Distributed Scheduling for Efficient HVAC Pre-cooling Operations ⋆

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Abstract: Supply fan takes up to 30% of Heating, Ventilation and Air Conditioning (HVAC) systems' in-building energy consumption. Since the fan power consumption is a cube function of the mass flow rate of supply air, it is thus possible to reduce the energy consumption significantly by elongating and scheduling the pre-cooling processes. To handle the computational complexity raised for large scale buildings, two distributed consensus-like scheduling algorithms are proposed in this paper. Simulation studies indicate higher energy efficiency compared to uncoordinated pre-cooling processes; the computation is faster than centralized optimization, although with performance sacrifice.

Keywords: HVAC; Pre-cooling; Distributed scheduling.

1. INTRODUCTION

HVAC systems consume 40% of the building energy in USA as estimated by Department of Energy. The efficiency of HVAC system design, operation and control has received extensive research attention for the purposes of resource conservation, cost reduction and etc. Through energy disaggregation analysis for variable air volume (VAV) systems (Englander [1990]), it was found that a large portion of energy use in HVAC systems is attributed to air movement devices. Therefore, an accurate estimation of fan power model and efficient operation of fans are critical for reducing energy consumptions.

The problem of energy consumption saving of the supply fan is studied in this paper. As reported in the literature, this problem can be discussed from different perspectives. Firstly, a good fan controller, regardless of the control structure and controlled variables, which produces more stable system performance is foundation for comfort guarantee and fan's routine operation (Roger [2005]). Secondly, it was noticed that energy is wasted at the terminal dampers or economizer dampers. The static pressure and correspondingly the fan power will not be unnecessarily high if these damper positions are chosen properly (Englander [1990]Nassif [2010]). Here another angle from temporal perspective is adopted. The cooling or heating tasks are properly scheduled along the time line such that the total energy is reduced. This approach is motivated by the fan's power function, which is proportional to the cube of the mass flow rate. Rooms have to be cooled down to predefined temperature setpoints before occupied according to the room occupancy schedule. This process is called pre-cooling. By elongating pre-cooling with a lower supply air mass flow rate, the energy consumption can be significantly reduced.

Pre-cooling strategies, in its various forms, have been proposed in the literature. The potential to reduce peak-period electrical demand by adjusting HVAC controls was validated in Xu [2004]. Morgan [2007] explored the effectiveness of different pre-cooling strategies in different locations. Optimization based pre-cooling strategies were investigated in Yin [2010]. All these proposals achieve either energy savings or peak cost reduction via pre-cooling the thermal masses.

However, it is common that groups of rooms require pre-cooling at similar times, for example in the morning around 08 : 00AM. If individual rooms operate independently without a central coordinator, an aggregated energy demand peak is often observed. Thus a pre-cooling scheduler, which coordinates the pre-cooling processes for all rooms supplied by a single supply fan, may provide potential energy saving. In fact, optimization based scheduling for HVAC systems has shown great potential for energy conservation and cost reduction. An approach called 'Green Scheduling' was proposed in Nghiem [2011] to reduce the peak demand. The approach was further extended and successfully applied to chiller plants and radiant systems in buildings (Behl [2012]Nghiem [2013]).

An optimal scheduling of the pre-cooling processes was proposed in previous study Nikitha [2014]. Different with the Xu [2004]Yin [2010]Morgan [2007], which either consider assignment of temperature setpoint and chiller performance or develop uniform pre-cooling strategy for the

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whole building, it determines the optimal air mass flow rate profiles for all rooms based on detailed system models and optimization. Substantial energy saving was observed via simulation. The method requires to solve a centralized optimization problem, which is computationally formidable if there are hundreds of rooms. In this paper, two distributed consensus-like algorithms are proposed for computational efficiency. The convergence of the optimization algorithms is investigated. The effectiveness on energy demand reduction and computation is illustrated via simulation studies.

The rest of the paper is organized as follows. The in-building part of HVAC systems is described in section 2; the centralized scheduling problem is formulated in section 3; two distributed scheduling algorithms are proposed in section 4; simulation results are presented in section 5; finally conclusions are drawn in section 6.

