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**UNIVERSITY OF CALIFORNIA,
IRVINE**

**Heterogeneity in Motorists' Preferences for Travel Time and Time Reliability:
Empirical Finding from Multiple Survey Data Sets and Its Policy Implications**

DISSERTATION

**Submitted in partial satisfaction of the requirements for the degree of
DOCTORAL OF PHILOSOPHY
in Economics**

by

Jia Yan

**Dissertation Committee:
Professor Kenneth A. Small, Chair
Professor David Brownstone
Professor Justin L. Tobias**

2002

The dissertation of Jia Yan
is approved and is acceptable in quality
and form for publication on microfilm:

Committee Chair

University of California, Irvine

2002

DEDICATION

Dedicated To

My Grandmother, Jing-Lan Xu

My parents, Da-Qi Yan, Shan-Shan Wang

My sister and brothers:

Ping Yan

Dan Yan

Jie Yan

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ABSTRACT OF THE DISSERTATION

Heterogeneity in Motorists' Preferences for Travel Time and Time Reliability: Empirical Finding from Multiple Survey Data Sets and Its Policy Implications

By

Jia Yan

Doctor of Philosophy in Economics

University of California, Irvine, 2002

Professor Kenneth A. Small, Chair

The deregulation experience in airline, banking, and telecommunication suggests that the heterogeneity in consumers' preferences has important policy significance. However, the varied nature in motorists' preferences has been hardly recognized in urban passenger transportation sector. In this public sector, the public authority generally offers a uniform class of services to all potential users. This dissertation employs the new advances in econometrics on survey data sets from road pricing experiment in Los Angeles area to study the diversity in motorists' preferences for travel time and travel time reliability. The empirical findings are used to explore the efficiency and distributional effects of road pricing that accounts for users' heterogeneity.

This dissertation found substantial heterogeneity in motorists' preferences for both travel time and travel time reliability. Furthermore, based on a simulation model, this dissertation found that road pricing policies catering to varying preferences can substantially increase efficiency while maintaining the same political feasibility as the current experiments. This dissertation also explores how to apply the recent developments in Bayesian econometrics to estimate the multinomial probit models combining different sources of data, which can be used to estimate the diversity in peoples' preferences with more flexibility in model specification.

PREFACE

This dissertation is based on the findings of several research projects in which I was involved. The first is the research project from 1998 to 1999 funded by University of California Transportation Center to study the viability of value pricing on roads, in which Prof. Kenneth Small is principle investigator. The main findings of this project were summarized as a co-authored paper with Prof. Small, “The Value of Value Pricing of Roads: Second-Best Pricing and Product Differentiation”, published in *Journal of Urban Economics*. Parts of contents of this paper appear in Chapter 4 of this dissertation. The second research project is the one from 1999 to 2001 funded by University of California Transportation Center and Brookings Institution to investigate the heterogeneity in motorists’ preferences, in which Prof. Small and Dr. Clifford Winston at Brookings Institution are principle investigators. This project is the basis for the paper, “Uncovering Motorists’ Preferences Using Revealed and Stated Preference Data”, which is co-authored with Prof. Small and Dr. Winston and appears in Chapters 1 and 2 of this dissertation.

During the revisions and modifications for this paper, I enriched the data sample by combining it with another data set collected by researchers at California Polytechnic State University at San Luis Obispo, under the leadership of Edward Sullivan, and with participation by Prof. Small and me. Additionally, I added some new policy simulations based on simulation model developed in the paper published in *Journal of Urban Economics*. The final version of the co-authored paper, with the title of “Uncovering the

Distribution of Motorists' Preferences for Travel Time and Reliability: Implications for Road Pricing", has been submitted for publication and parts of its contents appear in Chapters 2 and 4 of this dissertation.

Chapter 3 of this dissertation is drawn from work my own, in which I use Hierarchical Bayesian Analysis to estimate the heterogeneity in peoples' preferences on combined data sets.

INTRODUCTION

The experience with deregulation in airline, telecommunications, and banking has taught us the variety of consumers' preferences. Firms in these industries have learned to succeed in tough competition by offering a variety of prices and services that respond to different desires held by consumers (Winston, 1998). During this process, they have increased capacity, exploited niche markets, and learned to price discriminate. These results at least suggest that the heterogeneity in preferences has great policy significance in these industries.

Road transportation is not a typical market. In this public sector, the public authority generally offers a uniform class of service to all potential users. However, like customers in other industries, motorists may have different preferences. For example, some motorists might be willing to pay large amount to use express roadways that ensure little delay, while others may be willing to pay a modest fee to achieve this purpose. A uniform class of service in this case can not satisfy all road users.

Recently a few tentative steps have been taken toward catering to heterogeneity in motorists' preferences. In the Los Angeles, San Diego, and Houston areas some motorists now can make a choice between a free but congested roadway and a priced but free-flowing roadway. At the same time, researchers are increasingly recognizing the role of heterogeneity among people in understanding real transportation policies. For example, recent papers have highlighted the role of the heterogeneity in motorists' preferences for

time saving in understanding the politics and economics of road-pricing demonstration projects. Schemanske (1991, 1993), and Arnott et. al. (1992) show that with heterogeneous users, differential tolls on separate roadways may be superior to single toll. Verhoef and Small (2000), and Small and Yan (2001) show that the heterogeneity in value of time is important for evaluating congestion policies that offer pricing as an option, such as the three experiments in Los Angeles, San Diego, and Houston. Generally, the existence of heterogeneity favors these experiments because product differentiation then offers a greater advantage: those with high value of time reap more benefits from the high-priced option, while those with low value of time find it all the more important not to be subjected to policies aimed at the average users.

Very little research has focused on measuring the heterogeneity in motorists' preferences, including their willingness-to-pay for saving travel time and reducing travel time unreliability on roads, which are also called as value of time and of reliability respectively. Previous studies on measuring value of time used in travel concentrated on using data from peoples' actual choice (revealed preference (RP) data) for transportation mode. These measures can not represent motorists' value of time, because they capture the disutility that people attach to spending time on public transit. Additionally, although some of them have addressed the heterogeneity of people in value of time to some extent, they mainly focused on how the value of time varies with observed characteristics, such as income, trip purpose and travel mode (MVA consultancy et. al., 1987). Some recent studies have used hypothetical situations to obtain estimates more closely reflecting the value of time during automobile travel (Calfee and Winston, 1998; Calfee, Winston, and

Stempski, 2001); these are called stated preference (SP) studies. The stated preference studies are hampered by a doubt as to whether the behavior in hypothetical situations applies when motorists are confronted with similar choices in practice. Value of reliability is still a quite new topic and almost all the existing studies in this topic are based on stated preference data.

The three road pricing experiments in Los Angeles, San Diego, and Houston provide us the opportunity to investigate the heterogeneity in motorists' preferences in an automobile-dominate environment. Some researchers have used the revealed preference data from these experiments to investigate motorists' choice behavior on route, time-of-day, and mode choice. Parkany (1999) and Li (2001) used data collected in the 1997 and 1996 from Los Angeles road pricing experiment (California State Route 91—i.e. SR91) respectively to examine the determinants of SR91 toll lane use. Parkany especially focused on studying how travelers use traffic information to make route choice. Based on newly collected data in November of 1999, Yan et. al. (2002) modeled travelers' choices of route, time-of-day, and mode on SR91, and different travel demand elasticities were calculated based on estimate results. They also estimated SR91 commuters' value of time, which is in the range of \$13 - \$16 per hour. Lam and Small (2001) used data collected in the summer of 1998 on SR91 to measure travelers' willingness to pay to reduce both travel time and travel time uncertainty. Their estimated value of time is between \$5 and \$25 per hour, depending on different models. The most reliable model gives the estimate of \$23 per hour for value of time. The travel time unreliability in Lam and Small's research is defined as the difference between 90th percentile and median of

the distribution of travel time. Their best model shows that male would like to pay \$15 per hour and female would like to pay \$32 per hour to reduce this uncertainty in travel time.

Several researchers have modeled travel choice behavior using data from other locations. Using data from Interstate 15 (I15) road pricing experiment near San Diego, Brownstone et. al. (2000, 2001) estimated travelers' value of time and value of reliability. As in Lam and Small, travel-time unreliability is defined as the difference between 90th percentile and median of travel time distribution. Their data shows that the value of time is between \$25 and \$30 per hour, and value of reliability is about \$20 per hour. Ghosh (2001) compared the value of time estimates from I15 revealed preference data with ones from stated preference data also on I15 users. His results show that revealed preference data always gives higher estimates for both value of time and value of reliability.

Among above studies, only Ghosh (2001) investigated both the observed and unobserved heterogeneity in values of time and of reliability. He estimated heterogeneity in values of time and of reliability using RP and SP data respectively, and found substantial unobserved heterogeneity in SP data. Lam and Small (2001) addressed this issue to the extent that values of time and of reliability vary with income, trip distance, and gender.

This thesis is to investigate systematically the heterogeneity in motorists' values of time and of time reliability, as well as implications of the heterogeneity to road pricing

policies. It uses two survey data sets from SR91 road pricing experiment in estimation. The first one is a two-stage mail survey collected by Brookings Institution including both RP and SP surveys. The other one is a phone RP survey collected by researchers at California Polytechnic State University at San Luis Obispo (Cal Poly). Many econometric advances are used in this thesis, and they make it possible to address the following issues regarding the nature of motorists' preferences.

Unobserved Heterogeneity. Unobserved heterogeneity in preferences is the one that can not be explained by observed characteristics, and it can be captured by discrete choice models with random parameters. This thesis uses mixed-logit specification, which extends the stochastic part of random utility into two parts, one capturing unobserved heterogeneity in preferences, one with the extreme distribution standard for logit models. Mixed logit model can be estimated using Simulated Maximum Likelihood Estimation.

Revealed and Stated Preferences. Both RP and SP data have drawbacks. RP data from road pricing experiments is often hindered by strong correlation among key variables – travel time, money cost, and time reliability. SP data overcomes this problem, because the correlation among variables is controlled by researchers. However, SP data only represents peoples' choice behavior under hypothetical settings. The doubt about whether it can represent peoples' choice behavior in real world is reasonable. This thesis combines the advantages of these two types of data to investigate the heterogeneity in motorists' preferences.

Reliability. Travel time reliability is a critical influence on any mode and route choice, but it is hard to measure. In this thesis, the travel time reliability is specified in SP survey questions. Furthermore, this thesis uses non-parametric quantile regression on data from actual driving conditions to develop feasible travel time reliability measure for RP data.

Chapter 1 of this thesis is to summarize past literatures addressing above issues, describe SR91 road pricing experiment in Los Angeles area, the survey data used in estimation, as well as how to obtain measures of travel conditions facing survey respondents in revealed preference data.

In Chapter 2, this thesis uses the revealed preference and stated preference data collected by Brookings Institution, as well as the revealed preference data collected by Cal Poly to estimate the heterogeneity in both value of time and value of reliability. These data can be combined in estimation, as we will see, because they are from the same population and were collected at the same time. Binary choice models modeling motorists' route choice behavior—whether to take SR91 toll lanes, are used to uncover the distribution of their preferences for time saving and travel time unreliability. Both the observed heterogeneity, which can be explained by observable individual characteristics, and unobserved heterogeneity, which arises from unobserved personalities, are investigated.

In Chapter 3, this thesis proposes a Bayesian approach of combining RP and SP data sources to estimate the heterogeneity in peoples' preferences based on probit model. Up to now, all studies combining RP and SP data have been based on logit model and classical estimation method. The reasons are the computational conveniences associated with logit model and testing difference between RP and SP choice processes using classical statistic method. This chapter shows that how the recent developments in Bayesian analysis for multinomial probit model can be used in combining RP and SP data to estimate probit model capturing unobserved heterogeneity in peoples' preferences, and in testing difference between RP and SP choice processes.

Chapter 4 shows how the estimated heterogeneity in motorists' preferences affects both the efficiency and political feasibility of various congestion pricing policies. The studies in this chapter are based on a simulation model developed by Small and Yan (2000). This chapter extends Small and Yan's study in the following ways. First, it evaluates various proposed congestion pricing policies using "real" heterogeneity in travelers' preferences. Second, based on estimated heterogeneity in Chapter 2, it tries to find congestion policies which are both efficient and politically feasible. Efficiency and political feasibility are in the sense that these policies generate nontrivial social welfare gain on the one hand, and introduce tolerable direct loss in consumer surplus as well as eliminate distribution disparities among people with different preferences on the other hand.

CHAPTER 1

Literature Review, Description of California State Route 91 (SR91)

Road Pricing Experiment, and Data Used in Estimation

1.1: Literature Review

The literature review can be divided into three parts, literature on modeling peoples' heterogeneity, literature on combining different sources of data in discrete choice modeling, and literature on measuring travel time reliability on highway.

1.1.1: Literature on modeling heterogeneity in preferences

For most applications to measure consumers' heterogeneity, the data is limited and provides little individual-level information. For example, in travel demand modeling, it is much harder to observe one individual's choices over a long time period than to observe multiple individuals' choices in a short time range. As a result, the fixed-effects method to model heterogeneity is almost impossible, because it requires large size on individual level observations. Literature on modeling heterogeneity in preferences mostly relies on random-effects discrete-choice model developed by Heckman (1981), in which individual level parameters are viewed as stochastic, and their distributions may be conditional on individual characteristics. The part of distributions explained by observed individual characteristics is called observed heterogeneity in preferences, and the remaining pure random part is called unobserved heterogeneity in preferences.

Researchers have developed discrete choice models with random parameters based on both logit and probit model to investigate varied nature in consumers' preferences. The logit model with random coefficients or random components is also called mixed logit (as described by McFadden and Train (2001)). Procedures for estimating discrete choice models with random parameters have been developed in both classical and Bayesian framework. The classical method is based on Simulated Maximum Likelihood Estimation (SMLE) developed by Lee (1992), and Hajivassiliou and Ruud (1994). Revelt and Train (1998) began to employ SMLE for mixed-logit model. SMLE can be also used on multinomial probit model with random parameters based on GHK probit simulator developed by Geweke (1989), Hajivassiliou (1990) and Kean (1990). The Bayesian approach for estimating multinomial probit model has been developed by Albert and Chib (1993), McCulloch and Rossi (1994), Allenby and Rossi (1998), and McCulloch, Polson, and Rossi (2000). Their method can be easily extended to estimate multinomial probit models with random parameters by Hierarchical Bayesian analysis.

Based on developed estimation method for the mixed-logit model, Revelt and Train (1998) modeled households' choices of appliance efficiency level. Brownstone and Train (1999) forecast new product penetration with flexible substitution patterns. Bhat (2000) estimated both the observed and unobserved diversity in preferences in urban work travel mode choice modeling.

Based on Hierarchical Bayesian Analysis, Ainslie and Rossi (1998) investigated similarities in choice behavior across product categories. Allenby and Rossi (1999) measured the heterogeneity in consumers' brand choice.

1.1.2: Literature on Combining Different Sources of Data in Discrete Choice Modeling

Traditionally, empirical studies on individual's choice behavior rely on data from peoples' actual choice behavior, i.e., revealed preference data (RP). However, as Hensher(1999) summarized, there are compelling reasons to use data from peoples' choices under hypothetical settings, i.e., stated preference data (SP). For example, forecasting demand for new products with attributes unobserved in real market must use stated preference data. In travel demand modeling, the key variables, such as money cost, travel time, and travel time reliability, in revealed preference data are likely to be highly correlated, which leads to identification problem in estimation. SP data, in which the correlation between variables can be well controlled, is necessary in this case. The main drawback of SP data is that it can not overcome the doubt that the behavior exhibited in hypothetical situations may not apply to real choices.

Ben-Akiva and Morikawa (1991) proposed an estimation method combining RP and SP data from the same respondents to estimate multinomial logit model. This methodology can improve the efficiency in estimation, correct the possible bias in SP responses, and identify individual-level parameters that can hardly be identified from RP

data only. The basic idea of this methodology is to assume that some parameters can be combined to be the same across RP and SP data, and control for the difference in the two data generating processes at the same time. The differences in RP and SP data generating processes are represented by letting some parameters be different across RP and SP data, and assuming that the error terms in RP and SP random utility functions have different variances, although they are still independent Gumbel distributions. In estimation, the variance of SP error is re-scaled by a scale parameter to be the same as the variance of RP error. Since the variances themselves are not identified in discrete choice models, the variance of RP error is normalized and the scale parameter, along with other parameters representing peoples' preferences, are estimated.

The estimation methods for joint RP and SP multinomial logit model include simultaneous estimation developed by Ben-Akiva and Morikawa (1991), Hensher and Bradley (1993), and Bhat (1995), and sequential estimation developed by Swait and Louviere (1993). Morikawa (1994), and Bhat and Castelar (2002) also discussed incorporating and estimating the correlation between RP and SP observations from one individual in joint RP/SP modeling.

The methodology of combining RP and SP data can be used to combine, compare, and test process differences in sources of data, not only for RP and SP data, but also for RP and RP, as well as SP and SP data. The null hypothesis that parameters representing preferences are the same across data sources can be tested by likelihood-ratio test, as

proposed by Swait and Louviere (1993); Louviere, 1993; Bradley and Daly, 1994; Swait and Adamowicz (1997).

In practice, Adamowicz, Louviere, and Williams (1994) combined RP with SP data to study how people value environmental amenities. Brownstone, Bunch, and Train (2000) combined RP and SP to model consumers' choices for alternative-fuel vehicles.

1.1.3: Literature on Measuring Travel Time Reliability on Highway

The measures of travel time reliability here mean the measures for day-to-day variability of travel time. They are hard to get in most empirical studies, most of which have to rely on stated preference data to study the effects of travel time reliability on travelers' choice behavior. The new technology advancements make it possible to use data from real traffic conditions to construct measures for travel time reliability nowadays. Traffic data from loop-detectors embedded in highways provides useful information on travel time reliability. Kazimi et al. (2000), Lam (2000), and Ghosh (2001) both used loop detector data covering their research corridors within two months to estimate the sample percentiles of travel time on the roads. Different measures for travel time reliability can be constructed either using variance of travel time or using the distance between upper percentiles and the median. In their travel choice models, the difference between 90 percentile and median of travel time works well as measure of travel time reliability. However, as Brownstone et. al. (2000) pointed out, loop-detector data is likely to give inaccurate estimates for actual travel time on roads. To correct the

measurement error in loop detector data, they also collected floating-car data, which was collected by driving cars along the corridor for many times over 5 days and is thought as the most accurate measure for travel time. The limited size floating card data was then used to construct an imputation model, in which the relationship between floating car data and loop detector data was estimated. The floating car data outside the five-day period was predicted based on the imputation model, and measures of travel time reliability were then constructed using predicted floating car data.

Cohen and Southworth (1999) proposed another procedure of measuring travel time reliability. They first constructed a model based on queue theory representing the delays due to highway incidents, which are main factors of causing travel time uncertainty. The real incident data from highways was used to fit their model based on a microsimulation model. In their final results, both the mean and variance of travel time can be expressed as the functions of volume to capacity ratio, and the specific functional forms depend on designed capacity of hiways.

1.2: Empirical Setting

This thesis investigates the heterogeneity in motorists' preferences for travel time and travel time reliability based on California State Route 91 (SR91) road pricing experiment in Los Angeles area.

1.2.1: Brief Description of SR91 Road Pricing Experiment

SR91 connects rapidly growing residential areas in Riverside and San Bernadino Counties to job centers in Orange and Los Angeles Counties. It had been one of the most heavily congested freeway corridors of California. In December 1995, four new toll lanes opened in the median of the existing facility (16 kilometers in length), built and operated by a private company. This results in two toll lanes (called the 91 Express Lanes) and four free lanes in each direction. To use the Express Lanes, a vehicle must have a transponder to pay tolls electronically. Tolls vary over time based on a preset toll schedule and they are set by the private company to maximize its profit subject to a maximum rate-of-return constraint. Vehicles with three or more occupants pay only half the published toll. Table 1 and Table 2 show the toll schedules in 1999 and summer of 2000, which is the time periods covered by this thesis, on Westbound of SR91 (for morning commuters) over morning peak period because this thesis focuses on morning work trips. As showed, the toll schedules are almost the same across days except for some time slots on Monday and Friday

Table 1. Toll Schedule on Westbound in 1999

	Monday	Tuesday	Wednesday	Thursday	Friday
4-5 am			\$1.65		
5-6 am			\$2.90		
6-7 am			\$3.00		\$2.90
7-8 am			\$3.25		
8-9 am	\$3.00		\$2.90		
9-10 am			\$1.95		

Table 2. Toll Schedule on Westbound in Summer of 2000

	Monday	Tuesday	Wednesday	Thursday	Friday
4-5 am			\$1.65		
5-6 am			\$2.90		
6-7 am			\$3.00		\$2.90
7-8 am			\$3.30		\$3.00
8-9 am	\$3.10		\$3.00		\$2.90
9-10 am			\$2.25		

1.2.2: Survey Data

This thesis employs two survey data sets in analysis. The first one is a telephone RP survey composed of SR91 commuters obtained by random-digit dialing and observed license plates on the SR91 corridor. The survey was conducted in November of 1999 by researchers at California Polytechnic State University at San Luis Obispo. The respondents in this survey were asked about their most recent trip on a Monday through Thursday during the morning peak (4-10 am). These questions concern route choice (91 express lanes and free lanes), time of commute, trip distance, vehicle occupancy. The respondents were also asked to provide various personal and household characteristics, as well as whether they have flexible work arrival time. In latter analysis, this data is called as Cal Poly data and the sample size of this data is 438 respondents.

