabricated in TSMC 130 nm BCD technology with a dimension of $3.04 \text{ mm} \times 5.24 \text{ mm}$ and is mounted with two 10 nH inductors supplied by Coilcraft on two sides of the demonstration PCB. The captured peak efficiency is 85.24% when operating the converter at 9.6 V/2.31 V, 1.8 MHz switching frequency, 2.6 A load. The recorded peak output current is 5.2 A.

Chapter 7 Conclusion and Future Work

7.1 Conclusion

In this dissertation, it has discussed the increasingly demanding of high power but compact power delivery system for high performance computing systems. From the case study of some commercial products, it can be summarized that the magnetic devices, namely inductors and transformers, are the major obstacle of this trend. By analyzing several existing solutions, the issue can hardly be solved as none of these solutions achieves 10s nH inductance, fine regulation and high efficiency at the same time.

While the existing solutions are not satisfying, three works have been proposed from the perspective of circuit operation innovation, special circuit technique and novel circuit topology. The first one is a dual-inductor hybrid converter operated in multi-phases. Different from the conventional dual-phase operation for this dickson-star switched-capacitor converter derived hybrid converter, the proposed one can potentially reduce the output inductor by 3x and thus the overall volume of the converter is shrunk.

The second introduced method is zero-voltage-switching assisted by an auxiliary circuit. Detailed operation and optimization are presented in Chapter. 4. The demonstrated ZVS-3LB converter has shown great advantage at light load operation where normally the switching loss dominates. With the help of ZVS operation, the switching frequency can be boosted so that the inductor size will be reduced.

Further more, in the last work, a novel ITSAB converter is proposed. There are two demonstration prototype of this type of converter. The first chip is optimized, designed and fabricated in $1.7 \text{ mm} \times 1.9 \text{ mm}$ area of a 130 nm BCD process. The active die is flip-chipped on a 6.5 mm \times 6.5 mm package substrate together with power capacitors and two 10 nH IPD inductors for demonstration. The other chip, which targets higher output power, is fabricated in the same process but has considered more of 3D packaging during PCB prototyping. All of these attempts have demonstrated 10 nH inductors can be used on power converters that operate below 5 MHz switching frequency, while achieving high power density and heavy load capability.

In addition to the works presented in this dissertation, some other works can be done to enrich the proposed solutions.

7.2 Future Work

7.2.1 **Bi-directional Operation**

The ITSAB converter is derived from a Dual Active Bridge (DAB) converter, who has a feature of bi-directional operation as illustrated in Fig. 7.1. Basically, by swapping the leading phase and trailing phase of A and B, the current flows in different directions. The same concept can be applied to ITSAB converter. By moving the control signals of *Q*3 and *Q*4 from trailing phase to leading phase, the current could flow from lower voltage to higher voltage. Of course, to enable this operation, the power switches must be able to block both directions of voltage and current and the related gate drive signals should also be re-designed. Despite of the implementation difficulty, the bi-directional converter could potentially be found useful in applications of robot, EV, where a battery is used and recycling energy to charge battery is promising.



Figure 7.1. Bi-directional operation of DAB converter

7.2.2 Constant Current (CC) Mode for Battery charger

The regulation of ITSAB converter is different from the most step-down converter which is based on the modulation of the pulse width of control signals. As introduced in Chapter. 5, ITSAB converter regulates its output voltage by modulating the phase shift ϕ , as illustrated in Fig. 7.2. In fact, the converter doesn't directly modulate the output voltage but instead regulating its output current in response to different reference voltages.



Figure 7.2. Modulation of inductor current in ITSAB converter

How about directly feeding back the output current and modulating the phase shift time in response to that current? This is absolutely applicable, but the range of resulted output voltage needs some more analysis. This CC mode can be used on a battery charger. As commonly known, the charging of a battery must go through CC mode when the energy level is low and enter constant-voltage (CV) mode when it's charged to a certain power level. ITSAB converter, because of its special control method, would be a potential candidate for this application.

