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Authors

Sheikholeslam, Shahab
Desoer, Charles A.

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I: Linear Model

Shahab Sheikholeslam
Charles A. Desoer

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PATH Goal Statement

The research described in this report is part of the Program on Advanced Technology for the Highway (PATH). PATH research is being conducted at the Institute of Transportation Studies at the University of California at Berkeley, to develop more effective highways. The aim of PATH is to increase the capacity of the most used highways, to decrease traffic congestion, and to improve safety and air quality. PATH is a cooperative venture of the automobile and electronic industries, universities, and local, state, and federal governments.

Longitudinal Control of a Platoon of Vehicles
I: Linear Model

Shahab Sheikholeslam and Charles A. Desoer

PATH Project

August 18,1989

Abstract

This paper presents a systematic analysis of a longitudinal control law of a platoon of vehicles using a linear model to represent the vehicle dynamics of each vehicle within the platoon. The basic idea is to take full advantage of recent advances in communication and measurement and using these advances in longitudinal control of a platoon of vehicles: in particular, we assume that for i = 1,2,. . . vehicle i knows at all times v_l and a_l (the velocity and acceleration of the lead vehicle) in addition to the distance between vehicle i and the preceding vehicle, $i - 1$.

We have analyzed platoons of identicaland non-identicalvehicles. All the vehicles within the platoon are initially moving with a steady-state velocity of v_0. Then the successive vehicle spacings are computed as a result of lead vehicle's acceleration from its initial steady-state velocity (vo) to its final steady-state velocity. In the case of a platoon of ***identical vehicles, we*** have shown that through the appropriate choice of design parameters, deviations in the successive vehicle spacings get attenuated from the front to the back of the platoon. An important feature of the design **is** that such deviations do not exhibit oscillatory time-behavior.

Our analysis of a platoon of ***non-identical*** vehicles shows that deviation of the i-th vehicle position from its assigned position(i.e.,Δ_i) is affected by the corresponding deviations of all the vehicles in front of the i-th vehicle(i.e.,$\Delta_1, \ldots, \Delta_{i-1}$) together with the change (w_l) of the lead vehicle's velocity from its steady-state value. Since in the control law there are not enough parameters to cancel the terms involving $\Delta_1, \ldots, \Delta_{i-2}$, we have to investigate by simulation the magnitude of such effects on the i-th vehicle.

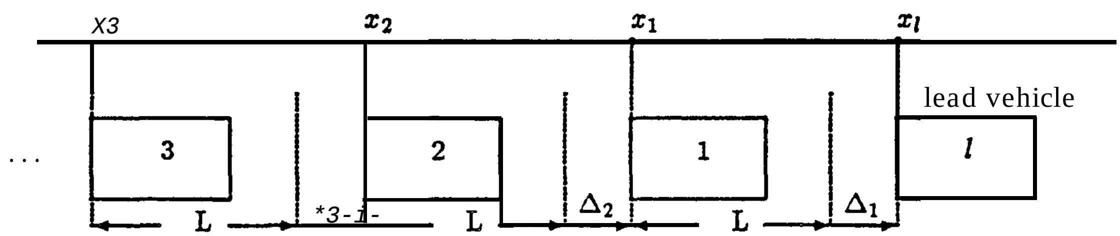
Longitudinal Control of a Platoon of Vehicles
I: Linear Model

1 Introduction

The subject of design and analysis of various longitudinal control laws has been studied extensively from late 1960's until mid 1970's. Throughout the literature numerous topics such as vehicle-follower controllers, nonlinear vehicle dynamics, entrainment-extrainment maneuvers, and automated guideway transit systems have been reported.[Chi.l,Hauk.l,Shla.l,Hob.l] Even though much effort has been spent on various control laws for longitudinal control of a platoon of vehides[Rous.l,Hob.2,Hauk.2,Tak.l,Caud.l,Fra.l,Shla.2], this paper presents a systematic analysis of the longitudinal control of such platoons.

The basic concept of this study is to take full advantage of recent advances in communication and measurement [Wal.l] and using these advances in longitudinal control of a platoon of vehicles: in particular, we assume that for i = 1,2,. . . vehicle **i** knows at all times v_l and a_l (the velocity and acceleration of the lead vehicle) in addition to the distance between vehicle $i - 1$ and vehicle i (see fig. 4).

In the case of a platoon of identical vehicles (see fig. 1) for **a** change in the lead vehicle's velocity, (see fig. 7), the resulting changes in the spacing of successive vehicles is shown in figure 11. These simulation results show that through the appropriate choice of coefficients in the control law for each vehicle in the platoon the deviations in vehicle spacings from their respective steady-state values do not get magnified from the front to the end of the platoon and in fact get attenuated as one goes down the platoon. An important feature of the design is that such deviations do not exhibit oscillatory time-behavior.



x; is the abcissa of the i-th vehicle
x_l is the abcissa of the lead vehicle
This figure defines $\Delta_1, \Delta_2, \ldots$, and L.

direction of motion

Figure 1: Platoon of 4 vehicles

2 Platoon Configuration

Figure 1 shows the assumed platoon configuration for a platoon of 4 vehicles. The platoon is assumed to move in a straight line. The position of the i-th vehicle's rear bumper with respect to a fixed reference point 0 on the roadside is denoted by x;. The position of the lead vehicle's rear bumper with respect to the same fixed reference point 0 is denoted by x_l. Each vehicle is assigned a slot of length L along the road. As shown, A; is the deviation of the i-th vehicle position from its assigned position. The subscript i is used because Δ_i is measured by the sensors located in the i-th vehicle.

Given the platoon configuration in figure 1, elementary geometry shows that: for $i = 2,3,\ldots$

$$\Delta_i(t) := x_{i-1}(t) - x_i(t) - L \tag{2.1}$$

$$\dot{\Delta}_i(t) := \dot{x}_{i-1}(t) - \dot{x}_i(t) \tag{2.2}$$

$$\ddot{\Delta}_i(t) := \ddot{x}_{i-1}(t) - \ddot{x}_i(t) \tag{2.3}$$

where, as usual, "." denotes the time derivative.

Kinematics The corresponding kinematic equations for the lead vehicle and the first vehicle are as follows:

$$\Delta_1(t) := x_l(t) - x_1(t) - L \tag{2.4}$$
$$\dot{\Delta}_1(t) := v_l(t) - \dot{x}_1(t) \tag{2.5}$$
$$\ddot{\Delta}_1(t) := a_l(t) - \ddot{x}_1(t) \tag{2.6}$$

From (2.2) and (2.5) we obtain the following useful relations

$$\dot{x}_2 = \dot{x}_1 - \dot{\Delta}_2 = v_l - \dot{\Delta}_1 - \dot{\Delta}_2 \tag{2.7}$$
$$\dot{x}_3 = \dot{x}_2 - \dot{\Delta}_3 = v_l - \dot{\Delta}_1 - \dot{\Delta}_2 - \dot{\Delta}_3 \tag{2.8}$$
$$\dot{x}_i = \dot{x}_{i-1} - \dot{\Delta}_i = v_l - \dot{\Delta}_1 - \cdots - \dot{\Delta}_i \tag{2-9}$$

for i = 2,3,...

Remark *Ai is measured in vehicle i and is used in its control law. We assume that we know the lead vehicle's velocity (v_l) and acceleration (a_l) for each vehicle in the platoon.(This requires a communication link from lead vehicle to each vehicle of the platoon.)

3 Vehicle Model

Figure 2 shows the vehicle model of the i-th vehicle in the platoon: x_i denotes the position of the i-th vehicle's rear bumper with respect to a fixed reference point on the road; the block $(m_i g \sin\theta)$ specifies the component of the i-th vehicle's weight parallel to the road surface, where m_i denotes the i-th vehicle's mass, g denotes the acceleration due to gravity, and θ denotes the angle between the road surface and a horizontal plane (θ positive corresponds to uphill travel); the block $(\frac{\rho A_i C_{di}}{2}(\dot{x}_i + V_{wind})^2 sgn(\dot{x}_i + V_{wind}))$ specifies the force due to the air resistance, where ρ denotes the specific mass of air, A_i denotes the cross-sectional area of the i-th vehicle, C_{di} denotes the i-th vehicle's drag coefficient, and V_{wind} denotes the velocity of the wind gust; constant d_{mi} denotes the mechanical drag of the i-th vehicle; the block $(\frac{1}{\tau_i s+1})$ models the i-th vehicle's engine dynamics, where τ_i denotes the i-th vehicle's engine time constant; u_i denotes the throttle input to the i-th vehicle's engine; c_i denotes the control input to the i-th vehicle's engine; F_i denotes the force produced by the i-th vehicle's engine.