The second sample is a two-stage survey collected by Brookings Institution (Brookings), including both RP and SP elements. For Brookings sample, a market research firm, Allison-Fisher, Inc., conducted the survey using members of two nationwide household panels, National Family Opinions and Market Facts. An advantage of this setup is that background information was already on file from these panels, so our questionnaires could be briefer and avoid sensitive questions like income. First, the firm sent a screener survey to identify people who used the road in question for its full length, so that they would have the option of using either route. Those respondents who answered positively were then sent a questionnaire which asked them to report on their daily commute for an entire week, on which they provided the same information as described in Cal Poly data. By asking about their choice on up to five weekdays, we create the opportunity to investigate whether commuters alter their route choices from day to day depending on travel conditions and their schedules.

People who returned the RP survey were also given a stated preference (SP) survey in which they were presented with eight hypothetical commuting “packages” that included the toll, travel time, and travel-time reliability of a trip both on the express lane and the free lane. In each case they were asked to indicate which lane they would choose. Figure 1 shows an illustrative scenario and totally there are eight hypothetical commuting scenarios. Respondents who indicated that their actual commute was less (more) than 45 minutes were given scenarios that involved trips ranging from 20-40 (50-70) minutes.

Free Lanes	Express Lanes
Usual Travel Time: 25 minutes	Usual Travel Time: 15 minutes
Toll: None	Toll: \$3.75
Frequency of Unexpected Delays of 10 minutes or more: 1 day in 5	Frequency of Unexpected Delays of 10 minutes or more: 1 day in 20
Your Choice (check one):	
Free Lanes <input type="checkbox"/>	Toll Lanes <input type="checkbox"/>

Figure 1. Illustrative Scenario of Brookings SP Survey

It turned out that many people who said they had a lane choice did not, and some others failed to complete the survey. Thus the survey had to be conducted three waves of potential respondents--in December 1999, July 2000, and September 2000--to assemble an adequate sample. The final Brookings sample consists of 89 respondents providing 385 daily observations about actual behavior (RP), and 74 respondents providing 577 separate observations about hypothetical behavior (SP). The subsamples of 89 RP and 74 SP respondents include 52 people in common who answered both surveys.

Table 3 shows the summary statistics of these survey samples. Values for the Brookings data are broadly consistent with population summary statistics, indicating that we have a representative sample.¹ The median household income (assigning midpoints to the

¹ The distributions of the RP sample's commuting times and route share are close to the ones in 1998 survey data collected by University of California at Irvine (Lam and Small (2001)) and 1999 survey data collected by California Polytechnic State University at San Luis Obispo (Sullivan et al. (2001)). The

income intervals) is \$46,250. We estimate the average wage rate to be about \$23 per hour.² The Brookings sample contains information for multiple days and indicates that inertia is a powerful force in route choice behavior because 87 percent of the RP respondents made the same choice every day during the survey week. In fact, about half of the Brookings RP respondents do not have a transponder and thus have committed to not choosing the express lanes on any of our survey days.

The Cal Poly sample's route shares, commuting patterns, respondents' age and sex, and so on are closely aligned with the Brookings sample. Respondents in the Cal Poly sample do have higher household incomes and shorter trip distances than the Brookings respondents; apparently the Brookings sample drew from a wider geographical area including people who reside in lower-priced housing.

socioeconomic data are consistent with Census information, and diverge where appropriate. For example, our median income (approximately \$46,250) is higher than the average income in the two counties where our respondents lived (\$36,189 in Riverside County and \$39,729 in San Bernardino County in 1995, as estimated by the Population Research Unit of the California Department of Finance). But this should be expected because our sample only includes people who are employed and commute to work by car. The median number of people per household (which can be expected to be stable across time) is 2.81 and 3.47 in our RP and SP subsamples respectively; these are not far from the 1990 Census figures of 2.85 for Riverside County and 3.15 for San Bernardino County.

² Data from the US Bureau of Labor Statistics (BLS) for the year 2000 record the mean hourly wage rate by occupation for residents of Riverside and San Bernardino Counties. We combine the BLS occupational categories into six groups that match our survey question about occupation, then assign to each person in our sample the average BLS wage rate for the appropriate occupational group. We then add 10 percent to reflect the higher wages likely to be attracting these people to jobs that are relatively far away.

Table 3. Descriptive Statistics of Survey Samples

	<i>Value or Fraction of Sample</i>		
	Cal Poly-RP	Brookings-RP	Brookings-SP
Route Share:			
91X	0.26	0.25	
91F	0.74	0.75	
One-Week Trip Pattern:			
Never Use 91X		0.68	
Sometimes Use 91X		0.13	
Always Use 91X		0.19	
Percent of Trips in Each Time Period:			
4:00am-5:00am	0.11	0.15	
5:00am-6:00am	0.22	0.13	
6:00am-7:00am	0.23	0.26	
7:00am-8:00am	0.20	0.21	
8:00am-9:00am	0.14	0.15	
9:00am-10:00am	0.10	0.10	
Age of Respondents:			
<30	0.11	0.12	0.10
30-50	0.62	0.62	0.64
>50	0.27	0.26	0.26
Sex of Respondents:			
Male	0.68	0.63	0.63
Female	0.32	0.37	0.37
Household Income (\$):			
<40,000	0.14	0.23	0.24
40,000-60,000	0.24	0.60	0.59
60,000-100,000	0.40	0.15	0.13
>100,000	0.22	0.02	0.04
Flexible Arrival Time:			
Yes	0.40	0.55	0.50
No	0.60	0.45	0.50
Trip Distance (Miles):			
Mean	34.23	44.76	42.56
Standard Deviation	14.19	28.40	26.85
Number of People in Household:			
Mean	3.53	2.91	3.44
Standard Deviation	1.51	1.63	1.55
Number of Respondents	438	89	81
Number of Observations	438	385	633

1.2.3: Estimating Travel Time and Reliability for RP Sample

To estimate the demand models using revealed preference (RP) data, we need the most accurate possible measures of the travel time and reliability faced by each traveler in our survey, for either route. These differ greatly, of course, by time of day. They may differ by day of the week as well, but that variation is smaller and would require extensive data collection to measure accurately, so it is not considered here. Our task is simplified by the findings of Sullivan et al. (2000), confirmed by our own observations, that travel time is essentially constant on the Express Lanes.

This thesis is to use actual field measurements of travel times taken at many different times of day over the six-hour morning period covered by survey data. Our strategy furthermore assumes that for any given time of day (such as 7:42 a.m.), the travel times observed are random draws from a distribution which is known to travelers through their past experience. By assuming that travelers' decisions depend on "travel time" and "reliability", what we mean is that travelers pay attention to the central tendency of this distribution and its dispersion. For central tendency, a plausible measure is either the mean or the median; the mean is most consistent with prior studies on value of time. For dispersion, there are a number of plausible measures; two that have been used in past studies are (a) the standard deviation and (b) the difference between the 90th and 50th percentiles. The latter difference is appropriate if what concerns travelers the most about reliability is the potential for occasional significant delays, meaning they care more about the right-hand side

of the distribution than its left-hand side; since the distribution turns out to be highly asymmetric, this seems *a priori* a better variable than the standard deviation.

The importance of reliability to the traveler depends critically on information. Noland and Small (1995) show that the dependence of utility on reliability can be derived from a more primitive formulation in which the traveler optimizes departure time in order to minimize the expected costs of travel and schedule mismatches. The more information the traveler has before choosing the departure time, the smaller the resulting expected costs. Furthermore, empirical estimates by Noland et al. (1998) suggest that these scheduling considerations account for virtually all the observed impact of reliability on choice. Thus if the traveler could learn the exact travel time early enough to reoptimize departure time accordingly, reliability would have little effect; however, empirical evidence suggests that in most situations travelers are far from having this capability.

On the facility in question, there is no sign before the express lane entrance with traffic information. Previous surveys described by Parkany (1999) suggest that whatever information travelers have about conditions that day is mostly acquired en route through radio reports, and thus has limited value to them. Furthermore, based on our experience in field measurements, congestion on the 10-mile trip is only weakly correlated with congestion encountered before the entrance of toll lanes. Rather, unexpected delays on the free lanes often occur within a one-mile segment just before the end of the toll lanes, due to a busy entrance there and a lot of lane changing just upstream of a major freeway intersection.

We therefore assume that travelers have no information, other than the distribution of travel times across days, about the travel time on a given day. To the extent this assumption is wrong, we will tend to overestimate VoR and underestimate VoT because some travelers who we think are deterred by unreliability may actually be deterred by the travel time itself, which unbeknown to us they can observe or estimate.

Our field measurements consist of floating-car data, collected by driving along the road with a stopwatch and clipboard.

We noticed that there was no congestion on the express lanes at any time during the ten days when data were collected by us. We therefore approximate the travel time on the express lanes at all times by the travel time we observe on the free lanes when there is no congestion (for example, at 4:00 am). For the location covered by our measurements, this travel time is 8 minutes, corresponding to a speed of 75 miles per hour.

Floating-car data measuring travel times on the free lanes were collected on eleven days. The first day's measurements were collected by the California Department of Transportation (Caltrans) on October 28, 1999. The other measurements were collected by us on July 10-14 and Sept. 18-22, 2000, which are exactly the time periods covered by two later waves of the Brookings survey.

Data were collected from 4:00 am to 10:00 am on each day, for a total of 210 observations y_i of the travel-time savings from using the express lanes at times of day denoted by x_i , $i=1,\dots,210$. Our objective is to estimate the mean and quantiles of the distribution (across days) of travel time y conditional on time of day x . To do so, we use non-parametric methods of the class of locally weighted regressions. In these methods, the range of the independent variables (in our case, just one variable) is divided arbitrarily into a grid, and a separate regression is estimated at each point of the grid. In our case, there is just one variable, x . For given x , the regression makes use only of observations with x_i near x , the importance of each being weighted in a manner that declines with $|x_i-x|$. The weights are based on a kernel function $K(\bullet)$, and how rapidly they decline is controlled by a *bandwidth* parameter h ; typically only observations within one bandwidth of x get any positive weight.

The specific form of locally weighted regression we use is known as *local linear fit*. For each value of x , it estimates a linear function $y_i = a + b(x_i-x) + e_i$ in the region $[x-h, x+h]$ by minimizing a loss function of the deviations between observed and predicted y . Denote the p -th quantile value of y , given x , by $q_p(x)$. Its estimator is then:

$$q_p(x) = \arg \min_a \sum_{i=1}^n g_p [y_i - a - b(x_i - x)] \cdot K[(x_i - x)/h] \quad (1.1)$$

where $g_p(t)$ is the loss function. Similarly, denoting the mean of y given x by $m(x)$, its estimate is given by the same formula but with subscript p replaced by m .

In the case of mean travel-time savings, we use a simple squared-error loss function, $g_m(t) = t^2$, in which case equation (1.1) becomes the *local linear least square regression*. In the case of percentiles of travel-time savings, including the median, we follow Koenker and Bassett's (1978) suggestion and use the following loss function, which is asymmetric except for the median ($p=0.5$):

$$g_p(t) = \{|t| + (2p - 1)t\} / 2 \quad (1.2)$$

With this loss function, equation (1.1) defines the *local linear quantile regression* (Yu and Jones, 1997). It can be shown that the estimated percentile values converge in probability to the actual percentile values as the number of observations n grows larger, provided the bandwidth h is allowed to shrink to zero in such a way that $nh \rightarrow \infty$. In the case of the median ($p=0.5$), this is a least-absolute-deviation loss function, and therefore the estimator can be thought of as a non-parametric least-absolute-deviation estimator.

The choice of kernel function has no significant effect on our results. We use the *biweight* kernel function, which has the following form:

$$K(u) = \frac{15}{16}(1 - u^2)^2, \quad |u| \leq 1 \quad (1.3)$$

and is zero for $|u| > 1$.

The choice of bandwidth, however, is important. We first tried the bandwidth proposed by Silverman (1985):

$$h = 0.9 \frac{\min_i \hat{\text{std}}(x)}{1.34 \hat{\sigma}} n^{-0.5} \quad (1.4)$$

where d is the difference between the 75th and 25th percentile of x . This bandwidth turns out to be about 0.5 hour for our data. However, there is rather extreme variation in our data at particular times of day, especially around 6:00 a.m., due to accidents that occurred on two days around that time. While these accidents are part of the genuine history and we want to include their effects, they produce an unlikely time pattern for reliability when used with the bandwidth defined by equation (1.4) -- namely, one with a sharp but narrow peak in the higher percentiles around 5:30 a.m., followed by the expected broader peak centered around 7:30 a.m. We therefore increased the bandwidth to 0.8 hour in order smooth out this first peak.

The estimate results are shown in Figures 2 and 3. Figure 2 shows the raw field observations of travel-time savings. The non-parametric estimates of mean, median, and 80th percentile are superimposed. Median time savings reach a peak of 5.6 minutes around 7:15 a.m.

Figure 3 shows the same raw observations after subtracting our non-parametric estimate of median time savings by time of day. An interesting pattern emerges. Up to 7:30 a.m., the scatter of points is reasonably symmetric around zero with the exception of

three data points. But after that time the scatter becomes highly asymmetric, with dispersion in the positive range (the upper half of the figure) continuing to increase until after 8:00 a.m. while dispersion in the negative range decreases. This feature is reflected in the three measures of dispersion, or unreliability, that are also shown in the figure: the standard deviation and the 80th-50th and 90th-50th percentile differences. The standard deviation peaks at roughly 7:45 a.m., the other two between 8:15 and 9:30. The reason for these differences is that traffic in the later part of the peak is affected by incidents occurring either then or earlier. This mostly affects the upper tails of the distribution of travel-time savings and so is most apparent in the percentile differences. The standard deviation, by contrast, is higher early in the rush hour because of days with little congestion—showing up as negative points in Figure 2. Such dispersion is probably less relevant to travelers than dispersion in the upper tails, leading us to prefer the percentile differences as reliability measures. These measures are also considerably less correlated with median travel time than is the standard deviation. In our estimations, we obtained the best statistical fits using the 80th-50th percentile difference.³

³ In our RP and joint RP/SP models, the 90th-50th percentile difference fit almost as well as the 80th-50th difference (in terms of log-likelihood) and resulted in similar coefficient estimates. The 75th-50th percentile difference, an additional measure, and the standard deviation fit noticeably less well and gave statistically insignificant results for the reliability measure.

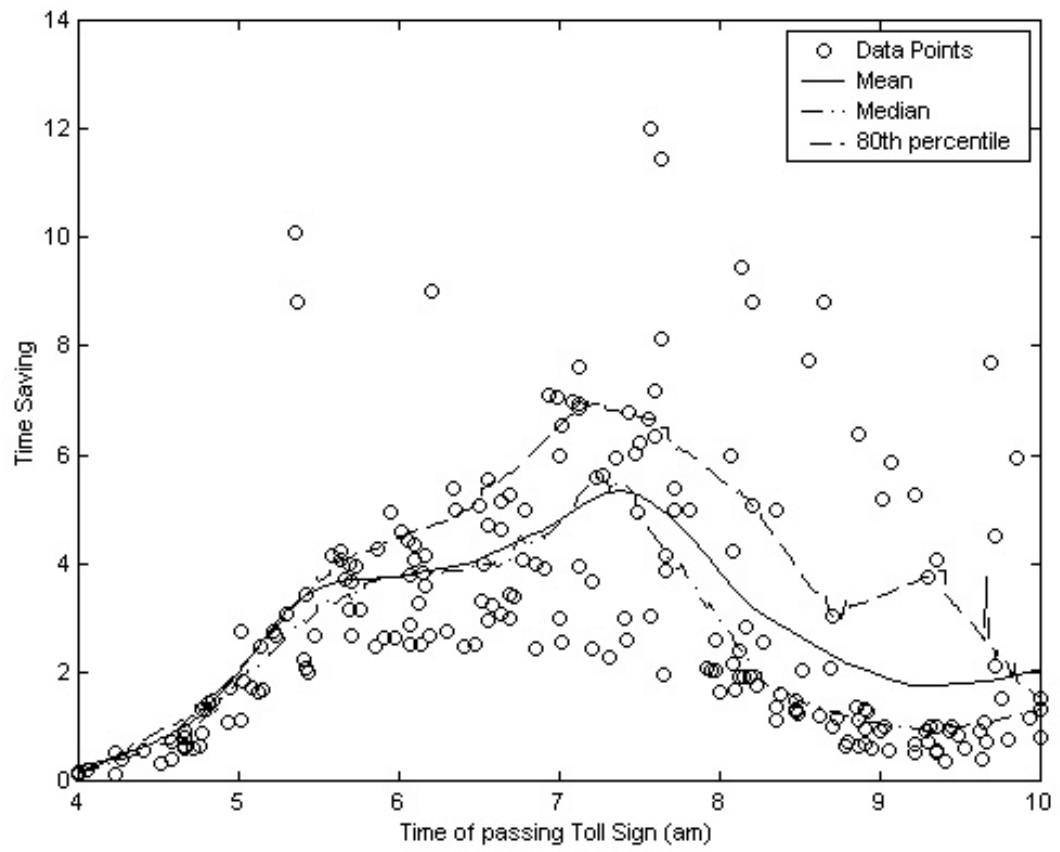


Figure 2. Time Saving

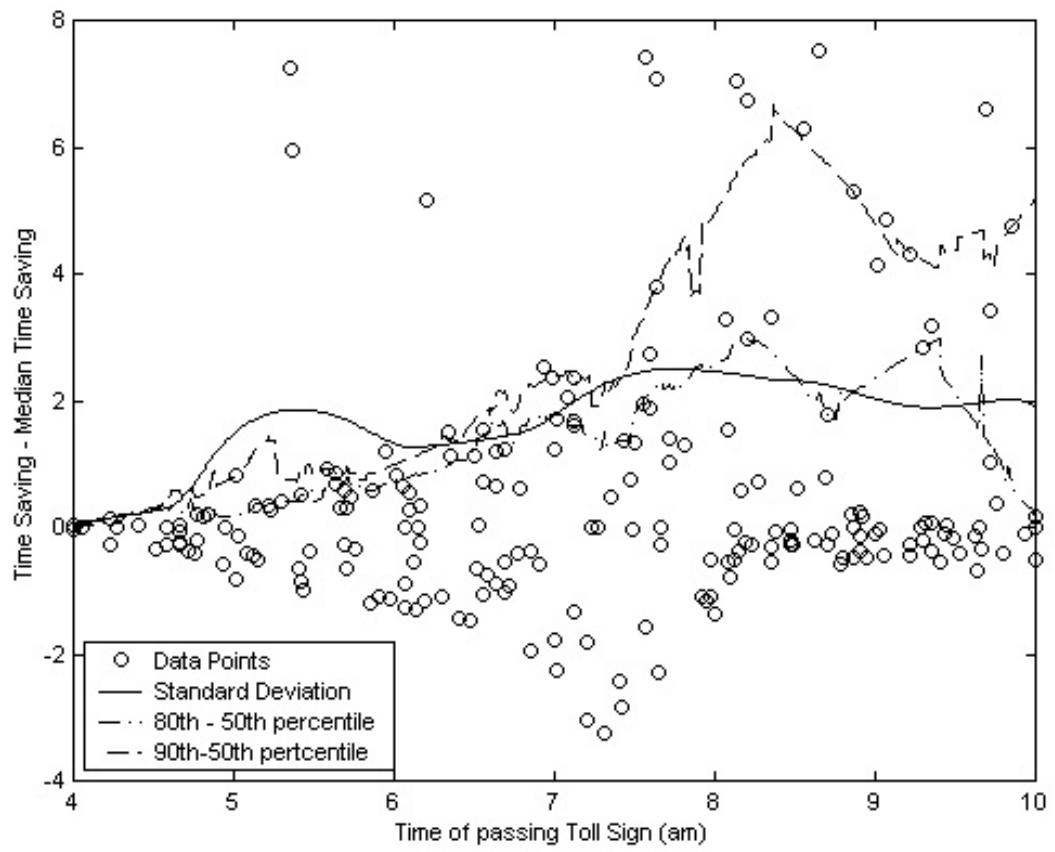


Figure 3. Dispersion of Time Saving

CHAPTER 2

Uncovering the Distribution of Motorists' Preferences for Travel Time and Time

Reliability

All empirical results in this thesis are based on discrete choice models, which can be derived from a conditional indirect utility function (which may represent expected utility after optimizing over choice of time of day to start the trip) for user i considering option j . This function is of the general form:

$$U_{ij} = \mathbf{b}x_{ij} + \mathbf{e}_{ij}$$

where \mathbf{b} is a coefficient vector, x_{ij} is a vector of independent variables (which may include alternative-specific constants and user characteristics interacted with travel characteristics), and \mathbf{e}_{ij} is a random term. Assuming that some measures of cost c , time t , and reliability r are included among the variables, the two measures for motorists' preferences for travel time and time reliability, value of time (VoT) and value of reliability (VoR) are defined as

$$VoT_{ij} = \frac{\partial U_{ij} / \partial t_{ij}}{\partial U_{ij} / \partial c_{ij}}; \quad VoR_{ij} = \frac{\partial U_{ij} / \partial r_{ij}}{\partial U_{ij} / \partial c_{ij}}$$

In the models of this thesis, these quantities are independent of the alternative label, j .

2.1: Results from Brookings Data

Brookings data includes one RP data, which records respondents' actual choice behavior up to 5 working days, and one SP data, which records respondents' choice behavior up to 8 hypothetical situations. We first model the RP data.

2.1.1: Revealed Preference Estimates

Table 4 defines the independent variables used.