2. HVAC SYSTEMS AND OPERATION

The in-building part of HVAC systems normally consists of outdoor dampers, air handling units(AHU), ducts, terminal units and etc. AHU is responsible to filter the air and to cool it to the setpoint temperature; the supply fan blows the cool air into the supply ducts. Based on the feedback from thermostat located in each room, the damper position in the terminal unit is adjusted to maintain the room temperature within a comfort bound. A return fan transports the exhaust to outside or recycles it within the building. The out-building part, including chillers etc., provides enough coolant to cool down the supply air. In the following part, the models of HVAC components relevant to this paper are presented first.

(1) Room thermal model
Assume that there are J rooms supplied by a supply fan. The temperature dynamic of each room is governed by a first order model:

$$\dot{T}_j + \frac{\dot{m}_j}{M_j} T_j = \frac{\dot{m}_j c_p T_{ca} + \dot{Q}_j}{M_j c_p}, \tag{1}$$

where T_j, $\dot{m}_j$, M_j, $\dot{Q}_j$ are the temperature, cool air mass flow rate, air mass and the cooling load of room j; T_{ca} is the temperature of the cool air and c_p is the specific heat capacity.

(2) Fan power model
According to Mitchell [2012], the fan power can be approximated by the cube function of the supply air mass flow rate:

$$P_{fan} = k_{fan} \dot{m}(t)^3, \tag{2}$$

where the constant k_{fan} is obtained via curve fitting.

(3) Distribution network
The distribution duct can be treated as a tree structure with rooms as the leaf nodes. The pressure drop over a piece of constant-area duct is given by

$$\Delta P = f_D \frac{\rho L v^2}{2D} = f_D \frac{8L}{\pi^2 D^5} \dot{m}(t)^2, \tag{3}$$

where f_D, L, D, ρ, v are the friction factor, duct length, duct diameter, fluid density, the average fluid velocity and $\dot{m}(t) = \frac{\pi}{4} D^2 \rho v$ is the air mass flow rate along the duct.

(4) Damper model
The pressure drop across an opposite blade damper is determined by

$$\Delta P = k_d \dot{m}(t)^2, \tag{4}$$

where the flow resistance coefficient k_d varies with the damper position θ (Nassif [2010]). Here the leakage is ignored such that k_d can vary between k_{min} to ∞, which correspond to the fully open and fully closed damper positions respectively.

3. OPTIMAL SCHEDULING

It is assumed that the cooling load $\dot{Q}_j(t)$ is predictable, which may be obtained from historical data. Each room has its own occupancy time $[t_j^s \; t_j^e]$ and the temperature setpoint T_j^s. Pre-cooling process is required to cool down the room temperature such that $T_j(t_j^s) \leq T_j^s$; during the occupancy time, assume that an ideal controller produces the exact mass flow rate $\dot{m}_j(t) = \frac{\dot{Q}_j(t)}{(T_j^s - T_{ca})c_\rho}$ such that $\dot{Q}_j$ is compensated and $T_j(t)$ always stays below the setpoint. A pre-cooling scheduling problem is formulated in Nikitha [2014] that the optimal mass flow rate profiles are determined such that the overall energy consumption is minimal and the temperature constraints are fulfilled. It is necessary to include all relevant components, such as water pumps and chillers, in the energy consumption function. Here for simplicity, only the supply fan is considered. The optimization problem is stated as $\mathcal{P}_c$:

$$\min_{\dot{m}_j(t),\ 1 \leq j \leq J} \quad E = \int_{t_{mor}}^{t_{eve}} P_{fan}(t) dt \tag{5}$$

subject to

(1) $P_{fan}(t) = k_{fan} \dot{m}(t)^3 = k_{fan} (\sum_{j=1}^{J} \dot{m}_j(t))^3$,
(2) $\dot{m}_j(t) = \frac{\dot{Q}_j(t)}{(T_j^s - T_{ca})c_\rho}, \quad t_j^s \leq t \leq t_j^e, \ 1 \leq j \leq J$,
(3) room thermal model (1) and $T_j(t_j^s) \leq T_j^s$.
(4) balancing constraint
The mass flow rates $\dot{m}_j$ have to be assigned in such a way that the corresponding fan's speed and dampers positions $\{\theta_1, \ldots, \theta_J\}$ are within their capacities. Otherwise, mass flow rate profiles are artificial since the resulted pressure distribution is not balanced. The mathematical formulation of this constraint is determined by the topology of the duct network as shown in Nikitha [2014], to which readers may refer.

