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# UNIVERSITY OF CALIFORNIA, IRVINE

Frictional Markets: A Labor Market Model and Two Monetary Experiments

### DISSERTATION

submitted in partial satisfaction of the requirements for the degree of

### DOCTOR OF PHILOSOPHY

in Economics

by

Francisco Klapp

Dissertation Committee: Profesor Guillaume Rocheteau, Chair Professor John Duffy Associate Professor Michael Choi

 $\bigodot$  2023 Francisco Klapp

# DEDICATION

To Lupe and Huilo May your wagging tails keep making the world a happier place...

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### ABSTRACT OF THE DISSERTATION

Frictional Markets: A Labor Market Model and Two Monetary Experiments

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While the usual paradigm of supply and demand in a frictionless market is useful for discussing many issues, plenty of important questions are not easily addressed with this approach. In this dissertation I aim to further our understanding of markets where trading frictions are relevant from two different perspectives: a search-theoretic model and laboratory experiments.

The first chapter of this dissertation focuses on a frictional labor market where firms can decide to replace current employees by randomly search for new candidates. I show that even in an economy where agents are risk neutral and face no liquidity constraints, severance payments emerge as part of an optimal contract when firms can search on-the-job. An optimal contract is composed of both the wage and a payment conditional on the worker being replaced. The size of the payment is chosen to allow the firm to internalize the externality associated so that firms take into account the total surplus effect of their replacement hire decisions. I add ex ante heterogeneity among workers which makes the pairwise optimal contract insufficient to achieve the constrained-efficient result (even under the Hosios condition) and then use a calibrated version of the model to illustrate that a lower on-the-job search cost can in fact reduce welfare. In the second chapter, I develop an experimental framework to investigate price, output and welfare consequences of implementing the optimal monetary policy in a version of the searchtheoretic money model of Lagos and Wright (2005): The Friedman Rule. I aim to further understand previous experimental results by Duffy and Puzzello (2022) which are somewhat at odds with the standard theory and contrary to the optimality of the rule vis-à-vis an inflationary scheme, which they suggest could be rationalized on the basis of liquidity constraints and/or precautionary motives due to future price uncertainty. For this, I request subjects to make predictions about market prices and include a novel treatment for the decentralized goods market to try and mitigate price uncertainty: prices are exogenously imposed on consummers so they can only select from a fixed menu of quantities (and prices) when making an offer. My results tend to be consistent with previous experimental findings, but no clear evidence for the Friedman Rule emerges. Even when prices are fixed exogenously in pairwise meetings, evidence in favor is still only mixed and high volatility of prices in the centralized market persists. When using subjects own predictions about the centralized market price to look at how they expected to rebalance their holdings conditional on their beliefs there seems to be a clear bias: subjects mostly want to increase their token holdings regardless of previous trades.

In the third and last chapter, I use a semi-unstructured bargaining approach to experimentally study the effects of liquidity constraints on the determination of terms of trade. This setting is specially relevant for search-theoretic models of money: trade surplus and its division are endogenously and simultaneously determined, with participants facing possible liquidity constraints due to buyer's endogenous ex-ante choice of costly money holdings. My aim is to test the empirical relevance of two widely used axiomatic bargaining solutions: generalized Nash bargaining and Kalai's proportional bargaining. Each bargaining solution predicts different outcomes and buyer's anticipating their decision's effect on the bargaining outcome may choose to additionally restrict their money holdings, which can prevent efficient outcomes from being achieved, even when money is costless to hold. A most relevant issue, since the protocol to determine terms of trade in monetary economies is critical for normative results, optimal monetary policy, and the welfare cost of inflation. By imposing different costs to holding money, I find strong evidence that costlier holdings do incentivize participants to economize on money holdings leading to a more constrained bargaining set resulting in less production and surplus, but find only mixed evidence in favor of any of the two bargaining solutions. Finally, I introduce two possible variations to the model that help better understand the data: myopic buyers and a sunk cost fallacy.

# Chapter 1

# On the Role of Severance Payments when Firms Search on the Job

### 1.1 Introduction

Among labor regulations there is a vast set of security provisions governing the dismissal of employees. Most of such diverse provisions impose a 'firing cost' to the firm composed of some combination of two separated elements: a pure transfer from the firm to the worker to be laid off, and a pure tax to be paid by the firm. One of these security provisions that is prevalent in most countries is mandated severance payments (henceforth SP's). According to a 2012 World Bank report some form of severance payment can be found in more than 150 countries. As a result the effects of severance payments (both government-mandated and freely bargained) on equilibrium allocations and welfare is a topic of keen interest in macro and labor economics. Now, ever since Lazear (1990) 's seminal "bonding critique", there's been vast debate regarding severance payments theoretical -and practical- impact, specially when households are risk neutral and there are no wage rigidities. This "bonding critique" states that in the absence of contractual and market frictions, any mandated pure transfer from the firm to the dismissed worker can be neutralized by an appropriately designed wage contract: firms reduce entry wage by an amount equal to the expected present value of the future transfer and then pay "interest" via a higher wage while the contract lasts, leaving the expected wage cost unchanged. This is as if the employee issued a bond to the employer at the moment of being hired, thus the name. More recently, models with risk averse agents, that care about consumption smoothing, and some sort of financial constrain or incomplete insurance markets have cast nuance into Lazear's neutrality of SP's result.

In this paper I show that even in a simple model with risk neutral agents and no borrowing constraints, SP's can emerge endogenously as part of the optimal contract when firms can search on-the-job. By search on-the-job I mean keeping or re-opening vacancies at a certain flow cost so to replace low productivity workers.

An employment contract is composed of the wage and a payment conditional on the worker being replaced (the SP) over which the firm and the worker bargain. Depending on the size of the SP firms can internalize the total surplus effect of their decision to replace a worker, which is expressed as a higher productivity/ability threshold before searching on-the-job. The model accounts for two facts observed in the data: the existence of non-governmentmandated SP's in a large number of US firms and the high fraction of replacement hires that can not be explained by quits. Moreover, the model allows to investigate the welfare implications of changes in the relative cost of on-the-job search with respect to search by idle firms under different assumptions regarding ex-ante and ex-post worker heterogeneity. For this last task I perform numerical calibrations on three different versions of the model using standard parameters in the literature and matching some to US data.

The main results of the paper are as follows. SP's are essential in this environment in the sense that they can emerge endogenously as part of the optimal contract. Moreover, this optimal contract ensures that voluntary on-the-job search by firms is welfare enhancing in the decentralized economy as long as the traditional congestion and thick market externalities are balanced out, which can be attained by the Hosios condition in the special case when there is no ex-ante worker heterogeneity. In the presence of heterogeneity in workers abilities the pairwise optimal contract is insufficient to achieve the constrained-efficient result (even under the Hosios condition). Thus, absent match idiosyncratic shocks, numerical exercises suggest that a relative lower on-the-job search would be welfare diminishing. Lastly, when both ex-ante and ex-post heterogeneity is present, a lower relative on-the-job search cost can have ambiguous effects.

### 1.1.1 Related Literature

**Risk Neutral Agents** Given this well-known neutrality result, a vast majority of early research in the labor search literature using linear utility functions (as in the textbook Pissarides (2000) model) decided to disregard the effect of severance payments (and other pure transfers) or just conceptualize all firing costs (including severance payments) as pure taxes. This second view, which Bertola and Rogerson (1997) called the "standard view of firing taxes", is mostly justified on two grounds: (1) If quantitatively the tax component is substantially larger than the transfer component -which empirically does not hold-<sup>1</sup>, then the pure tax approach would be a reasonable approximation, or (2) if contractual imperfections induce the transfer component to act exactly as a tax, for example by precluding certain wage adjustment (some limit bonding). Traditional examples of this strain of literature based on linear utility models with some sort of wage rigidity that induce non-neutrality of SP's include Garibaldi and Violante (2005), where the effect of the exogenous wage rigidity on unemployment depends on whether it is imposed on the insider or the outsider, while in Fella (2012) wage rigidity is endogenous because of labor unions with and SP's are even desirable as long as entry wages are flexible enough. Notable exceptions are Saint-Paul (1995)

<sup>&</sup>lt;sup>1</sup>Garibaldi and Violante (2005) and Bank (2012) show that the transfer portion is usually considerably larger than the pure tax component in most severance

and Fella (0200) who relay on efficiency wages and SP's as commitment devices that can play a desirable welfare enhancing role.

**Risk Averse Agents** A separate more recent string of the literature investigates SPs in non-linear models mostly arguing for the possibility of SP's being desirable or optimal and welfare enhancing. For example, Bertola (2004) shows how mandated SP's can have a positive welfare effect in the presence of risk aversion and uninsurable risk, while Pissarides (2004) introduces the idea that a voluntary SP can be a perfect substitute for insurance and superior to precautionary savings due to risk neutrality of firms. Fella and Tyson (2013) offer a different approach where SP's are periodically renegotiated and there is a government minimum which if higher than what parties agreed can be reverse via bonding which renders excess government-mandated SP's neutral under certain conditions for the unemployment benefits. More recently, Cozzi and Fella (2016) use the idea of non-insurable permanent lower earnings due to loss of tenure/experience while unemployed which strengthens the role of SP's as part of a desirable contract by the worker who has all the bargaining power in their model. In Lale (2018)'s model any government-mandated SP is partially reversed via imperfect bonding using a two tier contract, the partial nature of the bonding is due to the sources of incompleteness embedded in the model, moreover this partial bonding which implies a steeper wage profile has negative welfare implications because it runs counter to consumption smoothing.

Long Lasting Vacancies Closely related to this paper, Acharya and Wee (2018) include long lasting vacancies created at a fix initial cost that allow firms to keep temporarily searching for workers even after matched but this does not justify the existence of SP's because they assume a two tier wage to deal with the above mentioned excessive search by firms but this feature requires searching to be observable.Long lasting vacancies are can be analogous to this model where firms can always search on-the-job by paying a flow cost but does not allow to understand the role of the relative flow cost of on-the-job search compared to the cost of posting a vacancy by an idle firm. Moreover, the goal of Acharya and Wee is to capture the relationship between replacement hiring and the wage-productivity gap. The idea of firms posting long lasting vacancies is also present in Fujita and Ramey (2007) and Haefke and R. (2017), where job destruction is exogenous and endogenous, respectively, include this feature with the purpose of generating sluggishness in the response of the labor market to productivity shocks and allow firms to only actually search after the match as been destroyed.

**Optimal Employment Contract** <sup>2</sup> A distinctive feature of this model vis-à-vis the most standard Pissarides model, is the form of the employment contract which relates to the existence of the option to search on-the-job by firms. Early labor search models usually assume that the employment contract involves only a constant wage with no hiring/firing fee or state-dependent compensation. In most instances, these restrictions on the contract space are unimportant because the only thing that matters for the risk-neutral workers and firms is the division of the match surplus (e.g. Shimer (1996)). The above-mentioned 'bonding critique' by Lazaer can be actually tough of as an application of this idea.

Now, the exact form of the employment contract is more relevant when workers can take actions that affect the duration of the match, such as on-the-job search or crime opportunities. As pointed out by Shimer (2005a) and Stevens (2005), a constant wage may fail to achieve a pairwise Pareto-efficient outcome. Moreover, standard bargaining solutions cannot always be used when the contract is restricted to a constant wage since the bargaining set need not be convex (Bonilla and Burdet (2005), Shimer (2005a)).

 $<sup>^{2}</sup>$ This section borrows from the discussion in Engelhardt, Rocheteau, and Rupert (2008) who in the context of stochastic opportunities to commit crimes face a similar issue to the one arising form the option to replace workers by firms in this model.

### 1.1.2 Empirical Evidence

**Severance Payments** According to a recent survey of US firms 88% <sup>3</sup> of the interviewed companies offer some sort of severance package when termination is due to a reduction in force or corporate restructuring (even though it is not usually required by law) but only 6 percent provide severance on retirement (see Table 1.1). In the same spirit, Parsons (2013) documents that, 40 percent of workers in establishments with more than 100 employees,were covered by private SP clauses.

Circumstance	% offering SP
Reduction in force or corporate restructuring	88%
Involuntary termination	62%
Termination for cause	13%
Retirement	6%
Death	3%
Disability	3%

Table 1.1: Severance payments by cause in the US

According to Table 1.2 from the World Bank's "Reforming Severance Payments" report lower income countries tend to rely mostly on mandated SP's while higher income countries tend to have a mix of both, or as in the case of the US, Japan and Singapore fully rely on private agreements between parties.

	number	Examples
none	17	Haiti, D.R Congo, Brunei
agreement only	15	US, Japan, Norway, Singapore
both	27	Australia, Argentina, Germany
mandated only	30	Korea, Brazil, UK
mandated (no info agrmt.)	94	HK, Mexico, Spain

Table 1.2: Severance payments by contingency US and other countries

<sup>&</sup>lt;sup>3</sup>Lee Hecht Harrison and Compensation Resources Inc (2018)

**Replacement Hires in the US** According to Quarterly Workforce Indicators (QWI) replacement hires -defined as those in excess of net employment change- explain more than 40 of total hires in the US. Figure 1.1 shows the evolution of replacement hiring in the US.



Figure 1.1: Replacement hiring share

Replacement hiring can occur for two reasons: (1) firms may choose to re-fill positions vacated by a worker who has quit, (2) firms may choose to replace current workers with better applicants. Data suggests that not all of replacement hiring can be accounted for by quits. The second channel which has been less explored, accounts for a relevant portion of replacement hiring not explained by quits. Following Acharya and Wee (2018), if the primary reason for the occurrence of replacement hiring was just to replace workers that have quit, one would expect that the ratio of quits to total hires would follow the same trend as the replacement hiring share. Since the QWI does not distinguish between separations due to quits or layoffs, one can use information on the level of quits and hires from the Jobs Openings and Labor Turnover Survey (JOLTS) to compute the ratio of quits to total hires (Figure 1.2a).

Using the JOLTS microdata, Elsby, Michaels, and Ratner (2016) focus on firms with zero net employment change and measure the cumulative hires rate (solid blue line) and cumulative quits rate (dashed-red line) at such firms. While quits certainly affect the amount of replacement hiring, Figure 1.2b from Elsby, Michaels, and Ratner (2016) reveals the wedge



Figure 1.2: Replacement hires explanations

that exists between the cumulative hires rate and cumulative quits rates (plus other separations)<sup>4</sup>, suggesting that a significant portion of replacement hiring is indeed related to layoffs.

### 1.2 Environment

The model builds on Pisarides (2000) textbook model. Time is continuous and goes on forever. The economy is composed of a unit-measure of infinitely-lived workers and a large measure of firms. There is one final good produced by firms. Each worker is endowed with one indivisible unit of time that has two alternative, mutually exclusive uses: search for a job, work for a firm. So, there is no cost to searching for workers, but it is not compatible with holding a job.

Individuals are risk-neutral and discount at rate r >. They are not liquidity constrained and can borrow and lend at rate r. An unemployed worker who is looking for a job enjoys utility flow z, which we interpret as the utility from not working. Each worker is endowed with some

<sup>&</sup>lt;sup>4</sup>Other separations include retirement and disability

ability  $a \in [\underline{a}, \overline{a}]$  which is not observable by the firm until after matched. The distribution of abilities is F(a) with density f(a). I denote u(a) and e(a) the density of unemployed and employed workers with ability a, respectively. The unit-measure of individuals can then be written as

$$1 = \int_{\underline{a}}^{\overline{a}} f(\alpha) d\alpha = \int_{\underline{a}}^{\overline{a}} u(\alpha) d\alpha + \int_{\underline{a}}^{\overline{a}} e(\alpha) d\alpha$$

Firms are composed of a single job, either matched or vacant, and also discount future profits at rate r > 0. Vacant firms are free to enter and pay a flow cost,  $\sigma_v$ , to advertise a vacancy. One of the major innovations of this model is that now matched firms can also pay a flow cost,  $\sigma_f$ , to perform on-the-job search and match with unemployed and if profitable to replace their current worker. To ensure that on-the-job search can be attractive to firms I assume  $\sigma_v > \sigma_f$  which has to be the case since the expected gain of replacing a worker is always smaller than the expected gain for a vacant firm

.<sup>5</sup> Vacant firms produce no output while filled jobs produce y = g(a, x), where x is the above-mentioned match specific idiosyncratic productivity component. For simplicity, new matches start with maximum productivity  $x = \bar{x}$  and at a rate  $\lambda$  receive a productivity shock which implies they draw new x from distribution G(x),  $\underline{x} \leq x \leq \bar{x}$ .

The labor market is subject to search-matching frictions. The flow of hirings is given by the aggregate matching function M(U, V) where U is the measure of unemployed workers actively looking for jobs,  $U \equiv \int_{\underline{a}}^{\overline{a}} u(\alpha) d\alpha$ , and V is the effective number of positions offered by searching firms, either vacant or matched but searching. The matching function,  $M(\cdot, \cdot)$ , is continuous, strictly increasing, strictly concave with respect to each of its arguments and exhibits constant returns to scale. Furthermore,  $M(0, \cdot) = M(\cdot, 0) = 0$  and  $M(\infty, \cdot) =$  $M(\cdot, \infty) = \infty$ . Following Pissarides' terminology, we define  $\theta = V/U$  as labor market tightness. Each position (in a vacant or matched firm) is filled according to a Poisson process

<sup>&</sup>lt;sup>5</sup>One can think of this as assuming that firms who are already producing face a lower cost of searching because of economies of scope. An alternative modeling decision would be to assume that already matched firms are more efficient in terms of their meeting rate.

with arrival rate  $\frac{M(U,V)}{V} = q(\theta)$ . Similarly, each unemployed worker finds a job according to a Poisson process with arrival rate  $\frac{M(U,V)}{U} = f(\theta) = \theta q(\theta)$ .

### 1.3 Equilibrium

This paper focuses on steady-state equilibria where the distribution of individuals of different abilities across states, u(a) and e(a), and market tightness,  $\theta$ , are constant over time. As a consequence, matching probabilities are also time-invariant.

### **1.3.1** Bellman Equations

A worker receives a constant wage, w(a, x), that depends on his ability and a stochastic productivity component, x, to be defined below, and also receives a severance payment, SP(a, x), , conditional on being replaced by the firm. The pair (w(a, x), SP(a, x)) will be determined through some bargaining solution. As I will show below, an employment contract composed of a firing fee and a constant wage generates a pairwise optimal outcome in our context. Given that this is the type of contract the model calls for, it is the one I choose to adopt.<sup>6</sup>

The flow Bellman equation for an unemployed worker of ability a is:

$$rU(a) = z + \theta q(\theta) \{ W(a, \bar{x}) - U(a) \}^+$$
(1.1)

where  $\{x\}^+ = \max(x, 0)$ , since a worker would never accept a job with lower value than unemployment. The interpretation for (1.1) is as follows. An unemployed worker of ability *a* enjoys a utility flow from not working, *z*. A job is found with an instantaneous probability

<sup>&</sup>lt;sup>6</sup>Implicit in this formulation is that the firm commits to the terms of the employment contract. In particular, once the worker pays the hiring fee the firm does not renege on the promised future wage. Note also that firms have no incentive to fire their workers once the hiring fee has been paid since their expected profits from opening a new vacancy is zero.

 $\theta q(\theta)$ . Upon taking a job an individual enjoys the capital gain  $W(a, \bar{x}) - U(a)$ , since all new matches start with the highest possible idiosyncratic productivity,  $\bar{x}$ .

The flow Bellman equation for a vacant firm is:

$$rV = -\sigma_v + q(\theta) \int_{\underline{a}}^{\overline{a}} \Phi_u(\alpha) \{J(\alpha, \overline{x}) - V\}^+ d\alpha$$
(1.2)

where  $\Phi_u(a)$  represents the fraction of unemployed workers of ability a from the total pool of unemployed. From (1.2), a vacant firm incurs an advertising flow cost  $\sigma_v$ , finds an unemployed worker with an instantaneous probability  $q(\theta)$ , and enjoys and expected capital gain  $\int_{\underline{a}}^{\overline{a}} \Phi_u(\alpha) \{J(\alpha, \overline{x}) - V\}^+ d\alpha$ , which depends on the distribution of unemployment across abilities.

The flow Bellman equations for a matched firm with a worker of ability a and a current idiosyncratic productivity of x is:

$$rJ(a,x) = g(a,x) - w(a,x) + \lambda \int_{\underline{x}}^{\overline{x}} \max\{J(a,\epsilon) - J(a,x), V - J(a,x)\} dG(\epsilon)$$
  
+ 
$$\max\{q(\theta) \int_{\underline{a}}^{\overline{a}} \Phi_u(\alpha)\{J(\alpha,\overline{x}) - J(a,x) - SP(a,x)\}^+ d\alpha - \sigma_f, 0\}$$
(1.3)  
On-the-job search

According to (1.3) a matched firm enjoys a flow profit of g(x, a) - w(x, a) and with an instantaneous probability  $\lambda$  receives a idiosyncratic productivity shock, which is a new draw of x, that implies and expected capital loss (win) of  $\int_{\underline{x}}^{\overline{x}} \max\{J(a, \epsilon) - J(a, x), V - J(a, x)\} dG(\epsilon)$ . That is a capital gain (loss) that depends on the new realized value of x and the choice of the firm whether to remain open or dissolve. Additionally, the matched firm has always the option to engage into on-the-job-search. If the firm decides to search, it incurs a flow cost of  $\sigma_f$  and with flow probability  $q(\theta)$  it is matched with a new worker whose ability dependents on the current ability distribution among the unemployed, which means and expected capital gain of  $\int_{\underline{a}}^{\overline{a}} \Phi_u(\alpha) \{ J(\alpha, \overline{x}) - J(a, x) - SP(a, x) \}^+ d\alpha$  where SP(a, x) is the severance payment to the replaced worker.

**Lemma 1.** Since J(a, x) increasing in  $x \Rightarrow x = R(a)$  is the reservation productivity, such that J(a, R(a)) = 0.

**Lemma 2.**  $J(\alpha, \bar{x}) - J(a, x) - SP(a, x)$  is increasing in  $\alpha \Rightarrow \alpha = \hat{a}(a, x)$  is the minimum acceptable ability, such that  $J(\hat{a}(a, x), \bar{x}) - J(a, x) - SP(a, x)) = 0$ .  $\hat{a}(a, x)$  is the minimum acceptable ability level for a replacement hire.

**Lemma 3.** Since  $J(\alpha, \bar{x}) - J(a, x) - SP(a, x)$  is decreasing in  $x \Rightarrow x = S(a)$  is the cutoff productivity below which matched firms decide to search, such that  $J(\alpha, \bar{x}) - J(a, S(a)) - SP(a, S(a)) = 0$ .

Given Claims 1 to 3 I can rewrite (1.3) as a simpler Flow Bellman equation that depends only on the ability specific idiosyncratic productivity thresholds for both match dissolution, R(a), and on-the-job search by firms, S(a). Figure 1.3 represents illustrates those cutoff rules: for a firm matched with a worker of ability a, if idiosyncratic productivity x is below R(a) the match is dissolved, if x is between R(a) and S(a), the match continues but the firm decides to start searching for a new worker, and above S(a) it is not convenient for the firm to incur the cost,  $\sigma_f$ , of searching since the potential upside (net of the SP) of searching is not enough to compensate.



Figure 1.3: Idiosyncratic Productivity Cutoff Points

I rewrite (1.3) as:

$$rJ(a,x) = g(a,x) - w(a,x) + \lambda \int_{R(a)}^{\bar{x}} J(a,\epsilon) dG(\epsilon) - \lambda J(a,x) + \mathbb{1}_{\{x < S(a)\}} \left\{ \underbrace{q(\theta) \int_{a'_{a,x}}^{\bar{a}} \Phi_u(\alpha) \Big[ J(\alpha,\bar{x}) - J(a,x) - SP(a,x) \Big] d\alpha - \sigma_f \right\}}_{\text{On-the-job search}}$$
(1.4)

where  $\mathbb{1}_{\{x < (a)\}}$  is an indicator function, that takes the value of 1 if x < S(a) and 0 otherwise. From (1.4) we have now that with flow probability  $\lambda$  the matched firm draws a new value of x faces a capital loss of J(a, x), and conditional on the new draw of x being larger than R(a) the firm experiences an expected capital gain of  $\int_{R(a)}^{\bar{x}} J(a, \epsilon) dG(\epsilon)$ . Additionally, a firm matched with a worker of ability a is going to search on-the-job if x < S(a), which means it pays a flow cost of  $\sigma_f$  and with flow probability  $q(\theta)$  it meets a new worker and forms a new match if the worker ability is above  $a'_{a,x}$ , which depends on the distribution of ability among the unemployed. Thus, the expected capital gain is  $\int_{a'_{a,x}}^{\bar{a}} \Phi_u(\alpha) \left[ J(\alpha, \bar{x}) - J(a, x) - SP(a, x) \right] d\alpha$  which takes into account the current ability pool of unemployed and the SP to be paid by the firm.

From (1.4) a matched firm would choose S(a) to maximize the value of J(a, x), which solves:

$$\int_{a'_{a,x}}^{\bar{a}} \Phi_u(\alpha) J(\alpha, \bar{x}) d\alpha = \int_{a'_{a,x}}^{\bar{a}} \Phi_u(\alpha) [J(a, S(a)) + SP(a, S(a))] d\alpha + \frac{\sigma_f}{q(\theta)}$$
(1.5)

where the left-hand side is the expected benefit for the firm from hiring a replacement worker and the right-hand is the cost including the capital loss, the SP and the expected flow cost of searching,  $\frac{\sigma_f}{q(\theta)}$ . Finally, the flow Bellman equation for an employed worker is:

$$rW(a,x) = w(a,x) + \lambda \int_{R(a)}^{\bar{x}} W(a,\epsilon) dG(\epsilon) + \lambda G(R)U(a) - \lambda W(a,x) + \mathbb{1}_{\{x < S(a)\}} \left\{ \underbrace{q(\theta) \int_{a'_{a,x}}^{\bar{a}} \Phi_u(\alpha) [U(a) - W(a,x) + SP(a,x)] d\alpha}_{\text{On-the-job search}} \right\}$$
(1.6)

Equation (1.6) is analogous to (1.4): workers earn a wage, w(a, x), and with flow probability  $\lambda$  receive an idiosyncratic productivity shock which implies an expected capital loss of  $\int_{R(a)}^{\bar{x}} W(a,\epsilon) dG(\epsilon) - W(a,x)$  if the new realization of x is above R(a), which means the match continues, otherwise the capital loss is U(a) - W(a,x), which means the individual becomes unemployed again.