Table 4. Definitions of Independent Variables in Brookings RP models

<u>Generic Variables</u>	
(all conditional on time of day)	
MedTime	Median travel time in minutes
Cost	Posted toll for a solo vehicle in dollars, divided by 2 if car occupancy is 3 or more
Dmp80	Unreliability of travel time on free lanes, given as the <u>d</u> ifference between the <u>m</u> edian and <u>80%</u> percentile time
<u>Traveler and Trip Characteristics</u>	
Income	Annual per capita income in thousands of dollars, calculated as mid-point of the household-income interval divided by household size
Dist	Distance from origin to destination, in 10 miles
Fage	1 if female with age between 35 and 60, 0 otherwise
Dflex	1 if with flexible arrival time, 0 otherwise

Brookings RP data records respondents' route choice over up to 5 days. We begin with simple binary logit models on the entire sample of observations, including observations on the same person on different days but ignoring the correlation among them. These models are called as *trip-based*. The dependent variable is simply whether the person chose the toll road on that day. Robinson (1982) shows for the case of probit

that the maximum likelihood estimate of such a model is consistent, though inefficient. Although we use logit here, we find using probit instead makes little difference.⁴ We account for the correlation among multiple observations of a single individual by adopting a robust estimator for the standard errors of our estimates.

Let $L(B)$ be the likelihood function obtained on the assumption that all observations are independent. Because \hat{B} is consistent estimate of B , by delta method⁵, we have

$$\text{var}(\hat{B}) \approx \left(\frac{\partial^2 \ln L}{\partial \hat{B} \partial \hat{B}'} \right)^{-1} \text{var} \left(\frac{\partial \ln L}{\partial \hat{B}} \right) \left(\frac{\partial^2 \ln L}{\partial \hat{B} \partial \hat{B}'} \right)^{-1} \quad (2.1)$$

The middle factor on the right-hand side, variance of the gradient, is estimated empirically which makes equation (2.1) a “sandwich estimator”(Greene, 2000, pp. 490-491). Considering the panel structure of our data set, we have

$$\frac{\partial \ln L}{\partial \hat{B}} = \sum_{i=1}^N \frac{\partial \ln L_i}{\partial \hat{B}} = \sum_{j=1}^{NG} \left(\sum_{i=1}^{n_j} \frac{\partial \ln L_i}{\partial \hat{B}} \right) \equiv \sum_{j=1}^{NG} g_j \quad (2.2)$$

⁴ As a check on the influence of error correlation on our results, we also estimated pure cross-section models by randomly drawing one day's observation per person. Some efficiency is lost by doing this, but not too much because most respondents made the same choice each day. The process is repeated 500 times and averaged the coefficient estimates. The estimated parameters from this method were smaller in magnitude but had very similar ratios compared with the those from models on entire sample of observations, indicating that there is more random variation in utility across people than across observations for the same person. These results are not reported here.

⁵ Because, by delta method, we know $\text{var} \left(\frac{\partial \ln L}{\partial \hat{B}} \right) \approx \left(\frac{\partial^2 \ln L}{\partial \hat{B} \partial \hat{B}'} \right) \text{var}(\hat{B}) \left(\frac{\partial^2 \ln L}{\partial \hat{B} \partial \hat{B}'} \right)$

where NG is the total number of respondents, n_j is the number of observations from j th respondent, and $N \equiv \sum_{j=1}^{NG} n_j$ is the total number of observations. Because choice behavior is independent among respondents, equation (2.2) means that we represent the gradient of log-likelihood as the sum of NG independently and identically distributed (*iid*) random variables. The sample mean of these NG random variables is zero because of the condition for maximizing L .

A good empirical estimator for the variance of the sum of NG *iid* random variables g_j averaging to zero is simply the NG times the sample standard error,

$$\text{var}\left(\sum_{j=1}^{NG} \hat{g}_j\right) = \frac{NG}{NG-1} \sum_{j=1}^{NG} g_j g_j' \quad (2.3)$$

The robust estimator is then simply equation (2.1) with (2.3) used to calculate the middle factor on the right-hand side.

An alternative way to model RP data is to take multiple days' observations of a respondent as only one observation (the explanatory variables are averaged over days), and create a dependent variable describing the frequency of using toll lanes. These models are called as *person-based*. One advantage of a person-based model is that it captures the fact that the traveler's decision to get a transponder is not made daily, and thus presumably is based on some averages of time savings and money costs over a long

time period. Another advantage is that person-based RP models are easier to combine later with SP models because their correlation structure is simpler. For person-based models, we eliminate respondents who have a transponder but who travel less than three days in our data, because defining a frequency for them involves too much error.

The possible interval [0,1] for the frequency of choosing the express lanes is divided into two or more intervals j . For each individual I in our sample, let y_i be the choice variable, indicating which frequency category is observed. We can assume the following choice process:

$$y_i = j \text{ if } k_{j-1} \leq BX_{ij} + \mathbf{e}_i < k_j \quad (2.4)$$

where the k 's are threshold parameters to be estimated along with utility coefficients B . If \mathbf{e}_i has an extreme value distribution, the probability of individual i choosing category j is

$$P_{ij} = \frac{1}{1 + \exp(-k_j + BX_{ij})} - \frac{1}{1 + \exp(-k_{j-1} + BX_{ij})} \quad (2.5)$$

In general, this is the ordered logit model; when there are only two categories, it reduces to the simple binary logit model.

We first specify the following two types of ordered-logit models based on different ways of categorizing the interval [0,1]. In the first type, there are three categories: “never use toll lanes”, “sometimes using toll lanes” and “always using toll lanes”. In the second type, there are four categories, with the open interval (0,1) divided into (0,0.5) and [0.5, 1); that is, we use the frequency of 50% as an additional cut point.

For each type, we test whether some frequency categories can be combined. For type 1, we test whether the categories "sometimes" and "always" can be combined. For type 2, we test whether the four categories can be combined into two, “less than 0.5” and “greater than or equal to 0.5”. The test is that devised by Vuong (1989) for non-nested models.⁶ In order to use Vuong’s test, it is necessary to adjust the dependent variable of one model to the same definition as the other one. For example, when testing the competing models for type 1, we compute both likelihoods as the sum of probabilities of choosing "never" versus choosing "sometimes or always".

For both types, the results of Vuong’s test accept the null hypothesis that these competing models are equivalent. In other words, we cannot reject any of the four specifications in favor of a different one. However, for type 1 model, we prefer the 3-category specification (the first one) considering the fact that about 13% of our sample

⁶ Vuong shows that the likelihood ratio of two non-nested models, f and g , has limiting distribution of $N(0, nw^2)$ under the null hypothesis that these two models are equivalent, where n is the number of observations and w is estimated consistently by:

$$\hat{w}^2 = \frac{1}{n} \sum_{i=1}^n \left[\log \left(\frac{f_i}{g_i} \right) \right]^2 - \left[\frac{1}{n} \sum_{i=1}^n \log \left(\frac{f_i}{g_i} \right) \right]^2$$

where f_i, g_i are densities at observation i of two models respectively.

(11 respondents) chose the category of “sometimes” and we hate to lose that information. For type 2 model, we prefer the 2-category specification (the second one) because the other specification contains a rare case – only 5% of the sample (4 respondents) chose the frequency interval [0.5,1).

The estimated RP results are shown in Tables 5, 6, and 7. Table 5 shows the results of trip-based models (binary logit); Table 6 shows the results of person-based models with three choice categories (ordered logit), and Table 7 shows the results of person-based models with two choice categories (binary logit).

We found through experimentation that distance has a strong effect on the time coefficient (hence on VoT), but it is a nonlinear one and seems best captured by a cubic form with no intercept (i.e., "MedTime" is not entered by itself). In fact, as we show for a subsequent table, the relationship between VoT and distance is concave throughout the range of most of our data, and declines with distance for most of the sample. Following conventional specifications we also allowed VoT and VoR to depend on income (models 2 and 4), which they arguably do but in fact the models with income entered simply as a taste-shifter for express lanes (Models 1 and 3) fit slightly better

Given results from earlier studies such as Lam and Small (2001), we were surprised to find little clear-cut effect of gender on choice of express lanes. The strongest effect we found, which is shown in Models 3 and 4, is based on the hypothesis that

women aged 35-60, who are relatively likely to have children living at home, place a higher value on their time.

Table 5. Trip-Based Models

Variables	Model 1	Model 2	Model 3	Model 4
Constant	-0.9843 (1.1696)	0.0541 (0.0059)	-1.5371 (1.2483)	-0.6674 (1.2268)
Cost	-1.6741 (0.6606)	-2.0409 (0.6975)	-1.2578 (0.6835)	-1.5592 (0.7066)
Cost * Income		0.0154 (0.0059)		0.0132 (0.0056)
Fage * MedTime			-0.3515 (0.1935)	-0.3588 (0.1948)
Dist * MedTime	-0.4140 (0.1314)	-0.4226 (0.1320)	-0.2935 (0.1420)	-0.2983 (0.1419)
Dist ² * MedTime	0.0827 (0.0218)	0.0838 (0.0220)	0.0631 (0.0219)	0.0636 (0.0219)
Dist ³ * MedTime	-0.0036 (0.0009)	-0.0037 (0.0009)	-0.0028 (0.0009)	-0.0028 (0.0009)
Dmp80	-0.8442 (0.3178)	-0.8238 (0.3144)	-0.6047 (0.3261)	-0.5890 (0.3211)
Income	0.0455 (0.0163)		0.0397 (0.0157)	
Dflex	0.9384 (0.5412)	0.9551 (0.5349)	1.2218 (0.6428)	1.2390 (0.6378)
#of Obs.	385	385	385	385
# of Perosns	89	89	89	89
Log-likelihood	-174.16	-175.90	-167.76	-169.24
Pseudo R ²	0.1945	0.1865	0.2241	0.2173
<u>VoT:</u>				
<i>Estimated Mean in Sample</i>				
95%-ile	\$29.79/hr	\$33.95/hr	\$44.91/hr	\$56.26/hr
50%-ile	\$16.71/hr	\$17.63/hr	\$17.80/hr	\$18.84/hr
5%-ile	\$3.99/hr	\$3.21/hr	\$-10.36/hr	\$-21.83/hr
<i>Estimated SD Due to Observed Heterogeneity in Sample</i>				
95%-ile	\$33.28/hr	\$131.46/hr	\$123.30/hr	\$634.51/hr
50%-ile	\$11.16/hr	\$13.43/hr	\$16.04/hr	\$18.86/hr
5%-ile	\$5.65/hr	\$7.00/hr	\$6.43/hr	\$7.10/hr
<u>VOR:</u>				
<i>Estimated Mean in Sample</i>				
95%-ile	\$68.82/hr	\$71.56/hr	\$94.98/hr	\$85.43/hr
50%-ile	\$29.93/hr	\$29.65/hr	\$28.31/hr	\$26.93/hr
5%-ile	\$13.38/hr	\$11.15/hr	\$1.41/hr	\$-6.09/hr
<i>Estimated SD Due to Observed Heterogeneity in Sample</i>				
95%-ile	\$0/hr	\$162.68/hr	\$0/hr	\$531.79/hr
50%-ile	\$0/hr	\$4.46/hr	\$0/hr	\$5.24/hr
5%-ile	\$0/hr	\$1.09/hr	\$0/hr	\$0.81/hr

Note: Numbers in parentheses are robust standard errors.

Table 6. Person-Based Models (never, sometimes, or always)

Variables	Model 1	Model 2	Model 3	Model 4
Cost	-1.9240 (0.8985)	-2.1951 (0.9124)	-1.4296 (0.9413)	-1.6585 (0.9519)
Cost * Income		0.0128 (0.0063)		0.0111 (0.0063)
Fage * MedTime			-0.4116 (0.1962)	-0.4175 (0.1964)
Dist * MedTime	-0.4176 (0.1654)	-0.4227 (0.1654)	-0.2841 (0.1751)	-0.2865 (0.1749)
Dist ² * MedTime	0.0851 (0.0309)	0.0858 (0.0309)	0.0633 (0.0324)	0.0636 (0.0324)
Dist ³ * MedTime	-0.0037 (0.0013)	-0.0038 (0.013)	-0.0028 (0.0014)	-0.0028 (0.0014)
Dmp80	-0.8175 (0.4196)	-0.8045 (0.4163)	-0.5171 (0.4501)	-0.5064 (0.4459)
Income	0.0380 (0.0174)		0.0334 (0.0177)	
Dflex	1.0251 (0.5706)	1.0288 (0.5687)	1.3477 (0.6406)	1.3551 (0.6395)
threshold1	0.3464 (1.4547)	-1.1253 (1.4292)	0.3929 (1.5375)	-0.2748 (1.5097)
threshold2	0.5107 (1.4562)	-0.2758 (1.4237)	1.3045 (1.5504)	0.6300 (1.5157)
<i>Summary Statistics</i>				
#of Obs.	84	84	84	84
Log-likelihood	-61.25	-61.56	-58.95	-59.20
Pseudo R ²	0.1373	0.1329	0.1697	0.1662
<u>VoT:</u>				
<i>Estimated Mean in Sample</i>				
95%-ile	\$28.26/hr	\$29.24/hr	\$41.92/hr	\$63.30/hr
50%-ile	\$15.59/hr	\$16.03/hr	\$16.10/hr	\$16.80/hr
5%-ile	\$0.02/hr	\$0.29/hr	\$-17.53/hr	\$-21.75/hr
<i>Estimated SD Due to Observed Heterogeneity in Sample</i>				
95%-ile	\$70.57/hr	\$112.66/hr	\$194.95/hr	\$740.26/hr
50%-ile	\$14.01/hr	\$15.62/hr	\$21.88/hr	\$23.04/hr
5%-ile	\$6.18/hr	\$6.50/hr	\$7.88/hr	\$8.05/hr
<u>VoR:</u>				
<i>Estimated Mean in Sample</i>				
95%-ile	\$62.40/hr	\$54.48/hr	\$68.87/hr	\$67.33/hr
50%-ile	\$24.67/hr	\$25.11/hr	\$20.57/hr	\$20.72/hr
5%-ile	\$7.58/hr	\$6.30/hr	\$-26.46/hr	\$-24.80/hr
<i>Estimated SD Due to Observed Heterogeneity in Sample</i>				
95%-ile	0	\$49.39/hr	0	\$305.34/hr
50%-ile	0	\$2.46/hr	0	\$2.44/hr
5%-ile	0	\$0.33/hr	0	\$0.19/hr

Notes: 1. Numbers in parentheses are standard errors.

2. Independent variables of a respondent are averaged over different days.

Table 7. Person-Based Models (<0.5 or ≥0.5)

Variables	Model 1	Model 2	Model 3	Model 4
Constant	-0.2546 (1.6169)	0.7094 (1.5780)	-1.2284 (1.7946)	-0.4255 (1.7467)
Cost	-2.1684 (1.0232)	-2.5269 (1.0477)	-1.6063 (1.1023)	-1.9064 (1.1197)
Cost * Income		0.0152 (0.0068)		0.0129 (0.0071)
Fage * Median			-0.5009 (0.2403)	-0.5077 (0.2410)
Dist * Median	-0.4314 (0.1858)	-0.4391 (0.1858)	-0.2714 (0.2006)	-0.2762 (0.2006)
Dist ² * Median	0.0805 (0.0334)	0.0861 (0.0334)	0.0532 (0.0355)	0.0539 (0.0355)
Dist ³ * Median	-0.0034 (0.0015)	-0.0035 (0.0015)	-0.0023 (0.0016)	-0.0023 (0.0016)
Dmp80	-0.9494 (0.4794)	-0.9368 (0.4745)	-0.5800 (0.5318)	-0.5770 (0.5272)
Income	0.0434 (0.0187)		0.0372 (0.0195)	
Dflex	1.1756 (0.6723)	1.1999 (0.6731)	1.7509 (0.8493)	1.7763 (0.8506)
<i>Summary Statistics</i>				
#of Obs.	84	84	84	84
Log-likelihood	-37.52	-37.72	-35.14	-35.29
Pseudo R ²	0.1863	0.1819	0.2378	0.2345
<u>VoT:</u>				
Estimated Mean in Sample				
95%-ile	\$31.18/hr	\$34.25/hr	\$45.67/hr	\$50.22/hr
50%-ile	\$16.29/hr	\$17.38/hr	\$17.99/hr	\$18.51/hr
5%-ile	\$2.19/hr	\$1.48/hr	\$-11.88/hr	\$-36.07/hr
<i>Estimated SD Due to Observed Heterogeneity in Sample</i>				
95%-ile	\$59.70/hr	\$155.66/hr	\$155.12/hr	\$766.16/hr
50%-ile	\$11.64/hr	\$12.29/hr	\$17.96/hr	\$20.96/hr
5%-ile	\$5.02/hr	\$5.34/hr	\$6.54/hr	\$6.71/hr
<u>VoR:</u>				
Estimated Mean in Sample				
95%-ile	\$59.34/hr	\$58.06/hr	\$67.02/hr	\$66.33/hr
50%-ile	\$25.88/hr	\$25.29/hr	\$21.34/hr	\$20.60/hr
5%-ile	\$8.67/hr	\$5.64/hr	\$-22.87/hr	\$-33.34/hr
<i>Estimated SD Due to Observed Heterogeneity in Sample</i>				
95%-ile	0	\$110.22/hr	0	\$395.63/hr
50%-ile	0	\$2.64/hr	0	\$2.68/hr
5%-ile	0	\$0.41/hr	0	\$0.20/hr

Notes: 1. Numbers in parentheses are standard errors.

2. Independent variables of a respondent are averaged over different days.

Lam and Small (2001) also find that dividing cost by vehicle occupancy improves goodness of fit in their models. Our RP data provide limited information on vehicle occupancy, which is known for some respondents based on a question asked in the RP survey. For others, some answered a question on the SP survey that provides occupancy, but others did not. Because of our lack of certainty about occupancy, we do not use it as in Lam and Small. However, we do use it to calculate the 50% discount that applies to carpool of three or more people, applying that discount to those respondents who tell us the carpool with three or more in the car. We check the stability of models to this data problem by introducing a dummy for the uncertain observations (i.e. those people not answering the occupancy question) and by interacting this dummy with cost; its coefficient provides no indication that the results are influenced by these observations, so those results are not shown here.

According to our model specifications, VoT varies with trip distance in all four models and varies with income and gender/age categories in some models; VoR also varies with income in some models. Thus there is heterogeneity in VoT and VoR due to observable variables. This heterogeneity is of great interest, so we want to characterize the results in terms of not only the mean but also the standard deviation (SD) of VoT or of VoR across our sample. We do this by computing the quantities labeled "Estimated Mean in the Sample" and "Estimated SD Due to Observed Heterogeneity in the Sample."

Each of these quantities is subject to statistical uncertainty, which we characterize by giving the median, 5th percentile, and 95th percentile based on the statistical

uncertainty in our coefficient estimates. These are calculated by a bootstrapping method. In the trip-based models, we draw random values B^r for the coefficient vector B according to its estimated asymptotic distribution, which is multivariate normal with variance-covariance matrix defined in equation (2.1). For each draw r we compute the appropriate ratios of coefficients in order to compute the mean and sample standard deviation of VoT and VoR across our sample. In the person-based models bootstrap replicates are formed by drawing dependent variable y_{ij} ($y_{ij} = 1$ if $y_i = j$) from a Bernoulli distribution with probability of success given by P_{ij} , which is estimated by evaluating equation (2.5) at Maximum Likelihood Estimate (MLE) of B . We re-estimate B for each replicate and use it to calculate the sample means and standard deviations of VoT and VoR (This method could not be used in trip-based models because it cannot account for the correlation among multiple observations for a given individual.). For all models, we do bootstrapping for 1000 replications and report the 5th, 50th, and 95th percentiles across those replications.

We see the median estimate of mean VoT is quite stable across the twelve specifications shown in these three tables, falling between \$15.50 and \$18.84 per hour. Its standard deviation is of course higher in the specifications in which it varies with income, since that adds additional observed heterogeneity. VoR is not quite as stable, but still the median estimate of its mean lies between \$20.50 and \$28.00 per hour. The precision of estimation of these quantities falls off markedly when the dummy for middle-aged females (Fage) is included as a shifter on the coefficient of travel time. (This is also reflected in the lower t-statistics on the other variables containing cost and travel

time.) The precision is in fact so low that the 5%-ile values for both mean VoT and mean VoR are negative, indicating that in these models one cannot say with 95% confidence that VoT and VoR are positive. For this reason, we prefer Models 1 and 2 over Models 3 and 4.

Our estimation results suggest that VoT is the function of trip distance. In Brookings RP data, 10%-ile of distance is 20.5 miles and 90%-ile of distance is 68 miles. In Figure 4, we plot VoT with respect to distance within this range using results from the three alternate forms of Model 1 shown in Tables 5, 6, and 7. The relationship between VoT and distance for other models is similar.

We also tried to estimate the person-based models with random parameters in order to measure the unobserved heterogeneity in value-of-time and value-of-reliability. We could hardly get convergent results for these models because of small size of RP data, so they are not reported.