In Nikitha [2014], $\dot{m}_j(t)$ during pre-cooling phase is characterized by only two parameters, air flow rate amplitude and pre-cooling duration. Even though the number of decision variables is greatly reduced, the computational burden is still formidable for large scale scheduling problems which may consider hundreds of rooms in a building. In section 4, two distributed scheduling algorithms based on discrete-time system models are presented. It is expected that the proposed algorithms would ease the computation with an acceptable performance loss.

4. DISTRIBUTED SCHEDULING

To have a finite number of decision variables, the time interval $[t_{mor} \; t_{eve}]$ is divided into N partitions evenly with

δ minutes for each partition. The mass flow rate to room j is a constant, i.e. $\dot{m}_j(t) = \dot{m}_j^i \; \forall t$ within the partition i (hereafter the superscript and subscript of $\dot{m}$ refer to the partition and room indexes, respectively). The occupancy-start time t_j^s is approximated by an integer index i_j. The model (1) is discretized accordingly as

$$T_j^{i+1} = (1 - \frac{\dot{m}_j^i \delta}{M_j}) T_j^i + \frac{\dot{m}_j^i c_\rho T_{ca} + \dot{Q}_j^i}{M_j c_\rho} \delta. \qquad (6)$$

Assumption 1. There is a given profile of mass flow-rate $\dot{m}_j^i(0)$ for $1 \le j \le J$ and $1 \le i \le N$, which satisfy the physical constraints and temperature requirements.

$\dot{m}_j^i(0)$ may be obtained as an aggregation of each room's local scheduling result. It is common that the supply fan is designed to be powerful enough to cool the whole building quickly in peak time. Therefore the constraints violation may rarely occur. The distributed algorithms aims to improve the optimality of $\dot{m}_j^i(0)$ progressively in a distributed fashion. Simulation studies, as shown in section 5 later, indicate that the optimal $\dot{m}(t)$ tends to be flat. This observation suggests to replace the original energy cost with a 'flatness' function, which is adopted here. The scheduling problem is converted to a consensus-like one, which attempts to minimize the differences among all partitions while satisfying various constraints.

4.1 Distributed algorithm 1

(1) Consensus-like optimization
For simplicity, it is assumed that $N = 4J$. As shown in Figure 1, every adjacent 4 partitions are treated as a group in series. For example, group $G_j = \{P_q,\ P_{q+1},\ P_{q+2},\ P_{q+3}\}$. There are J groups in total. The roles of each partitions within a group are defined as below. The second partition labeled with slashes is called an active partition, which is to make a decision on how much to shift the mass flow rate forward and backward. The mass flow rate of the first and third partitions will change according to the decision of the active partition. The forth one is called idle partition, which is included to separate the adjacent groups.

The consensus algorithm is an iteration based algorithm. Correspondingly, the grouping is dynamically evolving with iterations. At iteration k, group $G_j = \{P_q,\ P_{q+1},\ P_{q+2},\ P_{q+3}\}$; at iteration $k+1$, G_j will shift one partition forward with P_{q+2} as the new active partition. The iteration is illustrated in Figure 1, as well. It is easy to see that each partition will become active once in a cycle of 4 iterations.

The adjustable mass flow rate variables associated with group G_j is defined as a set $\mathbb{M}_j$, for example $\mathbb{M}_j = \{\dot{m}_j^{q+1}\}$. During each iteration, every group will execute its own local optimization $\mathcal{P}_{a,j,k}$, where the subscripts j, a and k refer to the index of group G_j, the index of active partition in group G_j and iteration number. The designed mass flow rate to room j for partition i in the kth iteration is denoted as $\dot{m}_j^i(k)$. The optimization problem $\mathcal{P}_{a,j,k}$ is defined as below:

$$\min_{\Delta_b,\ \Delta_f} \sum_{i=-2}^{1} |\dot{m}^{a+i}(k) - \dot{m}^{a+i+1}(k)|^2 \qquad (7)$$