**Optimal Employment Contract** To determine the details of the employment contract we define  $S(a, x) \equiv W(a, x) - U(a) + J(a, x)$  as the total surplus of a match. From (4) and (6),

$$r\mathbb{S}(a,x) = g(a,x) + \lambda \int_{R(a)}^{\bar{x}} \mathbb{S}(a,\epsilon) dG(\epsilon) - \lambda \mathbb{S}(a,x) + \mathbb{1}_{\{x < S(a)\}} \Big\{ q(\theta) \int_{a'_{a,x}}^{\bar{a}} \Phi_u(\alpha) [J(\alpha,\bar{x}) - \mathbb{S}(a,x)] d\alpha - \sigma_f \Big\}$$
(1.7)

Equation (1.7) has the following interpretation. A match of an individual of with ability, a, and idiosyncratic productivity, x, generates a flow surplus, g(a, x) - rU(a), composed of the output of the job minus the permanent income of an unemployed person, rU(a). At a Poisson rate  $\lambda$  a new productivity shock occurs and if the new realization of x is below R(a)the match is destroyed, otherwise it continues with the new idiosyncratic productivity. If the idiosyncratic productivity is below the threshold, S(a), the there is on-the-job search with the corresponding flow cost,  $\sigma_f$ , and an expected capital gain of  $\int_{a'_{a,x}}^{\bar{a}} \Phi_u(\alpha) [J(\alpha, \bar{x}) - S(a, x)] d\alpha$ . Suppose a worker and a firm could jointly determine, S(a), the idiosyncratic threshold at which on-the-job search begins. It can be seen from (1.7), that the surplus of the match is maximized if:

$$\int_{a'_{a,x}}^{\bar{a}} \Phi_u(\alpha) J(\alpha, \bar{x}) d\alpha = \int_{a'_{a,x}}^{\bar{a}} \Phi_u(\alpha) [J(a, S(a)) + W(a, S(a)) - U(a)] d\alpha + \frac{\sigma_f}{q(\theta)}$$
(1.8)

Comparison of (1.5) and (1.8) reveals that if  $SP(a, S(a)) \neq W(a, S(s)) - U(a)$ , the firm's choice of when to start searching on-the-job and the choice that maximizes the match surplus differ, i.e. the total surplus of the match is not maximized. If SP(a, S(a)) < W(a, S(a)) - U(a) matched firms try to replace productive enough workers because they do not fully internalize the negative externality they impose on the workers they replace. If SP(a, S(a)) > W(a, S(a)) - U(a) then firms don' take full advantage of the lower flow cost of on-the-job search.

By allowing the employment contract to include a severance payment conditional on ability and idiosyncratic match productivity, SP(a, x), the worker and the firm can reach a pairwise-efficient outcome. The employment contract (w(a, x), SP(a, x)) is determined by the generalized Nash solution where the worker's bargaining power is  $\beta \in [0, 1]$ . The contract satisfies

$$\left(w(a,x), SP(a,x)\right) = \arg\max\left\{W(a,x) - U(a)\right)^{\beta} (J(a,x) - V)^{1-\beta}\right\}$$
(1.9)

**Lemma 4.** The employment contract solution to (1.9) is such that

$$w(a,x) = (r+\lambda)SP(a,x) + rU(a) - \lambda \int_{R(a)}^{1} SP(a,\epsilon)dG(\epsilon)$$
(1.10)

$$SP(a,x) = \beta \mathbb{S}(a,x) = W(a,x) - U(a,x)$$

$$(1.11)$$

Proofs of this lemma can be found in the Appendix. Wage is set so that the severance payment is exactly equal to the capital loss of the replaced worker, which is selected so to split the total surplus of the match according to each agent's bargaining power, so this wage setting guarantees that the firm internalizes the effect of his search decision on the total surplus of the match.

### **1.3.2** Characterization of the Equilibrium

Given the free entry condition, V = 0, the model can be reduced to five sets of equations and five sets of unknowns: market tightness,  $\theta$ , productivity thresholds for each ability level, R(a) and S(a), the number of unemployed by ability, u(a), and the minimum acceptable ability level for a replacement worker given the current worker ability and idiosyncratic productivity,  $a'_{a,x}$ . Detailed descriptions of these equations can be found in the Appendix. From now on, for the productivity of matches I assume g(a, x) = a + x, this simple linear technology of production implies that the size of the shock does not depend on the worker's ability.

#### i. Job Creation

$$J(a^e, \bar{x}) = \int_{\underline{a}}^{\bar{a}} \Phi_u(\alpha) \{ J(\alpha, \bar{x}) \}^+ d\alpha = \frac{\sigma_v}{q(\theta)}$$
(1.12)

The job creation condition, (1.12), is directly derived from the free entry of firms and the fact that due to random matching the expected value of a newly created vacancy is going to depend on the flow cost,  $\sigma_v$ , and the average time to match with an acceptable individual whose ability will depend on the current unemployment pool.

#### ii. Job Destruction

$$J(a, R(a)) = 0 (1.13)$$

The job destruction condition, (1.13), is an indifference condition between producing and dissolving the match, and thus transitioning to vacancy. This determines the value of the reservation productivity for a match with a worker of ability a, R(a).

#### iii. Search Condition

$$\int_{a'_{a,x}}^{\bar{a}} \Phi_u(\alpha) \Big[ J(\alpha, \bar{x}) - J(a, S(a)) - SP(a, S(a)) \Big] d\alpha = \frac{\sigma_f}{q(\theta)}$$
(1.14)

Equation (1.14) is an indifference condition between searching or not for a firm matched with an individual of ability a and idiosyncratic productivity S(a) obtained from (3). It helps pin down the productivity threshold below which a firm decides to start searching, S(a).

#### iv. Replacement Worker Minimum Ability

$$J(a'_{a,x},\bar{x}) = J(a,x) + SP(a,x)$$
(1.15)

Similar to (1.14), this is an indifference condition so that the capital loss faced by the firm (including the SP) when replacing a worker is equal to the capital gain due to the new hire.

#### v. Unemployment Rate

$$\frac{u(a)^{ss}}{f(a)} = \frac{\lambda G(R(a)) + q(\theta)e_s(a)^{ss}}{\lambda G(R(a)) + q(\theta)e_s(a)^{ss} + \theta q(\theta)}$$
(1.16)

where

$$e_s(a)^{ss} = \frac{\lambda(G(S(a)) - G(R(a)))}{\lambda + q(\theta)}$$

The steady state unemployment rate by ability comes from the law of motion of unemployment and firms searching on the job matched with workers of ability a. Here, it is important to notice that given an ability level, there are always individuals trying to be replaced by their firms which are those with idiosyncratic ability between R(a) and S(a),  $e_s(a)$ , and matches being dissolved, those with idiosyncratic productivity below R(a).

Now, I can define the steady state equilibrium.

**Definition.** A steady state is characterized by:

- market tightness,  $\theta$
- a set of productivity thresholds, R(a) and S(a)
- a set of minimum acceptable ability,  $a'_{a,x}$
- steady state unemployment by ability,  $u(a)^{ss}$

The random search assumption implies that the labor market tightness depends on the distribution of abilities among the unemployment pool, which itself depends on tightness.

**Proposition 1.** The following derivatives describe the equilibrium:

- $\partial R(a)/\partial a < 0$
- $\partial S(a)/\partial a < 0$
- $\partial SP(a, x)/\partial a > 0$ ,  $\partial SP(a, x)/\partial x > 0$

•  $\partial w(a,x)/\partial a > 0$ ,  $\partial w(a,x)/\partial x > 0$ 

The first two results mean that higher ability workers endure larger negative idiosyncratic productivity shocks before either the match is dissolved or the firm starts searching for a replacement. As is to be expected, the second two results show that higher ability worker and higher idiosyncratic productivity is compensated with a higher wage and bigger SP when replaced.

### 1.4 Special Cases

In this section I present three restricted versions of the model which are easier to solve but still reflect the most important ideas in the model.

### 1.4.1 No Ex-ante Heterogeneity

Assuming that all agents have the same level of ability, in particular a = 0. In this special case, the productivity threshold structure presented in Figure 1.3 remains but naturally there would be only one search idiosyncratic productivity threshold, S, and one reservation productivity, R. Given this major simplification the model can be expressed a set of three equations and three unknowns which then can be further reduced to two equations in order to show existence and uniqueness of an active<sup>7</sup> equilibrium.

#### **Decentralized Solution**

The three equations are simplified versions of the job destruction condition, (1.13) and the job creation condition, (1.12), the search condition, (1.14). As in the textbook Pissaride's model

<sup>&</sup>lt;sup>7</sup>An equilibrium with non-zero vacancies.

unemployment does not need to be calculated simultaneously to solve for market tightness or the productivity threshold, and thus can be obtained trough a modified Beveridge Curve afterwards.

The simplified versions are:

#### i. Search Equation

$$S - R = (\sigma_v - \sigma_f) \frac{r + \lambda + q(\theta)}{q(\theta)}$$
(1.17)

#### ii. Job Creation

$$\frac{\beta\sigma_v}{(1-\beta)q(\theta)} + \frac{\sigma_f}{q(\theta)} = \frac{1-S}{r+\lambda}$$
(1.18)

#### iii. Job Destruction

$$R + \frac{\lambda \Phi(R, S, \theta)}{1 - \beta} = z + \sigma_f - \sigma_v + \frac{\beta}{1 - \beta} \sigma_v \theta$$
(1.19)

where

$$\Phi(R, S, \theta) = \int_{R}^{1} J(\epsilon) dG(\epsilon)$$

Note from (1.17) that S - R, that is the productivity level above reservation productivity where firms search depends directly on  $\sigma_v - \sigma_f$ . Since the capital gain (expected capital gain in the model with ex-ante heterogeneity) is smaller for a producing firm than an idle firm, they will only engage in on-the-job search if the flow cost differential compensates<sup>8</sup>. If the cost is the same then there is no incentives to seach on-the-job for firms. Note that

 $<sup>^{8}</sup>$ An alternative to this would be a higher meeting efficiency for firms who are already producing

there is symmetry with models where workers are the ones searching on-the-job, in those even if there is no cost to search for both workers and the unemployed, not all workers would decide to search since the expected capital gain of the new job might not be high enough to compensate the loss of the current job.

Substituting (1.17) in (1.18) and (1.19) the equilibrium can be expressed as an  $(\theta, R)$  pair. Since the Job Destruction equation has a positive slope and the Job Creation equation has a negative slope for R < 1 then there exists a steady-state active equilibrium.



Figure 1.4: Steady state with on-the-job search by firms and no ex-ante heterogeneity

#### Planner's Problem

As a benchmark I compute the planners constrained-efficient solution. A planner would choose paths for  $\{\theta, S, R\}^9$ , that is market tightness and both productivity thresholds, to

<sup>&</sup>lt;sup>9</sup>Equivalently the planner could choose  $\{V_v, R, S\}$  where  $V_v$  is the number of positions created by vacant firms
maximize:

$$\int_{0}^{\infty} e^{-rt} \left\{ \underbrace{y_{h} + y_{l}}_{\text{home production}} + \underbrace{zU}_{\text{home production}} - \underbrace{\sigma_{v}(\theta u - e_{l})}_{\text{cost of on-the-job search}} - \underbrace{\sigma_{f}e_{l}}_{\text{cost of on-the-job search}} \right\} dt \quad (1.20)$$

subject to

$$\dot{u} = \lambda G(R)(1-u) + e_l q(\theta) + u\theta q(\theta)$$
(1.21)

$$\dot{e}_l = \lambda (1-u)[G(S) - G(R)] - \lambda e_l - q(\theta)e_l \tag{1.22}$$

$$\dot{y}_h = \theta q(\theta) u + \lambda (1 - u) \int_S^1 \epsilon dG(\epsilon) - \lambda y_h$$
(1.23)

$$\dot{y}_l = \lambda(1-u) \int_R^S \epsilon dG(\epsilon) - \lambda y_l - q(\theta) y_l$$
(1.24)

Equation (1.20) shows that the planner maximizes the discounted present value of output and home production minus the flow cost of vacancies and on-the-job search. Output is produced by individuals with idiosyncratic productivity above S,  $y_h$ , and those with productivity between S and R,  $y_l$ . The importance of this distinction that arises from the stochastic nature of productivity is clear when looking at the evolution of both variables, (1.23) and (1.24), respectively. Home production is just zU, while the flow cost of vacancies and and on-the-job search by firms is  $\sigma_v(\theta u - e - l)$  and  $\sigma_f e_l$ , respectively. The law of motion of unemployment is standard and considers the fact that at rate  $\lambda$  matches who draw below R are dissolved and that there is replacement of workers with productivity between S and R,  $e_Lq(\theta)$ . The number of searching firms (or equivalently, workers at searching firms),  $e_l$ , changes according to the number of draws between S and R after a productivity shock and decreases when a firm replaces a worker with a new higher productivity applicant.

From (1.23), all new matches, either between an idle firm and an unemployed individual or a producing firm and an unemployed,  $\theta q(\theta)$ , start at the maximum productivity,  $\bar{x} = 1$ . At rate  $\lambda$  matches are shocked and draw a new productivity, if it is above S, it remains or becomes part of  $y_h$ , that is  $\lambda(1-u) \int_S^1 \epsilon dG(\epsilon)$ . Equation (1.24) has a similar interpretation, at rate  $\lambda$  matches that draw productivity below S and above R become part of  $y_l$ , and firms who manage to replace a worker,  $q(\theta)y_l$  are not a part of  $y_l$  anymore.

Welfare and Unemployment The efficient-constrained solution to the planner's problem is exactly the same as the decentralized solution with optimal contracts if  $\eta = \beta$ , that is, the traditional Hosios Efficiency Condition still holds with on-the-job search by firms. A conditioned that will be lost once ex-ante heterogeneity is included.



Figure 1.5: Steady state with on-the-job search by firms and no ex-ante heterogeneity

A corollary of the equilibrium being the same as in the decentralized economy is that a planner would conduct at least some on-the-job search as long as  $\sigma_v > \sigma_f$ , and that the ability of firms to replace workers with better applicants is welfare enhancing (or at least not harmful if  $\sigma_v \leq \sigma_f$ ). All of this conditional on the optimal contract being available. Now, even though searching is welfare enhancing given that it allows to take advantage of a lower search cost to relocate workers, as shown in Figure 1.5 where net output is decreasing in the relative cost of on-the-job search,  $\sigma_f/\sigma_v$ , its effect on unemployment may still be a concern. From the Steady State versions of (1.21) and (1.22), I obtain

$$u^{ss} = \frac{\lambda G(R) + q(\theta) e_s^{ss}}{\lambda G(R) + q(\theta) e_s^{ss} + \theta q(\theta)}$$
(1.25)

with

$$e_s^{ss} = \frac{\lambda(G(S) - G(R))}{\lambda + q(\theta)}$$

which is a simplifies version of (1.16). Since  $R \leq S$ , given any value of  $\theta$ , unemployment could be higher compared to the case with no search (or  $\sigma_v = \sigma_f$ ), but in equilibrium changes in  $\theta$  lead to a decrease in unemployment. A lower relative cost of on-the-job, that is a lower  $\sigma_f/\sigma_v$  ratio leads to an increase in the fraction S/R, which indicates how much replacement there is, but this is more than offset by an increase in market tightness which leads to lower unemployment.

Figure 1.6 illustrates this point using standard parameters further discussed in the numerical exercise of the next section. Panel 1.6a shows both the positive sloped unemployment rate and the negative sloped S/R ratio, while panel 1.6b shows how market tightness is decreasing in  $\sigma_f/\sigma_v$ .



Figure 1.6: Unemployment and on-the-job search

# Exogenous Job Destruction Rate

For the second special case I keep the ex-ante heterogeneity given by the ability distribution but impose an exogenous job destruction rate, which is equivalent to assuming that all idiosyncratic productivity shock are negative enough so that at a Poisson rate  $\lambda$  matches are always dissolved.

**Proposition 2.** If the destruction rate is exogenous, such that matches are dissolved at a rate  $\lambda$ , then the threshold for on-the-job search is just in term of ability and so the unemployment rate by ability turns into:

$$\frac{u(a)}{f(a)} = u_{sa} = \frac{\lambda}{\lambda + \theta q(\theta)} \text{ if } a \ge S$$
$$u_{rs} = \frac{\lambda + q(\theta)}{\lambda + q(\theta) + \theta q(\theta)} \text{ if } S > a \ge R$$
$$u_{ar} = 1 \text{ if } a < R$$

1.4.2

Individuals face only three possible unemployment rates depending on their ability: low ability types, those below R, are always unemployed; medium ability types, those between S and R, face a rate of  $\frac{\lambda+q(\theta)}{\lambda+q(\theta)+\theta q(\theta)}$  since they are always trying to be replaced by their employers; and for high ability individuals, those with ability above S, the unemployment rate is  $\frac{\lambda}{\lambda+\theta q(\theta)}$ . Figure 1.7 represents these new cutoff rules in terms of ability.



Figure 1.7: Ability Cutoff Points and Unemployment Rate by Group

#### **Decentralized Solution**

In this special case I can reduce the model to 4 equations and 4 unknowns<sup>10</sup>: market tightness,  $\theta$ , ability thresholds, S and R, plus the unemployment rate  $\frac{u(a)}{f(a)}$ . The four equations are simplified versions of the job destruction condition, (1.13), the job creation condition, (1.12), the search condition, (1.14) and the new Steady State unemployment equations determined by Proposition 2.

#### **Planner's Problem**

Again, as a benchmark I compute the planners constrained-efficient solution. A planner would choose paths for  $\{\theta, S, R\}$ , that is market tightness and both ability thresholds, to maximize:

$$\int_{0}^{\infty} e^{-rt} \left\{ \underbrace{(1-u_{sa}) \int_{S}^{\bar{a}} f(\alpha) \alpha d\alpha}_{\text{home production}} + \underbrace{zU}_{\text{home production}} - \underbrace{\sigma_{v}(\theta U - (1-u_{rs}) \int_{R}^{S} f(\alpha) d\alpha}_{\text{output of matches with } S > a > R} \right\}$$
(1.26)

cost of vacancies from non-producing firms

 $^{10}$ Note that in the general model there are five sets of equations and five sets of unknowns

subject to

$$\dot{u}_{sa} = (1 - u_{sa})\lambda - \theta q(\theta)u_{sa} \tag{1.27}$$

$$\dot{u}_{rs} = (1 - u_{rs})(\lambda + q(\theta)) - \theta q(\theta)u_{rs}$$
(1.28)

Again, (1.26) shows that the planner maximizes the discounted present value of output considering the different ability levels and home production (which is independent of ability) minus the flow cost of vacancies and on-the-job search. Output is produced by matched individuals with ability above S,  $(1 - u_{sa}) \int_{S}^{\bar{a}} f(\alpha) \alpha d\alpha$ , plus matched individuals with ability between R and S,  $(1 - u_{rs}) \int_{R}^{S} f(\alpha)(\alpha) d\alpha$ . Home production is just zU, while the flow cost of vacancies and and on-the-job search by firms is  $\sigma_{v}(\theta U - (1 - u_{rs}) \int_{R}^{S} f(\alpha) d\alpha)$  and  $\sigma_{f}(1 - u_{rs}) \int_{R}^{S} f(\alpha) d\alpha$ , respectively. All of this while still constrained by the laws of motion for the unemployment rate, (1.27) and (1.28). Solving the Hamiltonian for the Steady State we obtain analogous equations that define the equilibrium.<sup>11</sup> Just to illustrate I reproduce the job destruction equation for both the planner's and the decentralized cases.

$$R_{planner} = \max(z + \sigma_f - (1 + \theta)\sigma_v, \underline{a})$$
(1.29)

$$R_{sp} = \max(z + \sigma_f - \sigma_v, \underline{\mathbf{a}}) \tag{1.30}$$

since it is clear that  $R_{planners} \leq R_{SP}$  for any value of  $\beta$ . This means that those permanently unemployed in the constrained-optimal solution are fewer than those in the decentralized

<sup>&</sup>lt;sup>11</sup>detail of the simplified equations can be found in the appendix

equilibrium (unless  $R_{sp} \leq \underline{a}$  in which case no one is ever permanently unemployed). Unfortunately, other questions do not have such clear cut comparison, and thus a numerical exercise can be helpful to showcase the differences.

#### Numerical Exercises

In the following section I present a simple numerical exercise to illustrate some of the properties of this restricted version of the model.

The unit of time corresponds to 1 year and the rate of time preference is set to r = 0.05. The output from a match is normalized to a value between 1 and 3 depending on individuals ability, a, coming from a General Pareto Distribution with position parameter,  $\mu = 1$ , shape parameter, k = -0.4 and scale parameter,  $\sigma = 0.8$ . The flow of utility when unemployed is z = 0.533, which is 40% of the average productivity given the ability distribution.<sup>12</sup> I assume a Cobb-Douglas matching function,  $M(u, V) = AV^{\eta}U^{1-\eta}$ , with constant returns to scale and I set  $\eta = 0.5$ , so that workers and firms' contributions to the matching process are symmetric. I set the bargaining power of the worker  $\beta = 0.5$  just to keep it standard even though in a model with ex-ante heterogeneity  $\eta = \beta$  does not assure that the division of the match surplus internalizes the externalities associated with firms' entry decision. For each vacancy every unemployed individual creates the same congestion externality, but since they differ in productivity their thick market externality differs across them (see Shimer and Smith (2001), Albrecht and Vroman (2002), Blázquez, Marcel, and J. (2008) and Mangin and Benoît (2021) for more detail on this issue). The parameters A and  $\sigma_v$  and  $\sigma_f$  are chosen to match the average job-finding rate and the average V-U ratio while  $\lambda$  is chosen to match the separation rate. For the years 1951–2003 the job-finding rate, taken from Shimer (2005b), is 0.45 per month, implying that the annualized expected number of job offers,  $\theta q(\theta)$ , is 5.40.

<sup>&</sup>lt;sup>12</sup>The choice of the value for z, is always controversial, Shimer (2005b) proposes y = 1 and z = 0.4 (see Hagedorn and Manovskii (2008) an alternative)

This yields A = 0.8 and  $\sigma_v = 0.015$  and  $\sigma_f = 0.010$  so that the above holds in the planner's solution. The monthly job separation rate, also taken from Shimer (2005b), is found to be 0.034, implying an annualized rate of 0.408, so jobs last on average about 2 years. Here I consider all of the separations to be non-replacement related. Table 1.3 recapitulates the parameters and functional forms used in the exercise.

Parameter	Value	Description
	Model	
r	0.05	Real interest rate
z	0.533	Unemployed utility flow
$\beta$	0.5	Bargaining power of workers
$\eta$	0.5	Elasticity of matching function
$\sigma_v$	0.015	Recruiting cost of vacant firm
$\sigma_{f}$	0.010	Recruiting cost of matched firm
$\lambda$	0.408	Job destruction rate
A	0.8	Efficiency of matching technology
	Generalize Pareto Distribution, $F(a)$	
<u>a</u>	1	Lowest ability level
$\bar{a}$	3	Highest ability level
$\mu$	1	Position of GPD
$\sigma$	0.8	Scale of GPD
k	-0.4	Shape of GPD

Table 1.3: Parameters and functional forms

Using the these functional forms and parameter values I compute the Steady State equilibrium for the optimal contract case, the planner's case and an ad-hoc surplus sharing rule where the worker gets a fraction  $\beta$  of the surplus and the firm gets  $1 - \beta$  using a flat wage conditional ability, w(a). As discussed above, a constant wage may fail to achieve a pairwise Pareto-efficient outcome and here standard bargaining cannot even be used (due to te lack of convexity), this is why this flat wage benchmark is only introduced as an unrealistic rule to compare.

Figure (1.8) shows the unemployment rate by ability for the three different cases. The blue



Figure 1.8: Unemployment rate by ability

continuous line corresponds to the optimal contract case, the orange dashed line is the ad-hoc flat wage rule and the black dotted line is the planner's constrained-efficient solution. As is to be expected, both the reservation ability, a = R, and the ability search-cutoff, a = S, are lower in the planner's solution. In particular, a planner would only use on-the-job search by firms as long as the flow cost advantage in searching is relevant compared to creating a new vacancy (which is not the case in this first example).

It is important to notice that even though the optimal contract helps lower S(a) compared to the ad-hoc rule, it is still too high, which means that excessive replacement hiring is taking place in this economy due to the above-mentioned discrepancy between the thick market and the congestion externalities. Panel 1.9a shows the separation rate by ability, which is what lies behind the different unemployment rates by ability. Matches of individuals with ability below S experiment a higher separation rate due to firms' on-the-job search. Panel 1.9b shows how for those individuals the replacement component explains a large share -around a third- of separations, as observed in the data.

The discrepancy in terms of the ability threshold, S, between the optimal contract and the planner's solution is decreasing in the relative cost of on-the-job search,  $\sigma_f/\sigma_v$ . As  $\sigma_f/\sigma_v$  converges to 1, S converges to the planner's solution as on-the-job search becomes an irrelevant option for firms. Figure 1.10 shows this inefficiency for this particular numerical exercise.



(b) Replacement to total separation ratioFigure 1.9: Separation and Replacement Rates

So far the planner's constrained-efficient solution implies no on-the-job search by firms but it does not have to be the case. As in the restricted model with no ex-ante heterogeneity it is possible that the planner optimally chooses S > 1, so that a group of low ability individuals are hired by firms to produce but also take advantage of the lower flow cost of on-the-job search. Just as an example, if  $\sigma_f$  is low enough relative to  $\sigma_v$ , and z is high enough (but still below 1 so to avoid permanently unemployed individuals) one can see the planner's decision to search on-the-job. Figure 1.11 with  $\sigma_v = 0.15$  and  $\sigma_f = 0.001$  is an illustration of this.

Welfare and Unemployment Taking net output as a measure of welfare one can compare it and other variables ,such as overall unemployment, to the efficient-constrained planner's solution. The following table summarizes such comparison, where on can see a decrease in both net output and unemployment for the decentralized economy with the pairwise optimal contract or the rule.



Figure 1.10: Inefficient search



Figure 1.11: Unemployment rate by ability, extreme case

Given the above-mentioned inefficiency a lower relative cost of on-the-job search is no longer necessarily welfare enhancing. Figure 1.12, which is the equivalent to Figure 1.5 excluding idiosyncratic shocks and introducing ex-ante heterogeneity, illustrates this point.

Using the current parameters values a decrease in the on-the-job search cost would be welfare reducing, since it increases the discrepancy between the planner's choice of S and the decentralized choice by enough to offset any possible gains from the possibility to search on-the-job at a lower cost for better candidates.

## 1.4.3 A Compressed Distribution of Skills

For the third restricted version of the model I impose a condition on the support of the ability distribution, F(a), and the idiosyncratic productivity distribution, G(x). In particular, that the distribution of abilities is narrow enough so that searching firms are always willing

	Planner	SP	Rule
$\theta$	45.6	16.1	15.1
f( heta)	5.40	3.21	3.11
q( heta)	0.12	0.20	0.21
$\overline{S}$	1	1.07	1.14
F(S)	0%	16.3%	30.4%
$u_{rs}$	—	15.9%	16.5%
$u_{sa}$	7.0%	11.3%	11.6%
U	7.0%	12.0%	13.1%
Net Output	1.23	1.21	1.20%

Table 1.4: Comparison of key variables under standard parameters



Figure 1.12: Steady state with on-the-job search by firms and no ex-ante heterogeneity

to accept any new match because of its higher initial idiosyncratic productivity. Note that now for every ability level, as in the unrestricted case, there is going to be both an idiosyncratic productivity search threshold and a reservation productivity level. This is the natural progression from the previous two restricted models.