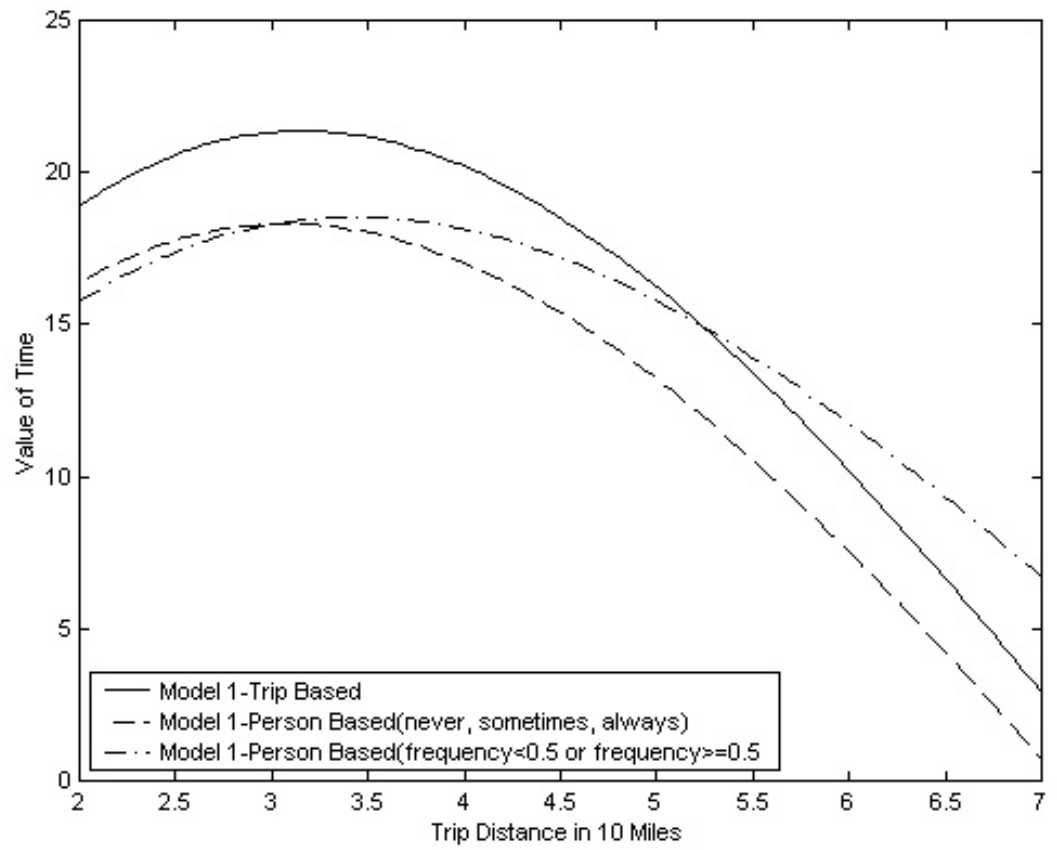


Figure 4. The relationship between VoT and trip distance

2.1.2: Stated Preference Estimates

Because the Brookings SP data are more numerous than its RP data and because the independent variables are less correlated with each other, we are able to go much further in distinguishing random from systematic effects. In particular, we estimate models with random parameters, which can be used to study the unobserved heterogeneity in people's preference. A special case of random parameters is a random alternative-specific constant, which is precisely the error-components model that enables one to estimate ordinary logit models more efficiently. In our empirical work, we first estimate this error-components model and then extend to a true random-parameters model. For purposes of explanation, it is easiest to derive them both at the same time.

In general, then, we consider the coefficient vector B (including the constant) as random with mean b and deviation \mathbf{h} . Then the binary choice model can be written as

$$y_{it} = \begin{cases} 1 & \text{if } b_{it}X_{it} + \mathbf{h}_i X_{it} + \mathbf{e}_{it} > 0 \\ 0 & \text{otherwise} \end{cases} \quad (2.6)$$

where η_i is the random part of the coefficient vector. This is an example of the "mixed logit" model described by Brownstone and Train (1999). When all the components of η_i are set to zero except that corresponding to the constant term, we have the same model as before but estimated with the more realistic assumption that the random terms are correlated across observations for a given survey respondent.

The error term of equation (2.6), which we denote $\mathbf{n}_{it} = \mathbf{h}_i X_{it} + \mathbf{e}_{it}$, implies a panel-type correlation structure, which can depend on variables X as follows:

$$E(\mathbf{n}_{it}) = 0 \quad (2.7a)$$

$$E(\mathbf{n}_{it}\mathbf{n}_{it}') = X_{it}'\Omega X_{it} + \mathbf{s}_e^2 \quad (2.7b)$$

$$E(\mathbf{n}_{it}\mathbf{n}_{is}') = X_{it}'\Omega X_{is} \quad \text{if } t \neq s \quad (2.7c)$$

$$E(\mathbf{n}_{it}\mathbf{n}_{js}') = 0 \quad \text{if } i \neq j \quad (2.7d)$$

where Ω is the variance-covariance matrix of \mathbf{h}_i and where \mathbf{s}_e^2 is normalized to $\pi^2/3$, just like the random term in a simple multinomial logit model.

The probability of respondent i choosing the toll lanes at situation t , conditional on η_i , is

$$P(y_{it} = 1 | \mathbf{h}_i) = \frac{1}{1 + \exp(-bX_{it} - \mathbf{h}_i X_{it})} \quad (2.8)$$

Then the unconditional joint probability of respondent i 's choice sequence (y_{it}) over several choice situations t is

$$P_i = \int_{\mathbf{h}_i} \left\{ \prod_t P(y_{it} | B, X_{it}, \mathbf{h}_i) \right\} f(\mathbf{h}_i | \Omega) d\mathbf{h}_i \quad (2.9)$$

where $f(\bullet)$ represents the joint density function of \mathbf{h}_i . The integration in equation (2.9) is calculated using Monte-Carlo simulation method. That is, we draw \mathbf{h}_i^r from the joint distribution $f(\bullet)$ and evaluate the probabilities conditional on \mathbf{h}_i^r , repeating for $r=1, \dots, R$. The simulated value of P_i in (2.9) is then:

$$SP_i = \frac{1}{R} \sum_{r=1}^R \left\{ \prod_t P(y_{it} / B, X_{it}, \eta_i^r, \Omega) \right\} \quad (2.10)$$

We estimate B, Ω by maximizing the following simulated log-likelihood function

$$SL(B, \Omega) = \sum_i \ln(SP_i) \quad (2.11)$$

Lee (1992) and Hajivassiliou (1994) show that under regularity conditions, the estimator is consistent and asymptotically normal, and when the number of replications rises faster than the square root of the number of observations, the estimator is asymptotically equivalent to the maximum likelihood estimator.

Table 8 presents the additional variables used in the SP models. In the case of occupancy, respondents were asked to declare whether they were answering the questions as solo drivers or as carpoolers, and in the latter case how many people they travel with if they carpool. The measure of unreliability here is entirely different from that in the RP

data: namely, it is the probability of being delayed by 10 minutes or more, a value provided in the questionnaire as one of the attributes of a route.⁷

Table 8. New Variables for SP models

Toll	The toll listed in survey questions
Unreliability	Unreliability of travel time
Workers	Number of workers at work site, in thousands
Transponder	1 if with transponder, 0 otherwise
Long	1 if respondent received the questionnaire designed for people whose commute time is more than 45 minutes; 0 otherwise
Occupancy	Number of people in vehicle

Table 9 presents some results of estimating SP models with random coefficients. We set the number of replications as 1000 for all these models. The model specification is chosen through testing using simple logit model.

In models 1 and 2 of Table 9, only the constant term is randomized, i.e. it simply adds an error-components panel structure to the simple logit model. In models 3 and 4, we also randomize the coefficients of time and unreliability, assuming they have independent normal distributions. This leads to some probability of a traveler having the "wrong" sign for these two coefficients; we tried unsuccessfully to use a log-normal distribution and truncated normal for them. (We tried randomizing toll coefficient, but this creates problems in calculating VoT and VoR because the toll coefficient, which appears in the denominator of those ratios, can take zero values.)

⁷ The probability was always stated for the trip as a whole. It was given as 0.05 for all trips using 91X, and either 0.05, 0.1, or 0.2 for trips using 91F.

We show models both with and without the variable "transponder" in the SP portion. This variable describes a choice made in real life that is closely related to the RP dependent variable. Thus it raises issues of the relationship between observed (RP) choices and hypothetical (SP) choices, as discussed by Morikawa (1994).⁸ Models that include "transponder" can be interpreted as allowing for "inertia", that is true state dependence; that is, the answers to SP questions are affected by the previously determined condition of the respondent. As in so many contexts, it is difficult to be sure whether one is measuring state dependence or unobserved heterogeneity, but we think the latter is controlled for adequately by the error structure. Thus including "transponder" may be viewed as estimating a short-run model conditional on transponder choice, whereas omitting it yields a more long-run model in which transponder choice is implicitly part of the decision to sometimes take the express lanes. The fact that it is highly significant is not surprising, although it does suggest that respondents ignored our instructions in the SP survey to "assume ... that you have a transponder."

Again, we estimate the distributions of estimated means and standard deviations of VoT and VoR by a Monte Carlo method, in which B is drawn from its estimated asymptotic multivariate normal distribution.

Models 3 and 4 achieve quite good precision, and indicate that indeed there is considerable heterogeneity in the valuation of time and reliability. When SP choice probabilities are conditioned on whether or not the person travels with a transponder

⁸ The reason we did not condition SP choices on the RP dependent variable, as does Morikawa, is that many of our SP respondents did not take the RP survey. Fortunately, we asked in the SP survey if they

(Model 3), the precision is greater. Value of time varies across the sample, but on average is about two-thirds as large as in the RP results of Section 2.1.1. This replicates findings of other studies, noted earlier, that SP surveys tend to produce lower values of time than RP surveys.

travel with a transponder, permitting us to use this proxy for the RP choice.

Table 9. SP Results: Binary Logit with Error Components

Variables	Model 1	Model 2	Model 3	Model 4
Constant				
Mean	-4.7142 (1.0617)	-4.0131 (1.2296)	-5.8212 (1.2596)	-4.9157 (1.3025)
Std. dev.	3.1178 (0.4598)	3.3339 (0.4569)	3.6901 (0.7160)	3.9547 (0.8450)
Toll	-1.0226 (0.2268)	-1.0272 (0.2289)	-1.2078 (0.2427)	-1.3109 (0.3155)
Long * Time	-0.1943 (0.0288)	-0.2001 (0.0309)	-0.2229 (0.0416)	-0.2572 (0.0590)
(1-Long) * Time	-0.2533 (0.0339)	-0.2435 (0.0352)	-0.3062 (0.0522)	-0.2954 (0.0579)
Std. dev. of coefficients of Time ^a			0.1089 (0.0414)	0.2031 (0.0526)
Unreliability				
Mean	-5.4261 (0.9993)	-5.4214 (0.9991)	-6.3574 (1.4368)	-6.8667 (1.5952)
Std. dev.			6.9483 (2.0556)	7.8940 (2.3341)
Transponder	2.5476 (0.7996)		3.1471 (1.0756)	
Occupancy	0.9560 (0.3568)	0.9975 (0.5573)	1.1478 (0.4918)	1.4135 (0.5735)
Workers	-0.7110 (0.2543)	-0.6359 (0.2983)	-0.8840 (0.3607)	-1.0022 (0.4121)
Dflex	0.8928 (0.7882)	1.3002 (0.7788)	1.4905 (1.0071)	2.1495 (1.0745)
<i>Summary Statistics</i>				
#of Obs.	577	577	577	577
# of Persons	74	74	74	74
Log-likelihood	-226.39	-231.36	-220.47	-223.28
Pseudo R ²	0.3979	0.3847	0.4137	0.4062
<i>VoT:</i>				
<i>Estimated Mean in Sample</i>				
95% -ile	\$20.65/hr	\$20.58/hr	\$19.90/hr	\$21.41/hr
50% -ile	\$12.70/hr	\$12.59/hr	\$12.61/hr	\$12.50/hr
5% -ile	\$8.72/hr	\$8.58/hr	\$8.54/hr	\$7.84/hr
<i>Estimated SD Due to Heterogeneity in Sample^b</i>				
95% -ile	\$4.19/hr	\$3.34/hr	\$17.59/hr	\$1945/hr
50% -ile	\$1.75/hr	\$1.15/hr	\$10.08/hr	\$10.98/hr
5% -ile	\$0.23/hr	\$0.10/hr	\$5.47/hr	\$5.81/hr
<i>VoR:</i>				
<i>Estimated Mean of VoR in Sample</i>				
95% -ile	\$8.57/incident	\$8.55/incident	\$8.43/incident	\$8.12/incident
50% -ile	\$5.32/incident	\$5.25/incident	\$5.11/incident	\$5.13/incident
5% -ile	\$3.48/incident	\$3.50/incident	\$2.95/incident	\$2.74/incident
<i>Estimated SD Due to Heterogeneity in Sample^b</i>				
95% -ile	0	0	\$10.32/incident	\$10.34/incident
50% -ile	0	0	\$6.11/incident	\$5.88/incident
5% -ile	0	0	\$3.15/incident	\$2.98/incident

Note: Numbers in parentheses are standard errors.

^a The coefficients of "Long*time" and "Short*time" are specified to have a single random error, whose standard error is given by this row.

^b In models 1 & 2, heterogeneity in VoT arises solely from observed heterogeneity, namely variation in distance; there is no heterogeneity in VoR. In models 3 & 4, heterogeneity in VoT and VoR also arises from unobserved heterogeneity, namely randomness in the coefficients of "time" and "unreliability".

2.1.3: Combined RP/SP Results

In this section, we combine RP and SP data together to estimate the parameters, following methods discussed by Ben-Akiva and Morikawa (1991) and Hensher and Bradley (1993). We account for the differences between RP and SP data in two ways. First, we allow them to have random terms with different variances; we do this by normalizing the RP variance as usual and estimating a separate scale parameter m for the ratio of standard deviations. Second, we allow some coefficients to have separate values across the two data sets, hoping to use the RP coefficient to correct any survey bias that may be in the SP.

We also account for serial correlation between the RP and SP error terms. Like Morikawa (1994), we do this by splitting the error terms into two components

$$\mathbf{e}_{it}^{RP} = \mathbf{l}_i + \mathbf{n}_{it}^{RP} \quad (2.12)$$

$$\mathbf{e}_{it}^{SP} = q\mathbf{l}_i + \mathbf{n}_{it}^{SP} \quad (2.13)$$

The first component, \mathbf{l}_i , is assumed to be a standard normal variate; it represents individual effects and therefore accounts for correlation among the responses for a given individual, including (to the extent that θ is positive) correlation between an individual's RP and SP responses. The other components, v_{it}^{RP} and v_{it}^{SP} , are assumed independently extreme value distributed with variances $(\pi^2/3)$ and $(1/\mu^2)(\pi^2/3)$, respectively. The combined model can then be written as follow:

$$y_{it}^{RP} = 1 \quad \text{if } B^{RP} X_{it}^{RP} + \mathbf{l}_i + \mathbf{n}_{it}^{RP} > 0 \quad (2.14)$$

$$y_{it}^{SP} = 1 \quad \text{if } \mathbf{m}B^{SP} X_{it}^{SP} + \mathbf{m}q\mathbf{l}_i + \mathbf{m}\mathbf{n}_{it}^{SP} > 0 \quad (2.15)$$

where the superscripts RP, SP represent the data source of the observation; y_{it} represents the dependent variable indicating toll lane choice by respondent i at situation t ; X_{it}^{RP}, X_{it}^{SP} are the vectors of exogenous variables, which include some common variables and some unique variables; B^{RP}, B^{SP} are the vectors of unknown parameters, some of which may be constrained to be equal across the RP and SP observations; and \mathbf{m} is a scale parameter to be estimated., defined as $\mathbf{m}^2 = \text{var}(\mathbf{n}_{it}^{RP}) / \text{var}(\mathbf{n}_{it}^{SP})$.

The purpose of multiplying the condition in (2.15) by \mathbf{m} is so that the final terms in the two equations are independently and identically distributed. Writing (2.15) in this way also reminds us that the estimated SP coefficients should be multiplied by \mathbf{m} before comparing them to estimates from ordinary logit equations such as in Table 9, or to B^{RP} from (2.14). This scale adjustment is automatic for any "pooled" variables, i.e. for variables that are common to the sets X_{it}^{RP}, X_{it}^{SP} . Even with these provisos, the magnitudes of the coefficients of each equation will adjust to reflect the other parts of the error terms (2.12) and (2.13).

We estimate unknown parameters B , q , and \mathbf{m} by Simulated Maximum Likelihood Estimation as described earlier. The results are shown in Tables 10-13. The

number of replications is 1000. In all these models we allow coefficients of time and reliability to differ between the RP and SP equations; whereas we constrain the coefficient of cost⁹ to be the same (except for scale factor m), a constraint not rejected by likelihood-ratio tests.

2.1.3.1 Fixed Coefficients

Tables 10 and 11 present estimated coefficients for models in which coefficients B are all fixed.

In Model 1 and Model 2 of Table 10, the RP choice is modeled as trip-based with one exception: when there are multiple RP observations from people without a transponder, we treat them as a single observation with the independent variables averaged over different days. The scale effect m is estimated to be very close to one. This is presumably because the rather large value estimated for q allows the variance of n_{it}^{RP} to exceed that of n_{it}^{SP} even with m near one. In Model 2, the variable "transponder", representing the effect of an actual travel choice on the hypothetical SP responses, is no longer statistically significant. This is probably because some of the previously observed effect of this variable in SP models (which of course is highly correlated with the RP responses) was actually due to correlation between the error terms of the RP and SP equations.

⁹ The cost variable in the SP data is here defined just as in the RP models, namely Toll if occupancy <3 and

In Model 3 and Model 4 of Table 10, we change the model specification for RP data to person-based with two choice categories (whether or not the observed frequency of toll road use is less than 0.5), as in Model 3 of Table 7. Thus there is only one RP observation for each respondent, so the assumption that I_i is standard normal is innocuous.

In these two models, the estimated scale factor is small, between 0.35 and 0.58, because the within-individual random variation in RP choices has been eliminated. This causes SP coefficients to be correspondingly larger. As in Model 2, the effect on the SP responses of having a transponder in real life is not quite statistically significant.

0.5*Toll otherwise; this is slightly different from the variable "Toll" used in the SP-only models.

Table 10. Joint RP/SP Results

Variables	Model 1	Model 2	Model 3	Model 4
<i>Pooled Variables</i>				
Constant	-0.5553 (1.0131)	-0.5108 (0.9308)	-0.4578 (1.6378)	0.0100 (1.8411)
Cost	-0.9781 (0.4933)	-1.0102 (0.4590)	-2.2689 (1.0189)	-2.6844 (1.0813)
Income	0.0242 (0.0136)	0.0237 (0.0138)	0.0356 (0.0211)	0.0369 (0.0184)
Dflex	0.5486 (0.4278)	0.5552 (0.3984)	2.2633 (0.7654)	2.3071 (0.9175)
Workers	-0.3145 (0.1476)	-0.3492 (0.1498)	-1.6072 (0.7964)	-2.2856 (1.2283)
<i>RP Variables</i>				
Dist * Time	-0.2683 (0.1135)	-0.2840 (0.1100)	-0.4460 (0.2117)	-0.5270 (0.2214)
Dist ² * Time	0.0647 (0.0226)	0.0672 (0.0220)	0.0907 (0.0388)	0.1040 (0.0392)
Dist ³ * Time	-0.0030 (0.0010)	-0.0031 (0.0010)	-0.0039 (0.0017)	-0.0044 (0.0017)
Fage * Time	-0.3839 (0.1469)	-0.3691 (0.1487)	-0.5932 (0.2598)	-0.5765 (0.2604)
dmp80	-0.5335 (0.2879)	-0.5730 (0.2740)	-0.6984 (0.5382)	-0.8860 (0.5502)
<i>SP Variables</i>				
Constant	-2.3056 (2.1613)	-3.3935 (2.3845)	-8.3108 (5.2998)	-13.553 (8.1356)
Long * Time	-0.1877 (0.0962)	-0.1877 (0.0899)	-0.4723 (0.2260)	-0.5651 (0.2550)
(1-Long) * Time	-0.2138 (0.1142)	-0.2339 (0.1124)	-0.5488 (0.2469)	-0.7091 (0.3379)
Unreliability	-4.9633 (2.6065)	-5.1614 (2.5019)	-12.383 (5.6130)	-15.191 (6.8418)
Occupancy	0.3277 (0.3755)	0.2398 (0.2712)	1.6538 (1.0982)	1.8258 (1.2805)
Transponder		2.2987 (1.2126)		7.2867 (3.6999)
<i>Scale Parameter</i>				
m	1.0916 (0.5727)	1.0504 (0.4940)	0.4338 (0.2073)	0.3493 (0.1656)
<i>Corr. Parameter</i>				
q	2.8327 (1.4914)	2.8072 (1.3269)	7.8384 (3.3080)	8.9635 (4.2287)
# of Obs.	802	802	660	660
# of Persons	109	109	108	108
Log-likelihood	-331.55	-326.03	-261.68	-257.68

Note: Numbers in parentheses are standard errors.

We consider Models 3 and 4 of Table 10 to best represent choice behavior of this set of survey respondents among the models with fixed coefficients on cost, time, and reliability. By combining data sets and accounting for plausible error structures, we have been able to measure the effects of the most important travel variables with quite good precision. It is encouraging that the relevant marginal rates of substitution, especially the value of time, are reasonably robust to variations in the model structure and the particular control variables included.

The estimated VoT and VoR based on models of Table 10 are summarized in Table 11. Compared with models using RP data alone, the precision of estimated VoT and VoR in the RP sample is improved significantly by combining RP data with SP data. The estimated mean VoT for RP observations (the second row of numbers in Table 11) is very stable across different model specifications, about \$17/hr in the RP sample and about 80 percent of this value in the SP sample.