Remark The summing node at the bottom of figure 2 represents Newton's second law for the i- th vehicle:

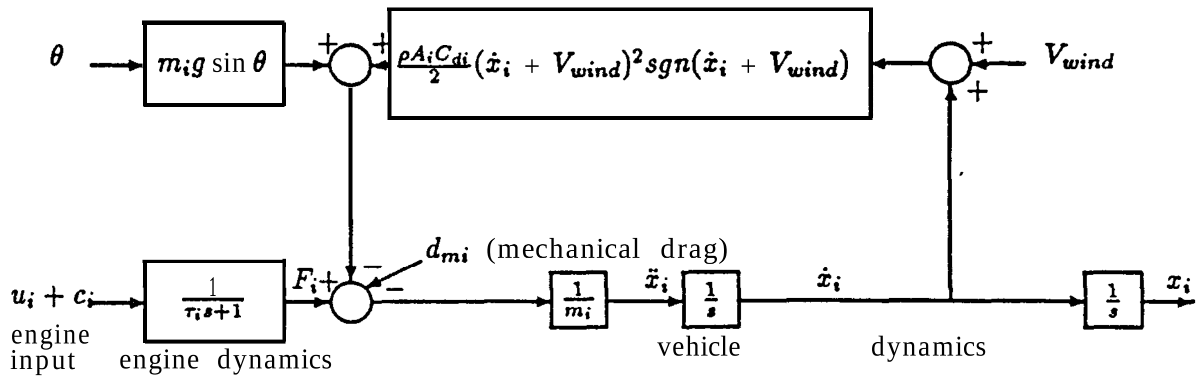


Figure 2: vehicle model of the i-th vehicle in the platoon

$$F_i - m_i g \sin\theta - \frac{\rho A_i C_{di}}{2}(\dot{x}_i + V_{wind})^2 sgn(\dot{x}_i + V_{wind}) - d_{mi} = m_i \ddot{x}_i \quad (3.1)$$

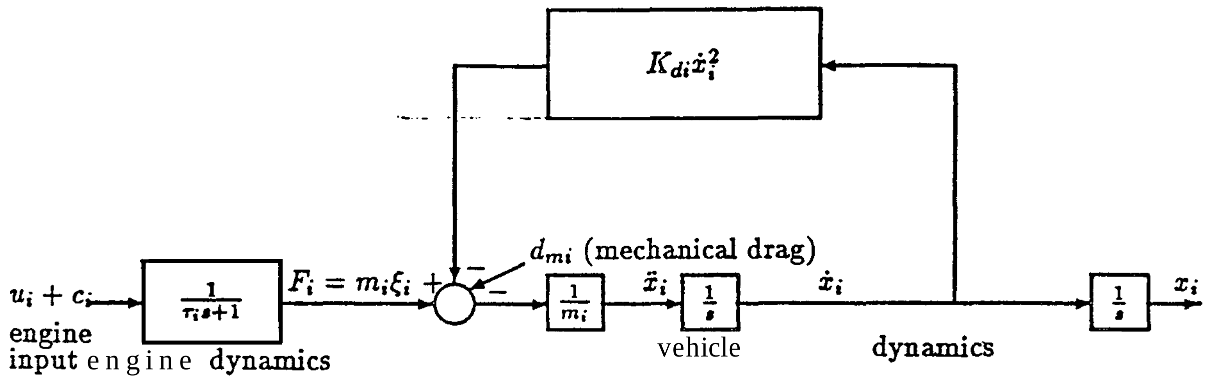
Working model In the preliminary study we assume that the road surface is horizontal (θ = 0) and there is no wind gust(V_{wind} = 0). Figure 3 shows the simplified vehicle model of the i-th vehicle in the platoon : K_{di} denotes $\frac{\rho A_i C_{di}}{2}$; since the vehicles are assumed to travel in the same direction at all times, $sgn(\dot{x}_i) = 1$ for $i = 1, 2, \ldots$

4 Proposed Control Law

We propose the following linear control law for longitudinal control of vehicles: for the first vehicle the control law is

$$c_1 := c_{p1}\Delta_1(t) + c_{v1}\dot{\Delta}_1(t) + c_{a1}\ddot{\Delta}_1(t) + k_{v1}v_l(t) \text{ t } k_{a1}a_l(t) \quad (4.1)$$

for vehicles 2,3,. . . the control law is



F_i = engine force applied to vehicle i
ξ_i = commanded acceleration of vehicle i

Figure 3: simplified model of the i-th vehicle in the platoon

$$c_i := c_{pi}\Delta_i(t) + c_{vi}\dot{\Delta}_i(t) + c_{ai}\ddot{\Delta}_i(t) + k_{vi}[v_l(t) - v_i(t)] + k_{ai}[a_l(t) - a_i(t)] \qquad (4.2)$$

where $c_{pi}, c_{vi}, c_{ai}, k_{vi},$ and k_{ai} are design constants. Note that the control law for the first vehicle differs from the control law for all the other vehicles in the two rightmost terms in (4.1). This is due to the fact that for the first vehicle $v_l - v_1 = \dot{\Delta}_1$ and $a_l - a_1 = \ddot{\Delta}_1$ which are already a part of the first vehicle's control law; whereas, for vehicle i (i $= 2, 3, \ldots$) $v_l - v_i = \dot{\Delta}_1 + \cdots + \dot{\Delta}_i$ and $a_l - a_i =$ i i * $+ \cdots + \ddot{\Delta}_i$ so that the i-th vehicle's control law contains terms relating to $\dot{\Delta}_1, \ldots, \dot{\Delta}_{i-1}$ and $\ddot{\Delta}_1, \ldots, \ddot{\Delta}_{i-1}$ in addition to terms relating to $\dot{\Delta}_i$ and $\ddot{\Delta}_i$.

Comparison of our control law (4.2) for the i-th vehicle with the control laws in the literature shows that using the lead vehicle's acceleration (a_l) in the i-th vehicle's control law is the new addition to the i-th vehicle's control laws considered in the literature. Shladover had used lead vehicle's velocity (v_l)[Shla.1] and $\ddot{\Delta}_i$ IShla.21 in the i-th vehicle's control law.

Implementation Issues Figure 4 shows the platoon configuration under

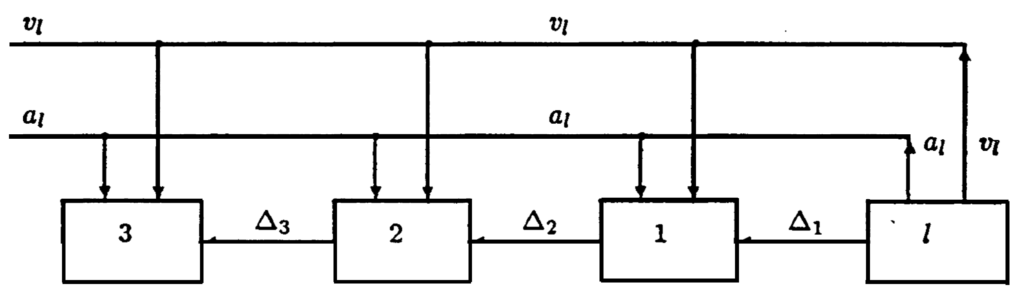


Figure 4: Platoon Configuration under the proposed control law for a platoon of 4 vehicles

the proposed control law for **a** platoon of 4 vehicles; As shown in figure 4, the lead vehicle's velocity (v_l) and acceleration (a_l) are transmitted to all the vehicles within the platoon. In addition, sensors on each vehicle, say i, measure the deviation of the i-th vehicle from its assigned position, namely Δ_i.

Communication of the position,velocity, and acceleration information is unidirectional: from the lead vehicle to each vehicle in the platoon. Communication speed and processing of the measured data should be fast compared to the time constants of the vehicle dynamics.Preliminary studies of Professor Warland [Wal.1] and his students suggest that such a requirement is feasible with the present communication and data processing technology.

Assumption(linearization of the air resistance) Note that in the steady state, $\Delta_i \equiv 0$, $a_l \equiv a_i \equiv 0$, and $v_l \equiv v_i \equiv v_0$ for $i = 1, 2, \ldots$

Considering figure 3 and writing the Taylor expansion of the drag force due to the air resistance for the i-th vehicle about the steady-state velocity(v_0), we obtain

$$K_{di}\dot{x}_i^2 = K_{di}\left[v_0^2 + 2v_0(\dot{x}_i - v_0)\right] + h.o.t. \tag{4.3}$$

Neglecting the **effect of the** higher order terms in (4.3) **and simplifying** the right hand side, we obtain the approximate drag force **due to the** air resistance for the i-th vehicle

$$K_{di}\dot{x}_i^2 \approx (-K_{di}v_0^2) + (2K_{di}v_0)\dot{x}_i \tag{4.4}$$

Notation In the sequel we will adopt the following notations:

for $i = 1, 2, \ldots$

$c_{pin} := \frac{c_{pi}}{m_i}$, $c_{vin} := \frac{c_{vi}}{m_i}$, $c_{ain} := \frac{c_{ai}}{m_i}$

$k_{vin} := \frac{k_{vi}}{m_i}$, $k_{ain} := \frac{k_{ai}}{m_i}$

$d_{0i} := -\frac{K_{di}}{m_i}v_0^2$, $d_{1i} := \frac{2K_{di}}{m_i}v_0$

$w_l(t) := v_l(t) - v_0$

thus w_l denotes the increment of velocity of the lead vehicle from its steady-state value.

Initial Conditions We assume that for all $t < 0$, the platoon is in steady-state: for $t < 0$, $\dot{x}_i(t) = \dot{x}_l(t) = v_0$, $\Delta_i(t) = 0$ and the u_i's are the corresponding appropriate constants. (The steady-state value of the function $f(.)$ is denoted by $f(0-)$)

4.1 First vehicle dynamics

We are going to use the proposed linear control law for the first vehicle (4.1) and the simplified vehicle model in figure 3 to finally obtain equ(4.15) relating Δ_1 **to** w_l:

From (2.6) and the summing node equation in figure 3 we obtain

$$\ddot{\Delta}_1(t) = q(t) - \frac{1}{m_1}(F_1(t) - d_{m1} - K_{d1}\dot{x}_1^2(t)) \tag{4.5}$$

Note that $a_l(t)$ is a command signal.

