Title
Utilization of Bandpass Filtering for the Matrix Pencil Method

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The necessary and sufficient condition that (A1) holds is

$$|g^i|_N = N - |g^{i+(N-1)/2}|_N.$$

That is

$$|g^i|_N + |g^{i+(N-1)/2}|_N = N \tag{A2}$$

where "g" is a primitive element. According to the number theory [7], we have

$$|g^{(N-1)/2}|_N = N - 1; \qquad |A \times B|_N = ||A|_N \times |B|_N|_N$$

then

$$\begin{aligned} |g^{i+(N-1)/2}|_N &= |g^i \times g^{(N-1)/2}|_N = ||g^i|_N \times |g^{(N-1)/2}|_N|_N \\ &= ||g^i|_N \times (N-1)|_N = |-|g^i|_N|_N \\ &= |N - |g^i|_N|_N \end{aligned}$$

as $0 < |g^i|_N \le N - 1$, $1 \le i \le N - 1$, we have

$$|N - |g^i|_N|_N = N - |g^i|_N.$$

It means that

$$|g^{i+(N-1)/2}|_N = |N - |g^i|_N|_N = N - |g^i|_N.$$

So

$$|g^i|_N + |g^{i+(N-1)/2}|_N = N.$$

Therefore, (11) is proved.

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Utilization of Bandpass Filtering for the Matrix Pencil Method

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Abstract—**This correspondence describes an algorithm named the bandpass matrix pencil (BPMP) method for estimating the parameters of an exponential data sequence. The matrix pencil (MP) method, along with a filtering technique, is used to estimate the complex exponentials of the signal. However, due to special requirements to the filtered data by the MP method, the prefiltering process is not trivial. The approach presented here utilizes the backward process for the IIR filtering and the circular convolution for the FIR filtering, respectively. Monte Carlo simulations are presented to illustrate the performance of the proposed filtering schemes.**

I. INTRODUCTION

The mathematical model of an observed signal can generally be formulated as

$$y(k) = x(k) + n(k) = \sum_{i=1}^{M} R_i Z_i^k + n(k), \quad k = 0, 1, \cdots, N-1 \tag{1}$$

where

$$z_i = \exp(-\alpha_i + j\omega_i) \tag{2}$$

and z_i's and R_i's are the poles and residues of the signal, respectively. M is the number of poles of the signal, and $n(k)$ is the background noise. α_i and ω_i are the damping factor and angular frequency of the ith sinusoid, respectively. Once the number of poles and their values have been determined, the residues at the poles can be found by the least squares method. Hence, only the problem of estimation of the poles is considered in this correspondence.

The most popular method for pole retrieval is Prony's method. However, Prony's method is notorious for its extreme sensitivity to noise. There are many modified versions of the Prony method. The most well known one is the principal eigenvector (PE) method [1]. Recently, Hua and Sarkar [2], [3] developed a new technique, named the matrix pencil (MP) method, for pole estimation. The advantage of using matrix pencil is that the signal poles can be found directly from the eigenvalues of the matrix contrast to the PE method, which generally requires two-step processes. In the first step one solves a matrix equation, and finds the roots of a polynomial equation in the second step.

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Even though the MP method can filter out part of the noise by using a singular value decomposition (SVD) of the data matrix and discarding the nonprincipal singular vectors similar to ESPIRT, some effects of noise still exist in the principal singular values and vectors. Therefore, to further combat noise, prefiltering can be used prior to the SVD filtering. To keep the underlying property of the exponentially damped or undamped sinusoidal signals after prefiltering, the backward process for the IIR filters is introduced. For the FIR filters, we suggest using the circular convolution instead of the linear convolution.

II. Pole Retrieval by the MP Method

In order to motivate the development of the bandpass matrix pencil (BPMP) method, the algorithm of the MP method for pole estimation is briefly reviewed in this section. For the noiseless data, we can define the matrix pair Y_1, Y_2 as

$$Y_1 = [X_1, X_2, \cdots, X_L]; \qquad Y_2 = [X_0, X_1, \cdots, X_{L-1}] \tag{3}$$

where $x_i = [x(i), x(i+1), \cdots, x(N-L+i-1)]^T$. The superscript T denotes the transpose. It can be shown that