**Proposition 3.** If the support of the ability distribution is narrow enough compared to the support of the idiosyncratic productivity component such that

$$J(\underline{a}, \overline{x}) \ge J(a, x) + SP(a, x) \quad \forall \quad (a, x) \quad with \quad x \le S(a)$$

$$(1.31)$$

then

$$a'_{a,x} = \underline{a} \tag{1.32}$$

Given Proposition 3 I can solve (1.15) and rewrite the model, that is (1.12), (1.13) and (1.14) in a simpler way, while (1.16) remains the same.

#### A numerical Exercise

As with the second restricted version of the model I present a simple numerical exercise to illustrate some of its properties. Again, I keep the output from a match normalized to a value between 1 and 3 but now depending on individuals' ability, a, and their idiosyncratic productivity, x. So, ability still comes from a General Pareto Distribution but with position parameter,  $\mu = 1$ , shape parameter, k = -0.4 and scale parameter,  $\sigma = 0.4$ . While x is drawn from Uniform Distribution with [0, 1] support. Remember that all new matches start at the highest idiosyncratic productivity level,  $\bar{x} = 1$ . The flow of utility when unemployed is z = 0.7143, which is 40% of the mean productivity of the individuals pool (considering average idiosyncratic productivity).

Here again, the parameters A and  $\sigma_v$  and  $\sigma_f$  are chosen to match the average job-finding rate and the average V - U while  $\lambda = 0.9$  is chosen to match the separation rate of 0.3, but now considering that only matches drawing a realization below R(a) of the idiosyncratic productivity shock are destroyed. This yields A = 2.225 and  $\sigma_v = 0.250$  and  $\sigma_f = 0.010$ . Table 1.5 recapitulates the parameters and functional forms that changed in for this particular exercise.

In what follows I compute the Steady State equilibrium for the optimal contract case and the above mentioned ad-hoc surplus sharing rule.

Parameter	Value	Description
2	0.7143	Unemployed utility flow
A	2.225	Efficiency of matching technology
$\lambda$	0.9	Productivity shock arrival rate
$\sigma_v$	0.250	Recruiting cost of vacant firm
	Uniform Distribution, G(x)	
$\underline{x}$	0	Lowest idiosyncratic productivity level
$ar{x}$	1	Highest idiosyncratic productivity level
	Generalize Pareto Distribution, $F(a)$	
<u>a</u>	1	lowest ability level
$\bar{a}$	2	highest ability level
$\mu$	1	Position of GPD
$\sigma$	0.4	Scale of GPD
k	-0.4	Shape of GPD

Table 1.5: Changes to parameters and functional forms

Figure (1.13) shows the unemployment rate by ability for optimal contract and the ad-hoc rule.

Panel 1.14a shows the separation rate by ability, which is what lies behind the different unemployment rates by ability. For each ability level, matches of individuals with idiosyncratic productivity below x = S(a) experiment a higher separation rate due to firms' on-the-job search. Panel 1.14b shows how for those individuals the replacement component explains an important share of the separations, around 1/3 independently of the ability level. It is important to notice that kinks, such as the one observed in Figure 1.13 and Panel 1.14a are common in this restricted version of the model, since S(a) and R(a) are bounded between zero and one. For example, here one can see that R(a) = 0 is reached for high ability levels, which implies that some high ability matches are never dissolved, regardless of their idiosyncratic productivity, and are only destroyed due to replacement hiring. Now, with extreme parameter values it is also possible to obtain S = 0 for some high ability matches, which means that those matches are never dissolved and those high ability workers are never replaced. One potential way to avoid this would be to assume that the productivity of a match, g(a, x), is ax instead of a + x, but this further complicates solving the model.



Figure 1.13: Unemployment rate by ability



Figure 1.14: Separation and Replacement Rates

Welfare and Unemployment Again, taking net output as a measure of welfare I compare some key variables. Table 1.6 shows how both the results for both unemployment and net output are more favorable using the rule instead of the optimal pairwise contract. This speaks of the importance of the externalities not being solved by the SP.

While in the first restricted model with idiosyncratic shocks and no ex-ante heterogeneity (whit the Hosios condition holding) a lower on-the-job search cost was always welfare enhancing, and in the second constrained model with no idiosyncratic shocks and ex-ante ability dispersion it could be welfare reducing, in this case it ca be either.

	SP	Flat
$\theta$	5.9	8.7
f( heta)	5.40	6.54
q( heta)	0.92	0.76
U	8.0	6.6
$u_{1/3}$	8.9	7.1
Net Output	2.05	2.1

Table 1.6: Comparison of key variables



Figure 1.15: Steady state with on-the-job search by firms and no ex-ante heterogeneity

Figure 1.15 illustrates this, starting from a relatively low on-the-job search cost an increase in it reduces welfare as is to be expected, but when  $\sigma_f/\sigma_v$  approaches 1 then an increase actually becomes welfare enhancing (a feature that holds for a wide variety of parameters). This result points to the difficulty of predicting outcomes (for example changes in policy) in a context of on-the-job search with ability heterogeneity with no additional instruments to deal with the asymmetry in the congestion and thick market externalities common to the presence of individuals with different ability levels.

## 1.5 Conclusions

I show that even in an economy where agents are risk neutral and face no liquidity constraints, severance payments emerge as part of an optimal contract when firms can search on-thejob. In this setting an optimal contract is composed of both the wage and a payment conditional on the worker being replaced. The size of the payment is chosen to allow the firm to internalize the externality associated so that firms take into account the total surplus effect of their replacement hire decisions. This relates two facts observed in the data: the existence of non-government-mandated SP's and the high fraction of replacement hires that are not explained by quits.

The fact that firms incur in a lower flow cost to search while producing implies that on-the-job search by firms is always welfare enhancing in the decentralized economy with the optimal pairwise contract (SP's) as long as the traditional congestion and thick market externalities are balanced out (for example by the Hosios condition). Nevertheless when ex-ante ability heterogeneity is introduced the optimal contract does not guarantee that a lower on-thejob search cost will be welfare enhancing in the decentralized solution since the mentioned externalities come into play and can offset any gains.

The model calibration with standard literature parameters tends to suggest that the externalities associated with ex-ante ability heterogeneity are far more relevant than the distortion created when an ad-hoc rule surplus splinting rule is used instead of the optimal contract.

The model could be extended to explicitly take into account policy tools, such as taxes and subsidies or other labor protections, to restore the efficient-constrained result with the pairwise optimal contract.

# Chapter 2

# An Experimental New Monetarist Exploration

# 2.1 Introduction

There are obvious good reasons why Central Banks do not typically run large-scale real life experiments to asses the impact of alternative monetary policies on prices and welfare and further our understanding of monetary policy. Though one could argue that maneuvers in the last decades such as the different QE's and forward guidance near the Zero Lower Bound are indeed policy experiments, those are only taken as last resort measures in desperate times (and the environment is all but controlled). All of this does not mean that monetary policy experiments would not be valuable or should only be done in times of crisis. On the contrary, it just follows that pursuing them regularly but in an alternative more feasible way such as a low cost, small-scale, controlled setting like a laboratory (or online platform) with paid subjects is specially appealing.

This is why for this project I implement an on-line monetary policy experiment that allows

me to test policies that would be prohibitively costly or just too risky to even be entertained by a Central Bank, but that are prevalent in the theoretical literature where holding money is costly and that highlight the main mechanisms and channels at play in our current models.

Throughout this project my framework for policy analysis is Lagos and Wright (2005) searchtheoretic money model where an intrinsically useless object (fiat money) can be valued in exchange and raise society's welfare. This framework is part of a tradition which has one common specially desirable feature to study monetary phenomena: pairwise meetings with no record-keeping, commitment or monitoring technology which makes money -in Wallace (1998)'s terminology- essential. Particular to Lagos-Wright is the combination of pairwise meetings (DM stage) and periodical access to a competitive market (CM stage) with quasi-linear preferences that enables agents to re-balances their portfolios ensuring that in steady state all potential buyers enter the DM stage with the same wealth, keeping the model tractable. This closed form solution plus the explicit two stage dynamic structure make it an ideal candidate for experimental purposes. Moreover, in the baseline version of this class of model with unrestricted money holdings, and where both goods and money are divisible<sup>1</sup>, money is usually neutral but not super-neutral and the deflationary Friedman (1969) Rule is the optimal welfare-enhancing policy.

According to Friedman's policy prescription, the policy maker must engineer a rate of return for money that compensates agents for the cost of holding money balances. This can be accomplished by contracting the money supply at a rate equal to the agent's rate of time preference or, equivalently, by having money pay sufficient interest. By doing this, the policy maker can drive tho cost of holding money to zero, which in turn implies agents will hold enough liquid wealth to maximize the surplus from trades. In terms of the Fisher equation,  $i = \pi + \rho$ , where *i* is the nominal interest rate,  $\pi$  the expected inflation and  $\rho$  the discount rate (which would be the return of an illiquid asset), then the Central Bank would set  $\pi = -\rho$ 

<sup>&</sup>lt;sup>1</sup>As opposed to second generation models where money holdings are binary.

so that i = 0.

The optimality of the Friedman rule is a robust finding across various kinds of monetary models in which money plays an explicit role (i.e cash in advance, money-in-the-utility func-(i.e. bargaining, price)taking, price posting) but it is rarely observed (ore even considered) in practice. One explanation is that even if you believe that monetary search models are a coherent framework to study monetary policy, the Friedman rule may not be incentive-feasible if the government does not have the enough coercive power to tax. Agents may simply choose not to participate in the market in order to avoid incurring the tax that is required to make the money supply contract at the optimal rate. An alternative explanation in the monetary search literature is that the Friedman Rule may be feasible but not optimal. Shi (1997), who first studied optimal monetary policy in a search theoretic model, showed that the Friedman rule is optimal when agents participation decision is exogenous but not necessarily if the participation decision is endogenous because of congestion externalities.<sup>3</sup> Alternatively, the presence of uninsurable idiosyncratic productivity shocks may require a growing money supply as part of the optimal policy so to generate some redistribution of real balances among buyers which acts as insurance.

Following Duffy and Puzzello (2020), henceforth DP(2022), my goal in this paper is to evaluate monetary policies and their mechanisms in a controlled setting, focusing on the well-known Friedman rule. DP(2022) experimental results are somewhat at odds with the theory and contrary to the optimality of the rule  $vis-\dot{a}-vis$  an inflationary scheme, which they suggest could be rationalized on the basis of liquidity constraints and money illusion or

 $<sup>^{2}</sup>$ The Friedman rule is not necessarily optimal in OLG models where money plays a role as intergenerational storage technology and most New Keynesian models that assume a cashless limit where the cost of holding money is irrelevant

<sup>&</sup>lt;sup>3</sup>Rocheteau and Wright (2005) study the optimal monetary policy in a model with free entry of sellers under alternative pricing mechanism and revisit Shi's findings under alternative trading mechanisms, respectively.

precautionary motives due to uncertainty about future prices.<sup>4</sup> The authors also report a high number of rejections in the decentralized market (where prices and quantities are determined under a take-it-or-leave-it (TIOLI) bargaining protocol) and subjects both producing and consuming in the centralized market which is contrary to the theoretical model where the CM should only be a means for agents to re-balance their money holdings. This discrepancy in previous studies is just not a detail or mere curiosity, since the mechanisms at play that enable cost-less money holdings -such as the Friedman rule- to be optimal (and inflation harmful) are at the very core of most models where money is essential. So, to understand whether previous results are explained by some behavioral bias, precautionary motives, a coordination failure or just participants' inattention is paramount when thinking about this family of models and their policy implications.

With this in mind I then depart from DP(2022) by modifying the implementation of the model so to create what I believe is an easier scenario for standard theoretical predictions to hold, and at the same time try to make more explicit why the theoretical predictions might be at odds with previous findings. In particular, I first replicate DP(2022)'s TIOLI benchmark with slight modifications so that producing or consuming in the CM are mutually exclusive and take advantage to the linear technology and utility, frame it as a pure *Centralized Token Exchange*. I also add sliders to help subjects make decisions and calculate payoffs based, which simplifies the participant's decision space (which might be a source of confusions). Moreover, I request subjects to make predictions about market prices in the *Centralized Token Exchange*, which is used to help them inform their decisions and is also an important new piece of information particular to this implementation. Most importantly, I add the following modification to the decentralized goods market to try and mitigate price uncertainty as a treatment: prices are exogenously imposed on consumers so they can only select from a fixed menu of quantities (and prices) when making an offer, which are always

 $<sup>{}^{4}\</sup>text{DP}(2022)$  suggest in their conclusions that to automate the centralized market to facilitate re-balancing of money holdings could be a way to help further understand the observed departures from theory predictions

consistent with the model predicted monetary equilibrium.

This project pretends aims be an alternative to the solution offered by Jiang, Zhang, and Puzzello (2019), henceforth JPZ(2019), who implement a version of the the closely related Rocheteau and Wright (2005) money search model whose main difference is that pairwise meetings are replaced by a second anonymous competitive market where only a fraction,  $\sigma$ , want to consume, while  $1 - \sigma$  can produce, which can render money useful. JPZ(2019) obtain results that are closer to the theoretical predictions for different implementations of inflationary schemes, thus solving some of the issues in DP(2022), but do not test directly for the Friedman Rule, which is the optimal welfare-enhancing policy in that model as well.<sup>5</sup> An advantage of this proposal is that I retain the explicit bilateral meetings structure which is crucial for generating certain policy results (beyond the Freedman Rule) and for the coexistence of assets with different rates of return (Rocheteau and Nosal (2017)). Moreover, staying closer to DP(2022) allows me, as mentioned above, to better understand which aspects of the model or implementation are not consistent with actual subject behavior generating the theoretical inconsistent results and thus gain a better understanding of monetary models in an experimental framework.

Results tend to be consistent with the previous experimental findings of DP(2022) with strong evidence in favor of the monetary equilibrium as the norm, price stability when the money supply is unchanged, and no clear evidence that either quantities or welfare are higher under the Friedman Rule, contrary to what is predicted by the theoretical model. Moreover, even when prices are fixed exogenously in pairwise meetings, as it is the case in two of my treatments, evidence in favor of the optimal monetary policy is still only mixed and high volatility of prices in the centralized market persists which would be consistent with the idea that there are incentives for subjects to engage in precautionary savings. This could be caused by the Shapley and Shubik (1977) market game implementation of

<sup>&</sup>lt;sup>5</sup>Using a centralized market instead of the DM stage in the laboratory is already proposed and implemented in DP(2022) in a smaller scale experiment with fewer subjects/sessions.

the centralized market, with only a few buyers and sellers, leading to this price volatility (and even market breakdowns at times). When using subjects own predictions about the centralized market price to look at how they expected to rebalance their holdings conditional on their beliefs there seems to be a clear bias: subjects mostly want to increase their token holdings regardless of previous trades. This is an interesting finding that further supports the idea of precautionary savings.

## 2.2 Related Literature

There's a growing literature studying macroeconomic policies in laboratory settings (see Kagel and A. Roth (2016) or Hommes (2020) for a review). This paper belongs to that category but particularly to the narrower family of experimental implementations with an explicit role for money.

Marimon1994 ; Marimon and S. Sunder (1993); Marimon and S. Sunder (1995), Lim, Prescott, and Shyam Sunder (1994), and Bernasconi and Kirchkamp (2000) implement an overlapping generations (OLG) model to study inflation where the role of money is as a store of value. Brown (1996), O. Ochs and Duffy (1999); Duffy and J. Ochs (2002), Berentsen, McBride, and Rocheteau (2017) analyze implementation where the role of money is as a medium of exchange.

Duffy and Puzzello (2014b) were the first to implement the Lagos-Wright model in the laboratory looking to understand the welfare consequences of the existence of fiat money in comparison to a gift-exchange economy sustained by a social norm (a contagious grimtrigger strategy played by all agents). Even though, theoretically, some of the gift-exchange equilibria Pareto dominate the monetary equilibrium with a finite number of agents, making money non-essential here as well, they found that subjects avoid non-monetary gift-exchange equilibria in favor of the monetary equilibrium. They find that welfare is significantly higher in environments with money, suggesting that money plays a key role as an efficiency enhancing coordination device. They do not consider alternative monetary policy regimes in this early work. Camera, Casari, and Bigoni (2013); Camera and Casari (2014) also compare outcomes across environments with and without fiat money and find that the introduction of fiat money fosters cooperation specially in larger groups.

Duffy and Puzzello (2014a) studied whether subjects would adopt a fiat money for exchange purposes if they initially participated in a gift-exchange only Lagos-Wright economy and the reverse scenario where subjects initially experienced a Lagos-Wright economy with a constant supply of fiat money and then fiat money was taken away. They find that When fiat money was introduced (without forcing its use), subjects adopted it in exchange, but with no improvement in output or welfare. When subjects began in the setting with fiat money, when it was taken away, activity and welfare markedly declined. They also address money neutrality and found that when the fixed supply of money was doubled, prices approximately doubled and quantities did not change -in line with neutrality. However, in the case where the fixed supply of money was halved, prices did not adjust downward and there were welfare losses.

Anbarci, Dutu, and Feltovich (2015) study the effects of inflation in a version of the Lagos and Wright (2005) model with price posting as in Burdett, Shi, and Wright (2001). While they find inflation has a negative effects on production and welfare, subjects in their experiment make static choices, and the inflation is implemented by having buyers borrow money from a bank at a specified interest rate.

Davis et al. (2019) study finite horizon environments where monetary exchange may or may not be supported. They find that fiat money tends to promote welfare in all environments, regardless of whether there is supposed to be a monetary equilibrium. As mentioned in the introduction, and as it will become clear in the following sections, this paper is closely related and builds on Duffy and Puzzello (2022) who implement the Lagos and Wright (2005) model to test the welfare, output and price implications of: two implementations of the Friedman Rule (interest on money holdings and decreasing money supply), Friedman's K-percent rule and a constant money supply. They find mixed experimental results, which on one hand further confirm the monetary equilibrium as the norm and the predictions of the quantitative theory of money, but on the other hand are at odds with the theory by being contrary to the optimality of the Friedman rule. They find that the inflationary K-percent rule seems to be the best in terms of welfare in the laboratory setting.

Conceptually this project aims to be an alternative to Jiang, Zhang, and Puzzello (2019), who address some of these concerns by implementing a version of the Rocheteau and Wright (2005) money search model where pairwise meetings are replaced by a second anonymous competitive market where only a fraction can consume, while the rest can only produce, which renders money useful. They obtain results that are closer to the theoretical predictions for different implementations of inflationary schemes.

## 2.3 Theoretical Framework

In this section I present a very abbreviated version of the theoretical framework of Lagos and Wright (2005) that guided this experiment focusing on the monetary equilibrium where money is valued. In what follows I describe the model environment and the decision problem faced by individuals. I'll focus in the Steady State predictions for both the baseline scenario with a constant money supply and the Friedman Rule achieved by interest paying money holdings. A detailed version of the model can be found in the original Lagos and Wright (2005) paper or in Rocheteau and Nosal (2017) textbook. In a discrete time setting there is an even number N of indefinitely lived agents who discount the future at a rate  $\beta \in (0, 1)$ . Each period has two parts. The first part is the Decentralized Market, or DM for short, where agents are randomly matched into pairs where one agent is randomly assigned to be a producer while the other a consumer of the DM good. For simplicity, there is an exhaustive matching rule so that no agent remains unmatched. Each consumer makes a take-it-or-leave offer (TIOLI) proposing terms of trade (quantity of DM good and price) and the producer then either accepts or rejects it. The second part of each period is the Centralized Market, or CM, where all agents participate choosing to be either a consumer or producer in a competitive market for the CM good (also referred as general good or numéraire).

Both DM and CM goods are perishable and there is no direct record keeping technology that allows for IOU's or any other form of credit to be issued but here is an intrinsically worthless object that is durable: fiat money. The money supply at the beginning of period tis  $M_t = \sum_{i=1}^{N} m_t^i$  where  $(m_t^1, m_t^2, ..., m_t^N)$  is the distribution of money holdings at said time. The money supply expands at a gross rate  $\mu$  so that  $M_{t+1} = \mu M_t$ . Money is injected (or withdrawn) by the Government trough a lump-sum transfer (or tax)  $\tau_t$  at the beginning of the CM. Government can also pay interest  $i_m$  on money holdings at beginning of the CM. Thus the Government budget constraint in period t would be  $N\tau_t + i_m M_t = (\mu - 1)M_t$ .

Period utility functions for buyers and sellers are:

$$U^{b}(q, x, y) = u(q) + x - y,$$
  $U^{s}(q, x, y) = -c(q) + x - y$ 

, respectively, where q is DM consumption or production, x is CM consumption and y CM production. I assume u'(q) > 0, u''(q) < 0,  $u'(0) = \infty$ , c'(q) > 0,  $c''(q) \ge 0$ , u(0) = c(0) = 0 and  $\bar{q} = u(\bar{q})$  for some  $\bar{q} > 0$ . There exist a  $q^* \in (0, \infty)$  such that  $u(q^*) = q^*$ , that is  $q^*$  maximizes the surplus of a DM match.

Let  $\phi_t$  be the price in period t of money in terms of the numéraire. Given the quasi-linearity of preferences and access to the CM market in every period, the agents problem can be state as follows:

$$\max_{m_{t+1}} \{ -[\phi_t - \beta(1+i_m)\phi_{t+1}]m_{t+1} + \beta \frac{1}{2} [u(q_{t+1}(m_{t+1})) - c(q_{t+1}(m_{t+1}))] \}$$
(2.1)

That is, the choice of real balances,  $m_{t+1}\phi_{t+1}$ , to bring into the next period is defined by the trade off between the opportunity cost of holding money,  $-[\phi_t - \beta(1+i_m)\phi_{t+1}]m_{t+1}$ , and the expected benefits of being able to trade in the DM due to holding liquidity,  $\beta \frac{1}{2}[u(q_{t+1}(m_{t+1})) - c(q_{t+1}(m_{t+1}))]$ . Using the assumed TIOLI bargaining protocol, which means the producer is compensated just so that they are indifferent and thus the buyers payment satisfies  $c(q_{t+1}) = \phi_{t+1}m_{t+1}(1+i_m)$  it is possible to obtain a closed form steady state monetary equilibrium:

$$\frac{u'(\tilde{q})}{c'(\tilde{q})} = 1 + \frac{\mu - \beta(1+i_m)}{\frac{1}{2}\beta(1+i_m)}$$
(2.2)

Note that any such solution requires that  $\phi_t \geq \beta(1+i_m)\phi_{t+1}$  or  $\mu \geq \beta(1+i_m)$  so  $\tilde{q} < q^*$ unless  $\mu = \beta(1+i_m)$  which corresponds to the Friedman Rule.

#### 2.3.1 Implementation

For the online experiment, I consider 4 implementations of the model.

- 1. Baseline BASE ENDO. In the baseline scenario, money supply is constant ( $\mu = 1$ ) and no interest is paid on money holdings ( $i_m = 0$ ). In this case  $\tilde{q} < q^*$  since  $\beta < 1$ , that is the first best would not be achieved.
- 2. Baseline Exogenous Price BASE EXO. Same as above,  $(\mu = 1)$  and  $(i_m = 0)$ , but in this case Consumers in the DM are constrained in the TIOLI offers they can make.

Given their token holdings,  $m_i^t$ , Consumers select from a menu the quantity they wish to consume but the price  $(\phi_t)$  is endogenously set at  $fracM_t/N\tilde{q}$ . Note that this is a price consistent with the theoretical steady state predictions of the model.

- 3. Friedman Rule FR EXO. In this scenario, the Friedman Rule is implemented by interest payment on money holdings  $(i_m > 0)$  which are financed trough lump-sum taxes and a constant money supply  $(\mu = 1)$ . Thus, in order to achieve  $\tilde{q} = q^*$  I set  $i_m = 1/\beta - 1$ . For the Government budget constraint to hold lump-sum taxes must equal the interest payments,  $-N\tau_t = i_m M_T$
- 4. Friedman Rule Exogenous Price FR ENDO. Same as FR ENDO, but again including a price constraint on DM offers consistent with the model theoretical predictions, that is  $\phi_t = \frac{M_t/N}{q^*}$ .

## 2.3.2 Hypotheses

Based on the theoretical model and conjectures about the experimental design and results from previous work, such as DP(2022) and JPZ(2019), I formulate the following hypotheses to test using my experimental data. First, that individuals coordinate on the monetary rather than the autarkic equilibrium:

**Hypothesis 1** Monetary equilibrium rather than the autarkic outcome better characterizes trading behavior.

Second, consistent with the optimality of the Friedman Rule:

Hypothesis 2 Quantities traded and welfare: higher under the FR (ENDO or EXO) v/s BASE (ENDO or EXO). Third, consistent with the quantity theory of money, the rate of change in the price level should equal the rate of change in the money supply, and fourth, price levels should reflect the monetary policy regime:

Hypothesis 3 Given the constant money supply there should not be inflation.

**Hypothesis 4** Prices in both the DM and CM should be lower in FR-EXO than in Base-EXO.

Finally, all of the above predictions require that subjects can make the proper calculations, coordinate on the monetary equilibrium and expect others to behave similarly rationally. Imposing equilibrium prices as a constrain for DM Consumers eliminates price risk in the DM and could nudge consumer to the model predicted equilibrium.

**Hypothesis 5** Hypotheses 1 through 4 should hold more accurately for BASE-EXO and FR-EXO than for BASE-ENDO and FR-ENDO.

## 2.4 Experimental Design

My experiment involves 4 treatments, all of which are based on the Lagos-Wright (2005) model in a laboratory setting. I first discuss the baseline scenario with constant money supply and no interest in money holdings (with endogenous DM price) and then move on to the FR and to the respective cases with exogenous DM price. Each of the 3 sessions of the baseline treatments involves N players and consist of a number of sequences (or supergames). At the start of each new sequences all subjects are endowed with M/N "tokens", which is the name for fiat money throughout the experiment, plus a fixed number of points, E. Subjects are clearly instructed (and repeatedly reminded) that tokens have no redemption value and

that only their total points at the end of each sequence matter for their final dollar payoff. Each sequence consist of an indefinite number of periods divided into two decision making parts: the decentralized market (DM) and the centralized market (CM). At the beginning



Figure 2.1: DM Consumer's Interface

of each DM part, all N subjects are randomly and anonymously paired with one another to form N/2 pairs. As instructed to the subjects, one person in each pair is selected to be the consumer while the other the producer, with both roles being equally likely and history independent in each period. Each consumer *i* moves first and makes a TIOLI offer consisting of a pair  $q_i, d_i$ , where  $q_i \in [0, \bar{q}]$  is the requested quantity of DM good to be produced by the producer and  $d_i \in [0, d_i^D M]$  the amount of fiat money offered by the consumer in exchange, with  $d_i^{DM}$  being subject *i* money holdings at that time. Then producer *i'* either accepts or rejects the proposal that subject *i* submitted. If the proposal is accepted, consumer *i* earns  $u(q_i)$  points that get added to their point total (which would be the initial endowment if it is the first period of the sequence) and gives up  $d_i$  tokens. Producer *i'* incurs a cost of  $c(q_i)$  that is subtracted from their point total, but receives  $d_i$  tokens as part of the exchange. If the producer rejects the offer then no production takes place and no money is exchanged, so both subjects in the pair leave the DM with the same total points and token holdings that they entered. In the second half of each period, the CM part, all N subjects participate together



Figure 2.2: DM Producer's Interface

in a centralized market for the CM good. Given the linear technology in the consumption and production of the numéraire this market serves as a device for re-balancing money holdings.