Using the linearized drag force due to the air resistance for the first vehicle from (4.4) in (4.5) we obtain

$$\ddot{x}_1(t) = a_l(t) - \frac{F_1(t)}{m_1} + \frac{d_{m1}}{m_1} + \frac{K_{d1}}{m_1}(-v_0^2 + 2v_0\dot{x}_1(t)) \quad (4.6)$$

Using the notation for ξ_1, d_{01}, and d_{11} we can rewrite (4.6) as

$$\ddot{\Delta}_1(t) = a_l(t) - \xi_1(t) + \frac{d_{m1}}{m_1} + d_{01} + d_{11}\dot{x}_1(t) \quad (4.7)$$

From equations (2.5) and (4.7) we obtain

$$\ddot{\Delta}_1(t) = a_l(t) - \xi_1(t) + \frac{d_{m1}}{m_1} + d_{01} + d_{11}v_l(t) - d_{11}\dot{\Delta}_1(t) \quad (4.8)$$

Note that (4.8) yields the following relationship in the steady state ,

$$\ddot{\Delta}_1(0-) = -\xi_1(0-) + \frac{d_{m1}}{m_1} + d_{01} + d_{11}v_l(0-) = 0 \quad (4.9)$$

Taking Laplace transforms from both sides of (4.8) and noting the initial conditions and (4.9) we obtain

$$(s^2 + d_{11}s)\hat{\Delta}_1(s) = -\hat{\xi}_1(s) + \frac{\xi_1(0-)}{s} + (s + d_{11})\hat{w}_l(s) \quad (4.10)$$

where we use the symbol "ˆ" to distinguish Laplace transforms from the corresponding time-domain functions.

The engine model for the first vehicle with control law ***(4.1) gives***

$$m_1\tau_1\dot{\xi}_1(t) + m_1\xi_1(t) = u_1 + c_{p1}\Delta_1(t) + c_{v1}\dot{\Delta}_1(t) + c_{a1}\ddot{\Delta}_1(t) + k_{v1}v_l(t) + k_{a1}a_l(t) \quad (4.11)$$

From (4.11) we note the following relationship in the steady state

$$m_1\xi_1(0-) = u_1 + k_{v1}v_l(0-) \quad (4.12)$$

Taking Laplace transforms of both sides of (4.11) and noting the steady-state relation given by (4.12) we obtain

$$\hat{\xi}_1(s) = \frac{\xi_1(0-)}{s} + \frac{c_{p1} + c_{v1}s + c_{a1}s^2}{m_1(\tau_1 s + 1)}\hat{\Delta}_1(s) + \frac{k_{v1} + k_{a1}s}{m_1(\tau_1 s + 1)}\hat{w}_l(s) \quad (4.13)$$

Substituting the expression for $\hat{\xi}_1$ from (4.13) into (4.10) and reordering the terms corresponding to $\hat{\Delta}_1(s)$ and $\hat{w}_l(s)$ we obtain

$$\left\{(m_1\tau_1)s^3 + [m_1(1 + \tau_1 d_{11}) + c_{a1}]\, s^2 + (m_1 d_{11} + c_{v1})s + c_{p1}\right\}\hat{\Delta}_1(s)$$
$$= \left\{(m_1\tau_1)s^2 + [m_1(1 + \tau_1 d_{11}) - k_{a1}]\, s + (m_1 d_{11} - k_{v1})\right\}\hat{w}_l(s) \quad (4.14)$$

Dividing both sides of (4.14) by m_1 we obtain

$$\left\{\tau_1 s^3 + [1 + \tau_1 d_{11} + c_{a1n}]\, s^2 + (d_{11} + c_{v1n})s + c_{p1n}\right\}\hat{\Delta}_1(s)$$
$$= \left\{\tau_1 s^2 + [1 + \tau_1 d_{11} - k_{a1n}]\, s + (d_{11} - k_{v1n})\right\}\hat{w}_l(s) \qquad (4.15)$$

4.2 Second vehicle dynamics

We are going to use the proposed linear control law for the second vehicle (4.2) and the simplified vehicle model in figure 3 to finally obtain equ.(4.25) relating Δ_2 to Δ_1 and w_l:

From (2.3) and the summing node equation in figure 3 for the first and second vehicle we obtain

$$\ddot{\Delta}_2(t) = \frac{1}{m_1}(F_1(t) - d_{m1} - K_{d1}\dot{x}_1^2(t)) - \frac{1}{m_2}(F_2(t) - d_{m2} - K_{d2}\dot{x}_2^2(t)) \quad (4.16)$$

Using the linearized drag force due to the air resistance for the first and second vehicle from (4.4) in (4.16) we obtain

$$\ddot{\Delta}_2(t) = \frac{F_1(t)}{m_1} - \frac{d_{m1}}{m_1} - \frac{K_{d1}}{m_1}(-v_0^2 + 2v_0\dot{x}_1(t)) - \frac{F_2(t)}{m_2} + \frac{d_{m2}}{m_2} + \frac{K_{d2}}{m_2}(-v_0^2 + 2v_0\dot{x}_2(t)) \quad (4.17)$$

Using the notation for $\xi_1, d_{01}, d_{11}, \xi_2, d_{02}$, and d_{12} we can rewrite (4.17) as

$$\ddot{\Delta}_2(t) = \xi_1(t) - \frac{d_{m1}}{m_1} - d_{01} - d_{11}\dot{x}_1(t) - \xi_2(t) + \frac{d_{m2}}{m_2} + d_{02} + d_{12}\dot{x}_2(t) \quad (4.18)$$

From equations (2.5), (2.7) and (4.15) we obtain

$$\text{ii@)} = \xi_1(t) - \xi_2(t) + \left(\frac{d_{m2}}{m_2} + d_{02} - \frac{d_{m1}}{m_1} - d_{01}\right) + (d_{12} - d_{11}) v_l(t) + (-d_{12} + d_{11}) \dot{\Delta}_1(t) - d_{12} \dot{\Delta}_2(t) \quad (4.19)$$

Note that (4.19) yields the following relationship in the steady state

$$\ddot{\Delta}_2(0-) = \xi_1(0-) - \xi_2(0-) + \left(\frac{d_{m2}}{m_2} + d_{02} - \frac{d_{m1}}{m_1} - d_{01}\right) + (d_{12} - d_{11}) v_l(0-) = 0 \quad (4.20)$$

Taking Laplace transforms from both sides of (4.19) and noting the initial conditions gives

$$\begin{aligned} (s^2 + d_{12}s)\hat{\Delta}_2(s) &= \hat{\xi}_1(s) - \hat{\xi}_2(s) + \frac{\left(\frac{d_{m2}}{m_2} + d_{02} - \frac{d_{m1}}{m_1} - d_{01}\right) + (d_{12} - d_{11}) v_l(0-)}{s} \\ &+ (d_{12} - d_{11}) \hat{w}_l(s) + (-d_{12} + d_{11}) s \hat{\Delta}_1(s) \end{aligned} \quad (4.21)$$

The engine model for the second vehicle with control law (4.2) gives

$$m_2 \tau_2 \dot{\xi}_2(t) + m_2 \xi_2(t) = u_2 + c_{p2} \Delta_2(t) + c_{v2} \dot{\Delta}_2(t) + c_{a2} \ddot{\Delta}_2(t) + k_{v2}(v_l(t) - v_2(t)) + k_{a2}(a_l(t) - a_2(t)) \quad (4.22)$$

From (4.22) we note the following relationship in the steady state

$$m_2 \xi_2(0-) = u_2 \quad (4.23)$$

Taking Laplace transforms of both sides of (4.22) and noting the steady-state relation given by (4.23) we obtain

$$\begin{aligned} \hat{\xi}_2(s) &= \frac{\xi_2(0-)}{s} + \frac{c_{p2} + (c_{v2} + k_{v2})s + (c_{a2} \text{ t } k_{a2})s^2}{m_2(\tau_2 s + 1)} \hat{\Delta}_2(s) \\ &+ \frac{k_{v2}s + k_{a2}s^2}{m_2(\tau_2 s + 1)} \hat{\Delta}_1(s) \end{aligned} \quad (4.24)$$

Substituting the expressions for $\hat{\xi}_1$ from (4.13) and $\hat{\xi}_2$ from (4.24) into (4.21) and reordering the terms corresponding to $\hat{\Delta}_2(s), \hat{\Delta}_1(s)$ and $\hat{w}_l(s)$ after normalization we obtain

$$\left\{\tau_2 s^3 + [1+\tau_2 d_{12} + c_{a2n} + k_{a2n}]\, s^2 + (4\,2 + c_{v2n} + k_{v2n})s + c_{p2n}\right\} \hat{\Delta}_2(s)$$
$$= \left\{(\frac{\tau_2 s+1}{\tau_1 s+1})(c_{p1n} + c_{v1n}s + c_{a1n}s^2) - (k_{v2n}s + k_{a2n}s^2) + (\tau_2 s+1)(-d_{12}+d_{11})s\right\} \hat{\Delta}_1(s)$$
$$+ \left\{(\frac{\tau_2 s+1}{\tau_1 s+1})(k_{v1n} + k_{a1n}s) + (\tau_2 s + 1)(d_{12} - d_{11})\right\} \hat{w}_l(s) \qquad (4.25)$$

4.3 i-th vehicle dynamics ($i = 3, 4, \ldots$)

We are going to use the proposed linear control law for the i-th vehicle (4.2) and the simplified vehicle model in figure 3 to finally obtain equ.(4:35) relating Δ_i to $\Delta_{i-1}, \ldots, \Delta_1$, and w_l:

From ***(2.3)*** and the summing node equation in figure ***3*** for the $i-$ 1-st and i-th vehicles we obtain

$$\ddot{\Delta}_i(t) = \frac{1}{m_{i-1}}(F_{i-1}(t) - d_{mi-1} - K_{di-1}\dot{x}_{i-1}^2(t)) - \frac{1}{m_i}(F_i(t) - d_{mi} - K_{di}\dot{x}_i^2(t)) \qquad (4.26)$$

Using the linearized drag force due to the air resistance for the $i-1$-st and i-th vehicles from ***(4.4)*** in ***(4.26)*** we obtain

$$\ddot{\Delta}_i(t) = \frac{F_{i-1}(t)}{m_{i-1}} - \frac{d_{mi-1}}{m_{i-1}} - \frac{K_{di-1}}{m_{i-1}}(-v_0^2 + 2v_0\dot{x}_{i-1}(t)) - \frac{F_i(t)}{m_i} + \frac{d_{mi}}{m_i} + \frac{K_{di}}{m_i}(-v_0^2 + 2v_0\dot{x}_i(t)) \qquad (4.27)$$

