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Publication Date

1966-05-01

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UCRL-16692

Lawrence Radiation Laboratory
and
Department of Chemistry
Berkeley, California

AEC Contract No. W-7405-eng-48

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May 1966

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ABSTRACT

A model is proposed for odd-odd nuclei in which the independent excitations are quasi-particles defined by a special Bogoliubov transformation and in which a finite range Gaussian central interaction acts as the residual interaction. The model is applied to La^{140} where energy levels, wavefunctions, (d,p) spectroscopic factors, and M1 transition rates are computed and compared with experiment.

I. INTRODUCTION

In recent years many models of the nuclear many body problem have been described in terms of independent excitations called quasi-particles. Kisslinger and Sorenson¹ studied systems of single closed shell nuclei in 1960. Since that time, several attempts have been made to extend these techniques in heavier nuclei to systems that contain both neutrons and protons outside magic configurations.^{2,3,4,5} The neutron-proton interaction was assumed to affect the single particle energies in some systematic way or was taken explicitly into account in first order perturbation theory. Although Kisslinger and Sorenson⁵ concluded that their calculations were consistent with the limited amount of information on odd-odd nuclei that was available at that time, no detailed calculations using an odd-odd quasi-particle model were attempted.

In the previous paper⁶ we analyzed data from measurements on the reaction $\text{La}^{139}(\text{d},\text{p})\text{La}^{140}$. These data and data from other experiments^{7,8} give information about (d,p) cross sections and gamma-ray branching ratios, as well as the spins and parities of eleven states. Therefore we decided to make a quantitative comparison between experiment and theory for an odd-odd quasi-particle model.

In this model we assume that the 50 proton 82 neutron system forms an inert core. The self-consistent single particle energies for the 83rd neutron are determined from experimental data. For the seven valence protons we make a special Bogoliubov transformation to define quasi-particles. The energies and wavefunctions for these independent excitations are determined by solving the self-consistent problem for minimizing the energy of each quasi-particle state. Thus effects of core polarization by the unpaired proton are considered. Finally the neutron-proton interaction

is considered both in first order perturbation theory and by diagonalizing it in a truncated quasi-particle basis. Since none of the experimental data^{6,7,8} indicates any phonon admixture in states which have been characterized, we neglect those terms in the Hamiltonian which introduce the quasi-particle correlations that generate collective vibrations. In sections II and III we give the mathematical details of the model, discuss how we fit the parameters of the model to the experimental data and give the results of the calculation. In section IV we compute (d,p) spectroscopic factors, gamma transition probabilities and compare the results with experiment.

II. THEORY

We consider that the system $^{82}_{50}\text{Sn}^{132}$ forms an inert core with respect to single particle and collective excitations. It contributes only to the self-consistent potential for nucleons outside this core. With this approximation, the m-scheme Hamiltonian for the system may be written in the occupation number representation as

$$H = H_p + H_n + H_{np} \quad (1)$$

where

$$H_p = \sum_{\alpha} \epsilon_{\alpha} C_{\alpha}^{\dagger} C_{\alpha} + \frac{1}{4} \sum_{\substack{\alpha\beta \\ \gamma\delta}} \langle \alpha\beta | V_{pp} | \gamma\delta \rangle C_{\alpha}^{\dagger} C_{\beta}^{\dagger} C_{\delta} C_{\gamma}$$

$$H_n = \sum_a \epsilon_a C_a^{\dagger} C_a$$

$$H_{np} = \sum_{\substack{\alpha a \\ \beta b}} \langle \alpha a | V_{np} | \beta b \rangle C_{\alpha}^{\dagger} C_a^{\dagger} C_{\beta} C_b$$

In this equation ϵ 's are the single particle energies, C 's are annihilation operators for shell model particles, Greek letters designate the quantum numbers for a complete set of compatible observables in the assumed self-consistent potential for protons and Roman letters designate these quantum numbers for the neutron.

If we assume that the single particle states are generated from the usual shell model harmonic oscillator potential with a single particle spin orbit term, then $|\alpha\rangle = |\Psi(p)_{n_{\alpha} s_{\alpha} l_{\alpha} j_{\alpha} m_{\alpha}}\rangle$,⁹ and if we assume a truncated space of only one oscillator shell, the

proton summation is performed over the $1g_{7/2}$, $2d_{5/2}$, $3s_{1/2}$, $2d_{3/2}$, and $1h_{11/2}$ orbitals while the sum for the neutron is over the $2f_{7/2}$, $3p_{3/2}$, $1h_{9/2}$, $3p_{1/2}$, $2f_{5/2}$, and $1i_{13/2}$ orbitals. Even this highly truncated shell model calculation would be very laborious and probably not physically illuminating. However, since in this region of the nuclear periodic table the description of even and odd-A systems in terms of independent quasi-particles which are defined by a special Bogoliubov transformation is very good,⁵ we make the transformation

$$\xi_{\alpha}^{+} = u_{\alpha} C_{\alpha}^{+} - s_{\alpha} v_{\alpha} C_{-\alpha} \quad (2)$$

Here ξ_{α}^{+} is a creation operator for quasi-particles. The transformation coefficients are chosen so that the quasi-particle operators obey the anticommutation relations for fermions and that they transform under rotations in the same manner that shell model particles do. A set of sufficient conditions for the transformation to be canonical and for the system to be spherically symmetric are that u_{α} and v_{α} are real and that

$$\begin{aligned} v_{\alpha} &> 0 \\ u_{\alpha} &= u_{j_{\alpha}} \\ v_{\alpha} &= v_{j_{\alpha}} \\ u_{\alpha}^2 + v_{\alpha}^2 &= 1 \\ s_{\alpha} &= (-1)^{j_{\alpha} - m_{\alpha}} \end{aligned} \quad (3)$$

By defining s_{α} this way, we have adopted the Condon and Shortley phase conventions.¹⁰

We will briefly outline the treatment of H_p under a special Bogoliubov transformation. (For a more extensive description, see Lane.)¹¹ We consider the

auxiliary Hamiltonian

$$H'_p = H_p - \lambda N_p \quad (4)$$

where N_p is the counting operator for shell model protons and λ is a c-number.