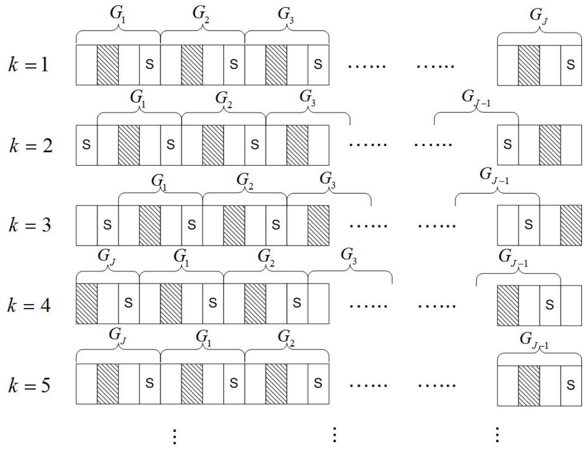

Fig. 1. Grouping and shifting

subject to

$$\dot{m}^{a-2}(k) = \dot{m}^{a-2}(k-1), \qquad (8)$$
$$\dot{m}^{a+2}(k) = \dot{m}^{a+2}(k-1), \qquad (9)$$
$$\dot{m}^{a-1}(k) = \dot{m}^{a-1}(k-1) + \Delta_b, \qquad (10)$$
$$\dot{m}^{a}(k) = \dot{m}^{a}(k-1) - \Delta_b - \Delta_f, \qquad (11)$$
$$\dot{m}^{a+1}(k) = \dot{m}^{a+1}(k-1) + \Delta_f, \qquad (12)$$
$$\dot{m}_j^{a}(k+1) = \dot{m}_j^{a}(k) - \Delta_b - \Delta_f \ge 0, \qquad (13)$$
$$\dot{m}_j^{a-1}(k) = \dot{m}_j^{a-1}(k-1) + \Delta_b, \qquad (14)$$
$$\dot{m}_j^{a+1}(k) = \dot{m}_j^{a+1}(k-1) + \Delta_f, \qquad (15)$$
$$\Delta_b \ge 0, \Delta_f \ge 0, \qquad (16)$$
$$T_j(i_j) \le T_j^s, \qquad (17)$$
$$\textit{balancing constraints}. \qquad (18)$$

(2) Convergence analysis
The problem of interest is the convergence of the overall cost function along the consensus iterations, which is defined as:

$$J_k = \sum_{i=1}^{N-1} (\dot{m}^i(k) - \dot{m}^{i+1}(k))^2, \qquad (19)$$

which is the sum of the cost function of all J groups. Since $\sum_{i=-2}^{1} |\dot{m}^{a+i}(k) - \dot{m}^{a+i+1}(k)|^2 \le \sum_{i=-2}^{1} |\dot{m}^{a+i}(k-1) - \dot{m}^{a+i+1}(k-1)|^2$ as in (7) holds for each group, $J_k \le J_{k-1}$. It is easy to see that the series J_k is lower bounded. Therefore according to the monotone convergence theorem, J_k will converge to a constant.

(3) Variants
(a) more or less rooms
In case there are more rooms, i.e. $N \le 4J$, it is advantageous to allocate each group more than 1 decision variable, i.e. $\mathbb{M}_j = \{\dot{m}_r^{q+1} | r \in \mathbb{R}_j\}$, where $\mathbb{R}_j \subset \{1,\ \ldots,\ J\}$ and $\mathbb{R}_j \cap \mathbb{R}_v = \emptyset$ if $j \ne v$. The condition $\mathbb{R}_j \cap \mathbb{R}_v = \emptyset$ implies at anytime, at most 1 group is allowed to modify the mass flow rate profile for a particular room.

In case of $N \ge 4J$, it may be advantageous to divide the whole time-span into a few segments. Each segment has less than $4J$ partitions. Then it is possible to apply the above proposed approach to each segment. If necessary, it is also possible to change the segmentation online.

(b) free adjustment, no active partition

In fact, it is not necessary to define an active partition P_a and to allow only P_a to shift its mass flow rate forward and backward. It is more general to allow partitions $P_{a-1},\ P_a,\ P_{a+1}$ to shift mass flow rate among them freely. It is also possible to reduce the total mass flow rate of the three partitions, while remaining the constraints satisfied.