Appendix A

FPGA Embedded System Design for ZVS-3LB Converter

In the ZVS-3LB converter design, the embedded system on FPGA as shown in Fig. 4.7 is designed based on NiosII system. The blocks added and the connection between them are captured in Fig. A.1. The system core is Nios II Processor, while its supporting system clock is generated from a PLL IP. On-chip memory is to stored the data received from the master PC. PWM generators are the blocks to output gate control signals for each power switch and its source code is attached in the following pages. Another important block is MATLAB AXI bus IP for communicating between the master PC, where a MATLAB software is used to sweep parameters, and the slave FPGA which is built on Nios II processor. By programming to the specific address of each PWM generator through the MATLAB AXI bus, each PWM signal would have its duty ratio and phase completely controlled by the master PC, so that the automation testing can be launched.



Figure A.1. Embedded system design of the ZVS-3LB controller based on NiosII system

One of the key blocks is the PWM signal generator as the verilog source code shown below. This block can read the data memory in the NiosII core. Therefore, by sending data from a PC to the FPGA, the PWM generator block can fetch the required duty ratio and phase information and thus generates the desired PWM control signals. The verilog source code for the up counter is also listed following the PWM generator block.

PWM Generator

```
module PWM_Gen_v0_5(
```

```
input
                     clk,
2
      input
                    reset_n,
3
      input
                     slave_chip_select_n,
4
                     slave_read,
      input
5
      output reg [31:0] slave_readdata,
6
      input
                     slave_write,
              [31:0] slave_writedata,
      input
8
      input sync,
9
      input [15:0] period_minus2,
10
      output reg pwm_out,
      output reg S_reg
13 );
14 wire [15:0] counter_out;
16 // avalon slave
17 reg R_reg;
18 reg [15:0] duty;
19 reg [15:0] phase;
20 reg writedone;
21 always @(posedge clk or negedge reset_n)
22 begin
    if (~reset_n)
23
24
    begin
      duty <= 16'b0;
25
```

```
phase <= 16'b0;</pre>
26
      writedone <= 1'b0;</pre>
27
    end
28
    else if (~slave_chip_select_n & slave_read)
29
      slave_readdata <= {16'hffff, 16'h0};</pre>
30
    else if (~slave_chip_select_n & slave_write)
31
    begin
32
    //load in duty and phase setup of the PWM signal
33
      duty <= slave_writedata[15:0];</pre>
34
      phase <= slave_writedata[31:16];</pre>
35
      writedone <= 1'b1:
36
    end
37
    else writedone <= 1'b0;</pre>
38
39 end
40
41 always @(posedge clk or negedge reset_n) begin
42 if (~reset_n) begin
    R_reg <= 1'b0;
43
    end
44
    //when counter counts to duty number, trigger falling edge of PWM
45
    else if(counter_out == duty) begin
46
    R_reg <= 1'b1;
47
    end
48
   else begin
49
    R_reg <= 1'b0;
50
    end
51
52 end
53 always @(posedge clk or negedge reset_n) begin
54 if (~reset_n) begin
    S_reg <= 1'b0;
55
    end
56
    //when counter counts to 0, initiate PWM rising edge
57
```

```
else if(counter_out == 16'b0) begin
58
    S_reg <= 1'b1;</pre>
59
   end
60
   else begin
61
    S_reg <= 1'b0;
62
    end
63
64 end
65 Cnt_Up16_1 Cnt16_inst(
    .clk(clk),
               //Counter clock
66
   .period_minus2(period_minus2),
67
                                 // 8 bit output from the counter
   .counter_out(counter_out),
68
   .load(~sync), //command of loading phase
69
   //load in starting counter number, which is phase shift
70
   .phase_loadin(phase)
71
72);
73 SR_latch_gate SR_latch_inst(
.R(R_reg),
75 .S(S_reg),
  .Q(pwm_out)
76
77);
78 endmodule
```

```
Up Counter
```

```
1 'timescale 1ps / 1ps
2 module Cnt_Up16_1 (
    clk,
                //Counter clock
3
    period_minus2,
4
   counter_out,
                    // 8 bit output from the counter
5
   phase_loadin,
6
    load
                //command of loading phase
7
8 );
9
    input clk; // clock declared as an input port
10
    input [15:0] period_minus2;
    input load;
12
    output reg [15:0] counter_out; // counter_out declared as an 8 bit
     output register
    input [15:0] phase_loadin;
14
15
    always @(posedge clk)
16
      begin
17
      if (~load) begin
18
        counter_out <= (phase_loadin+1);</pre>
19
        end
20
      else begin
21
        if (counter_out > (period_minus2) ) begin//FFE=-2//period-2
22
        counter_out <= 0;</pre>
23
        end
24
        else begin
25
        counter_out <= counter_out + 1'b1;</pre>
26
        end
27
      end
28
      end
29
                   // identifies the end of the module
30 endmodule
```

