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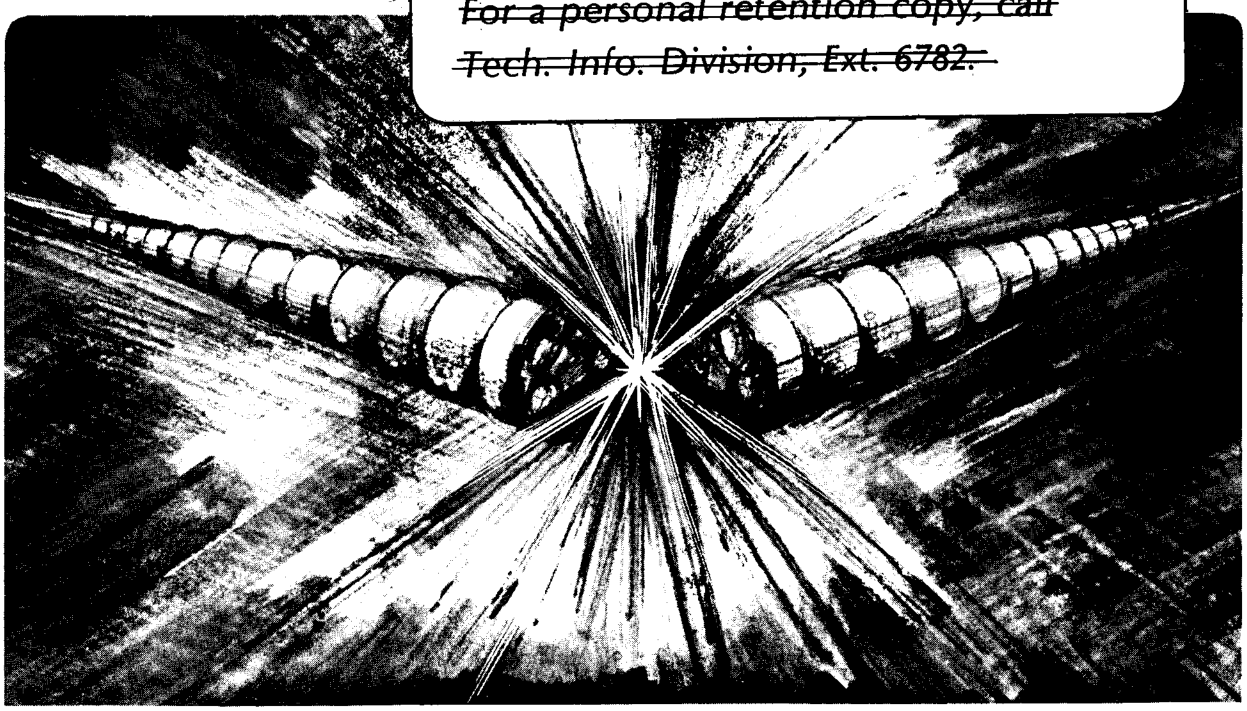

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Introduction

Beams consisting of closely spaced bunches, crossing at a small angle, represent the simplest way to achieve high luminosity with a moderate beam current and an acceptable peak event rate without resorting to the use of dipole magnets at, or very close, to the interaction point (IP). The elements of a procedure to derive beam parameters for this particular configuration form the content of this note, following closely the treatment of refs. 1,2,3.

The Quantities of Interest

We consider the energy E and the luminosity $\mathcal{L}$ to be the primary input variables. As a measure for the peak event rate we use the average number of events, $\langle n \rangle$, resulting from the collision of two beam bunches, calculated for a total cross section $\sigma_T \simeq 2 \times 10^{-25} cm^2$. This quantity is related to the luminosity and the bunch spacing S_B as follows:

$$\langle n \rangle = \sigma_T L = \sigma_T \frac{S_B}{c} \mathcal{L} \qquad (1)$$

where c is the velocity of light, thus showing the merits of closely spaced bunches with regard to duty factor. Another quantity of central importance is β^*, the value of the β-function at the IP. Lower limits on β^* derive mostly from considerations of chromatic effects, and are presently estimated to be in the range of 1 m to 2 m.