After the transformation to quasi-particles, equation (4) may be written

$$H'_p = H_{00} + H_{11} + H_{20} + H_I$$

where

$$H_{00} = \sum_{\alpha} [v_{\alpha}^2 (\epsilon_{\alpha} - \lambda - \frac{1}{2} \mu_{\alpha}) - \frac{1}{2} u_{\alpha} v_{\alpha} \Delta_{\alpha}] \quad (5)$$

$$H_{11} = \sum_{\alpha} [(u_{\alpha}^2 - v_{\alpha}^2) (\epsilon_{\alpha} - \lambda - \mu_{\alpha}) + 2u_{\alpha} v_{\alpha} \Delta_{\alpha}] \xi_{\alpha}^{\dagger} \xi_{\alpha}$$

$$H_{20} = \sum_{\alpha} s_{\alpha} [u_{\alpha} v_{\alpha} (\epsilon_{\alpha} - \lambda - \mu_{\alpha}) + \frac{1}{2} (v_{\alpha}^2 - u_{\alpha}^2) \Delta_{\alpha}] (\xi_{\alpha}^{\dagger} \xi_{-\alpha}^{\dagger} + \xi_{-\alpha} \xi_{\alpha})$$

$$H_I = \frac{1}{4} \sum_{\substack{\alpha\beta \\ \gamma\delta}} \langle \alpha\beta | v_{pp} | \gamma\delta \rangle : c_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\delta} c_{\gamma} :$$

$$\Delta_{\alpha} = -\frac{1}{2} \sum_{\beta} s_{\alpha} s_{\beta} \langle \alpha - \alpha | v_{pp} | \beta - \beta \rangle u_{\beta} v_{\beta}$$

$$\mu_{\alpha} = -\frac{1}{2} \sum_{\beta} [\langle \alpha - \beta | v_{pp} | \alpha - \beta \rangle + \langle \alpha\beta | v_{pp} | \alpha\beta \rangle] v_{\beta}^2$$

Here the symbol :AB: indicates that the product of operators is normal ordered.¹² One can generate the Hamiltonian for a set of independent quasi-particles and at the same time conserve the average number of shell model particles by choosing the v_α 's so that H_{20} vanishes identically and by choosing λ so that

$$\langle \gamma | N_p | \gamma \rangle = n \quad (6)$$

where n is the number of protons and γ is an arbitrary quasi-particle configuration. This requires that a set of $r + 1$ coupled equations must be solved for each $\bar{j}_\gamma$ where r is the number of orbitals considered in the problem. This is accomplished by an iterative technique. For numerical simplicity and in order to reduce the number of parameters in the theory, we use a pairing force, i.e.

$$\langle \alpha \beta | V_{pp} | \gamma \delta \rangle = G(-1)^{j_\alpha + j_\gamma + \ell_\alpha + \ell_\gamma + m_\alpha + m_\gamma} \delta_{\beta - \alpha} \delta_{\delta - \gamma} \quad (7)$$

In order to minimize the energy of the ground state, the use of the pairing force requires that

$$u_\alpha = (-1)^{\ell_\alpha} \sqrt{1 - v_\alpha^2} \quad (8)$$

Equations (3) and (8) specify all sign conventions. It has been shown that this is an acceptable approximation for the actual two body force in pairing type matrix elements.¹³ We further assume that this independent quasi-particle representation is good in the sense that the matrix elements of H_I are small and that we can

neglect the correlations between the quasi-particles that the terms of H_I introduce. It is known that the lowest energy states that arise from such correlations are collective and vibrational in character.⁵ It is also known¹⁴ that for single closed shell even-even nuclei in the mass 140 region the energy of this state is about 1.5 MeV but as neutrons are added, the neutron-proton long range quadrupole force causes the energy of the lowest vibrational state to drop sharply. However since La^{140} is a single closed shell plus one nucleus and because there are experimental assignments for only the first 0.5 MeV of excitation, we assume that we can neglect vibrational particle mixing.

The state vectors for the independent quasi-particles are

$$|\gamma^\circ\rangle \equiv \xi_\gamma^+ |\tilde{0}\rangle = c_\gamma^+ \prod_{\substack{\alpha > 0 \\ \alpha \neq \gamma}} (u_{\gamma\alpha} + s_{\alpha\gamma} v_{\gamma\alpha} c_\alpha^+ c_{-\alpha}^+) |0_p\rangle \quad (9)$$

where $|\tilde{0}\rangle$ is the quasi-particle vacuum and $|0_p\rangle$ is the shell model particle vacuum. In this case it represents the fifty protons in the inert $^{82}_{50}\text{Sn}^{132}$ core. The transformation coefficients are now labeled with respect to the single quasi-particle state in order to indicate that blocking is now considered explicitly. Physically we have effected a binding of six of the valence protons into a new core so that we are now dealing with a two particle (one quasi-particle and one shell model particle) system. The basic states in such a system are given by

$$|\bar{j}_\alpha^\circ \bar{j}_a^\circ JM\rangle = [\xi_\alpha^+ c_a^+]_M^J |0'\rangle \quad (10)$$

where the bracket notation denotes vector coupling and $|0'\rangle$ is the direct product $|\tilde{0}\rangle |0_n\rangle$.

The matrix elements of H_{np} in this basis are given by

$$\begin{aligned}
 \langle \bar{j}_\alpha \bar{j}_a J | H_{np} | \bar{j}_\beta \bar{j}_b J \rangle = & \\
 F(\bar{j}_\alpha \bar{j}_\beta) \left\{ u_{\beta\alpha} u_{\alpha\beta} \langle \bar{j}_\alpha \bar{j}_a J | V_{np} | \bar{j}_\beta \bar{j}_b J \rangle \right. & \quad (11) \\
 - v_{\beta\alpha} v_{\alpha\beta} \sum_{J'} \hat{J}' \left\{ \begin{matrix} j_\beta j_a & J \\ j_\alpha j_b & J \end{matrix} \right\} \langle \bar{j}_\alpha \bar{j}_b J' | V_{np} | \bar{j}_\beta \bar{j}_a J' \rangle \Big\} & \\
 + \delta_{\bar{j}_\alpha \bar{j}_\beta} \delta_{j_a j_b} \sum_{J'} \frac{\hat{J}'}{\hat{j}_a} \langle \bar{j}_\gamma \bar{j}_a J' | V_{np} | \bar{j}_\gamma \bar{j}_b J' \rangle v_{\gamma\gamma}^2 &
 \end{aligned}$$

where

$$\hat{t} \equiv 2t+1 \quad (12)$$

and

$$F(\bar{j}_\alpha \bar{j}_\beta) = \frac{(2j_\alpha-1)(2j_\beta-1)}{4} \pi \frac{\hat{j}_\gamma}{2} (u_{\alpha\gamma} u_{\beta\gamma} + v_{\alpha\gamma} v_{\beta\gamma}) \quad (13)$$