Table 11. Estimated VoT and VoR from Models in Table 10

	Model 1	Model 2	Model 3	Model 4
<u>VoT:</u>				
<i>Estimated Mean in Sample</i>				
<i>RP Results</i>				
95% -ile	\$40.34/hr	\$40.73/hr	\$29.83/hr	\$28.19/hr
50% -ile	\$16.40/hr	\$17.16/hr	\$16.83/hr	\$16.59/hr
5% -ile	\$-1.38/hr	\$0.67/hr	\$3.61/hr	\$5.37/hr
<i>SP Results</i>				
95% -ile	\$19.62/hr	\$17.96/hr	\$19.99/hr	\$21.84/hr
50% -ile	\$12.05/hr	\$12.07/hr	\$13.23/hr	\$13.67/hr
5% -ile	\$5.77/hr	\$7.10/hr	\$8.02/hr	\$6.72/hr
<i>Estimated SD Due to Observed Heterogeneity in Sample</i>				
<i>RP Results</i>				
95% -ile	\$126.08/hr	\$109.96/hr	\$48.95/hr	\$37.45/hr
50% -ile	\$24.94/hr	\$24.42/hr	\$13.39/hr	\$11.87/hr
5% -ile	\$11.70/hr	\$11.63/hr	\$6.79/hr	\$6.37/hr
<i>SP Results</i>				
95% -ile	\$4.26/hr	\$4.22/hr	\$3.32/hr	\$3.02/hr
50% -ile	\$1.04/hr	\$1.42/hr	\$1.09/hr	\$1.61/hr
5% -ile	\$0.10/hr	\$0.17/hr	\$0.14/hr	\$0.33/hr
<u>VoR:</u>				
<i>Estimated Mean in Sample</i>				
<i>RP Results</i>				
95% -ile	\$84.18/hr	\$83.86/hr	\$43.02/hr	\$42.15/hr
50% -ile	\$32.01/hr	\$33.39/hr	\$18.52/hr	\$19.54/hr
5% -ile	\$0.47/hr	\$8.00/hr	\$-7.10/hr	\$-0.87/hr
<i>SP Results</i>				
95% -ile	\$8.19/incident	\$7.83/hr	\$7.44/incident	\$9.00/incident
50% -ile	\$5.08/incident	\$5.09/hr	\$5.47/incident	\$5.65/incident
5% -ile	\$2.29/incident	\$2.65/hr	\$3.83/incident	\$2.82/incident

Value of time varies by trip distance. When measured from RP data it declines noticeably with trip length for actual trips over 30 miles; when measured from SP data, it is smaller for those people who had longer actual trips (and hence who received a survey depicting long trips) than for others. The estimated standard deviation of VoT due to observed heterogeneity in the RP sample is not quite stable. The median of estimated standard deviation of VoT in the RP sample from Model 1 and Model 2 is almost twice

as big as the one from Model 3 and Model 4. We prefer the results from Model 3 and Model 4, because the estimated relationship between VoT and trip distance in Model 1 and Model 2 implies that about 15 percent of the RP sample have negative value of time.

When unreliability is measured as the 80th-50th percentile difference (RP data), its value is slightly larger than value of travel time in Model 3 and Model 4, and twice as large as value of travel time in Model 1 and Model 2; when measured as frequency of delays of 10 minutes or more (SP data), its value is between \$5 and \$6 per incident of such a delay and stable across models.

Finally, conditioning SP choice probabilities on transponder choice has little effect on estimated results, but improves the precision of estimates slightly.

2.1.3.2 Random Coefficients

In the next step, we randomize the coefficients of time and unreliability to measure unobserved heterogeneity in VoT and VoR. First, we use Model 2 in Table 10 as the base model. However, we can not randomize RP coefficients in this model because our model specification in equation (2.9) and (2.10) restricts the correlation among RP errors at 0.233. The estimated standard errors for the random terms of RP coefficients would capture part of correlation across RP observations, and hence bias the estimated unobserved heterogeneity in VoT and VoR. Considering this, we only randomize time and unreliability coefficients in the SP sample, and the results are shown in Model 1 of Table 12.

We can randomize the coefficients of time and unreliability both in the RP sample and SP sample when using person-based models like Model 3 or Model 4 in Table 10. We assume that the coefficients of time in the RP sample and in the SP sample have the same random component, that is, the model specification can be written as

$$B_{Time}^{RP} = \mathbf{a}_1 dist + \mathbf{a}_2 dist^2 + \mathbf{a}_3 dist^3 + \mathbf{a}_4 fage + \mathbf{d} \quad (2.16)$$

$$B_{Time}^{SP} = c_1 Long + c_2 (1 - Long) + \mathbf{d} \quad (2.17)$$

where \mathbf{d} is a normal variate with mean zero and variance \mathbf{s}_d^2 .

Because the measures of unreliability have different meanings in RP and SP data, we can not assume that the coefficients of them have the same random component. However, we assume that the ratio between standard deviation and mean of the unreliability coefficient is the same across RP and SP data. That is, we reparameterize the model specification as

$$B_{Unreliability}^{RP} = \bar{b}^{RP} + \mathbf{m}^{RP} = \bar{b}^{RP} \left(1 + \frac{\mathbf{m}^{RP}}{\bar{b}^{RP}} \right) = \bar{b}^{RP} (1 + \mathbf{v}) \quad (2.18)$$

$$B_{Unreliability}^{SP} = \bar{b}^{SP} + \mathbf{m}^{SP} = \bar{b}^{SP} \left(1 + \frac{\mathbf{m}^{SP}}{\bar{b}^{SP}} \right) = \bar{b}^{SP} (1 + \mathbf{v}) \quad (2.19)$$

where $\mathbf{m}^k \sim N(0, \mathbf{s}_{m^k}^2)$, $k = RP, SP$; and $\mathbf{v} \sim N(0, \mathbf{s}_v^2)$. The estimated results from this model are shown as Model 2 of Table 12. Here we use Model 3 rather than Model 4 of

Table 10 as base model because otherwise we encounter problems achieving convergence; we guess the problem arises from the interaction between true state dependence and spurious state dependence. Additionally, we delete the variable “occupancy” in order to get more precise estimates.

Table 12. Joint RP/SP Model – with random time and unreliability coefficients

Variables	Model 1	Model 2
<i>Pooled Variables</i>		
Constant	-0.5145 (1.0162)	-2.4635 (1.2947)
Cost	-1.0171 (0.5131)	-2.7725 (1.6302)
Income	0.0251 (0.0147)	0.0601 (0.0311)
Dflex	0.5431 (0.4384)	4.2402 (1.9901)
Workers	-0.3286 (0.1520)	-1.4872 (0.8383)
Std. dev. Of Time		0.3541 (0.1938)
Ratio between Std. dev. and mean of unreliability		1.2445 (0.3631)
<i>RP Variables</i>		
Dist * Time	-0.2759 (0.1119)	-0.5038 (0.2805)
Dist ² * Time	0.0642 (0.0221)	0.1108 (0.0556)
Dist ³ * Time	-0.0030 (0.0010)	-0.0049 (0.0024)
Fage * Time	-0.3496 (0.1529)	-1.0295 (0.4609)
dmp80	-0.5654 (0.2853)	-1.0503 (0.5489)
<i>SP Variables</i>		
Constant	-3.4980 (2.6178)	-4.6442 (4.1386)
Long * Time	-0.1859 (0.0967)	-0.5232 (0.3112)
(1-Long) * Time	-0.2338 (0.1228)	-0.5697 (0.3408)
Std. dev. of Time	0.1138 (0.0634)	
Unreliability Mean	-5.1326 (2.7243)	-12.418 (7.1422)
Std. dev.	4.4131 (2.5701)	
Occupancy	0.2849 (0.2657)	
Transponder	2.2687 (1.3044)	
<i>Scale Parameter</i>		
m	1.2637 (0.7190)	0.5271 (0.3248)
<i>Corr. Parameter</i>		
q	2.9838 (1.6308)	8.2660 (4.5338)
# of Obs.	802	660
# of Persons	109	108
Log-likelihood	-320.95	-253.42

Note: Numbers in parentheses are standard errors.

The estimated VoT and VoR from models in Table 12 are summarized in Table 13. The results in Model 1 are similar to those of Model 2 in Table 11, except that we get measures for unobserved heterogeneity of VoT and VoR in the SP sample. When using results from this model, we may consider using the estimated means of VoT and VoR in the RP sample as the measures for mean VoT and VoR; using the estimated *observed*

heterogeneity of VoT in the RP sample as the measure of heterogeneity in VoT; and using the estimated heterogeneity of VoR in the SP sample as the measure of heterogeneity in VoR.

Table 13. Estimated VoT and VoR from Models in Table 10

	Model 1	Model 2
<u>VoT:</u>		
<i>Estimated Mean in Sample</i>		
<i>RP Results</i>		
95% -ile	\$45.13/hr	\$35.22/hr
50% -ile	\$17.84/hr	\$15.32/hr
5% -ile	\$-0.52/hr	\$- 1.30/hr
<i>SP Results</i>		
95% -ile	\$19.69/hr	\$17.10/hr
50% -ile	\$11.96/hr	\$11.63/hr
5% -ile	\$5.83/hr	\$6.61/hr
<i>Estimated SD Due to Heterogeneity in Sample^a</i>		
<i>RP Results</i>		
95% -ile	\$113.71/hr	\$82.58/hr
50% -ile	\$24.24/hr	\$23.21/hr
5% -ile	\$10.69/hr	\$16.08/hr
<i>SP Results</i>		
95% -ile	\$15.25/hr	\$15.42/hr
50% -ile	\$8.73/hr	\$8.62/hr
5% -ile	\$2.88/hr	\$3.22/hr
<u>VoR:</u>		
<i>Estimated Mean in Sample</i>		
<i>RP Results</i>		
95% -ile	\$87.76/hr	\$50.86/hr
50% -ile	\$32.41/hr	\$22.14/hr
5% -ile	\$1.84/hr	\$4.61/hr
<i>SP Results</i>		
95% -ile	\$8.55/incident	\$8.77/incident
50% -ile	\$5.02/incident	\$4.42/incident
5% -ile	\$2.04/incident	\$1.99/incident
<i>Estimated SD Due to Heterogeneity in Sample</i>		
<i>RP Results</i>		
95% -ile	\$0/hr	\$64.64/hr
50% -ile	\$0/hr	\$26.98/hr
5% -ile	\$0/hr	\$5.10/hr
<i>SP Results</i>		
95% -ile	\$7.89/incident	\$8.57/incident
50% -ile	\$4.32/incident	\$5.48/incident
5% -ile	\$0.91/incident	\$1.93/incident

^a In model 1, heterogeneity in VoT in the RP sample arises only from observed heterogeneity; in model 2, it also arises from unobserved heterogeneity.

Model 2 of Table 12 gives us nice results for heterogeneity in VoT and VoR. The median estimated mean of VoT in the RP sample from this model is slightly lower than the one from other models.

2.1.4: Summary of Results from Brookings Data

In Brookings data, the estimated value of time is stable across different model specifications; its median value is in the range of \$16-\$18 per hour in the RP sample, and in the range of \$11-\$13 per hour in the SP sample. These results confirm the results from other studies, that is, the estimated mean of VoT using RP data is higher than the one using SP data, although the difference between them in this paper is much smaller than in Ghosh (2000a). The difference is probably due to misperceptions of travel-time savings, as noted before, causing travelers to perceive the time differences in the SP questions as indicating a smaller actual difference in service quality than they really do. As argued before, it is the RP value that is most germane to congestion modeling and cost-benefit analyses.

We use different measures for time unreliability in the RP and SP sample. In the RP sample, VoR is imprecisely measured and not too stable; its estimated value is in the range \$19-\$30 per hour. In the SP sample, the estimate VoR is robust, with value about \$5 per incident (of 10 minutes or more unexpected delay) for all models.

We find large heterogeneity in value of time. Our RP data show observed heterogeneity in VoT which can be explained by trip and individual characteristics, with estimated standard deviation across our sample between \$13 and \$25 per hour depending on model specification. We are unable to extract a satisfactory measure of the unobserved heterogeneity (i.e. that unexplained by objective variables) in our RP data alone. In the SP data, by contrast, the ability of trip distance to account for heterogeneity in VoT is tenuous, but we find substantial unobserved heterogeneity. Overall, the estimated standard deviation of VoT is between \$7 and \$10 per hour across individuals in the SP sample.

We did not find obvious evidence that value of reliability varies with trip or individual characteristics. Additionally, our RP data alone cannot give us a precise measure of unobserved heterogeneity in VoR. The SP sample, in contrast, shows that there is significant unobserved heterogeneity in VoR.

By combining RP data with SP data, we can overcome some of the limitations in the RP data because certain coefficients can plausibly be constrained to be the same in the RP and SP portions of the combined data set. This results in two major improvements to our RP results. First, the precision of all estimated coefficients in common with the RP models is improved substantially compared to those using RP data alone. Second, with combined data we can measure unobserved heterogeneity in both VoT and VoR across the RP sample.

2.2: Combining Cal Poly Data with Brookings Data

In order to get more reliable estimates for the heterogeneity in values of travel time and travel time reliability, we combine the RP data collected by Cal Poly with Brookings data in this section. The feasibility of combining these data together is that they are from the same population and collected at almost the same time.

2.2.1: Revealed Preference Estimates

We first combine two RP data together. Based on the modeling experience in above section, we choose the *person-based* model with two choice categories to model Brookings RP data. Cal Poly data is a simple cross-section. We account for the differences between these two RP data sources by letting their random terms have different variances, and by specifying different alternative-specific constants in them. Our rationale is that other determinants for commuters' route choice are the same across these two data sets. In estimation, the variance of Brookings RP error terms is normalized, and the one of Cal Poly error terms is scaled by a scale parameter. Brookings survey and Cal Poly survey have different categories for household income. To keep the consistent of income variable, we define the following two dummies instead of using numerical income calculated as median point of each category in modeling:

1. *High Income Dummy*: 1 if household annual income is greater than \$100,000, and 0 otherwise.

2. *Medium Income Dummy*: 1 if house hold annual income is between \$60,000 and \$100,000, and 0 otherwise.

The estimate results are shown in Table 14.

Table 14. Results from Joint Model Combining Two RP Data

Variables	Coefficients
Brookings Constant	-0.5150 (0.9674)
Cal Poly Constant	-1.7157 (0.7827)
Cost	-1.3443 (0.5312)
Cost × High Income Dummy	0.9047 (0.3096)
Cost × Medium Income Dummy	0.4693 (0.2149)
Dist × Time	-0.2618 (0.0917)
Dist ² × Time	0.0412 (0.0162)
Dist ³ × Time	-0.0017 (0.0007)
Dmp80	-0.5989 (0.2298)
Female	1.1294 (0.3904)
Age30-50	1.1951 (0.4465)
Household Size	-0.3874 (0.1846)
Dflex	0.2428 (0.3774)
Scale Parameter for Cal Poly sample	0.5028 (0.1977)
# of Observations	522
# of Persons	522
Log-Likelihood	-267.84

Note: Numbers in parentheses are standard errors.

Even with much larger data size, mixed-logit model capturing unobserved heterogeneity in motorists' preferences is still unidentified using RP only data. Compared with results using only Brookings RP data, the efficiency of estimates is improved significantly. The parameter estimates have the expected signs. Same as earlier studies, females and middle-aged motorists are more likely to choose SR91 toll lanes. These effects can not be identified solely in Brookings RP only model. The effect of household size is combined with household income in models with Brookings data. We do not

model like that here because we use income dummies instead of numerical values. The results show that motorists with smaller household size are more likely to choose toll lanes.

By specification, the value time is nonlinear with respect to both household income and trip distance, and the value of reliability is nonlinear with respect to income. Consistent with expectations, motorists with higher incomes are less responsive to the toll. Same as specifications in above section, the effect of distance on the time coefficient is captured well by a cubic form with no intercept, but the pattern is slightly different. When graphed, the dependence of the value of time on distance is characterized by an inverted U, initially rising but then falling for trips greater than 45 miles and all respondents in our sample have positive value of time. Using Brookings RP data only, the value of time falls for trips greater than 30 miles and about 10 percent respondents have negative value of time.

We tested whether Brookings and Cal Poly respondents react differently to the cost, time, and unreliability variables and found that there were no statistically significant differences.

2.2.2: Joint RP/SP Estimates

In this section, we combine all the three data sets — Brookings RP (BR), Brookings SP (BS), and Cal Poly RP (C) to estimate the heterogeneity in motorists'

preferences for both travel time and travel time reliability. The latent utility differences corresponding to these three data are like:

$$U_i^{BR} \equiv \mathbf{q}_i^{BR} + \mathbf{b}_i^{BR} X_i^{BR} + \mathbf{n}_i^{BR} + \mathbf{h}_i^{BR} \quad (2.20)$$

$$U_{it}^{BS} \equiv \mathbf{q}_i^{BS} + \mathbf{b}_i^{BS} X_{it}^{BS} + \mathbf{r} \mathbf{n}_i^{BR} + \mathbf{h}_{it}^{BS} \quad (2.21)$$

$$U_i^C \equiv \mathbf{q}_i^C + \mathbf{b}_i^C X_i^C + \mathbf{h}_i^C. \quad (2.22)$$

where the superscripts BR, BS, and C represent different data sources; index i runs through all individuals in the data sets; X is the vector including toll, travel time, and travel time unreliability; \mathbf{n}_i^{BR} is a random term with standard normal, thus the parameter \mathbf{r} captures the correlation between RP and SP observations from the same individual in Brookings data; \mathbf{h}_i^{BR} , \mathbf{h}_{it}^{BS} , and \mathbf{h}_i^C are independently logistic distributed; and

$$\mathbf{q}_i^k = \bar{\mathbf{q}}^k + \mathbf{f}^k W_i^k + \mathbf{x}_i^k \quad (2.23)$$

$$\mathbf{b}_i^k = \bar{\mathbf{b}}^k + \mathbf{g}^k Z_i^k + \mathbf{V}_i^k \quad (2.24)$$

where the superscript $k = BR, BS, C$; W_i^k and Z_i^k are individual characteristics capturing observed heterogeneity in preferences, while unobserved heterogeneity is captured by the random terms \mathbf{x}_i^k and \mathbf{V}_i^k . The term \mathbf{x}_i^k indicates an individual's unobserved alternative specific preferences, and \mathbf{V}_i^k represents an individual's unobserved preferences regarding

travel characteristics. Because the two RP data sets have only one observation for each individual, \mathbf{x}_i^{BR} and \mathbf{x}_i^C are redundant given \mathbf{h}_i^{BR} and \mathbf{h}_i^C .

In estimation, it is the variance of \mathbf{h}_i^{BR} that is normalized, and two scale parameters \mathbf{m}^{BS} and \mathbf{m}^C are estimated:

$$\mathbf{m}^{BS} \equiv \mathbf{s}^{BR} / \mathbf{s}^{BS} \quad (2.25)$$

$$\mathbf{m}^C \equiv \mathbf{s}^{BR} / \mathbf{s}^C \quad (2.26)$$

where each \mathbf{s} is the standard deviation of the corresponding \mathbf{h} .

Some parameters are assumed to be identical in two or three of above choice processes in order to combine the advantages of RP and SP data. Especially, like joint RP/SP model in Brookings only data, we assume that cost and time coefficients have the same random components across RP and SP samples. The measures for travel time unreliability in RP and SP samples are different, instead of assuming the same random components for the their coefficients, we assume that the ratio of standard deviation to the mean of the unreliability coefficient is the same across samples. Thus we can have:

$$\mathbf{b}_i^k = \bar{\mathbf{b}}^k + \mathbf{g}^k Z_i^k + \mathbf{V}_i \quad (2.27)$$

$$r_i^k = \bar{r}^k + \mathbf{p}_i^k = \bar{r}^k \left(1 + \frac{\mathbf{p}_i^k}{\bar{r}^k} \right) \equiv \bar{r}^k (1 + \mathbf{v}_i) \quad (2.28)$$

where \mathbf{b}_i refers to the vector of cost and time coefficients; r_i is the unreliability coefficient; V_i is independent with \mathbf{v}_i , and $V_i \sim N(0, \Omega)$ with Ω diagonal, $\mathbf{v}_i \sim N(0, \mathbf{S}_v)$.

The estimate results of this joint model are shown in Table 15. The coefficients of all the travel characteristics relevant to the RP choice are estimated with greater precision than before. The parameters capturing unobserved heterogeneity in the coefficients of cost, time and unreliability are also precisely estimated, as are the scale and correlation parameters describing error structure.

Table 15. Results from Joint Model Combining Three Data Sets

Variables	Coefficients
<i>RP Variables</i>	
Brookings Constant (\bar{q}^{BR})	0.2473 (0.7799)
Cal Poly Constant (\bar{q}^{BS})	-1.8389 (0.6860)
Cost	-2.2682 (0.3589)
Cost \times High Income Dummy	1.3147 (0.2794)
Cost \times Medium Income Dummy	0.6566 (0.2088)
Dist \times Time	-0.4933 (0.1009)
Dist ² \times Time	0.0868 (0.0189)
Dist ³ \times Time	-0.0037 (0.0009)
Dmp80	-0.7049 (0.2550)
<i>SP Variables</i>	
Constant (\bar{q}^C)	-1.2246 (0.8856)
Standard deviation of constant (\mathbf{s}_x)	0.1284 (0.6669)
Cost	-1.0986 (0.3128)
Cost \times High Income Dummy	0.1915 (0.6469)
Cost \times Medium Income Dummy	-0.0827 (0.2948)
Long \times Time	-0.1834 (0.0394)
(1-Long) \times Time	-0.2127 (0.0590)
Unreliability	-5.1686 (1.1195)
<i>Pooled Variables</i>	
Female	1.3849 (0.4046)
Age30-50	1.3021 (0.3856)
Household Size	-0.5902 (0.1738)
Dflex	0.7481 (0.4179)
Standard deviation of coefficient of cost (part of Ω)	0.6577 (0.1826)
Standard deviation of coefficient of time (part of Ω)	0.1268 (0.0471)
Ratio of standard deviation to the mean for coefficients of dmp80 and unreliability (\mathbf{s}_v)	0.9886 (0.3136)
<i>Other Parameters</i>	
Scale parameter for Cal Poly sample (\mathbf{m}^C)	0.3743 (0.0981)
Scale parameter for Brookings SP sample (\mathbf{m}^{BS})	1.4723 (0.3585)
Correlation parameter between RP and SP (\mathbf{r})	2.5493 (0.4969)
<i>Summary Statistics</i>	
# of Observations	1155
# of Persons	548
Log-Likelihood	-501.28

Note: Numbers in parentheses are standard errors.

The estimate results in Table 15 are used to calculate motorists' implied values of time and reliability and indicate the extent of their heterogeneity. The results are shown in Table 16.