Appendix B

Automatic Parameter Sweeping Source Code in Matlab for ZVS-3LB Converter

```
1 clc
2 clear all
3 close all
4 %% create a seperate folder to store mat, data and screenshot
5 datename=date;
6 mkdir(datename);
7 %% parameters to include in the file name
8 L=['IHLP-5050_'];
9 Cfly=['10_1_0p1uF_'];
10 Laux=['IHLP-1616BZ_'];
11 %% Instrument Connection
12 % Find a VISA-USB object.
rigol_load = instrfind('Type', 'visa-usb', 'RsrcName', 'USB0::0x1AB1::0
     xOE11::DL3A201300268::0::INSTR', 'Tag', '');
14 % Create the VISA-USB object if it does not exist
15 % otherwise use the object that was found.
16 if isempty(rigol_load)
     rigol_load = visa('NI', 'USB0::0x1AB1::0x0E11::DL3A201300268::0::
     INSTR');
18 else
```

```
fclose(rigol_load);
19
     rigol_load = rigol_load(1);
20
21 end
22 %
23 % Find a VISA-USB object.
24 ks_ps = instrfind('Type', 'visa-usb', 'RsrcName', 'USB0::0x2A8D::0x0102
     ::MY56003339::0::INSTR', 'Tag', '');
25
26 % Create the VISA-USB object if it does not exist
27 % otherwise use the object that was found.
28 if isempty(ks_ps)
     ks_ps = visa('NI', 'USB0::0x2A8D::0x0102::MY56003339::0::INSTR');
29
30 else
    fclose(ks_ps);
31
     ks_ps = ks_ps(1);
32
33 end
34 % Find a VISA-USB object.
35 RIGOL_PS = instrfind('Type', 'visa-usb', 'RsrcName', 'USB0::0x1AB1::0
     x0E11::DP8B213700918::0::INSTR', 'Tag', '');
36 % Create the VISA-USB object if it does not exist
37 % otherwise use the object that was found.
38 if isempty(RIGOL_PS)
     RIGOL_PS = visa('NI', 'USB0::0x1AB1::0x0E11::DP8B213700918::0::INSTR
39
     ');
40 else
     fclose(RIGOL_PS);
41
      RIGOL_{PS} = RIGOL_{PS}(1);
42
43 end
44 %% input parameters
45 fsw=1e6;%switching frequency
46 fclk=200e6;%Embedded system clk
47 period=fclk/fsw;
```

```
48 vref=3.75;%target output voltage
49 vg=15;%input voltage
50 vdri=12;%gate driving voltage
51 idri=0.5;%gate driving supply current setup
52 duty0=round(vref*period/vg);
53 QF_QbarR=4;
54 \text{ ddmax}=12;
55 ddmin=5;
56 dd0=round(mean([ddmax ddmin]));
57 \text{ cd0} = -3;
58 duty=duty0+cd0;%duty ratio
59 duty10=duty+dd0;%duty ratio for phase 1
60 duty30=duty-dd0;%duty ratio for phase 2
61 duty_Aux10=1;%duty ratio of Auxa
62 duty_Aux30=1;%duty ratio of Auxb
63 QbarF_AuxR10=2;
64 QbarF_AuxR30=QbarF_AuxR10;
AuxF_QR10=3;
66 AuxF_QR30=3;
67 Auxa_ext10=5;
68 Auxa_ext30=5;
69 %% initialize pwm