$\bar{j}_\gamma \neq \bar{j}_\alpha, \bar{j}_\beta.$

Equation (11) is a special case of the matrix element for the neutron-proton interaction in the case where both proton and neutron are quasi-particles.

$$\begin{aligned}
 & \langle \bar{j}_\alpha \bar{j}_a J | H_{np} | \bar{j}_\beta \bar{j}_b J \rangle = \\
 & F(\bar{j}_\alpha \bar{j}_\beta) F(\bar{j}_a \bar{j}_b) \left\{ (u_{\beta\alpha} u_{\alpha\beta} u_{ba} u_{ab} + v_{\beta\alpha} v_{\alpha\beta} v_{ba} v_{ab}) \right. \\
 & \times \langle \bar{j}_\alpha \bar{j}_a J | V_{np} | \bar{j}_\beta \bar{j}_b J \rangle \\
 & - (u_{\beta\alpha} u_{\alpha\beta} v_{ba} v_{ab} + u_{ba} u_{ab} v_{\beta\alpha} v_{\alpha\beta}) \\
 & \times \sum_{J'} \hat{J}' \begin{Bmatrix} j_\beta j_a J' \\ j_\alpha j_b J \end{Bmatrix} \langle \bar{j}_\alpha \bar{j}_b J' | V_{np} | \bar{j}_\beta \bar{j}_a J' \rangle \Big\} \\
 & + \delta_{j_\alpha j_\beta} \delta_{\bar{j}_a \bar{j}_b} \sum_{j_c} \frac{\hat{J}'}{j_c} \langle \bar{j}_\alpha \bar{j}_c J' | V_{np} | \bar{j}_\beta \bar{j}_c J' \rangle (u_{\beta\alpha} u_{\alpha\beta} - v_{\beta\alpha} v_{\alpha\beta}) v_{cc}^2 \\
 & + \delta_{j_\alpha j_\beta} \delta_{j_a j_b} \sum_{j_\gamma} \frac{\hat{J}'}{j_\gamma} \langle \bar{j}_\gamma \bar{j}_a J' | V_{np} | \bar{j}_\gamma \bar{j}_b J' \rangle (u_{ba} u_{ab} - v_{ba} v_{ab}) v_{\gamma\gamma}^2
 \end{aligned} \tag{14}$$

The formulas that we have presented assume only that H_{np} is a two body interaction. At present there are three different methods for treating this interaction. One may take a highly realistic interaction, i.e., one which explains two nucleon scattering, the deuteron problem, and gives the correct results in nuclear matter calculations, and then attempt to explain differences between experiment and theory by systematic corrections to the model. The second approach is to invent an interaction which has desirable analytic properties and can explain certain systematic features observed in many nuclear spectra. The well-known pairing plus quadrupole model is an example of this. Finally, one may choose an interaction which is sufficiently simple to allow extensive calculations and even parameter fitting but which is complicated enough to simulate the properties of the realistic force. Such an interaction will be model dependent, for it will depend on the assumptions of the

model (e.g., the stability of the inert core), the extent to which the basic states approximate the Hartree-Fock self-consistent orbitals and the truncation of the Hartree-Fock single particle orbitals. The latter will depend on the region of the nuclear periodic table, the number of valence particles and the size of the computer. If the model is untested and there is only a limited amount of experimental data, the third approach is perhaps the most useful since it not only allows one to draw conclusions about the physical assumptions built into the model but also makes predictions which can be tested experimentally.

For the neutron-proton interaction, the Pauli principle does not exclude any states of relative motion. Therefore the number of parameters for a reasonably sophisticated interaction soon exceeds the number of available data. We therefore choose the simplest possible force capable of explaining the data. The simplest reasonable neutron-proton interactions is the δ -function. It, however, fails to reproduce the experimental levels in La^{140} . The next simplest force is the finite range central interaction with Gaussian shape. We chose this interaction but restricted it so that all exchange components of the force have the same range parameters. With this approximation

$$\begin{aligned}
 \langle \bar{j}_\alpha \bar{j}_a J | V_{np} | \bar{j}_\beta \bar{j}_b J \rangle &= \frac{1}{2} \{ (V_{TE} + V_{TO}) + (V_{TE} - V_{TO})(-1)^{j_\beta + j_b + J} P_{\beta b} \} \\
 &\times \langle \bar{j}_\alpha \bar{j}_a J | e^{-r_{pn}^2/r_0^2} | \bar{j}_\beta \bar{j}_b J \rangle \quad (15) \\
 &- \frac{1}{2} \{ (V_{TE} - V_{SE} + V_{TO} - V_{SO}) + (V_{TE} + V_{SE} - V_{TO} - V_{SO})(-1)^{j_\beta + j_b + J} P_{\beta b} \} \\
 &\times \langle \bar{j}_\alpha \bar{j}_a J | e^{-r_{pn}^2/r_0^2} | \bar{j}_\beta \bar{j}_b J \rangle
 \end{aligned}$$

where for notational convenience

$$|\bar{j}_\alpha \bar{j}_a J\rangle \equiv |n_\alpha n_a (\frac{1}{2}l_\alpha) j_\alpha (\frac{1}{2}l_a) j_a JM\rangle \quad (16)$$

In this equation, π_S is the singlet projection operator, $P_{\beta b}$ is an operator which exchanges the proton and neutron quantum numbers in the ket vector, V 's are the strengths of the two body potential in the triplet even, triplet odd, singlet even and singlet odd states, and r_0 is the range of the force.