Table 16. Values of Time and Reliability from Results in Table 15

	Median Estimate	90% Confidence Interval^a [5%-ile, 95%-ile]
RP Estimates		
<i>Value of time (\$/hour)</i>		
Median in sample	20.20	[14.72, 25.54]
Unobserved heterogeneity ^b	11.01	[6.48, 16.74]
Total heterogeneity in sample ^b	12.60	[8.30, 18.12]
<i>Value of reliability (\$/hour)</i>		
Median in sample	19.56	[8.03, 31.17]
Unobserved heterogeneity ^b	27.67	[11.56, 47.64]
Total heterogeneity in sample ^b	28.13	[11.56, 48.58]
SP Estimates		
<i>Value of time (\$/hour)</i>		
Median in sample	9.46	[6.18, 13.53]
Unobserved heterogeneity ^b	13.46	[7.41, 22.02]
Total heterogeneity in sample ^b	13.56	[7.52, 22.99]
<i>Value of reliability (\$/incident)</i>		
Median in sample	4.17	[2.37, 6.30]
Unobserved heterogeneity in sample ^b	7.78	[4.36, 12.64]
Total heterogeneity ^b	7.79	[4.36, 12.66]

All estimates in Table 14 are significantly different from zero at a 5% significance level. The median value of time base don commuters' revealed preferences is \$20.20/hour. In our data, median time savings at the height of rush hour are 5.6 minutes; thus, the average commuter would pay \$1.89 to realize these savings. The median value of reliability is \$19.56/hour. Unreliability peaks at 3 minutes; thus, the average commuter

would pay \$0.98 to avoid this possibility of unanticipated delay. Given these estimates, the actual peak toll of \$3.30 would be expected to attract somewhat fewer than half of the total peak traffic—which, in fact, it does.

We are also interested in how much motorists' preferences vary. We use the interquartile difference (the difference between 75th and 25th percentile values) as our heterogeneity measure because it is unaffected by high upper-tail values occasionally found in the calculations of ratios. This measure of heterogeneity exceeds 60% of the median value of time and is greater than the median value of unreliability, indicating that commuters exhibit a wide distribution of preferences for speedy and reliable travel.

It is interesting that the heterogeneity is almost all from unobserved sources, verifying the importance of “taste variation” in motorists' behavior and our attempt to capture it. To be sure, unobserved heterogeneity reflects limitations on empirical work and presumably could be reduced if it were possible to measure all variables that underlie individuals' preferences.

The implied SP values of time are smaller on average than the RP values. This finding may reflect the aforementioned tendency of travelers to overstate the travel time they lose or would lose in congestion. For example, suppose a motorist is in the habit of paying \$1.56 to save 10 minutes, but perceives that saving as 15 minutes. That motorist may then answer SP questions as if he or she would pay \$1.56 to save 15 minutes—yielding an SP value of time that understates the value used in actual decisions. The SP

value of unreliability may be similarly biased, but we have no point of comparison. The median value of \$4.17 per incident means that the median motorist in our sample would pay \$0.42 per trip to reduce the frequency of 10-minute delays from 0.2 to 0.1.

CHAPTER 3

Bayesian Analysis of Combining RP and SP Data in Discrete Choice Modeling

This chapter is to show how the recently developments in Bayesian approach for estimating the multinomial probit model can be used in joint RP and SP analysis. Most literatures on combining revealed preference data (RP) with stated preference data (SP) are based on logit model. The reason is the computational convenience of logit model. The multinomial probit model is hard to compute, especially when the number of choice alternatives is large and the correlation between RP and SP observations is not negligible. However, multinomial probit model has advantages in modeling flexibility, especially in dealing with correlation over choice alternatives and between RP and SP observations. The recent papers by Albert and Chib (1993), and by McCulloch and Rossi (1994) developed a Bayesian approach for estimating the multinomial probit model. Bayesian approach has theoretical advantages in interpreting results from finite sample, as well as in testing and model selection. In practice, Geweke, Keane, and Runkle (1997) found that given sample size of data, Bayesian approach performs better than simulated maximum likelihood estimation for multinomial probit model in the sense that the estimates have smaller RMSE. Bolduc (1996) found Bayesian approach to be about twice as fast as classical method in run time for their specifications on multinomial probit model.

How is the idea of using Bayesian approach to estimate multinomial probit model combining RP and SP observations? The method developed by Albert and Chib (1993), and McCulloch (1994) has limitation for this problem, because their method solves the

identification problem associated with probit model (as we will describe later) by introducing restriction via prior, that is, specifying prior for the full parameter space but only report the marginal posterior of identified parameters. In this case, it is very possible that the analytic forms for both the marginal prior and posterior on identified parameters are hard to get. As a result, the method developed by Chib (1995) for calculating Bayes factor can not be used, which makes it is difficult to test the difference between RP and SP choice processes.

McCulloch, Polson, and Rossi (2000) proposed a Bayesian approach for multinomial probit model with fully identified parameters. This method can be easily extended to multinomial probit models combining RP and SP data. We begin with the binary choice case.

3.1: Binary Choice Case

3.1.1: Models without Hierarchy

In this section, we discuss models with all coefficients to be fixed across individual. When RP and SP data sources are combined, we expect that at least some parameters do not vary with sources of data. At the same time, we need account for the their differences by letting other parameters be different and letting the error terms have different variances. Additionally, we need account for the correlation between RP and SP

choices from the same individual. The latent variable z_i for individual i , can be written

as

$$z_i^r = X_i^r B^r + \mathbf{e}_i^r \quad (3.1a)$$

$$z_i^s = X_i^s B^s + \mathbf{e}_i^s \quad (3.1b)$$

$$\mathbf{e}_i \equiv \begin{bmatrix} \mathbf{e}_i^r & \mathbf{e}_i^s \end{bmatrix} \sim IIDN(\mathbf{0}, \mathbf{S}) \quad (3.1c)$$

where the superscripts r and s indicate RP and SP data respectively; $IIDN(\cdot)$ represents identical independently normal density function; the coefficient vector B includes those varying with data sources and those common across data sources, and where

$$\mathbf{S} = \begin{bmatrix} \mathbf{s}_r^2 & \mathbf{s}_{rs} \\ \mathbf{s}_{rs} & \mathbf{s}_s^2 \end{bmatrix} \quad (3.1d)$$

The observed individual's choice

$$y_i^k = 1 \text{ if } z_i^k > 0, \quad k = r, s \quad (3.2)$$

In the following analysis, we let $y_i = (y_i^r \quad y_i^s)'$, $z_i = (z_i^r \quad z_i^s)'$,

$X_i = \text{diag}(X_i^r \quad X_i^s)$, and $B = (B^r \quad B^s)'$. This model is unidentified because the

likelihood function $\prod_i L_i(B, \mathbf{S})$ will not change if we multiply both B and \mathbf{S} by a

constant. The classical method is to normalize the variance of one error term, say \mathbf{s}_r^2 , as constant and maximize the reparameterized likelihood function with respect to identified parameters $(B/\mathbf{s}_r, \Sigma/\mathbf{s}_r^2)$. Bayesian approach has option of introducing restrictions via prior distribution, and it can also define priors on identified parameters directly. Here we employ the latter method to discuss how to estimate probit model combining RP and SP observations.

Because \mathbf{e}_i^r and \mathbf{e}_i^s have a joint normal distribution, we can derive the following conditional distribution

$$\mathbf{e}_i^s | \mathbf{e}_i^r \sim IIDN(\mathbf{l} \mathbf{e}_i^r, \mathbf{d}) \quad (3)$$

where $\mathbf{l} = \mathbf{s}_{rs}/\mathbf{s}_r^2$, and $\mathbf{d} = \mathbf{s}_s^2 - \mathbf{s}_{rs}^2/\mathbf{s}_r^2$. Let $\mathbf{S} = \tilde{\mathbf{S}}/\mathbf{s}_r^2$, then

$$\tilde{\mathbf{S}} = \begin{bmatrix} 1 & \mathbf{l} \\ \mathbf{l} & \mathbf{m} \end{bmatrix} \quad (3.4a)$$

The square root of $\mathbf{m} = \mathbf{s}_s/\mathbf{s}_r$ is called as scaled parameter in the literature of combining RP and SP observations. By normalizing \mathbf{s}_r^2 as 1, $\tilde{\mathbf{S}}$ can be written as

$$\tilde{\mathbf{S}} = \begin{bmatrix} 1 & \mathbf{l} \\ \mathbf{l} & \mathbf{d} + \mathbf{l}^2 \end{bmatrix} \quad (3.4b)$$

The model is estimated by drawing random numbers from the following posterior distribution:

$$p(B, \tilde{\mathbf{S}}, z|y) \propto \left(\prod_i p(y_i|z_i) p(z_i|B, \tilde{\mathbf{S}}) \right) \cdot p(B, \tilde{\mathbf{S}}) \quad (3.5)$$

The random draws are taken by Gibbs sampling, which draws in turn from the following complete conditionals:

$$1. z_i^r | z_i^s, B, \tilde{\mathbf{S}}, y_i^r$$

Because $z_i \sim IIDN(X_i B, \tilde{\Sigma})$, we can get $z_i^r | z_i^s, B, \tilde{\mathbf{S}} \sim IIDN(\mathbf{q}^r, \mathbf{f}^r)$, and

$$\mathbf{q}^r = X_i^r B + \frac{\mathbf{I}}{\mathbf{d} + \mathbf{I}^2} (z_i^s - X_i^s B), \quad \mathbf{f}^r = 1 - \frac{\mathbf{I}^2}{\mathbf{d} + \mathbf{I}^2}. \text{ Then,}$$

$$z_i^r | z_i^s, B, \tilde{\Sigma}, y_i^r \sim \begin{cases} TN_{(0, \infty)}(\mathbf{q}^r, \mathbf{f}^r), & \text{if } y_i^r = 1 \\ TN_{(-\infty, 0)}(\mathbf{q}^r, \mathbf{f}^r), & \text{if } y_i^r = 0 \end{cases} \quad (3.6)$$

where $TN_{(a,b)}(\cdot)$ represents truncated normal distribution which is truncated below a and above b .

$$2. z_i^s | z_i^r, B, \tilde{\mathbf{S}}, y_i^s$$

Similar to step 1, we can know that

$$z_i^s \Big| z_i^r, B, \tilde{\Sigma}, y_i^s \sim \begin{cases} TN_{(0,\infty)}(\mathbf{q}^s, \mathbf{d}), & \text{if } y_i^s = 1 \\ TN_{(-\infty,0)}(\mathbf{q}^s, \mathbf{d}), & \text{if } y_i^s = 0 \end{cases} \quad (3.7)$$

where $\mathbf{q}^s = X_i^s B + \mathbf{I}(z_i^r - X_i^r B)$.

3. $B \Big| z, \tilde{\Sigma}, y$

Let $z = \{z_i\}_{i=1}^N$, $y = \{y_i\}_{i=1}^N$, $X = \{X_i\}_{i=1}^N$, and define a conjugate normal prior on B , that is,

$B \sim N(B_0, L_B)$, we can get

$$B \Big| z, \tilde{\Sigma}, y \sim N\left(\left(X \mathbf{W}^{-1} X + L_B^{-1}\right)^{-1} \left(X \mathbf{W}^{-1} z + L_B^{-1} B_0\right), \left(X \mathbf{W}^{-1} X + L_B^{-1}\right)^{-1}\right) \quad (3.8)$$

where $\mathbf{W} = \mathbf{I}_N \otimes \tilde{\Sigma}$.

4. $\tilde{\Sigma} \Big| z, B, y$

The draws for $\tilde{\Sigma}$ are taken by constructing $\mathbf{e}_i^s = \mathbf{I} \mathbf{e}_i^r + \mathbf{n}_i$, then $\mathbf{n}_i \sim IIDN(0, \mathbf{d})$, and we

can draw \mathbf{I} and \mathbf{d} from the following univariate regression model

$$\mathbf{e}^s = \mathbf{I} \mathbf{e}^r + \mathbf{v} \quad (3.9)$$

where $\mathbf{e}^s = \{\mathbf{e}_i^s\}_{i=1}^N$, $\mathbf{e}^r = \{\mathbf{e}_i^r\}_{i=1}^N$, and $\mathbf{n} = \{\mathbf{n}_i\}_{i=1}^N$. Define conjugate Normal — Inverse

Gamma prior on \mathbf{I} and \mathbf{d} , that is, $(\mathbf{I} \quad \mathbf{d}) \sim N(\mathbf{I}_0, \mathbf{L}_1) \cdot IG\left(\frac{k}{2}, \frac{ks^2}{2}\right)$, we then can have

$$\mathbf{I} | z, B, \mathbf{d}, y \sim N(\mathbf{t}\mathbf{v}, \mathbf{t}) \quad (3.10)$$

$$\mathbf{d} | z, B, \mathbf{I}, y \sim IG(\mathbf{a}, \mathbf{b}) \quad (3.11)$$

In equations (3.10) and (3.11),

$$\mathbf{t} = \left(\frac{\sum_{i=1}^N (z_i^r - X_i^r B)^2}{\mathbf{d}} + \frac{1}{\mathbf{L}_1} \right)^{-1}, \quad \mathbf{v} = \frac{\sum_{i=1}^N (z_i^r - X_i^r B)(z_i^s - X_i^s B)}{\mathbf{d}} + \mathbf{L}_1 \mathbf{I}_0, \quad \mathbf{a} = \frac{N+k}{2},$$

$$\mathbf{b} = \left(\frac{1}{ks^2} + 0.5 \sum_{i=1}^N [(z_i^s - X_i^s B) - \mathbf{I}(z_i^r - X_i^r B)]^2 \right)^{-1}.$$

Simulation Example 3.1

The simulation example is designed as:

$$z_i^r = -0.8 - 1.2 * x_i^r + \mathbf{e}_i^r$$

$$z_i^s = -0.4 - 1.2 * x_i^s + \mathbf{e}_i^s$$

The variance of \mathbf{e}_i^r is 2, and the one of \mathbf{e}_i^s is 3. The correlation between \mathbf{e}_i^r and \mathbf{e}_i^s is set as 0.5. By this setting, the true values of identified parameters are $B/\mathbf{s}_r \approx (-0.57, -0.28, -0.85)$, $\mathbf{I} \equiv \mathbf{s}_{rs}/\mathbf{s}_r^2 \approx 0.61$, $\mathbf{d} = (\mathbf{s}_s^2 - \mathbf{s}_{rs}^2 / \mathbf{s}_r^2) / \mathbf{s}_r^2 \approx 1.13$. The independent variable x_i^r and x_i^s are generated independently from a uniform distribution with support $[-2, 2]$. We use last 5000 Gibbs draws from a total of 6000 to form our posterior distributions of estimated parameters.

We employ two priors: the first is proper but diffuse, and the second is improper on \mathbf{d} . Specifically, under prior 1, $p(\mathbf{B}) = N(\mathbf{0}, \mathbf{10}^7 \mathbf{I})$, and $p(\mathbf{I}, \mathbf{d}) = N(1, 4)IG\left(\frac{5}{2}, \frac{5}{2}\right)$. Under prior 2, $p(\mathbf{B}) = N(\mathbf{0}, \mathbf{10}^7 \mathbf{I})$, and $p(\mathbf{I}, \mathbf{d}) = N(0, 10^3)IG(0, 0)$. Figure 5, from top to bottom, shows the estimated results for B , \mathbf{I} , and \mathbf{d} respectively under prior 1 with sample size of 3000. The results are reasonable.

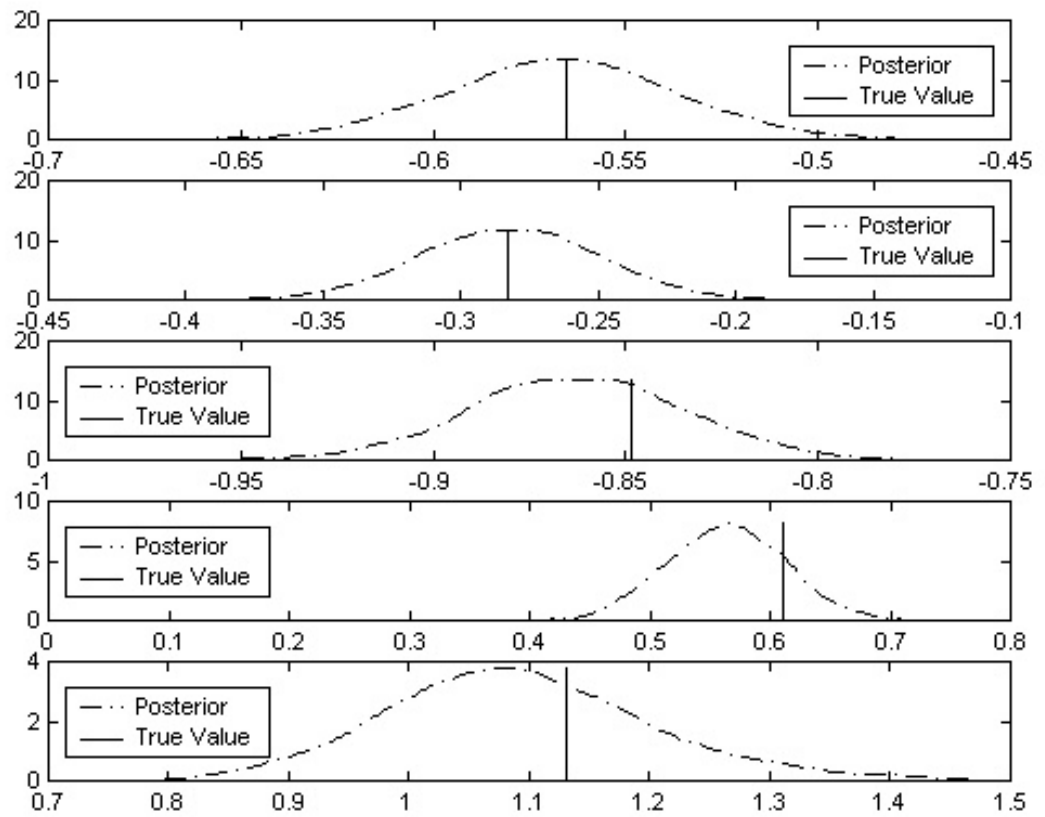


Figure 5. Simulation Exmapple 3.1 with Prior 1 and Sample Size 3000

Figure 6, in the same order as figure 5, shows the estimated results under prior 2 with sample size of 2000. This algorithm still works well.

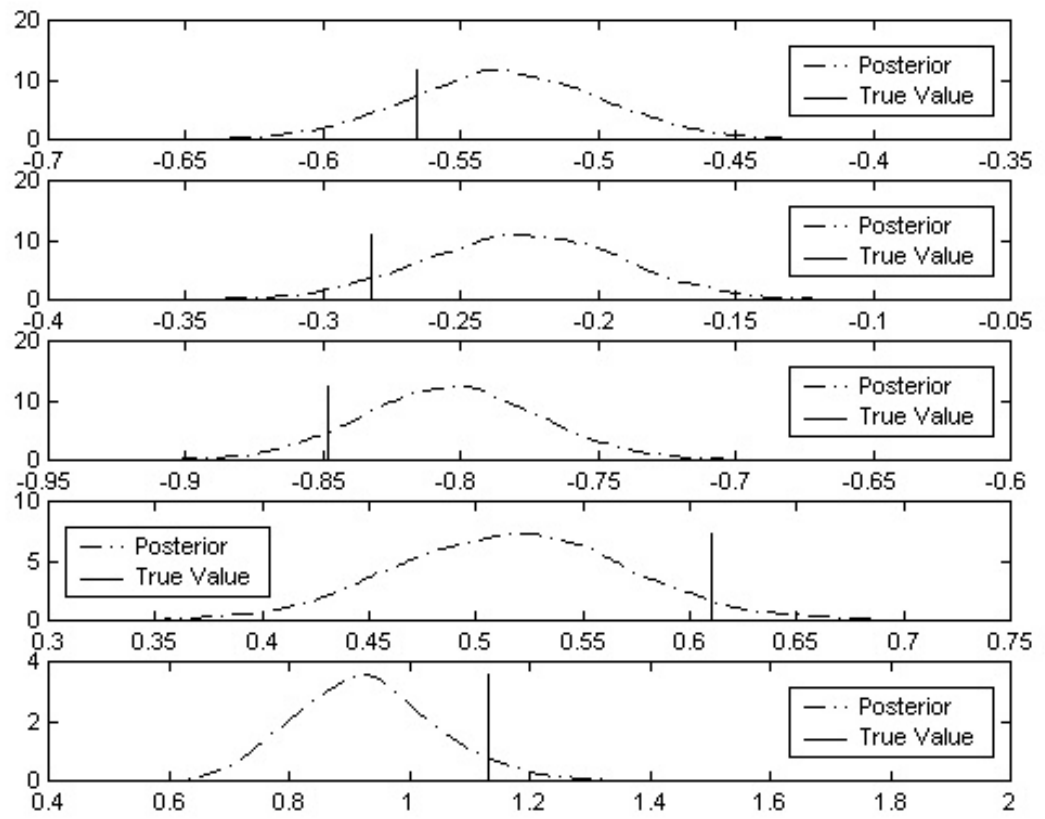


Figure 6. Simulation Example 3.1 with Prior 2 and Sample Size 2000

3.1.2: Hierarchical Analysis

The above algorithm can be easily extended to hierarchical analysis combining RP and SP data, which can be used to uncover individual's preference. In hierarchical model, the latent utilities in equation (3.1a) and (3.1b) are rewritten as

$$z_i^r = X_i^r B_i^r + e_i^r \quad (3.12a)$$

$$z_i^s = X_i^s B_i^s + e_i^s \quad (3.12b)$$

The parameters $B_i \equiv (B_i^r \ B_i^s)'$ representing individuals' preferences are conditional on individuals' characteristics W_i , and include a stochastic part e_i capturing unobserved heterogeneity, that is,

$$B_i = W_i \mathbf{g} + e_i \quad (3.13)$$

The error term can be modeled generally as $e_i \equiv (e_i^r \ e_i^s)' \sim IIDN(\mathbf{0}, \Psi)$. Thus if both B_i^r and B_i^s are $k \times 1$ vectors, Ψ is a $2k \times 2k$ matrix. In above model, some parameters in $\mathbf{g} \equiv (\mathbf{g}^r \ \mathbf{g}^s)'$ and Ψ are constrained to be the same across data sources. The Gibbs sampling for this model adds one more layer, specifically the random draws are drawn from the following steps:

The first layer:

$$z_i^r | z_i^s, B_i, \tilde{\Sigma}, y_i^r \quad (3.14a)$$

$$z_i^s | z_i^r, B_i, \tilde{\Sigma}, y_i^s \quad (3.14b)$$

$$B_i | z_i, \tilde{\Sigma}, y_i \quad (3.14c)$$

$$\tilde{\Sigma} | z, B, y \quad (3.14d)$$

where $B = \{B_i\}_{i=1}^N$. In the first layer, draws from (3.14a), (3.14b), and (3.14d) are the same as step 1, 2, and 4 respectively in the model without hierarchy. For (3.14c), if

$$B_i \sim IIDN(B_{i0}, \Lambda_{B_i}), \quad B_i | z_i, \tilde{\Sigma}, y_i \sim N(Dd, D), \quad \text{and} \quad D = \left(X_i' \tilde{\Sigma}^{-1} X_i + \Lambda_{B_i}^{-1} \right)^{-1},$$

$$d = X_i' \tilde{\Sigma}^{-1} z_i + \Lambda_{B_i} B_{i0}.$$

The second layer:

$$\mathbf{g} | \Psi, B \quad (3.15a)$$

$$\Psi | \mathbf{g}, B \quad (3.15b)$$

We define conjugate Normal — Inverse Wishart prior for \mathbf{g} and Ψ ,

$$(\mathbf{g} \quad \Psi) \sim N(\mathbf{g}_0, \Lambda_g) \cdot IW(\mathbf{r}, \mathbf{r}R)$$

then

$$\mathbf{g}|\Psi, B \sim N\left(\left(W'\Phi^{-1}W + \Lambda_g^{-1}\right)^{-1}\left(W'\Phi^{-1}B + \Lambda_g\mathbf{g}_0\right), \left(W'\Phi^{-1}W + \Lambda_g^{-1}\right)^{-1}\right) \quad (3.16a)$$

$$\Psi|\mathbf{g}, B \sim IW(\mathbf{r} + K, \mathbf{r}R + V) \quad (3.16b)$$

In equations (3.16a) and (3.16b), $W = \{W_i\}_{i=1}^N$, K is the dimension of B and $K > N$,

$$\Phi = \mathbf{I}_K \otimes \Psi, \text{ and } V = \sum_{j=1}^K (B_j - W_j\mathbf{g})(B_j - W_j\mathbf{g})'.$$

Simulation Example 3.2

We design the following simulation example:

$$z_i^r = \mathbf{a}x_i^r + \mathbf{b}_i^r w_i^r + \mathbf{e}_i^r$$

$$z_i^s = \mathbf{a}x_i^s + \mathbf{b}_i^s w_i^s + \mathbf{e}_i^s$$

$$\mathbf{b}_i^r = \bar{\mathbf{b}}^r + \mathbf{m}_i$$

$$\mathbf{b}_i^s = \bar{\mathbf{b}}^s + \mathbf{m}_i$$

$$\mathbf{e}_i \sim IIDN(\mathbf{0}, \Sigma)$$

$$\mathbf{m}_i \sim IIDN(0, \mathbf{s}_m^2)$$

where $a = -1.2$, $\bar{\mathbf{b}}^r = -0.8$, $\bar{\mathbf{b}}^s = -0.4$, $\Sigma = \begin{pmatrix} 2 & 0.5\sqrt{2}\sqrt{3} \\ 0.5\sqrt{2}\sqrt{3} & 3 \end{pmatrix}$, $\mathbf{s}_m^2 = 0.6$. Note

that we assume that \mathbf{b}_i^r and \mathbf{b}_i^s have common random term in this example. The purpose of doing this is that in some applications, SP data is combined with RP data to estimate the unobserved heterogeneity in consumers' preferences, which can not be identified using RP data only. Because of this assumption, we modified the Gibbs sampling like

The first layer:

$$z_i^r | z_i^s, (a, \bar{\mathbf{b}}^r, \bar{\mathbf{b}}^s), \mathbf{m}_i, y_i^r$$

$$z_i^s | z_i^r, (a, \bar{\mathbf{b}}^r, \bar{\mathbf{b}}^s), \mathbf{m}_i, y_i^s$$

$$(a, \bar{\mathbf{b}}^r, \bar{\mathbf{b}}^s) | z, \mathbf{m}, y$$

$$\mathbf{m}_i | z_i, (a, \bar{\mathbf{b}}^r, \bar{\mathbf{b}}^s), y_i$$

The second layer:

$$\mathbf{s}_m^2 | \mathbf{m}$$

We choose the following proper but diffuse priors: $(\mathbf{a}, \bar{\mathbf{b}}^r, \bar{\mathbf{b}}^s)' \sim N(\mathbf{0}, 10^7 \mathbf{I})$,

$\mathbf{I} \sim N(1, 10)$, $\mathbf{d} \sim IG\left(\frac{5}{2}, \frac{5}{2}\right)$, $\mathbf{s}_m^2 \sim IG\left(\frac{5}{2}, \frac{5}{2}\right)$, and the sample size is set as 3000. The

results are shown in figures 7 and 8, and we get good results.

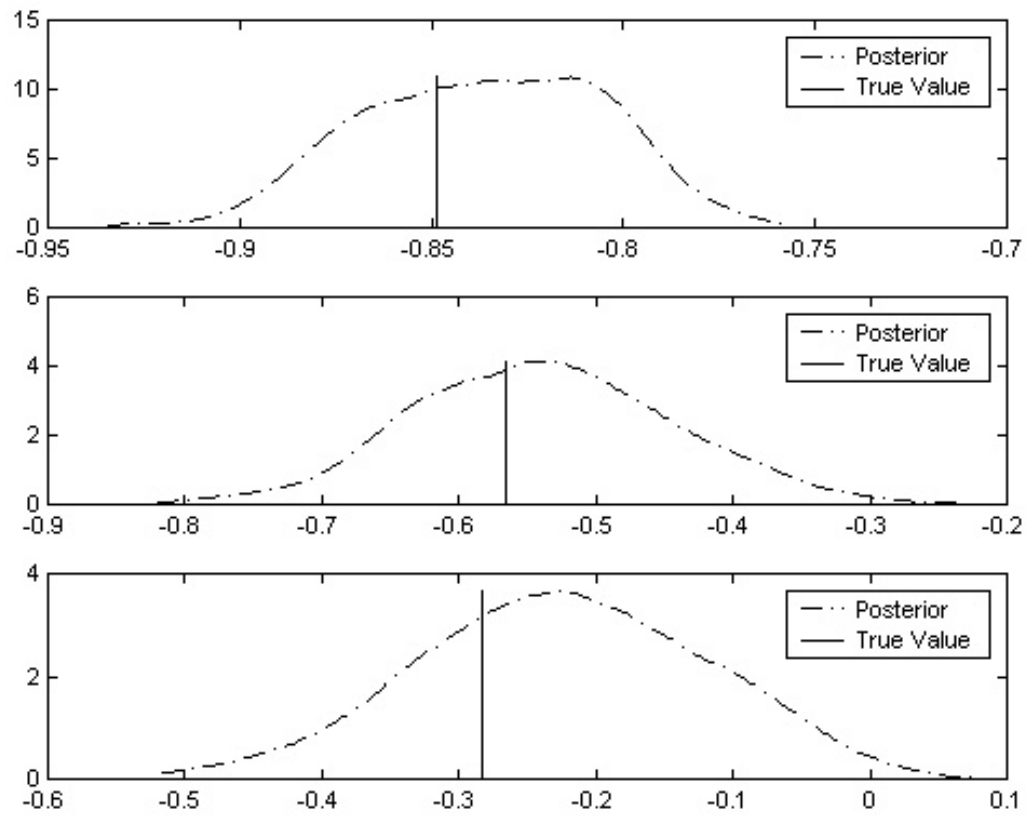


Figure 7. Estimate Results for Simulation Example 3.2 (\mathbf{a} , $\bar{\mathbf{b}}^r$, $\bar{\mathbf{b}}^s$ from top to bottom)

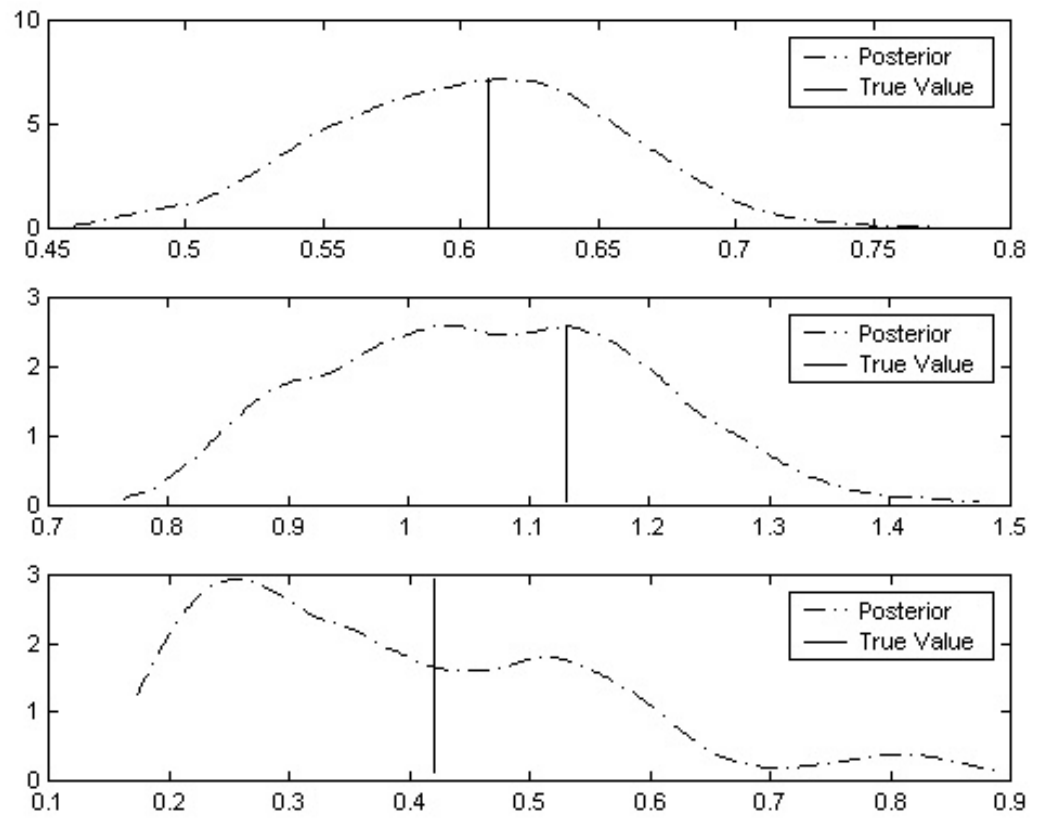


Figure 8. Estimated Results for Simulation Example 3.2 ($\mathbf{l}, \mathbf{d}, \mathbf{s}_m^2$, from top to bottom)

3.2: Multinomial Case

Extending above analysis to multinomial probit model is also straightforward.

Suppose there are J choice alternatives in RP data and K choice alternatives in SP data,

that is, $y_i^r \in \{0,1,\dots,J-1\}$, and $y_i^s \in \{0,1,\dots,K-1\}$. Since choices only depend on utility

difference, the multinomial probit model can be described by the following utility

differences with respect to choice alternative 0:

$$z_{ij}^r = \mathbf{X}_{ij}^r \mathbf{B}^r + \varepsilon_{ij}^r, \quad j = 1, \dots, J-1 \quad (3.17a)$$

$$z_{ik}^s = \mathbf{X}_{ik}^s \mathbf{B}^s + \varepsilon_{ik}^s, \quad k = 1, \dots, K-1 \quad (3.17b)$$

and

$$\varepsilon_i \equiv \{\varepsilon_{i1}^r, \dots, \varepsilon_{iJ-1}^r, \varepsilon_{i1}^s, \dots, \varepsilon_{iK-1}^s\}' \sim \text{IIDN}(\mathbf{0}, \Sigma) \quad (3.17c)$$

The observed individual's choice

$$y_i^t = \begin{cases} 0 & \text{if } \max(z_{ij}^t) < 0 \\ h & \text{if } \max(z_{ij}^t) = z_{ih}^t > 0 \end{cases} \quad (3.18)$$

where $t = r, s$, and if $t = r$, $j, h \in \{1, \dots, J-1\}$; if $t = s$, $j, h \in \{1, \dots, K-1\}$. Still, to get identification, the first diagonal element of Σ is normalized as 1, and similar to binary case, the identified variance matrix can be written as

$$\tilde{\Sigma} = \begin{bmatrix} 1 & \mathbf{I} \\ \mathbf{I} & \Delta + \mathbf{I}\mathbf{I}' \end{bmatrix} \quad (3.19)$$

where λ is a 1 by $(J-2+K-1)$ vector representing normalized covariance between ε_{i1}^r and other elements in ε_i ; Δ is the variance matrix of the conditional distribution

$\left\{ \varepsilon_{i2}^r, \dots, \varepsilon_{iJ-1}^r, \varepsilon_{i1}^s, \dots, \varepsilon_{iK-1}^s \right\} \mid \text{var}(\varepsilon_{i1}^r) = 1$. The Gibbs sampling of estimating this model is like:

$$1. z_{ij}^r \mid \left\{ z_{ih}^r \right\}_{h \neq j}, \left\{ z_{ik}^s \right\}_{k=1}^{K-1}, \mathbf{B}_i, \tilde{\Sigma}, y_i^r$$

This draw is taken from univariate truncated normal which is truncated below $\max(z_{ih}^r)$ if $y_i^r = j$ and is truncated above $\max(0, \max(z_{ih}^r))$ if $y_i^r \neq j$, where $j \neq h$.

$$2. z_{ik}^s \mid \left\{ z_{ij}^r \right\}_{j=1}^{J-1}, \left\{ z_{ih}^s \right\}_{h \neq k}, \mathbf{B}_i, \tilde{\Sigma}, y_i^s$$

Similarly, this draw is taken from univariate truncated normal which is truncated below $\max(z_{ih}^s)$ if $y_i^s = k$ and is truncated above $\max(0, \max(z_{ih}^s))$ if $y_i^s \neq k$, where $k \neq h$.

$$3. \mathbf{B} \mid \mathbf{z}, \tilde{\Sigma}, \mathbf{y}$$

This step is the same as in binary case.

4. $\tilde{\Sigma}|z, B, y$

The only difference between this step and the one in binary case is that instead of drawing from a univariate normal regression model, we draw $\tilde{\Sigma}$ from the following multivariate normal regression model

$$\eta = \xi\lambda' + v$$

where $\eta = (\eta_1, \dots, \eta_N)'$, and $\eta_i = (\epsilon_{i2}^r, \dots, \epsilon_{iJ-1}^r, \epsilon_{i1}^s, \dots, \epsilon_{iK-1}^s)'$; $\xi = (\xi_1, \dots, \xi_N)'$, and $\xi_i = \text{diag}(\epsilon_{i1}^r)$ with dimension of $J - 2 + K - 1$; $\mathbf{n}_i \sim IIDN(\mathbf{0}, \Delta)$.

3.3: Summary

By simulation examples, this chapter shows how the newly advances in Bayesian econometrics for estimating the multinomial probit model can be extended with no trouble to analyze probit models combing RP and SP data sources. The Bayesian approach works well on these simulation examples, and by combining with method for calculating Bayes factor proposed by Chib (1995), it can be employed to estimate joint RP/SP model, compare and test the differences between different choice processes. Bayesian approach can provides us more flexibility in model specification, and more advantages in interpreting estimate results, testing hypothesis and model selection.

CHAPTER 4

Policy Implications of Heterogeneity in Motorists' Preferences

In this chapter, we combine the estimate results in former chapters and the simulation model developed by Small and Yan (2001) to investigate the policy implications of heterogeneity in motorists' preferences.

4.1: The Model

This model considers two roadways, A and B, connecting the same origin and destination. Both have the same length L and the same free-flow travel-time $T_f L$. A user of type i ($i=1,2$) traveling on road r ($r=A,B$) incurs travel cost c_{ir} which consists of operating cost \mathbf{b} plus a time cost $\mathbf{a}_i T_r$ per unit distance. The parameter \mathbf{a}_i is the value of time, and it is this parameter for which we introduce heterogeneity, by assuming that $\mathbf{a}_1 > \mathbf{a}_2$. Unit travel time T_r (the inverse of speed) is represented by flow congestion of a standard type, depending on volume-capacity ratio N_r/K_r so that:

$$c_{ir}(N_r) = \mathbf{b}L + \mathbf{a}_i T_f L \left[1 + \mathbf{g} \left(N_r / K_r \right)^k \right] \quad i=1,2; r = A, B \quad (4.1)$$

where \mathbf{g} and k are parameters. The congestion-dependent part of cost, $d_{ir} \equiv \mathbf{a}_i T_f L \mathbf{g} (N_r / K_r)^k$, is what we call *delay cost*. This particular functional form has the property that the marginal external cost is k times the average delay cost:

$$MEC_r \equiv \sum_i N_{ir} \partial c_{ir} / \partial N_r = k \cdot \left(\sum_i N_{ir} d_{ir} \right) / N_r, \text{ where } N_{ir} \text{ is the number of type-}i \text{ users on}$$

road r . We use values $g=0.15$ and $k=4$, following common practice.¹⁰

Demand by each group has the linear form

$$N_i(P_i) = a_i - b_i P_i \quad (4.2)$$

where a_i and b_i are positive parameters and P_i is the "inclusive price" or "full price", defined as the minimum combination of travel cost plus toll (τ) for this user group:

$$P_i = \underset{r}{\text{Min}} \{ c_{ir} + \tau_r \} . \quad (4.3)$$

The inverse demand function corresponding to (4.2) is denoted $P_i(N_i)$.

The social welfare function is defined as the area under the inverse demand curve, less total cost:

$$W = \sum_{i=1}^2 \int_0^{N_i} P_i(t) dt - \sum_{i=1}^2 \sum_{r=A}^B N_{ir} C_{ir} \quad (4.4)$$

This function is strictly concave in the four variables N_{ir} .

¹⁰ See Small (1992), pp. 69-72, for a discussion of empirical evidence for this functional form. These particular parameters are known as the Bureau of Public Roads formula.

4.1.1: Types of Solution

The equilibrium conditions are those of Wardrop (1952), stating (i) that users of a given type choose the road or roads that minimize inclusive price, and (ii) that inclusive price be equalized across the two roads for any user group that uses both roads. We assume that if the roads are differentiated it is road A that offers faster travel, so that $N_{1A} > 0$ and $N_{2B} > 0$. (This is a substantive assumption if the roads are of unequal capacity.) Wardrop's conditions can then be written:

$$c_{1A}(N_A) + t_A \leq c_{1B}(N_B) + t_B \quad (4.5a)$$

$$c_{2A}(N_A) + t_A \geq c_{2B}(N_B) + t_B \quad (4.5b)$$

$$N_{1B} \cdot (c_{1A} + t_A - c_{1B} - t_B) = 0 \quad (4.5c)$$

$$N_{2A} \cdot (c_{2A} + t_A - c_{2B} - t_B) = 0 \quad (4.5d)$$

$$N_{1B}, N_{2A} \geq 0 \quad (4.5e)$$

It is useful to distinguish four possible cases, depending on whether each of (4.5a) and (4.5b) is an inequality or an equality.

Case SE: fully separated equilibrium. Both (4.5a) and (4.5b) are inequalities, i.e., each group strictly prefers a different roadway. Because we assumed $a_1 > a_2$, these

conditions require that road A be more expensive but less congested than road B, ¹¹ i.e., $\tau_A > \tau_B$ and $(N_A/K_A) < (N_B/K_B)$.

Case SE1: partially separated equilibrium with group 1 separated. Group 1 strictly prefers road A but group 2 is indifferent: that is, (4.5a) is an inequality but (4.5b) an equality. Like the fully separated equilibrium, SE1 requires that road A have higher toll but lower travel time. Note it is not impossible that $N_{2A}=0$, if this conditions happens to yield indifference for group 2; but we would expect this only by coincidence.

Case SE2: partially separated equilibrium with group 2 separated. Group 2 strictly prefers road B, but group 1 is indifferent: (4.5a) is an equality, (4.5b) an inequality. Again, road A must have a higher toll but is faster. The boundary solution $N_{1B}=0$ can occur, but again only by chance.

Case IE: fully integrated equilibrium. Both groups are indifferent between the two roads; (4.5a-b) both hold with equalities. Since the two groups have different values of time, this can occur only if the roads have equal tolls and equal speeds.

¹¹ Subtracting (4.5b) from (4.5a) and applying (4.1) yields $(\mathbf{a}_1 - \mathbf{a}_2)(N_A / K_A)^k < (\mathbf{a}_1 - \mathbf{a}_2)(N_B / K_B)^k$, which (given $\mathbf{a}_1 > \mathbf{a}_2$ and $k > 0$) implies $N_A / K_A < N_B / K_B$. This in turn implies $c_{2A} < c_{2B}$, so (4.5b) requires $t_A > t_B$

4.1.2: Pricing Regimes

We consider the following alternative pricing regimes, also called policies.

First-best regime (FB): a public operator charges tolls on both roads that maximize welfare (4.4). It can be shown that this policy yields conventional marginal-cost pricing on each road.