(c) updating the set $\mathbb{M}_j$ in a more intelligent way

For some groups, the values of the adjustable variables in $\mathbb{M}_j$ may be quite small, so the cost reduction may be marginal. It is reasonable to choose other variables of large value to adjust, which means to choose the adjustable variables dynamically. However a local decision may violate the requirement $\mathbb{R}_j \cap \mathbb{R}_v = \emptyset$. Therefore an intelligent approach for adjustable variable selection may be preferred. Otherwise, the constraint (17) may be replaced with a local temperature constraint.

(d) original energy cost function

If the energy cost function is adopted in local optimization, there is no need to insert an idle partition since there is no coupling between groups.

4.2 Distributed algorithm 2

(1) Consensus-like optimization

Instead of insertion of an idle partition to decouple the cost functions among groups as in section 4.1, a different consensus-like strategy is proposed here. For iteration k, assume that each partition i only has the information of partitions $\{i-1,\ i,\ i+1\}$. Each partition will make a decision whether to shift some amount of air to left, right or not by comparing the 3 variables $\{\dot{m}^{i-1}(k),\ \dot{m}^{i}(k),\ \dot{m}^{i+1}(k)\}$. Let $d^i(k) = \arg\max_{j\in\{i-1,i,i+1\}}(\dot{m}^i(k) - \dot{m}^j(k)) - i$. The values of $d^i(k)$, -1, 0, 1, represent left, till, and right, respectively. An example of the shifting direction decisions is depicted in Figure 2.

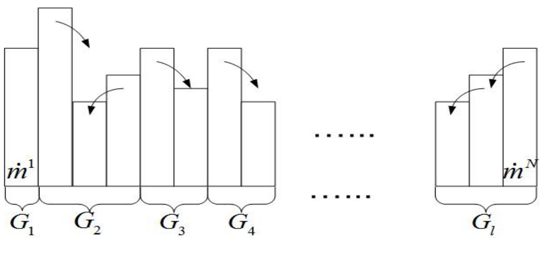

Fig. 2. Shifting directions and grouping

Based on the shifting directions, it is possible to divide the partitions into several groups(for analysis purpose only). The principle of grouping is the connection of the shifting directions. It is possible to assign certain adjustable variables $\mathbb{M}_i$ to each partition as in Algorithm 1, but without the restriction of $\mathbb{R}_j \cap \mathbb{R}_v = \emptyset$. It is also possible to rotate(or dynamically change) the assignment along iterations.

Take partition i, $d^i(k) = 1$ and $\mathbb{M}_i = \{\dot{m}^i_j\}$ as example. The amount of shifting air is determined by the optimization $\mathcal{P}^i(\dot{m}^i(k-1), \dot{m}^{i+1}(k-1))$:

$$\min_{\hat{\Delta}^i} |\hat{\dot{m}}^i(k) - \hat{\dot{m}}^{i+1}(k)|^2 \tag{20}$$

subject to

$$\hat{\dot{m}}^i(k) = \dot{m}^i(k-1) - \hat{\Delta}^i, \tag{21}$$

$$\hat{\dot{m}}^{i+1}(k) = \dot{m}^{i+1}(k-1) + \hat{\Delta}^i, \tag{22}$$

$$\hat{\dot{m}}^i_j(k) = \dot{m}^i_j(k-1) - \hat{\Delta}^i \geq 0, \tag{23}$$

$$\hat{\dot{m}}^{i+1}_j(k) = \dot{m}^{i+1}_j(k-1) + \hat{\Delta}^i \tag{24}$$

$$\hat{\Delta}^i \geq 0. \tag{25}$$

The objective of $\mathcal{P}^i$ is solely to flatten the mass flow rate profile without consideration of temperature requirement and balancing condition, therefore the solution of $\mathcal{P}^i$ may not be feasible. Obviously, the constraint violation is due to the unconscious large adjustments. To enforce these constraints, the adjustments are scaled by a scalar α, which is determined by a central optimization $\mathcal{P}_g$:

