University of California

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THE CP-NONCONSERVING DECAY $K_1^0 \to \pi^+ \pi^- \pi^0$

Berkeley, California

UCRL-11938
Euratum

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

Sept. 21, 1965

ERRATUM

TO: All recipients of UCRL-11938

FROM: Technical Information Division

Subject: UCRL-11938, "THE CP-NONCONSERVING DECAY $K_1^0 \to \pi^+ \pi^- \pi^0$" by Jared A. Anderson, Frank S. Crawford, Jr., Robert L. Golden, Donald Stern, Thomas O. Binford, and V. Gordon Lind, dated February 5, 1965. [Phys. Rev. Letters 14, 475 (1965)].

Our paper contains an internal inconsistency in sign convention. Our corrected results for $y = [(m_2 - m_1)/|m_2 - m_1|]\ \mathrm{Im}\,(a_1/a_2)$ in Eqs. (2) and (3) are $y = -1.00 \pm 0.65$ and -0.80 ± 0.55, respectively. The sign of y should also be reversed in footnotes 7 and 10, and in the labeling of Figs. 1 and 2. We are indebted to Y. Tomozawa for his observation.

ADDENDUM

The CP-Nonconserving Decay $K_1^0 \rightarrow \pi^+\pi^-\pi^0$

Jared A. Anderson, Frank S. Crawford, Jr., Robert L. Golden,
Donald Stern, Thomas O. Binford, and V. Gordon Lind

[Phys. Rev. Letters 14, 475 (1965)]

April 8, 1965

Glashow and Weinberg have pointed out to us that if the decay $K_1^0 \rightarrow (+-0)$ is allowed, its amplitude should be imaginary, relative to that for $K_2^0 \rightarrow (+-0)$.[1] Thus we expect $x = 0$ in $a_1(+-0)/a_2(+-0) \equiv x + iy$. Our result is $x = +0.25 \pm 0.65$; $y' \equiv y(m_2 - m_1)/|m_2 - m_1| = +1.00 \pm 0.65$.[2] Thus $x = 0$ is consistent with our result.

Imposing the constraint $x = 0$ and reanalyzing our 18 events, we find

$$y' = +0.90 \pm 0.50. \quad (1)$$

The corresponding intensity ratio is

$$\Gamma_1(+-0)/\Gamma_2(+-0) = y^2 = 0.81^{+1.15}_{-0.65}. \quad (2)$$

We find odds of 10 to 1 that $\Gamma_1(+-0)/\Gamma_2(+-0)$ is less than 2.5, and 100 to 1 that it is less than 5. The effect of the observation of Glashow and Weinberg is to reduce our upper limit on $\Gamma_1(+-0)/\Gamma_2(+-0)$ by a factor of two.

We still cannot rule out $\Gamma_1(+-0)/\Gamma_2(+-0) = 0$.

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1. Sheldon L. Glashow and Steven Weinberg, accompanying paper.

2. J. A. Anderson et al., Phys. Rev. Letters 14, 475 (1965).

Review Letters

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February 5, 1965

The CP-Nonconserving Decay $K_1^0 \rightarrow \pi^+\pi^-\pi^0$ †

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February 5, 1965

In our paper[1] on the absolute decay rate $\Gamma_2(+-0)$ for $K_2^0 \rightarrow \pi^+ \pi^- \pi^0$, we made the observation that the time distribution of our sixteen $\pi^+ \pi^- \pi^0$ events is completely compatible with $\Gamma_1(+-0) = 0$, where $\Gamma_1(+-0)$ is the rate for $K_1^0 \rightarrow \pi^+ \pi^- \pi^0$. Thus our results are consistent with CP invariance.[2] In reference 1 we imposed the constraint $\Gamma_1(+-0) = 0$ in obtaining the result $\Gamma_2(+-0) = (2.90 \pm 0.72) \times 10^6$ sec^{-1}.

We have discovered that two good events were inadvertently omitted from that paper.[3] Adding these two events to the sample of reference 1, we find that $\Gamma_1(+-0)$ is still consistent with zero. Our corrected result is $\Gamma_2(+-0) = (3.26 \pm 0.77) \times 10^6$ sec^{-1}, still in good agreement with the prediction $\Gamma_2(+-0) = (2.87 \pm 0.23) \times 10^6$ sec^{-1} of the $\Delta I = 1/2$ rule.