Second-best regime (SB): the same objective is pursued but subject to the constraint $t_B=0$.

Profit-maximizing regime (PM): t_A is chosen to maximize revenues on road A subject to the constraint $t_B=0$. (By calling this “profit-maximizing”, we implicitly assume there are no variable costs to the road owner of serving traffic.)

No-toll regime (NT): $t_A = t_B = 0$.

The no-toll regime is determined by solving (4.1)-(4.3) and (4.5) with equalities in (4.5a) and (4.5b); the solution is assumed to be of the integrated equilibrium (IE) type, since there is nothing to distinguish the two roadways from each other. (This is in fact the only regime where IE can occur, due to our assumption of strictly unequal values of time.) Each of the other regimes calls for maximizing either welfare, as given by (4.4), or revenues $R = \sum_r t_r N_r$, while imposing constraints (4.5)

4.1.3: Solutions

We assume that at least some type 1 users use road A and at least some type 2 users use road B. We consider a congested traffic condition, so the toll charged under a policy regime is strictly greater than zero. The general form of the first-best (FB) problem is therefore:

$$\max W = \int_0^{N_{1A}+N_{1B}} P_1(t)dt + \int_0^{N_{2A}+N_{2B}} P_2(t)dt - \sum_i \sum_r N_{ir} c_{ir}$$

$$s.t. h_1 \equiv P_1(N_{1A} + N_{1B}) - c_{1A}(N_{1A} + N_{2A}) - \mathbf{t}_A = 0 \quad (4.6a)$$

$$h_2 \equiv P_2(N_{2A} + N_{2B}) - c_{2B}(N_{1B} + N_{2B}) - \mathbf{t}_B = 0 \quad (4.6b)$$

$$h_3 \equiv N_{1B} \cdot (P_1 - c_{1B} - \mathbf{t}_B) = 0 \quad (4.6c)$$

$$h_4 \equiv N_{2A} \cdot (P_2 - c_{2A} - \mathbf{t}_A) = 0 \quad (4.6d)$$

$$g_1 \equiv P_1(N_{1A} + N_{1B}) - c_{1B}(N_{1B} + N_{2B}) - \mathbf{t}_B \leq 0 \quad (4.6e)$$

$$g_2 \equiv P_2(N_{1A} + N_{2A}) - c_{2A}(N_{1A} + N_{2A}) - \mathbf{t}_A \leq 0 \quad (4.6f)$$

$$g_3 \equiv -N_{1B} \leq 0 \quad (4.6g)$$

$$g_4 \equiv -N_{2A} \leq 0 \quad (4.6h)$$

where $P(\cdot)$ and $c(\cdot)$ are the functions defined by (4.2) and (4.1). Certain constraints are added for the SB, TB, and PM policy, and the objective function is replaced by toll revenues in PM policy. Because we assume $N_{1A}, N_{2B} > 0$, (4.6a-b) are the same as (4.3) of the paper; (4.6c-d) are equivalent to (4.5c-d); (4.6e-f) to (4.5a-b); and (4.6g-h) to (4.5e).

Suppose I_1, I_2, I_3, I_4 are the Lagrangian multipliers for the four equality constraints, and g_1, g_2, g_3, g_4 are those for the inequality constraints. According to the Kuhn-Tucker theorem, the necessary condition for the optimal solution $N^* = (N_{1A}^*, N_{1B}^*, N_{2A}^*, N_{2B}^*)$, $I^* = (I_1^*, I_2^*, I_3^*, I_4^*)$, $g^* = (g_1^*, g_2^*, g_3^*, g_4^*)$ are:

$$\nabla W(N^*) - \sum_{i=1}^4 I_i^* \nabla h_i(N^*) - \sum_{j=1}^4 g_j^* \nabla g_j(N^*) = 0 \quad (4.7a)$$

$$g_j^* g_j(N^*) = 0, j = 1, 2, 3, 4 \quad (4.7b)$$

$$g_j^* \geq 0, j = 1, 2, 3, 4 \quad (4.7c)$$

$$g_j \leq 0, j = 1, 2, 3, 4 \quad (4.7d)$$

If constraints (4.6e) and (4.6f) are binding at the same time, the tolls on both routes must be equal as shown in section 2. This is impossible for SB, TB and PM policy and our numerical results also show that this case is never optimal for FB policy. As a result, the possible solution cases for the programming problem are only three:

1. $g_1^* = 0, g_2^* > 0$ (SE1);

In this case, (4.7c) $\Rightarrow g_2 = 0$, i.e., (4.6f) must be binding. This means type 2 users are indifferent for two routes. Then (4.6e) can't be binding, i.e., type 1 users strictly prefer road A and, from (4.6c), $N_{1B}^* = 0$.

2. $g_1^* > 0, g_2^* = 0$ (SE2);

In this case, constraint (4.6e) is binding and constraint (4.6f) is not binding, and $N_{2A}^* = 0$.

3. $g_1^* = 0$ and $g_2^* = 0$;

In this case, we can only say (from the argument above) that (4.6e) or (4.6f) or both must be non-binding, therefore N_{1B}^* or N_{2A}^* or both must be zero. Thus there are three solution cases:

3a. (4.6f) is binding and (4.6e) is not; $N_{1B}^* = 0$ (SE1).

3b. (4.6e) is binding and (4.6f) is not. $N_{2A}^* = 0$ (SE2).

3c. Both (4.6e) and (4.6f) are non-binding. $N_{1B}^* = N_{2A}^* = 0$ (SE).

In the paper, we divide the programming problem into different cases (SE, SE1, SE2) and solve each case under each policy. The above classification shows that the solutions from these cases include all of the possible solutions for the whole problem.

a). FB Policy

Case SE. Substituting $N_{1B} = 0$ and $N_{2A} = 0$ into the welfare function, the welfare maximizing problem can be written as:

$$\max W = \int_0^{N_{1A}} P_1(t) dt + \int_0^{N_{2B}} P_2(t) dt - N_{1A} \cdot c_{1A}(N_{1A}) - N_{2B} \cdot c_{2B}(N_{2B})$$

The objective function is strictly concave because it equals the sum of four strictly concave functions. Therefore, the solution to the first-order conditions must be unique. The optimal

traffic (N_{1A}^*, N_{2B}^*) in this case can be solved out from those first-order conditions. The corresponding tolls on the two routes, determined by (4.6a-b), are:

$$t_A = P_1 - c_{1A} = N_{1A} \cdot c'_{1A}(N_{1A}) \equiv MEC_{1A}$$

$$t_B = P_2 - c_{2B} = N_{2B} \cdot c'_{2B}(N_{2B}) \equiv MEC_{2B}$$

The optimal toll on each road is equal to the difference between social and private marginal cost on that road, known as "marginal external cost" MEC , just as in a single-route model.

Case SE1. Substituting $N_{1B} = 0$ into the welfare function, we get:

$$\max W = \int_0^{N_{1A}} P_1(t) dt + \int_0^{N_{2A}+N_{2B}} P_2(t) dt - N_{1A} \cdot c_{1A}(N_{1A} + N_{2A}) - N_{2A} \cdot c_{2A}(N_{1A} + N_{2A}) - N_{2B} c_{2B}(N_{2B})$$

This objective function is also strictly concave because it equals the sum of five strictly concave functions. The corresponding tolls are:

$$t_A = P_1(N_{1A}) - c_{1A} = N_{1A} c'_{1A}(N_{1A} + N_{2A}) + N_{2A} c'_{2A}(N_{1A} + N_{2A}) \equiv MEC_A = P_2 - c_{2A}$$

$$t_B = P_2(N_{2A} + N_{2B}) - c_{2B}(N_{2B}) = N_{2B} c'_{2B}(N_{2B}) \equiv MEC_{2B}$$

The tolls are again the differences between social and private marginal costs on each route.

The social cost on route A includes the users of both groups; the social cost on route B

includes just the users of group 2. We also check the corner solution of $N_{2A} = 0$ in the simulation study.

Case SE2: This case is symmetric to SE1.

b). *SB Policy*

Case SE. The welfare maximizing problem under second-best pricing policy for the fully separated equilibrium case can be written as:

$$\max W = \int_0^{N_{1A}} P_1(t)dt + \int_0^{N_{2B}} P_2(t)dt - N_{1A}c_{1A}(N_{1A}) - N_{2B}c_{2B}(N_{2B})$$

$$s.t. P_2(N_{2B}) = c_{2B}(N_{2B})$$

N_{2B} is determined solely by the constraint and numerical results in the paper show that there is only one positive real solution for N_{2B} . The objective function is a strictly concave function of N_{1A} , so if this case can occur, the solution is unique. The corresponding toll on route A is:

$$t_A = N_{1A}c'_{1A}(N_{1A}) \equiv MEC_{1A}$$

This toll is just the difference of social and private marginal cost on that road, the social cost including just the users of group 1. There are no route spill-overs in fully separated equilibrium: that is, road A is treated just as in the FB policy.

Case SE1. The corresponding Lagrangian is:

$$\begin{aligned}
L = & \int_0^{N_{1A}} P_1(t) dt + \int_0^{N_{2A}+N_{2B}} P_2(t) dt - N_{1A} c_{1A}(N_{1A} + N_{2A}) - N_{2A} c_{2A}(N_{1A} + N_{2A}) - N_{2B} c_{2B}(N_{2B}) \\
& - \mathbf{I}_1 [P_1(N_{1A}) - c_{1A}(N_{1A} + N_{2A}) - P_2 + c_{2A}(N_{1A} + N_{2A})] \\
& - \mathbf{I}_2 [P_2(N_{2A} + N_{2B}) - c_{2B}(N_{2B})]
\end{aligned}$$

where the constraints (4.6a-b) have been rewritten using (4.6f) as an equality in order to eliminate t_A as a variable. The Lagrangian Multiplier \mathbf{I}_1 represents the shadow price of not price discriminating on road A, that is, it represents the increase of social welfare that could be achieved by charging type-1 users more than type-2 users, since the latter have a sub-optimally priced substitute (road B). This problem can be solved for $N_{1A}, N_{2A}, N_{2B}, \lambda_1$, and λ_2 . The toll which decentralizes the solution allocation is then determined by (4.6a) as:

$$t_A = N_{1A} c'_{1A} + N_{2A} c'_{2A} - \left[\frac{P'_2 N_{2B} c'_{2B} \cdot (P'_1 - c'_{1A} + c'_{2A})}{P'_1 P'_2 - P'_1 c'_{1B} - P'_2 c'_{2B}} \right]$$

The toll on route A equals to marginal external cost minus a positive adjustment term which depends on the slope of demand function and cost function.

Case SE2. The Lagrangian is:

$$L = W - \mathbf{I}_2 [P_2(N_{2B}) - c_{2B}(N_{1B} + N_{2B})] - \mathbf{g}_1 [P_1(N_{1A} + N_{1B}) - c_{1B}(N_{1B} + N_{2B})]$$

where (4.6e) has been used as an equality with Lagrangian multiplier g_1 which represents the "shadow price" of not being able to price discriminate on road B.

Again, we solve and use (4.6a) to determine the toll on route A as:

$$t_A = N_{1A} c'_{1A} - \left[\frac{(N_{1B} c'_{1B} + N_{2B} c'_{2B}) P'_2 P'_1}{P'_1 P'_2 - P'_1 c'_{2B} - P'_2 c'_{1B}} \right]$$

The toll here equals to the marginal congestion cost plus an adjustment term which depends on the slopes of the demand and cost functions. When the users are identical, so that $c_{1B} = c_{2B}$ and $P_1 = P_2$, this formula reduces to equation (4.2) of Verhoef et al. (1996).

It is difficult to judge analytically whether these solutions for cases SE1 and SE2 are unique, because of the non-linear form of the constraints. In the simulation study, we use different initial values to show that in these cases no more than one equilibrium solution can be found.

c). PM Policy

The maximizing problem here has the same constraints as the ones in the SB policy. The only difference is that the objective function now is:

$$R = (N_{1A}) [P_1(N_{1A}) - c_{1A}(N_{1A} + N_{2A})] + N_{2A} [P_2(N_{2A} + N_{2B}) - c_{2A}(N_{1A} + N_{2A})]$$

Case SE. The solution of this case must be unique because the same reason as SE case in SB policy. The toll which maximizes revenue is found to be:

$$t_A = N_{1A} [c'_{1A}(N_{1A}) - P'_1]$$

The toll is set at marginal social cost plus a monopolistic mark-up which is inversely related to the demand elasticity of group 1 (compare Small (1992, eq. (4.41))). Equivalently, this equation can be written as $t_A + N_{1A} P'_1 = N_{1A} c'_{1A}$, that is, marginal revenue equals marginal cost.

Case SE1. The toll is found to be:

$$t_A = N_{1A} c'_{1A} + N_{2A} c'_{2A} - N_{1A} P'_1 + \left[\frac{(N_{2A} P'_2 c'_{2B} + N_{1A} P'_1 P'_2 - N_{1A} P'_1 c'_{2B})(P'_1 - c'_{1A} + c'_{2A})}{P'_1 P'_2 - P'_1 c'_{2B} - 2(P'_2)^2 + P'_2 c'_{2B}} \right]$$

Again the toll equals marginal congestion cost plus a monopolistic mark-up.

Case SE2. The revenue-maximizing toll on route A is:

$$t_A = N_{1A} c'_{1A} - N_{1A} P'_1 + \left[\frac{N_{1A} (P'_1)^2 (P'_2 - c'_{2B})}{P'_1 (P'_2 - c'_{2B}) - c'_{1B} P'_2} \right]$$

Again, the uniqueness of equilibrium solution for case SE1 and SE2 is proved numerically.

4.2: Simulation Results

In above model, time and unreliability are not distinguished, but can be assumed to be functionally related. To use the model with the estimate results in chapter 2, we specify the full price p_{ir} for a user of type i on roadway r to be $p_{ir} = \mathbf{t}_r + \mathbf{j}_i T_r + \mathbf{d}_i R_r$, where \mathbf{t} is toll, T is travel-time delay (time less free-flow time), and R is unreliability. We assume that for each roadway, R_r / T_r is fixed at a value $s=0.3785$, which is the ratio of the average R to average T over the 4-hour peak period (5-9 a.m.) in the unpriced lanes in our floating car data. Thus $p_{ir} = \mathbf{t}_r + \mathbf{a}_i T_r$, where $\mathbf{a}_i = \mathbf{j}_i + s\mathbf{d}_i$. For \mathbf{j}_i and \mathbf{d}_i we use the VOT and VOR estimates in table 16 based on RP behavior, taking the two user groups to be represented by the 75th and 25th percentiles.¹² This yields values of $\mathbf{a}_1 = \$40.86/hr$, and $\mathbf{a}_2 = \$17.62/hr$.

The other parameters in cost and demand functions are calibrated to reproduce real traffic conditions observed on SR91 in fall 1999. In the cost function, the length of two routes is 10 miles, and the capacity of toll lanes (route A) is 2000, half of the one of free lanes (route B). The free-flow travel time 0.9231 minute per mile given the speed limit of 65 miles per hour. In the demand functions, the parameters are calibrated so that the price elasticity for the two groups is -0.58 , based on the estimate by Yan, Small, and Sullivan (2001) using Cal Poly data, and the time difference between the toll lanes and the free lanes is 6 minutes under profit maximizing toll. As a result, the intercepts of demand functions for

¹² The third and sixth rows of table 14 show the difference between 75th and 25th percentiles. The percentiles themselves are: \$27.70 and \$15.10 for VOT, and \$34.79 and \$6.66 for VOR.

group 1 and group 2 people are both 7200, and the slopes of the demand functions for group 1 and group 2 are about -1.66 and -2.75 respectively. Given these settings, the profit-maximizing toll is \$4, which is quite plausible.

Table 17 shows the simulation results. The first column in this table is the base case, that is, there are no tolls on both two routes. The fourth column shows the results for first-best pricing policy. Substantial social welfare gain can be gained by pricing roads optimally, however the direct loss in consumers' surplus in this case is also big especially for people with lower value of time. This creates a political barrier to implement first-best pricing on roads. Given the estimated heterogeneity in value of time, a politically feasible policy — second-best pricing (the second column) only charging toll on rout A improves social welfare by \$0.16 per vehicle, which is much less efficient than first-best pricing. Heterogeneity in preferences increases efficiency of second-best pricing. As shown in the third column, without heterogeneity, the social welfare gain from second-best pricing is negligible.

Can we find a pricing policy which is as politically feasible as second-best pricing but with much larger efficiency? To answer this question, we resolve the non-linear programming problem for first-best pricing (equation (4.6)) but adding a political feasibility constraint, that is, the largest consumer surplus loss no greater than that in the second-best pricing policy shown in the second column. We solve this complicated problem numerically by searching in 2-dimensional space for the tolls on route A and route B within the range between zero and first-best tolls. The results are shown in the fifth column of Table 17, and

the toll is called as limited differentiated toll. Compared with the first-best toll, the toll is lower but more sharply differentiated, and it causes substantially smaller losses in consumer surplus for both groups. Furthermore, it narrows the gap between losses in consumer surplus for the two groups. The social welfare gain from this policy is more than one third that of first-best pricing and much larger than that of second-best pricing.

Catering to heterogeneity is apparently the key to softening the distributional effects of more efficient road pricing. This is indicated by a “limited uniform toll” policy shown in the last column of the table, defined to generate the same efficiency gain as the limited differentiated toll. It harms the low-VOT group far more than the high-VOT group. Thus if analysts consider only uniform tolls, they are likely to find that policymakers pay little attention to the efficiency gains because of large distributional disparities.

Traffic on SR91 has increased considerably since 1999. We show the effects of differentiated pricing with greater congestion by recalibrating the simulation model to double the time difference between the lanes that existed in the fall of 1999 (again, assuming that the operator’s toll maximizes profit). The results, shown in table 18, indicate that the welfare gains from all the policies are more than doubled with increased congestion, yet the consumer-surplus losses in constrained policies are only about 50 percent greater. If we ignore heterogeneity, distributional concerns also increase as evidenced by the greater disparity among users groups with the limited uniform toll (last column). But this disparity is virtually eliminated by the limited differential toll. As congestion on major highways continues to grow, the case for accounting for heterogeneity will only strengthen.

Table 17. Simulation Results— Fall 1999 Traffic Conditions

PRICING REGIME	Base case: no toll	Second-best toll: heterogeneity present	Second-best toll: heterogeneity not present	First-best differentiated toll	Limited differentiated toll	Limited uniform toll
Toll:						
Express lanes	0	\$1.80	\$0.97	\$4.51	\$1.34	\$0.78
Regular lanes	0	0	0	\$4.18	\$0.47	\$0.78
Travel time (minutes):						
Express lanes	14	11	12	10	12	13
Regular lanes	14	15	14	11	14	13
Consumer surplus: ^a						
High-VOT users	0	-\$0.45		-\$2.41	-\$0.44	-\$0.40
Low-VOT users	0	-\$0.26		-\$2.82	-\$0.45	-\$0.55
Homogeneous users	0		-\$0.23			
Social welfare ^a						
All users	0	\$0.16	\$0.06	\$0.86	\$0.28	\$0.28

^a Consumer surplus and social welfare are measured relative to the no-toll scenario and divided by the number of users in the no-toll scenario to put them in per capita terms. Social welfare is equal to the sum of the two groups' consumer surplus plus revenue, divided by total number of users in the no-toll scenario.

Table 18. Simulation Results—Increased Congestion

PRICING REGIME	Base case: no toll	Second-best toll: heterogeneity present	Second-best toll: heterogeneity not present	First-best differentiated toll	Limited differentiated toll	Limited uniform toll
Toll:						
Express lanes	0	\$4.42	\$2.68	\$8.51	\$2.81	\$1.43
Regular lanes	0	0	0	\$7.93	\$0.77	\$1.43
Travel time (minutes):						
Express lanes	20	14	15	12	16	18
Regular lanes	20	21	20	13	19	18
Consumer surplus: ^a						
High-VOT users	0	-\$0.71		-\$2.66	-\$0.68	-\$0.54
Low-VOT users	0	-\$0.42		-\$3.31	-\$0.71	-\$0.89
Homogeneous users	0		-\$0.38			
Social welfare ^a	0	\$0.48	\$0.23	\$2.18	\$0.67	\$0.67

^a See the footnotes of Table 5.

CONCLUSION

This dissertation has applied recent econometric advances to analyze the behavior of commuters in Southern California and found substantial heterogeneity in commuters' preferences for both travel time and travel time reliability. As expected, commuters with higher household income have higher values of time and of reliability. Additionally, commuters with long trip distance have lower values of time, which is consistent with residential selectivity. However, most of the heterogeneity in commuters' preferences can not be explained by observed characteristics. One possible explanation is that in very expensive and congested metropolitan areas such as Southern California, consumers face significant constraints in trading off housing expense for commuting time.

Based on a simulation model and the uncovered heterogeneity, this dissertation found pricing policies with a greater chance of public acceptance by catering to varying preferences. Recent "value pricing" experiments have made a start to account for varying preferences by letting motorists make an option between priced and unpriced roads. However, as shown in the simulation results of this dissertation, leaving part of roadway unpriced severely reduces the efficiency. Differentiated pricing taking preference heterogeneity into account can realize substantial efficiency gains on the one hand, and ameliorate distributional concerns on the other hand. Differentiated pricing is also politically feasible by reducing the direct loss in consumer surplus. This policy may thus be the key to break the impasse in efforts to relieve highway congestion.

This dissertation also investigated how to employ the new advances in Bayesian approach for estimating the multinomial probit model in travel demand analysis combining different sources of data. Multinomial probit model has advantages to model the correlation across choice alternatives and across observations of different data from the same individual, and Bayesian approach, also with theoretical advantages in interpreting results, makes the multinomial probit model more feasible to handle in practice. Bayesian approach provides us with a new tool to measure commuters' behavior based on more flexible model specifications.

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