$$Y_1 = Z_1 R Z_0 Z_2; \qquad Y_2 = Z_1 R Z_2 \tag{4}$$

where $z_0 = \text{diag}[z_1, z_2, \cdots, z_M]$, $R = \text{diag}\,[R_1, R_2, \cdots, R_M]$. Z_1 and Z_2 are full rank Vandermonde matrices [3]. Then we generate the matrix pencil as

$$Y_1 - \lambda Y_2 = Z_1 R(Z_0 - \lambda I) Z_2. \tag{5}$$

One can show in general that the rank of $Y_1 - \lambda Y_2$ will be M, provided $M \leq L \geq N - M$ where L is the matrix pencil parameter. However, if $\lambda = z_i$, $i = 1, 2, \cdots, M$, the ith row of $Z_0 - \lambda I$ is zero, thus, the rank of $Z_0 - \lambda I$ will be $M - 1$. Therefore, the matrix pencil $Y_1 - \lambda Y_2$ will also reduce in rank to $M - 1$. By definition, z_i's are exactly the generalized eigenvalues of the matrix pair $\{Y_1, Y_2\}$. Furthermore, it can be shown that the generalized eigenvalues of $Y_1 - \lambda Y_2$ can be found from the nonzero eigenvalues of $Y_2^+ Y_1$, where Y_2^+ is the Moore–Penrose pseudoinverse of Y_2.

III. The Infinite Impulse Response (IIR) Filter Processing

The backward process is proposed for the IIR filtering. The input signal is required to be a linear combination of exponentially damped sinusoidal signals. This requirement can be satisfied by most signals encountered in practice.

The backward process in the time domain can be written as

$$y(k) = \sum_{j=0}^{m} b_j x(j+k) - \sum_{i=1}^{n} a_i y(i+k). \tag{6}$$

If all poles of the signal are different from those of the IIR filter, then $y[k]$ can be divided into two parts. One part is generated by means of the poles of the signal, denoted as $y_x(k)$, while the other part arises from the poles of the filter, denoted as $y_h(k)$. It can be shown that $y_x(k)$ satisfies the following difference equation

$$y_x(k) = \sum_{j=0}^{m} b_j x(j+K) - \sum_{i=j}^{n} a_i y_x(i+k) \tag{7}$$

where

$$y_x(k) = \sum_{i=1}^{m} R_i H(z_i) z_i^k. \tag{8}$$

Since the backward process starts at the endpoint of the data sequence, it is required that the initial conditions be zero, that is, $x(k) \approx 0$, $y_x(k) \approx 0$, for $k \leq N$. Therefore, the input signal must be an exponentially damped signal! To secure the backward process without involving the poles of the filter, $y_h(k)$ must not be zero for $k \geq N$. This can be achieved by placing the poles of the filter outside the unit circle. Because the backward process does not involve the poles of the filter, placement of the poles of the IIR filter outside the unit circle would not induce the process unstable. Also, such a procedure guarantees that the poles of the signal would be different from those of the filter.

To simplify the filter design procedure, we consider the second-order IIR bandpass filter. The transfer function $H(z)$ of the bandpass filter is defined as

$$H(z) = K \frac{(z+1)(z-1)}{(z - re^{j\omega_0})(z - re^{-j\omega_0})} \tag{9}$$

where K is a constant controlling the gain of the filter. To find the difference equation of the filter, we rewriter $H(z)$ in the canonical form. That is,

$$H(z) = K' \frac{1 - z^2}{1 - q_1 z - q_2 z^2} \tag{10}$$

where

$$q_1 = \frac{2 \cos \omega_0}{r}; \qquad q_2 = -\frac{1}{r^2}. \tag{11}$$

K' is chosen so that the filter has a unit gain at $\omega = \omega_0$. To control the sharpness or wideness of the passband of the filter, we introduce another parameter BW such that

$$r = \frac{1}{1 - \text{BW}}. \tag{12}$$

Substituting (14) into (13), we have

$$q_1 = 2(1 - \text{BW}) \cos \omega_0; \qquad q_2 = -(1 - \text{BW})^2. \tag{13}$$

Provided that N and the damping factors are large enough, $x(k)$ and $y_x(k)$ are approximately zero when k is larger than N. Using (9), we get

$$y_x(N-1) \approx K' x(N-1)$$

$$y_x(N-2) \approx q_1 y_x(N-1) + K' x(N-2) \tag{14}$$

$$y_x(k) = q_2 y_x(k+2) + q_1 y_x(k+1) + K' \tag{15}$$

$$\cdot [x(k) - x(k+2)], \qquad \text{for } 0 \leq k \leq N-3. \tag{16}$$

To get the desired filtering effect, the filtering process can be done by using a series of cascaded bandpass filters. Assuming that the number of the filters is d and denoting the output of the dth filter as $y_x^{(d)}(k)$, one can use $y_x^{(d)}(k)$ to form the matrix pair Y_1 and Y_2. The poles of the signal can be obtained from the eigenvalues of $Y_2^+ Y_1$.