70 fun_PWM_setting(period, duty10, duty30, QF_QbarR, duty_Aux10, duty_Aux30,
      QbarF_AuxR10, QbarF_AuxR30, AuxF_QR10, AuxF_QR30, Auxa_ext10,
     Auxa_ext30);
71 %% file name
72 file=['ZVS_3LB_', L, Laux, Cfly, datename];
73 %% instruments parameters
74 ival = [0.5 \ 0.8 \ 1 \ 1.2 \ 1.5 \ 1.8 \ 2 \ 3 \ 4];
75 pts=length(ival);
76 i=1; %init array pointer
```

```
77 k = 1;
```

```
78 ramp_down = 0; %ramp down flag. If set, then ramp down. If cleared, ramp
       up.
79 delay = 1; %time delay between current steps while ramping
80 %pre-form various measurement/calc arrays
81 vbus_meas = zeros(1,pts);
82 ibus_meas = zeros(1,pts);
83 vo_meas = zeros(1,pts);
84 io_meas = zeros(1,pts);
85 pout_meas = zeros(1,pts);
86 pin_meas = zeros(1,pts);
87 eff_per = zeros(1,pts);
88 %% initialize test
89 % connect to instrument objects
90 fopen(rigol_load);
91 fopen(ks_ps);
92 fopen(RIGOL_PS);
93 % communicating with instrument objects
94 %driver supply
95 fprintf(RIGOL_PS, ':INST CH1');
96 fprintf(RIGOL_PS, ':CURR 0.5');
97 fprintf(RIGOL_PS, ':CURR:PROT 1');
98 fprintf(RIGOL_PS, ':CURR:PROT:STAT ON');
99 fprintf(RIGOL_PS, ':VOLT 12');
100 fprintf(RIGOL_PS, ':OUTP CH1,ON');
101 pause(delay);
102 %load
103 fwrite(rigol_load,'SOUR:SENS 1'); %enable voltage sense inputs
104 scpi_str = ['SOUR:CURR:LEV:IMM ' num2str(ival(1,i))];
105 fwrite(rigol_load,scpi_str); %set initial current value to imin
106 fwrite(rigol_load, 'SOUR:INP:STAT 1');
107 pause(delay);
108 %turn on power supply
```

```
111
```

```
109 fwrite(ks_ps, ['VOLT ', num2str(vg),', (@2)']);
110 fwrite(ks_ps,'OUTP ON, (@2)');
111 %create spreadsheet for data
112 for Iout=ival
H3 Row1=["Num" "duty1" "duty3" "duty_Aux1" "duty_Aux3" "QbarF_AuxR1" "
      QbarF_AuxR3" "AuxF_QR1" ...
               "AuxF_QR3" "Auxa_ext1" "Auxa_ext3" "QF_QbarR" "Vg" "Vout"
114
      . . .
               "Iout" "Eff"];
115
H16 writematrix(Row1, ['./', datename, '/', [file, '_', num2str(Iout), 'A'],
       '.xls'], 'Sheet',1,'Range', 'A1:P1');
117 end
118 disp('Ramping up current and measuring efficiency');
119 while ramp_down==0
120 %% set pwm
121 for duty1=[duty10 duty10+1 duty10+2 duty10+3]
      for duty3=[duty1-ddmax:1:duty1-ddmin]
           duty = duty0 + cd;
123
           for duty_Aux3=[duty_Aux30]
124
               for duty_Aux1=[duty_Aux10 duty_Aux10+1]
125
                   for QbarF_AuxR1=[QbarF_AuxR10 QbarF_AuxR10+1]
126
                       for QbarF_AuxR3=[QbarF_AuxR1]
