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STRENGTH OF CONCRETE UNDER COMBINED STRESSES

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STRENGTH OF CONCRETE UNDER COMBINED STRESSES

A Report of an Investigation
by

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to

THE REINFORCED CONCRETE RESEARCH COUNCIL
OF
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SYNOPSIS

A criterion for failure of plain concrete subjected to combined stresses was established from tests of sixty-five tubular specimens tested to failure under various combinations of shearing and compressive stress. A procedure for determining the shearing strength of rectangular reinforced concrete beams without web reinforcement was developed. Excellent correlation was obtained between calculated and observed shearing strength of a limited group of beams for which the mechanism of failure had been defined by other investigators.

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ii	19	head	load
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18	12	a	σ a
22	19	Couard	Mr. Couard
	26	tan	tan ϕ
24	23	6 to 10	6 to 10, subjected to concentrated loads.
25	23	average stresses	average stresses
27	6	Eq. 20	Eq. 14
	10	beams were selected	beams, subjected to concentrated loads, were selected
	21	0.96	0.94
32	9	figure	value
35	5 & 7	Nielson	Nielsen
39	16	that the quantity	that in Figure 14 the quantity
41	10,11	(Title of Table should be placed above the tabulation)	
References page 2	Ref. 16	Hanson	Hanson, N.W.
Table 6		(All σ and τ should read σ_a and τ_a)	

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NOTATION

The letter symbols used in this report are generally defined when they are introduced. The most frequently used symbols are listed below for convenient reference.

A - cross sectional area, or numerical coefficient

A_e - effective area resisting shear

A_s - area of longitudinal reinforcement

b, d - width and effective depth of rectangular beam, respectively

C - compression force due to internal moment

f'_c - compressive strength of 6 x 12 in concrete test cylinders

f''_c - compressive strength of 3 x 6 in. concrete test cylinders

f_y - yield strength of longitudinal reinforcement

I_1, I_2, I_3 - principal stress invariants

j - ratio of internal moment arm to effective depth d.

J - polar moment of inertia

k - ratio of depth of compression zone to effective depth d

M - bending moment

p - (As/bd)

P - axial head

T - tension force due to internal moment

V - total shearing force

σ - normal stress

τ - shearing stress

I INTRODUCTION

A. Objectives

Failure of structural elements usually occurs under complex loading conditions, hence the state of stress at failure is generally complex. For this reason, knowledge of criteria of failure for materials subjected to combined stresses is important. Reinforced concrete elements, such as rigid frames, plate floor slabs, shell roofs, and similar prestressed concrete elements, are subjected to loads producing combinations of normal and shearing stresses. Therefore, investigations of strength limitations for concrete subjected to combined stresses becomes particularly important if the ultimate strength theories are to be accepted.

Investigations of failure in plain concrete is complicated by the nonhomogeneity and nonisotropy of the material, and in reinforced concrete it is further complicated by the presence of reinforcement. It has been often stated that presence of reinforcement in concrete alters the nature of the material and, therefore, changes the criteria for failure. Indeed, in considering the mechanism of failure, that is, the progress from the initial local failure to the ultimate collapse, bond between concrete and reinforcement, and deformation or failure in the reinforcement plays a very important role. It seems reasonable, however, that the conditions producing local failure in concrete are essentially the same for both plain and reinforced concrete, and therefore knowledge of failure criteria for plain concrete would be valuable for predicting collapse strength of reinforced or prestressed concrete structural elements of any size or shape.

The study reported here had two principal objectives. The first was to verify the validity of a failure criterion for plain concrete based on a relationship between normal and shearing mean stresses. Such a hypothesis had been formulated by the writers as a result of a preliminary investigation, but in view of the limited data obtained at that time, further verification was deemed desirable. The test program carried out as a part of this investigation is described in Part II of this report and the formulation of the failure criterion is described in Part III.

The second objective was to apply the failure criteria to the determination of shearing strength of structural elements. All previous methods for determining shearing strength of structural elements were largely empirical, based on tests of the particular kind of structural elements, such as beams, slabs, or footings, and attempts to correlate the strength properties of the material with the shearing strength of the structural elements were unsuccessful. In this study an excellent correlation between the failure criterion and the shearing strength of a particular class of rectangular reinforced concrete beams without web reinforcement was obtained. This correlation is described in Part IV of this report.

Effects of specimen size and shape, and of stress distribution in the test specimen, are of considerable importance in all studies of material properties, particularly for such a non-homogeneous material as concrete. Some considerations of these effects, from a statistical point of view, are discussed in Part V of this report.

B. Acknowledgements

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A task committee, appointed by the Reinforced Concrete Research Council, to provide general supervision of the investigation was constituted as follows: E. Hognestad, chairman and R. Archibald, P. M. Ferguson, and A. M. Freudenthal, members.

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II TEST PROGRAM

A. Scope of Test Program

The experimental investigation was limited to the study of strength of plain concrete subjected to various combinations of compression and shear. All test specimens were of the same tubular shape: 9 in. outside diameter, 6 in. inside diameter, and 30 in. long. Stresses at failure were determined for specimens subjected to pure axial compression, pure torsion, and torsion combined with compression of 20, 40, 60, and 80 percent of pure compressive strength. Three concrete mixes were used in this investigation with nominal compressive strengths of 3000, 4500, and 6000 psi at the age of 28 days. All specimens were tested at approximately 28 days after casting.

The test program was divided into three series. The objective of tests in series I was to determine the effect of end conditions imposed by a special torque band on the pure compressive strength of specimens. For each mix one specimen was tested without the torque band and one with the torque band clamped on the ends. In all subsequent tests, series II and series III, all pure compression test specimens had torque bands clamped on the ends.

Series II included tests of duplicate specimens for each of the loading conditions in the program. At the conclusion of the tests in this series it became evident that due to normal variations in the performance of plain concrete and due to initial difficulties in the test procedure, some of the tests of series II should be repeated to verify the results. In series II "sequence" loading was used in which a predetermined amount of compression was applied to the specimen before producing failure in torsion. It seemed desirable to investigate the effect of "proportional" loading in which compression and torsion were increased in proportional steps. Therefore, series III included a number of tests, some of which were designed to verify the results of series II, and some to determine the effect of proportional loading on the strength of concrete.

In all, 65 specimens were tested, 6 in series I, 36 in series II and 23 in series III. The specimens were designated by a code intended to identify the series number, the nominal strength of the mix, and the loading conditions. Thus, code II 4.5 TC.4A refers to a specimen in series II, made of 4500 psi concrete mix, subjected to torsion T, combined with compression C of 0.4 of pure compressive strength; A designates the first specimen of two made from a given batch. Specimens subjected to sequence loading were marked A and B - first or second, and specimens subjected to proportional loading were marked P.

B. <u>Materials and Mix Proportions</u>

Type I Portland Cement of Santa Cruz brand was used in this investigation. The cement was purchased in a single lot, blended for uniformity and stored in steel drums. A blend of three sands was used as fine aggregate, the blend having been chosen for its soundness, uniformity, and gradation. Petrographic description of the sands and the results of sieve analyses are shown in Table 1. The sand was air-dried and stored in steel drums. The proportion of each sand was weighed out separately for each batch to insure better uniformity of gradation. Local river gravel was used as coarse aggregate, varying in size from sieve No. 4 to 1/2 inch maximum. Gravel was stored in bins and was air-dried prior to its use. Petrographic description of the mineral content of the coarse gravel and the results of sieve analysis are also shown in Table 2. Mix proportions were selected to give desired strength of concrete, and several trial batch mixes were made before the proportions were finally selected. In preparing specimens for the test series I, it was found that mixes with nominal strength of 4500 and 6000 psi appeared to be too wet, and the mixes were redesigned slightly so that specimens in tests series II and III with nominal strength of 4500 and 6000 psi were made with a mix differing slightly from those in series I. Mix proportions, shown in

Table 3, are based on corrected quantities for saturated surface dry conditions of all aggregates.

C. Fabrication and Curing of Specimens

Two cubic foot batches were mixed in a horizontal non-tilting drum Lancaster mixer. All batching was done by weight, the materials being used in air-dry condition. The dry materials were first blended in dampened mixer for one minute, then water was added and the batch mixed for three minutes, and the slump measured usually about seven minutes after starting the mixing operation.

All tubular specimens were cast in a vertical position in a split metal mold with a wooden core consisting of four wedged pieces held in a rubber tube, figure 1. The mold was mounted on a vibrating table and the concrete was placed in it continuously through a funnel. During placing the concrete was continuously rodded and vibrated on a special table. A high frequency, 8000 to 10000 cycles per second, Viber unit was used on the base of the vibrating table, and the placing was completed in about 4 to 5 minutes. This casting procedure was selected after several trials as one which resulted in the most uniform concrete without any noticeable segregation or excessive voids. Several different casting procedures were tried, the specimens were sawed longitudinally in halves, and the structure of concrete examined visually. With the casting procedure described above no segregation could be observed in the specimen, except in the bottom 1 inch, where due to continuous vibration, some compaction of coarse aggregate was noted.

Two tubular test specimens, six 3 x 6 in. control cylinders, and two 6 x 12 in. control cylinders were made from each batch. Standard steel molds were used for the control specimens and were filled in a standard manner in three layers compacted by rodding and then by an internal vibrator. The tubular specimens and the control cylinders were left in the laboratory covered by wet burlap for 24 hours. After this period the specimens were stripped and placed in the fog-room at 100 per cent humidity and 70°F for about 27 days.

Flexure beams, 6 x 6 x 20 in., were used to determine moduli of rupture of the mixes used. One set of three beams, six 3 x 6 in. and two 6 x 12 in. control cylinders were made for each of the mixes used. These were cast in oiled wooden molds, air cured under wet burlap for 24 hours, and then cured in the fog-room for 27 days prior to testing.

D. Loading Apparatus

The loading apparatus consisted of a 400,000 lb. Baldwin Southwark Universal Testing Machine to apply axial compression, and a special frame assembly to apply torsional loads. The torsion loading frame assembly, shown in Figure 2, consisted of two end-beams held together by four 3/4 inch diameter tie rods, two cross-beams which transmitted the loads to torque bands clamped on the ends of the specimens, a hydraulic jack to apply the load to one of the cross-beams, and a proving ring to measure the applied load accurately. The torque load at each end of the specimen was produced by loading the diagonally opposite cross-beams at midspan and transmitting the end reactions from the cross-beams to the torque bands. These reactions formed opposite couples at each end of the specimen. Details of the frame assembly are shown in Figure 3, a test set-up in the testing machine is shown in Figure 4, and the general experimental set-up is shown in Figure 5. The proving ring load was measured by means of SR-4 gages. Several calibrations of the proving ring showed a constant sensitivity of 2.8 lb. per microinch.

To eliminate frictional restraint to rotation at the ends of the specimen special thrust bearing assemblies were used. Each thrust bearing assembly consisted of a Rollway Precision T-46 thrust bearing mounted on a base with a short centering shaft, and modified by replacing the rollers with 96 chrome alloy steel 3/4 in. diameter bearing balls. The amount of frictional resistance to rotation offered by the thrust bearings was measured in several calibration tests and the calibration curve is shown in Figure 6.

E. Test Procedure

Specimens were scheduled for test at the age of 28 days. In some cases due to unforeseen experimental difficulties testing was delayed from 1 to 4 days as shown in the table below

Age at test, days	No. of specimens
28	39
29	7
30	12
31	1
32	6

On the day scheduled for testing, the specimens were removed from the fog room and kept damp by wet burlap covers. The ends of test specimens and control cylinders were capped with one inch thick steel plates using hydrostone, care being taken to square the ends with the longitudinal axes. In all cases test specimens and control cylinders were tested at the same age.

1. Pure compression: Specimens were tested in a 400 kip Baldwin Southwark Universal Testing Machine using a loading rate of approximately 15 kips per minute. The specimen was placed on the thrust bearing assembly with 11-3/4 in. diameter one inch thick steel plate inserted between the capping plate and the bearing assembly to obtain a fairly uniform distribution of the load on the bearings. The specimen was centered in the machine, and another one inch steel plate and a thrust bearing assembly

was placed on top of the specimen. A spherical block was an integral part of the upper head of the machine.

Torsion bands were positioned by means of a spacer 17-1/2 inches apart, and the bands were aligned in a horizontal plane so that the diagonal cross-beams bearing on the bands would be positioned in planes parallel to the longitudinal axis of the specimen. Four alloy steel bolts, one half inch in diameter, were used to clamp the bands in place. To prevent slipping of the bands the bolts were tightened until the tension in them was approximately equal to yield strength; after several uses the bolts exhibited necking and had to be replaced. The yield load for the steel bolts was determined as 5.6 kips per bolt.

2. Pure torsion: The torsion loading assembly was supported on steel tables placed adjacent to the testing machine, see figure 5. As the loading assembly was not rigidly attached to outside supports, any mis-alignment in position of either the hydraulic jack or the proving ring would result in tipping of the assembly, with the result that ultimate loads could not be developed on first loading. Some difficulty with alignment was experienced when the first few specimens under torsional loads were tested in series II. This was eliminated by providing special alignment strings, and using a plumb-bob, a level, and a combination-square to adjust the orientation of the jack, proving ring, cross-beams, and end-beams. This procedure resulted in satisfactory alignment and no further trouble was experienced. The load was applied gradually by pumping the hydraulic jack manually, the total time of loading to failure being approximately five minutes.

3. Compression combined with torsion: Specimens were placed in the 400 kip Universal Testing Machine and the torsional loading frame was aligned as described above. Specimens were first subjected to a predetermined amount of an axial compressive load -- a portion of nominal compressive

strength of the specimen -- and then subjected to torsion until failure took place. The total time to produce failure was approximately 5 minutes. This type of loading was designated as "sequence" loading, indicating that compression was applied first followed by torsional loads. It was considered desirable to investigate the effect of so-called "proportional" loading, in which compression and torsion would be increased in alternate steps more or less in direct proportion. Four specimens in series III were subjected to this type of loading. The load increments are shown in the table which follows.

Load increments, per cent of load at failure

III 3TC.4P		III 3TC.4P		III 4.5TC.4P		III 4.5TC.8P	
Compr.	Torsion	Compr.	Torsion	Compr.	Torsion	Compr.	Torsion
18	12	18	11	21	14	21	12
37	25	37	22	41	28	41	24
56	38	56	33	62	42	62	36
74	63	74	55	83	70	83	59
100	100	100	100	100	100	100	100

Five increments of loading were selected arbitrarily. Since increase in torsional capacity is not proportional to compressive loads, torsional load increments were made relatively smaller than compressive load increments, in order to prevent failures before the predetermined amount of compression could be developed.

4. <u>Control specimens</u>: For each batch of concrete, six 3 x 6 in. cylinders and two 6 x 12 in. cylinders were prepared. The compressive strength of all the control cylinders was determined in a standard manner. One of the 6 x 12 in. cylinders was used to determine the stress-strain relationship up to about 80 percent of its ultimate load and then the specimen was tested for compressive strength. To correlate the compressive strength of the mixes with the modulus rupture, three 6 x 6 x 20 in. flexure prisms were made for each mix and tested at the age of 28 days

under third-point loading. Results of control specimen tests are shown in Table 4.

F. Measurements and Stress Calculations

Outside and inside diameters of the tubular specimens were determined at several points at each end of the specimen and the average values were calculated for the specimen. The total torque applied to the specimen at failure was determined as the couple formed by the cross-beam reactions to the jack load. The couple's lever arm was 12 inches, so that it was equal to half the jack load times 12. The torque ΔT resisted by friction at each end was assumed to be equal to the slipping torque at a given compression load P, and was determined by calibration. The net torque on the specimen was defined as $T_n = T - \Delta T$.

Compressive and shearing stresses were calculated from conventional equations:

$$\sigma = P/A \quad \text{and} \quad \tau = T_n \, r_o/J \qquad (1)$$

where σ = average compressive stress, and τ = maximum shearing stress at the extreme fibers. Strain measurements made in a previous study indicated that linear shearing stress distribution is valid for pure torsion specimens. It is assumed that linear shearing stress distribution is also valid for the combined loading conditions. Also, the ratio of average shearing stress to the maximum shearing stress is 0.83:1.00, indicating a reasonably uniform stress distribution across the thickness. Therefore, the maximum nominal shearing stress τ was used as an indication of shearing strength of concrete. Calculated values of normal and shearing stresses at failure are shown in Table 5.