Using the notation for $\xi_{i-1}, d_{0i-1}, d_{1i-1}, \xi_i, d_{0i}$, and d_{1i} we can rewrite ***(4.27) as***

$$\ddot{\Delta}_i(t) = \xi_{i-1}(t) - \frac{d_{mi-1}}{m_{i-1}} - d_{0i-1} - d_{1i-1}\dot{x}_{i-1}(t) - \xi_i(t) + \frac{d_{mi}}{m_i} + d_{0i} + d_{1i}\dot{x}_i(t) \qquad (4.28)$$

From equations (2.9) and (4.28) we obtain

$$\ddot{\Delta}_i(t) = \xi_{i-1}(t) - \xi_i(t) + (\frac{d_{mi}}{m_i} + d_{0i} - \frac{d_{mi-1}}{m_{i-1}} - d_{0i-1}) + (d_{1i} - d_{1i-1})v_l(t)$$
$$\text{t} \quad (-d_{1i} + d_{1i-1})(\dot{\Delta}_1(t) \;\text{t}\; \cdots \;\text{t}\; \dot{\Delta}_{i-1}(t)) - d_{1i}\dot{\Delta}_i(t) \qquad (4.29)$$

Note that (4.29) yields the following relationship in the steady state

$$\tilde{\Delta}_i(0-) = \xi_{i-1}(0-) - \xi_i(0-) + \left(\frac{d_{mi}}{m_i} + d_{0i} - \frac{d_{mi-1}}{m_{i-1}} - d_{0i-1}\right) + (d_{1i} - d_{1i-1})v_l(0-) = 0 \tag{4.30}$$

Taking Laplace transforms from both sides of (4.29) and noting the initial conditions gives

$$\begin{aligned}(s^2 + d_{1i}s)\hat{\Delta}_i(s) &= \hat{\xi}_{i-1}(s) - \hat{\xi}_i(s) + \frac{\left(\frac{d_{mi}}{m_i} + d_{0i} - \frac{d_{mi-1}}{m_{i-1}} - d_{0i-1}\right) + (d_{1i} - d_{1i-1})v_l(0-)}{s} \\ &+ (d_{1i} - d_{1i-1})\hat{w}_l(s) + (-d_{1i} + d_{1i-1})s(\hat{\Delta}_1(s) + \cdots + \hat{\Delta}_{i-1}(s))\end{aligned} \tag{4.31}$$

The engine model for the i-th vehicle with control law **(4.2)** gives

$$m_i\tau_i\dot{\xi}_i(t) + m_i\xi_i(t) = u_i + c_{pi}\Delta_i(t) + c_{vi}\dot{\Delta}_i(t) + c_{ai}\ddot{\Delta}_i(t) + k_{vi}(v_l(t) - v_i(t)) + k_{ai}(a_l(t) - a_i(t)) \tag{4.32}$$

From **(4.32)** we note the following relationship in the steady state

$$m_i\xi_i(0-) = u_i \tag{4.33}$$

Taking Laplace transforms of both sides of (4.32) and noting the steady-state relation given by (4.33) we obtain

$$\begin{aligned}\hat{\xi}_i(s) &= \frac{\xi_i(0-)}{s} + \frac{c_{pi} + (c_{vi} + k_{vi})s + (c_{ai} + k_{ai})s^2}{m_i(\tau_i s + 1)}\hat{\Delta}_i(s) \\ &+ \frac{k_{vi}s + k_{ai}s^2}{m_i(\tau_i s + 1)}(\hat{\Delta}_1(s) + \cdots + \hat{\Delta}_{i-1}(s))\end{aligned} \tag{4.34}$$

Substituting the expressions for $\hat{\xi}_{i-1}$ and $\hat{\xi}_i$ from (4.34) into (4.31) and reordering the terms corresponding to $\hat{\Delta}_i(s), \ldots, \hat{\Delta}_1(s)$ and $\hat{w}_l(s)$ after normalization we obtain

$$\begin{aligned}&\left\{\tau_i s^3 + [1 + \tau_i d_{1i} + c_{ain} + k_{ain}]s^2 + (d_{1i} + c_{vin} + k_{vin})s + c_{pin}\right\} \hat{\Delta}_i(s) \\ &= \left\{\begin{array}{c}\left(\frac{\tau_i s+1}{\tau_{i-1}s+1}\right)[c_{pi-1n} + (c_{vi-1n} + k_{vi-1n})s + (c_{ai-1n} + k_{ai-1n})s^2] - (k_{vin}s + k_{ain}s^2) \\ + (\tau_i s + 1)(-d_{1i} + d_{1i-1})s\end{array}\right\}\hat{\Delta}_{i-1}(s\end{aligned}$$

$$+ \left\{ \begin{array}{c} (\frac{\tau_i s+1}{\tau_{i-1}s+1})(k_{vi-1n}s + k_{ai-1n}s^2) - (k_{vin}s + k_{ain}s^2) \\ + (\tau_i s + 1)(-d_{1i} + d_{1i-1})s \end{array} \right\} (\hat{\Delta}_1(s) + \cdots + \hat{\Delta}_{i-2}(s))$$
$$+ \quad \{(\tau_i s + 1)(d_{1i} - d_{1i-1})\} \, \hat{w}_l(s) \qquad (4.35$$

Our analysis of the longitudinal control of a platoon of vehicles using the control laws (4.1) and (4.2) shows that: for the first vehicle, (4.15) shows the dependence of Δ_1 on w_l; for the second vehicle, (4.25) shows the dependence of Δ_2 on Δ_1 and w_l; for the i-th vehicle $(i=3,4,\ldots)$, (4.35) shows the dependence of Δ_i on $\Delta_{i-1}, \Delta_1 + \cdots + \Delta_{i-2}$, and w_l.

5 Special case: identical vehicles

In the simulation studies of subsection 5.4 below, we consider a platoon of identicalvehicles. Applying the proposed linear control laws to a platoon of identical vehicles, one notes the following simplifications:

The control law is the same for vehicles 2,3,. . . hence, we choose to drop the subscript i from the constant coefficients in the i-th vehicle's control law (4.2) for $i = 2,3,\ldots$

Since $d_{11} = d_{12} = \cdots = d_{1i}$ for identical vehicles, we write $d_1 := d_{1i}$ for $i = 1,2,\ldots$

Since the engine time constant is the same for all the vehicles in the platoon, we let $\tau := \tau_i$ for $i=1,2,\ldots$

5.1 Computation of $\hat{h}_{\Delta_1 w_l}(s)$- the transfer function from w_l to Δ_1

From (4.15) and using the simplified notation we obtain

$$\begin{aligned} &\left\{\tau s^3 + (1 + \tau d_1 + c_{a1n})s^2 + (d_1 + c_{v1n})s + c_{p1n}\right\} \hat{\Delta}_1(s) \\ &\quad = \left\{\tau s^2 + (1 + \tau d_1 - k_{a1n})s + (d_1 - k_{v1n})\right\} \hat{w}_l(s). \end{aligned} \tag{5.1}$$

Thus:

$$\hat{h}_{\Delta_1 w_l}(s) = \frac{\tau s^2 + (1 + \tau d_1 - k_{a1n})s + d_1 - k_{v1n}}{\tau s^3 + (1 + \tau d_1 + c_{a1n})s^2 + (d_1 + c_{v1n})s + c_{p1n}} \tag{5.2}$$

Equ. (5.2) is the first basic design equation. From (5.2),we note that we can independently select all the zeros and all the poles of $\hat{h}_{\Delta_1 w_l}$ by choosing the design parameters $c_{a1n}, c_{v1n}, c_{p1n}, k_{a1n}$, and k_{v1n}. It is crucial to note that ***the selection of zeros and poles are independent of one another.***

5.2 Computation of $\hat{h}_{\Delta_2\Delta_1}$- the transfer function from Δ_1 to Δ_2

From (4.25) and using the simplified notation we obtain

$$\begin{aligned} & \left\{\tau s^3 + [1 + \tau d_1 + c_{an} + km] \; s^2 + (d_1 + c_{vn} \; t \; k_{vn})s + c_{pn}\right\} \; \hat{\Delta}_2(s) \\ = \;& \left\{(c_{a1n} - k_{an})s^2 + (c_{v1n} - k_{vn})s + c_{p1n}\right\} \hat{\Delta}_1(s) \\ t \;& \left\{k_{v1n} + k_{a1n}s\right\} \hat{w}_l(s) \end{aligned} \qquad (5.3)$$

Thus:

$$\hat{h}_{\Delta_2\Delta_1}(s) = \frac{(c_{a1n} - k_{an})s^2 \; t \; (c_{v1n} - k_{vn})s + c_{p1n}}{\tau s^3 + (1 + \tau d_1 + c_{an} \; t \; k_{an})s^2 \; t \; (d_1 \; t \; c_{vn} \; t \; k_{vn})s \; t \; c_{pn}} \qquad (5.4)$$

From (5.3),we note that in addition to the transfer function from **A** $_1$ to Δ_2 there is a transfer function from w_l to Δ_2.