$$\langle \bar{j}_\alpha \bar{j}_a J | e^{-r_{pn}^2/r_0^2} | \bar{j}_\beta \bar{j}_b J \rangle = \sum_k f_k F_k \quad (17)$$

where

$$F_k = \frac{1}{k} \iint R_{n_\alpha l_\alpha}(r_p) R_{n_a l_a}(r_n) v_k(r_p, r_n) R_{n_\beta l_\beta}(r_p) R_{n_b l_b}(r_n) \times r_p^2 r_n^2 dr_p dr_n \quad (18)$$

and

$$f_k = (-1)^{j_\alpha + j_\beta + J} (\hat{j}_\alpha \hat{j}_a \hat{j}_\beta \hat{j}_b)^{\frac{1}{2}k} \times \begin{Bmatrix} j_\beta & j_a & J \\ j_b & j_\beta & k \end{Bmatrix} \begin{pmatrix} j_\beta & k & j_\beta \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} j_a & k & j_b \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix} \quad (19)$$

with the restriction that both $(l_\alpha + k + l_\beta)$ and $(l_a + k + l_b)$ must be even, otherwise $f_k = 0$. The $R(r)$'s are the radial wavefunctions¹⁵ and $v_k(r_p, r_n)$ is the Legendre transform of the Gaussian.

$$\langle \bar{j}_\alpha \bar{j}_a J | e^{-r_{pn}^2/r_0^2} \pi_S | \bar{j}_\beta \bar{j}_b J \rangle = \frac{1}{4} \sum_k q_k F_k \quad (20)$$

where

$$q_k = 2(-1)^{j_\alpha + j_\beta + J+1} \hat{k} (j_\alpha j_\beta j_a j_b \bar{l}_\alpha \bar{l}_\beta \bar{l}_a \bar{l}_b)^{\frac{1}{2}} \times \begin{Bmatrix} J & j_\beta & j_b \\ \frac{1}{2} & l_b & l_\beta \end{Bmatrix} \begin{Bmatrix} l & j_\alpha & \frac{1}{2} \\ j_a & l_a & J \end{Bmatrix} \begin{Bmatrix} l & j_\beta & J \\ l_a & k & l_\alpha \end{Bmatrix} \quad (21)$$

Finally let us consider the problem of conservation of particle number in the proton quasi-particle wavefunctions. Equation (9) corresponds to an ensemble of nuclei each with a different number of protons. However the average number of protons taken over the ensemble is equal to the number of valence protons. In order for this to be true for each proton quasi-particle state, the BCS equations must be solved for each state with proper regard for conserving the average number of particles. In doing this one blocks,¹¹ i.e., neglects, conjugate states one of which is occupied by the odd particle. In general, the v 's for the various single particle states depend on the quasi-particle state, but since we consider states with only one quasi-particle, their different spins and parities automatically make them orthogonal. For many nuclear properties simpler treatments than blocking are justified, but the errors in the eigenvalues are not small if the neutron-proton interaction is treated with basis states generated by operating with the various quasi-particle creation operators on the same BCS solution. If the v 's are chosen so that the expectation value of the number operator is n in the ground state, then a

one quasi-particle state for an orbital well below the Fermi surface gives an expectation value $\sim(n+1)$. Those states for orbitals well above the Fermi surface have an expectation value of $\sim(n+1)$. Therefore, the neutron in La^{140} interacts with a different average number of protons depending on the state of the quasi-proton. Thus when blocking is not considered, states which are predominantly hole have a higher energy relative to particle states. In model calculations, Nilsson¹⁶ has shown that the blocking method may unduly reduce the pairing correlation and that corrections due to blocking are often spurious when states with different seniority are being compared. However, since we are considering only seniority one states in the proton system, these considerations are not important.

III. NUMERICAL SOLUTIONS FOR La^{140}

Before evaluating the n-p force matrix elements, we seek a self-consistent iterative solution for the coupled BCS equations in order to determine the quasi-proton energies and wavefunctions. To solve these equations it is necessary to specify a p-p interaction (we have chosen a pairing force) and a set of single particle levels. For single particle levels we choose the set used by Tamura and Udagawa¹⁷ but we have altered $1g_{7/2} - 2d_{5/2}$ spacing to agree with the value obtained by Kisslinger and Sorensen.¹ The value of the pairing force matrix element was chosen so that it reproduces the experimental $1g_{7/2} - 2d_{5/2}$ separation in La^{139} . These parameters, the quasi-particle energies, and the occupation probabilities are given in Table 1.

We would like to take both single particle proton energies and the single particle neutron energies from experimental data for one nucleon outside the Sn^{132} core. Unfortunately this is not now possible and so in the case of protons, we have resorted to the use of calculated values for these parameters. However for the neutrons we have taken the position of the single particle levels from the experimental work of Fulmer et al.¹⁸ for Ce^{141} . Since this nucleus has one neutron and eight protons outside the Sn^{132} core, it is necessary to correct for the effect of n-p residual interaction on the spacing of these levels. Considering only first order effects, we find that

$$\epsilon_{j_a}^- = \epsilon_{j_a}^+ - \sum_{j_\gamma J'} \frac{\hat{J}'}{\hat{j}_a} \langle \bar{j}_\gamma \bar{j}_a J' | V_{np} | \bar{j}_\gamma \bar{j}_a J' \rangle v_\gamma^2 \quad (22)$$

where $\epsilon_{j_a}^-$ is the value for Sn^{133} and $\epsilon_{j_a}^+$ is the experimental value for Ce^{141} . The

v_γ 's represent the unblocked solution of Ce^{140} (see Table 1). Finally for the theory as formulated, we need to specify five parameters for the n-p interaction. Four of these are the strengths of the interaction in the triplet-even, triplet-odd, singlet-even, and singlet-odd states of relative motion. The fifth parameter is denoted by X and is given by

$$X = vr_0^2 \quad (23)$$

Although v and r_0 have different physical meanings, only X has mathematical relevance. As we explained in the last section, the values of these parameters are sensitive to the truncation of the shell model and the quasi-particle states. Since we expect that the model will have validity only for the fourteen negative parity states that are predominately from the configurations generated by the $1g_{7/2}$ and $2d_{5/2}$ proton quasi-particle states and the $2f_{7/2}$ neutron state, we have truncated the calculation to include only the $1g_{7/2}$, $2d_{5/2}$, and the $2d_{3/2}$ quasi-proton states and the $2f_{7/2}$, $3p_{3/2}$, $2f_{5/2}$ neutron shell model states. Each of these three quasi-proton states is specified by the five shell model orbitals that are listed in Table 1. To specify these five n-p force parameters we have used a least squares search technique to find the minimum of X^2 in this five dimensional parameter space, where X^2 is the sum of the squares of deviations in energy after the theoretical spectrum has been normalized to the experimental 7- state at 284 keV. We chose this state rather than the ground state because in our theoretical model this state has no configuration mixing and experimentally this also seems to be true.⁶ However experimentally it is found that the 5- ground state is strongly admixed between the