The discovery[4] that CP invariance may not hold in neutral kaon decay admits the possibility that $\Gamma_1(+-0)$ is of the same order of magnitude as $\Gamma_2(+-0)$.[5] In this paper we reanalyze our eighteen events without the assumption that $\Gamma_1(+-0)$ is zero, and thus without the assumption of CP invariance.

Let a_1 and a_2 denote the complex amplitudes for K_1^0 and K_2^0 decay into $\pi^+ \pi^- \pi^0$, where K_1^0 and K_2^0 refer to the short- and long-lived decay eigenstates; let x and y denote the real and imaginary parts of $a_1/a_2 = x + iy$. Then for K^0 produced at time t = 0 via the reaction $\pi^- + p \rightarrow \Lambda + K^0$, the total decay rate into $\pi^+ \pi^- \pi^0$ has the form[6]

$$\Gamma(+-0) = 1/2 |a_2|^2 \ |1 + (x + iy) \exp(-t/2\tau_1 + imt)|^2, \qquad (1)$$

where $|a_2|^2 = \Gamma_2(+-0)$, $|a_1|^2 = \Gamma_1(+-0)$, $m = m_2 - m_1$, and where we can (for our experiment) take the K_2^0 lifetime to be effectively infinite as far as the time dependence of (1) is concerned. For each event we construct an a priori decay probability p_i based on Eq. (1)[7] and normalized to unity for decay between t = 0 and t = T_i, where T_i is the potential time for the event.[8] We then construct the likelihood function $L(x, y) = \prod_i p_i$. From a contour plot of L(x, y) we obtain the results:[9, 10, 11]

$$x = + 0.25 \pm 0.65, \qquad y = + 1.00 \pm 0.65. \qquad (2)$$

Figure 1 shows a comparison of the data with the time distribution corresponding to the result (2).[12]

In the above analysis we made use of only the time distribution of the 18 events. We now reanalyze these events with the additional hypothesis that $\Gamma_2(+-0)$ satisfies the $\Delta I = 1/2$ rule, which predicts $\Gamma_2(+-0) = (2.87 \pm 0.23) \times 10^6$ sec^{-1}.[13] We construct a likelihood function $L_1(x, y)$ by multiplying the likelihood L(x, y) by the Poisson probability $e^{-\bar{n}}\bar{n}^n/n!$; here n = 18 is our observed total number of events, and $\bar{n} = \bar{n}(x, y)$ is the total predicted number of events calculated by combining the $\Delta I = 1/2$ rule, the size of our sample of K^0, the time distribution (1), and our geometrical detection efficiency $\epsilon(t)$, which is the smooth curve plotted in Fig. 1. In Fig. 2 we show a contour plot of $L_1(x, y)$. From this plot we obtain the results

$$x = + 0.25 \pm 0.55, \qquad y = + 0.80 \pm 0.55. \qquad (3)$$

The most-likely value for $x^2 + y^2 \equiv \Gamma_1(+-0)/\Gamma_2(+-0)$ is 0.70. If we integrate over the relative phase of a_1 and a_2 in the likelihood function we obtain a probability distribution for $\Gamma_1(+-0)/\Gamma_2(+-0)$.

We conclude that the odds are 9 to 1 that $\Gamma_1(+-0)/\Gamma_2(+-0)$ is less than 5. Our best estimate for the amplitude ratio $a_1(+-0)/a_2(+-0) \equiv x + iy$ is given by Eq. (3). We cannot rule out $a_1(+-0)/a_2(+-0) = 0$.

We are grateful to Sheldon L. Glashow for stimulating discussions, and to Luis W. Alvarez for his interest and support, and for valuable comments.

Footnotes and References

† Work performed under auspices of the U. S. Atomic Energy Commission.

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1. D. Stern, T. O. Binford, V. G. Lind, J. A. Anderson, F. S. Crawford, Jr., and R. L. Golden, Phys. Rev. Letters 12, 459 (1964).

2. In K(neutral) $\rightarrow \pi^+ \pi^- \pi^0$, pion angular-momentum states higher than S states are strongly suppressed by angular-momentum barrier-penetration factors. If the pions are in S states, $\pi^+ \pi^- \pi^0$ has CP = -1; hence $K_1^0 \rightarrow \pi^+ \pi^- \pi^0$ is forbidden by CP invariance.