G. Modes of Failure

After completion of each test the specimen was photographed, taking two or more views of the failure, so that a complete record of the types of failure was obtained. A study of these records revealed five principal modes of failure, illustrated in Figure 7.

The specimens loaded in pure compression exhibited two modes of failure: one was splitting along vertical planes, indicated by vertical cracks in Figure 7a, and the other was failure along somewhat inclined planes as shown in Figure 7b. No consistent correlation between type of failure and nominal strength of concrete could be noted. In some cases, due to the presence of torque bands, a secondary failure was observed such as a transverse circumferential splitting, accompanied by crushing of concrete and spalling along a somewhat inclined plane within the thickness of the tube.

The specimens loaded in pure torsion failed by splitting along the helicoidal surface making an angle somewhat less than 45^{o} with a longitudinal axis, Figure 7c.

Two types of failure occurred in specimens subjected to combined compression and torsion. One, shown in Figure 7d, consisted of two principal cracks: a steep one inclined approximately 15^{o} to the longitudinal axis, and another making an angle of approximately 45^{o} with that axis. The inclinations of the cracks varied with the relative magnitudes of the compressive load, but no consistent relationship between angle of inclination of the crack and the loading condition or nominal concrete strength could be noted. Furthermore, it was not apparent which of the two cracks formed first and which was a secondary crack. Another prevalent type of failure was that of complete and sudden breakdown along several planes, as shown in Figure 7e. The specimen broke down in numerous pieces and the principal crack causing this type of failure could not be observed. This latter type of failure was particularly prevalent for the 6000 psi concrete under practically all combinations of compression and torsion. It was not limited to the 6000 psi concrete, however, as other concretes with higher proportions of compressive load also exhibited a similar type of failure.

III. DETERMINATION OF A FAILURE CRITERION FOR PLAIN CONCRETE

A. Introduction

In formulating a law of strength under combined states of stress, some agreement must be reached as to what constitutes failure. Criteria such as yielding, initiation of cracking, load carrying capacity, and extent of deformation have been used to define failures. Thus, the type of specimen and type of test have an appreciable influence on the correlation obtained among tests in which the various definitions of failure have been employed. Furthermore, the application of criteria based on test specimens to full size structural elements is made difficult by the necessity of a definition of failure for both test specimen and full size element, which definition need not be the same for each. In this investigation failure is defined as the ultimate load carrying capacity of the test specimen. This seems reasonable in view of the statically determinate nature of the stress distribution up to and including failure. On the other hand, determination of micro-crack formation, yielding, or measurement of limiting strain (difficulty intensified by creep and plastic flow) all seem to be less desirable indicators of failure for concrete. Ideally, in any program defined to establish the effect of state of stress on strength of concrete, the law defining failure should be capable of predicting strengths of different types and sizes of test specimens. This problem involves the nature of loading, boundary conditions at loaded surfaces, stress distribution at critical sections, stress or strain rate, and stress history of specimen including effects of curing and shrinkage. Although these factors per se may not affect the actual application of the failure law in practice when applied to structural elements, they are determinative in explaining and correlating test data taken from various sources.

The nature and structure of the material must be identified. In concrete the physical and mechanical properties of the aggregate and of the cement paste, and the nature of deformations and of volume changes due to temperature or shrinkage are significant factors affecting the behavior of the material. Obviously the mix proportions must also be included.

In this report the effect of size and shape of specimen, effect of manner of loading and application of the failure law to structural elements is discussed elsewhere. With regard to the material properties, the standard compressive strengths of three different mixes are used as parameters to identify a particular concrete mix.

B. Review of Failure Theories

A review of theories of failure applicable to concrete may be found in references (1, 2)*. In addition to the work discussed there the following experimental investigations may be cited.

A series of biaxial compression tests in 15 cm. concrete cubes conducted by Wastlund (3) indicate that the intermediate principal stress is determinative in the failure of the specimen, thus contradicting the Mohr theory for biaxial states of stress in the compression-compression quadrant.

Blakey and Beresford (4) have conducted tests on plain concrete beams, discs, and slabs loaded and supported in such a manner that failure under unaxial tension, equal tension at right angles, and equal compression and tension at right angles could be obtained. For these biaxial states of stress, a failure law based upon distortion energy and volumetric strain was tentatively proposed. However, test results do not agree with the law in the present form in the tension-compression quadrant (1).

C. Invariant Formulation of Failure Laws

A failure criterion based upon state of stress must be an invariant

* Numbers in parentheses indicate references listed at the end of the report.

function of the state of stress, that is, independent of the choice of the coordinate system by which stress is defined. One method of representing such a function is to utilize the principal stresses. Thus, $F(\sigma_1, \sigma_2, \sigma_3,) = 0$ is frequently used to indicate the general functional form of a failure law, the actual determination of the function being left to theoretical conjecture of experimentation. However, unless some restriction is placed on the function, considerable difficulty is experienced in accomplishing such an experimental determination. In the past, to give a few examples, the function has been associated with the normal and shear stresses on the plane of failure neglecting the influence of the intermediate principal stress, or with effective stress and mean stress, or with the total strain energy and mean stress, or with assumed expressions based upon experimental data.

In the general case of a polydimensional state of stress this method of establishing a failure law is particularly difficult to pursue since three independent parameters, the principal stresses, are involved. Furthermore, it is difficult to supply a physical explanation of failure on this basis.

An alternative way of describing a failure law has proved usefu. It is known that any invariant symmetric function of the state stress (e.g. a criterion of failure) can be expressed in terms of the three principal stress invariants. Thus, one can write

$$F\ (I_1,\ I_2,\ I_3) = 0 \qquad (2)$$

where

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3, \qquad I_2 = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1, \text{ and } I_3 = \sigma_1\sigma_2\sigma_3 \qquad (3)$$

In formulating criteria for yielding of ductile materials subjected to combined states of stress it has been found that other invariants of the state of stress, which of course can be formed by proper combinations of the principal invariants, are more susceptible of physical interpretation. An explanation of such invariants which is independent of the properties of a specific material or class of materials is contained in a paper by Novozhilov (5). It seems reasonable, therefore, to utilize these invariants in formulating a law of failure of a brittle material such as concrete.

D. Mean Shearing Stress and Mean Normal Stress

The concept of mean stress at a point can be approached as follows: consider an infinitesimal spherical element of volume. On the surface of this element the state of stress can be expressed in terms of a shearing stress τ_s and normal stress σ_s. The mean value of the shearing stress can be based upon stresses existing on all possible planes of orientation through the point by carrying out the averaging process over the spherical surface. Although shearing stress can be either positive or negative, the sign has no significance with respect to physical mechanism of failure, and it is expedient to take the average in the sense of the root mean. Thus,

$$\tau_a = \mathrm{Lim}_{S \to o} \left[\frac{1}{S} \int_S \tau^2 dS \right]^{1/2} \qquad (4)$$

where S denotes the surface of the spherical element of volume. Carrying out the indicated operations leads to Eq. 5.

$$\tau_a = \frac{1}{\sqrt{15}} \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_1)^2 + (\sigma_3 - \sigma_1)^2 \right]^{1/2} \qquad (5)$$

or in terms of the principal invariants,

$$\tau_a = \sqrt{\frac{2}{15}} \, (I_1^2 - 3I_2)^{1/2} \qquad (6)$$

In the case of the normal stress σ_s on the surface of the spherical element the sign of the stress is significant, i.e., whether the stress is tensile or compressive certainly influences the mechanism of failure. According, the mean normal stress can be defined by Eq. 7.

$$\sigma_a = \mathrm{Lim}_{S \to o} \frac{1}{S} \int_S \sigma_s dS \qquad (7)$$

Evaluation of this expression give

$$\sigma_a = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = \frac{I_1}{3} \tag{8}$$

The third "plasticity" invariant has been shown by Novozhilov to represent the ratio of the mean shearing stress to the maximum shearing stress, which varies over very narrow limits. Therefore, its effect on the formulation of a failure law would be of secondary importance. Consequently, it is possible to write a failure law in the form

$$F(\sigma_a, \tau_a) = 0 \quad \text{or} \quad \tau_a = f(\sigma_a) \tag{9}$$

For an isotropic, elastic, ductile material following Hooke's Law it is well know that experimental evidence discloses an independence of σ_a and τ_a for the condition of yielding: furthermore, τ_a is directly proportional to the energy of distortion. However, for materials such as concrete, it does not appear to be expedient to interpret τ_a in terms of an elastic energy.

It is more suitable to seek a physical interpretation of the parameters σ_a and τ_a that is independent of the properties of a specific material or class of materials.

Finally, the failure criterion in the form $\tau_a = f(\sigma_a)$ accounts for the effect of the intermediate principal stress, σ_2, and thus is a natural generalization of the Mohr theory of failure which neglects its effect, and whose importance for concrete has been shown in the experiments of Wastlund, among others.

The form of the function $f(\sigma_a)$ is left to experiment. In this investigation two trial functions have been utilized, a linear equation and a quadratic equation. Which form is preferable is a question that additional substantiating tests and applications of the theory to structural elements will ultimately have to answer.

E. Test Results

As noted elsewhere most of the specimens tested in this program were subjected to sequence loading. As seen from Table 5 the specimens tested under conditions of proportional loading (designated by P) did not show any significant variation from those tested under the sequence loading. Therefore, no distinction is made between the two types of loading. Mean stresses defined by Eqs. 5 and 8 were calculated and are shown in Table 6.

The experimental data showing the relationship between mean normal and shearing stress-ratios is shown in Fig. 8. Points represent average values for a group of similar specimens and the shaded area represents the scatter of the data. It can be seen that the experimental data can be closely approximated by a quadratic parabola of the form:

$$\left(\frac{\tau_a}{f_c}\right) = A + B\left(\frac{\sigma_a}{f_c}\right) + C\left(\frac{\sigma_a}{f_c}\right)^2 \qquad (10)$$

where $_a$ and f_c are taken positive when compressive. The lower bound of the test data is approximated by a straight line of the form

$$\left(\frac{\tau_a}{f_c}\right) = A + B\left(\frac{\sigma_a}{f_c}\right) \qquad (11)$$

Numerical values of coefficients A, B, and C in Eqs. 10 and 11 were determined empirically from data shown in Fig. 8, and are given in the table below.

	Parabolic Criterion		Straight-Line Criterion	
	6 x 12 in cyl. f'_c	3 x 6 in cyl. f''_c	6 x 12 in cyl. f'_c	3 x 6 in cyl. f''_c
A	0.050	0.045	0.050	0.045
B	1.224	1.274	0.949	0.941
C	0.826	1.160	0	0

The failure criteria defined by Eqs. 10 and 11 can be transformed into equations relating conventional shearing and normal stress-ratios by substituting the following

$$\sigma_a = \frac{\sigma}{3} \quad \text{and} \quad \tau_a = 0.366\left[\sigma^2 + 3\tau^2\right]^{1/2} \qquad (12)$$

The transformation leads to an equation of the form:

$$\left(\frac{\tau}{f_c}\right) = 0.1 \left[A'+B' \left(\frac{\sigma}{f_c}\right)+C'\left(\frac{\sigma}{f_c}\right)^2+D'\left(\frac{\sigma}{f_c}\right)^3+E'\left(\frac{\sigma}{f_c}\right)^4 \right]^{1/2} \quad (13)$$

where the numerical coefficients for the failure criteria in terms of conventional stresses are given in the table below.

	Parabolic Criterion		Straight-Line Criterion	
	6 x 12 in cyl. f'_c	3 x 6 in cyl. f''_c	6 x 12 in cyl. f'_c	3 x 6 in cyl. f''_c
A'	0.622	0.504	0.622	0.504
B'	10.1	9.50	7.86	7.01
C'	5.8	8.63	-8.46	-8.85
D'	-18.6	-27.2	0	0
E'	+ 2.09	4.13	0	0

The test data in terms of conventional stresses σ and τ and the empirical curves are shown in Fig. 9.

Taking into account the scatter of the data it is desirable to select a conservative criterion for predicting failure under various conditions of stress; for the test specimens tested in this investigation a reasonably conservative failure criterion is provided by the "straight line" criterion, which can be expressed in terms of σ, τ, and f'_c as follows:

$$(\tau/f'_c) = 0.1 \left[0.622 + 7.86 \left(\frac{\sigma}{f'_c}\right) -8.46 \left(\frac{\sigma}{f'_c}\right)^2\right]^{1/2} \quad (14)$$

Where σ and τ are normal and shearing stresses at failure, and f'_c is the nominal compressive strength of 6 x 12 in. control cylinders.

For specific states of stress this criterion gives the following values:

Pure compression: $\sigma_c = f'_c$

Pure shear: $\tau_o = 0.079 f'_c$

Maximum shear strength: $\tau_{max} = 0.156 f'_c$ when $\sigma = 0.5 f'_c$ (15)

Assuming that the criterion is valid for the case of pure tension, the tensile strength of concrete is σ_t:

$$\text{Pure tension:} \qquad \sigma_t = 0.076 f'_c \qquad (16)$$

The effect of size, shape of specimen, and other variables will be discussed in Part V of this report. Nevertheless, because Eq. 14 conservatively represents the lower bound of test data, this criterion may be considered applicable, within certain limits, to reinforced or prestressed concrete structural elements. Applicability of this criterion to some concrete structural elements is considered in part IV of this report.

IV SHEARING STRENGTH OF REINFORCED CONCRETE BEAMS

A. Review of Present Knowledge

Excellent reviews of the present knowledge of strength of reinforced concrete beams in shear have been published recently in the Bulletins of the Reinforced Concrete Research Council (6, 7, 8) and of the University of Illinois (9). These reviews contain a historical survey of the problem and extensive data on recent tests carried out primarily at the University of Illinois. The principal factors in the diagonal tension failures brought out in these bulletins are:

(a) diagonal tension cracking, followed by

(b) a readjustment of internal stresses, and

(c) final failure in the compression zone which has been referred to as shear-compression failure.

Several recent American and European developments are not included in the above references and are reviewed below.

In 1951 a memorandum entitled "Some Notes on Shear Strength of Reinforced Concrete Beams" (10) was submitted to the newly formed joint ASCE-ACI Committee on Shear and Diagonal Tension. This memorandum discussed the inadequacy of existing theories on shear failure in reinforced concrete beams, and the need for formulation of a more realistic mechanism of failure in shear in terms of:

(a) strength of concrete under various conditions of combined stress, and

(b) stress distribution in reinforced concrete beams during various stages of loading.

It was proposed in this memorandum that the ultimate shear load V carried by the shear-compression zone of the beam be calculated as

$$V = \tau_i bkd \qquad (17)$$

where τ is the ultimate shearing strength taken as a function of the state of stress, $\tau = f(\sigma)$, b is the width of the rectangular beam, and kd is the depth of the shear-compression zone where failure occurs.

Stress distribution in the cracked zone of reinforced concrete beams with web reinforcement was briefly discussed in this memorandum. This study showed how some shear can be carried by the cracked zone as a consequence of the development of secondary bi-axial compression stresses.

A limited experimental study of strength of concrete subjected to various amounts of torsion and compression was carried out at the University of California in 1952. Further study of the 1952 test results led to the development of a failure criterion in terms of octahedral or mean stresses, published in 1955 (1). In order to apply this failure criterion to practical problems it seemed necessary to verify its validity for different concrete strengths and mixes. This constituted one of the objectives of the study reported here.

In a series of articles published in Genie Civil (11) during 1952-1954, A. Couard presented his ideas on mechanism and criteria of failure in concrete and their applications to the determination of shearing strength of reinforced concrete beams. Couard postulated that a failure criterion in concrete is defined by a modified internal friction relationship.

Consider an element subjected to a plane state of stress, σ_x, σ_y, τ, Fig. 10, which results in failure along a rupture plane defined by angle β. Then, according to Couard:

$$\tau_r = R' + \sigma_r \tan \qquad (18)$$

where σ_r and τ_r are normal and shearing stresses respectively on the rupture plane, R' is intergranular resistance to sliding when

$\sigma_r = 0$, and tan ϕ is the coefficient of internal friction.