$1g_{7/2}$ and $2d_{5/2}$ proton states and so its energy position will be sensitive to the magnitude of an off-diagonal matrix element and also to the relative energy spacing of the $1g_{7/2}$ and $2d_{5/2}$ quasi-proton states. This latter quantity might be expected to change slightly from La^{139} to La^{140} because the neutron-proton quadrupole interaction will lower the energy of the vibrational phonon. In Figure 1 we compare the experimental level scheme with the theoretical predictions for these states. In Table 2 we list the energies and wavefunctions for all levels below 600 keV in excitation energy. The mean deviation between the experimental and theoretical spectra is 52 keV. This spectrum was generated with a neutron-proton force having $V_{TE} = -37.00$ MeV, $V_{TO} = -11.10$ MeV, $V_{SE} = -37.00$ MeV, $V_{SO} = 11.10$ MeV, and $X = 1.0$.

IV. WAVEFUNCTIONS

A. (d,p) Spectroscopic Factors

Under certain general assumptions,¹⁹ the differential cross section for the (d,p) reaction may be written as

$$\frac{d\sigma^\gamma}{d\Omega} = \frac{\hat{J}_f}{J_i} \sum_l S_l^\gamma \phi(l, Q, \theta) \quad (24)$$

where J_i is the spin of the target nucleus, J_f is the spin of the residual nucleus, and S_l^γ is the spectroscopic factor. The last factor, called the single particle cross section, is assumed to depend on the Q-value, the orbital angular momentum of the stripped neutron, and the angle of the outgoing proton. The spectroscopic factor is a sum of overlap integrals

$$S_l^\gamma = \sum_{j=l \pm \frac{1}{2}} I_\gamma^2(l, j) \quad (25)$$

If we write a state vector in La^{140} as

$$\Psi_{JM}^\gamma \equiv |\gamma JM\rangle = \sum_{j_\alpha j_a} c_{j_\alpha j_a}^\gamma |\bar{j}_\alpha^\circ \bar{j}_a^\circ JM\rangle \quad (26)$$

then the overlap integral for the state labeled γ is

$$I_\gamma(l, j) = \langle \Psi_{JM}^\gamma | [\Psi_\beta \psi_b]^J_M \rangle = c_{j_\beta j_b}^\gamma \quad (27)$$

Here Ψ_β denotes the target ground state which we assume to be a pure $1g_{7/2}$ quasi-proton state, and ψ_b represents the spin angle part of the stripped neutron's wavefunction.

No experimental evidence exists from the (d,p) reaction⁶ for l -mixing in the states observed to 580 keV excitation. Therefore experimental spectroscopic factors are extracted by assuming that the $2f_{7/2}$ state is the only single particle neutron component in the wavefunctions and by assuming that the 7^- state at 284 keV is entirely from the $1g_{7/2}$ quasi-proton configuration.

In Table 3 the experimental S_3 values are compared with the calculated values from the wavefunctions that are listed in Table 2. Also in Table 3 we compare the theoretical and experimental differential cross sections normalized to the value for the 7^- state at 284 keV. In computing the theoretical cross sections, we have used the DWBA single particle cross sections obtained from fitting angular distributions for 65° in the laboratory frame.⁶ The experimental intensities for the 31 keV, the 43 keV, and the 38 keV states were deduced by assuming that the $1g_{7/2}$ and $2d_{5/2}$ proton states and the $2f_{7/2}$ neutron states were the only components in the state vectors. This was consistent with the total unresolved intensity of these three states. This calculation underestimates configuration mixing between the $1g_{7/2}$ and $2d_{5/2}$ proton quasi-particle states for spins 1, 2, and 3 and this is consistent with the incorrect energy ordering in the lowest states. If one allows the spacing between the $1g_{7/2}$ and $2d_{5/2}$ quasi-proton states to be an additional parameter, then a slightly improved energy fit and much better (d,p) predictions can be obtained. Insofar as this parameter has any physical significance, it probably simulates the first order effects of particle phonon coupling. The predictions for the other states seem to be in good agreement with the (d,p) data.

B. M1 Transition Probabilities

This discussion of gamma transition probabilities is based on chapter 17 of de-Shalit and Talmi.¹⁰

In the long wave approximation, the transition probability is given by

$$T_t(L) = \frac{8\pi(L+1)}{L[(2L+1)!!]} \frac{1}{\hbar} \left(\frac{\Delta E}{\hbar c} \right)^{2\lambda+1} B_t(L) \quad (28)$$

where L is the multipolarity of the transition, ΔE is the energy of the transition, t is an index so that $t = e$ denotes an electric multipole and $t = m$ denotes a magnetic multipole. The reduced transition rate is given by

$$B_t(L) = \sum_{M, M_f} |\langle \alpha_f J_f M_f | O_M^{tL}(p) + O_M^{tL}(n) | \alpha_i J_i M_i \rangle|^2 \quad (29)$$

Here $O_M^{tL}(\tau)$ is the one body multipole moment operator where $\tau = 1$ denotes a proton and $\tau = 2$ denotes a neutron. In general, for any one body operator, we have

$$\begin{aligned} & \langle \bar{j}_\alpha \bar{j}_\beta J_\beta M_\beta | O_M^{tL}(\tau) | \bar{j}_\alpha \bar{j}_\beta J_\beta M_\beta \rangle = \\ & R(\bar{j}_\alpha \bar{j}_\beta \tau) \langle \bar{j}_\alpha \bar{j}_\beta J_\beta M_\beta | O_M^{tL}(\tau) | \bar{j}_\alpha \bar{j}_\beta J_\beta M_\beta \rangle \end{aligned} \quad (30)$$