Figure 2.3: CM Role Selection Interface

As in Duffy and Puzzello (2014a, 2020) and JPZ(2019), the CM is implemented using a Shapley and Shubik (1977) market game but framed in different but equivalent way. So, while in previous experimental work dealing with the Lagos and Wright model each subject decided on how many units of the CM good to produce and sell,  $y_i$ , or tokens to bid in exchange for CM good  $b_i$ , thus determining the market prices that clears all exchanges as  $P = \sum_{i} \frac{b_i}{y_i}$  and then (due to linearity) obtaining 1 point for each purchased unit  $(b_i/P)$  or losing 1 point for each produced one,  $-y_i$ , in this paper subjects directly decide whether to be *Token Sellers* or *Token Buyers* in what I call the *Centralized Token Exchange*. If player *i* decides to be a Token Seller, they specify an amount of tokens to sell in this Token Exchange,  $b_i \in [0, b_i^{CM}]$ , where  $b_i^{CM}$  is *i's* money holdings at that time. If player *i* chooses to be a Token Buyer they specify the number of points to bid for tokens in this exchange,  $p_i \in [0, PT_i^{CM}]$ , where  $PT_i^{CM}$  is the subjects point total. The market price that clears all exchanges is then,  $P = \sum_{i} \frac{b_i}{p_i}$ . If there are no points bids or tokens being sold, then there is no market price and no token exchange. Following the completion if the CM, money holdings and points are adjusted: Token Sellers receive  $b_i/P$  points (and give up  $b_i$  tokens), while Token Buyers give



Note: if you don't enter a bid, you will be assigned a bid of zero Tokens

Figure 2.4: CM Token Seller Interface

are in the <b>Centralized Market</b> now with: 0.02 Tokens 14.97 Points average players hold 10 Tokens each	
low many <b>Points</b> would you like to use to bid for <b>okens</b> :	Market Price History =
REMEMBER: How many <b>Tokens</b> you get will depend on Market Price, you can use the 1st slider to inform your decision	3.5 3.1 3 2.53
Your Market Price Forecast (Tokens per Point): 2.2	2 2.4 2 1.9 1e8
Your <b>Point</b> offer: 4.7	1.5 Round 1 Round 2 Round 3 Round 4 → Your Prediction → Actual Market Price
Tokens to earn given the predicted10.34Market Price:	Highcharts.com

Next

Note: if you don't enter a quantity, you'll be assigned a zero Point offer

Figure 2.5: CM Token Buyer Interface

up  $p_i$  points and receive  $P * p_i$  tokens.

Even though it is equivalent, the advantage of framing the centralized market this way is that now subjects only deal with one good, the DM good, which is referred as the "Special Good" and a Centralized Token Exchange, where they can trade points for tokens directly at a the emerging market price, P. This is intended as a way to make clearer that the role of the CM is to re-balance token holdings and not to create any trade surplus since only trading the good in pairwise meetings accomplishes it.

Following the completion of the CM, the random termination scheme is implemented with a random integer is drawn from the set 1, 2, 3, 4, 5, 6 to determine whether the sequences continues for another period. If the random number is different from 6, then the sequences continues for another period, point balances and money holdings carry over to the next period. Otherwise, the current sequence ends, point balances are final (and recorded) and token holdings are eliminated (which is clearly communicated to subjects). This random termination scheme with continuation probability  $\beta = 5/6$  to implement an infinite horizon model with discounting. Depending on the remaining time in the session, a new sequence may start, where subjects again star with M/N tokens and E tokens. After the last sequence, two sequences are selected at random and subjects are paid based on their combined final token holdings for those two.

#### 2.4.1 Friedman Rule Treatment

In addition to the above discussed baseline treatment, I consider an implementation of the Friedman Rule for this environment via payment of interest on money holding, which I refer to as FR-ENDO. In this implementation interest rate  $i_m$  is paid on individual money holdings at the beginning of the CM after any DM exchanges, so if subject *i* left the DM with  $d_i^{DM'}$  money holdings then those would be increased to  $(1 + i_m)d_i^{DM'}$ . Achieving the first best

requires that  $\mu = \beta(1+i_m)$  and since I wish to achieve it without changing the money supply, so to keep things simple for participants, then  $\mu = 1$  and the interest on money holdings must be financed trough a lump-sum tax,  $\tau$ . The policy rule is then  $1 = \beta(1+i_m)$ , or  $1/\beta - 1 = i_m$ , which together with the governments budget constrain implies that  $\tau = (1/\beta - 1) * M/N$ which is imposed on participants right after the interest payment. So again, if subject *i* left the DM with  $d_i^{DM'}$  money holdings, then after the interest payment and taxation they would face their CM decisions with  $b_i^{CM} = (1 + i_m)d_i^{DM'} - (1/\beta - 1)M/N$ . The precise details of the FR-ENDO rules and timing of taxes and interests are clearly communicated to subjects with examples and reminders.

## 2.4.2 Exogenous Pricing Restrictions Treatments

For the treatments with exogenous DM prices, BASE-EXO and FR-ENDO, the only difference in terms of implementation is that subjects are instructed that throughout their entire session whenever they were selected to be consumers in a pairwise meeting they would only select the quantity requested in their offer and the price would be given and fixed at  $fracM_t/N\tilde{q}$  and  $\frac{M_t/N}{q^*}$ , respectively. IS is also clarified that when selected to be a producer in the DM their counterpart would be bind by the same restriction. Finally, participants are told that the price is fixed and does not depend on their actions but there is no reference to a notion of equilibrium being made.

#### 2.4.3 Parametrization and Procedures

The model parametrization follows prior work in the area by DP(2022)to keep comparisons a easy as possible. Thus, I set the discount factor (which is the continuation probability) at  $\beta = 5/6$  and use a CRRA utility function for the DM,  $u(q) = Aq^{(1-B)}/(1-B)$ , with A = 1.635 and B = 0.224. This function paired with the DM linear cost implies that the
first best solution is:

$$q^{\star}: u'(q^{\star}) = c'(q^{\star}) = 1 \Rightarrow q^{\star} = 9.$$

On the other hand, the equilibrium solution for the baseline model (with no interest on money holdings and a constant money supply) is:

$$q: u'(\tilde{q}) = 1 + 2(1 - \beta)(\beta) \Rightarrow q^* = 2.$$

This is a particularly convenient parametrization since it makes the difference between  $q^*$ and  $\tilde{q}$  sufficiently large so that they are distinguishable even with the noise associated with laboratory experiments. As explained in the previous section the CM has a 1-1 linear utility and production technology for simplicity which also allows me to frame it a simple *Token Exchange*.

I set the number of participants<sup>6</sup> to N = 10 for every session<sup>7</sup> and each individual is endowed with 10 tokens so, since  $\mu = 1$ ,  $M_1 = M = 100$  for all treatments and across all periods. For the Friedman Rule (both FR-ENDO and FR-EXO) I set  $i_m = 0.2$ , which is the solution to  $\mu = 1 = \beta(1 + i_m)$ , so that subjects receive a 20% interest on their money holdings at the beginning of the CM,  $b_i^{CM}$ .

The key values for this standard parametrization of the model plus the associated steady state predictions regarding quantities traded in the DM and prices are provided in Table ??, where the welfare maximizing first best is attained in the Friedman Rule treatments. Last columns presents welfare relative to first best, measured as u(q) - c(q).

 $<sup>^6\</sup>mathrm{DP}(2022)$  set the number of participants at 14 while JPZ (2019) go with smaller groups of 8 to 10 participants

<sup>&</sup>lt;sup>7</sup>The exception being the first session with 16 participants which lead to a slower than expected passed online experiment and thus smaller groups in all of the latter sessions

Treatment	q	$P_{DM} = d/q$	$P_{CM}$	i	$\frac{M_{t+1}}{M_t}$	Welfare Relative to 1st Best
Baseline	2	10/2 = 5	10/2 = 5	0%	1	62%
Friedman Rule	9	10/9 = 1.11	10/9 = 1.11	20%	1	100%

Table 2.1: Equilibrium predictions given parametrization

#### 2.4.4 Procedures

This experiment was conducted fully online using the Otree Python framework (Chen, Schonger, and Wickens, 2016). For each sessions all 10 recruited subjects had no prior experience with this experiment and were drawn from the undergrad population of UC Irvine and were paid on the basis of their performance in the experiment. Each session lasted between 90 and 130 minutes.

I employ a between subjects design where a single combination of monetary policy regime and DM pricing treatment is in effect fir the entire duration of a session. At the start of each session, subjects were given interactive instructions with exercises and questions (that they needed to complete to advance) so that they would familiarize themselves with the user interface and decisions they would face. Subjects were required to be logged to a Zoom meeting throughout the duration of the session so they could ask any questions directly. Screenshots for all the instructions can be found in Appendix A. The instructional time took around 30 minutes and the experiment only stated after all participants had completed them. Subjects were informed that each session consisted of a number of sequences, with each sequence consisting of an indefinite number of periods, and that after each period the continuation probability was  $\beta 1/6$  and the termination probability  $1 - \beta = 5/6$ . Since the length and duration of each sequence was unknown to subjects, the number of sequence to be played was undisclosed as well. participants were only told that "when a sequence ends, then depending on the available time, another would immediately start". Subjects were told that a *virtual dice*, which was displayed on the screen, was used to determine continuation of sequences, but in practice I drew 3 realizations of sequence lengths a priori using a discrete uniform distribution and used those realizations for the 3 sessions of each of the 4 treatments to facilitate comparisons. The number of sequences and lengths are included in Table ?? below.

Regardless of the treatment, each participants was endowed at the beginning of each sequence with 40 points and 10 tokens. Both token holdings and point balances carried over from period to period but not from sequence to sequence. Point balances at the end of two sequence were selected at random to pay subjects at a fixed rate of 1 point = 0.25 In practice, the first and third sequences of each session were selected at random during the first session and applied as the rule (without telling subjects) to all subsequent sessions. As stated above, each period consisted of two parts. The first one, the DM, which was presented to subjects as the "Goods Market" were subjects were anonymously and randomly paired and in each paired one subject was selected to be the consumer and the other the producer. As shown in Figure 2.1, the consumer would move first, using one slider to request a certain amount of DM good, referred as the "Special Good" and a second slider to offer a certain amount of his or her tokens in exchange. DM good requests could range between 0 and 20 for treatments with no constrains on DM price, while token offers could go from zero to their current token holdings. For the FR-EXO and BASE-EXO, the maximum DM request would depended on the proposers token balance and the exogenous price: if the exogenous price was 5 tokens per unit (as in the BASE-ENDO case) and the subject had 15 tokens, then they could request 0,1,2 or 3 units only. The DM consumer's interface provided real time information for each regarding potential point benefit, producer's point cost, trade surplus and implicit unit price for each possible offer. The history of past trades with term's of trade and outcome was also displayed for users.

Again, common to all treatments is that after viewing the consumer's proposal, the producer

decides whether to accept it or not (see Figure 2.2). Information regarding the effect on points and tokens of each decision is provided. If accepted the proposal is implemented immediately production takes place and tokens are exchanged. Adjusted balances are carried overt to the CM part and results screen is showed to both participants indicating the outcome and changes to their token and points balances.

In the CM part of the period, referred to subjects as the "Token Exchange", subjects first choose to become either Token Sellers or Buyers (see Figure 2.3)<sup>8</sup> with a reminder of what each role implies. Token Sellers use a slider to decide how many of their tokens to bring to the exchange to be sold and are encourage to make a prediction about the Market Price, P, which helps inform their decision in terms of how many points they would get conditional on that price. Subjects are instructed that the market price is only determined after everyone has make a decision and information regarding their own past prediction and past market prices are provided (see Figure 2.4). As shown in Figure 2.5 Token Buyer are encourage to start by making a market price prediction and then select the amount of points they would to bid in exchange of tokens. Expected point earnings conditional on their prediction are shown to them as well as information regarding their past prediction and realized market prices.

At the end of each CM, subjects learn about the realized market price, P, and changes to their tokens and point balances. Then the realization of the random draw was revealed. If the number drawn was less or equal than 5, the sequence continued for another period with subjects' points and tokens balances carrying over to the next DM. Otherwise, if a 6 was drawn the sequence ended and subjects were again informed that their token holdings were now useless and discarded but that their points (if the sequence was randomly selected) would translate into a dollar payment.

 $<sup>^{8}</sup>$ If a participant reaches the CM without any tokens they can only choose to be Token Buyers and if they arrive with no Points left they can only choose to be Token Sellers

Particular to both Friedman Rule treatments (FR-EXO and FR-ENDO) is that the proportional 20% interest on token holdings as well as the 2 lump-sum token tax, which is consistent with a constant supply of money, is paid at the beginning of the CM before subjects choose whether to be token sellers or buyers.<sup>9</sup>

### 2.5 Experimental Results

I report on data from 3 sessions of each of my four treatments. Each treatment involved 10 inexperienced subjects (with the exception of the first session with 14 subjects), thus data comes from  $4 \times 3 \times 10 + 4 = 124$  subjects. A summary with information regarding the sessions can be found in Table ??.

Treatment	Obs. No.	No. Seq.	Seq. Lengths	No. Round	Avg. Earnings	Max. Earnings	Min. Earnings
Baseline - Endogenous	1	3	8, 3, 5	16	\$23.62	\$30.83	\$13.92
Baseline - Endogenous	2	4	5, 3, 8, 6	22	\$24.32	\$29.12	\$21.48
Baseline - Endogenous	3	4	4, 8, 9, 3	24	\$23.85	\$27.11	\$20.41
Baseline - Exogenous	1	3	8, 3, 5	16	\$23.73	\$29.66	\$18.16
Baseline - Exogenous	2	4	5, 3, 8, 6	22	\$23.72	\$28.86	\$14.46
Baseline - Exogenous	3	4	4, 8, 9, 3	24	\$23.95	\$29.72	\$20.16
FR - Endogenous	1	3	8, 3, 5	16	\$24.16	\$29.07	\$19.07
FR - Endogenous	2	4	5, 3, 8, 6	22	\$23.81	\$34.00	\$19.82
FR - Endogenous	3	4	4, 8, 9, 3	24	\$23.40	\$28.06	\$18.21
FR - Exogenous	1	3	8, 3, 5	16	\$24.32	\$29.14	\$18.16
FR - Exogenous	2	4	5, 3, 8, 6	22	\$24.30	\$53.62	\$9.49
FR - Exogenous	3	4	4, 8, 9, 3	24	\$23.76	\$31.36	\$18.48

Table 2.2: Characteristics of Sessions

#### 2.5.1 Proposals and Acceptance Rate

Figure 2.6 shows the percentage of non-zero token offers in the DM that were accepted by treatment including 95% confidence intervals. Zero token offers were uncommon across all

 $<sup>^{9}</sup>$ In the event that subjects don't have enough tokens to pay the tax, then they become indebted to the government and are forced to become *Token Buyers* in the CM (and forthcoming CM's) until they can afford the tax

treatments for subjects entering the DM with positive token holdings, which as can be seen in Figure 2.7 is the majority of subjects, accounts only for less than 5% of all offers (additional detail on the distribution of token holdings can be found in Appendix A). Those two facts, that subjects tend to enter the DM holding the fiat object and trade using those tokens, strongly supports the idea that a monetary equilibrium rather than autarky is the norm (Hypothesis 1). The acceptance rate of such offers varies between 35% and 80% depending on the treatment with higher acceptance rates in both BASE-ENDO and FR-ENDO where the DM price is set endogenously. More detail regarding the acceptance rate for each session can be found on Appendix A. These finding are consistent and reinforce what is found in



Figure 2.6: Accepted Offers as Fraction of Total Offers Across Treatments with 95% Confidence Intervals

DP(2014a,b) and DP(2022) but contrary to what is argued there, that monetary equilibrium should be consistent with an acceptance rate of a 100%, it is not necessary the case due to

the selected bargaining protocol (TIOLI) which would make producers indifferent when the predicted monetary equilibrium is being played. Thus an acceptance rate around 50% should not come as a surprise and could be considered as evidence of producers actually being indifferent between accepting and rejecting. Moreover, some rejection could be a natural part of subjects trying out offers that are just not convenient to producers while they are learning the equilibrium, which would explain the higher acceptance rate in the BASE-EXO and FR-EXO treatments. More formally, Table 2.3 reports producers acceptance of money offers using a random effects logit and probit regressions with standard errors clustered at the subject level<sup>10</sup>. This provides additional evidence for tokens being valued as a means of exchange: acceptance probability goes up with d, the token offer, and decreases with q, the requested amount. Interest payments on token holdings, in both the FR-EXO and FR-ENDO treatments, increase the acceptance rate but there is no effect for the BASE-EXO treatment. Given the above discussion regarding the predicted acceptance rate, is not reasonable to looks for evidence in favor or against Hypothesis 5 (EXO treatments leading to results closer to theoretical predictions) in this section, nevertheless the higher acceptance rate in Friedman rule implementations conditional on all other factors (such as q and d) could also be interpreted as evidence in favor of subjects responding to the incentives as is predicted in the model. There is also a strong decay effect in acceptance over time within sequences (seq\_period), but not at the start of a new sequence (new\_seq), which has been also observed in previous experiments.

#### 2.5.2 DM Traded Output

Figure 2.8 shows the average over all sessions of DM output traded from accepted proposals across treatments, where the red triangles are the model predictions. With the exception the BASE-EXO treatment traded quantities depart considerably from the model steady

<sup>&</sup>lt;sup>10</sup>Most of the regression analysis in the rest of the paper relies on simpler subject and time fixed effects but due to the non-linearity of the logit and probit models it is not feasible in this case



Figure 2.7: Subjects Entering DM with 0 Tokens as Fraction of Total Subjects Across Treatment with 95% Confidence Intervals

	logit	probit
(Intercept)	$-0.595^{*}$	$-0.367^{*}$
	(0.297)	(0.176)
$baseline\_exo$	0.457	0.352
	(0.318)	(0.186)
fr_endo	$1.025^{***}$	$0.602^{***}$
	(0.279)	(0.165)
fr_exo	$1.402^{***}$	$0.872^{***}$
	(0.286)	(0.169)
new_seq	0.271	0.175
	(0.214)	(0.127)
$seq_period$	$-0.207^{***}$	$-0.124^{***}$
	(0.043)	(0.025)
q	$-0.184^{***}$	$-0.093^{***}$
	(0.034)	(0.018)
d	$0.212^{***}$	$0.097^{***}$
	(0.029)	(0.012)
$token_holdings$	-0.007	-0.003
	(0.005)	(0.003)
Num. obs.	1288	1288
Log Likelihood	-687.259	-690.689

 Table 2.3: Random Effects Probit and Logit Regression Analysis of Acceptance of Money Offers

\*\*\*p < 0.001; \*\*p < 0.01; \*p < 0.05

state values. However, it is still valuable to consider whether traded quantities and DM prices are consistent with the comparative statics predictions across treatments. Here it is important to highlight the fact that for BASE-EXO treatment the DM price is endogenously set at 5 tokens per unit, which rules out requests for large quantities and considerably limits the range of possible results, which might be also driving its higher acceptance rate observed in the previous section. This could be consider as a first piece of evidence in supporting Hypothesis 5, that is imposing the steady state prices nudges subjects in the he predicted monetary equilibrium and fosters trade. Regarding Hypothesis 2, that quantities traded should be larger when the Friedman Rule is in place, I found mixed evidence: the BASE-ENDO treatment shows the highest quantities being trade significantly above both

Friedman Rule treatments, but on the other hand, both Friedman Rule treatments show higher traded quantities than BASE-EXO. So at least when comparing BASE-EXO to FR-EXO the comparative statics regarding output hold. This results can also be seen in Table 2.4, where I use the panel structure of the data to regress the traded quantities on treatment factors and controlling for fixed effects on subjects, time, and both subjects and time. In this case BASE-ENDO is taken as the standard (constant term) so coefficients on other dummies are to be understand as changes with respect to it. The online result that holds across specifications is that the quantity being trade in BASE-EXO is lower than in BASE-ENDO, and when controlling for both time and subject effect, which seems like the most reasonable, evidence is again mixed as discussed above. Further detail regarding DM traded output for each session can be found on Appendix A.



Figure 2.8: Average DM Traded Output Across Treatments with 95% Confidence Intervals

	DM Traded Output		
	Fixed Effect: Time and ID	Fixed Effect: ID	Fixed Effect: Time
	(1)	(2)	(3)
Constant	3.720***	3.500***	4.289***
	(0.420)	(1.246)	(0.682)
$baseline\_exo$	$-2.738^{***}$	$-2.250^{*}$	$-2.197^{***}$
	(0.392)	(1.272)	(0.609)
fr_endo	$-0.738^{**}$	-0.500	0.614
	(0.304)	(1.597)	(0.692)
fr_exo	$0.638^{**}$	0.667	-0.231
	(0.269)	(1.907)	(0.669)
Observations	488	488	488
$\mathbb{R}^2$	0.698	0.672	0.202
Adjusted $\mathbb{R}^2$	0.573	0.566	0.157

Table 2.4: Regression Analysis of Traded Quantities on Treatment Dummies

#### **2.5.3** Prices

In this section the effect of the different treatments on DM and CM price levels. Figure 2.9 shows the average price for trades that took place in the DM including 95% confidence intervals, where it is important to remember that in both the BASE-EXO and FR-EXO treatments prices are set endogenously, so the only relevant comparison is between BASE-ENDO and FR-ENDO. When doing so, it is clear that prices in BASE-ENDO are significantly lower than what is predicted by the model (red triangle) but relatively closer for the FR-ENDO treatments, this provides only relative support for Hypothesis 4: price level (expressed in terms of tokens per unit of output) in the FR-ENDO treatment should be lower than in BASE-ENDO. In Table 2.5 I present a regression analysis of DM prices on different treatments controlling for fixed effects: there's no significant difference between treatments with unrestricted prices. Note that I included both the BASE-EXO and FR-EXO treatments even though their DM price is fixed just for completeness in the comparisons with the baseline (constant term) which again corresponds to BASE-ENDO, an alternative which gives similar



results is to just exclude those observations.

Figure 2.9: Average DM Price Across Treatments with 95% Confidence Intervals

Regarding CM prices, one can observe that there's a a big dispersion and the average price seems to be heavily influences by outliers (see Figure 2.10) and with no distinguishable differences between treatments (see Table 2.6). This is likely driven by a very thin market with few sellers and buyers, so even though there are only 8 periods out of a total of 248 where no trade takes place in the CM (due to a lack of sellers or buyers) in many cases there's only one party on either side of the market game implementation. This is a major issue, since the CM is supposed to be the instance where subjects can re-balance their money holdings at a known stable price, and the introduction of price risk in this stage could have major implications in the willingness to accept fiat currency in the DM. Having a larger

	DM Price	
	Fixed Effect: Time and ID	Fixed Effect: ID
	(1)	(2)
Constant	0.929***	0.916***
	(0.119)	(0.134)
$treatment baseline\_exo$	4.067***	4.020***
	(0.056)	(0.146)
$treatment fr_endo$	-0.023	-0.232
	(0.053)	(0.161)
$treatment fr_exo$	0.123**	0.124
	(0.050)	(0.148)
Observations	454	454
$\mathbb{R}^2$	0.986	0.963
Adjusted R <sup>2</sup>	0.980	0.961

Table 2.5: Regression Analysis of DM Price on Treatment Dummies

group of subjects in each session could be a solution, since as shown by Duffy, Matros, and Temzelides (2011) groups of size 20 act like price takers and the resulting outcome resembles a competitive equilibrium. JPZ(2019) seem to have founded a solution by keeping the roles of buyers and sellers fixed trough out every session in their experimental Rocheteau-Wright (2005) experiment, since having too many participants is not only expensive but tends to slow down the progress of the experiment in a non linear way.

Regarding the evolution of prices across times, there no systematic evidence for an upward (or downward) trend in prices within sequences, which is evidence in favor of Hypothesis 3: price levels should be stable since the money supply is kept constant . Figure 2.10 shows the DM unit price for each sessions of both treatments where the that price is allowed to be determined endogenously, while Figures 12 and 13 depict the CM market price for each session of all 4 treatments. More formally, adding a time trend (or controlling for time effects) to regression analysis in Tables ?? and ?? are not significant and do not alter results. Though there's no evidence of a clear trend there's clear dispersion in CM prices which I

	CM Price		
	Fixed Effect: Time	OLS	
	(1)	(2)	
Constant	2.685	9.546***	
	(7.013)	(2.906)	
$treatment baseline\_exo$	-3.662	-3.681	
	(4.151)	(4.181)	
$treatment fr_endo$	-2.308	-2.375	
	(4.094)	(4.127)	
$treatment fr_exo$	-2.263	-2.748	
	(4.133)	(4.162)	
Observations	240	240	
$\mathbb{R}^2$	0.117	0.004	
Adjusted $\mathbb{R}^2$	0.009	-0.009	
Residual Std. Error	$22.681 \ (df = 213)$	22.886 (df = 236)	
	*p<0.1; *	**p<0.05; ***p<0.01	

Table 2.6: Regression Analysis of CM Price on Treatment Dummies

further analysed. When looking at subjects predictions about their price expectations at the beginning of each CM (which they voluntarily submit to help inform their decisions while using the interface slider calculator) there is no evidence of a no evidence of a price trend either.

#### 2.5.4 Welfare

Taking int account that utility and production are linear in the CM with no trade surplus and thus should only be used to rebalance money holdings, thus one natural way of measuring overall period welfare is just adding up the trade surplus u(q) - q of each pair of each DM round. However as noted in Figure 6, depending on the treatment only 35% to 80% of non-zero token proposals are accepted. The theory does not suggest different acceptance rates under different monetary policy schemes, that is the extensive margin should not be affected. Instead, different monetary regimes should work trough the intensive margin, that



Figure 2.10: DM Price for BASE-ENDO (left) and FR-ENDO (right) with period average (black line) and predicted price (red line)

is the quantity traded in each individual DM meeting. Given the above mentioned empirical differences in acceptance rates across treatments, I include the total welfare measure -including both the extensive and intensive margin- with an intensive margin only measure. Both measures are compared to their respective potential first best, every pair trading  $u(q^*) - q^*$  for the total welfare measure, and pairs with accepted proposals only trading  $u(q^*) - q^*$ . Figure 2.11 shows the average of both measures across all sequences in all sessions for the different treatments. Regarding the intensive margin, which the most adequate measure in this experiment, I found no evidence for higher welfare under the Friedman Rule when DM prices are left to be decided endogenously, quite the contrary. When imposing equilibrium prices exogenously, this welfare measure looks more according to the model predictions , but still way below the first best. When looking at the total welfare measure, there is only very week evidence for the Friedman Rule and it mostly explained by different acceptance rates as previously discussed. Table 2.7 shows the regression analysis of the intensive welfare measure on treatment dummies and confirms the same idea: results are indistinguishable from BASE-ENDO (our baseline) with the exception of BASE-EXO. Table 2.8 which is equivalent to Table ?? but for the total welfare measure reinforces the idea of welfare in treatments being equivalent to the baseline with the exception of the lower level in BASE-EXO.