3. In the notation of Table I of reference 1, they are event 1845161: χ^2(prod) = 3.4, χ^2(dec) = 1.7, p_{K^0} (lab) = 590 ± 9, t_{K^0} = 5.31; T_{K^0} = 14.7; event 1849320: 1.1, 1.1, 628 ± 8, 21.1, 31.1.

4. J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters 13, 138 (1964); see also A. Abashian, R. J. Abrams, D. W. Carpenter, G. P. Fisher, B. M. K. Nefkens, and J. H. Smith, Phys. Rev. Letters 13, 243 (1964).

5. See, for example, S. L. Glashow, Phys. Rev. Letters 14, 35 (1965).

6. Equation (1) is not exact; it is based on the approximation a = 1 and b = 0 in

$$K^0 = a(|K_1\rangle + |K_2\rangle)/\sqrt{2} + b(|K_1\rangle - |K_2\rangle)/\sqrt{2} ,$$

whereas actually a - 1 and b are each of order 10^{-3} according to reference 4. For the experiment reported here this contributes a negligible correction to Eq. (1), because we can determine a_1/a_2 only to about ±1, not to $\pm 10^{-3}$.

7. We use $\tau_1 = 0.89 \times 10^{-10}$ sec, and $|m| = 0.75 \times 10^{10}$ sec^{-1} (which is $0.67/\tau_1$). The choice 0.75 is our weighted average of the values summarized in Table I of T. Fujii, J. V. Jovanovich, F. Turkot, and G. T. Zorn, Phys. Rev. Letters 13, 253 (1964). Our result (2) is however, quite insensitive to the precise value we choose for $|m|$, for $|m|$ between 0.4 and $1.1 \times 10^{+10}$ sec^{-1}. For example, for $|m| = 0.50$, we obtain $x = +0.6 \pm 0.7$, $y = +1.1 \pm 0.7$; for $|m| = 1.00$ we find $x = 0.1 \pm 0.7$, $y = +0.9 \pm 0.7$.

8. The decay times t_i are listed in Table I of reference 1. The potential times T_i for the 18 events are as follows (in the order of that table, and in units of 10^{-10} sec): 11.88, 24.24, 15.65, 8.12, 7.72, 4.13, 6.92, 17.62, 13.06, 11.76, 9.83, 8.59, 14.20, 3.99, 153.0, 22.4; 14.7, and 31.1.

9. The quoted errors correspond to a decrease of the likelihood function L (x, y) by a factor $e^{-1/2}$ from its maximum value. We prefer to give our results in terms of x and y rather than in terms of $\Gamma_1/\Gamma_2 = x^2 + y^2$ and the phase $\phi = \arg(a_1/a_2)$, because the likelihood function L (x, y) is to a fair approximation given by L = f(x) f(y), where f(x) and f(y) are nearly Gaussian in shape. The probability distribution for Γ_1/Γ_2 is, on the contrary, very non-Gaussian.

10. The sign of x is determined (in principle) by this experiment, but the sign of y is not separable from that of $m_2 - m_1$. Thus our result (2) for y is actually $[(m_2 - m_1)/|m_2 - m_1|]\ y = +1.00 \pm 0.65$. In writing (2) we take $m_2 - m_1$ to be positive.

11. If the result (2) were known to be exact, we would have to assign 18% of the observed counts to K_1^0 decay. Then our measured value of $\Gamma_2(+-0)$ would be corrected by a factor of 0.82 to $\Gamma_2(+-0) = 0.82 \times (3.26 \pm 0.77)$ $= (2.65 \pm 0.63) \times 10^6$ sec^{-1}.

12. Inspection of Fig. 1 suggests that (within the large statistical uncertainties) $x = y = 0$ fits the data slightly <u>better</u> than the maximum-likelihood result (2). This slight apparent inconsistency is mainly due to the fact that in $L(x, y)$ we make use of the individual decay times t_i and potential times T_i of the 18 events; each t_i is correlated with its own T_i in the factor p_i. The function $\epsilon(t)$ in Fig. 1 is on the contrary based on a smoothed distribution of potential times obtained from several thousand associated-production events.

13. The prediction $\Gamma_2(+-0) = 2.87 \times 10^6$ sec^{-1} is based on a weighted average of results for $\Gamma_+(+00)$ compiled in Table I of G. Alexander and F. S. Crawford, Jr., Phys. Rev. Letters 9, 68 (1962).