The factor $R(\bar{j}_\alpha \bar{j}_\beta \tau)$ is given by

$$\begin{aligned} R(\bar{j}_\alpha \bar{j}_\beta \tau) &= F(\bar{j}_\alpha \bar{j}_\beta) [u_{\beta\alpha} u_{\alpha\beta} - (-1)^T v_{\beta\alpha} v_{\alpha\beta}] \delta_{\tau,1} \\ &+ (1 - \delta_{\tau,1}) \end{aligned} \quad (31)$$

where $(-1)^T = -1$ for operators which are odd under time reversal and $(-1)^T = 1$ for operators which are even. Equation(29) can then be written as

$$B_t(L) = \frac{1}{J_i} \sum_{\substack{J_\alpha J_a \\ J_\beta J_b \\ \tau}} C_{J_\alpha J_a J_f}^{\alpha_f} C_{J_\beta J_b J_i}^{\alpha_i} R(\bar{J}_\alpha \bar{J}_\beta \tau) \langle \bar{J}_\alpha \bar{J}_a J_f || O^{tL}(\tau) || \bar{J}_\beta \bar{J}_b J_i \rangle^2 \quad (32)$$

After decoupling the reduced matrix element, equation (32) may be written as

$$B_t(L) = \hat{J}_f \sum_{\substack{J_\alpha J_a \\ J_\beta J_b}} C_{J_\alpha J_a J_f}^{\alpha_f} C_{J_\beta J_b J_i}^{\alpha_i} \quad (33)$$

$$\times [(-1)^{J_\alpha + J_a + J_i + L} R(\bar{J}_\alpha \bar{J}_\beta 1) \left\{ \begin{matrix} J_\alpha J_f J_a \\ J_i J_\beta L \end{matrix} \right\} \langle \bar{J}_\alpha || O^{tL}(p) || \bar{J}_\beta \rangle \delta_{\bar{J}_a, \bar{J}_b}]$$

$$+ (-1)^{J_\alpha + J_b + J_f + L} \left\{ \begin{matrix} J_a J_f J_\alpha \\ J_i J_b L \end{matrix} \right\} \langle \bar{J}_a || O^{tL}(n) || \bar{J}_b \rangle \delta_{\bar{J}_\alpha, \bar{J}_\beta}]^2$$

Experimental information exists only for M1 transition probabilities.⁶ The reduced single particle diagonal matrix element for the M1 multipole operator is

$$\langle n \frac{1}{2} l j || O^{m1} || n \frac{1}{2} l j \rangle = [(3/4\pi) j(j+1)(2j+1)]^{\frac{1}{2}} g_{lj} (e\hbar/2Mc) \quad (34)$$

where g_{lj} is the gyromagnetic ratio for the state that has orbital angular momentum l and spin j . The non-diagonal elements with our phase conventions are given by

$$\langle n' \frac{1}{2} l' j' \| Q^{ml} \| n \frac{1}{2} l j \rangle = (-1)^{j-l+\frac{1}{2}} (g_l - g_s) \left[\frac{3}{4\pi} \frac{2l(l+1)}{(2l+1)} \right]^{\frac{1}{2}} \frac{e\hbar}{2Mc} \delta_{l,l'} \delta_{n,n'} \quad (35)$$

where g_s and g_l are the spin and orbital angular momentum gyromagnetic ratios.

For convenience we define dimensionless reduced transition probabilities B' by

$$B'_m(1) = (4\pi/3) (e\hbar/2Mc)^{-1} B_m(1) \quad (36)$$

If the transition energy is expressed in MeV, then

$$T_m(1) = 4.25 \times 10^{12} (\Delta E)^3 B'_m(1) \text{ sec}^{-1} \quad (37)$$

Since details of the interaction (e.g., neglect of tensor force) might considerably influence the amount of configuration mixing, we computed transition probabilities not only for the theoretical wavefunctions but also for experimental wavefunctions extracted from the (d,p) experiment.⁶ In Table 4 we tabulated the reduced transition probabilities and the transition probabilities for all possible M1 transitions between states below 579 keV in excitation. In theory I we use the wavefunctions from Table 2 and Schmidt limit gyromagnetic ratios. In theories II and III we used wavefunctions extracted from the (d,p) experiment.⁶ Of course only absolute values of the state vector's coefficients can be obtained. However since there are only two components in the expansion (see reference 6, Table 5), we can specify the vectors if we assume a sign for the off-diagonal matrix element of the Hamiltonian matrices of angular momentum one, two, and three. We choose this sign to agree with that of the theoretical

matrix element. Theory II uses Schmidt limit gyromagnetic ratios and theory III uses effective gyromagnetic ratios. These were deduced from magnetic moment data²⁰ on the nuclei La¹³⁹(1g_{7/2}), Pr¹⁴¹(2d_{5/2}), and Ce¹⁴¹(2f_{7/2}). The relative rates are normalized to the most intense gamma ray that depopulates each parent state. The experimental data are taken from the work of J. S. Geiger, et. al.⁷ The very limited experimental data do not offer a very good test of the theoretical wavefunctions or of the consistency of our model with (d,p) and gamma ray data. However both theories II and III give acceptable results. Underestimating the configuration mixing probably also explains the discrepancies that occur in theory I.

V. DISCUSSION

In this paper we have investigated the possibility of describing La^{140} in terms of two interacting particles, a shell model neutron and a quasi-proton. A quasi-proton is defined by searching for the vector which gives a minimum expectation value for the energy of each spin and parity in a class of vectors which are more general than those given in the Hartree-Fock approximation. Indeed the quasi-proton describes not only independent excitations of the Hartree-Fock type but also seniority zero correlations between pairs of protons. Thus in the shell model representation, the quasi-particle is partly particle in nature and partly hole. The diagonal neutron-proton matrix elements in the La^{140} problem (see equation (11)) reflect this by not only taking into account the energy of interaction between the neutron and conjugate pairs of protons but also by forming a linear combination of two body particle-particle and particle-hole matrix elements. Thus the calculation predicts that the lowest energy member of the $|(1g_{7/2})^0 2f_{7/2}; JM\rangle$ configuration is 6-. This is consistent with the Brennan and Bernstein rule²¹ for the ground state of particle hole odd-odd nuclei. The lowest member of the $|(2d_{5/2})^0 2f_{7/2}; JM\rangle$ configuration is also 6- which is consistent with the rules for particle-particle configurations. These results occur because the BCS calculations show that for 57 protons the $1g_{7/2}$ orbital is predominantly full and the $2d_{5/2}$ orbital is predominantly empty (see Table 1). The only way a 3- state could be the ground state would be by extensive configuration interactions. Experimentally⁶ the ground state appears to have large configuration admixtures but theoretically we get a very pure wave-function for the lowest 3- state in particular with respect to mixing of the $2d_{5/2}$ proton orbital.