Figure 2.11: Average Intensive Margin (left blue bar) and Overall (right blue bar) Welfare Compared to First Best Across Treatments with 95% Confidence Interval

#### 2.5.5 Money Holdings and Rebalancing

One distinctive feature of this model is the clear degenerate distribution of token holdings and the role of the CM as a means for subjects to rebalance their holdings in-between pairwise meetings: DM consumers should leave the pairwise meetings with zero holdings and then everyone should leave the CM with the same amount of tokens. In this section

Fixed Effect: Time and ID		
Fixed Effect. Time and ID	Fixed Effect: ID	Fixed Effect: Time
(1)	(2)	(3)
$0.775^{***}$	0.763***	0.610***
(0.033)	(0.118)	(0.081)
$-0.319^{***}$	$-0.287^{**}$	-0.061
(0.036)	(0.128)	(0.081)
-0.025	-0.026	0.210**
(0.021)	(0.159)	(0.084)
-0.010	-0.026	$0.153^{*}$
(0.022)	(0.153)	(0.083)
488	488	488
0.812	0.789	0.225
0.735	0.721	0.181
	Fixed Effect: Time and ID $(1)$ $0.775^{***}$ $(0.033)$ $-0.319^{***}$ $(0.036)$ $-0.025$ $(0.021)$ $-0.010$ $(0.022)$ $488$ $0.812$ $0.735$	Fixed Effect: Time and IDFixed Effect: ID $(1)$ $(2)$ $0.775^{***}$ $0.763^{***}$ $(0.033)$ $(0.118)$ $-0.319^{***}$ $-0.287^{**}$ $(0.036)$ $(0.128)$ $-0.025$ $-0.026$ $(0.021)$ $(0.159)$ $-0.010$ $-0.026$ $(0.022)$ $(0.153)$ 488488 $0.812$ $0.789$ $0.735$ $0.721$

 Table 2.7: Regression Analysis of Welfare on Treatment Dummies

I further explore the experimental results in this regard. I first look at the evidence that subjects are actually using the CM to rebalance their holdings after exiting the DM. For this I look at individual's changes in money holdings during the DM of each period,  $\Delta DM^M$ (which only depends on inflows and outflows due to pairwise trading), and changes that occur in the CM,  $\Delta$  (which would include both interest and taxes for the Friedmann Rule treatments). Table 2.9 presents the results for a simple regression of  $\Delta CM^M$  on  $\Delta DM^M$ for the 4 different treatments. The negative significant coefficient on the constant money supply and no interest on money holding treatments is an indicator of subjects using the CM to rebalance after trading in the DM, in an imperfect manner though since the coefficient is different from -1. This is not the case for both Friedman Rule treatments where the coefficient is not significantly different from zero, which would indicate that subjects where not rebalancing in a systematic way. The constant term is not different from zero (at least not at a high confidence level) which indicating that at least there is no bias, such as always triving to increase or decrease holdings regardless of DM outcome.

	Welfare Extensive and Intensive Margins			
	Fixed Effect: Time and ID	Fixed Effect: ID	Fixed Effect: Time	
	(1)	(2)	(3)	
Constant	0.393***	0.218	0.363***	
	(0.049)	(0.143)	(0.057)	
baseline_exo	0.061***	-0.007	$0.065^{**}$	
	(0.018)	(0.196)	(0.032)	
fr_endo	0.243***	0.224	0.099***	
	(0.018)	(0.208)	(0.034)	
fr_exo	0.299***	0.224	0.176***	
	(0.021)	(0.186)	(0.035)	
Observations	1,288	1,288	1,288	
$\mathbb{R}^2$	0.234	0.177	0.095	
Adjusted R <sup>2</sup>	0.134	0.088	0.076	

 Table 2.8: Regression Analysis of Welfare on Treatment Dummies

Figure ?? and 2.13 shows the predicted CM prices reported by each subject at the beginning of the CM (which they supply to help to inform their decision), plus the average of the predictions (colored line) and the realized market clearing price (bars). As expected, there's less -but still significant- dispersion in predicted prices under the EXO treatments, and one can observe increased variance in predictions in periods that follow unusual realizations of the market price, which are usually explain by low participation with in some cases only one subject trading in either side of the market.

Given the important differences between individual predictions, average predictions and realized prices it is interesting to take a second look at the regression analysis of Table 2.9 but with  $\Delta CM^M$  being calculated using the predicted price, that is what was the rebalancing the subject expected to achieve conditional on their own price prediction. Table 2.10 shows the results for this specification for the 4 different treatments, with the slope coefficient relatively close to -1 now for both treatments where DM prices are endogenous. And again, as in Table 2.9, a negative significant coefficient for BASE-EXO and a negative

		$\Delta C M$	$I^M$	
	Baseline Endo	FR Endo	FR Exo	Baseline Exo
	(1)	(2)	(3)	(4)
$\Delta DM^M$	$-0.871^{***}$	-0.385	-0.202	$-0.557^{***}$
	(0.118)	(0.260)	(0.351)	(0.087)
Constant	-0.271	-1.543	0.223	$-1.501^{*}$
	(0.570)	(1.682)	(1.382)	(0.830)
Observations	716	610	590	580
$\mathbb{R}^2$	0.092	0.071	0.028	0.163
Adjusted $\mathbb{R}^2$	0.011	-0.017	-0.068	0.079

Table 2.9: Regression Analysis of CM Rebalancing

non-significant coefficient for FR-EXO. More interestingly and contrary to my findings in Table 2.9, I found a significant positive value for the constant in all 4 treatments, which means that when taking into account subjects predictions about the CM price there seems to be a clear bias: subjects mostly wanted to increase their token holdings regardless of what occurred in the previous DM. This can also be related with the idea of subjects incurring in precautionary savings in the face of price uncertainty and even their future ability to re-balance their portfolio after pairwise meetings in the future.

Table 2.10: Regression Analysis of Predicted CM Rebalancing Using Predicted Price

	$\Delta CM^M$ Predicted Price			
	Baseline Endo	FR Endo	FR Exo	Baseline Exo
	(1)	(2)	(3)	(4)
$\Delta DM^M$	$-0.795^{***}$	$-0.879^{***}$	-0.431	$-0.475^{***}$
	(0.168)	(0.260)	(0.351)	(0.087)
Constant	$5.714^{**}$	$16.712^{***}$	$12.627^{***}$	$3.347^{***}$
	(2.574)	(1.682)	(1.382)	(0.830)
Observations	716	620	620	620
$\mathbf{R}^2$	0.127	0.258	0.138	0.166
Adjusted $\mathbb{R}^2$	0.049	0.189	0.057	0.087

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01



Figure 2.12: CM price (bars) with individual predictions (circles) and average period predictions (solid line) for BASE-ENDO (left) and BASE-EXO treatments (right)

# 2.6 Conclusions

This on line experimental implementation of the Lagos and Wright (2005) model mostly confirms some of the earlier results of DP(2022): surplus generating trade instead of autarky is the norm in this economy with the fiat object being valued but even though the Friedman Rule is the optimal theoretical welfare enhancing policy it does not seem to outperform a constant money supply. Quantities traded in the and welfare are not systematically higher when the Friedman Rule is implemented via interest being paid on money holdings, a result that is only partially overturned when imposing exogenous prices consistent with the theoretical predictions on all DM offers, which is the main innovation in this paper.

As discussed in DP(2022), subjects face uncertainty about the prices they would face in the DM, until they are matched and made an offer, as well as the CM, until after the market has cleared, which could explain precautionary saving motives and be behind the mixed



Figure 2.13: CM price (bars) with individual predictions (circles) and average period predictions (solid line) for FR-ENDO (left) and FR-EXO treatments (right)

experimental results. This (tremendous) uncertainty, even when completely eliminated from the DM via the BASE-EXO and FR-EXO treatments, as it is done in this paper, still persists in the CM which would be consistent with the idea of precautionary savings. Figure 2.14 offers some support for this conjecture: contrary to the model prediction, there's only a small number of subjects willing to spend all of their tokens, even when prices are fixed exogenously in the DM. For treatments when the Friedman Rule is in place the number of subjects leaving the DM with no token holdings is even lower which could be partially explained by the lump-sum tax levied on subjects at the beginning of the CM to finance the interest payment. When taking into account subjects predictions about the CM price (which is a second innovation of this paper) and looking at how they expected to rebalance their holdings conditional on their beliefs there seems to be a clear bias: subjects mostly wanted to increase their token holdings regardless of what occured in the previous DM. This is an interesting finding that can also be related with the idea of precautionary savings in the face of price uncertanty and a future inability of rebalancing their portfolio after pairwise





Figure 2.14: Consumers Spending all of their Money Holdings as Fraction of Total Meetings Across Treatments with 95% Confidence Intervals

In future research, before adding even more complexity into the search and match framework such as credit markets, coexistence of multiple currencies and real assets it would be valuable to look further into which of the basic elements of the model are explaining this departure from theory. Is it mostly uncertainty about future prices and even access to the CM market, is there a failure of subjects to understand the environment (such as discounting v/s return of assets), or subject's trust in their counterparts rationality in pairwise meetings (and thus coordination)? Those are all questions where laboratory experiments have an advantage, allowing for example to isolate individual decisions regarding investments in the liquid asset given a real rate of interest from the coordination problem and pairwise bargaining, and thus a natural complement to monetary theory and non-experimental data analysis.

# Chapter 3

# (Semi) Unstructured Bargaining with Costly Money Holdings

# 3.1 Introduction

A long standing question in economics is how self-interested people bargain to determine the allocation of something valuable in a non-cooperative way, e.g Nash (1950), Rubinstein (1982). This subject lends itself perfectly to the controlled small scale environment of the laboratory, and thus there's a long tradition of bargaining experiments<sup>1</sup>, where the standard framework involves splitting a fixed pie, with a fairly structured bargaining process, and mostly the absence of liquidity constraints, which if present are imposed exogenously.

In this paper, I focus on bargaining solutions in a setting that is specially relevant for monetary theory: the pie size (i.e output) is determined endogenously and simultaneously with the division of the pie (i.e payment from buyer to seller), with participants facing possible liquidity constraints due to buyer's endogenous decision of possibly costly money holdings.

<sup>&</sup>lt;sup>1</sup>For surveys see Camerer (2003) and Güth and Kocher (2014).

This framework allows me to empirically distinguish between two axiomatic solutions, while retaining one important feature in monetary models such as Lagos and Wright (2005) and Aruoba, Rocheteau, and Waller (2007), which is that liquidity constraints arise naturally from the fact that money can be costly to hold, but more interestingly that buyers anticipating their decision's effect on the bargaining outcome may choose to economize on money holdings, preventing efficient outcomes, even when money is costless to hold.

Thus, the main contribution of this paper is that I can empirically test the relevance of two axiomatic bargaining solutions that have been widely applied in monetary theory using an experiment that maintains one key element in that tradition: buyer's ex-ante money holdings decision. The first solution is Nash's generalized bargaining solution (Nash (1950), Nash (1953)) while the second is Kalai's proportional solution (Kalai (1977)), which are the most widely used in applied work, and result in different predictions in the two-dimensional bargaining problem that I consider here, depending on buyer's money balance decisions, and thus whether liquidity constraints are binding or not. The innovation in this approach is that by varying the cost of money holdings I can identify which solution better characterizes the experimental data not just by looking at the bargaining outcome but also at the liquidity constraint itself (i.e how much money buyer's decide to bring to the bargaining stage). This is a key departure from the closely related work of Duffy, Lebeau, and Puzzello (2021) -henceforth DLP(2021)- where liquidity constrains are exogenous, and any money not used in trades is completely worthless, in fact theory predicts that welfare result qualitatively change when the liquidity constraints are endogenized through costly ex-ante choice of balances (see Rocheteau, Hu, et al. (2021).

Both Nash and Kalai's bargaining solutions are axiomatic in that they each follow from a particular set of assumptions. In the case of Nash solutions are assumed to be individually rational, Pareto efficient, independent of irrelevant alternatives and invariant to scale changes in utility representations. While Kalai replaces invariance with a strong monotonicity axiom, which implies that any trade surplus expansion is to be proportionally split between parties. As hinted above, this difference plays an important role in this experiment: since buyer's decide their money holdings taking into account how this affects the bargaining outcome, the proportionality feature of Kalai allows the first best to be achieved when money is not costly to hold, while Nash's non-monotonicity prevents it.

As a benchmark to compare my experimental data, I show that even in the case when money is not costly to hold (which is implemented in the experiment by setting the interest rate equal to zero) buyers should still choose to be liquidity constrained under Nash but not under Kalai, regardless of the bargaining weight<sup>2</sup>, enabling a direct test as to which bargaining solution best characterizes the experimental data. I also test for the notion that the buyers, regardless of the bargaining solution, should economize more on money holdings when the interest is higher. Based on previous literature, I select a bargaining weight of 1/2to further compare my experimental data with theoretical predictions regarding quantity, payments and surplus split<sup>3</sup>.

My experiment consists of three interest rate treatments, 0%, 30% and 60%. Subjects participate in independent rounds, as either buyers or sellers, and are confronted with one interest rate value for the first half of the experiment and another one for the remaining half. All six possible combination are implemented, with half of them taking place online and half in person.

To preview my results, I first find strong support that higher interest rates (which can be interpreted in this context as inflation) do incentivize participants to economize on money holdings leading to a more constrained bargaining set resulting in less production and surplus. Second, and contrary to theory, even when money is costly to hold buyers do not spend all

<sup>&</sup>lt;sup>2</sup>The exception being the case where all of the trade surplus goes to the buyer.

<sup>&</sup>lt;sup>3</sup>An alternative approach would be to directly test the axioms underlying Nash and Kalai, but since I can directly observe trading outcomes and procedure, it is beyond the scope of this paper. More on this in Section 2.

of their holdings in the bargaining stage. Third, I only find mixed evidence in favor of either bargaining solution. While money holdings are closer to the predictions under Kalai (and thus considerably higher than what Nash implies), quantities traded and payments do not comport systematically according to any of the bargaining solutions predictions. Moreover, in favor of the Nash bargaining hypothesis, I find that buyer's surplus share is increasing in the interest rate, which is a distinctive feature of that bargaining solution. Interestingly, an extreme alternative model where buyers are myopic in deciding their money holdings, so liquidity constraints becomes effectively exogenous, provides evidence that is entirely consistent with Nash bargaining, which is contrary to DLP's (2021) findings in support of Kalai. A second more plausible model, where buyer's directly consider their interest cost when bargaining, which is a sunk cost, better fits the data and provides some additional evidence in support of Kalai. This behavioral trait opens interesting possibilities for future research in monetary theory.

## 3.2 Related Literature

This paper relates to a few different traditions: Bargaining Experiments, Consumption/Saving Experiments, Joint Production Experiments, Experimental Macroeconomics with an Explicit Role for Money, and Applied Monetary with Explicit Price Formation Mechanisms.

#### **3.2.1** Bargaining Experiments

There's a long history of experiments on bargaining with the first implementations relying on mostly unstructured designs. Early examples of this unstructured face-to-face approach include Nydegger and Owen (1974), A. E. Roth and Malouf (1974) and A. Roth and Murnighan (1982), who were already concerned about testing the axioms underlying different solutions and the role of information. Rubinstein's (1982) alternating offers extensive game, as well as ?? ultimatum game, had a major influence on experiments about bargaining, moving the research towards games with a much more structure.

Traditional bargaining experiments, whether structured or not, tend to focus on splitting a fixed amount. For example Binmore, Shaked, and Sutton (1989) and Binmore, Proulx, et al. (2001) look at the role of outside options given a fixed pie to split in an structured setting<sup>4</sup>, while more recently Feltovich and Swierzbinski (2011), Anbarci and Feltovich (2013) and Anbarci and Feltovich (2018) study the role of outside options and disagreement values, unstructured bargaining games as well as in structured games. An application to the labor context is Korenok and Munro (2021), who use unstructured wage bargaining in a dynamic labor-search model where the match surplus is exogenous.

Departing from the strict split the pie framework, Galeotti, Montero, and Poulsen (2019) use an unstructured bargaining setting where participants chat to decide upon one of the predefined allocations, some of which may be fairer but not efficient. They find evidence inconsistent with both the Nash (1950) and Kalai (1977) solutions, do not address the Kalai (1977) proportional solution. Navarro and Veszteg (2020) test the axioms of several bargaining solutions using unstructured bargaining and provide evidence against the scale invariance axiom, with the Nash bargaining solution and the Kalai-Smorodinsky solutions as poor predictors of their data.

In all of these unstructured bargaining studies, participants freely negotiate an outcome before a time limit, and bargaining typically takes place in a single dimension, how to split a fixed pie. An exception is DLP(2021), which as mentioned above is the closest to this paper, where subjects simultaneously decide on both the size of the pie and how to divide it under exogenous liquidity constraint treatments.

<sup>&</sup>lt;sup>4</sup>Binmore (2007) for a defense of the structured approach and a summary of his extensive work.

#### **3.2.2** Joint Production Experiments

Literature on joint-production is undoubtedly related, since the main idea is that noncooperative bargaining occurs among players who have first incur some cost to jointly produce the pie to be latter split (Karagözoğlu (2012) for a survey). Nevertheless, there are important differences between the two-dimensional bargaining in this experiment and the joint-production framework: subjects in this experiment decide on the size of the pie and the division of that pie simultaneously, that is, there is no sequential structure. This is a key difference since given the sequential structure, once the size of the pie is determined, the costs are sunk so it should not theoretically matter, with research questions in that literature focusing on behavioral explanations (such as norms or equality concerns) of why differences in effort or cost to produce do empirically matter. The latter seems to differs considerably from the focus of this paper but since in this experiment buyer's decide on their costly money holdings before entering the bargaining stage, there is a sequential element and a sunk cost element, which makes the link with this literature more relevant.

#### 3.2.3 Consumption/Saving Experiments,

The above mentioned sequential element, of buyer's deciding on how much money to borrow before entering the bargaining stage, relates with the expansive literature on intertemporal dynamic (often stochastic) optimization problems, such as consumption/savings problems, which has has been repeatedly tested in economic experiments. An early reference is HEY and Dardanoni (1988). Prominent work in this area are Ballinger et al. (2011), Carbone and Hey (2004), as well as Carbone and Duffy (2014). See Kagel and A. Roth (2016) for a detailed survey of intertemporal consumption/savings experiments.

Somewhat unsurprisingly, the main finding of this literature is that subjects systematically deviate from optimal behavior, even when confronted with relatively simple tasks.

# 3.2.4 Experimental Macroeconomics with an Explicit Role for Money

This paper also belongs - although more due to its inspiration and implications than mechanicsto the family of experimental implementations with an explicit role for money.

Marimon and S. Sunder (1993); Marimon and S. Sunder (1994); Marimon and S. Sunder (1995), Lim, Prescott, and Shyam Sunder (1994), and Bernasconi and Kirchkamp (2000) implement an overlapping generations (OLG) model to study inflation where the role of money is as a store of value. O. Ochs and Duffy (1999); Duffy and J. Ochs (2002), and Berentsen, McBride, and Rocheteau (2017) analyze implementation where the role of money is as a medium of exchange.

Duffy and Puzzello (2014b); Duffy and Puzzello (2014a) were the first to implement a complete versions of Lagos-Wright model in the laboratory looking to understand the welfare consequences of the existence or introduction of fiat money in comparison to a gift-exchange economy sustained by a social norm (a contagious grim-trigger strategy played by all agents). Duffy and Puzzello (2022) investigate inflation and the Friedman Rule in in a similar setting. Note that they rely on a fairly structured approach: buyers make a take-it-or-leave offer which theoretically allows the first best to be achieved in that model (under the Friedman Rule). One way to think about my paper, is that it corresponds to the missing bargaining stage that would make for a more general version of the Lagos-Wright implementations of Duffy and Puzzello.

Anbarci, Dutu, and Feltovich (2015) study the effects of inflation in a version of the Lagos and Wright (2005) model with price posting as in Burdett, Shi, and Wright (2001). While they find inflation has a negative effects on production and welfare, subjects in their experiment make static choices, and the inflation is implemented by having buyers borrow money from a bank at a specified interest rate. In this paper I borrow the idea of buyer's having to borrow money at a certain interest rate (which can be thought of as inflation) before entering the trading stage which in Anbarci et al.(2015) corresponds to visiting one of the seller's that posted a price, while here trading is implemented via pairwise bargaining.

Jiang, Zhang, and Puzzello (2019) address some of these concerns by implementing a version of the Rocheteau and Wright (2005) money search model where pairwise meetings are replaced by a second anonymous competitive market where only a fraction can consume, while the rest can only produce, which renders money useful.

# 3.2.5 Applied Monetary with Explicit Price Formation Mechanisms

Last but not least, there is a relevant applied theoretical literature employing different price formation mechanism in monetary economics including *inter alia* generalized Nash and Kalai bargaining. Classical examples of this are Lagos and Wright (2005), Aruoba, Rocheteau, and Waller (2007) and Rocheteau and Wright (2005). These studies show that the efficiency of the monetary equilibrium, the welfare costs of inflation, and the impact of monetary policy greatly depend on the trading protocol and the bargaining weights. This experiments aims to provide additional evidence on which bargaining solution seems to better represent subjects behavior in a controlled setting with some of the main characteristics of the above mentioned studies such as costly money holdings and endogenous liquidity constrains.

#### 3.2.6 This Paper's Contribution

My paper aims to adds to the literature reviewed here in the following ways:

1) In this experiment, players endogenously and simultaneously determine both the size of

the pie (produced output), and how to divide that pie (payment from buyer to seller). Thus, departing from the one-dimensional paradigm, which to the best of my knowledge is only been attempt in an unstructured manner by DLP(2021).

2) I consider the role of endogenous liquidity constraints via costly money holdings. Before each pairwise meeting, buyer's must first decide how much money to bring to the bargaining stage, which then becomes a constraint on the payments they can make to their counterpart. The upside of this innovation is twofold: i) it allows me to differentiate between the bargaining solutions of Nash and Kalai using more than one metric, and ii) this setting retains one of the key elements of the applied monetary literature, which is costly money holdings.

## 3.3 Theoretical Framework

This simple bargaining model is inspired by both the "bargaining stage" between buyers and sellers and the notion that (under most conditions) it is costly to bring money balances to said stage in the monetary model of Lagos and Wright (2005).<sup>5</sup> This framework is part of a tradition which has one common specially desirable feature to study monetary phenomena: pairwise meetings with no record-keeping, commitment or monitoring technology which makes money -in Wallace (1998) terminology- essential. While in this version money is not strictly essential since it will have a redemption value, the spirit of anonymous pairwise meetings and costly money holdings is retained. Thus, it still captures many interesting bargaining situations where the outcome of the bargaining problem includes the size of the surplus, in addition to the surplus division between the two parties, with one key feature to monetary models: buyers decide their money holdings before the meeting, which is deemed as a sunk cost, and alters the bargaining space.

<sup>&</sup>lt;sup>5</sup>The most well known model of the third generation where money holdings are unbounded and goods are divisible, as opposed to second generation models where money holdings are binary.

The outline of the simple one period model<sup>6</sup> is as follows. The are two types of active agents: buyers (consumers) and sellers (producers), who meet anonymously in pairs and bargain over the quantity, q, to produce and trade of the only good in exchange for a certain payment, P, of tokens which can be thought of as money, and will at as the numéraire. As in third generation monetary models, units of the good and tokens are perfectly divisible. Information about utility and cost functions is complete.

Buyers, get utility u(q) from consuming q which they can't produce themselves. Since buyers can only work and get tokens at the end of the period, before each meeting, they can borrow  $m \leq \bar{m}$  tokens (money) at an exogenous interest rate r, which has to be paid back in full at the end of the period. I assume perfect enforcement for these loans (but not in pairwise meetings) and buyers work enough (with linear technology) at the end of the period, so there's no defaulting on debts. Buyers can offer  $P \leq m$  tokens in exchange for some amount q, produced by the seller. Seller incurs a cost c(q) from producing quantity q of the good, which is made on the spot and is conditional on the two parties reaching an agreement. Seller can then trade their tokens for their own consumption good at a one-to-one ratio and get linear utility out of it.

If an exchange of P tokens for q units takes place: the B=buyer's payoff is  $S^b = u(q) - y$ and seller's payoff is  $S^s = y - c(q)$ , such that the total surplus of the trade is equal to S = u(q) - c(q). If no agreement is reached, then the buyer and the seller get a payoff of zero. Utility and cost functions satisfy: u' > 0, u'' < 0, c' > 0, c'' > 0, u(0) = c(0) = 0. There exists  $\bar{q}$  such that  $u(\bar{q}) = c(\bar{q})$ , and  $q^*$  such that  $u'(q^*) = c'(q^*)$ . So,  $q^*$  maximizes the joint surplus in a pair. Figure 3.1 shows the the timeline of the period described above. Figure 3.2 shows the utility function of the buyer against the cost function of the seller using the parameterization of the experiment The bargaining problem is to choose  $q \in [0, \bar{q}]$ , and to choose  $P \leq m$ . A proposal is a (P, q) pair offered by either the buyer or the seller.

<sup>&</sup>lt;sup>6</sup>One could think of time as being discrete and both types being infinitely lived, but since there's no discounting or record keeping technology between sellers and buyers it is equivalent to a one period model.

Borrowing Stage	Bargaining Stage	Repayment Stage
- Buyers decide their money holdings, $m$	- Pairwise meetings b/w buyers and sellers	- Buyer works to payback loan: $\Rightarrow -m(1+r)$
- By borrowing up to $\bar{m}$ at an interest rate $r$	- Bargain over $\{q, P\}$ . If agreement is reached:	
	* Seller produces $q$ and receives $\mathbf{P} \Rightarrow \mathbf{P} + c(q)$	
	* Buyer pays $\mathbf{P} \leq m$ and consumes $q: \Rightarrow u(q)$	

Figure 3.1: Round Timeline

#### 3.3.1 Two Commonly Used Solutions to the Bargaining Problem

Given the environment, the equilibrium predictions depend on the bargaining protocol assumed. Here, I particularly focus on the axiomatic approaches of Nash and Kalai, where the main difference depends on one of the axioms<sup>7</sup>: Kalai's solution satisfies strong monotonicity (each player's surplus increases with the bargaining set) while Nash's does not.

As mentioned above, this difference is one of the features to be experimentally tested as it generates distinct bargaining outcomes when the token holdings the buyer brings to the pairwise meeting is an endogenous variable (even if money is not costly to hold) or if the buyer endogenously liquidity-constrained. In the following two sections I describe the outcomes under both different bargaining solutions and how it impacts the buyers decision on money holdings before the pairwise meeting.