Figure Captions

Fig. 1. Time distribution. The smooth curve is the geometrical detection efficiency $\epsilon(t)$, normalized so that it represents dN/dt for the 18 events, if they are due to K_2^0 only, i.e., $x = y = 0$. The histogram corresponds to dN/dt predicted by the maximum-likelihood result (2). The points with error flags are the observed events.

Fig. 2. Contours of equal likelihood for $x = \mathrm{Re}(a_1/a_2)$ and $y = \mathrm{Im}(a_1/a_2)$, where a_1 and a_2 are the amplitudes for K_1^0 and K_2^0 decay into $\pi^+ \pi^- \pi^0$. The contours labeled 1 std dev, 2 std dev, and 3 std dev correspond to a decrease in the likelihood function $L_1(x, y)$ by factors $e^{-1/2}$, $e^{-4/2}$, and $e^{-9/2}$ from L_1 (max).

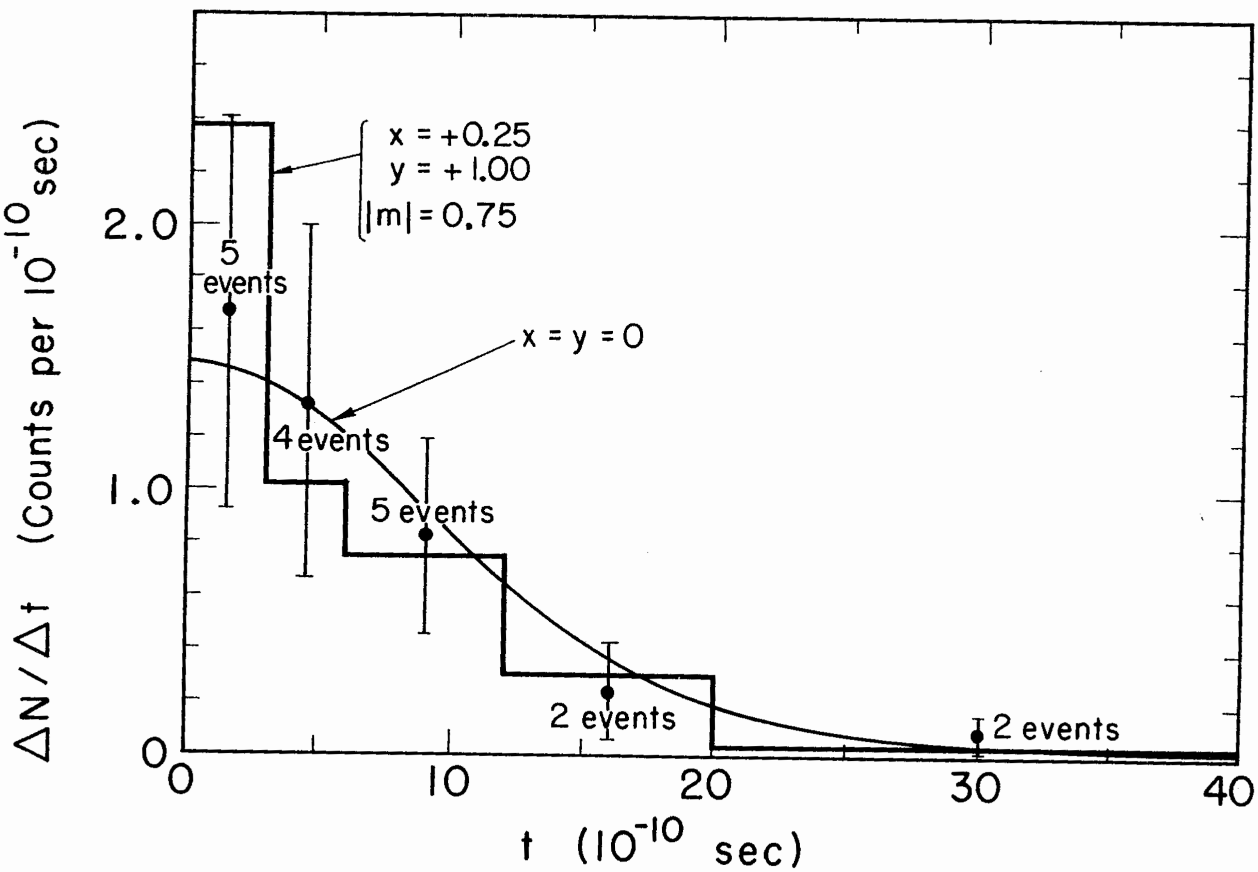
x = +0.25
y = + 1.00
|m| = 0.75
x = y = 0
5 events
4 events
5 events
2 events
2 events
2.0
1.0
0
0
10
20
30
40
ΔN/Δt (Counts per 10^-10 sec)
t (10^-10 sec)
MUB-5203