The major discrepancies in the wavefunctions seem to be in states of spin 1, 2, and 3-. In all cases we appear to underestimate the amount of $1g_{7/2}$ and $2d_{5/2}$ proton mixing. This is reflected in the 1- and 3- states which have relatively poorer fits to the energy levels.

Attempts to explain La^{140} using a simple particle hole shell model failed as did the odd-odd quasi-particle model with a delta function for neutron-proton force. Use of a central force gives reasonable agreement between the theory and the known experimental levels. Although the wavefunctions for the lower spin states seem to underestimate configuration mixing between the $1g_{7/2}$ and $2d_{5/2}$ proton orbitals, use of the model wavefunctions obtained from the (d,p) experiment give results not inconsistent with the very limited gamma ray data. The theoretical model predicts that there are appreciable admixtures of higher neutron configurations in some of the low energy states. The energy resolution in the (d,p) experiment⁶ was sufficiently poor that angular distributions had to be made on groups of peaks. Therefore even appreciable admixtures in the lower spin states would not be detected in the angular distributions. The present calculation also predicts the location of the unobserved negative parity states of spin 4, 5, 6. It will be particularly interesting to find the second 6- state since it is predicted to be 200 keV below the 3- state and this prediction is quite insensitive to the neutron-proton interaction that was chosen. Whether the underestimate of configuration mixing is due to the choice of force, the truncation of basic states, or the neglect of phonon admixtures is problematic. The answer to this question must wait until realistic interactions and the serious study of the entire mass 140 region with Hartree-Fock Bogoliubov theory are undertaken.

VI. ACKNOWLEDGEMENTS

The author wishes to thank Professor John O. Rasmussen for many helpful suggestions and critically reading the manuscript. The advice and assistance of Dr. Jean Kern throughout the course of this work is also greatly acknowledged.

Footnotes and References

* This work was performed under the auspices of the U. S. Atomic Energy Commission.

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9. We adopt the following conventions for notational convenience:

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15. We adopt the convention that n equals the number of nodes (excluding $r = 0$ but including $r = \infty$) and that the nuclear size parameter ν equals the

m_0/n which is twice the value used by de-Shalit and Talmi.¹⁰

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Table 1.

Quasi- Particle State	Single Particle Energy	Quasi- Particle Energy	λ_p	$ \Delta_p $	V Coefficients				
					$1g_{7/2}$	$2d_{5/2}$	$1h_{11/2}$	$2d_{3/2}$	$3s_{1/2}$
Vacuum	-	-	0.441	0.884	0.8505	0.4823	0.2328	0.1728	0.1543
$1g_{7/2}$	0.00	0.0	0.386	0.713	0.8590	0.4165	0.1879	0.1387	0.1237
$2d_{5/2}$	1.00	0.166	0.194	0.718	0.7939	0.3557	0.1725	0.1302	0.1169
$1h_{11/2}$	2.18	1.091	0.156	0.763	0.7747	0.3591	0.1791	0.1360	0.1224
$2d_{3/2}$	2.88	1.727	0.149	0.774	0.7712	0.3607	0.1810	0.1376	0.1239
$3s_{1/2}$	3.20	1.931	0.147	0.778	0.7701	0.3613	0.1816	0.1381	0.

Table 2.

Spin	Energy (MeV)	STATE VECTORS								
		$1g_{7/2}$	$1g_{7/2}$	$1g_{7/2}$	$2d_{5/2}$	$2d_{5/2}$	$2d_{5/2}$	$2d_{3/2}$	$2d_{3/2}$	$2d_{3/2}$
Parity		$2f_{7/2}$	$3p_{3/2}$	$2f_{5/2}$	$2f_{7/2}$	$3p_{3/2}$	$2f_{5/2}$	$2f_{7/2}$	$3p_{3/2}$	$2f_{5/2}$
0-	0.620	0.9530					-0.2130		0.2152	
1-	0.027	0.2037		0.0049	-0.9387	-0.2393	0.0940		0.1056	-0.0030
1-	0.366	0.9530		0.1594	0.1723	0.1741	0.0274		0.0576	0.0480
2-	0.050	0.2362	-0.0195	0.0024	0.8962	0.1444	-0.0946	0.3192	0.0202	-0.0921
2-	0.155	0.9611	-0.1088	0.0503	-0.2426	-0.0137	0.0236	-0.0158	0.0292	0.0332
3-	0.069	0.9892	-0.1261	0.0474	0.0295	0.0121	0.0028	0.0390	0.0289	0.0050
3-	0.391	0.0325	0.0506	0.0188	-0.9760	-0.1833	0.0095	0.0589	0.0733	-0.0293
4-	0.025	0.9938	-0.0926	0.0266	0.0248	0.0304	-0.0058	0.0356		0.0131
4-	0.196	0.0351	-0.0250	0.0061	-0.9353	-0.3064	0.0805	-0.1424		0.0515
5-	0.015	0.9900	-0.1270	0.0280	0.0106		0.0231	0.0491		
5-	0.437	0.0149	0.1236	0.0040	-0.9533		-0.0835	0.2621		
6-	-0.165	0.0867		-0.0017	0.9962					
6-	0.032	0.9962		-0.0000	-0.0867					
7-	0.284	1.0								

Table 3.