#### 3.3.2 Generalized Nash Bargaining

Nash's (1950) solution satisfies the three following axioms: Pareto Optimality, Scale Invariance, and Independence of Irrelevant Alternatives. A Pareto Optimal solution implies that there is no attainable outcome that makes one player better off without making the other

<sup>&</sup>lt;sup>7</sup>There's also the issue of scale invariance in Nash which does not play a role in this experimental approach.



Figure 3.2: Buyer's consumption utility and seller's production cost

player worse off. Scale Invariance means that the solution is invariant to affine transformations of player's surplus. And Independence of Irrelevant Alternatives, implies that if some possible outcomes other than the solution are removed from the bargaining set, the solution remains the same.

The Nash solution is given by:

$$[q(m), P(m)] = \underset{q, P \le m}{\operatorname{argmax}} [u(q) - P]^{\theta_N} [P - c(q)]^{1 - \theta_N}$$

If  $P \leq m$  does NOT bind (i.e there's no liquidity constraint):

$$q = q^{\star}$$

$$P = m^{\star} = (1 - \theta_N)u(q^{\star}) + \theta_N c(q^{\star})$$

However, if  $m < m^*$ , then  $P \leq m$  does bind, P = m and q is the implicit solution to:

$$m \equiv z_N(q) = [1 - \Theta(q)]u(q) + \Theta(q)c(q)$$

where

$$\Theta(q) = \frac{\theta_N u'(q)}{\theta_N u'(q) + (1 - \theta_N)c'(q)}$$

Thus, the buyer's choice of balances is:

$$\max_{m} \left\{ -rm + u(q(m)) - P(m) \right\}$$

If r > 0 buyer will never bring more balances than his offer,  $P = m < m^*$ . So, using m = z(q), we have:

$$\max_{q} \left\{ -rz_{N}(q) + u(q(m)) - P(m) \right\}$$

So, the first order condition that determine the solution is:

$$\frac{u'(q)}{z'_N(q)} = 1 + r \tag{3.1}$$

Thus, when deciding their money holdings, buyers take into account the cost of holding money via the interest rate, but also that as q approaches  $q^*$  there are two opposite effects from the buyer's perspective: First, that the total surplus, u(q) - c(q), increases, and second that the buyers surplus share decreases.

#### 3.3.3 Kalai's Proportional Solution

Kalai's (1977) solution satisfies Pareto Optimality and Independence of Irrelevant Alternatives, same as Nash's, but imposes the additional Strong Monotonicity axiom and does not satisfy Scale Invariance. Strong monotonicity means no one can be made worse off as the bargaining set expands and total surplus increases, which as discussed previous sections, is an important result of the Nash solution in this context. The Kalai solution is given by the following optimization problem:

$$[q, P] = \underset{q, P \le m}{\operatorname{argmax}} \left[ u(q) - P \right]$$

subject to:  $u(q) - P = \frac{\theta}{1-\theta}[P - c(q)]$ 

Which can be rewritten as:

$$q = \operatorname*{argmax}_{q} \theta_{K}[u(q) - c(q)]$$

subject to:  $(1 - \theta_K)u(q) + \theta_K c(q) \le P$ 

If  $P \leq m$  does not bind (i.e there's no liquidity constraint) the solution would be the same as in Nash:

$$q = q^{\star}$$
$$P = m^{\star} = (1 - \theta_K)u(q^{\star}) + \theta_K c(q^{\star})$$

However, if  $m < m^*$ , then  $P \leq m$  does bind, P = m and q is the implicit solution to:

$$m \equiv z_K(q) = [1 - \theta]u(q) + \theta c(q)$$

Using  $P = m = z_K(q)$ , the buyer's choice of balances before the pairwise meeting in this case is:

$$\max_{q} \left\{ -rz_{K}(q) + u(q) - z_{K}(q) \right\}$$

So, the first order conditions that determine the solution are:
$$\frac{u'(q)}{z'_K(q)} = 1 + r \quad \leftrightarrow \quad \frac{u'(q) - c'(q)}{z'_K(q)} = \frac{r}{\theta}$$

$$(3.2)$$

Thus, similar to the Nash solution case, when deciding their money holdings, buyers take into account the cost of holding money via the interest rate, but here there are no opposite effects as q approaches  $q^*$  since total surplus, u(q) - c(q), increases but the buyers' fraction of surplus remains constant.

#### 3.3.4 Comparison

When money holdings are exogenous and the buyer is not liquidity constrained, as in Duffy et al. (2022), the bargaining parameters  $\theta_K$  and  $\theta_N$  can have the same interpretation as the fraction of the total surplus assigned to the buyer. However, when the buyer is liquidity constrained the two solutions will differ. Under Nash bargaining, the buyer spends all their money, so:

$$m = [1 - \Theta(q)]u(q) + \Theta(q)c(q), \text{ where } \Theta(q) = \frac{\theta_N u'(q)}{\theta_N u'(q) + (1 - \theta_N)c'(q)}$$

Under Kalai bargaining, we also have P = m, but q is determined by  $m = (1-\theta)u(q) + \theta c(q)$ . The two solutions differ so long as  $q < q^*$ , since  $\Theta(q)$  is a function of q while  $\theta$  is a constant.

Now in this model, where money holdings are decided by the buyer, the first equality result above does not really apply, because even when it is cost-less to hold money (i.e when the interest rate is zero) the buyer under the Nash solutions is aware that by bringing more money to the meeting they are increasing the bargaining set and thus decreasing their share of the surplus. The latter is not true under the Kalai solution where the buyer's share is a constant. This important difference implies that  $q^*$  is attainable when r = 0 under the Proportional Solution but not under Generalized Nash, since buyer's will never decide to bring enough balances in the latter.

This becomes clear when looking at the optimality conditions of the buyer's problem when r = 0. Under Nash bargaining their first order condition from ?? would be:

$$u'(q) = z'_N(q)$$

where

$$z_N(q) = [1 - \Theta(q)]u(q) + \Theta(q)c(q)$$

with

$$\Theta = \frac{\theta_N u'(q)}{\theta_N u'(q) + (1 - \theta_N)c'(q)}$$

Since  $\Theta(q^{\star}) = \theta$  and  $\Theta'(q) < 0 \,\forall q$  we have that

$$z'(q\star) = u'(q\star) - \Theta(q\star)[u(q\star) - c(q\star)] > u'(q\star)$$

which means that  $q < q^*$  is required for the condition to hold and the first best is not achieved and the buyer's surplus (not only the share of surplus) decreases as q approaches  $q^*$ 

Under Kalai, when r = 0, the FOC condition ?? reduces to:

$$u'(q) - c'(q)$$

which is solved by  $q = q^{\star}$ .

Figure 3.3 highlight the different predictions of the two solutions, which are the main focus of these experimental implementation (more on this in the parameterization section). The



Figure 3.3: Predictions about surplus distribution (left), terms of trade (middle) and output (right) under Nash and Kalai barganing for  $\theta = 1/2$ 

left panel shows the surplus split predictions given the selected parameterization for the three different interest rate levels I study: 0%, 30% and 60%. Notice that under the Kalai solution, when  $\theta = 1/2$ , the buyer's and seller's surpluses monotonically increase and the split is constant following  $\theta$ . While under the Nash solution, buyer's surplus is non-monotone and is always grater than that of the seller, but the decreasing portion is never reached under positive interest rates, since the buyer would never choose such high money balances. The center panel illustrates the counterpart in the payment-quantity space of the left, where for a given output the payment from the buyer to the seller is always grater under Kalai, and the quantities being produce and traded are bigger as well. The right panel depicts the money holdings decision of the buyer for different interest rates which underlay the quantities and payment that are then made in the bargaining stage. For all the interest rate levels relevant to this experiment, money holdings are considerably smaller under Nash.

## 3.4 Experimental Design

My experiment involves 3 treatments (3X1 experimental design), where the treatment variable is the interest rate at which buyers can borrow money before pairwise meetings, either

0%, 30% or 60%. I adopt a within subject design, that is each participant in exposed to a combination of 2 different rates, one for the first half of the session and one for the second half. The main outcome variables are the amount borrowed by the buyer, *m*, the quantities being trade, *q*, and the payments, *P*, over which buyers and sellers bargain.

#### 3.4.1 Parameterization

Following the relevant literature, there are multiple objectives and considerations when selecting the parameters for the model. Among those I considered the most relevant: (1) Choice associated with first best should not be too focal, so  $q^*$  should be off-center between 0 and  $q^*$ . (2) Significant slope on both sides of  $u(q^*) - c(q^*)$  so that the first best is salient. (3) Large enough differences between the Nash and Kalai solutions so that there's some chance of detecting those solutions in terms of the quantities, payments and/or money holdings for the different interest rate treatments. (4) Simple functional forms for u(q) and c(q). (5) Appropriate scale so that payoffs and tokens can be expressed in US\$, thus avoiding the use of points that need to be latter converted into US\$.

With these in mind, and keeping close to the related work of DLP(2021) to allow for easier comparisons, I chose:

$$u(q) = Aq^{a}$$
 with  $A = 7.41752$  and  $a = 0.6$ 

$$c(q) = Bq^{b}$$
 with  $B = 0.82324$  and  $b = 0.151678$ 

So for  $q^*, \bar{q}$  and  $u(q^*) - c(q^*)$  we have:

$$q^{\star} = 4, \bar{q} = 11$$
 and  $S^{\star} = u(q^{\star}) - c(q^{\star}) = 10.3$ 

So, under this parameterization, participants bargain over a surplus of up to \$10.3 in each

round.

#### 3.4.2 Treatments and Procedure

To determine the maximum amount of money,  $\bar{m}$ , a buyer would be allow to borrow and thus how much they can "work" (or are endowed) at the end of the period to repay their loan plus any interest, I consider the payment that a buyer would make to a seller if the first best  $q^*$  is reached and all of the surplus goes to that seller. That is,  $\bar{m} = u(q^*)$ , which equals to \$17.02. So, in order for buyer's to be able to carry enough balances to reach  $q^*$ , if they so decided, without making any prior assumptions about the bargaining solution or bargaining power, I set their decision space as  $m \in [0,32]$ . Moreover, this is enough so that even in the extreme case where the seller takes all of the surplus,  $\bar{q}$  is attainable.

Now, since the interest rate for the treatments, r, is either 0%, 30% or 60%, I set the endowment to be received<sup>8</sup> by buyers at the end of the period to 32\*r to avoid default. Note that this will always be enough given that for both buyers and sellers' proposals are restricted to avoid negative surpluses to either parties (more about this below)<sup>9</sup>. One appealing feature of having buyer's endowments be a function of interest rates is that it imperfectly counters the negative wealth effect of higher rates.

Other than the interest rate and and endowment, the only element changing between sessions is the number of participants, ranging form 8 to 12 depending on turnout. At the beginning of each 30 round session (28 paying rounds plus 2 practice rounds) participants are randomly

<sup>&</sup>lt;sup>8</sup>An alternative way of thinking about this would be that buyers inelastically supply a certain amount of work at the end of the period, which is priced depending on the interest rate

<sup>&</sup>lt;sup>9</sup>Two alternative approaches, which I quickly discarded during trial sessions, is (1) to keep the end of period endowment constant and restrict the borrowing space accordingly so that default is avoided or (2) to have a constant but large enough endowment so that regardless of the interest rate buyers can always repay their loans. The problem with (1) is that it mechanically restricts the borrowing space when the rate is higher and this may lead to results that are not explained by participants actual decisions, while (2) is a costly option since the endowment it applies to all treatments and there would be massive disparities in buyer's wealth between treatments

selected to be either buyers or sellers, and maintained that role for the entire session to make sure they are familiarized with their task.

At the beginning of each round, before entering the bargaining stage, buyers are tasked with deciding their money holdings for the round. As shown in Figure 3.4, which is a screenshot of the buyers interface, participants are informed about the prevailing interest rate <sup>10</sup>, the endowment they'll receive at the end of the round and are instructed to use a slider bar to decide on their money holdings<sup>11</sup>

#### **Round 1 of 28: Choose your Money Holdings**

You are a Buyer, for this round, you can borrow up to \$32.0 to bring to the bargaining stage where you'll meet with a Seller.

Remember, you'll have to pay the loan plus interests back using your payoff from trading, whatever you have left from your loan after paying the seller and your round endowment of \$19.2, which you receive only after the meeting.



Figure 3.4: Money holdings decision screen for buyers with an interest rate of 60%

By moving the slider, buyers decide their money holding, m, and receive information about the maximum amount of output they could request with it,  $\hat{q}$ , which corresponds to  $m = c(\hat{q})$ given that proposals are constrained to avoid negative surplus outcomes. They are also informed about the gross payoff,  $u(\hat{q})$ , they would receive if  $\hat{q}$  is accepted and their net payoff ,  $u(\hat{q}) - (1+r)m$ .

 $<sup>^{10}</sup>$ An alert pop-up window would appear before the first practice round, the first paying round and the  $15^{th}$  round as a reminder of the prevailing rate for the upcoming rounds

<sup>&</sup>lt;sup>11</sup>Figure 3.4 corresponds to the 60% interest rate case, other cases are similar with only the information regarding the rate and end of period endowment changing accordingly.

#### Time left to complete this page: 0:56

#### Round 1 of 28: You are a Buyer make a deal with a Seller

You are a **Buyer**. You can offer up to **\$10.0** to the Seller.

Make a New Proposal

Ente	Enter the <b>Quantity</b> you'd like to buy and the <b>Payment</b> that you'd like to offer: (see Payoff and Cost Graph)										
Use	this first s	slider to m	ake your	Output re	quest: 1.8 units						
											5.1 units
And this slider to select the <b>Total Payment</b> you'd like to offer for those units: \$3.2											
											\$10.0
			Outp	ut:	Buyer Payoff (you):	Sel Co:	ler st:	Total Paymen	ıt:	Buyer Payo Cost:	ff - Seller
Your	new prop	oosal:	• 1 L	l.8 Inits	• \$10.6	•	\$2.0	• \$3.:	2	• \$8.6	
Your propo	most rece osal:	ent	• 1	.8 units	• \$10.6	•	\$2.0	• \$3.2	2	• \$8.6	
					Send new	v propo	sal				
Activ	o Dror	ocale									
	Re	eceived	Proposa	ls from 9	Seller			M	lv Prop	osals	
	Output:	Buyer Payoff:	Seller Cost:	Total Payment	Buyer Payoff - Seller Cost:		Output:	Buyer Payoff:	Seller Cost:	Total Payment:	Buyer Payoff -Seller Cost:
#3	2.7 units	\$13.5	\$3.7	\$4.2	\$9.8 Accept	#2	1.8 units	\$10.6	\$2.0	\$3.2	\$8.6
#2	2.4 units	\$12.5	\$3.1	\$4.2	\$9.4 Accept	#1	1.8 units	\$10.6	\$2.0	\$3.1	\$8.6
#1	2.2	\$11.9	\$2.7	\$4.2	\$9.2	My ne	w proposals	stack at the to	p		

Figure 3.5: Bargaining screen for buyer with m =\$10

After every buyer has decided their money holding for the round, buyers and sellers are randomly and anonymously paired and tasked with bargaining over q and P within T = 2minutes. Bargaining was largely unstructured, with the the time limit and the requirement that proposals consist of (P, q) pairs, such that their surplus is non-negative for both buyer and seller<sup>12</sup>. Because of those constraints I use the term semi-unstructured bargaining. Figure 3.5 shows the interface for a buyer which is exactly the same regardless of the interest rate (the seller's interface is almost identical). There a are two sliders at the top of the screen, one for output request, y, and one for the payment offered, P. By moving these sliders subjects are informed about the the payoffs associated with their offer if accepted,

ved proposals stack at the top

<sup>&</sup>lt;sup>12</sup>That is (P,q) are restricted so that  $u(q) - P \ge 0$  and  $P - c(q) \ge 0$ 

in particular they would see the buyers payoff, u(q), the seller's cost, c(q), the payment, P, and total surplus, u(q) - c(q).

All payoffs were denoted in dollars and subjects understood that their monetary payoffs were directly reflected by those values (which avoids the need to use "points", which are latter converted<sup>13</sup>). Note that any money that a buyer brings to bargaining stage and is not used as payment still has value at the end of the round, where it can be then used to repay their loan and add to their payoff. So, the interest rate is the unique device in this setup that tries to capture the idea that money is costly to hold given the redemption value, and thus it is not really a fiat object.

Once a participant decides on a proposal, they would click on 'Send new proposal" to make it available to their counterpart. Once a proposal was sent, it could not be withdrawn and it would be add to the top of the received proposals list, so a player's counterpart can accept a proposal at any time by clicking on the green "Accept" button next to it. Note that the q slider is constrained so that the highest output request that can be made by a buyer is c(q) = m, while the P slider has a maximum value of m for both sellers and buyers. Moreover, as previously mentioned, proposals are constrained so that player's surplus is nonnegative, which is implemented by a pop-up message suggesting to either increase (decrease) P or decrease (increase) q, whenever a player attempts such a proposal. (see Appendix B)

If many proposals were sent in a round, a scroll-bar allowed subjects to review them all. Proposals are only relevant to the round/counterpart context in which they were sent, so naturally the set of proposals was cleared out for each new round. A round ends when either a proposal was accepted or the 2 minute went by, whichever came first.

 $<sup>^{13}</sup>$ Given the model's parameterization this means participants bargain for a pie of up to \$10.3

#### 3.4.3 Hypotheses

Table 3.1 provides predictions for m (and thus P), q, seller and buyer's surpluses,  $S_s$  and  $S_b$ , the total surplus, S, and  $S_b/S$  under the Nash and Kalai bargaining solutions for all three interest rate values: r = 0%, r = 30% and r = 60%.

Theoretical predictions of the model which are at the heart of my hypotheses will depend on the bargaining weight,  $\theta$ . Following the applied monetary literature that inspired this experimental implementation, I assume  $\theta$  to be a primitive of the model which is exogenous and fixed, so it does not enter explicitly as a parameter affecting choices anywhere in the game. An additional reason for not pursuing a model with a varying value for  $\theta$  is that it would involve adding more structure to the bargaining game and complicate interpretations across liquidity treatments.

In particular I decide to base all of my predictions on  $\theta = 1/2$ , that is, assuming symmetry. This decision is based first on the results obtain by DLP(2021) who, as mentioned in the literature review section, have a similar unstructured bargaining experiment with exogenous money holdings and find that when participants are not liquidity constraint -and thus the Nash and Kalai solution coincide- the data is best described by theta = 1/2. Moreover, as suggested by existing theories, there's no reason to believe that  $\theta$  would change as liquidity constraints (either endogenous or exogenous) become binding Thus, DLP(2021) results should apply to this setting as well. And second, the literature that targets retail markups (ratio of price to marginal cost) using United States data, where a bargaining weight of 1/2 can be thought of as compromise between different results. Lagos and Wright (2005), whose bargaining setup inspires this experiment, uses the latter method in a model imposing the Nash bargaining solution and estimate the consumer's bargaining power to be between 0.315 and 0.404. In a closely related model under Nash, Aruoba, Waller and Wright (2011) obtains a value of 0.92. Using Kalai bargaining, Bethune, Choi and Wright. (20120) obtain an estimate of 0.72, while Venkateswaran and Wright (2013) get between 0.68 and 0.86, while Davoodalhosseini (2021) between 0.75 and 0.87.

r = 0%	m, P	q	$S_b$	$S_s$	S	$\frac{S_b}{S}$
Nash	5.96	2.16	5.82	3.31	9.13	64%
Kalai	11.87	4.00	5.15	5.15	10.30	50%
r = 30%	m, P	q	$S_b$	$S_s$	S	$\frac{S_b}{S}$
Nash	3.08	1.27	5.46	3.08	7.37	74%
Kalai	6.89	2.04	4.47	4.47	8.94	50%
r = 60%	m, P	q	$S_b$	$S_s$	S	$\frac{S_b}{S}$
Nash	1.95	0.89	4.98	1.26	6.23	80%
Kalai	3.78	0.88	3.10	3.10	6.20	50%

Table 3.1: Theoretical predictions for Nash and Kalai with  $\theta = 1/2$ 

Based on the theoretical model my hypotheses are as follows:

Hypothesis 1: As r increase, m decreases.

Money holdings should decrease as they become more costly to hold.

Hypothesis 2: If r > 0 then P = m.

Whenever money is costly to hold, buyers should spend all of their holdings in each round.

**Hypothesis 3:** As r increases, traded output q, payments P, total surplus S, and both seller's and buyer's surplus  $S_s$  and  $S_b$ , all decrease.

**Hypothesis 3 alternative:** As m increases, regardless of r traded output q, payments P, total surplus S, and both seller's and buyer's surplus  $S_s$  and  $S_b$ , all increase.

Hypotheses 1, 2 and 3 should hold for both bargaining solutions and are independent of *theta*. While Hypotheses 1 and 2 aim to check whether buyers respond to the interest rate, by economizing on money holdings, Hypothesis 3 goal is to more generally test if the bargaining results are consistent with the predictions for a more constrained situation<sup>14</sup>.

 $<sup>^{14}\</sup>mathrm{Note}$  that the first 3 hypotheses do not depend on a particular value for  $\theta.$ 

The following hypotheses are based on the particulars of each bargaining solution, and thus help disentangle which is the more relevant empirically in this context. The are more predictions for the variables that could be tested, but the ones below present the clearest opportunities to test in the data.

Hypothesis 4a: m (and P) under Nash are equal to  $m = \{5.96, 3.08, 1.95\}$  for r = 0%, r = 30% and r = 60%, respectively.

Hypothesis 4b: m (and P) under Kalai are equal to  $m = \{11.87, 6.89, 3.78\}$  for r = 0%, r = 30% and r = 60%, respectively.

Money holdings (and Payments) under Nash are lower than under Kalai for all three values of r. Buyers under Nash should take the non-monotonicity under account and thus carry fewer balances even when it is not costly to hold them.

Hypothesis 5a: q under Nash is equal to  $q = \{2.16, 1.27, 0.89\}$  for r = 0%, r = 30% and r = 60%, respectively.

**Hypothesis 5b:** q under Kalai is equal to  $q = \{4.0, 2.04, 0.88\}$  for r = 0%, r = 30%and r = 60%, respectively. Output is lower under Nash for r = 0% and r = 30%than Kalai and practically indistinguishable for r = 60% given my parameterization. Buyers under Nash carry fewer balances but output is almost the same under r = 60%due to the change in surplus that goes to the buyer.

Hypothesis 6a:  $S_b/S$  equals  $S_b/S = \{64\%, 74\%, 80\%\}$  under Nash for r = 0%, r = 30% and r = 60%, respectively.

Hypothesis 6b:  $S_b/S$  equals  $S_b/S = 50\%$  under Kalai for all three values of r.  $S_b/S$  is higher under Nash than Kalai for all three values of r, and though the absolute surplus is decreasing with r for both bargaining solutions the fraction increases under Nash while remaining constant under Kalai. Besides the hypotheses above, I present some additional questions that don't directly arise from the theoretical model but that are relevant for experiments on bargaining and experimental economics in general, and for which the data here generated may be useful. Since I don't have a model to formally address them I just pose them as interesting inquiries:

- 1. Additional Question 1: How is the bargaining result achieved in practice? Since I'm able to record all of the proposals leading to an agreement, I can explore this in the data. For example, do participants start out with proposals that are highly favorable to them and slowly converge to an agreement or do they go straight to making an offer that is acceptable to their counterpart? If the bargaining is more gradual, do participants make proposals for a certain output (the optimal q) and test different surplus splits or is there another pattern?
- 2. Additional Question 2: Are results from a fully remote implementation any different than an in-person implementation? Half of the sessions for this experiment where conducted in person, while the other half was conducted fully remotely trough Zoom.

## 3.5 Experimental Results

In this section I present general information about the sessions and subjects, and then comment on the results in relation to my hypotheses and questions.

## 3.5.1 Procedure and Sessions

This experiment was conducted both online and in person using the Otree Python framework (Chen, Schonger, and Wickens, 2016). In person person sessions were conducted over the local network of computers at the Experimental Social Science Laboratory at UC Irvine, while online session where conducted trough Zoom. Recruited subjects had no prior experience with this experiment and were drawn from the undergrad population of UC Irvine, using the Sona Systems software. At the beginning of each 90 minute session, subjects had to go over web based instructions which were also read out loud. After that they had to successfully complete a brief comprehension test before proceeding to two practice rounds, which were identical to actual paying rounds but did not count towards the participants compensation, explanations regarding some of the features of the interface were given and questions were allowed<sup>15</sup>. Instructions and comprehension test questions are found in Appendix B.

Session	Treatment	Format	Participants	Avg. Earnings	Max. Earnings	Min. Earnings
1-	r = 0%, r = 60%	ESSL	10	\$21.82	\$30.53	\$15.00
2-	r = 60%, r = 0%	ESSL	12	\$20.10	\$34.71	\$10.00
3-	r = 0%, r = 30%	ESSL	10	\$20.86	\$26.63	\$14.33
4-	r = 30%, r = 0%	ESSL	8	\$16.43	\$18.70	\$14.93
5-	r = 30%, r = 60%	ESSL	10	\$22.13	\$34.80	\$15.44
6-	r = 60%, r = 30%	ESSL	10	\$22.16	\$36.62	\$15.00
7-	r = 0%, r = 60%	Online	12	\$20.43	\$35.30	\$14.79
8-	r = 60%, r = 0%	Online	8	\$18.25	\$28.76	\$10.00
9-	r = 0%, r = 30%	Online	10	\$18.91	\$23.72	\$10.68
10-	r = 30%, r = 0%	Online	10	\$16.78	\$24.21	\$10.00
11-	r = 30%, r = 60%	Online	12	\$21.79	\$34.94	\$15.00
12-	r = 60%, r = 30%	Online	12	\$19.51	\$33.26	\$14.50

Table 3.2: Characteristics of Sessions

Following the instruction, comprehension test and practice rounds, which took about 30 minutes, the remaining time was devoted to the 28 actual paying rounds. Subjects were explained that their payoff in one of their paying rounds would be selected at random and count toward their monetary compensation, plus a show up payment of \$10. Table 3.2 is a general summary of the sessions including their size, interest rate treatment, payment information and whether it was conducted in person or online. Each session involved between 8 and 12 inexperienced subjects, thus data comes from 124 subjects in 28 rounds.

<sup>&</sup>lt;sup>15</sup>Regardless of the interest rate treatment I set r = 10% for all practice rounds

#### 3.5.2 Results Overview

Table 3.3 and 3.4 show the mean and median values, respectively, for the 1736 money holdings decisions and 1464 agreements, which implies an overall agreement rate of  $84.3\%^{16}$ . A first comparison of those two tables with Table 3.1, which reports the theoretical predictions for  $\theta = 1/2$ , is useful as a starting point for the overall analysis, which tends to show only mixed support for my hypotheses.