5.3 Computation of $\hat{h}_{\Delta_i\Delta_{i-1}}$- the transfer function from Δ_{i-1} to A; for $i = 3, 4, \ldots$

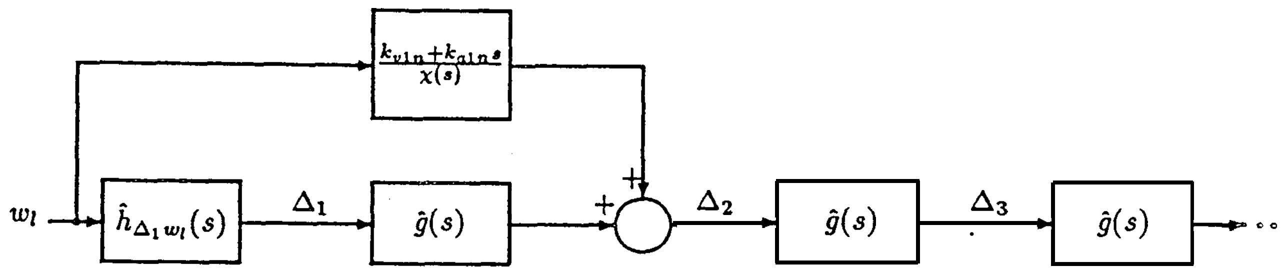
From (4.35) and using the simplified notation we obtain

$$\begin{aligned} & \left\{\tau s^3 \; t \; (1 \; t \; \tau d_1 \; t \; c_{an} + k_{an})s^2 + (d_1 + c_{vn} + k_{vn})s + c_{pn}\right\} \; \hat{\Delta}_i(s) \\ & = \left\{:_{an}s^2 + c_{vn}s \; t \; c_{pi}\right\} \hat{\Delta}_{i-1}(s) \end{aligned} \qquad (5.5)$$

thus,for $i = 3, 4, \ldots$

$$\hat{g}(s) := \hat{h}_{\Delta_i\Delta_{i-1}}(s) = \frac{c_{an}s^2 \; t \; c_{vn}s \; t \; c_{pn}}{\tau s^3 \; t \; (1 \; t \; \tau d_1 \; t \; c_{an} \; t \; k_{an})s^2 \; t \; (d_1 \; t \; c_{vn} \; t \; k_{vn})s \; t \; c_{pn}} \qquad (5.6)$$

Equ. (5.6) is the second basic design equation. From (5.6), we note that we can select the pole constellation for $\hat{g}(s)$ by choosing the appropriate



$\chi(s) = \tau s^3 + (1 + \tau d_1 + c_{an} + k_{an})s^2 + (d_1 + c_{vn} + k_{vn})s + c_{pn}$

Figure 5: Block diagram of the control law for a platoon of identical vehicles

design parameters $(c_{an} + k_{an})$, (c,, + k_{vn}), and c_{pn}. The zeros of $\hat{g}(s)$ are then chosen via the selection of the appropriate c_{an} and c_{vn}.

Note that if we choose $\hat{h}_{\Delta_1 w_l}$ to have the same pole constellation as $\hat{g}$(i.e., $c_{a1n} = c_{an} + k_{an}, c_{v1n} = c_{vn} + k_{vn}$, and $c_{p1n} = c_{pn}$), from (5.4) we obtain $\hat{h}_{\Delta_2\Delta_1} = \hat{g}$.

We consider the block diagram in figure 5 for analyzing the platoon. Some consideration of figure 5 suggests the main design objectives for the longitudinal control law: from (5.1), (5.3), and (5.5) we have for $i = 2, 3, \ldots$

$$\hat{h}_{\Delta_i w_l} = (\hat{g}(s))^{i-2}\left[\hat{h}_{\Delta_1 w_l}(s)\hat{g}(s) + \frac{k_{v1n} + k_{a1n}s}{\chi(s)}\right] \qquad (5.7)$$

1. Since the perturbations in Δ_i due to changes (w_l) in the lead vehicle's velocity from its steady-state value should not get magnified from one vehicle to the next as one goes down the platoon, we require that $|\hat{g}(j\omega)| \leq 1$ for all w > 0 and w $\mapsto |\hat{g}(j\omega)|$ to be a non-increasing function of w.

2. Since the inverse Laplace transform of $[\hat{g}(s)]^2$ is the convolution of the impulse response of $\hat{g}(s)$ with itself (i.e.,(g $*$ g)(t)), to avoid oscillatory behavior down the platoon it is desirable to have g(t) > 0 for all t.

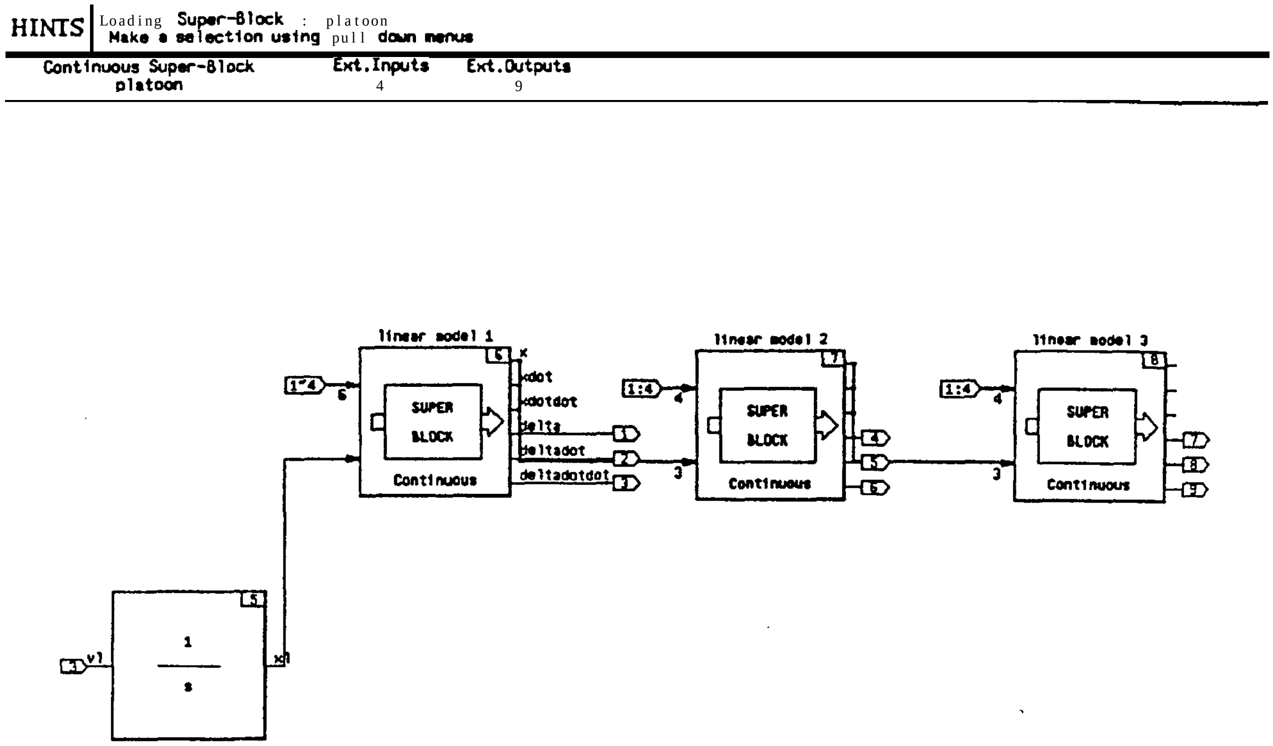


Figure 6: Simulation diagram for a platoon of 4 identical vehicles

5.4 Simulation Results

To examine the behavior of a platoon of identical vehicles under the proposed control law, simulations for platoons consisting of 4,11, and 16 vehicles were run using the System Build software package within MATRIXx. As an illustration, figure 6 shows the simulation diagram for a platoon of 4 identical vehicles. The dynamic blocks associated with the first, second, and third vehicle following the lead vehicle are denoted by linear model 1, linear model 2, and linear model 3, respectively. In all the simulations conducted, all the vehicles were assumed to be initially traveling at the steady-state velocity of V_e = 17.9 $m.sec^{-1}$ (i.e.,40 m.p.h.). Beginning at time $t = 0$ sec, the lead vehicle's velocity was increased from its steady-state value of 17.9 $m.sec^{-1}$ until it reached its final value of 32.0 $m.sec^{-1}$(i.e.,72 m.p.h.). Figure 7 shows the change (w_l) of the lead vehicle's velocity from its steady-state value as