The parameters that will be derived from the performance objectives are the crossing angle α, the normalized transverse rms emittance ε_N, the number of particles per bunch N_B, the rms bunch length σ_ℓ and the momentum spread in terms of $\sigma_\gamma = \sigma_E / m_p$.

Determination of these parameters, or rather a range of self-consistent sets of values for them, is essential to rationally designing the collider. The emittance, e.g., enters directly any discussion of required magnetic field quality, σ_ℓ and σ_γ determine essentially the RF-system parameters, and taken together they determine injector phase space density requirements.

Constraints Imposed by the Beam-Beam Interaction

The transverse phase space density and therefore the luminosity achievable with a given value of N_B are limited be the highly nonlinear beam-beam interaction. The linear, i.e., small amplitude, beam-beam tune shift, ΔQ_{bb}, may be taken as a measure for the strength of the interaction, and it has long become customary to express the luminosity in terms of ΔQ for purposes of performance estimates. For bunched, "round beams" ($\varepsilon_x = \varepsilon_y$, $\beta_x = \beta_y$) colliding head-on ($\alpha = 0$)

$$\Delta Q_x = \Delta Q_y = -\frac{1}{4\pi} \frac{r_0 N_B}{\varepsilon_N} = -\xi_0 \qquad (2)$$

$$\mathcal{L} = \frac{c}{S_B} \frac{1}{4\pi} \frac{\gamma}{\beta^*} \frac{N_B^2}{\varepsilon_N} = \frac{c}{S_B r_0} \cdot \frac{\gamma}{\beta^*} N_B \, \xi_0 \qquad (3)$$

where $\gamma = E/m_p$ and r_0 = classical proton radius. In this case $\mathcal{L}$ is directly proportional to ξ_0 and obviously (2) and (3) can be solved for ε_N and N_B in terms of $\mathcal{L}$ and ξ_0.

For $\alpha \neq 0$ these equations must be modified. For $\sigma_\ell < \beta^*$, justifying it to keep β constant over the length of the interaction, the following equations are easily derived:

$$\Delta Q_x + \Delta Q_y = -2\xi_0 \left(1+r^2\right)^{-1/2} \qquad (4)$$

$$\mathcal{L} = \frac{c}{S_B} \frac{\gamma}{\beta^* r_0} N_B \, \xi_0 \left(1+r^2\right)^{-1/2} \qquad (5)$$

with

$$r = \frac{\alpha \sigma_\ell}{2\sigma^*}, \quad \sigma^* = \left(\frac{\varepsilon_N \beta^*}{\gamma}\right)^{1/2} .$$

Equation (5) follows directly from the definition of the luminosity, while (4) is essentially a consequence of $\operatorname{div} \vec{E} = \rho/\varepsilon_0$. In the case of crossing in the (x-s)-plane, ΔQ_x tends to 0 for large values of r and Eq. (4) expresses ΔQ_y. As r tends to zero (4,5) go over into (3,4) since in that case $\Delta Q_x = \Delta Q_y$. We further note that $|\xi_0|$ as given in (2), for given ε_N, N_B, represents an upper bound for both $|\Delta Q_x|$, $|\Delta Q_y|$ in the case of $\alpha \neq 0$. An expression for ΔQ_y for arbitrary r has been derived[1]

$$\Delta Q_y = -\xi_0 \, 2 \, \frac{(1+r^2)^{1/2} - 1}{r^2} \qquad (6)$$

from which together with (4)

$$\Delta Q_x = -\xi_0 \, 2 \left[(1+r^2)^{-1/2} - \frac{(1+r^2)^{1/2} - 1}{r^2} \right] \qquad (7)$$

obviously follows.

The question as to what numerical value of ΔQ to assume as safe, or even whether the sum $\Delta Q_x + \Delta Q_y$ or the larger of the two, is the most relevant quantity for an arbitrary value of r goes beyond the scope of this note. For very small r (~0.1 or 0.2), i.e. almost in the regime of head-on collisions $\Delta Q_x \simeq \Delta Q_y$, one might tentatively extrapolate present experience suggesting that $|\Delta Q| \simeq 0.003$ is safe although pertaining to a weak-strong, rather than the present strong-strong configuration.[4] For larger r two key differences become signficant:

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first, a strong modulation of the beam-beam force as particles undergo synchrotron oscillations and move from head to tail of the bunch, and second, the appearance of odd resonances, excluded by symmetry in the head-on case. As we go to quite large values of r essentially only one-dimensional, vertical resonances will be important. Careful analysis of nonlinear detuning, resonance widths and overlap, will be required to shed more light on this question.