Applying his theory to the determination of the shape of the shear crack in a reinforced concrete beam, Mr. Couard assumed the terminal point of the crack at some section, say that of maximum moment for example, and then using stress distribution conventional in reinforced concrete analysis proceeded to calculate the angle of inclination of the rupture plane over a small increment of length. Proceeding backwards from the terminal point of the crack and assuming that the successive segments of the crack follow the rupture planes, the shape of the crack was determined. This method is not entirely rational because the shape of the crack is determined by tracing its shape from an assumed terminus to the initial point of the crack, rather than by tracing the propogation of the crack from its point of initiation to the end point.

A paper entitled "Some Implications of Recent Diagonal Tension Tests" by Professor P. M. Ferguson (12) was published just at the time of writing this report. In this paper Professor Ferguson clarified some aspects of the mechanism of diagonal tension failure described in the Illinois studies. The principal points emphasized by Professor Ferguson are:

(a) An initial diagonal crack usually forms near mid-depth of the beam and extends into both the compression and tension zones.

(b) With increase in load the crack extension into the compression zone is relatively flat.

(c) A general cracking in the zone around the tensile steel may develop simultaneously with (b).

(d) A sudden final failure may occur by shear-compression or by an extension of the flat crack into the compression zone. This is sometimes accompanied by a secondary failure in splitting and bond along the tensile reinforcement. In some cases splitting failure may develop into primary failure.

Professor Ferguson also discussed the formation of an initial diagonal tension crack either in a zone where moment cracks do not exist or as an extension of a moment crack.

One of the most important contributions of this paper is an experimental demonstration of the effect of local state of stress on failure, such as the effects of the manner of loading and of the type of support.

B. Shear Strength - A Rational Approach

1. Simple beams without web reinforcement: Several investigations of the shear strength of reinforced concrete beam without web reinforcement suggested that the total shearing force resisted by the compression zone is $V_u = \tau bkd$, (10, 7, 9). However, all previous attempts to define procedures for evaluating τ were unsuccessful, and usually empirical equations of the form $\frac{M}{bd^2f'_c} = k\,F(f'_c, p)$ were proposed. Based on the results of the tests described in part II of this report a simple rational procedure for evaluating V_u in terms of a variable shearing strength is presented below.

Consider simply supported rectangular beams without web reinforcement, of intermediate span/depth ratio, say 6 to 10. Such beams often fail due to formation of a critical shear crack, Figure 11, followed by shear-compression failure, described by numerous previous investigators (8, 9, 12).

The stresses at a point in the zone of shear-compression failure at loads approaching the ultimate are σ and τ, varying throughout the depth kd of this zone. The stress in the steel reinforcement at the critical crack is f_s. Determination of these stresses cannot be accomplished by the conventional flexure theory and very little work has been done to define these more accurately. Westergaard's work (13) is limited to stresses around a vertical crack due to pure flexure, Wastlund's work (14) dealing with cracking of reinforced concrete beams is not primarily concerned with stress distributions, and Saliger's work (15) deals primarily with bond stresses.

Assuming that the stresses σ, τ, and f_s can be defined in terms of external forces, geometry of cross-section, and material properties, and that the failure criterion $\tau = F(\sigma)$ is known, three hypotheses of shear-compression failure are possible.

(a) It is possible to assume that failure occurs when σ and τ at some critical point satisfy the failure criterion. In this case the criterion $\tau = f(\sigma)$, may be dependent on the nature of stress distribution, and the critical point would be defined for particular stress distribution.

(b) It is also possible to assume that ultimate failure occurs when σ and τ satisfy the failure criterion at all points. In this case, the failure criterion may also be dependent on the nature of stress distribution.

(c) Finally, a simple assumption can be made that failure occurs when the average stresses σ and τ satisfy the failure criterion, where σ and τ are defined as:

$$\sigma = \frac{C}{bkd} \quad \text{and} \quad \tau = \frac{V}{bkd} \tag{19}$$

In this case, if average normal stress σ at loads approaching failure is known, then τ can be determined from failure criterion $\tau = f(\sigma)$, and shearing strength can be determined as $V = \tau bkd$.

The ultimate shearing strength of a simply supported rectangular reinforced concrete beam without web reinforcement can be determined on the basis of the following additional assumptions.

I. Average compressive stress at failure is the same as proposed in the recent experimental study by Hognestad, Hansen, and McHenry (16) where

$$\sigma = \frac{3900 + 0.35 f'_c}{3200 + f'_c} f'_c \qquad (20)$$

This assumption appears to be valid for beams with concrete strength greater than 3000 psi. For low values of f'_c the value of σ approaches f'_c and may even exceed this value, hence Eq. 20 does not appear to be reasonable for f'_c less than 3000 psi.

II. Tensile stress f_s in the longitudinal steel reinforcement at the section of the critical crack is approximately equal to the yield strength f_y. This assumption appears to be valid for beams with moderate amounts of reinforcement made of fairly ductile steel with a well defined yield point, and a large yield strain range. For small amounts of reinforcement the stress f_s may be in strain hardening region, where f_s is greater than f_y. For high percentage of reinforcement, or for steel without well defined yield strength, the stress f_s may be indeterminate, i.e., f_s may be greater or less than f_y.

From calculations described below, the assumption that $f_s = f_y$ appears to be valid in the range of (pf_y) between 600 and 1000, where f_y is taken in psi.

The depth of the compression zone can be determined by equating C=T, as follows:

$$C = \sigma bkd = T = f_s A_s = f_y pbd \qquad (21)$$

Therefore

$$k = \frac{f_y pbd}{\sigma bd} = \frac{pf_y}{\sigma} \tag{22}$$

where σ is given by Eq. 20.

The shear force V at failure is given by Eq. 17, as follows:

$$V = \tau bkd$$

where $\tau = f(\sigma)$ is determined either from Eq. 20 or from Figure 9, and k is defined by Eq. 22.

This procedure was used to calculate values of ultimate shearing strength V_u for 54 beams tested in the recent studies at the University of Illinois. These beams were selected on the basis of span/depth ratio being sufficiently large so that the beams exhibited typical shear-compression failures. The results of these calculations are shown in Tables 7 and 8. It can be seen that the method is valid for beams having concrete strength f'_c greater than 3000 psi., and percent and type of reinforcement characterized by values of (pf_y) in the range of 600 to 1000. Good agreement is obtained for all the 21 beams which meet above limitations. The remaining beams fall outside the range of validity of the method proposed here, either because of amount of reinforcement or low concrete strength. For the 21 beams for which the method of analysis applies, the average ratio of calculated shear strength to that observed in the test is 0.96 and 90 percent of these ratios fall in the range of 0.8 to 1.05.

2. <u>Continuous beams without web reinforcement</u>: The problem of shear failures in relatively long, deep, reinforced concrete rectangular continuous beams without web reinforcement has received a great deal of attention in recent months. Failure of the frame girders at the Wilkins Air Force Depot, Ohio, (17) and the difficulties at other locations involving similar designs caused a great deal of concern about

the minimum requirement of the building codes then in effect. According to calculations failure occurred in the region of the point of inflection at nominal shearing stress $v = \frac{V}{bjd}$ approximately equal to 0.02 f'_c. Laboratory investigations (18) of beams modeled on the girders which failed at the Wilkins Depot showed that a shear failure, similar to that observed in the field, occurred at a nominal shearing stress $v = 0.038\ f'_c$. Furthermore, with superposition of axial tension on the test girder, such as may have been present in the field due to restraint to shrinkage, the girder failed at a shearing stress $v = 0.02\ f'_c$.

This fact caused considerable consternation as the values of shearing stress at failure were considered to be less than normal shearing strength of concrete, estimated as $\tau = 0.08\ f'_c$. This anomaly points out very clearly two fallacies in the present method of design for shearing strength.

One is that in a cracked beam $v = \frac{V}{bjd}$ is not a true measure of the shearing stress, because under conditions approaching failure the area available for shear resistance may be considerably less than bjd. Indeed, bkd would be a much better measure of the shearing area at loads approaching ultimate.

Another anomaly is the neglect of axial forces in estimating ultimate shearing strength. While shearing strength for failure in a shear-compression zone may be as high as 0.2 f'_c, shearing strength for failure in pure shear is only 0.08 f'_c, and with very little axial tension rapidly drops to very low stresses indeed, perhaps as low as 0.01 f'_c or 0.02 f'_c. It is apparent that shearing failures of a continuous beam in the zone of a point of inflection can occur when the shearing stress is of the order of magnitude of 0.08 f'_c. If prior to shear failure flexural or shrinkage cracks occurred on both sides and at some small distance from the point of inflexion, Fig. 12, it becomes clear that the shearing force must be transmitted by the sound concrete zone which may have a depth approximately equal to 0.4 d. In this case the average shear stress would be defined as

$$\tau = \frac{V}{0.4bd} \tag{23}$$

Using Equation 23 and $\tau = 0.08 f'_c$, corresponding to zero axial stress, would readily predict the failures observed in the laboratory tests (18). Allowing for nominal tensile shrinkage stress $\sigma = .008 f'_c$ (17) could reduce the shearing strength to approximately $0.02 f'_c$, explaining the failures at the Wilkins Air Force Depot, and of similar model tests referred to above.

3. Discussion: The proposed method for determining shearing strength of reinforced concrete structural elements is based on two basic considerations:

(a) the ultimate average shearing stress τ is not a constant value characteristic of the material, but must be determined as a function of known average normal stress σ at failure, and

(b) the concrete area A_e effective in resisting these stresses σ and τ, can be determined on the basis of assumed mechanism of failure. Hence:

$$V_c = \Sigma \tau A_e \tag{24}$$

The validity of Eq. 24 and of the method proposed above was demonstrated for simply supported and continuous rectangular reinforced concrete beams without web reinforcement within the following limitations:

(a) Shear-span-depth ratio (a/d) greater than 3.0

(b) Concrete strength f'_c greater than 3000 psi, and

(c) Amount and type of longitudinal reinforcement characterized by a value of (pf_y) ranging from 600 to 1000.

The generality of Eq. 24 and the method proposed above is limited by the following:

(a) Shear carried by longitudinal or special web reinforcement has not been considered.

(b) The magnitude of average normal stress σ at failure has been determined on the basis of a particular series of tests, and may not be generally applicable.

(c) The determination of A_e depends to a great extent on the amount of cracking in the beam due to external loads as well as on effect of shrinkage and temperature. Procedures for determining A_e proposed above are limited to certain types of simply supported and continuous concrete beams with certain restrictions on amount and yield strength of steel reinforcement.

These limitations clearly point to the problems which must be solved before a fully rational procedure can be developed. These are:

(a) Stress distribution in uncracked reinforced concrete structural elements of various shapes, with varying arrangements and types of reinforcement or prestressing.

(b) Criteria for cracking in plain concrete subjected to various states of stress and various stress histories.

(c) Stress distribution around arbitrary cracks in concrete structures, and

(d) Determination of critical cracks which determine the failure mechanism in various types of structural elements under different loading conditions.

Solution of these problems would lead to a rational definition of σ and A_e at loads approaching ultimate, and hence would lead to a rational method for determining the shearing strength of concrete structural elements.

V STATISTICAL EFFECTS IN THE FAILURE OF CONCRETE

Introduction

It is a well known fact that any simple test, such as a compression test, repeated a sufficient number of times with specimens that are nominally identical, shows a scatter of strength about a mean value. According to the fundamental assumption of the statistical theory of failure, this scatter constitutes an intrinsic quality of a material and must be included in any consideration of strength of the material. Thus, strength is not a single figure such as "ultimate strength in compression" but a quantity that must be defined by a statistical distribution function. This strength distribution function, considered simultaneously with the actual stress distribution in a specimen of prescribed material and size, may be used to determine the probability of failure of the specimen. It may be observed that the probability of failure of a test specimen depends upon three factors: size of specimen, stress distribution and state of stress within the specimen, and type of material. The first two factors determine the stressed volume of the specimen while the latter directly affects the parameters of the distribution function itself. The degree of difference between the classical definition of strength and the statistical definition depends to a large degree, then, on the nature of the distribution function. For example, for materials exhibiting a relatively large dispersion of strength, a marked volume effect would be expected, inasmuch as the probability of occurrence of weak elements of material increases with volume. This effect is demonstrated by "size effect" of concrete compression cylinders. Furthermore, stress distribution would be significant insofar as stressed volume is concerned, i.e., the larger the volume subjected

6 x 12 cylinders tested at ages of 7, 28, and 90 days, the ratio of 3 x 6 compressive strength to 6 x 12 compressive strength varied from 1.01 to 1.11; however, no consistent trends were noted. Nominal strength of these cylinders was in the three thousand to four thousand lb. per sq. in. range.

An extensive series of tests reported by Gonnerman (20) demonstrated the dependence of the size effect on such factors as maximum size of aggregate, type of aggregate, gradation of aggregate, mix proportion and consistency, factors which were not separated in the work of Blanks and McNamara. It is significant to note that the size effect was found to be more pronounced for higher strength concrete than for lower strength, (see Table 4, Page 241, loc. cit.)

Experiment related to shape effect

It is known that a change in the height-diameter ratio of cylinders affects the compressive strength. Results for a range of ratios from 0.5 to 4.0 are reported in the same study by Gonnerman. Further shape effects for cylinders, prisms, and cubes were also reported. A necessary condition for the existence of a shape effect is a non-uniform stress distribution. Such distributions are usually difficult to ascertain and are further complicated by the existence of combined stresses at critical sections. For circular cylinders, the relative importance of shape effect is most important for short cylinders as could be expected from the consideration of accompanying non-uniform stress distribution, or in other words, the greater significance of the so-called end effect.

Experiments related to stress distribution

Experiment concerned with the effect of distribution of stress often involve at the same time the effect of size of specimen. For example, flexure tests for varying span depth ratios using various ratios of width to depth, and tests involving center-point loading or third-point loading have been conducted. In such tests both size of specimen and the distribution of stress within a given stressed volume were varied. The recent work of Nielson (21) provides an excellent summary and critical examination of flexure tests on concrete. Among other things, it was pointed out by Nielson that in many instances variation in concrete quality with the size of specimen due to placement and compaction methods can be more significant than the effect of stress distribution or size effect in a statistical sense. In this connection the work of Wright and Garwood (22) should also be noted. It was further noted that the effect of method of curing is closely related to the size of specimen inasmuch as the drying process, and its accompanying surface shrinkage stresses, is dependent upon the size of the specimen and is proportionately more important for smaller specimens. In applying statistical theories in the case of non-uniform stress distribution to assess the influence of size of specimen it must be recognized that insufficient evidence exists at present to conclude definitely what measure of size of specimen is correct. For example, in applying the theory to rupture tests of beams it may be assumed that the entire tensile volume of beams subjected to bending may be considered or that only the surface layer in tension should be considered, in which case depth has no influence on modulus of rupture.

The importance of state of stress

The experience that combined stresses have an effect on a material different from that of uniaxial stress has been a prominent factor in promoting research on the problem of establishing a general criterion of failure. This problem exists, of course, both in classical and statistical formulation of theories of failure. From a statistical viewpoint the problem is to find the distribution function for a generalized function of the state of stress expressing failure. At present no such research has been reported.

Finally, it may be mentioned for material such as concrete the variation in statistical parameters must ultimately be expressible in terms of the mix proportions, water-cement ratio, aggregate gradation, casting and curing conditions, etc. It must be further noted that in testing various sizes cylinders in compression, a number of factors exist which are not operative to the same extent in all sizes. Among these factors are end restraint, axial stress due to drying, state of stress, and existence of tri-axial states of stress at the ends of the cylinder.

While the examples of research work discussed above indicate the significance of so-called statistical effects, they also demonstrate that there is presently an insufficient amount of experimental evidence available to define clearly the manner in which the strength dispersion of materials affects their behavior under load. Consequently, it does not appear feasible in this report to attempt any extensive quantitative analysis of statistical effects. Rather, a brief discussion of a commonly used statistical model with application to the control cylinders in this investigation will be made.

The statistical model

As pointed out by Epstein (23) in a paper on statistical aspects of fracture problems, the essence of the statistical models proposed in the study of failure of materials is the Griffith theory, which accounts for the difference between the calculated and observed strength of material in the existence of weakening flaws in the material. The random distribution of flaws in many real materials suggests immediately statistical formulation of the problem. On the basis of the flaw concept, the strength of a given specimen is determined by the weakest point in the specimen. Such a theory, called the "Weakest Link Theory", forms the basis of most of the research done to date. Of fundamental importance in this theory is the distribution function for the value of the weakest link as a function of size of specimen or alternatively the distribution function for strengths of a series of nominally identical specimens. Determination of the distribution function for a particular material for a given specimen configuration and method of loading has occupied a position of importance in many researches in this field. At the present time, however, whether the distribution function should be Gaussian or whether some other function is more appropriate seems to be a question that still remains unanswered. Because of the relative ease of application, the distribution function suggested by Weibull (24) has found favor with many. This theory has been adequately discussed elsewhere and will only be briefly reviewed here.