                       for AuxF_QR3=[AuxF_QR30 AuxF_QR30+1 AuxF_QR30+2]
128
                            for AuxF_QR1=[AuxF_QR3+2 AuxF_QR3+3 AuxF_QR3+4]
129
                                for Auxa_ext3=[duty_Aux3*5 duty_Aux3*6]
130
                                    Auxa_ext1=Auxa_ext3;
                                    fun_PWM_setting(period, duty1, duty3,
      QF_QbarR, ...
133
                                         duty_Aux1, duty_Aux3, QbarF_AuxR1,
      QbarF_AuxR3, AuxF_QR1, AuxF_QR3, ...
                                         Auxa_ext1, Auxa_ext3);
134
           %display current parametes
135
```

```
disp(['duty1=',num2str(duty1),', duty3=',num2str(duty3),',
136
     duty_Aux1=',num2str(duty_Aux1), ...
               ', duty_Aux3=',num2str(duty_Aux3),', QbarF_AuxR1=',num2str(
     QbarF_AuxR1),', QbarF_AuxR3=',num2str(QbarF_AuxR3), ...
               ', AuxF_QR1=', num2str(AuxF_QR1),', AuxF_QR3=', num2str(
138
     AuxF_QR3),', Auxa_ext1=',num2str(Auxa_ext1), ...
               ', Auxa_ext3=', num2str(Auxa_ext3)]);
139
          %turn on load here
140
           scpi_str = ['SOUR:CURR:LEV:IMM ' num2str(ival(1,i))];
141
           fwrite(rigol_load,scpi_str); %increment current
142
          fwrite(rigol_load, 'SOUR:INP:STAT 1');
143
          pause(delay);
144
           vbus_meas(i) = str2num(query(ks_ps, 'meas:volt:dc? (@2)')); %
145
     measure vbus voltage w/ tek_dmm
           ibus_meas(i) = str2num(query(ks_ps, 'meas:curr:dc? (@2)')); %
146
     measure ibus current w/ ks_ps
           vo_meas(i) = str2num(query(rigol_load, 'meas:volt:dc?'));
147
           io_meas(i) = str2num(query(rigol_load, 'meas:curr:dc?'));
148
          pout_meas(i) = vo_meas(i)*io_meas(i);
149
          pin_meas(i) = vbus_meas(i)*ibus_meas(i);
150
          eff_per_new = (pout_meas(i)/pin_meas(i))*100;
          M=[k duty1 duty3 duty_Aux1 duty_Aux3 QbarF_AuxR1 QbarF_AuxR3
152
     AuxF_QR1 ...
               AuxF_QR3 Auxa_ext1 Auxa_ext3 QF_QbarR vbus_meas(i) vo_meas(i
153
     ) ...
               io_meas(i) eff_per_new];
154
           writematrix(M, ['./', datename, '/', [file, '_', num2str(ival(1,
155
     i)), 'A'], '.xls'], 'Sheet',1, 'Range', ['A', num2str(k+1), ':P', num2str
     (k+1)]);
          k = k + 1;
156
           disp(['Vout=',num2str(vo_meas(i)),', Iout=',num2str(io_meas(i)),
157
      ', Efficiency=', num2str(eff_per_new)]);
```