For the present purpose we assume simplistically a tolerable maximum ΔQ_y, independent of r. Modifying this assumption will not change the structure of our procedure but modify numerical values and the functional dependence on r of the calculated quantities.

Inverting (5) and (6) and introducing the functions

$$f(r) = \left(1+r^2\right)^{-1/2} \quad \text{and} \quad g(r) = 2\,\frac{(1+r^2)^{1/2} - 1}{r^2}$$

we obtain:

$$N_B = \beta^* S_B \frac{r_o}{c\gamma} \frac{\mathcal{L}}{|\Delta Q_y|} \frac{g(r)}{f(r)} \qquad (8)$$

$$\epsilon_N = \beta^* S_B \frac{r_o^2}{c\gamma} \frac{\mathcal{L}}{|\Delta Q_y|^2} \frac{g^2(r)}{4\pi f(r)} \qquad (9)$$

Thus for any r, ϵ_N and N_B are determined. Note that N_B is monotonically increasing, ϵ_N monotonically decreasing with increasing r. Larger r therefore requires both a brighter beam as well as a larger total current.

To narow down the choice of α we must consider long range beam-beam effects. For bunches spaced S_B apart, close encounters occur at distances $S = \pm n(S_B/2)$ from the IP. We will not address the closed orbit effects but comment on the long-range beam-beam tune shift.

With the arrangements envisioned for the IR optics (at least for low β^*) most of the close encounters will occur in the quadrupoles where the beams are no longer round. Different formulations for the fields of the resulting bi-Gaussian distribution exist. Calculations based on an expansion of the form $B_{r,\phi}(r,\phi) = \sum_n A^{(n)}_{r,\phi}(r)\, e^{2in\phi}$ show that for the values of σ_x/σ_y expected and for beam separations $r \gtrsim 10\, \mathrm{Max}(\sigma_x,\sigma_y)$ the fields are well represented by a simple $1/r$ dependence.[3]

Furthermore, most close encounters occur at locations where the phase advance ψ from the IP is $\sim\pi/2$, due to the large values of β in the insertion quadrupoles. Exact calculations have shown that, at least for antisymmetric insertion design the total tune shift $|\Delta Q_{LR}|$ is approximated within 10% by substituting a drift length D on either side for the actual optical elements. This leads to

$$\Delta Q_y(LR) \cong -\Delta Q_x(LR) \cong -\xi_o \frac{8D}{n^2 S_B} \qquad (10)$$

where

$$n = d(s)/\sigma(s) = \alpha s/\sigma(s) \ .$$

From (10) together with the definition of r we obtain:

$$\alpha(r) = D^{1/2} \frac{r_o}{c\gamma} \left(\frac{2c}{\pi} \frac{\mathcal{L}}{|\Delta Q(0)|\,|\Delta Q(LR)|} \frac{g(r)}{f(r)} \right)^{1/2} \qquad (11)$$

and

$$\sigma(r) = \beta^* \left(\frac{S_B}{D}\right)^{1/2} 2r \left(\frac{|\Delta Q(LR)|}{8\,|\Delta Q(0)|}\, g(r) \right)^{1/2} \qquad (12)$$

Figure (1) shows $\epsilon_N(r)$, $N_B(r)$, $\alpha(r)$, $\sigma_\ell(r)$ normalized to allow scaling for different valus β^*, S_B, D, for $\mathcal{L} = 10^{33}\mathrm{cm}^{-2}\mathrm{s}$, $\gamma = 2.13\times10^4$, $|\Delta Q(0)| = 0.003$ and $|\Delta Q(LR)| = 0.001$.

A few caveats seem in order. First, again the question as to a tolerable magnitude of $\Delta Q(LR)$ arises, second it should be noted that this limit cannot be independent of S_B over a very large range of this variable. In contrast to the beam-beam interaction at the IP the nonlinear detuning, e.g., for the long range interaction is not proportional to the linear tune shift but decreases more rapidly with increasing beam separation. In other words, the long range effects from many close encounters at fairly large beam separation is expected to be more innocuous than those from a few encounters at smaller separation although $\Delta Q(LR)$ might be the same in both cases. In that sense the equations derived above, at least with regard to variations of S_B, are most useful as a first guide or if it is decided to keep the long range forces at essentially insignificant levels.