Table 3.3: Average money holdings of buyers and agreed outcomes by interest rate

	m	P	m-P	q	S	$S_b$	$S_s$	$\frac{S_b}{S}$
r = 0%	11.53	8.72	2.80	3.16	9.28	5.68	3.60	60.89%
r = 30%	7.99	6.32	1.62	2.49	8.75	6.08	2.67	69.15%
r=60%	7.50	6.23	1.23	2.41	8.53	5.83	2.70	68.62%

On the one hand, I find relative strong support for the more general Hypotheses 1 and 2, which are related to the borrowing cost. The costlier it is to bring money holdings to the meeting, the fewer balances subjects are going to carry, and subjects tend to bring costly balances only if they intend on using them, which are patterns that can be both observed in the average and median data. The same roughly applies to Hypothesis 3, which states that higher interest rate should negatively correlate with output being trade, total surplus and both buyer's and seller's surplus.

On the other hand, while money holdings for different interest rates tend to be closer to the Kalai Bargaining predictions, which is evidence in favor of Hypothesis 4a, the surplus share of buyers is closer to what one would expect to see under Nash since it appears to be increasing in r. In what follows I provide a more rigorous analysis of the data.

 $<sup>^{16}</sup>$ Agreement rate for r = 0%, r = 30% and r = 60% is 86.8%, 80.7% and 85.5\%, respectively.

	m	P	m - P	q	S	$S_b$	$S_s$	$\frac{S_b}{S}$
r = 0%	9.00	7.50	0.6	3.00	9.75	5.78	3.17	64%
r = 30%	5.80	5.00	0.4	2.20	9.18	6.11	2.27	73%
r = 60%	5.60	4.50	0.4	2.20	9.04	6.08	2.05	72%

Table 3.4: Median money holdings of buyers and agreed outcomes by interest rate

#### 3.5.3 Money Holdings and the Interest Rate

Figures 3.6 compares the average (and median) money holdings for all three interest rate treatments. A first conclusion to be drawn is that the interest rate does affect money holdings as theory suggests, though the magnitude, especially when r = 60% is above what is to be expected (for any of the two possible bargaining solutions studied here). Figure ?? left panel illustrates the amount of dispersion for money holdings decisions, as is to be expected in any experimental design, but also provides additional support for the idea that at least the order of average and mean money holdings comports according to predictions. Something supported by the probability distribution kernel estimated from the data, on the left panel of Figure 3.7.

**Finding 1:** Consistent with Hypothesis 1, as r increases buyers economize on the money holdings they bring to the bargaining stage.

More rigorous support for the negative effect of higher interest rates on money holdings is found on Table 3.5 which reports in its top half the results from the non-parametric Jonckheere Test for ordered alternatives and the Wilcoxon Rank Sum Test to compare pairs of distribution medians, using treatment-level data over all periods. Jonckheere tests for the null hypothesis that population medians for money holdings for each treatment of r are the same, that is,  $H_0: \tilde{m}_{r0} = \tilde{m}_{r30} = \tilde{m}_{r60}$ , against the ordered alternative hypothesis predicted by theory:  $H_A: \tilde{m}_{r0} \leq \tilde{m}_{r30} \leq \tilde{m}_{r60}$ , with at least one strict inequality. Wilcoxon Rank Sum (also known as Mann Whitney) tests for comparisons of medians in pairs of interest



Figure 3.6: Average (and median) money holdings with 95% confidence intervals for all three interest rate treatments plus predicted values for both the Nash and Kalai solutions, with letters N and K, respectively.

rate treatments at treatment-level data over all periods, that is  $H_0 : \tilde{m}_{r0} = m_{r30}$ , against  $H_A : \tilde{m}_{r0} \leq \tilde{m}_{r30}$ , and  $H_0 : \tilde{m}_{r30} = \tilde{m}_{r60}$ , against  $H_A : \tilde{m}_{r30} \leq \tilde{m}_{r30}$ . I can reject the null hypothesis in favor of the alternative for both tests, supporting the idea of the effect of r on m.

In the bottom half of Table 3.5 I present a results for the following fixed effects regression with dummy variables for the treatments,  $\beta_{r=30\%}$  and  $\beta_{r=60\%}$ :

$$m_{it} = C + \beta_1 \mathbb{1}_{r=30\%} + \beta_2 \mathbb{1}_{r=60\%} + \alpha_i + \omega_t + \varepsilon_i t$$

where *i* indexes for the individual buyer and *t* for the round number, so  $alpha_i$  is the individual effect,  $\omega_t$  is the time effect and  $\varepsilon_{it}$  the error term. Estimations for both dummy variables, which are significant and negative ( $\beta_1 = -3.08$  and  $\beta_2 = -4.178$ ), again support for the idea that money holdings decrease with  $r^{17}$  <sup>18</sup>.

<sup>&</sup>lt;sup>17</sup>Pairwise comparison of treatment level means using simple t-tests provides only partial support: the null  $\mu_{r=0\%} = \mu_{r=30\%}$  can be easily rejected, while  $\mu_{r=30\%} = \mu_{r=60\%}$  can not.

<sup>&</sup>lt;sup>18</sup>Results hold for regressions with individual buyer fixed effects only, time effects only and for random



Figure 3.7: Left panel: Money holdings distribution with predictions by interest rate for Nash and Kalai with letters N and K, respectively. Right panel: Probability distribution kernel estimation for money holdings.

A related point is whether buyers spend all of their money holdings in the bargaining stage when an agreement is reached, specially when money is costly to hold, that is when r > 0%. Tables 3.3 and 3.4 suggest that for r = 30% and r = 60% it is not the case, though they magnitude would decrease with r.

**Finding 2:** Contrary to hypothesis 2, even if r > 0 buyers do no spent all of their money holdings in the bargaining stage when they reach an agreement.

Table 3.6 presents some test results for the distribution of non-spent money holdings,  $m^e = m - P$ , focusing on the cases where r > 0%. On the non-parametric side it includes a Jonckheere test of ordered medians, to test whether non-spent money holdings decrease with r, and the Wilcox Sum Rank test for the null  $\tilde{m}^e_{r30\%} = 0$  versus the alternative  $\tilde{m}^e_{r30\%} \ge 0$  and  $\tilde{m}^e_{r60\%} = 0$ . I can reject the null for the ordered median test and the individual tests of medians comparing with zero, which is evidence that money holdings are not completely spent and that they are affected by r.

effects as well.

Test	Hypotheses: $H_0 - H_A$	Results
Ionakhooro	$H_0: \tilde{m}_{r0} = \tilde{m}_{r30} = \tilde{m}_{r60}$	Reject $H_0$
JOHCKHEELE	$H_A: \tilde{m}_{r0} \ge \tilde{m}_{r30} \ge \tilde{m}_{r60}$	p= .0000
	$H_0: \tilde{m}_{r0} = m_{r30}$	Reject $H_0$
Wilcox	$H_A: \tilde{m}_{r0} \ge \tilde{m}_{r30}$	p=.0000
Rank Sum	$H_0: \tilde{m}_{r30} = m_{r60}$	Reject $H_0$
	$H_A: \tilde{m}_{r30} \ge \tilde{m}_{r60}$	p=.0044
	$H_0: C = 0$	Reject $H_0$
	$H_A: C \neq 0$	p=.0000
Rogrossion	$H_0:\beta_1 \mathbb{1}_{r30}=0$	Reject $H_0$
Regression	$H_A:\beta_1 \mathbb{1}_{r30} \neq 0$	p=.0000
	$H_0: \beta_2 \mathbb{1}_{r60} = 0$	Reject $H_0$
	$H_A:\beta_2 \mathbb{1}_{r60} \neq 0$	p=.0000

Table 3.5: Results and p-values for buyer's money holdings tests

On the parametric side, it includes a regression similar to the one used for money holdings but with non-spent money holdings,  $m^e$ , on the right-hand-side and restricting to observation where an agreement is reached. The intercept in the regression is significant and positive while both  $\beta_1 = -1.07$  and  $\beta_2 = 1.62$  are negative and significant<sup>1920</sup>.

Test	Hypotheses: $H_0 - H_A$	Results
Ionalthaara	$H_0: \tilde{m^e}_{r0} = \tilde{m^e}_{r30} = \tilde{m^e}_{r60}$	Reject $H_0$
Jonekneere	$H_A: \tilde{m^e}_{r0} \ge \tilde{m^e}_{r30} \ge \tilde{m^e}_{r60}$	p = .0003
	$H_0: \tilde{m^e}_{r0} = 0$	Reject $H_0$
Wilcox	$H_A: \tilde{m^e}_{r0} \ge 0$	p=.0000
Rank Sum	$H_0: \tilde{m^e}_{r30} = 0$	Reject $H_0$
	$H_A: \tilde{m^e}_{r30} \ge 0$	p=.0000
	$H_0: C = 0$	Reject $H_0$
	$H_A: C \neq 0$	p = .0004
Regression	$H_0: \beta_1 \mathbb{1}_{r30} = 0$	Reject $H_0$
rtegression	$H_A:\beta_1 \mathbb{1}_{r30} \neq 0$	p=.0000
	$H_0: \beta_2 \mathbb{1}_{r60} = 0$	Reject $H_0$
	$H_A:\beta_2 \mathbb{1}_{r60} \neq 0$	p=.0000

Table 3.6: Results and p-values for buyer's non-spent money holdings tests

<sup>&</sup>lt;sup>19</sup>Pairwise comparison of treatment level means using simple t-tests provides additional evidence for positive non-spent money holdings: both null hypotheses  $\mu_{r=30\%} = 0$   $\mu_{r=60\%} = 0$  can not be rejected.

 $<sup>^{20}\</sup>mathrm{Results}$  hold for regressions with individual buyer fixed effects only, time effects only and for random effects as well.

### 3.5.4 Output, Payments, Surplus and the Interest Rate

Figure 3.8 shows the average (and median) agreed-upon output q, payment P, and buyer's and seller's surplus for all three interest rate treatment (the complete data distribution for all variables can be found in Appendix B. The top two panels of Figure 3.8, as well as Table 3.3 and 3.4 suggest that while both agreed quantities and payments decrease with higher interest rates, the magnitude of the decrease does not conform to theory predictions. This does not come as a surprise, since as I showed in the previous section, money holdings brought by buyer's to the bargaining stage in each round when r > 0 tend to be above what both the Nash and Kalai solutions predict. The bottom panels of Figure 3.8 paint a less favorable picture: buyer's surplus is clearly not decreasing in r as theory predicts, while seller's surplus is indeed decreasing but way below what any of the two presented solutions anticipate.

**Finding 3:** Consistent with Hypothesis 3, traded quantity, payments and seller's surplus decrease with r, but contrary to theory buyer surplus increases with r



Figure 3.8: Average (and median) agreed output, payment, buyer and seller surplus with 95% confidence intervals for all three interest rate treatments plus predicted values for both the Nash and Kalai solutions, with letters N and K, respectively.

As in the previous subsections, in what follows I present more detailed arguments for **Finding 3**, by first applying the same non-parametric test of ordered medians to treatment aggregated data for each of the variables and then by running the time and individual fixed effects regression for each. Results for both tests are presented in Tables 3.7 and 3.8, where I can reject the null for agreed-upon output, payments and seller's surplus but fail to reject in the case of the buyer's surplus.

Table 3.7: Results and p-values for Jonckheere test for agreed output, payment, buyer's surplus, seller's surplus and buyer's surplus share

Variable	Hypotheses: $H_0 - H_A$	Results
Output a	$H_0: \tilde{q}_{r0} = \tilde{q}_{r30} = \tilde{q}_{r60}$	Reject $H_0$
	$H_A: \tilde{q}_{r0} \ge \tilde{q}_{r30} \le \tilde{q}_{r60}$	p=.0000
Paymont P	$H_0: \tilde{P}_{r0} = \tilde{P}_{r30} = \tilde{P}_{r60}$	Reject $H_0$
1 ayinent, 1	$H_A: \tilde{P}_{r0} \ge \tilde{P}_{r30} \le \tilde{P}_{r60}$	p=.0000
Buyor Surplus S.	$H_0: \tilde{S}_{br0} = \tilde{S}_{br30} = \tilde{S}_{br60}$	Fail to Reject $H_0$
Duyer Surplus, $D_b$	$H_A: \tilde{S}_{br0} \ge \tilde{S}_{br30} \le \tilde{S}_{br60}$	p = .7018
Sollor Surplus S	$H_0: \tilde{S}_{sr0} = \tilde{S}_{sr30} = \tilde{S}_{sr60}$	Reject $H_0$
Seller Surplus, $S_s$	$H_A: \tilde{S}_{sr0} \ge \tilde{S}_{sr30} \le \tilde{S}_{sr60}$	p=.0000
Buyer Surplus	$H_0: \tilde{S_b/S_{r0}} = \tilde{S_b/S_{r30}} = \tilde{S_b/S_{r60}}$	Reject $H_0$
Share, $S_b/S$	$H_A: \tilde{S_b/S_{r0}} \le \tilde{S_s/S_{r30}} \le \tilde{S_s/S_{r60}}$	p=.0000

Table 3.8: Results and p-values for fixed effects regression for agreed output, payment, buyers surplus, seller surplus and total surplus

Varial	ple Estimated Value	$H_0: C = 0$ $H_A: C \neq 0$	$H_0: \beta_1 1_{r30} = 0 H_A: \beta_1 1_{r30} \neq 0$	$H_0: \beta_2 \mathbb{1}_{r60} = 0$ $H_A: \beta_2 \mathbb{1}_{r60} \neq$
q	$\hat{C} = 3.21,  \hat{\beta_1} = -0.54  ,  \hat{\beta_2} = -0.74$	Reject $H_0$ p=.0000	Reject $H_0$ p=.0000	Reject $H_0$ p=.0000
Р	$\hat{C} = 6.07,  \hat{\beta_1} = -1.84  ,  \hat{\beta_2} = -2.46$	Reject $H_0$ p=.0000	Reject $H_0$ p=.0000	Reject $H_0$ p=.0000
$S_b$	$\hat{C} = 8.85,  \hat{\beta_1} = 0.33  ,  \hat{\beta_2} = 0.29$	Reject $H_0$ p=.0000	Reject $H_0$ p=.0185	Reject $H_0$ p=.0265
$S_s$	$\hat{C} = 1.13,  \hat{\beta_1} = -0.58  ,  \hat{\beta_2} = -0.79$	Reject $H_0$ p=.0025	Reject $H_0$ p=.0000	Reject $H_0$ p=.0000
$S_b/S$	$\hat{C} = 0.89,  \hat{\beta_1} = 0.06  ,  \hat{\beta_2} = 0.07$	Reject $H_0$ p=.0000	Reject $H_0$ p=.00000	Reject $H_0$ p=.0000

#### 3.5.5 Bargaining Solution and the Data

All of the above still leaves a key issue unanswered: which bargaining solution best describes the data? The answer is not clear though: while average money holdings brought by buyers to the bargaining stage appear somewhat closer to the Kalai predictions (see Subsection 5.3), specially for r = 0% and r = 30%, payments and output (and thus surplus) are more ambiguous. This can be partially explained by buyers deciding to bring way more balances than either of the bargaining solutions would predict, specially when r = 60%, but then not spending all of them (specially when r = 0%). This leads to payments and quantities that are in-between both predictions for r = 0%, but above when r = 60%. Tables 3.9 and 3.10 illustrate this using non-parametric (Wilcox Rank Sum) and parametric (t-test) one-sample tests at treatment data level taking the Nash and Kalai predictions as null hypothesis against the two sided-alternative. Most of the tests tend to reject the null hypothesis (i.e the model predictions) with only a few exceptions

**Finding 4:** Contrary to Hypothesis 4a (Nash) and partially in favor of Hypothesis 4b (Kalai), money holdings are relatively closer to Kalai predictions, specially for r = 0% and r = 30%.

**Finding 5:** Contrary to either Hypothesis 5a and 5b quantities and payments (and participants surplus) tend to significantly differ from model predictions.

One last interesting piece of evidence in favor of the Nash solution is, as shown in Figure 3.9, buyer's share of surplus seems to be increasing in r%, which is a distinctive feature under this parameterization when compared with the constant share of surplus expected to be observed under Kalai.

**Finding 6:** Consistent with Hypothesis 6a (Nash), and contrary to Hypothesis 6b (Kalai), buyer's surplus share increases with r.

The bottom row of Table 3.7 and 3.8 provide more rigorous support: for both the test of

Table 3.9: Results and p-values for Wilcox Rank Sum test for money holdings and agreed output, payment, and buyer's surplus share

Varial	r = 0	0%	r =	= 30%	r =	60%
varia	Nash $(H_0)$	Kalai $(H_0)$	Nash $(H_0)$	Kalai $(H_0)$	Nash $(H_0)$	Kalai $(H_0)$
	m = 5.96	m = 11.87	m = 3.08	m = 6.89	m = 1.95	m = 3.78
m	Reject $H_0$	Reject $H_0$	Reject $H_0$	Fail to rej. $H_0$	Reject $H_0$	Reject $H_0$
	p = .0000	p = .0013	p = .0000	p = .363	p = .0000	p = .0000
	P = 5.96	P11.87 =	P = 3.09	P = 6.89	P = 1.95	P = 3.78
P	Reject $H_0$	Reject $H_0$	Reject $H_0$	Reject $H_0$	Reject $H_0$	Reject $H_0$
	p = .0000	p = .0000	p = .000	p = .0000	p = .0000	p = .0000
	q = 2.16	q = 4.00	q = 1.27	q = 2.04	q = 0.89	q = 0.88
q	Reject $H_0$	Reject $H_0$	Reject $H_0$	Reject $H_0$	Reject $H_0$	Reject $H_0$
	p = .0000	p = .0000	p = .0000	p = .0000	p = .0000	p = .0000
	$S_b/S = 0.64$	$S_b/S = 0.50$	$S_b/S = 0.74$	$S_b/S = 0.50$	$S_b/S = 0.80$	$S_b/S = 0.50$
$S_b/S$	Fail to rej. $H_0$	Reject $H_0$	Reject $H_0$	Reject $H_0$	Reject $H_0$	Reject $H_0$
	p = .1462.	p = .0000	p = .0006	p = .0000	p = .0000	p = .0000

ordered medians and the fixed effects regression the null hypothesis of the buyer's surplus being constant is easily rejected.

# 3.6 Two Alternative Models

In this section I present two alternative models that can help understand why experimental results seam to deviate from the standard theoretical predictions.

## 3.6.1 Money Holdings as Given - Extremely Myopic Buyer's

As stated in the standard model, the costly decision of how much money to bring to the bargaining stage is inseparable from the bargaining predictions, as r is part of the optimality conditions. In other words, when buyer's decide on their marginal money holdings they take into account their expected bargaining results (which are in turn affected by the buyers money holdings). As a thought experiment, that can help shed some light on the data, one

Varial	r	= 0%	r =	30%	r = 60%		
varia	Nash $(H_0)$	Kalai $(H_0)$	Nash $(H_0)$	Kalai $(H_0)$	Nash $(H_0)$	Kalai $(H_0)$	
	m = 5.96	m = 11.87	m = 3.08	m = 6.89	m = 1.95	m = 3.78	
m	Reject $H_0$	Fail to Rej. $H_0$	Reject $H_0$	Reject $H_0$	Reject $H_0$	Reject $H_0$	
	p = .0000	p = .0000	p = .0000	p = .0000	p = .0000	p = .0000	
	P = 5.96	P11.87 =	P = 3.09	P = 6.89	P = 1.95	P = 3.78	
P	Reject $H_0$	Reject $H_0$	Reject $H_0$	Reject $H_0$	Reject $H_0$	Reject $H_0$	
	p = .0000	p = .0000	p = .0000	p = .0018	p = .0000	p = .0000	
	q = 2.16	q = 4.00	q = 1.27	q = 2.04	q = 0.89	q = 0.88	
q	Reject $H_0$	Reject $H_0$	Reject $H_0$	Reject $H_0$	Reject $H_0$	Reject $H_0$	
	p = .0000	p = .0000	p = .0000	p = .0000	p = .0000	p = .0000	
	$S_b/S = 0.64$	$S_b/S = 0.50$	$S_b/S = 0.74$	$S_b/S = 0.50$	$S_b/S = 0.80$	$S_b/S = 0.50$	
$S_b/S$	Reject $H_0$	Reject $H_0$	Reject $H_0$	Reject $H_0$	Reject $H_0$	Reject $H_0$	
	p = .0063	p = .0000	p = .0000	p = .0000	p = .0000	p = .0000	

Table 3.10: Results and p-values for t-tests for money holdings and agreed output, payment, and buyer's surplus share

can think of the following alternative model: buyers decide their money holdings with the expectation of later trading a certain amount and considering the interest rate<sup>21</sup>, but without taking into account their impact on the bargaining set and its distribution, because they are extremely shortsighted, misinformed or don't understand the relationship, and thus money holdings effectively act as an exogenous constraint in the bargaining stage.

Thus, when m is taken as exogenous and using the same notation as in **Subsection 3.2**, both the Kalai and Nash solutions coincide for the unconstrained case, that is, when  $m > m^* = (1 - \theta)u(q^*) + \theta c(q^*)$ . When  $m < m^*$  money holdings are totally spent, P = m, and solutions do differ. So when using the Nash bargaining solution, q is such that it satisfies:

$$m \equiv z_N(q) = [1 - \Theta(q)]u(q) + \Theta(q)c(q)$$

where

$$\Theta(q) = \frac{\theta_N u'(q)}{\theta_N u'(q) + (1 - \theta_N)c'(q)}$$

<sup>&</sup>lt;sup>21</sup>One way of formalizing this would be to assume that buyers incorrectly think there will meet with a certain trading partner from a distribution with different fixed amounts to bargain over but after meeting find themselves bargaining over both  $\{q, P\}$ .



Figure 3.9: Average (and median) buyer's surplus share with 95% confidence intervals for all three interest rate treatments plus predicted values for both the Nash and Kalai solutions, with letters N and K, respectively.

While for the Kalai proportional solution, q satisfies:

$$m \equiv z_K(q) = [1 - \theta(q)]u(q) + \theta(q)c(q)$$

Figure 3.10 shows the distribution of agreed quantities, buyer and seller's surplus and buyer's surplus share conditional on buyer's money holdings including the predictions for this alternative model. While traded quantities and payments are expected to be increasing in money holdings for both Nash and Kalai, buyer's surplus is expected to be non-monotonic for Nash while strictly increasing for Kalai while the buyer's surplus share should be strictly decreasing with r, thus allowing for additional comparison between both bargaining solutions. A  $3^{rd}$  order polynomial regression to fit the data plus a 30-points moving average tend to suggest that for m < 11.53, that is when participant are liquidity constrained, the Nash solution is closer to the data.

A simple regression model with fixed effects for time and individual confirms that both quantities and seller's surplus increase with money holdings, while buyer's surplus share



Figure 3.10: Distribution of agreed-upon quantities, seller and buyer's surplus, and buyer's share of surplus with polynomial regression (continuous line) and moving average (dotted line) plus predicted values for both the Nash and Kalai solutions of the alternative model, as gray circles and orange triangles, respectively.

is indeed decreasing. For the buyer's surplus I run the same regression but including an interaction dummy to allow the money holdings coefficient to change according to theory, and capture the non-monotonicity, which also confirms.

**Finding 7:** The alternative setting, where money holdings are taken as given, provides additional evidence in favor of Nash bargaining and contrary to Kalai: buyer's surplus is non-monotonic, while buyer's surplus share is decreasing.

## 3.6.2 Sunk Cost Fallacy Model

A second more interesting (and plausible) alternative model is as follows: buyers correctly anticipate their money holdings' decision impact on the bargaining set and split but incur in a sunk cost fallacy when bargaining, that is, they still factor in the cost of holding money when already in the bargaining stage with their counterpart.<sup>22</sup> In terms of the model, and again keeping the notation from *Subsection 3.2*, this translates into a modified bargaining problem, that directly includes the money holding cost.

For the Nash bargaining solution, the buyer's bargaining problem is now:

$$[q(m), P(m)] = \underset{q, P \le m}{\operatorname{argmax}} [u(q) - P - rP]^{\theta_N} [P - c(q)]^{1 - \theta_N}$$

In the interesting case where  $P \leq m$  does bind, P = m and q is the implicit solution to:

$$m \equiv z_N(q, r) = [1 - \Theta(q, r)] \frac{u(q)}{1+r} + \Theta(q, r)c(q)$$

where

$$\Theta(q,r) = \frac{\theta_N u'(q)}{\theta_N u'(q) + (1+r)(1-\theta_N)c'(q)}$$

So, the first order condition that determine the solution is:

$$\frac{u'(q)}{z'_N(q,r)} = 1 + r \tag{3.3}$$

In the case of the Kalai bargaining solution:

$$[q, P] = \underset{q, P \le m}{\operatorname{argmax}} [u(q) - P - rP]$$

 $<sup>^{22}</sup>$ The sunk cost fallacy is a topic long studied in behavioral economics, e.g Arkes and Blumer (1985) and Thaler (1999), who find it is prevalent phenomenon.

subject to:  $u(q) - P - rP = \frac{\theta}{1 - \theta_K} [P - c(q)]$ 

Thus, when  $P \leq m$  binds, P = m and q is the implicit solution to:

$$m \equiv z_K(q, r) = \frac{(1 - \theta_K)u(q) + \theta_K c(q)}{(1 + r)(1 - \theta_K) + \theta_K}$$

Using  $P = m = z_K(q, r)$ , the buyer's choice of balances before the pairwise meeting in this case is:

$$\max_{q} \{ -rz_{K}(q,r) + u(q) - z_{K}(q,r) \}$$

So, the first order conditions that determine the solution are:

$$\frac{u'(q)}{z'_K(q,r)} = 1 + r \tag{3.4}$$

Equations 3.1 and 3.2 from the standard model are analogous to 3.3 and 3.4, but with marginal money holdings effect, z'(q, r), now directly depending on the interest rate. Figure 3.11 compares money holdings, output, buyer's surplus and share of surplus data means and medians with the corresponding alternative model predictions under Nash and Kalai for r =0%, r = 30% and r = 60%. The top two panels show that while money holdings predictions fit the data slightly better than the standard model, output predictions (and payments, not included in Figure 3.11) are still in between the two models and only marginally more favourable towards Kalai. In the bottom row, buyers' surplus experimental data is closely approximated by the alternative model, including the characteristic non-monotonicity of the experimental data, thus mildly supporting the Nash solution. In terms of the buyer's surplus share it can be seen that on of the most recognizable features of the Kalai solution, i.e the constant share of surplus, is no longer present, this makes it more consistent with my data and undermines one of the strongest arguments in favor of Nash in the traditional model predictions. Even though this alternative model still only offers partial evidence in favor



Kalai bargaining, it does tend to better represent the experimental data, and thus suggest that the sunk cost fallacy might be playing a role in participants decisions.

Figure 3.11: Average (and median) money holdings, output, buyer's surplus and surplus share with 95% confidence intervals for all three interest rate treatments plus predicted values for both the Nash and Kalai solutions in the alternative model, with letters N and K, respectively.