158	<pre>if(abs(vo_meas(i)-vref)<0.1)%only keep the data with Vo=vref</pre>
159	<pre>if(eff_per(i)<eff_per_new) %save="" data="" eff="" max="" points<="" pre="" the=""></eff_per_new)></pre>
160	<pre>opt.eff_per(i)=eff_per_new;</pre>
161	<pre>opt.vo(i)=vo_meas(i);</pre>
162	%
163	<pre>opt.duty1(i)=duty1;</pre>
164	<pre>opt.duty3(i)=duty3;</pre>
165	<pre>opt.duty_Aux1(i)=duty_Aux1;</pre>
166	<pre>opt.duty_Aux3(i)=duty_Aux3;</pre>
167	<pre>opt.QbarF_AuxR1(i)=QbarF_AuxR1;</pre>
168	<pre>opt.QbarF_AuxR3(i)=QbarF_AuxR3;</pre>
169	<pre>opt.AuxF_QR1(i)=AuxF_QR1;</pre>
170	<pre>opt.AuxF_QR3(i)=AuxF_QR3;</pre>
171	<pre>opt.Auxa_ext1(i)=Auxa_ext1;</pre>
172	<pre>opt.Auxa_ext3(i)=Auxa_ext3;</pre>
173	end
174	end
175	
176	
177	end
178	end
179	end
180	end
181	end
182	end
183	end
184	end
185	end
186	%update starting point for next load current
187	<pre>dd0=0.5*(opt.duty1(i)-opt.duty3(i));</pre>
188	<pre>cd0=0.5*(opt.duty1(i)+opt.duty3(i))-duty0;</pre>
189	<pre>duty_Aux10=opt.duty_Aux1(i);</pre>

```
duty_Aux30=opt.duty_Aux3(i);
190
           QbarF_AuxR10=opt.QbarF_AuxR1(i);
191
           QbarF_AuxR30=opt.QbarF_AuxR3(i);
192
           AuxF_QR10=opt.AuxF_QR1(i);
193
           AuxF_QR30=opt.AuxF_QR3(i);
194
           Auxa_ext10=opt.Auxa_ext1(i);
195
           Auxa_ext30=opt.Auxa_ext3(i);
196
197
           i=i+1;%load current to next value
           k=1;
199
           if(i>pts)
200
               ramp_down = 1;
201
               i=pts;
202
           end
203
204 end
205
206 disp('Ramping down current.');
  while ramp_down==1 %check if in ramp down mode
207
      if i<1 % if in ramp down mode and at the imin value, clear ramp down
208
      flag
           ramp_down = 0;
209
           i=1; %initialize i pointer to first position in ival array to
      prep for current ramp up
           hold off; %turn off plot hold so plot is wiped on next current
      ramp up sweep
      else %otherwise continue to decrement the current value
           scpi_str = ['SOUR:CURR:LEV:IMM ' num2str(ival(1,i))];
213
           fwrite(rigol_load,scpi_str); %decrement current
214
           pause(delay);
215
            i = i - 1; %decrement to next pointer value in ival array
216
      end
217
218 end
```

```
219 fwrite(ks_ps, 'OUTP OFF, (@2)');
220 fclose(ks_ps);
221 delete(ks_ps);
222 clear ks_ps;
223
224 fwrite(rigol_load, 'SOUR:CURR:LEV:IMM 0');
225 fwrite(rigol_load,'SOUR:INP:STAT 0');
226 fwrite(rigol_load,'SOUR:SENS 1'); %keep sense input enabled voltage
      sense inputs
227 fclose(rigol_load);
228 delete(rigol_load);
229 clear rigol_load;
230
231 fprintf(RIGOL_PS, ':OUTP CH1,OFF');
232 fclose(RIGOL_PS);
233 delete(RIGOL_PS);
234 clear RIGOL_PS;
235 %% save workspace
236 mat_name=['./', datename,'/', file, '.mat'];
237 save(mat_name);
238 %
239 disp('Efficiency sweep complete');
```