If values of $|\Delta Q(LR)|$ approaching or exceeding $|\Delta Q(0)|$ are envisaged a more detailed study, encompassing both aspects of the beam-beam interaction, are called for. It may be pointed out that in this case the total amplitude dependent detuning will be significantly modified since $\Delta Q_x(LR) \cong -\Delta Q_y(LR)$, while $\Delta Q_x(0)$ and $\Delta Q_y(0)$ have the same sign.

Choice of r, $\sigma_\ell(r)$ and Estimates of σ_γ

The choice of $\sigma_\ell(r)$, and therefore r, is to some extent arbitrary. Low values seem desirable for several reasons, such as lowered synchrotron radiation power and lessened demands on injector brightness. More importantly, perhaps, the lower r, the closer to the head-on regime the collider operates and the more credible are our estimates of a tolerable $\Delta Q(0)$ at the present level of understanding. For very short bunches eddy current losses begin to add to the cryogenic system power load:

$$P_{Eddy} \simeq \frac{1.23}{b} \left(\frac{ceN_B}{2\pi}\right)^2 n_B \left(\frac{Z_o}{2\sigma}\right)^{1/2} \sigma_\ell^{-3/2} \qquad (13)$$

where b is the vacuum chamber radius, n_B the number of bunches and σ the conductivity of the chamber wall. For a Cu wall at 4°K and values of $S_B \simeq 15$ m ($N_B \simeq 3\times10^{10}$) this becomes appreciable only for σ_z of a few centimeters or less.

RF-system considerations will play a dominant role in selecting σ_ℓ. Expecting a bucket/bunch area ratio of ~3 from RF-noise considerations $\sigma_\ell \cong 0.089\,\lambda_{RF}$ or 13.3, 7.6, 3.3 cm for f_{Rf} = 200 MHz, 350 MHz and 800 MHz, respectively. Considerations of collective stability, most prominantly the transverse mode coupling (or fast head-tail) instability, require a minimum ΔE, or equivalently σ_γ.[5] In most cases, however, the most severe lower limit on σ_γ derives from considerations of intrabeam scattering (IBS). The role of IBS