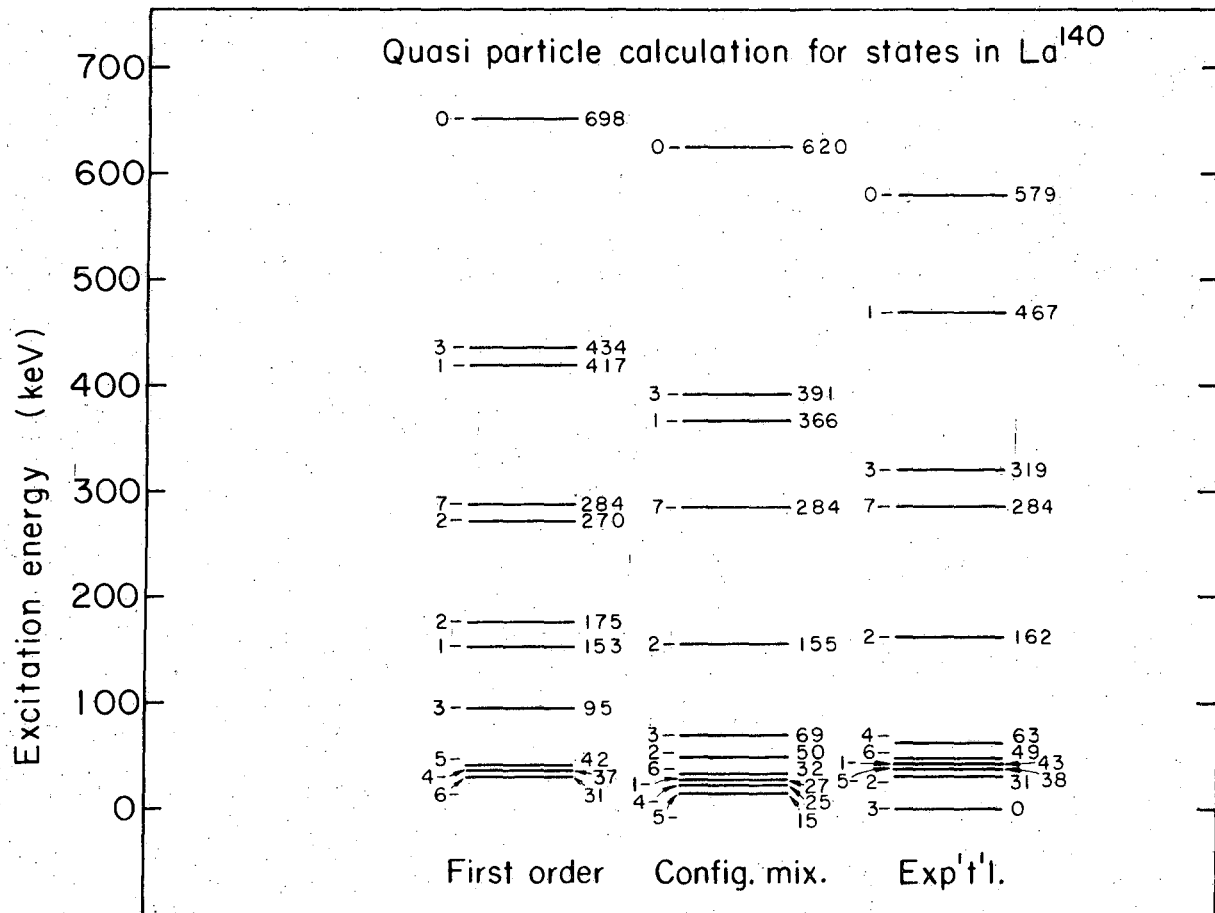
Energy keV	Spin π	$[S_3]_{th}$	$[S_3]_{exp}$	Relative Intensity Theoretical	Relative Intensity Experimentally
579	0-	0.91	1.0	0.06	0.05
43	1-	0.04	0.66	0.01	0.13
467	1-	0.93	0.35	0.19	0.07
31	2-	0.06	0.48	0.02	0.16
162	2-	0.93	0.52	0.32	0.17
0	3-	0.98	0.81	0.47	0.38
319	3-	0.001	0.18	0.00	0.09
63	4-	0.99	1.0	0.60	0.58
38	5-	0.98	1.0	0.74	0.73
49	6-	0.99	1.0	0.86	0.85
284	7-	1.0	1.0	1.0	1.0

Table 4.

Spin Parity Initial	Energy (MeV)	Spin Parity Final	Energy (MeV)	Transi- tion Energy (MeV)	Type	Rate TH.-I Sec ⁻¹	Rela- tive Rate TH.-I	Rate TH.-II Sec ⁻¹	Rela- tive Rate TH.-II	Rate TH.-III Sec ⁻¹	Rela- tive Rate TH.-III	Experi- mental Branching Ratios
0-	0.580	1-	0.466	0.114	M1	8.80×10^{10}	0.23	3.70×10^{10}	0.0051	3.98×10^{10}	0.0051	0.00069
		1-	0.043	0.537	M1	3.90×10^{11}	1.0	7.28×10^{12}	1.0	7.84×10^{12}	1.0	1.0
1-	0.466	2-	0.162	0.304	M1	1.00×10^{12}	1.0	2.59×10^{12}	1.0	1.86×10^{12}	1.0	1.0
		1-	0.043	0.423	M1	7.87×10^{10}	0.079	1.90×10^{12}	0.73	1.24×10^{12}	0.67	0.56
		2-	0.030	0.436	M1	1.24×10^{12}	1.2	1.52×10^{12}	0.59	5.34×10^{11}	0.29	0.53
2-	0.162	1-	0.043	0.119	M1	1.75×10^{10}	0.12	9.54×10^8	0.004	1.04×10^8	0.0005	0.011
		2-	0.030	0.132	M1	2.22×10^9	0.016	7.56×10^9	0.032	6.98×10^9	0.036	0.047
		3-	0.	0.162	M1	1.40×10^{11}	1.0	2.36×10^{11}	1.0	1.96×10^{11}	1.0	1.0
1-	0.043	2-	0.030	0.013	M1	2.45×10^8	1.0	1.74×10^8	1.0	1.35×10^8	1.0	1.0
2-	0.030	3-	0.	0.030	M1	9.05×10^7	1.0	8.03×10^5	1.0	3.65×10^7	1.0	1.0

Figure Captions

Fig. 1. Comparison of energies between calculated and experimentally determined states in ^{140}La .



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