Appendix C

ITSAB Converter Optimization Source Code in Matlab

```
1 %% topology independent constants
2 clear
_{3} L0=10e-9;
_{4} f0=100e6;
5 \text{ ALO} = 0.7;
 Kc = 1.4 e - 6; \% F / mm2 
7 Iout = 1;
8 Vin=12;
9 Vout=Vin/4;
10
11 %% power switches parameters
12 %process related, critical numbers are hided for confidential reason
13 Ron=1e-3*1.5;%on resistance unit
14 Vg=3.6;%gate drive voltage
15 %NMOS, for Q1~6
16 AsN=x;%Ron density mohm*mm2
17 CggN_den=x;%Cgg of NMOS density per mm2
18 CddN_den=x;%Cdd of NMOS density per mm2
19 CssN_den=x;%Css of NMOS density per mm2
20 %Total parasitic caps used in loss analysis
```

```
21 CdsN_den=CddN_den+CssN_den;
<sup>22</sup> CggN_AsN=CggN_den*AsN;
23 CdsN_AsN=CdsN_den*AsN;
24 %PMOS, for Q7,8
As P = x;
26 CggP_den=x;%Cgg of PMOS density per mm2
27 CddP_den=x;%Cdd of PMOS density per mm2
28 CssP_den=x;%Css of PMOS density per mm2
29 %Total parasitic caps used in loss analysis
30 CdsP_den=CddP_den+CssP_den;
31 CggP_AsP=CggP_den*AsP;
32 CdsP_AsP=CdsP_den*AsP;
33 %x1:Q1-Q2: 5V NMOS
34 %x2:Q3,Q4: 5V NMOS
35 %x3:Q5,Q6:5V NMOS
36 %x4:Q7,Q8:5V PMOS
37 %% Total capacitor area as one constrain
ACmax = 10/3;
39 %% Total switch area as one constrain
40 Asmax = 10;
41 %% CVX tool starts optimizing
42 cvx_clear
43 cvx_solver mosek
44 for i1=1:length(Asmax)
          for i2=1:length(ACmax)
45
               cvx_begin gp quiet
46
               variables x1 x2 x3 x4 Ro fs
47
               alpha=1.1;
48
               %inductor RL model
49
               RL = (32+382*(fs/f0)^3.78);%mohm
50
               %loop resistance
51
               Rs = (Ron/x3+Ron/x1+Ron/x2)*1e3+RL; \%mohm
52
```

```
%k vs. Q, from minimum RMS current evaluation
53
               kopt=25.22*Ro^-1.14*(Rs)^(1.14-0.51)+1.44;
54
               L1=kopt*Ro*1e-3/2/pi/fs;
55
               C1 = kopt / (Ro * 1e - 3 * 2 * pi * fs);
56
               L2 = L1;
57
               C2 = C1;
58
               C3 = 2 * C1;
59
               %conduction loss
60
               Pcond = (Ron/x1+Ron/x3+Ron/x4)*(alpha*0.5*Iout)^{2+...}
61
                    Ron/x2*(alpha*Iout)^2;%W
62
               %gate switching loss
63
               Pgate=Vg^2*fs*2*(CggN_AsN*x1+CggN_AsN*x2+CggN_AsN*x3+
64
     CggP_AsP*x4)*2;%W
               %Cds charging/discharging loss
65
               Psw=0.5*(Vout)^2*fs*(CdsN_AsN*x1*2+CdsN_AsN*x3+CdsP_AsP*x4);
66
               %inductor loss
67
               PL=(alpha*Iout*0.5)^2*(RL)*1e-3*2;%W
68
               %capacitors and switch area constrain
69
               AC = (C1 + C2 + C3) / Kc;
               As = 2*(x1*AsN+x2*AsN+x3*AsN+x4*AsP);
71
               Q = Ro/Rs;
               minimize(Pcond+Psw+PL+Pgate)
               subject to
74
                    AC <= ACmax(i2);
75
                    As <= Asmax(i1);
76
                    L1 <= 5e - 9 * 2;
77
                    1/Q <=1;
78
79
               cvx_end
 %% the rest is for printing results
80
               fprintf( 'loss = %3.2f\n', cvx_optval );