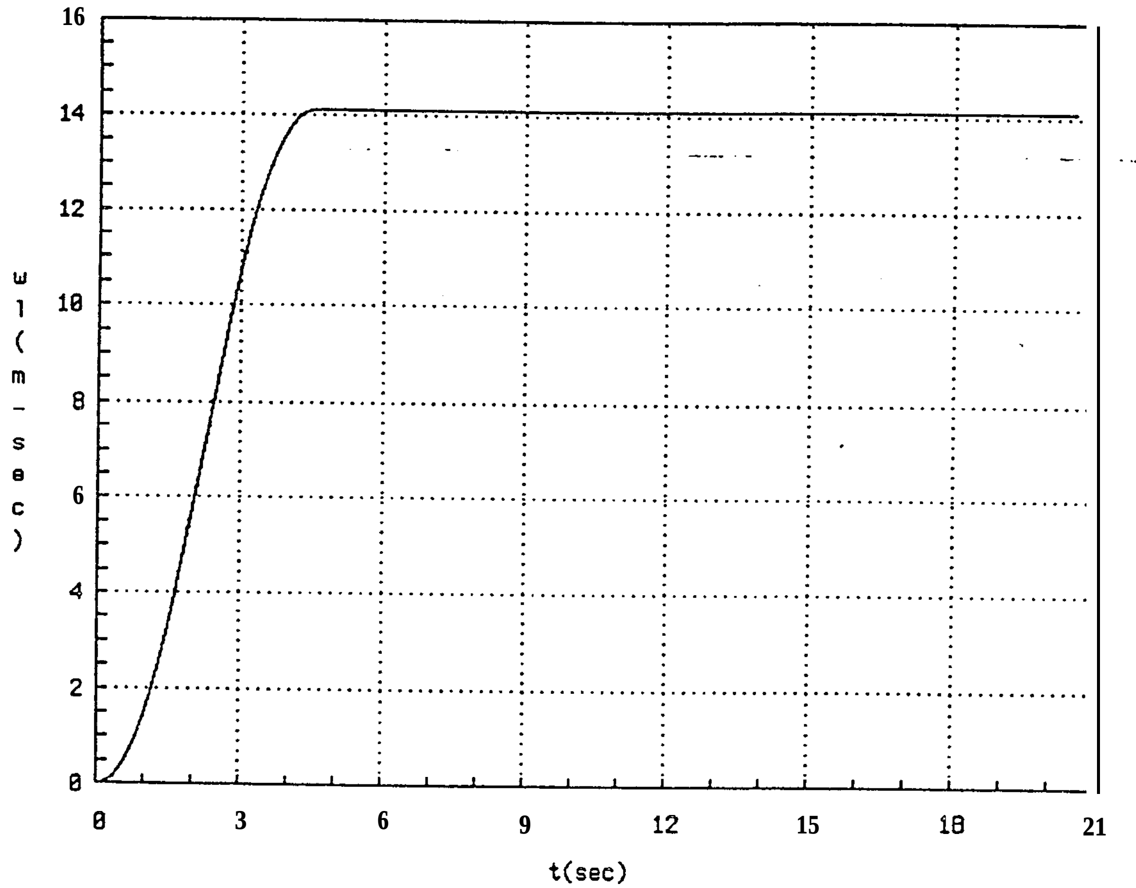


Figure 7: Increment of the lead vehicle's velocity from steady-state value (w_l)

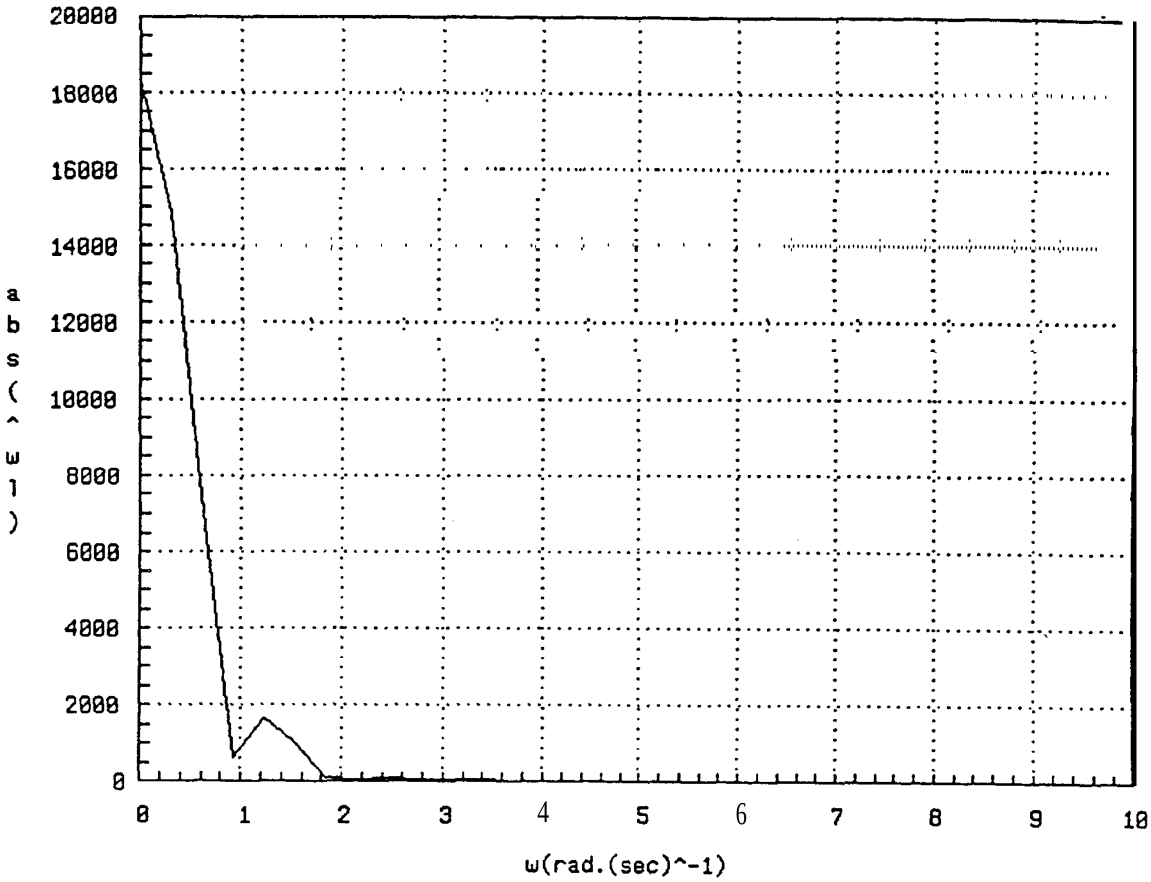


Figure 8: Magnitude of the Fast Fourier Transform of $w_l(|\hat{w}_l|)$

a function of time(t): the curve $w_l(t)$ corresponds to a maximum jerk of 3.0 $m.sec^{-3}$ and peak acceleration of 5.0 $m.sec^{-2}$ (i.e.,0.5g).

To get an idea of the frequency band in which most of the energy associated with the signal w_l lies, we computed the fast fourier transform of a periodic signal with period T = 20.48 sec; within each period, the value of the signal within the initial time intenal of 5.5 seconds of the period was identical to the value of w_l between t = 0 and t = 5.5; the value of the signal within the next time interval of 5.5 seconds of the period was the mirror image of the corresponding value in the first half of the period; the value of the signal in the remainder of the period was identically zero. This method for computing the fourier transform of w_l was employed so as to eliminate the high frequency componets in $\hat{w}_l$ due to the jumps in the periodic signal at the end of each period. Figure 8 shows the magnitude of the fast fourier transform of w_l. Note that most of the energy associated with w_l lies within

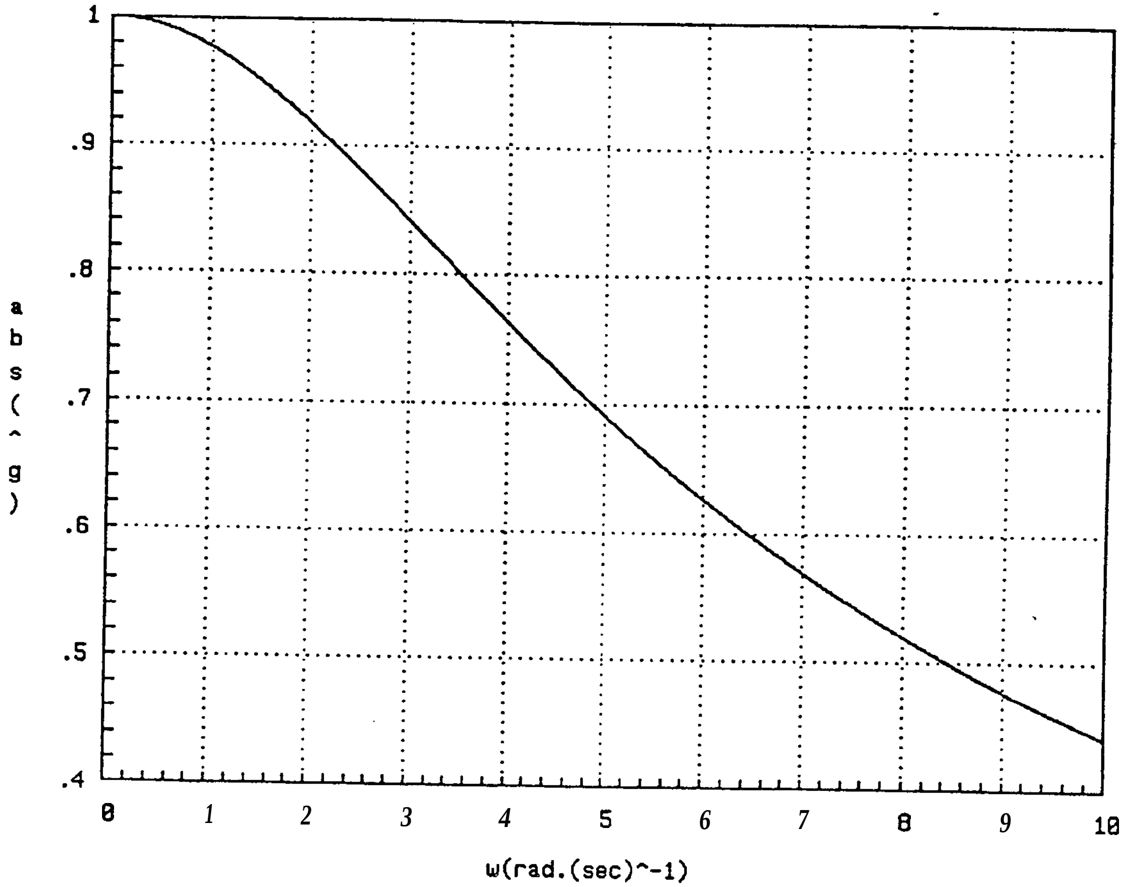


Figure 9: $|\hat{g}(j\omega)|$ vs. w

the interval from 0 to 2 radians per second.