The probability of rupture S for a given specimen volume V subjected to an arbitrary stress distribution σ is given by

$$S = 1 - e^{-B}, \text{where} \tag{25}$$

$$B = \int_V f(\sigma)\, dv. \tag{26}$$

The function B = -ln (1-S) is called the risk of rupture. The expression f (σ), the material function, expresses the strength properties of the material and must be determined experimentally. For a uniformly stressed specimen f (σ) is constant and we have $B = V f(\sigma)$. In general σ represents a generalized stress, that is, some function of the state of stress similar to the failure criterion of classical theory. Weibull and others have found that satisfactory agreement between results of tests and theory for uniaxial states of stress (tension or bending) in which case σ is the maximum stress, can be obtained by choosing

$$F(\sigma) = \left[\frac{\sigma - \sigma_1}{\sigma_o}\right]^m \tag{27}$$

where m, σ_1, and σ_o are material constants determined experimentally. Substituting Eq. (27) into Eq. (26) gives

$$B = \int_V \left(\frac{\sigma - \sigma_1}{\sigma_o}\right)^m dv \tag{28}$$

It is clear from the above equation that the risk of rupture for a given material is a function of the stressed volume of the specimen as well as the spatial distribution of stresses within. For specimens of different volumes subjected to the same state of stress, the relative strengths can be computed by equating the risks of rupture. Thus,

$$\frac{(\sigma - \sigma_1)_1}{(\sigma - \sigma_1)_2} = \left(\frac{V_2}{V_1}\right)^{\frac{1}{m}} \tag{29}$$

where subscripts 1 and 2 designate the two sizes of test specimen.

In the case of a test series with a large number of specimens the value of σ_1 approaches a lower limiting value. For a homogeneous material of relatively small strength dispersion, $m \rightarrow \infty$ and Eq. (29) leads to

$$(\sigma - \sigma_1)_1 = (\sigma - \sigma_1)_2$$

in which case the size effect disappears.

Application to control cylinders

In evaluating the material constants, Weibull has suggested the following procedure: for a uniformly stressed specimen we have from

Due to the limited amount of test data (in a statistical sense) the reliability of the constants σ_1 and m for the different mixes must be questioned. For example, the constant m, which is a measure of the dispersion of strength about a mean, is independent of the type and size of specimen, depending only upon material properties. The indicated difference in the test values of m for the two cylinder sizes may be due to differences in the concrete as a result of curing or in quality of concrete as a result of placement in the molds. No definite conclusion can be made at present.

The relationship between the average values of strength of 3 x 6 and 6 x 12 control cylinders for the three different mixes used in this investigation is shown in Figure 13. It is seen that for nominal concrete strength of 3,000 lbs. per square inch, the ratio of 3 x 6 to 6 x 12 strength is 1.07 for the 4,500 lbs. per square inch, this ratio is 1.11 and for the 6 x 12 inch cylinder, this ratio is 1.20.

The ratios for 3000 and 4500 psi concrete fall within the range of values reported by Blanks and McNamara (19). The value for 6,000 psi concrete is higher than previously reported values. However, it should be noted that prior work has been limited to lower strength concrete. Furthermore, the general trend of increase in the ratio with strength is consistent with earlier results obtained by Gonnerman (20).

Quantitative prediction of size effect for the control cylinders used in this investigation using Weibull's method is not satisfactory. For example, using the strength of 6 x 12 cylinders as a base, the strengths of the 3 x 6 cylinders can be computed from Eqn. (29). Designating the 3 x 6 cylinder by 1 and 6 x 12 cylinders by 2 we have

$$\frac{(\sigma - \sigma_1)_1}{(\sigma - \sigma_1)_2} = 8^{\frac{1}{m}} \tag{32}$$

Based upon the values of m obtained from 6 x 12 cylinders and values of σ_1 shown in the previous table, predicted strengths of 3 x 6 cylinders were obtained using Eqn. (32). Results are shown in the table below.

Nominal Mix Strength	Expt. Strength, psi	Predicted Strength, psi
3000	3120	3840
4500	4820	5270
6000	6240	6250

Comparison of Experimental and Predicted Strengths of 3 x 6 Cylinders based on test results from 6 x 12 Cylinders

Lack of agreement between experimental and predicted values of strength is attributable to several sources. As previously mentioned the values of m and σ_1 obtained are not sufficiently reliable to justify quantitive comparisons of data. In addition, it is possible that effects other than volume of specimen are incorporated in the tests. For example, the size effect due to curing and the differences in states of stress due to end restraint are not included, although they may be significant. No methods of including these effects are available at present.

Discussion

1. Size Effect: Experiments related to the determination of size of cylinder on compressive strength as yet have not conclusively separated the importance of several factors: end restraint (stress distribution effect) and volume of specimen (number of weakest links). In addition to these, the effect of curing conditions and age of test

must be considered. It seems to be the consensus that cylinder diameter should be at least 3 to 4 times as large as maximum aggregate size to eliminate aggregate size effect. From a statistical viewpoint the dependence of the material parameters (such as m, σ_1 , and σ_0 in Weibull's method) on the mix proportions has not been established. Generally, the averaging processes used in most prior investigations have tended to suppress important effects noted above.

2. Stress distribution and states of stress effects: Ideally, tests should be conducted on the same size and type of specimen to eliminate volume and shape effects, but this often poses severe experimental difficulties. For uniaxial states of stress such as flexure, a number of tests have been performed but results are not conclusive. The important matter of combined stresses has not received any attention from a statistical point of view. The consideration of polydimensional stress by Weibull (25) is not applicable to concrete in view of the importance of compressive stress in failure under combined stresses as demonstrated by classical theory of failure. Additional research is definitely needed in this area.

Relatively little research has been done for concrete of high strength, a material which is increasingly more important. It seems possible that the behavior will be somewhat different due to the increasing importance of the role of the aggregate in the behavior. Some tentative evidence is shown by the compressive tests in this investigation, in which case the relative strengths of 6,000 psi cylinders were much larger than the 3,000 psi cylinders.

3. Application: Research has largely been limited to laboratory test specimens subjected to uniaxial states of stress, such as tension, compression, and flexure. Relatively little attention has

been focused on the extrapolation of specimen results to full-sized structural elements. In the latter case, it seems obvious that the definition of failure needs attention when applied to full-sized elements, inasmuch as the weakest link theory predicts unreasonably low allowable stresses for large members unless a suitable modification is made. In view of the difficulty in developing a statistical theory of design of members it appears more feasible to concentrate on the correlation of the various test methods for laboratory controlled specimens in an effort to obtain a basis for comparison and correlation of results independent of the unique features of a particular test.

VI SUMMARY

The objectives of this investigation were to establish a criterion for failure in plain concrete subjected to shear-compression state of stress and to apply this criterion to the determination of shearing strength of structural elements. Sixty-five tubular specimens were tested to failure under various combinations of shearing and compressive stresses. The major variables were: three nominal compressive strengths of concrete, and five states of stress, ranging from pure shear to pure compression.

It was found that shearing strength was a function of the applied compressive stress varying from 0.08 with zero compression, and 0.2 f'_c with compressive stress of about 0.5 to 0.6 f'_c.

Formulation of a failure criterion in terms of mean stresses has an advantage in that it accounts for the intermediate principal stress, and is independent of the orientation of the reference planes used in calculating the stresses. The mean normal and shearing stresses at failure showed a definite correlation, and for the range of data in this investigation, showed an almost linear increase in mean shearing stress with increase of mean compressive stress.

The failure criterion obtained from the test program was applied to the determination of shearing strength of rectangular reinforced concrete beams without web reinforcement. Excellent correlation was obtained between calculated and observed shearing strength of a limited group of simply supported beams for which the mechanism of failure has been defined by other investigators. Also, excellent correlation was obtained between calculated and observed shearing strength of continuous reinforced concrete beams without web reinforcement, similar to the type in which recent failures have been

reported.

The relationship between shearing strength and normal stress demonstrated in this investigation has two important implications for design of reinforced concrete structures.

Shearing strength depends on the normal stress developed on the cross sectional area resisting the shear. A value of shearing strength which is defined independently of the normal stress is either exceedingly conservative for sections subjected to compression, or unsafe for sections with tensile normal stress, such as at inflection points, with tension induced by shrinkage, restraint to contraction, or settlement.

Evaluation of shearing stresses and normal stresses must be made on the basis of a realistic mechanism of failure, and the areas effective in resistive shearing and normal stresses must be defined rationally. Thus, the conventional definition of nominal shearing stress $\tau = V/bjd$ is quite unsatisfactory for evaluation of ultimate shearing strength of a beam.

A method for determining ultimate shearing strength V_u of reinforced concrete structural elements is proposed as follows:

$$V_u = \tau A_e$$

where effective area A_e depends on the mechanism of failure of the structural element, and average shearing strength τ depends on the average normal stress σ acting on the effective area A_e.

The method outlined above, and the particular definitions of τ, σ, and A_e, show satisfactory agreement with tests and field observations for a limited group of reinforced concrete beams without web reinforcement. Generalization of the proposed method and its application to various type of structural elements with varying types of reinforcement or prestressing, and to members subjected to various loading conditions are outside the scope of the investigation reported here.

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TABLE 1A MINERAL COMPOSITIONS OF SANDS

Mineral	Per Cent		
	Antioch	Monterey #8	Monterey #20
Quartz	79.5	87.0	81.4
Green Horneblende	7.9		5.9
Feldspar	5.4	10.8	4.9
Magnetite	2.6	1.6	1.9
Miscellaneous	4.6	0.6	5.9

TABLE 1B SIEVE ANALYSIS OF SANDS

Sieve Size	Per Cent Retained			
	Antioch	Monterey #8	Monterey #20	Blend*
No. 4				
8			2.2	1.0
16			87.5	41.2
30	12.5	6.3	99.5	51.1
50	35.9	75.3	99.9	81.0
100	69.4	99.7	100.0	95.3
Fineness Modulus	1.18	1.81	3.89	2.70

*Blend: 15 per cent Antioch; 38 per cent Monterey #20; 47 per cent Monterey #8

TABLE 2A MINERAL COMPOSITION OF GRAVEL

Mineral	Fair Oaks Gravel Per Cent
Basalt, Andesite, Dacite	43
Quartz	35
Schist (Amphibolite & Quartz Mica)	9
Horneblende	9
Chert	4
	100

TABLE 2B SIEVE ANALYSIS OF GRAVEL

Sieve Size	Per Cent Retained
1/2 in.	0
3/8 in.	24.4
No. 4	99.4
No. 8	100
Fineness Modulus	6.24

TABLE 3 MIX PROPORTIONS

Series	I,II,III	I	II,III	I	II,III
Nominal Compressive Strength psi	3000	4500	4500	6000	6000
Water/Cement Ratio, by Weight	0.69	0.51	0.52	0.42	0.41
Water/Cement Ratio, gal/sack	7.8	5.8	5.9	4.7	4.6
Sand/Cement Ratio, by Weight	4.03	2.84	2.92	1.98	1.98
Gravel/Cement Ratio, by Weight	4.36	3.46	3.57	2.72	2.73
Cement Content, Sacks/ cu. yd.	4.26	5.58	5.58	7.22	7.22

TABLE 4A STRENGTH OF CONTROL SPECIMENS

3000 psi concrete

Batch No.	3 x 6 in. cylinders: Ultimate load, kips			3 x 6 in. cylinders: Ave. stress f''_c ksi	6 x 12 in. cylinders: Ult. Load kips	6 x 12 in. cylinders: Ave. stress f'_c ksi
I 3C	22.1	21.7	22.0	3.14	85.9	2.91
	23.3	21.2	22.7		83.0	
II 3C	19.8	20.6	20.8	2.89	86.8	3.02
	20.2	20.7	20.6		84.6	
II 3T	23.4	21.4	22.6	3.27	84.5	3.07
	24.0	22.7	24.6		90.6	
III 3T	24.2	25.4	23.5	3.44	84.9	2.96
	23.6	24.7	24.5		83.6	
II 3TC.2	22.1	-	21.7	3.13	83.7	2.99
	22.1	22.2	22.6		86.6	
III 3TC.2	19.8	18.4	21.3	2.83	81.0	2.82
	19.8	20.0	20.7		79.8	
II 3TC.4	22.1	22.5	22.3	3.13	76.8	2.84
	22.4	22.1	21.3		84.9	
III 3TC.4	23.1	22.8	23.2	3.22	87.0	3.06
	22.6	22.3	22.8		87.0	
II 3TC.6	21.3	20.1	19.9	2.94	78.2	2.74
	20.5	22.3	20.5		78.4	
III 3TC.6	22.8	22.3	21.7	3.13	83.7	2.95
	21.5	22.2	22.5		84.2	
II 3TC.8	21.5	22.1	21.8	3.07	85.1	2.90
	21.6	21.4	21.8		80.3	
III 3TC.8	23.1	22.8	23.2	3.22	87.0	3.06
	22.6	22.3	22.8		87.0	
Average				3.12		2.94

6 x 6 x 20 in. Beams

Beam	Modulus of rupture, ksi
B3FA	0.438
B3FB	0.432
B3FC	0.431
Average	0.434

$$\text{Modulus of rupture} = \frac{0.434}{2.94} = 0.148$$

TABLE 4B STRENGTH OF CONTROL SPECIMENS

4500 psi concrete

Batch No.	3 x 6 in. cylinders Ultimate load, kips			3 x 6 in. cylinders Ave. stress f''_c ksi	6 x 12 in. cylinders Ult. Load kips	6 x 12 in. cylinders Ave. stress f'_c ksi
I 4.5C	34.9 34.4	33.5 34.4	34.3 34.7	4.85	119.8 115.2	4.07
II 4.5C	36.3 37.0	36.6 36.2	35.7 34.0	5.08	131.0 133.8	4.61
II 4.5T	35.1 35.7	35.5 36.2	35.1 35.7	5.03	124.5 124.4	4.35
III 4.5T	34.0 34.3	34.3 35.4	35.3 35.0	4.91	128.8 127.5	4.51
II 4.5TC.2	33.9 34.6	35.4 33.6	33.2 35.0	4.85	117.7 126.2	4.25
II 4.5TC.4	34.3 34.4	35.1 35.1	35.6 34.6	4.23	123.8 122.2	4.31
III 4.5TC.4	32.2 33.0	32.5 32.7	32.6 33.0	4.62	125.6 122.7	4.36
III 4.5TC.4	34.5 34.9	33.7 35.5	33.5 35.9	4.90	122.2 124.0	4.33
II 4.5TC.6	32.4 32.5	32.0 31.8	33.2 32.2	4.58	116.6 117.4	4.10
III 4.5TC.6	35.6 34.7	35.3 35.1	35.8 35.0	4.99	125.6 128.0	4.45
II 4.5TC.8	34.9 35.8	35.9 34.8	35.1 34.9	4.98	117.0 119.8	4.15
III 4.5TC.8	35.6 34.7	35.3 35.1	35.8 35.0	4.99	125.6 128.0	4.45
III 4.5TC.8P	32.2 33.0	32.5 32.7	32.6 33.0	4.62	125.6 122.7	4.36
Average				4.82		4.33