81
               fprintf( 'fs = \%3.2f kHz\n', fs/1e3 );
82
               fprintf( 'Q= %3.2f\n', Ro/Rs);
83
```

```
fprintf( 'kopt= %3.2f\n', kopt );
84
                fprintf( 'L= %3.2f nH\n', L1*1e9);
85
                fprintf( 'C= %3.2f uF\n', C1*1e6);
86
                fprintf( 'As= %3.2f\n', As );
87
                fprintf( 'Rs= %3.2f mohm\n', Rs );
88
                min_loss(i2)=cvx_optval;
89
                Pcond_opt(i2)=Pcond;
90
                Psw_opt(i2)=Psw;
91
                PL_opt(i2)=PL;
92
                As_opt(i2)=As;
93
                Q_opt(i2)=Ro/Rs;
94
                k_opt(i2)=kopt;
95
                C1_opt(i2)=C1;
96
                fs_opt(i2)=fs;
97
                Pgate_opt(i2)=Pgate;
98
                Rs_opt(i2)=Rs;
99
                As1_opt(i2)=AsN*x1;
100
                As2_opt(i2) = AsN * x2;
101
                As3_opt(i2)=AsN*x3;
102
                As4_opt(i2) = AsP * x4;
103
           end
104
105 end
106 figure
107 plot(C1_opt*1e6, min_loss,'LineWidth',2)
108 ax = gca;
109 set(ax, 'FontSize', 20)
110 xlabel('C_1 [\mu F]')
uu ylabel('Loss [W]')
112 grid on
113
114 figure
ns plot(C1_opt*1e6, fs_opt/1e6, 'LineWidth',2)
```

```
116 ax = gca;
117 set(ax, 'FontSize', 20)
118 xlabel('C_1 [\mu F]')
119 ylabel('Switching Frequency [MHz]')
120 grid on
121
122 figure
123 plot(C1_opt*1e6, As_opt, 'LineWidth',2)
124 ax = gca;
125 set(ax, 'FontSize', 20)
126 xlabel('C_1 [\mu F]')
127 ylabel('Switch size [mm<sup>2</sup>]')
128 grid on
129
130 figure
131 plot(C1_opt*1e6, k_opt,'LineWidth',2)
132 ax = gca;
133 set(ax, 'FontSize', 20)
134 xlabel('C_1 [\mu F]')
135 ylabel('Optimal k=f_s/f_r')
136 grid on
137
138 figure
139 plot(C1_opt*1e6, Q_opt,'LineWidth',2)
140 ax = gca;
141 set(ax,'FontSize',20)
142 xlabel('C_1 [\mu F]')
143 ylabel('Optimal Q=R_o/R_s')
144 grid on
145
146 Io_vec=logspace(log10(0.5),log10(3),20);
147 Pcond=(Ron/x1+Ron/x3+Ron/x4)*(alpha*0.5*Io_vec).^2+...
```

```
148 Ron/x2*(alpha*Io_vec).^2;%W
149 Pgate=Vg^2*fs*2*(CggN_AsN*x1+CggN_AsN*x2+CggN_AsN*x3+CggP_AsP*x4)*2;%W
150 Psw=0.5*(Vout)^2*fs*(CdsN_AsN*x1*2+CdsN_AsN*x3+CdsP_AsP*x4);
151 PL=(alpha*Io_vec*0.5).^2*(RL)*1e-3*2;%W
152 figure
153 Ptot=Pcond+Pgate+Psw+PL;
154 hold on
155 plot(Io_vec, Vout*Io_vec./(Vout*Io_vec+Ptot)*100,...
      'LineWidth',2)
156
157 ylim([90 97])
158 max(Vout*Io_vec./(Vout*Io_vec+Ptot))*100
159 xlabel('output current')
160 ylabel('Efficiency [%]')
161 set(gca,'FontSize',12)
162 grid on
163
164 figure
165 p=pie([Pcond_opt Psw_opt Pgate_opt PL_opt]./min_loss)
166 pText = findobj(p,'Type','text');
167 percentValues = get(pText,'String');
168 txt = {'Pcond: ';'Psw: ';'Pgate: ';'Pind:'};
169 combinedtxt = strcat(txt,percentValues);
170 pText(1).String = combinedtxt(1);
171 pText(2).String = combinedtxt(2);
172 pText(3).String = combinedtxt(3);
173 pText(4).String = combinedtxt(4);
174 set(gca,'FontSize',30)
```

```
175 title('Loss breadown at 1.5A')
```

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