The following values were chosen for the relevent parameters in the simulation:

$\tau = 0.2$ sec

$d_1 = 0.03$

$c_{a1n} = 1.994$, $c_{v1n} = 14.77$, $c_{p1n} = 24$, $k_{a1n} = 0.4$, $k_{v1n} = 0.02$

$c_{an} = 1$, $c_{vn} = 9.77$, $c_{pn} = 24$, $k_{an} = 0.994$, $k_{vn} = 1$

Using the above values for the parameters,we obtain:

$$\hat{h}_{\Delta_1 w_l}(s) = \frac{0.2s^2 + 0.606s + 0.01}{0.2(s+4)(s+5)(s+6)}$$

$$\hat{g}(s) = \frac{s^2 + 9.8s + 24}{0.2(s+4)(s+5)(s+6)}$$

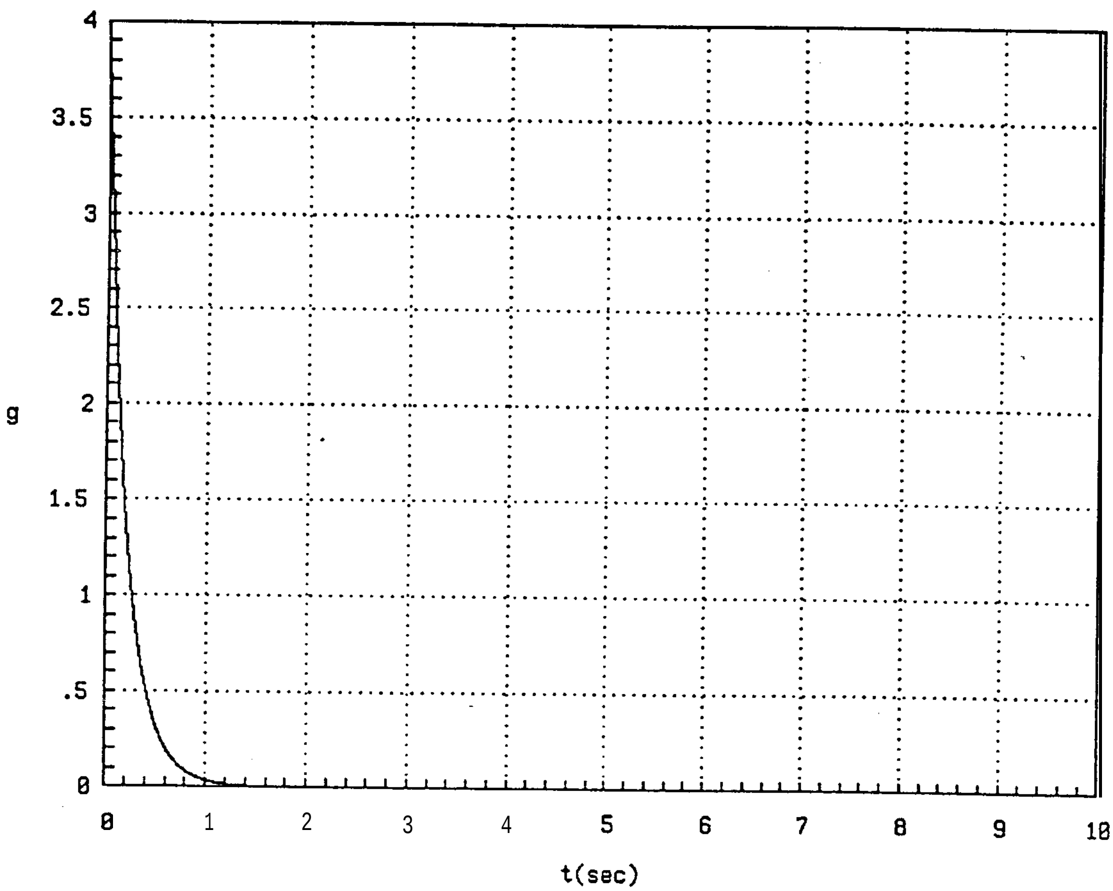


Figure 10: Impulse response of $\hat{g}$-i.e.,$g(t)$

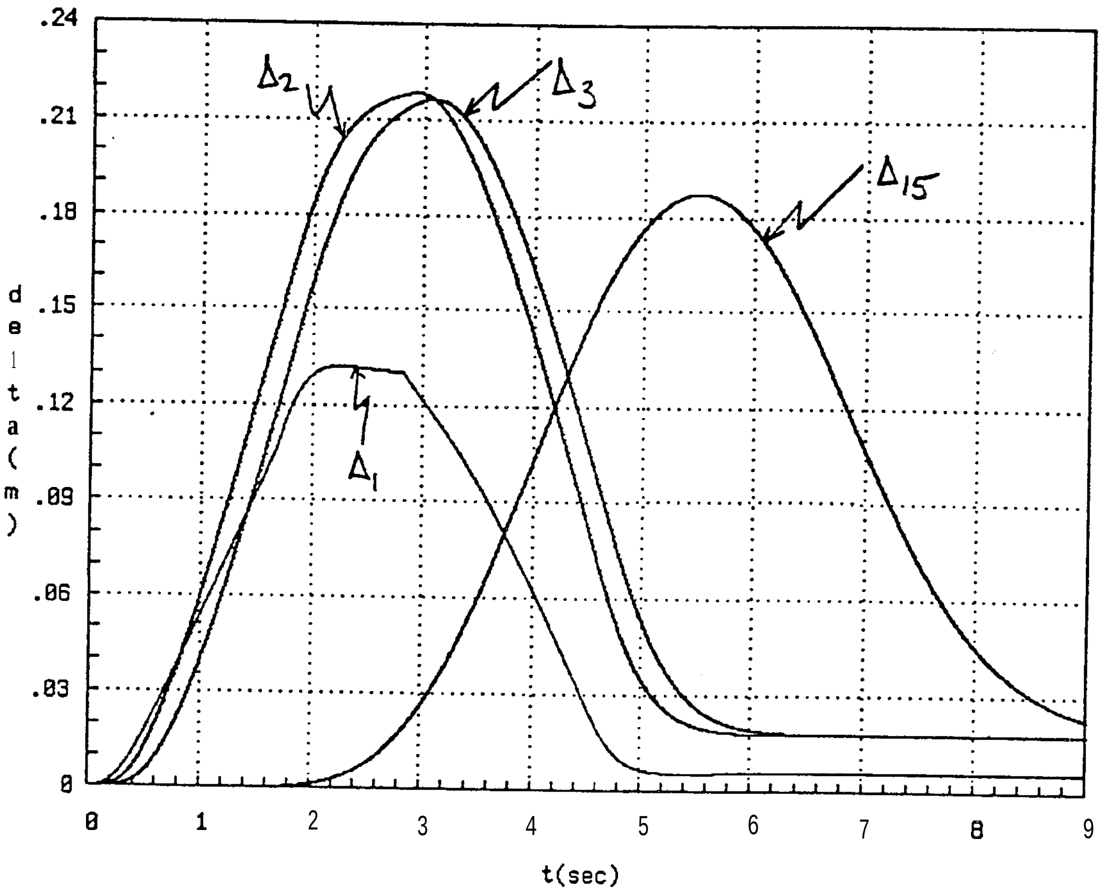


Figure 11: $\Delta_1, \Delta_2, \Delta_3, \Delta_{15}$ **vs.** t

Figure 9 illustrates the plot of $\omega \mapsto |\hat{g}(j\omega)|$. Note that $|\hat{g}(j\omega)| \leq 1$ for w > 0. The impulse response of $\hat{g}$ is shown in figure 10. Note that g(t) > 0 for all $t > 0$ and $t \mapsto$ g(f) is a decreasing function.

Figure 11 illustrates the resulting $\Delta_1, \Delta_2, \Delta_3,$ and Δ_{15} with the above choices of parameters.

Conclusions For the simulation studies above (figs. 6 to 11) all the vehicles were assumed to be identical and to be traveling initially at the steady-state velocity of 17.9 $m.sec^{-1}$(i.e.,40 m.p.h.). Beginning at time $t = 0$ *sec*, the lead vehicle's velocity was increased from its steady-state value of 17.9 $m.sec^{-1}$ until it reached its final value of 32.0 $m.sec^{-1}$(i.e.,72 *m.p.h.*).(see fig. 7) This change (w_l) in the lead vehicle's velocity from its steady-state value corresponded to a maximum jerk of 3.0 $m.sec^{-3}$ and peak acceleration of 5.0 $m.sec^{-2}$(i.e.,0.5g).

Simulation results show that the deviations of the vehicles from their preassigned positions do not exceed 0.22 m (i.e., $\frac{2}{3}$ of a foot) and decrease to values which are less than 2 cm. Such deviations decrease in magnitude from the second vehicle to the fifteenth vehicle in the platoon and exhibit no oscillatory behavior.

Robustness study Still considering a platoon of identical vehicles, further simulation studies were conducted to check the effect of delays in communicating the lead-vehicle-velocity (v_l) and acceleration (a_l) to all the other vehicles in the platoon. Figure 12 shows the deviations of the first (Δ_1), second (Δ_2), third (As), and tenth (Arc) vehicles' positions from their preassigned positions as a result of the change (w_l) in the lead vehicle's velocity from its steady-state value (figure 7) assuming:

1. communication of the lead vehicle's velocity (v_l) and acceleration (a_l) to each vehicle following the lead vehicle, say the i-th vehicle, is delayed by 20 *msec;*

2. let $\Delta_i^u(t)$ denote the value used in place of $\Delta_i(t)$ in the *i*-th vehicle's control law at time t (i.e.,$c_i(t)$). We assume that $\Delta_i^u(t) = \Delta_i(t - 0.005)(1+\eta)$ for $t \geq 0.005$ *sec* and $\Delta_i^u(t) = 0$ for $t < 0.005$ *sec*; η is a Gaussian random variable with mean zero and standard deviation of 0.1.