6 x 6 x 20 in. Beams

Beam	Modulus of rupture, ksi
B4.5FA	0.650
B4.5FB	0.702
B4.5FC	0.619
Average	0.660

$$\text{Modulus of rupture} = \frac{0.660}{4.33} = 0.152$$

TABLE 4C STRENGTH OF CONTROL SPECIMENS

6000 psi concrete

Batch No.	3 x 6 in. cylinders Ultimate load, kips			Ave.stress f''_c ksi	6 x 12 in. cylinders Ult.load kips	Ave.stress f'_c ksi
I 6C	46.1 43.3	44.4 46.9	47.8 48.0	6.52	152.7 149.2	5.28
II 6C	48.0 48.2	46.0 43.0	46.3 46.6	6.56	166.3 165.3	5.77
III 6C	42.5 44.3	41.9 44.5	43.1 43.6	6.13	151.7 150.8	5.30
II 6T	46.7 45.2	45.0 45.8	46.5 46.5	6.50	149.5 157.3	5.38
III 6T	44.5 44.7	42.9 44.8	45.3 45.7	6.31	140.0 148.7	5.06
II 6TC.2	44.2 44.9	44.3 44.4	46.4 45.3	6.35	142.6 144.0	5.02
II 6TC.4	42.9 43.8	42.9 44.7	43.9 45.2	6.21	138.5 143.8	4.96
III 6TC.4	39.6 41.1	42.3 42.1	43.4 43.5	5.94	147.9 150.4	5.23
II 6TC.6	42.4 43.0	40.3 43.5	42.7 41.6	5.98	140.9 146.3	5.04
III 6TC.6	43.9 42.0	43.4 42.4	41.7 41.1	6.01	140.0 135.7	4.84
II 6TC.8	44.1 44.7	44.8 45.4	44.9 45.2	6.34	155.9 152.8	5.42
III 6TC.8	43.9 42.0	43.9 42.4	41.7 41.1	6.01	140.0 135.7	4.84
Average				6.24		5.18

6 x 6 x 20 in. Beams

Beam	Modulus of rupture, ksi
B6FA	0.792
B6FB	0.748
B6FC	0.743
Average	0.761

$$\text{Modulus of rupture} = \frac{0.761}{5.18} = 0.147$$

TABLE 5 CONVENTIONAL COMPRESSIVE AND SHEARING STRESS AT FAILURE

Specimen		σ ksi	τ ksi	Specimen		σ ksi	τ ksi	Specimen		σ ksi	τ ksi
I 3C	A	2.95		I 4.5C	A	4.24		I 6C	A	5.17	
	B	2.79			B	4.45			B	4.43	
II 3C	A	3.03		II 4.5C	A	4.08		II 6C	A	5.57	
	B	3.00			B	4.32			B	6.21	
								III 6C	A	5.98	
									B	5.62	
II 3T	A		0.20	II 4.5T	A		0.21	II 6T	A		0.34
	B		0.20		B		0.28		B		0.27
III 3T	A		0.27	III 4.5T	A		0.40	III 6T	A		0.46
	B		0.24		B		0.34		B		0.46
II 3TC.2	A	0.61	0.44	II 4.5TC.2	A	0.83	0.43	II 6TC.2	A	1.18	0.85
	B	0.61	0.43		B	0.83	0.49		B	1.18	0.79
III 3TC.2	A	0.61	0.44								
	B	0.61	0.56								
II 3TC.4	A	1.22	0.66	II 4.5TC.4	A			II 6TC.4*	A	1.66	0.86
	B	1.22	0.66		B	1.66	0.92		B	1.66	0.87
III 3TC.4	P	1.22	0.66	III 4.5TC.4	P	1.67	0.83	III 6TC.4	A	2.35	1.08
					A	1.66	0.99		B	2.35	1.20
II 3TC.6	A	1.83	0.57	II 4.5TC.6	A	2.49	0.92	II 6TC.6	A	3.50	0.88
	B	1.84	0.74		B	2.51	1.15		B	3.52	1.08
III 3TC.6	A	1.83	0.68	III 4.5TC.6	A	2.49	1.07	III 6TC.6	A	3.54	1.17
	B	1.84	0.83								
II 3TC.8	A	2.46	0.66	II 4.5TC.8	A	3.37	0.80	II 6TC.8	A	4.74	0.80
	B	2.44	0.72		B	3.34	1.02		B	4.70	1.00
III 3TC.8	P	2.44	0.79	III 4.5TC.8	A	3.35	1.13	III 6TC.8	B	4.71	1.07
					P	3.32	0.94				

* Normal stress = 0.35 x nominal compressive strength

TABLE 6 MEAN COMPRESSIVE AND SHEARING STRESS AT FAILURE

		σ ksi	τ ksi			σ ksi	τ ksi			σ ksi	τ ksi
I 3C	A	0.98	1.04	I 4.5C	A	1.41	1.56	I 6C	A	1.72	1.90
	B	0.93	1.03		B	1.48	1.64		B	1.48	1.63
II 3C	A	1.01	1.12	II 4.5C	A	1.36	1.50	II 6C	A	1.86	2.05
	B	1.00	1.10		B	1.44	1.59		B	2.07	2.29
								III 6C	A	1.99	2.20
									B	1.87	2.07
II 3T	A		0.13	II 4.5T	A		0.14	II 6T	A		0.22
	B		0.13		B		0.17		B		0.17
III 3T	A		0.17	III 4.5T	A		0.26	III 6T	A		0.29
	B		0.15		B		0.22		B		0.30
II 3TC.2	A	0.20	0.36	II 4.5TC.2	A	0.28	0.41	II 6TC.2	A	0.39	0.67
	B	0.20	0.36		B	0.28	0.44		B	0.39	0.64
III 3TC.2	A	0.20	0.36								
	B	0.20	0.42								
II 3TC.4	A	0.40	0.62	II 4.5TC.4				II 6TC.4	A	0.55	0.83
	B	0.40	0.62		B	0.55	0.85		B	0.55	0.83
III 3TC.4	P	0.40	0.62	III 4.5TC.4	P	0.56	0.82	III 6TC.4	A	0.78	1.10
					A	0.55	0.88		B	0.78	1.16
II 3TC.6	A	0.61	0.76	II 4.5TC.6	A	0.83	1.09	II 6TC.6	A	1.17	1.41
	B	0.61	0.83		B	0.84	1.17		B	1.17	1.47
III 3TC.6	A	0.61	0.80	III 4.5TC.6	A	0.83	1.14	III 6TC.6	A	1.18	1.49
	B	0.61	0.86								
II 3TC.8	A	0.82	1.00	II 4.5TC.8	A	1.12	1.34	II 6TC.8	A	1.58	1.82
	B	0.81	1.01		B	1.11	1.39		B	1.57	1.86
III 3TC.8	P	0.81	1.03	III 4.5TC.8	A	1.12	1.42	III 6TC.8	B	1.57	1.86
					P	1.11	1.36				

TABLE 7A SELECTED BEAM TEST DATA

Beam	f'_c psi	f_y ksi	p	d in	a/d	b in	Mode of Failure	V_T kips	pf_y ksi	σ/f'_c	k	τ/f'_c	V_c kips	V_c/V_T
TESTS BY GASTON, 1952, REF. 9 Loads at third points Age at test about 28 days														
T2Ma	4320	47.7	.0138	10.6	3.4	6	S	9.25	.658	0.72	.212	.139	8.0	0.87
T2Mb	4020	48.3	.0138	10.6	3.4	"	"	9.75	.666	0.74	.224	.135	7.7	0.79
T2Mc	4470	46.8	.0190	10.6	3.4	"	"	12.50	.889	0.71	.280	.140	11.1	0.89
TESTS BY LAUPA, 1953, REF. 9 Center load through 6 x 12 in. column stub Age at test about 28 days														
S-2	3900	41.2	.0208	10.6	4.54	6	S	9.5	.857	0.74	.297	.135	9.9	1.04
S-3	4690	59.4	.0252	10.4	4.60	"	"	11.9	1.497	0.70	.458	.143	19.2	1.61
S-4	4470	44.8	.0321	10.4	4.63	"	"	12.5	1.438	0.71	.456	.141	17.9	1.43
S-5	4330	45.7	.0411	10.3	4.66	"	"	11.2	1.878	0.72	.604	.140	22.7	2.03
S-11	2140	47.5	.0190	10.5	4.57	"	"	7.6	0.903	0.88	.481	.093	6.1	0.80
S-13	3800	44.1	.0411	10.3	4.66	"	"	11.2	1.813	0.75	.638	.133	20.0	1.79
S-1	3940	44.6	.0146	10.7	4.79	"	T-S	8.4	0.651	0.74	.224	.135	7.6	0.91
S-9	2140	44.3	.0093	10.7	4.48	"	"	5.8	0.411	0.88	.220	.093	2.8	0.49
S-10	2280	44.8	.0139	10.6	4.54	"	"	7.7	0.623	0.86	.317	.092	4.2	0.55

* S - Shear-compression failure, T-S - Flexure-shear failure

TABLE 7B SELECTED BEAM TEST DATA

Beam	f'_c psi	f_y ksi	p	d in	a/d	b in	Mode of Failure	V_T kips	pf_y ksi	σ/f'_c	k	τ/f'_c	V_c kips	V_c/V_T
TESTS BY MOODY, SERIES A, 1953, REF. 7 Center load at midspan Age at test about 28 days														
A 1	4400	45.0	.0217	10.3	3.06	7	S	13.5	.976	.71	.312	.140	13.9	1.03
2	4500	45.0	.0215	10.5	3.00	7	S	15.0	.967	.71	.304	.140	14.1	.94
3	4500	45.0	.0222	10.6	2.99	7	S	17.0	.99	.71	.314	.140	14.6	.86
4	4570	45.0	.0237	10.6	2.96	7	S	16.0	1.08	.70	.332	.141	15.9	.99
B 1	3060	45.0	.0162	10.5	3.00	7	S	12.6	.72	.80	.298	.120	8.1	.64
2	3125	45.0	.0163	10.6	2.99	7	S	13.5	.72	.79	.296	.120	8.2	.61
3	2785	45.0	.0160	10.6	2.96	7	S	12.5	.72	.82	.317	.113	7.4	.59
4	2430	45.0	.0166	10.7	2.95	7	S	12.5	.72	.85	.362	.102	6.7	.54
C 1	920	45.0	.0081	10.6	2.99	7	S	4.5	.365	1.14	.348	-	-	-
2	880	45.0	.0083	10.7	2.94	7	S	5.5	.373	1.20	.354	-	-	-
3	1000	45.0	.0080	10.7	2.93	7	S	5.7	.360	1.00	.360	-	-	-
4	980	45.0	.0082	10.8	2.92	7	S	5.6	.369	1.05	.359	-	-	-

TABLE 7C SELECTED BEAM TEST DATA

TESTS BY MOODY, SERIES B, 1953, REF. 7
Loads at third points,
Age at test about 28 days

Beam	f'_c psi	f_y ksi	p	d in	a/d	b in	Mode of Failure	V_T kips	pf_y ksi	σ/f'_c	k	τ/f'_c	V_c kips	V_c/V_T
1	5320	45	.019	10.6	36	6	S	13.0	.855	.67	.240	.147	12.3	0.95
2	2420	"	"	"	"	"	"	8.0	"	.85	.417	.103	6.6	0.82
3	3730	"	"	"	"	"	"	11.7	"	.75	.305	.132	9.5	0.81
4	2230	"	"	"	"	"	"	9.9	"	.87	.442	.095	5.9	0.60
5	4450	"	"	"	"	"	"	11.7	"	.71	.271	.140	10.7	0.92
6	2290	"	"	"	"	"	"	7.9	"	.86	.433	.098	6.1	0.88
7	4480	"	"	"	"	"	"	11.5	"	.71	.269	.141	10.7	0.93
8	1770	"	"	"	"	"	"	7.0	"	.92	.528	.075	2.9	0.42
9	5970	"	"	"	"	"	"	12.0	"	.66	.219	.150	12.4	1.04
10	3470	"	"	"	"	"	"	11.0	"	.77	.321	.128	9.3	0.84
11	5530	"	"	"	"	"	"	13.5	"	.66	.234	.149	12.2	0.91
12	2925	"	"	"	"	"	"	10.6	"	.81	.363	.116	7.8	0.74
13	5480	"	"	"	"	"	"	12.5	"	.66	.235	.149	12.2	0.98
14	3270	"	"	"	"	"	"	9.7	"	.78	.334	.123	8.5	0.88
15	5420	"	"	"	"	"	"	11.5	"	.67	.238	.148	12.1	1.05
16	2370	"	"	"	"	"	"	8.5	"	.85	.422	.100	6.3	0.75

TABLE 7D SELECTED BEAM TEST DATA

Beam	f'_c	f_y ksi	p	d in	a/d	b in	Mode of Failure	V_T kips	pf_y ksi	σ/f'_c	k	τ/f'_c	V_c kips	V_c/V_T
TESTS BY VIEST-MORROW, 1955, REF. 8 Center load through 12 x 14 in. column stub Age at test about 28 days														
B40B4	5040	54.8	.0185	14.5	2.76	12	S	35.0	1.38	0.68	0.296	.145	37.6	1.07
B56B2	2130	68.3	"	"	3.86	"	"	22.5	1.26	0.88	0.671	.090	22.6	1.01
E2	2130	67.0	.0058	"	"	"	"	17.9	0.39	0.88	0.207	.090	6.9	0.39
A4	3620	47.8	.0241	14.8	3.80	"	"	31.0	1.15	0.76	0.419	.130	35.0	1.13
B4	3950	63.9	.0185	14.5	3.86	"	"	27.5	1.18	0.74	0.404	.135	37.6	1.37
E4	4120	62.2	.0124	"	"	"	"	24.5	0.77	0.73	0.256	.136	24.9	1.02
A6	5780	63.6	.0379	14.0	4.00	"	"	40.0	2.41	0.66	0.632	.148	40.8	2.27
B6	6630	67.6	.0183	14.6	3.82	"	"	30.7	1.24	0.63	0.285	.150	49.8	1.62
B70B2	2370	67.0	.0186	14.4	4.87	"	"	20.0	1.25	0.86	0.611	.098	24.6	1.23
A4	3950	63.2	.0246	14.5	4.83	"	"	29.8	1.55	0.74	0.532	.134	49.1	1.65
A6	6520	63.1	.0383	14.0	5.00	"	"	40.0	2.41	0.63	0.567	.150	92.8	2.32
B84B4	3950	67.4	.0188	14.3	5.86	"	"	25.0	1.27	0.74	0.433	.134	39.4	1.58
B113B4	4730	68.0	.0186	14.4	7.86	"	"	23.5	1.26	0.70	0.382	.141	44.0	1.87
B4R	4160	50.0	.0186	14.5	7.79	"	F	19.0	0.93	0.73	0.307	.136	30.2	1.59

TABLE 8 SELECTED BEAM TEST DATA

Criteria: (1) $1.00 > pf_y > 0.60$
(2) $f'_c > 3000$ psi

Beam	Ref. Table	pfy ksi	f'_c psi	V_c/V_T	Beam	Ref. Table	pfy ksi	f'_c psi	V_c/V_T
S-1	7A	0.65	3940	0.90	1	7C	0.86	5320	0.95
T2Ma	"	0.66	4320	0.87	15	"	"	5420	1.05
T2Mb	"	0.67	4020	0.79	13	"	"	5480	0.98
B-1	7B	0.72	3060	0.64	11	"	"	5530	0.91
B-2	"	0.72	3125	0.61	7	"	"	4480	0.93
E-4	7D	0.77	4120	1.02	9	"	"	5970	1.04
14	7C	0.86	3270	0.88	T2Mc	7A	0.89	4470	0.89
10	"	"	3470	0.84	B113 B4R	7D	0.93	4160	1.59
3	"	"	3730	0.81	A2	7B	0.97	4500	0.94
S-2	7A	"	3900	1.04	A1	7B	0.98	4400	1.02
5	7C	"	4450	0.92	A3	7B	0.99	4500	0.86

Total number of beams = 22

(V_c/V_T)	Lowest value	0.61
(V_c/V_T)	Highest value	1.59
(V_c/V_T)	Average value	0.94