Fig. 1

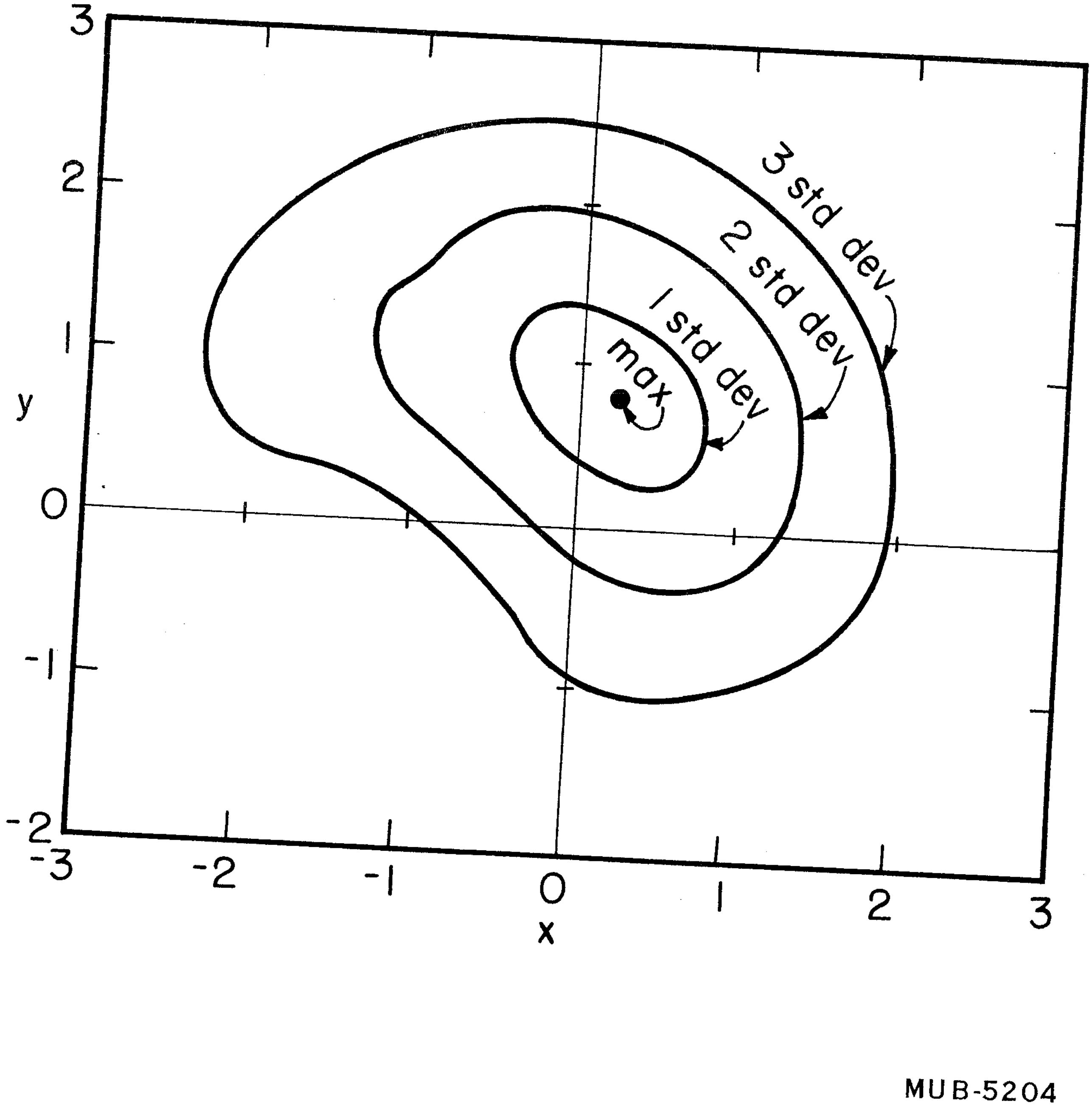

Fig. 2