Figure 12 shows that the deviations in positions of the first, second, third, and tenth vehicles from their respective preassigned positions are less than 0.29 m and decrease to values less than 2 *cm.*

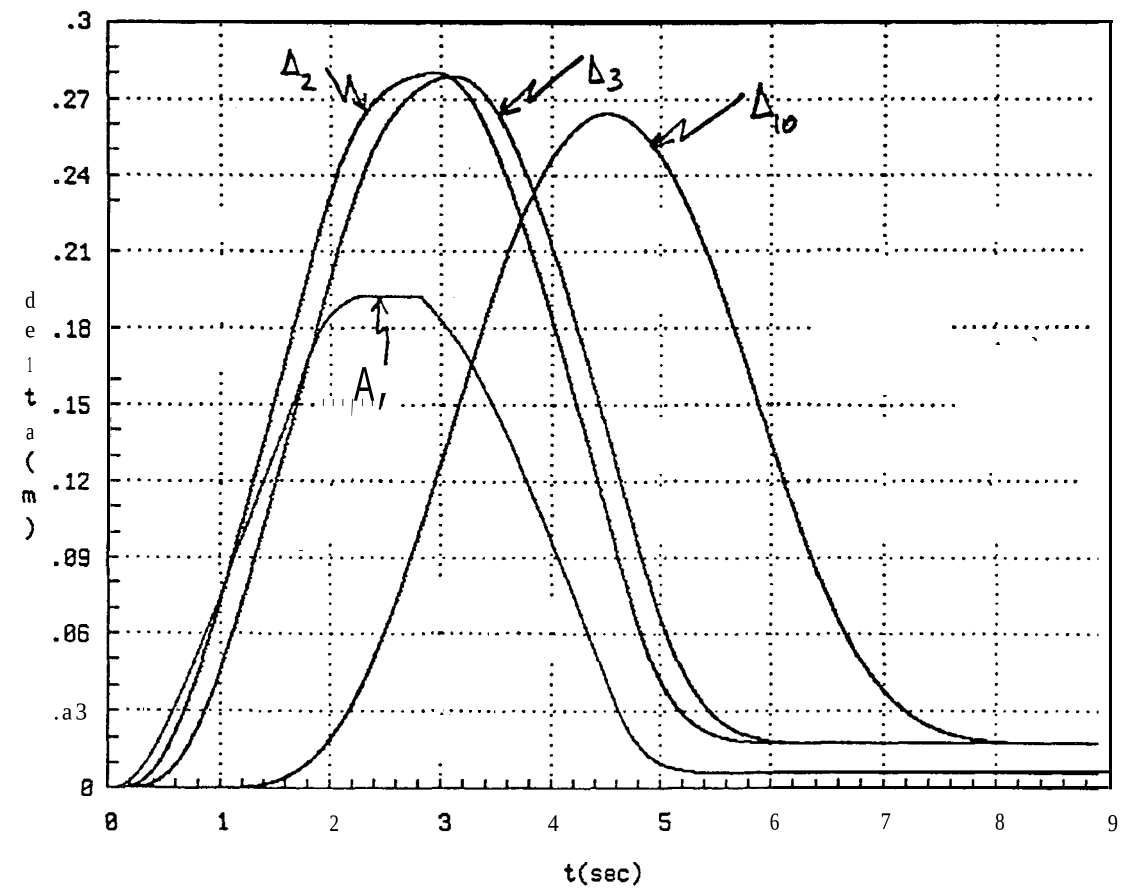


Figure 12: $\Delta_1, \Delta_2, \Delta_3, \Delta_{10}$ vs. t in the presence of noisy measurements and including communication delays

Such deviations do not get magnified from the second vehicle to the tenth vehicle in the platoon and do not exhibit oscillatory **behavior.**

Remark:platoon of non-identical vehicles Equ.(4.35) shows that in the case of non-identical vehicles $\hat{\Delta}_i$ depends on not only $\hat{\Delta}_{i-1}$ but also $\hat{\Delta}_1, \ldots, \hat{\Delta}_{i-2}$, and $\hat{w}_l$ for $i = 3, 4, \ldots$ (Note that the polynomial multiplier of $\hat{w}_l$ in (4.35) vanishes if and only if $d_{1i} = d_{1i-1}$; in general, d_{1i} varies from vehicle to vehicle because d_{1i} depends on cross-sectional area(A_i), drag coefficient(C_{di}), and vehicle mass(m_i)).

To eliminate the dependence of $\hat{\Delta}_i$ on $\hat{\Delta}_1, \ldots, \hat{\Delta}_{i-2}$ for $i = 3, 4, \ldots$, one might require that the multiplier of $(\hat{\Delta}_1 + \cdots + \hat{\Delta}_{i-2})$ in (4.35) vanishes for all values of s. Such a requirement will impose the following constraints on the choice of $k_{vi-1n}, k_{ai-1n}, k_{vin}$, and k_{ain} for $i = 3, 4, \ldots$

$$\tau_i k_{ai-1n} - \tau_{i-1} k_{ain} + \tau_i \tau_{i-1}(-d_{1i} + d_{1i-1}) = 0 \quad (5.8)$$

$$\tau_i k_{vi-1n} + k_{ai-1n} - \tau_{i-1} k_{vin} - k_{ain} + (\tau_i + \tau_{i-1})(-d_{1i} + d_{1i-1}) = 0 \quad (5.9)$$

$$k_{vi-1n} - k_{vin} + (-d_{1i} + d_{1i-1}) = 0 \quad (5.10)$$

Note that in order to satisfy the constraints given in (5.8), (5.9), and (5.10) for a platoon of n ($n = 3, 4, \ldots$) non-identical vehicles, one will have to solve a system of $(3n - 6)$ linear algebraic equations in $(2n - 2)$ unknowns. Thus, such a system of linear equations will generically have no solution for $n = 4, 5, \ldots$ This implies that $\hat{\Delta}_i$ will be affected by $\hat{\Delta}_1, \ldots, \hat{\Delta}_{i-2}$ for $i = 4, 5, \ldots$

The next step is to evaluate the effect of these differences in the vehicle models of the platoon.

6 Conclusion

We have shown that through the appropriate choice of design parameters, deviations in the successive vehicle spacings do not get magnified from the front to the back of a platoon of identical vehicles as a result of lead vehicle's acceleration from its initial steady-state velocity(v_0) to its final steady-state velocity. Furthermore, the deviations in the successive vehicle spacings do not exhibit any oscillatory time-behavior and the magnitude of such deviations is well within one foot for a platoon of 16 vehicles.

Our analysis of a platoon of non-identical vehicles shows that deviation of the i-th vehicle position from its assigned position(i.e.,Δ_i) **is** affected by the corresponding deviations of all the vehicles in front of the i-th **vehicle(i.e.,$\Delta_1, \ldots, \Delta_{i-1}$)** together with the **change (w_l)** of the lead vehicle's velocity from its steady-state value. Since elimination of the terms relating **to** $\Delta_1, \ldots, \Delta_{i-2}$ requires a system of linear algebraic equations with more equations than unknowns, we have to investigate by simulation the magnitude of such effects on the i-th vehicle.

7 References

[Caud.l] Caudill,R.J.,Garrard,W.L.,"Vehicle-Follower Longitudinal Control for Automated Transit Vehicles," ***Journal of Dynamic Systems, Measurement and Control,*** vo1.99, Dec.1977, pp.241-248.

[Chi.l] Chiu,H.Y.,Stupp,G.B.,Jr.,aud Brown,S.J.,Jr.,"Vehicle-Follower Control with variable gains for short headway automated guideway transit systems," ***Journal of Dynamic Systems, Measurement and Control,*** vo1.99, N0.3, Sept.1977, pp.183-189.

[Fra.l] Frank,A.A.,Liu,S.J.,Liang,S.C.,"Longitudinal Control Concepts for Automated Automobiles and Trucks Operating on a Cooperative Highway," Preprint for SAE Technical Papers, Aug.1989.

[Hauk.l] Hauksdottir,A.S.,Fenton,R.E.,"On the design of a vehicle longitudinal controller,"IEEE Trans. ***l&h. Tech.,*** vol.VT-34, No.4, Nov.1985, pp.182-187.

[Hauk.2] Hauksdottir,A.S.,Fenton,R.E.,UOn vehicle Longitudinal Controller Design,"1988 ***IEEE Workshop on Automotive Applications of Electronics,*** 1988, pp.77-83.

[Hob.l] Hoberock,L.L.,"A Survey of Longitudinal Acceleration Comfort Studies in Ground Transportation Vehicles," ***Journal of Dynamic Systems, Mea= surement and Control,*** vo1.99, June 1977, pp.76-84.

[Hob.2] Hoberock,L.L.,Rouse,R.J.,Jr,"Emergency ControlofVehicle Platoons:System Operation and Platoon Leader Control," ***Journal of Dynamic Systems, Measurement and Controk*** vol.98, No.3, Sept.1976, pp.245-251.

[Rous.l] Rouse,R.J.,Jr,Hoberock,L.L.,"Emergency Control of Vehicle Platoons:Control of Following-Law Vehicles,"*Jouurnal of Dynamic Systems, Measurement and Control,* vol.98 No.3, Sept.1976, pp.239-244.

[Shoal] Shiadover,S.E.,"Operation of automated guideway transit vehicles in dynamically reconfigured trains and platoons,"*Automated Guidewuy Transit Technology Progmm,* UMTA-MA-06-0085579-1, April 1979.

[ShIa2] ShIadover,S.E.,"Longitudinal Control of Automotive Vehicles in Close-Formation Platoons," Preprint for Joint Symposium on Advanced Automotive Technologies 1989 ASME Winter Annual Meeting, Dec.1989

[Tak.l] Takasaki,G.M.,Fenton,R.E.,"*On the Identification of Vehicle Longitudinal Dynamics,"IEEE Tmns.Auto.Cont.,* vol.AC-22, No.-4 Aug.1977, pp.610-615.

[Wal.l] rivate Communication at UC Berkeley, EECS Dept., Mar.-Aug.1989.