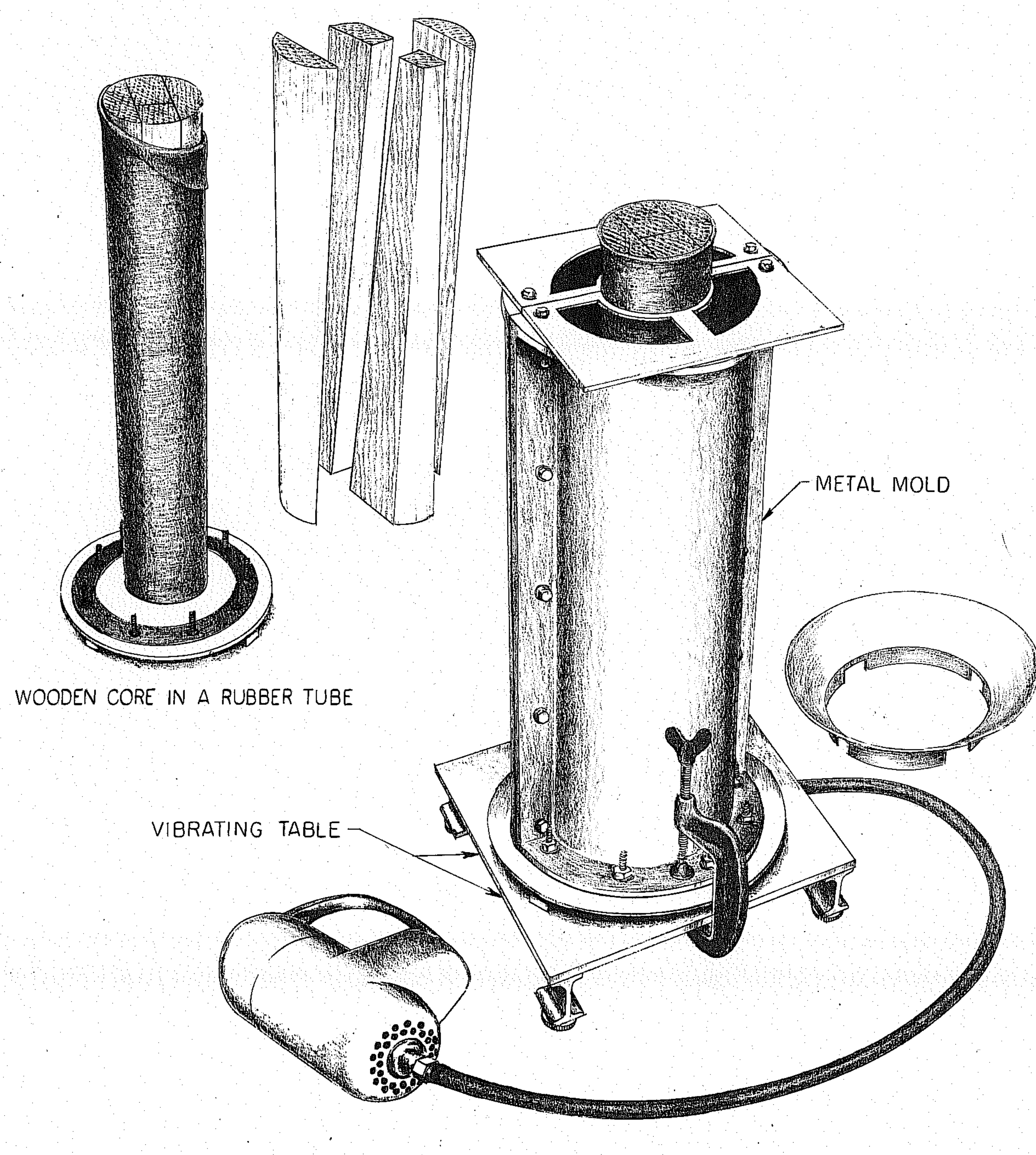


FIG. I SPECIMEN CASTING SET-UP

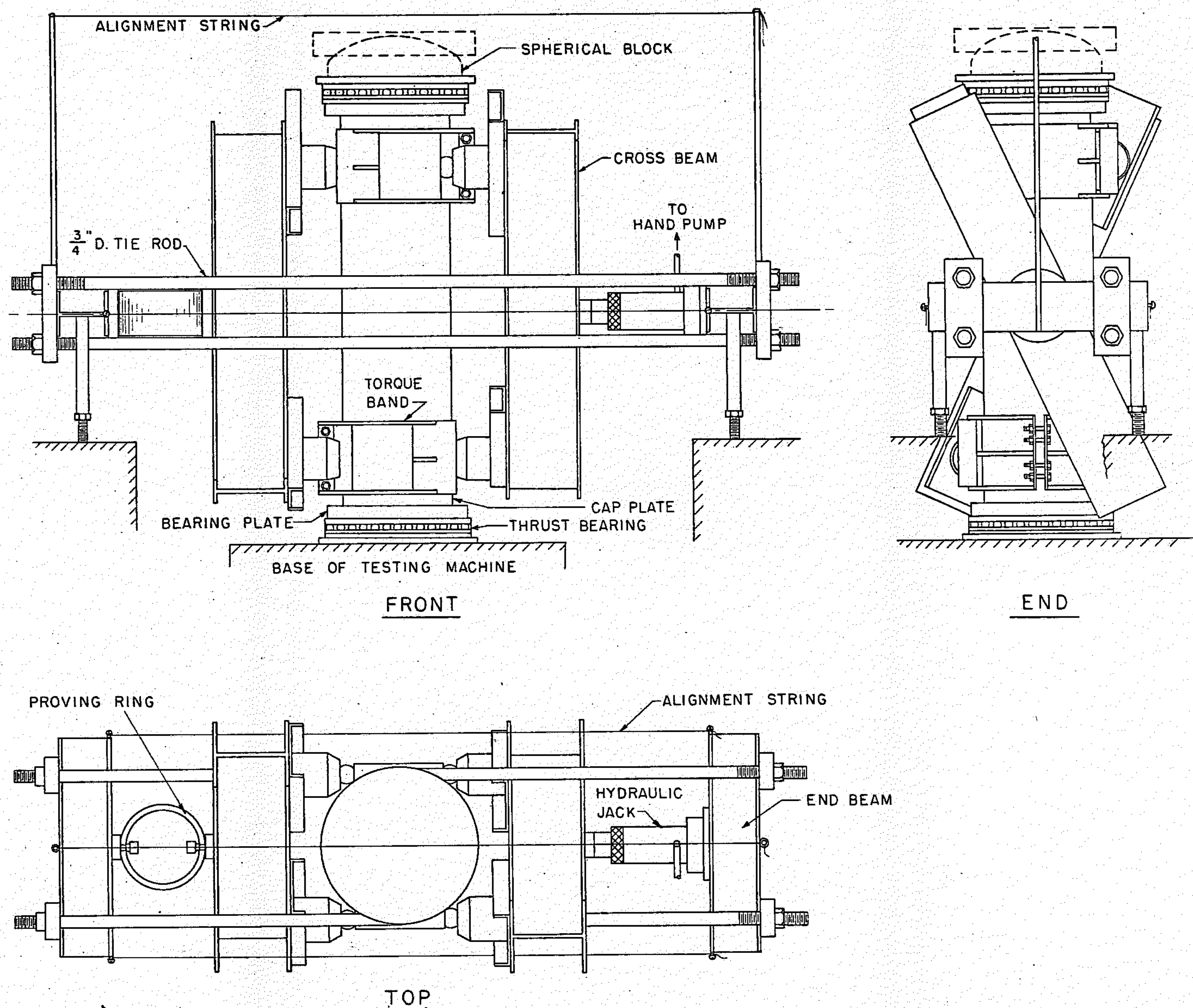


FIG. 2 LOADING FRAME ASSEMBLY

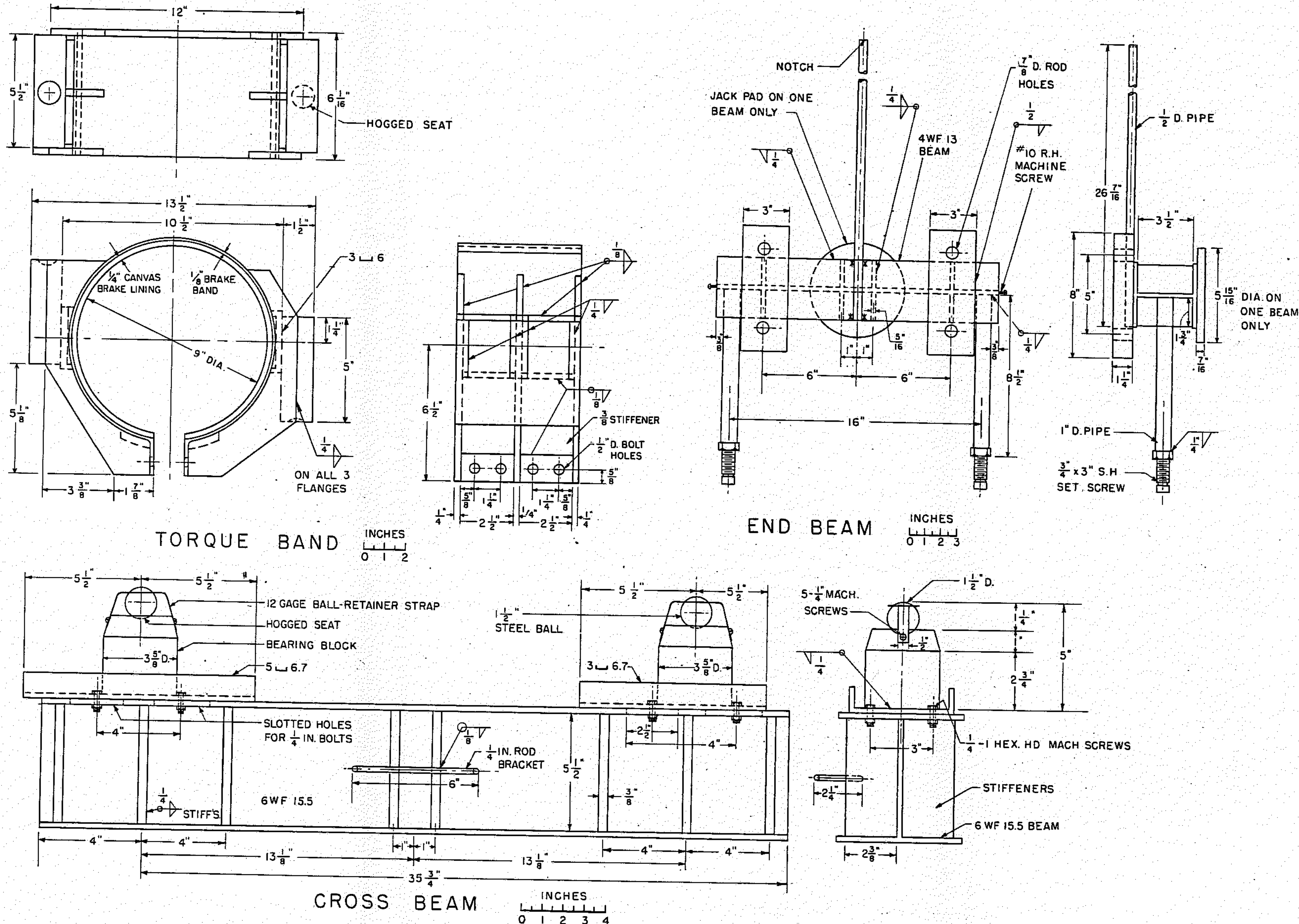


FIG. 3 LOADING FRAME DETAILS

FIG. 3

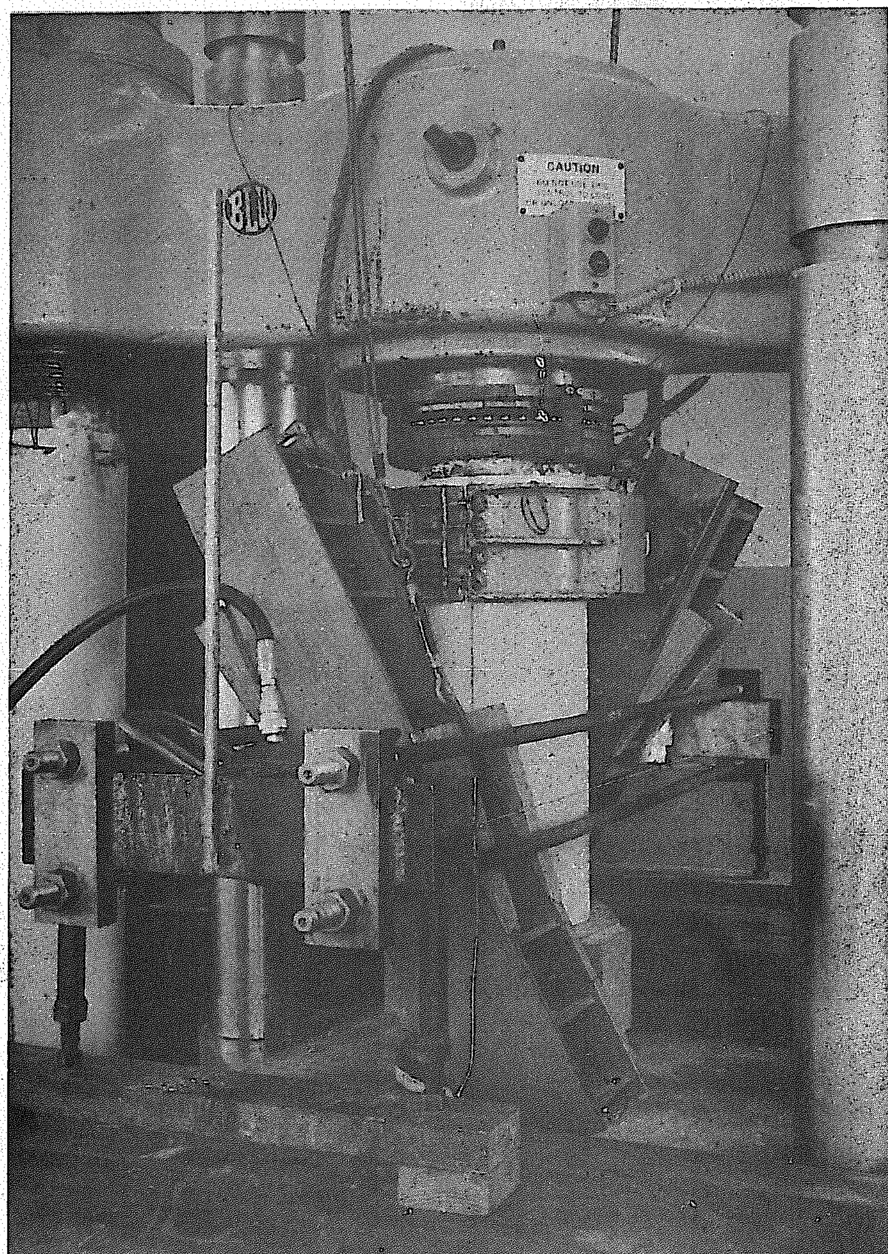


FIG. 4. SET UP IN TESTING MACHINE

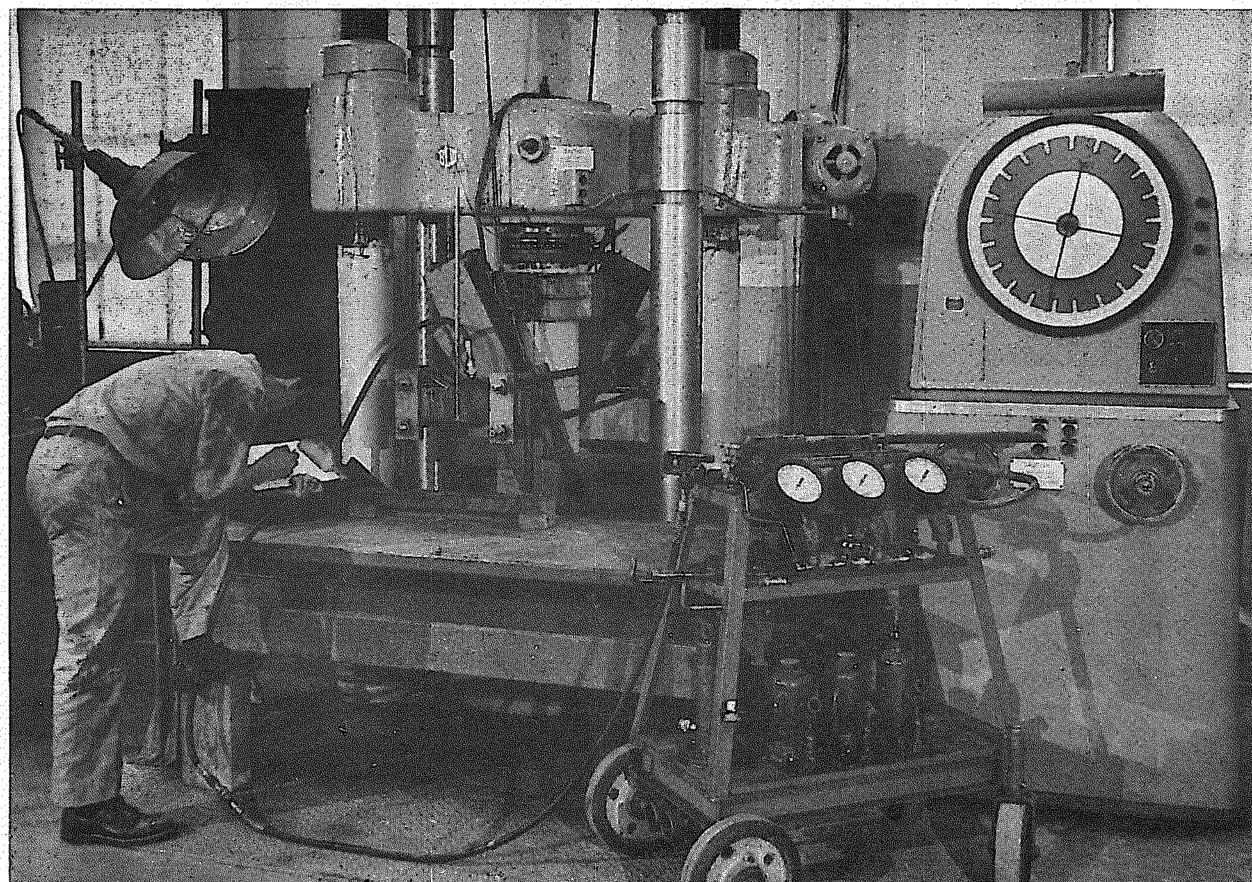


FIG. 5. GENERAL EXPERIMENTAL SET UP