The way to decide the central frequency ω_0 and the bandwidth parameter BW of the filter is to do a coarse FFT analysis of the data. Based on our simulations, we have found that the choices of ω_0 and BW are not critical when SNR is not too low.

IV. The Finite Impulse Response (FIR) Filter Processing

It is well known that IIR filtering is usually faster than FIR filtering due to the utilization of the recursive equation in the IIR filtering. However, IIR filtering can only be used for the signals

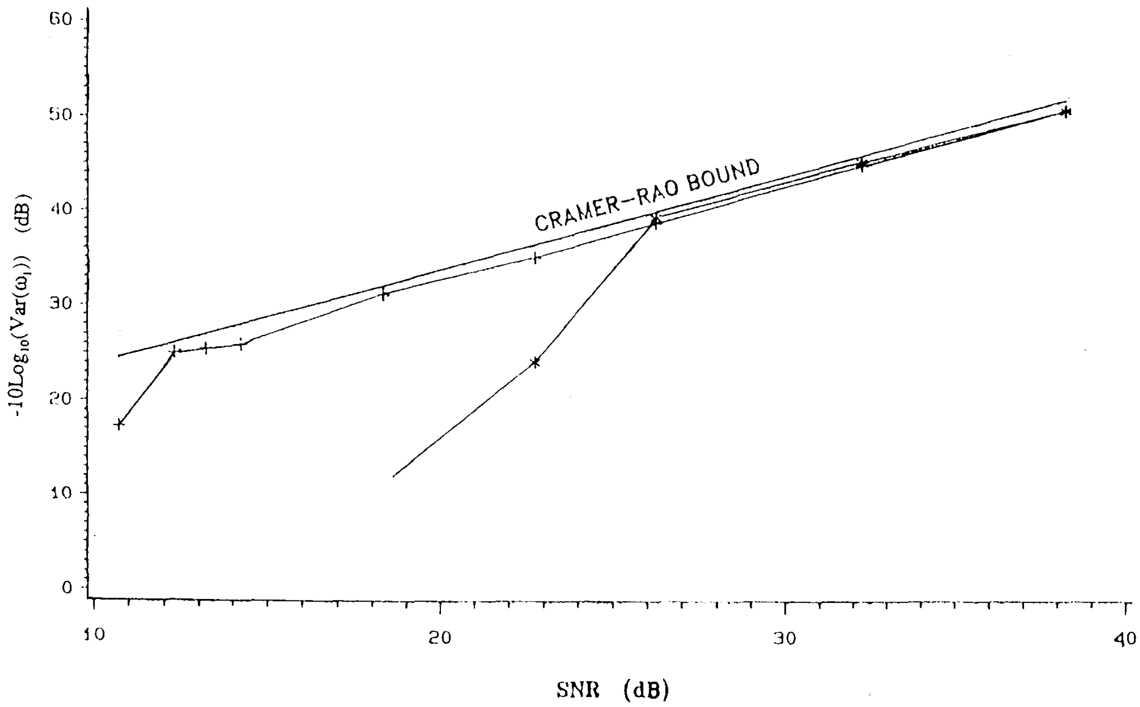

Fig. 1. The inverted sample variance of ω_1 versus SNR. BW = 0.1, $\omega_0 = 0.8796$, $N = 50$, $L = 20$, $d = 1$. *: MP, +: BPMP.

which contain exponentially damped sinusoidal signals as mentioned in the previous section.