$$\max_{\alpha} \alpha \tag{26}$$

subject to

$$\Delta^i = \alpha\hat{\Delta}^i,,\ 1 \leq i \leq N, \tag{27}$$

$$\dot{m}^i_j(k) = \dot{m}^i_j(k-1) - \Delta^i + |\frac{d^{i+1}(k)-1}{2}|\Delta^{i+1} + |\frac{d^{i-1}(k)+1}{2}|\Delta^{i-1},\ 1 \leq i \leq N, \tag{28}$$

$$T_j(i_j) \leq T^s_j,\ 1 \leq j \leq J, \tag{29}$$

$$\textit{balancing constraints},\ 1 \leq i \leq N. \tag{30}$$

It is noted that $\mathcal{P}_g$ may involve a large number of constraints due to (30). However, if it is assumed that there exists a $\alpha^i, 1 \leq i \leq N$ such that the balancing constraint for partition i is satisfied for any $0 \leq \alpha \leq \alpha^i$, (30) can be checked in a distributed manner. Then a bound of α is chosen as $\min_i\ \alpha^i$. Similarly for constraint (29), the J inequalities can be checked in parallel. Therefore, $\mathcal{P}_g$ would not increase computation burden significantly.

(2) Convergence analysis

Except for the groups of till partitions($\dot{m}^1$ for example), there are only 2 patterns of shifting direction connection within a group, as shown in Figure 3 and 4. We will analyze the energy cost within each group.

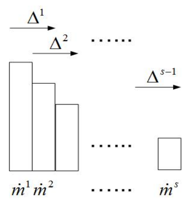

Fig. 3. Pattern 1

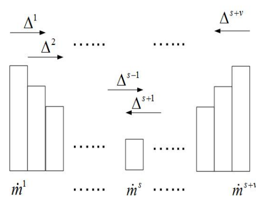

Fig. 4. Pattern 2

For groups of pattern 1 shown in Figure 3, a special case as shown in Figure 5 is examined first. Partition 1 moves Δ^1 to partition 2 at iteration k. The cost function at $k+1$ is $J_{k+1} = f(\dot{m}^1(k+1)) + f(\dot{m}^2(k+1))$.

Assumption 2. $f(x)$ is a differentiable function and $f'(x)$ is a monotonically increasing function.

$$J_{k+1} = f(\dot{m}^1(k) - \Delta^1) + f(\dot{m}^2(k) + \Delta^1) \quad (31)$$
$$= f(\dot{m}^1(k)) + \int_{\dot{m}^1(k)}^{\dot{m}^1(k)-\Delta^1} f'(x)dx$$
$$+ f(\dot{m}^2(k)) + \int_{\dot{m}^2(k)}^{\dot{m}^2(k)+\Delta^1} f'(x)dx$$
$$= f(\dot{m}^1(k)) + f(\dot{m}^2(k))$$
$$+ \int_0^{\Delta^1} f'(\dot{m}^2(k) + x) - f'(\dot{m}^1(k) - \Delta^1 + x)dx$$

Since $\dot{m}^2(k) \le \dot{m}^1(k) - \Delta^1$, $J_{k+1} \le J_k$. Now consider the case as shown in Figure 3. Assume that for a group of $s-1$ partitions $J_{k+1} \le J_k$ holds. Then for a group of s partitions,

$$J_{k+1} = f(\dot{m}^1(k) - \Delta^1) + \sum_{i=1}^{s-2} f(\dot{m}^{i+1}(k) + \Delta^i - \Delta^{i+1})$$
$$+ f(\dot{m}^s(k) + \Delta^{s-1}) \le f(\dot{m}^1(k) - \Delta^1)$$
$$+ f(\dot{m}^2(k) + \Delta^1) + \sum_{i=3}^{s} f(\dot{m}^i(k)) \le J_k. \quad (32)$$

For groups of pattern 2 shown in Figure 4, consider the special case shown in Figure 6 first.