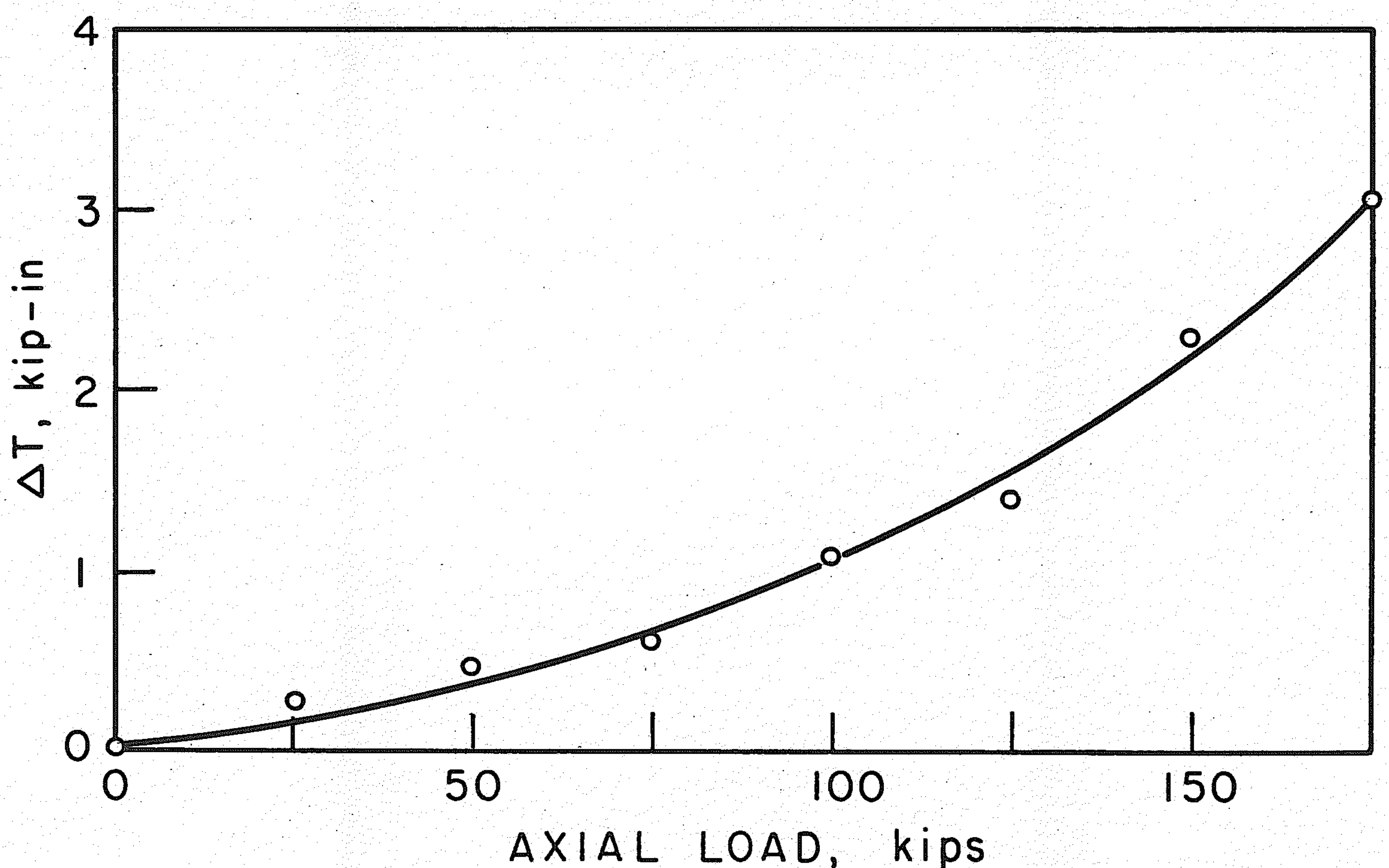


FIG. 6 THRUST BEARING FRICTION AT FIRST SLIP

FIG. 6

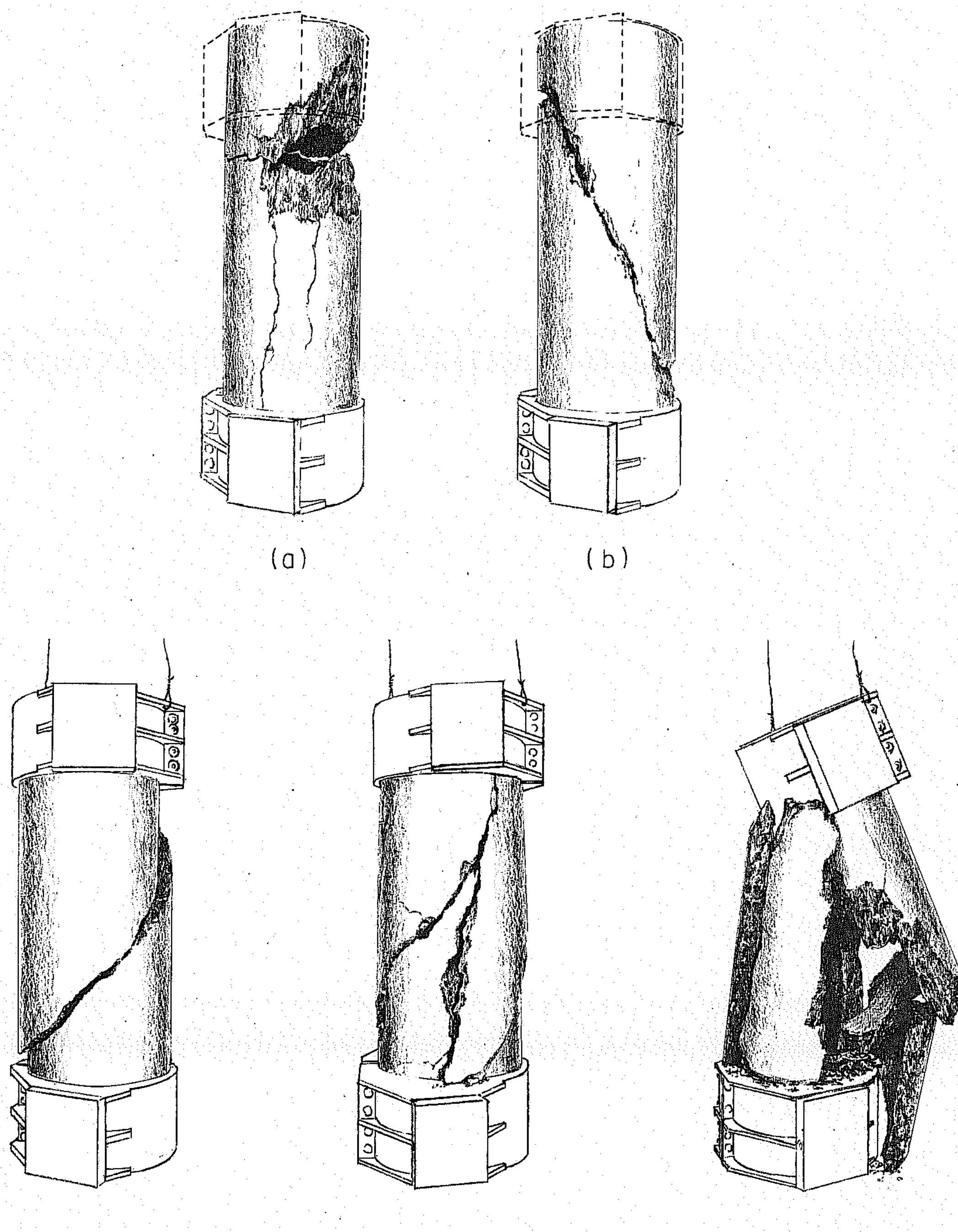


FIG.7 MODES OF FAILURE

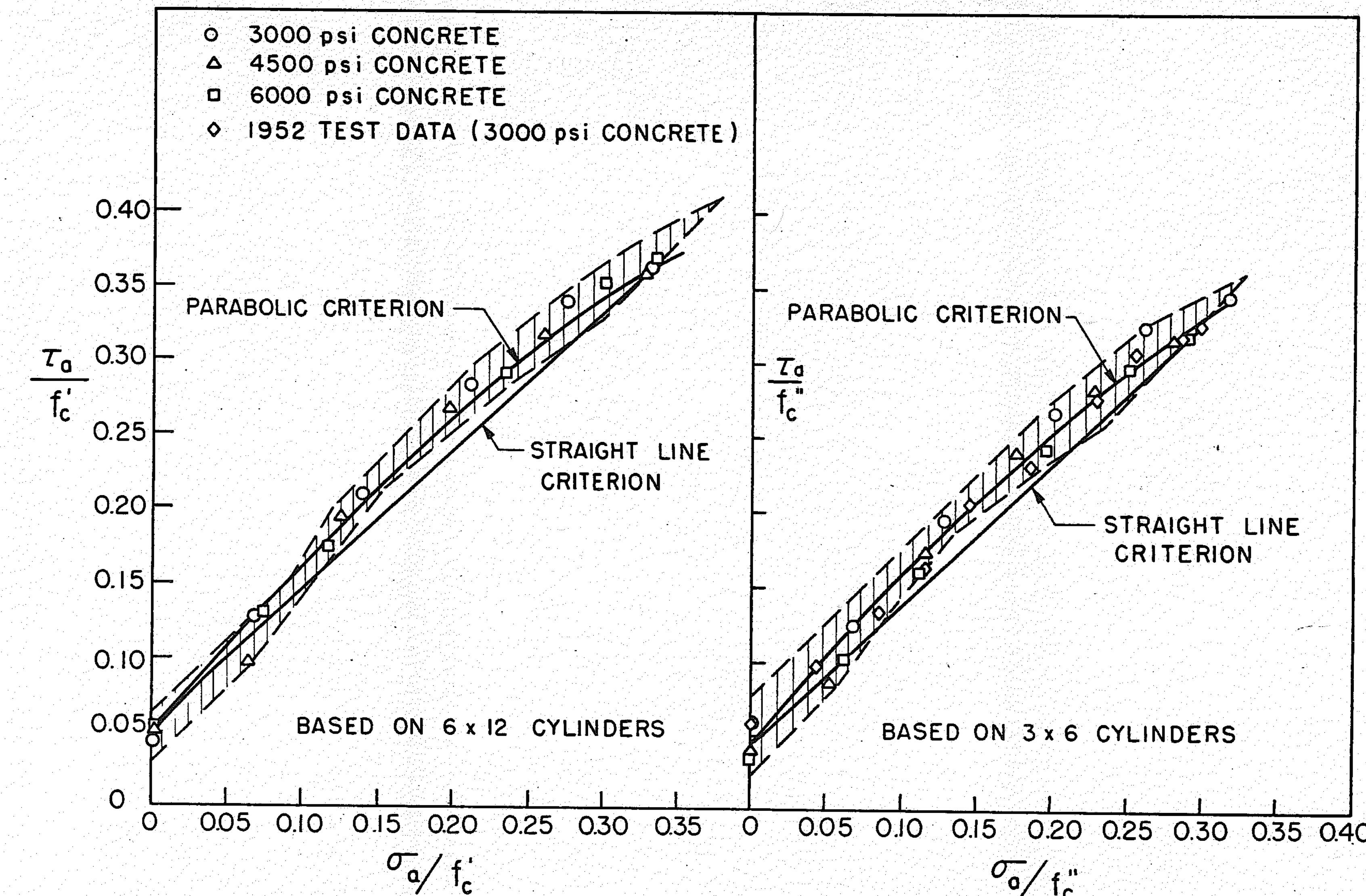


FIG. 8 RELATIONSHIP BETWEEN NORMAL AND SHEARING MEAN STRESSES AT FAILURE

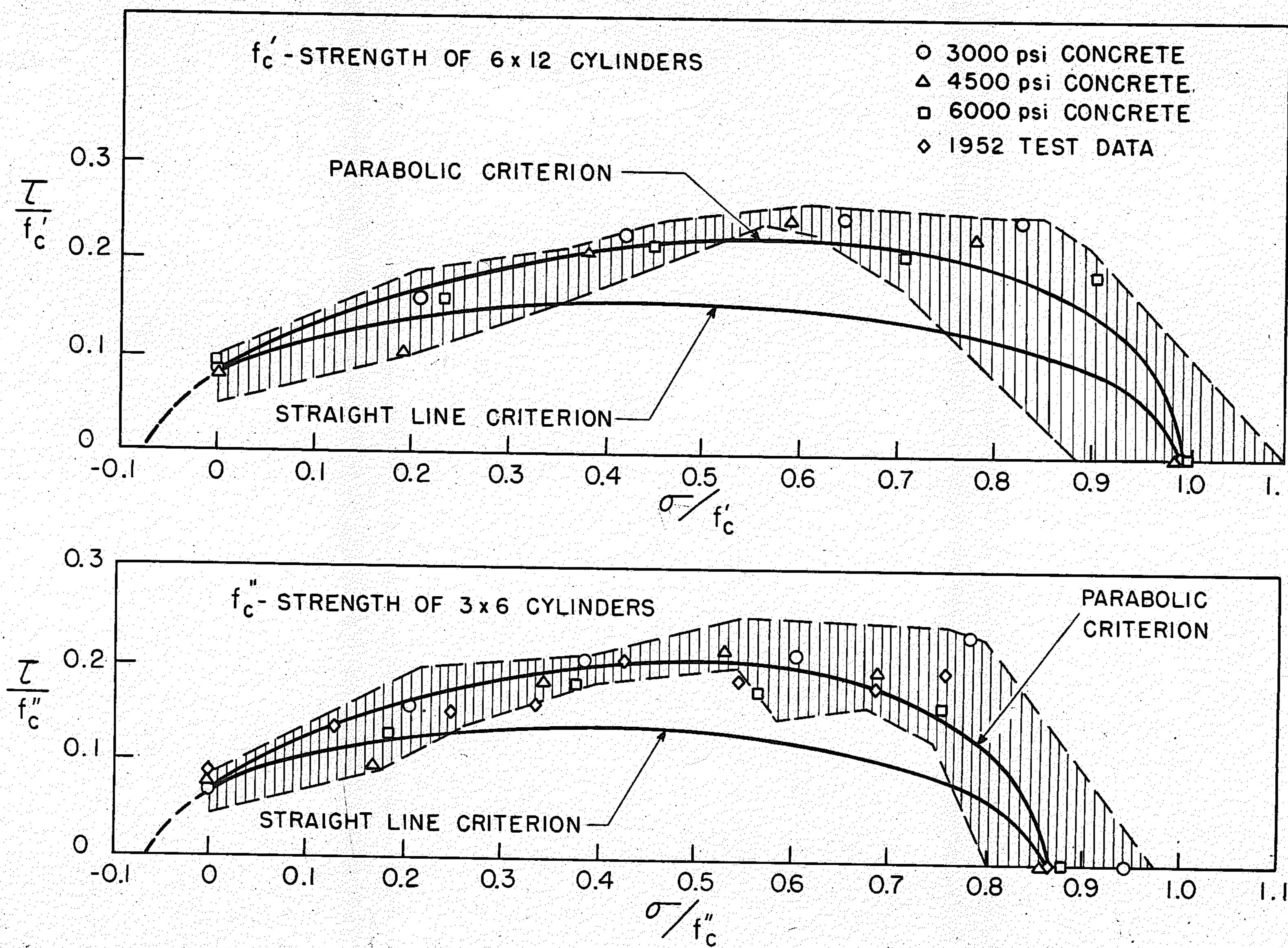
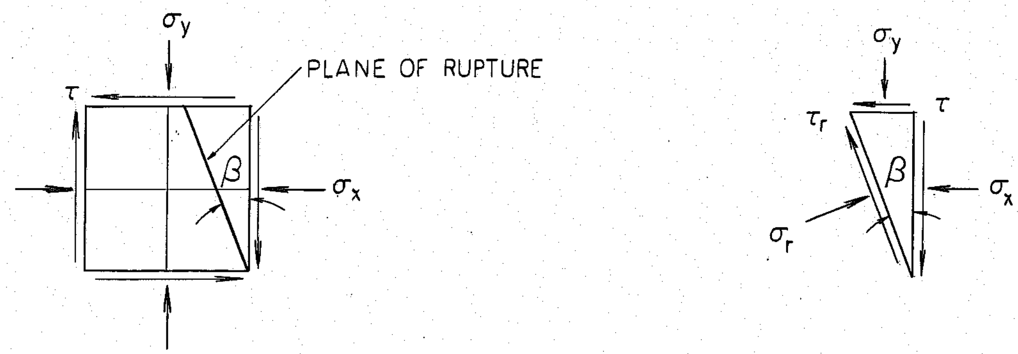