The first step for the FIR filtering is to find the impulse response of the filter. From the magnitude of the FFT of the signal, the cutoff frequencies of the bandpass filter ω_L, ω_U ($\omega_L < \omega_U$) can be obtained. The truncated finite impulse response is given by

$$h(n) = h_d(n)w(n), \qquad n = 0, \pm 1, \cdots, \pm \frac{q-1}{2} \tag{17}$$

where q is the length of the bandpass filter and odd $w(n)$ is the window function. The FIR filtering then is achieved by using the linear convolution. Even though the FIR filtering would not increase the number of poles of the filtered data, the number of the filtered data available to form the matrix pair Y_1 and Y_2 will be reduced because of the linear convolution. Now we introduce Lemma 1.

Lemma 1: If the length of the input signal is N and of the impulse response of the filter is q, then there are $N - q + 1$ data samples of the filter output. The output can be expressed as

$$y(k) = \sum_{i=1}^{M} R_i' Z_i^{k-q+1}, \qquad \text{for } q - 1 \le k \le N - 1. \tag{18}$$

Lemma 1 indicates that the longer the length of the filter, the shorter the filtered data that can be used by the MP method. To maintain the performance of the SVD filtering, we introduce a prefiltering approach by using the circular convolution.

The crux of the circular convolution is to generate a $(N - L) \times (N - L)$ filtering matrix by using the impulse response of the filter. That is,

$$H = \begin{bmatrix} h\left(\frac{q-1}{2}\right) & h\left(\frac{q-3}{2}\right) & \cdots & h\left(-\frac{q-1}{2}\right) \\ h\left(-\frac{q-1}{2}\right) & h\left(\frac{q-1}{2}\right) & \cdots & h\left(-\frac{q-3}{2}\right) \\ \vdots & \vdots & & \vdots \\ h\left(\frac{q-3}{2}\right) & h\left(\frac{q-5}{2}\right) & \cdots & h\left(\frac{q-1}{2}\right) \end{bmatrix} \tag{19}$$

If $N - L$ is even, the filter length is chosen as $N - L - 1$ and one zero is padded to the sequence. Premultiplying both Y_1 and Y_2 by H, we have

$$\begin{aligned} Y_1' &= HY_1 = [y_1', y_2', \cdots, y_L'], \\ Y_2' &= HY_2 = [y_0', y_1', \cdots, y_{L-1}'] \end{aligned} \tag{20}$$

where $y_i' = Hy_i$. Since H is full rank, for the no noise case, substituting (4) into (23) results in

$$Y_1' = HZ_1RZ_0Z_2, \qquad Y_2' = HZ_1RZ_2. \tag{21}$$

Therefore,

$$Y_2'^{+} Y_1' = Z_2^{+} Z_0 Z_2. \tag{22}$$

The signal poles are still the nonzero eigenvalues of $Y_2'^{+} Y_1'$.