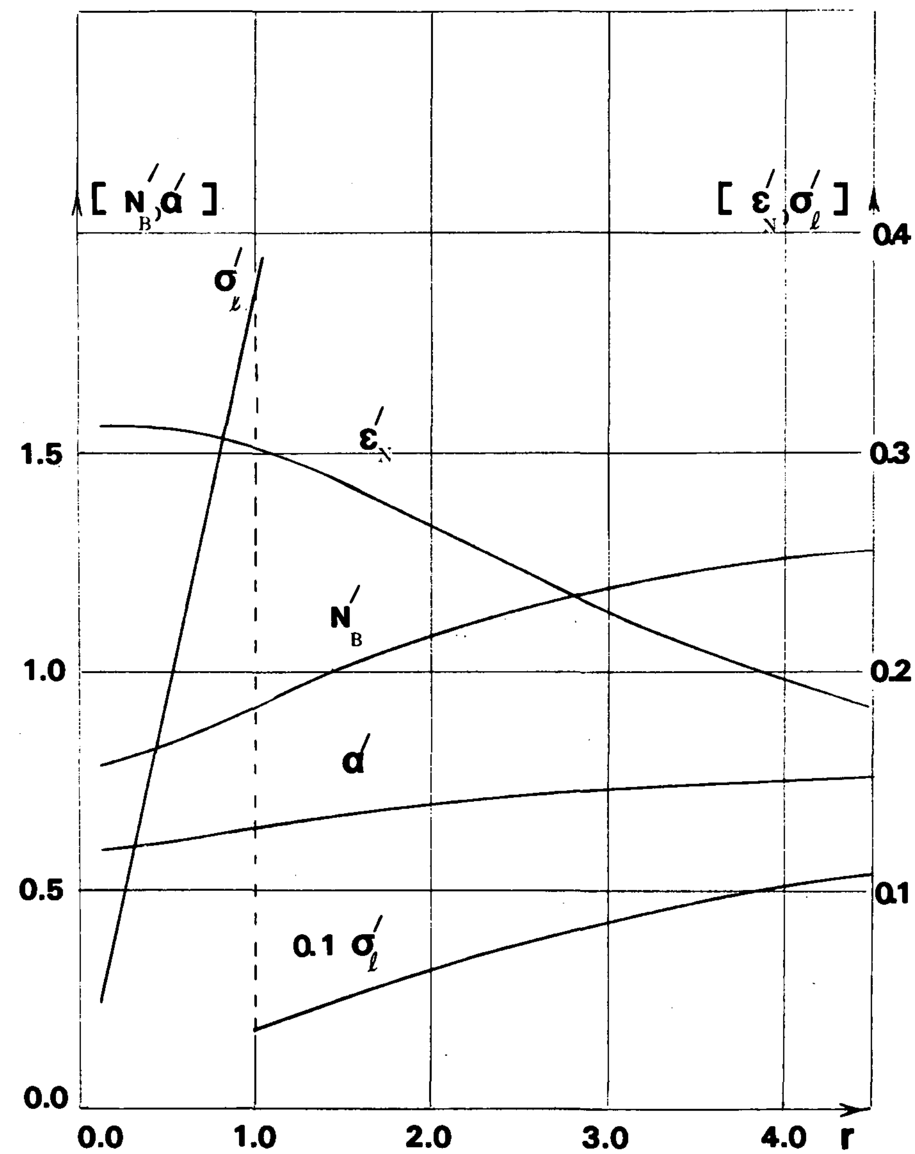

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Fig. 1. Shown are, assuming $\gamma = 2.13\text{x}10^4$, $\mathscr{L} = 10^{33}\text{cm}^{-2}\text{s}^{-1}$, $\Delta Q_y(0) = -0.003$, $\Delta Q_y(\text{LR}) = -0.001$, the following quantities as functions of $r = \alpha\ \sigma\ /2\sigma^*$:

$$\epsilon_N' = \epsilon_N \left(\frac{10^{-7}}{\beta^* S_B}\right)$$

$$N_B' = N_B \left(\frac{10^{-9}}{\beta^* S_B}\right)$$

$$\sigma_\ell' = \sigma_\ell \left(\beta^* \sqrt{S_B/D}\right)$$

$$\alpha' = \alpha\ 10^5/D^{1/2}$$

The normalizing factors indicate the functional dependence on the parameters β^*, S_B, D.