$$J_{k+1} = f(\dot{m}^1(k) - \Delta^1) + f(\dot{m}^2(k) + \Delta^1 + \Delta^3)$$
$$+ f(\dot{m}^3(k) - \Delta^3) = f(\dot{m}^1(k)) + \int_{\dot{m}^1(k)}^{\dot{m}^1(k)-\Delta^1} f'(x)dx$$
$$+ f(\dot{m}^2(k)) + \int_{\dot{m}^2(k)}^{\dot{m}^2(k)+\Delta^1+\Delta^3} f'(x)dx + f(\dot{m}^3(k))$$
$$+ \int_{\dot{m}^3(k)}^{\dot{m}^3(k)-\Delta^3} f'(x)dx = J_k \quad (33)$$
$$+ \int_0^{\Delta^1} -f'(\dot{m}^1(k) - \Delta^1 + x) + f'(\dot{m}^2(k) + x)dx$$
$$+ \int_0^{\Delta^3} f'(\dot{m}^2(k) + \Delta^1 + x) - f'(\dot{m}^3(k) - \Delta^3 + x)dx$$

Assume that $\dot{m}^3(k) \ge \dot{m}^1(k) \ge \dot{m}^2(k)$, $\Delta^1 \le \frac{\dot{m}^1(k) - \dot{m}^2(k)}{2}$ and $\Delta^3 \le \frac{\dot{m}^3(k) - \dot{m}^2(k)}{2}$ (generally it requires Δ^i is smaller than half of the difference between partition i and its neighbor partition which it is moving mass to). It is easy to see that

$$\dot{m}^2(k) \le \dot{m}^1(k) - \Delta^1 \quad (34)$$
$$\dot{m}^2(k) + \Delta^1 \le \dot{m}^3(k) - \Delta^3, \quad (35)$$

therefore $J_{k+1} \le J_k$. Now assume that for a group of $s+v-1$ partitions $J_{k+1} \le J_k$ holds. Consider the case as shown in Figure 4.

$$J_{k+1} = f(\dot{m}^1(k) - \Delta^1) + \sum_{i=2}^{s+v} f(\dot{m}^i(k+1))$$
$$\le f(\dot{m}^1(k) - \Delta^1) + f(\dot{m}^2(k) + \Delta^1) + \sum_{i=3}^{s+v} f(\dot{m}^i(k))$$
$$\le J_k \quad (36)$$

The above analysis indicates that the energy cost in each group is non-increasing. Therefore the overall cost function is non-increasing, which will converge to a constant. The fan power function P_{fan} obviously satisfies Assumption 2. Since other equipments, such as water pumps and chillers, consume much energy as well, it is advantageous to use a composite power function in a single optimization problem. If it still satisfies Assumption 2, the proposed scheduling algorithm can also be applied.

Table 1. Room specifications

	1	2	3	4
t_j^s	09:30	10:00	10:00	09:30
T_j^s	22°C	21°C	20°C	20°C

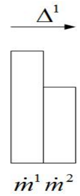

Fig. 5. Special case of pattern 1

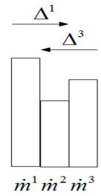

Fig. 6. Special case of pattern 2

5. SIMULATION STUDIES

A small scale and simplified simulation is conducted to illustrate the proposed algorithms. Four rooms are considered with details presented in Table 1. The period for pre-cooling scheduling starts at 08 : 00AM and ends at 10 : 00AM. The temperature at 08 : 00AM is 24°C and $M_j = 100kg$ for all rooms. The data in Causone [2010] is taken as cooling load $\dot{Q}(t)$. For simplicity, the distribution network is not included in simulation. A sampling interval $\delta = 0.1$hour is adopted for optimization in discrete domain. Firstly, the optimal profiles obtained from central optimization $\mathcal{P}_c$, which is solved by MATLAB® optimization toolbox are plotted in Figure 7. The optimal energy cost is 1.7×10^6(dimensionless); the optimization took 9.12s to complete. The trend of $\dot{m}$ empirically justifies the 'flatness' function as a reasonable alternate for distributed optimization, as chosen in section 4.