On an anecdotal level, at the end of the experiment participants were asked about any strategy/criteria that they were following while bargaining. Ideas related to "recovering the cost of borrowing" or "making up for the interest" where not the majority but nevertheless present.

**Finding 8:** Including the sunk interest cost in the bargaining problem provides some evidence in favor of the Kalai bargaining solution and brings predictions closer to the experimental data, suggesting that this might be a behavioral concern to take into account.

## 3.7 Additional Questions

The data collected in this experiment can also be used to address some questions that don't directly arise from the theoretical model but that are relevant for experiments on bargaining and experimental economics in general.

#### 3.7.1 Bargaining Process

In this section I describe some of the features of the process by which buyers and sellers reached agreements in each round. Note that an agreement, that is one party accepting one of their counterparts offer before two minutes, occurred in 84.3% of the rounds, and also differed across treatments but without a clear trend: 86.8%, 80.7% and 85.5%, for r = 0%, r = 30% and r = 60%, respectively<sup>23</sup>.

			Negotiation	Outcome
		All	Agreement	No Agreement
A 11	Seller	2.98	2.82	3.84
All	Buyer	3.11	2.98	3.83
treatments	Pair	6.10	5.80	7.66
	Seller	2.81	2.77	3.09
r = 0%	Buyer	2.93	2.79	3.24
	Pair	5.66	5.56	6.34
	Seller	2.97	3.15	2.93
r = 30%	Buyer	3.10	2.94	3.77
	Pair	6.06	5.86	6.92
	Seller	3.15	2.78	5.36
r = 60%	Buyer	3.37	3.19	4.40
	Pair	6.52	5.98	9.76

Table 3.11: Average number of proposals per round

Pairs of participants made an average of  $6.1 \ unique \ proposals^{24}$  in each round, with buyers

 $<sup>^{23}</sup>$ A simple probit model with random effects suggest that buyer's money holdings are not a significant predictor of whether an agreement is reached or not

<sup>&</sup>lt;sup>24</sup>Participants were not restricted to submitting the same proposal many times during the negotiation, which would translate to the same proposal appearing multiple times in their counterparts feed. A small number of participants opted to 'flood' their counterparts with hundreds of copies of the same proposal as

making an average of 3.11 and sellers 2.98. Note that I define a *unique proposal* as a one that is different from that participant's immediate previous proposal, either by having a different value of P or q. Thus, two non-consecutive identical proposals would still qualify as unique proposals. Table ?? provides details on the number of proposals by by treatment for buyers, sellers or pair. A random effects poison count model with the interest rate treatment, role (i.e buyer or seller), amount borrowed and whether an agreement was reaches as explanatory variables, confirms what can be seen in Table 3.11: interest rate has no significant effect on the number of proposals, buyers tend to make more proposals than sellers, and pairs where no agreement is reached tend to have more offers (agreement dummy =.0000 and participant's role dummy p-value=.0000 ). Interestingly, buyer's money holdings significantly decrease the number of offers (p-value=.0003), suggesting that agreement (or lack thereof) is harder to achieve in a more constraint setting.



Figure 3.12: Buyer's share of surplus implied in their offers ordered from  $4^{th}$  to last to last by treatment. Median offers are represented by a cross, average by a square and the dotted line is the average agreed surplus share.

Another interesting issue is the evolution of the surplus split in the negotiation. Figure 3.12 shows the share of the surplus that buyers assign to themselves in their own offers regardless of whether and agreement was reached or not, with each of the panels corresponding to one  $\overline{a \text{ strategy}}$ 

interest rate treatment. In each panel, offers are ranked by their order relative to the last offer made from right to left. Thus, the last offer made by a buyer before an agreement was reached or time run out will be on the right hand side of the panel (in blue), while the fourth-to-last offer will be on the left (in yellow). As is to be expected, buyers tend start with a proposal that is more favorable to them and gradually increase their counterparts share of surplus until an agreement is reached or they run out of time. That is, subjects do not jump immediately to the agreed upon result or just make one offer and stick with it (in a take-it-or-leave-it manner), which is reflected in that in 80% of the interactions buyer's make more than 1 proposal. The difference between a buyer's first proposal and the agreed-upon result in terms of their share of surplus is 16.9% points (the average first proposal for buyer's implies an 84% surplus for themselves). A similar analysis can be performed for sellers, who also tend to make more than one proposal and who's difference between first proposals' and agreed-upon surplus is 18.6% points starting from an average surplus for themselves of 52.9%.

Table 3.12: Average and median buyer's surplus share implied in buyer's first proposal and difference with agreed-upon results

	$1^{st}$ Pi	roposal	Difference		
Treatment	mean	median	mean	median	
r = 0%	82.8%	90.1%	21.5%	17.2%	
r = 30%	85.0%	87.5%	15.1%	11.4%	
r = 60%	84.2%	91.2%	14.1%	10.0%	

Table 3.12 shows the buyer's surplus share implied in each round's first proposal and the difference with the actual agreed-upon result by interest rate treatment, and though the first proposal's surplus split does not seem to vary with r, the calculated difference does. This is confirmed by a test of ordered medians (p - value = .0000) and a simple fixed effects regression with treatment dummies ( with positive significant coefficients). This relates to **Finding 6**, the increasing buyer's surplus share consistent with Nash bargaining: buyer's do not change their initial surplus position but tend to reach more favorable agreements under

higher interest rates (needing more proposals to do so).

**Finding 9:** Distinctive features of the bargaining process are: Participants start with more favorable proposals to themselves, agreement requires more proposals under greater constraints and changes in surplus split across treatment do not depend on participants initial proposals.

#### 3.7.2 Online vs In Person

As shown in Table 3.2, since half of the session where conducted in person at UCI's ESSL while the other half were conducted remotely via Zoom, but using the same subject, it is only natural to check for any systematic differences in outcomes. Table 3.13 summarizes the difference of means and medians, number of stars (from 1 to 3) next to the value represent whether the change is significant at 5%, 1% or 0.1%. Systematic patterns across treatments or in the aggregate are hard to observe, which suggests that online sessions are equivalent to in person. The online exception seems to arise when r = 0%, with buyer's holding more money and trading higher quantities with a lower buyer's surplus and share. Another interesting difference is that the agreement rate in person is significantly higher in person with an increase of 6.1pp, with 5.0pp, 7.3pp and 5.6pp, for r = 0%, r = 30% and r = 60%, respectively. Explanations for these phenomenon are beyond the scope of this paper.

Finding 10: There appears to be no major systematic difference between results for online and in person sessions at aggregated level, though there are some puzzling differences when r = 0%.

Table 3.13: Difference (in person minus online) in average and median values for money holdings, payments, quantities, number of proposals, surplus, buyer's surplus and buyer's surplus share

Mean										
Treatment	m	P	q	# props.	S	$S_b$	$S_b/S$			
r = 0%	$-1.31^{*}$	-0.51	$-0.51^{***}$	0.49**	-0.11	$-0.85^{***}$	$-7.23pp^{**}$			
r = 30%	$1.65^{***}$	0.44	0.07	0.19	0.19	-0.15	-2.09pp			
r = 60%	-0.35	0.25	0.08	-0.21	$0.40^{**}$	0.18	0.45pp			
All	0.07	0.09	-0.11	0.15	$0.17^{*}$	-0.28*	$-3.04pp^{**}$			
Median										
Treatment	m	P	q	# props.	S	$S_b$	$S_b/S$			
r = 0%	$-1.20^{*}$	-0.60	$-0.50^{***}$	0.00	$-0.15^{**}$	$-0.95^{*}$	$-7.98pp^{***}$			
r = 30%	$0.60^{*}$	0.20	0.10	0.00	0.05	-0.23	-2.98pp			
r = 60%	0.00	0.10	0.10	0.00	$0.25^{*}$	0.16	6.48pp			
All	0.00	0.10	0.00	0.00	0.00	-0.32	$-3.26pp^{**}$			

## 3.8 Conclusions

In this experimental implementation, I have studied bargaining solutions in a setting that is specially relevant for monetary theory: trade surplus is determined endogenously and simultaneously with its division, with participants facing possible liquidity constraints due to buyer's endogenous ex-ante choice of costly money holdings.

My experimental evidence supports that higher interest rates (which can be interpreted as inflation) incentivze participants to economize on money holdings leading to a more constrained bargaining set resulting in less production and surplus. My evidence in favor of a particular bargaining solution is only mixed: while money holdings are closer to the predictions under Kalai (and thus considerably higher than what Nash implies), quantities traded and payments do not comport systematically according to any of the bargaining solutions predictions. One explanation is that subjects do not spend all of their money holdings which reduces output, payments and surplus. On the other hand, I find that buyer's surplus share is increasing in the interest rate, which is a distinctive feature of the Nash bargaining solution.

Interestingly, an alternative model where buyer's are myopic in deciding their money hold-

ings, so the liquidity constraint becomes effectively exogenous and thus closer to the model of DLP(2021) provides evidence that is entirely consistent with Nash and contrary to Kalai bargaining (e.g non-monotonic buyer's surplus and a increasing buyer's share), which is contrary to DLP(2021)'s findings. This invites further research on which bargaining solution is more relevant in the monetary context, the impact of subjects inter-temporal problem solving abilities, and its welfare implications. A second more interesting (and plausible) alternative model where buyers correctly anticipate their money holdings' decision impact on the bargaining outcome but incur in a sunk cost fallacy when bargaining, that is, they still factor in the cost of holding money when already in the bargaining stage with their counterpart, makes predictions somewhat closer tho the data and provides support for Kalai. This behavioral trait opens interesting possibilities for future research in monetary theory.

Another promising avenue for future research is to incorporate unstructured bargaining in fully dynamic experimental settings of monetary economics, where money is actually a fiat object with no redemption value, such as those by Duffy and Puzzello (2022) or Jiang, Zhang, and Puzzello (2019). I leave that extension to future research.

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# Appendix A

# Appendix to Chapter 2

### A.1 Additional Results

This appendix includes additional details about traded output, distribution of money holdings and acceptance rate in the DM.

#### A.1.1 DM traded Output

Here I report the output traded in pairwise meetings in the DM for every period in each of the 3 sessions of the 4 different treatments including the average (black line) and the model theoretical predictions. The vertical gray dotted lines correspond to the beginning of a new sequence. Figure 15 includes data for the BASE-ENDO treatment on the left hand side and the BASE-EXO treatment on the right, while Figure 16 does the same for FR-ENDO and FR-EXO, respectively.



Figure A.1: DM Traded Output for BASE-ENDO (left) and BASE-EXO (right) with period average (black line) and predicted price (red line)

#### A.1.2 Distribution of Money Holdings

In this section I report on the distribution of money holdings at the beginning of the DM across all four treatments, excluding for the first period of each sequence, where money holdings are fixed at 10 tokens per subject. According to the model there should be a degenerated distribution of tokens at 10 units per subject, which is clearly different from what is observed in Figures 17 and 18, which show the frequency of token holdings, with a lot of mass centered around 10 tokens but a big dispersion of holdings.

#### A.1.3 DM Acceptance Rate

Lastly, I include the acceptance rate (for non-zero token offers) for every period in each of the 3 sessions of the 4 different treatments including the grand average (black dotted line). The



Figure A.2: DM Traded Output for FR-ENDO (left) and FR-EXO (right) with period average (black line) and predicted price (red line)

vertical gray dotted lines correspond to the beginning of a new sequence. Figure 19 includes data for the BASE-ENDO treatment on the left hand side and the BASE-EXO treatment on the right, while Figure 20 does the same for FR-ENDO and FR-EXO, respectively.

### A.2 Experimental Instructions

Here I provide the experimental instructions and quizzes used for the Friedman rule (FR-ENDO) treatment, note that all of the instructions are in the form of interactive slides, no further instructions about the particular experiment were given. Other instructions are similar. The complete set of instructions used in all experimental treatments are available upon request.



Figure A.3: Distribution of Money Holdings for BASE-ENDO (left) and FR-ENDO (right) treatments



Figure A.4: Distribution of Money Holdings for FR-ENDO (left) and FR-EXO (right) treatments



Figure A.5: Acceptance Rate (bars) and overall average for BASE-ENDO (left) and BASE-EXO treatments (right)



Figure A.6: Acceptance Rate (bars) and overall average for BASE-ENDO (left) and BASE-EXO treatments (right)

### Welcome!

#### Welcome to this experiment in the economics of decision making:

- During today's session, you and **9 other participants** will be called upon to make a series of consuming, producing, buying and selling decisions
- On top of your US\$7 show up payment you can earn more if you follow the instructions carefully and make good decisions
- The amount will be paid to you electronically at the end of the experiment
- Please plan on staying on your computer for the next 90 to 120 minutes
- Avoid closing this current tab though you can always use the provided link to return to the experiment



General Overview II - Points, Tokens, Interests and Taxes
• You will start each of your sequences with an endowment of <b>40 Points</b> and <b>10 Tokens</b> :
Points (click here)
Your <b>Points</b> do NOT carry over from one sequence to the next but
• at the end of the experiment <b>two sequences will be selected at random</b> and you'll get <b>US\$0.2 for each Point</b> you had (plus your US\$7 show up payment)
• so the more <b>Points</b> you have, the more US\$ you'll receive
Tokens (click here)     Taxes and Interests (click here)

# Instructions:

General Overview II - Points, Tokens, Interests and Taxes
You will start each of your sequences with an endowment of 40 Points and 10 Tokens:
Points (click here)
Tokens (click here)
Your <b>Tokens</b> do NOT carry over from one sequence to the next either
and they have NO value in terms of US\$ (so when a sequence ends they become worthless)
•however even though they have no redemption value they may help you earn <b>Points</b> when used for exchange (trading) purposes
Taxes and Interests (click here)



## Instructions:



We will now describe in detail what happens in each of these two parts of a round

#### **Decentralized Goods Market** —first part of each round:

**Goods Market** — Overview

• Pairs of participants are randomly formed with the option to trade the Special Good

• In each pair, one participant is randomly chosen to be a Consumer and the other a Producer

• In each round you'll have a 50/50 chance of being either a Producer or a Consumer



Next we'll see how many Points Consumers get from the Special Good and how many Points it costs Producers

Goods Market — Benefits and Costs
• If selected to be a <b>Consumer</b> you'll use a slider (like the one below) to make your <b>Special Good</b> request.
If selected to be Producer you'll just decide whether to Accept or Reject
• The <b>Consumer's</b> benefit and <b>Producer's</b> cost (in <b>Points</b> ) associated to a request -if accepted- will be always automatically displayed below
Check this example using the slider (click here)
A <b>Consumer</b> buying 5 units of <b>Special Good</b> would get a benefit of <b>7.35 Points</b> while the <b>Producer</b> incurs a cost of <b>5</b> <b>Points</b> . Thus, the Surplus from that trade is <b>2.35 Points</b> .
Did you get the same numbers?
Next
Your <b>Special Good request</b> : 6
Consumer's Point Benefit: Producer's Point Cost: Trade Surplus (Benefit minus Cost):
• 8.46 • 6.00 • 2.46

# **Practice Quiz:**



# Practice Quiz - Question 1

Goods Market Quiz: Suppose yo	u got to be a Consun	ner	
or this <b>Practice Goods Market</b> round su	ppose you are a <b>Consume</b> i	and you just met with a potential	Producer:
• Imagine you want to buy <b>5 units of S</b>	pecial Good		
• And you think the Producer won't acc	ept anything less than <b>10 T</b>	<b>okens</b> (you don't want to pay mor	e than that either)
<b>Tip:</b> There is now a second slider to mak	e your <b>Total Token</b> offer		
How many units should you request	and how many <b>Tokens</b> sh	ould you offer?:	
Use this first slider to make your <b>Specie</b>	<b>al Good</b> request:		
Your <b>Special Good request</b> : 4			
And this slider to select the total numb	er of <b>Tokens</b> you'd like to c	offer for those units:	
Your <b>Total Token offer</b> : 10			
Your Point Benefit as a Consumer:	Producer's Point Cost:	Point Surplus (Benefit-Cost):	Tokens per Unit:
• 6.18	• 4.00	• 2.18	• 2.50
l <b>ote:</b> If you don't choose your default off	er will be zero <b>Tokens</b> for z	ero Special Good	

# **Practice Quiz - Question 1: Results**

ioods Market			
•You offered <b>10.0 Tokens</b> for <b>4.0</b>	units of Special Good		
That is NOT correct			
u should have requested <b>5 units o</b> <b>kens per unit</b>	of <i>Special Good</i> in exchange for	10 Tokens to get 7.35 Points and	achieve a <b>Price of 2</b>
	Try again! (click	here)	
Use this first slider to make your a	Special Good request:		
Your Special Good request:			
And this slider to select the total       Your Total Token offer:	number of <b>Tokens</b> you'd like to o	ffer for those units:	
Your Point Benefit as a Consu	mer: Producer's Point Cost:	Point Surplus (Benefit-Cost):	Tokens per Unit:
•	•	•	•
Next			

Practic	e Quiz	- Ques	tion 2
---------	--------	--------	--------

this new <b>Practice Goods M</b>	<b>arket</b> round suppose again you a	are a <b>Consumer</b> and you just met with	a potential <b>Produce</b>
Imagine now you want to e	arn 9.54 Points		
And you think the Produce	won't accept anything less than	21 Tokens (you don't want to pay mo	re than that either)
		· · · ·	
How many units should yo	u request and how many <b>Toke</b>	ns should you offer?:	
Use this first slider to make	our <b>Special Good</b> request:		
Your Special Good request	: 7		
And this slider to select the	otal number of <b>Tokens</b> you'd like	e to offer for those units:	
Your Total Token offer: 21		•	
Your Point Benefit as a C	onsumer: Producer's Point Co	ost: Point Surplus (Benefit-Cost):	Tokens per Unit:
• 9.54	• 7.00	• 2.54	• 3.00

# **Practice Quiz - Question 2: Results**



Token Exc	hange — Overview
	5
• All 10 partic	ipants have the opportunity to interact in a single Centralized Token Exchange
• Each particip	pant <b>decides</b> in each round whether to become a:
- Token	Buyer: Use some of your Points to bid for Tokens
- Token	Seller: Sell some of your Tokens to get Points
How many <b>To</b>	wkens you get as Token Buyer or how many Points as a Token Seller depends on the resulting Market
Price (Token:	s per Point)
• Market Pric	$m{x}$ depends on current demand and supply and is only known to you after you make your decision
	Next
REMEMBE	<b>R:</b> You <b>DO NOT KNOW</b> what the <i>Market Price</i> will be when you are deciding how much to bid or sell. It
	is only determined afterwards, so you'd want to think about what it will be.

# Instructions:





Next

### **Practice Quiz:**



oosing a role
round:
on <b>Tokens</b> and want to increase your holdings (to use them to trade in the next <b>Goods</b> ifice some <b>Points</b> . Which role should you choose?
l you choose to be a:
Token Seller       (sell some of your Tokens to earn Points)
Token Buyer       (use some of your Points to bid for Tokens)

# **Practice Quiz 2 - Question 1: Results**



# Practice Quiz 2 - Question 2

Token Exchange Quiz: Choosing a role
For this <b>Practice Token Exchange</b> round:
• You think you are holding too many Tokens and want to trade them for Points. Which role should you choose?
For this round would you like to be a:
Token Seller       (sell some of your Tokens to earn Points)
Token Buyer (use some of your Points to bid for Tokens)

### Practice Quiz 2 - Question 2: Results



### Practice Quiz 2 - Question 3



Next

# Practice Quiz 2 - Question 3: Results



#### Practice Quiz 2 - Question 4



Next

### **Practice Quiz 2 - Question 4: Results**



# Lets get started!

Time left to complete this page: 1:23

You will now enter the first part (Goods Market) of the first round of your 1st Sequence:
Remember: You'll now start with: • 10 Tokens • 40 Points
You'll meet someone at random and then become either a <b>Consumer</b> or a <b>Producer</b> : • <b>Consumers</b> make an offer to their counterpart (offer some <b>Tokens</b> in exchange for some <b>Special Good</b> ) • <b>Producers</b> either <b>Accept</b> or <b>Reject</b> said offer

# Appendix B

# Appendix to Chapter 3

## **B.1** Additional Figures and Tables

### **B.2** Experimental Instructions

Here I provide the experimental instructions and quizzes used for the sessions with the r = 30 and r = 60% treatments. Instructions were read out loud and questions were taken at the end of the instructions and during the practice rounds. Instructions for other treatment combinations are similar. The complete set of instructions used in all experimental treatments are available upon request.



Figure B.1: Payments and output distributions with average value  $(\bar{x})$  and predictions by interest rate for Nash and Kalai with letters N and K, respectively.



Figure B.2: Buyer and seller surplus distributions with average value  $(\bar{x})$  and predictions by interest rate for Nash and Kalai with letters N and K, respectively.



Figure B.3: Total surplus and buyer's share of surplus distributions with average value  $(\bar{x})$  and predictions by interest rate for Nash and Kalai with letters N and K, respectively.

#### **Summary of Instructions**

#### Welcome to this experiment in decision-making.

#### General Information:

- You will participate in 2 Practice Rounds and then 28 independent Paying Rounds with 9 other participants.
- At the beginning of the Experiment you'll be randomly assign to be either a Seller or a Buyer for the entire session.
- Instructions will remain the same for every round throughout the entire experiment, the only thing changing will be interest rate at which money can be borrowed.
- At the start of each round, each **Seller** will be paired with one **Buyer** for that round.
- Chances of being paired with the exact same participant in 2 or more consecutive rounds are extremely low.
- In each round, each Seller-Buyer pair will bargain over the amount of **Output** the Seller provides (at a cost to them) and the **Payment** the Buyer makes for it (out of their money holdings).
- The bargaining takes the following form: both participants can make as many offers as they like regarding Output and Payment until one of them accepts an offer or 2 minutes go by. Once it is made, an offer can not be withdrawn, so there will be a list and any previous offer can always be latter accepted.

#### • Paying Rounds:

- After the Practice Rounds, you'll enter the Paying Rounds.
- At the end of the experiment, your earnings from one randomly selected round will be paid to you. Paying Round.

Please notice: Throughout the experiment, payoffs, costs and payments will always be expressed in US Dollars, \$.

#### Next

Figure B.4: First page of instructions for treatment with r = 30% and r = 60%.

#### **Summary of Instructions: Seller**

#### • If you are selected to be a Seller for the experiment:

- In each round you'll be randomly paired with a Buyer, who carries a certain amount of money, known to both Seller and Buyer.
- You can make as many offers as you'd like regarding how much Output to produce and sell, and the Payment you'd like to receive for them.
- You will also receive offers from the Buyer regarding how much Output they'd like to buy, and the Payment they'd like to make for them.
- The cost of producing said **Output**, q, for you as a Seller (expressed in \$) is: C(q)=0.82q<sup>1.51678</sup>



- The bargaining concludes if you accept an offer from the Buyer, they accept one of your offers, or you run out of time.
- If an agreement is reached you'll incur the cost of production and receive the agreed **Payment**, if NO agreement is reached there will be no production or **Payment**.
- $\circ\,$  At the end of each round, regardless of whether an agreement is reached, you will receive a fix amount of \$5.
- $\circ~$  Thus, your payoff for the round will be calculated as follows:
  - Agreed Payment minus the above-mentioned Output production cost plus your fix amount:

Payoff = Payment - Production Cost + Fix Amount

#### Next

Figure B.5: Second page of instructions for treatment with r = 30% and r = 60%.

#### **Summary of Instructions: Buyer**

#### • If you are selected to be a Buyer:

• In each round you'll be randomly paired with a Seller, but before that you'll have to decide how much money to bring to the meeting by taking a loan that you'll have to repay at the end of the round.

- The interest rate for borrowing money will be 30% for the first 14 rounds and 60% for the last 14 rounds (and 10% for both practice rounds)
- Once you meet with the Seller, you can make as many offers as you'd like regarding how much Output you want to purchase, and the Payment you are willing to make for them.
- You will also receive offers from the Seller regarding how much **Output** they'd like to produce and sell, and the **Payment** they'd wish to receive for them.
- Your payoff for buying said **Output**, q, for you as a Buyer (expressed in \$) is:  $U(q)=7.41q^{0.6}$



- The bargaining concludes if you accept an offer from the Seller, they accept one of your offers, or you run out of time.
- If an agreement is reached you'll receive your payoff from buying that **Output** and make the agreed **Payment**, if NO agreement is reached there will be no production or **Payment**.
- At the end of the round, regardless of whether an agreement is reached, you will receive a fix amount of \$9.6 for the first 14 rounds and \$19.2 thereafter, that will always be enough to help repay the amount you borrowed plus any interest.
- Thus, your payoff for the round will be calculated as follows:
  - Payoff for purchased Output minus the agreed Payment plus your fix amount minus repayment of your loan:

Payoff = Purchase Payoff - Payment + Fix Amount - Loan Repayment

Next

Figure B.6: Third page of instructions for treatment with r = 30% and r = 60%.

#### **Instructions Quiz**

Before proceeding to the Practice Rounds, please answer the following questions regarding the instructions you just read:

1. In each round you'll be paired with another participant. Is it likely that you'll be paired with the exact same participant in many consecutive rounds?

 $\odot$  Extremely likely, I'll be paired with the exact same participant in most consecutive rounds

 $\odot$  Fairly likely, I'll be paired with the exact same participant every other round

Inlikely, I'll probably won't be paired with the same participant in consecutive rounds ---> Correct! Chances of being paired with the same other participant in many consecutive rounds are very low

2. Are instructions and roles (Seller or Buyer) going to be changing throughout the experiment?

No, instructions stay the same throughout the experiment

- --> Correct! Instructions stay the same and are repeated in most decision screens
- $\bigcirc$  Yes, new instructions will be provided every other round
- $\bigcirc$  Yes, instructions change in every round

#### 3. The Seller in each pair:

 $\bigcirc$  Receives offers from the Buyer and decides whether to accept one of them or not

- $\bigcirc$  Sends proposals to the Buyer for them to decide whether to accept one or not
- $\bigcirc$  Both of the above

Figure B.7: Instructions questions first half.

#### **Instructions Quiz**

Before proceeding to the Practice Rounds, please answer the following questions regarding the instructions you just read:

4. Barganing between each Buyer and Seller has a maximum duration of 2 minutes. What happens if no agreement is reached before that:

The last proposal made by the Buyer must be accepted
No trade takes place so there is no production and no payment
--> Correct! The asset value is unknown to potential Buyers, but always lies between \$0 and \$100
The last proposal made by the Seller must be accepted

5. In each round, Buyers can borrow money before meeting with a Seller? Why would they want to do that?

To earn interest according to the prevailing interest rate
To produce output by themselves
To make proposals to Sellers who can produce output
-> Correct! Money borrowed by Buyers can be used for proposals and then payments to Sellers

6. Can a Buyer default on his loan (fail to repay his loan)?:

No, Buyers will always be able to repay their loan

--> Correct! Buyers receive a fix amount at the end of the period that is enough to repay their obligations

- $\bigcirc$  Yes, but there can be a penalty for not repaying a loan
- $\bigcirc$  Yes, but only if no agreement is reached during the barganing stage

Next

Figure B.8: Instructions questions second half.