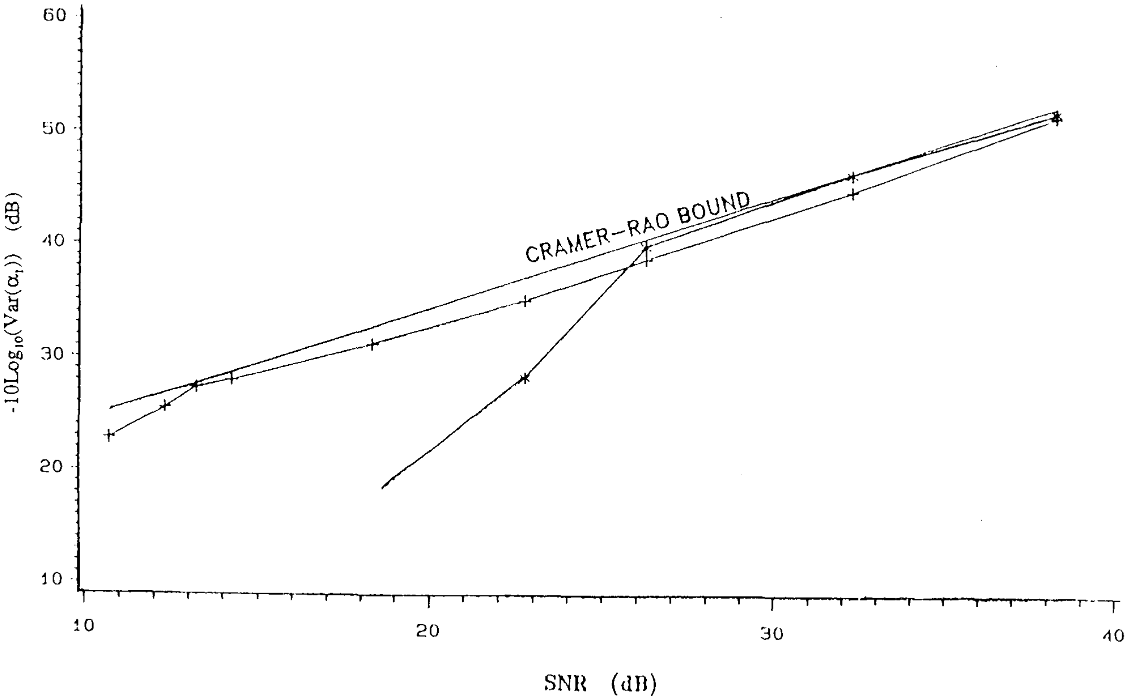

Fig. 2. The inverted sample variance of α_1 versus SNR. BW = 0.1, $\omega_0 = 0.8796$, $N = 50$, $L = 20$, $d = 1$. *: MP, +: BPMP.

TABLE I
THE MEAN-SQUARED ERROR AND THE BIAS OF ω_1 FOR DIFFERENT BW (SNR = 18.3 dB)

BW	0.001	0.01	0.05	0.1	0.3
MSE	6.43×10^{-4}	6.46×10^{-4}	6.98×10^{-4}	7.56×10^{-4}	1.32×10^{-3}
Bias	0.0016	0.0006	−0.0019	−0.0044	−0.0097

IV. THE PERFORMANCE STUDY OF THE BPMP METHOD

In this section, the computer simulations are carried out to illustrate the performance of the BPMP method. In the following simulations, the exponentially damped sinusoidal signals are used. Since the signals are real, (1) can be written as

$$y(k) = x(k) + n(k) = \sum_{i=1}^{M/2} b_i e^{-\alpha_i k} \sin(\omega_i k + \phi_i) + n(k), \quad k = 0, 1, \cdots, N-1 \tag{23}$$

where $n(k)$ is a zero-mean variance σ^2 Gaussian white noise. The signal-to-noise ratio (SNR) is defined as

$$\text{SNR (dB)} = 10 \log_{10} \frac{\sum_{k=0}^{N-1} x^2(k)}{N\sigma^2}. \tag{24}$$

In the first example, we consider the signal with the following parameters: $M = 4$, $\omega_1 = 0.26\pi$, $\omega_2 = 0.3\pi$, $\alpha_1 = \alpha_2 = 0.1$, $b_1 = b_2 = 1$, $\phi_1 = \phi_2 = 0$. The number of data N is 50. The IIR bandpass filter is used for filtering. The central frequency of the bandpass filter ω_0 is set as the average of ω_1 and ω_2. The parameters of the bandpass filter BW is chosen as 0.1. The number of filters d is 1. The matrix pencil parameter L equals 20.

To get the sample variance of the estimate of ω_1, 200 runs are performed. The noise used in each run is independent of the others. For comparison to the BPMP method, we have computed the Cramer-Rao lower bound (CRLB) for the variance of the angular frequency estimate. It can be seen that the BPMP method is slightly poorer than the MP method at high SNR as shown in Fig. 1. This is due to the truncation error of the backward process. However, the threshold is extended downward from 26 to 12 dB. The plot of the inverted sample variance of α_1 in Fig. 2 further confirms that the BPMP method is better than the MP method especially at low SNR.

In Table I, we tabulate the mean-squared error and the bias of ω_1 for BW changing from 0.001 to 0.3. One can see that the mean-squared error and the bias increases with the bandwidth of the filter as more noise pass through the filter. The mean-squared error and the bias in estimating ω_1 for the different choices of ω_0 are given in Table II. It is obvious that the mean-squared error and the bias of ω_1 become smaller when ω_0 is closer to ω_1.