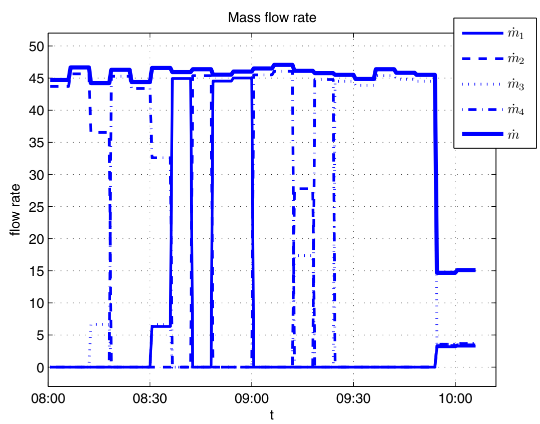

Fig. 7. Optimal mass flow rate profile by $\mathcal{P}_c$

Next, examine the performance of the proposed algorithms. For the sake of limited space, only the results of

Table 2. Scheduling results(C_i: cost of initial profile, C_o: cost of optimized profile, t_c: computation time.)

ipp/mins	12	18	30	42	60	90	120
$C_i/10^6$	36.5	17.6	7.03	6.36	4.17	2.29	2.02
$C_o/10^6$	17.8	10.9	6.4	4.62	3.08	1.88	1.86
t_c/s	2.12	2.70	3.52	2.35	2.98	3.40	2.48

Algorithm 1 are reported. The initial mass flow rate profile, as required in Assumption 1, is obtained as the aggregation of all the rooms' initial local scheduling results. Similar to Xu [2004]Morgan [2007]Yin [2010], in which a fixed pre-cooling duration and a constant temperature setpoint are adopted during pre-cooling phase, here each room's local scheduler fixes a pre-cooling period(called initial pre-cooling period, i.e. ipp, hereafter) and determines the minimal constant mass flow rate to satisfy the temperature requirement. Then distributed scheduling algorithms start to improve its optimality. Therefore, the adopted procedure may also be interpreted as an add-on component to improve the uncoordinated pre-cooling processes.

A set of simulations with different ipp values are performed. The energy costs are summarized in Table 2. Although the energy efficiency is worse than centralized optimization especially for small ipp values, the distributed optimized profiles can improve the initial profile up to 7% to 37%. The computation time is at least less than 50% of it for centralized optimization. Take $ipp = 30mins$ as example. The comparison of individual room and aggregated mass flow rates between the initial one and optimized one are presented in Figure 8 and 9, respectively. It is noted that the selection of ipp is critical, which is not addressed in this paper. It may be obtained by some optimization or based on historical experience.

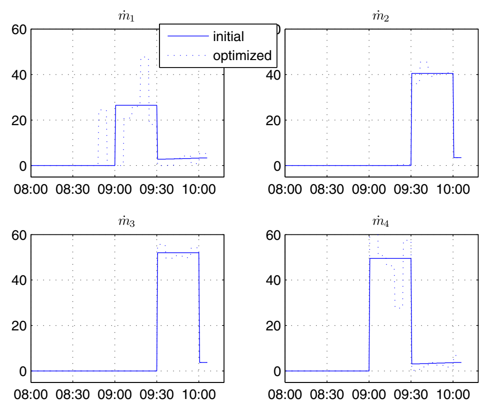

Fig. 8. Comparison of individual mass flow rates $\dot{m}_j$ for the case of $ipp = 30mins$

6. CONCLUSION

Distributed algorithms are proposed for scheduling the pre-cooling processes in HVAC systems. Their perfor-mance is examined via a small scale simulation. A more realistic simulation with detailed model information is the next step to verify their applicability. Other possible future works may include realizing the scheduling result via feedback control and experimental case study.

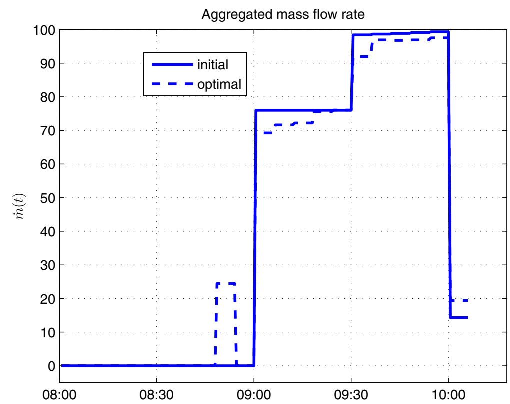

Fig. 9. Comparison of $\dot{m}$ for the case of $ipp = 30mins$

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