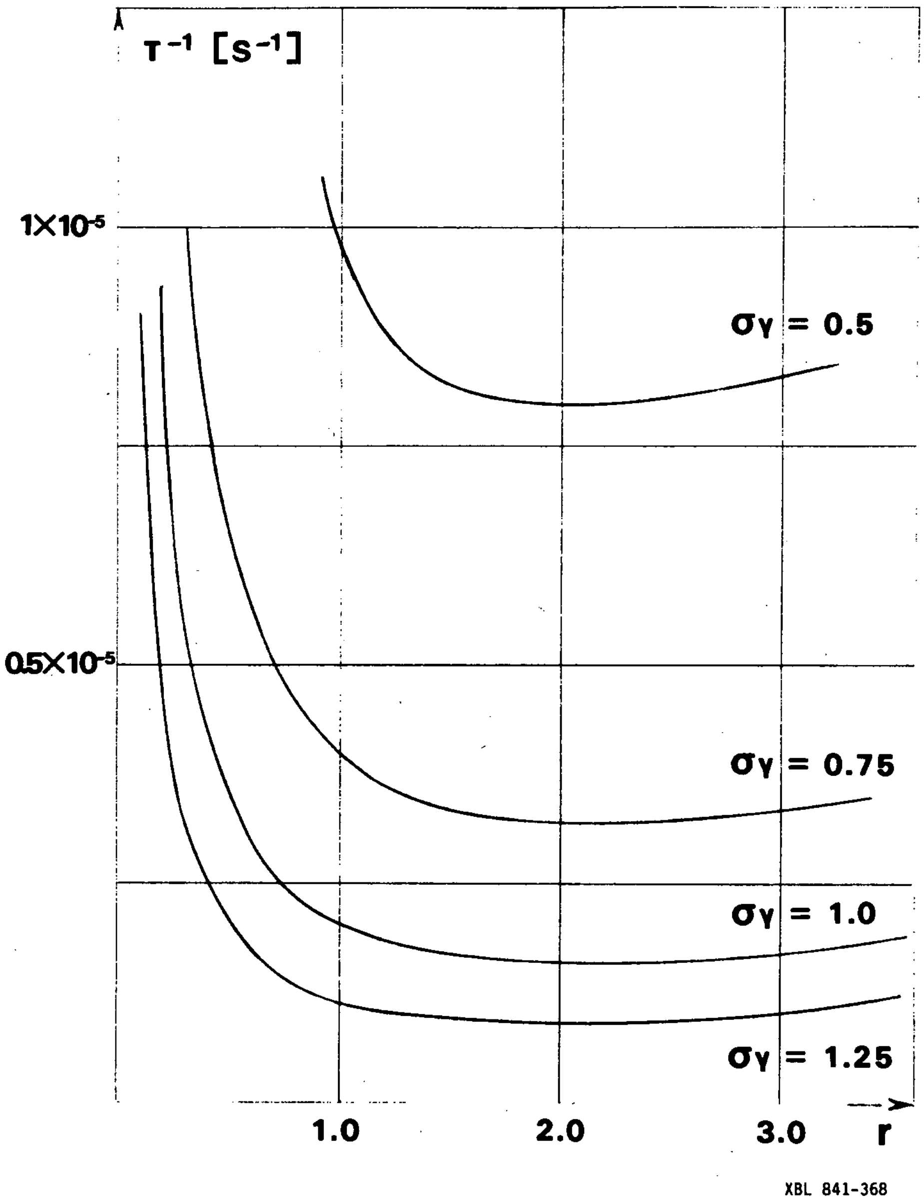

Fig. 2. Longitudinal intra beam scattering growth rates for different values of momentum spread σ_γ. Calculations are based on the same set of values $\gamma, \mathcal{L}, \Delta Q(0), \Delta Q(LR)$ as used in Fig. 1, and $\beta^* = 2$ m is assumed. Values as shown are based on the regular cells of the lattice only, assuming $L_c = 160$ m, $\mu = =60^\circ$ and a bend angle of $\sim 0.7^\circ$ per cell. In the parameter range covered the horizontal rate τ_x^{-1} is in the range of $0.3\,\tau_{\|}^{-1}$ to $\sim\tau_{\|}^{-1}$. Synchrotron radiation damping rates for $B = 6.5$ T are $\sim 2\times10^{-5}s^{-1}$ and $1\times10^{-5}s^{-1}$ for the longitudinal and horizontal planes, respectively.

is illustrated, based on the theory of Bjorken and Mtingwa,[6] in Fig. 2.

Example Parameter Set

Assuming $\gamma = 2.13 \times 10^4$, $\mathcal{L} = 10^{33} cm^{-2} s^{-1}$, $\Delta Q_y(0) = -0.003$, $\Delta Q_y(LR) = -0.001$, and $\beta^* = 2$ m, $S_B = 15$ m, $D = 75$ m, we construct the following example parameter set:

N_B	2.42×10^{10}
ϵ_N [m]	9.3×10^{-7}
σ [m]	0.133*
α_ℓ	5.2×10^{-5}
r	0.37
σ_γ	1.0
$\tau_{\parallel,IBS}$ [hours]	63
$\tau_{x,IBS}$ [hours]	110
P_s [kW]	~12.5**
P_{Eddy} [KW]	< 0.6***

* corresponding to $A_{bucket}/A_{bunch} \approx 3$ at 200 MHz

** synchrotron radiation power in 6.5 T ring, for one beam

*** upper limit, calculated with room temperature conductivity of Cu; 4°K value 3 to 10 times less

Conclusions

A procedure is demonstrated that allows the quick determination of beam parameters from the performance objectives together with some input from the lattice design. The greatest uncertainty remains in the area of estimating a safe, allowable strength of the beam-beam interaction, and care must be exercised not to specify values exceeding the beam brightness that can be delivered by the injector.

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