To ascertain the performance of the FIR filtering, the exponentially damped sinusoid signal with a small N and small damping factors is used for simulations. The parameters are $N = 30$, $M = 4$, $\omega_1 = 0.2\pi \simeq 0.628$, $\omega_2 = 0.35\pi \simeq 1.1$, $\alpha_1 = 0.02\pi$, $\alpha_2 = 0.035\pi$, $b_1 = b_2 = 1$, and $\phi_1 = \phi_2 = 0$. L is chosen as 10. ω_L and ω_U are set at 0.5 and 1.2, respectively. The window function is the

TABLE II
THE MEAN-SQUARED ERROR AND THE BIAS OF ω_1 WITH DIFFERENT ω_0 (SNR = 18.3 dB)

ω_0	0.7796	0.8168	0.8796	0.9424	0.9796
MSE	1.32×10^{-3}	9.21×10^{-4}	7.56×10^{-4}	8.92×10^{-4}	1.21×10^{-3}
Bias	0.0104	0.0038	−0.0044	−0.0137	−0.0222

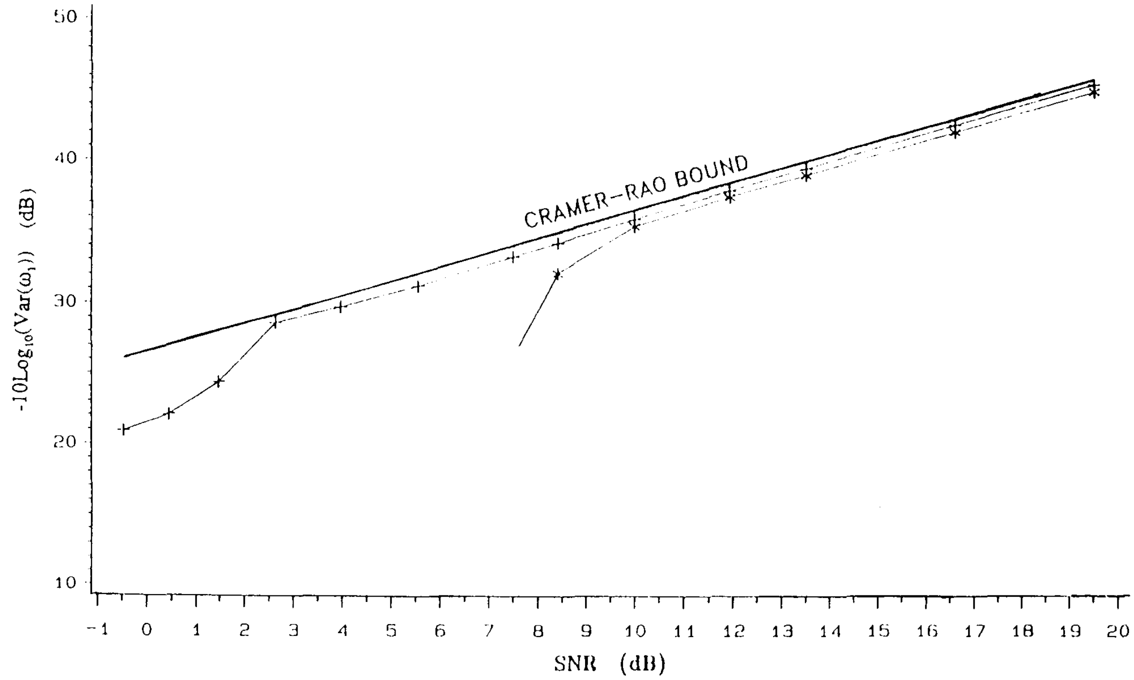

Fig. 3. The inverted sample variance of ω_1 versus SNR. $L = 10$, $q = 19$, $\omega_L = 0.5$, $\omega_U = 1.2$, $\beta = 5.658$, Kaiser window. *: MP, +: BPMP.

Kaiser window with $\beta = 5.658$ [5]. The length of the FIR filter is chosen as 19. From the plot of the sample variance of the estimate of ω_1 in Fig. 3, it is evident that the BPMP method performs better than the MP method.

V. CONCLUSION

The combination of MP method and the prefiltering provides a new robust technique for estimating parameters of exponential signals in noise. Since the MP method is based on the underlying property of the exponential signals, we proposed the backward process and the circular convolution for the IIR and the FIR filtering, respectively. The IIR filtering is especially suitable for long damped sinusoidal signals, while FIR filtering is more efficient for short damped or undamped sinusoidal signals. Finally, simulation results have been presented which confirm the expected performance of the proposed filtering techniques.

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A Simplified Derivation of the Performance of Edge-Connected Crossed-Electrode Arrays for Two-Dimensional Projection and Beamforming

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***Abstract*—This correspondence presents a theoretical analysis of the possible performance of edge-connected crossed arrays for two-dimensional beamforming introduced by Schau. The approach utilizes stan-**

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