FIG. 9 SHEAR - COMPRESSION STRENGTH



$$\begin{cases} \sigma_r = \sigma_y \sin^2\beta + \sigma_x \cos^2\beta + \tau \sin 2\beta \\ \tau_r = \sigma_y \sin\beta\cos\beta - \sigma_x \sin\beta\cos\beta + \tau\,(\cos^2\beta - \sin^2\beta) \end{cases}$$

FIG. 10 STRESSES ON THE RUPTURE PLANE

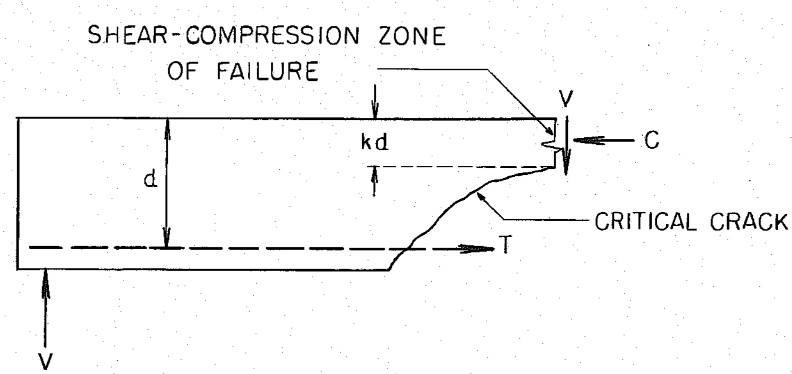
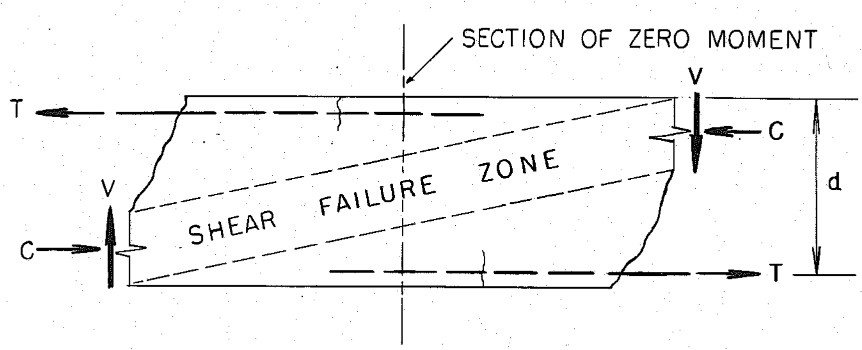


FIG. 11 SHEAR FAILURE IN A SIMPLY SUPPORTED BEAM



CONTINUOUS BEAM

FIG. 12 SHEAR FAILURE IN A CONTINUOUS BEAM

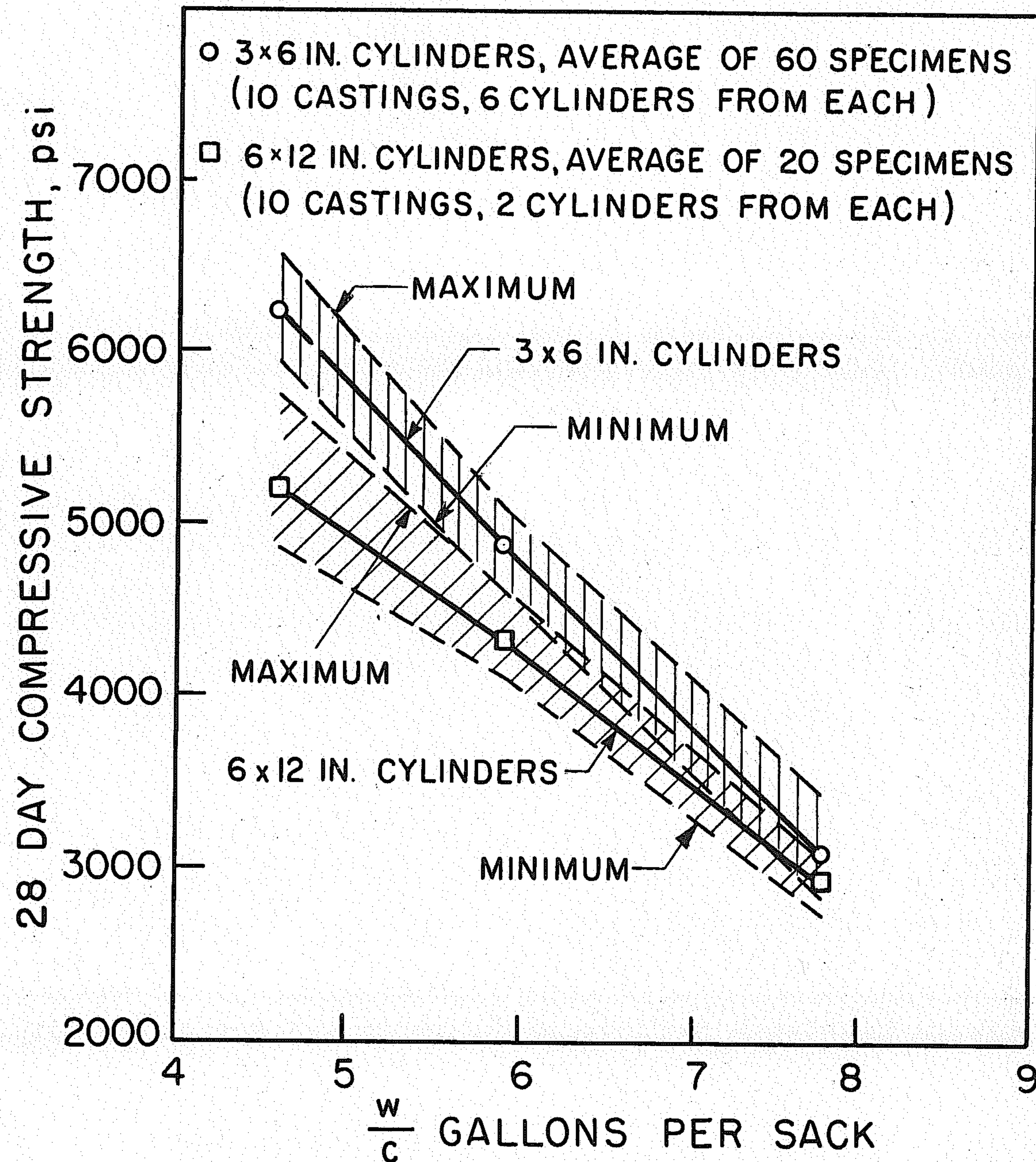


FIG. 13 COMPRESSIVE STRENGTH OF 3 x 6 IN. AND 6 x 12 IN. CONCRETE CONTROL CYLINDERS

FIG. 13

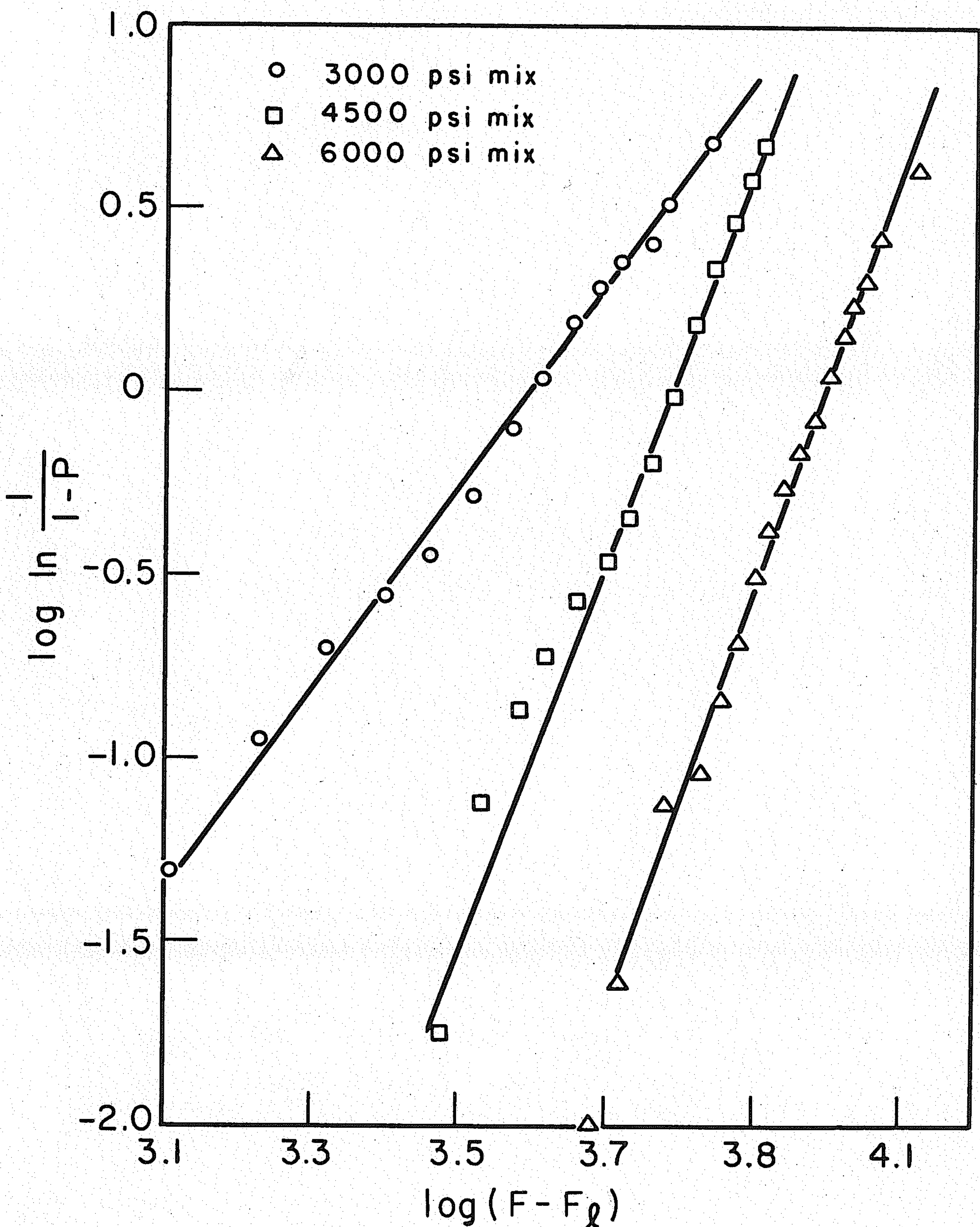


FIG. 14 STATISTICAL PLOT OF COMPRESSION TEST DATA FOR 3 x 6 CYLINDERS

FIG. 14

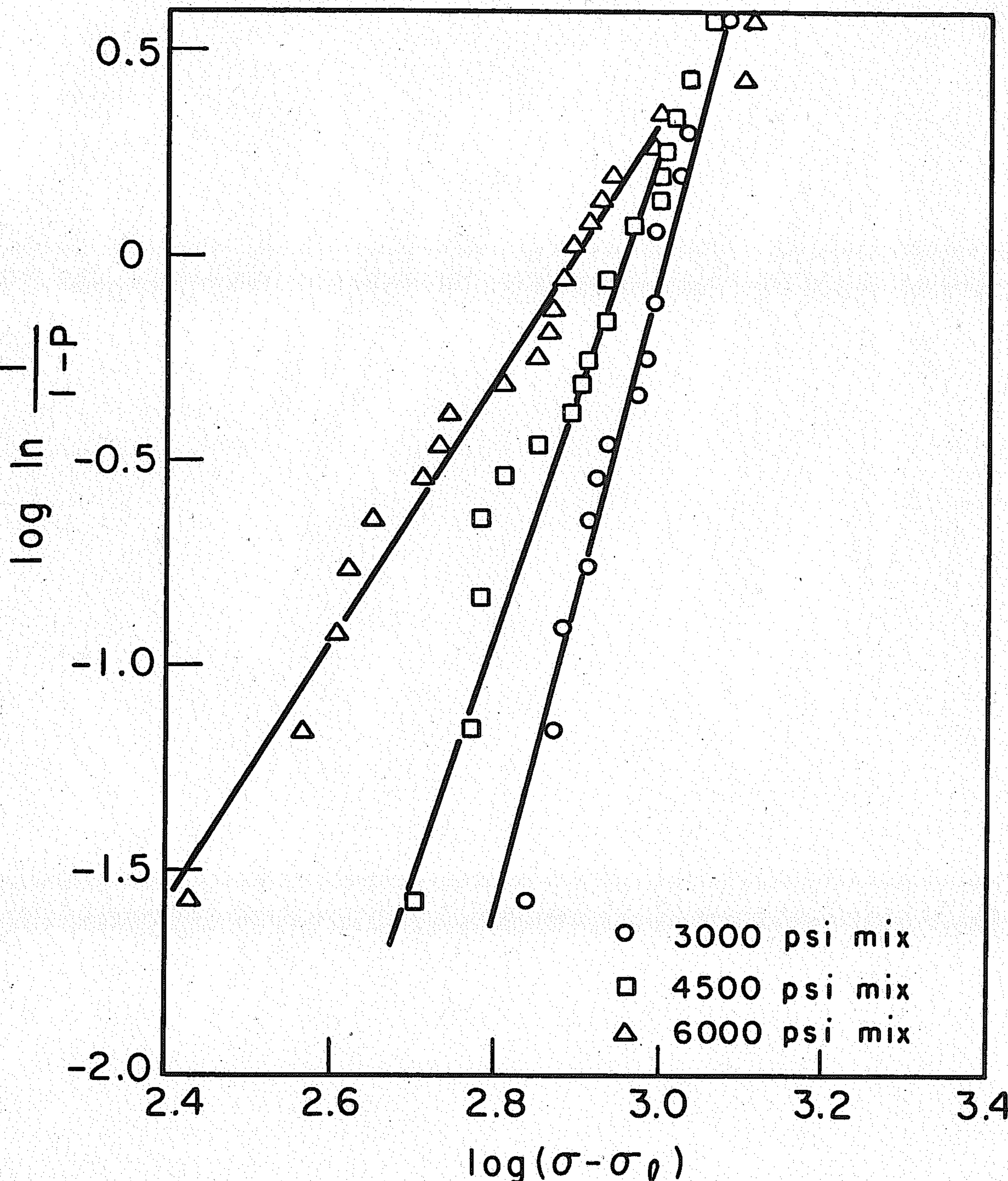


FIG. 15 STATISTICAL PLOT OF COMPRESSION TEST DATA FOR 6 x 12 CYLINDERS