

UNIVERSITY OF CALIFORNIA

Los Angeles

Demonstrating the Effects
of Interplanetary Secular Resonances
in Exoplanetary Systems Under the Influence
of the Stellar Gravitational Quadrupole Moment

A dissertation submitted in partial satisfaction
of the requirements for the degree
Doctor of Philosophy in Astronomy and Astrophysics

by

Thea Hendon Faridani

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ABSTRACT OF THE DISSERTATION

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Professor Smadar Naoz, Chair

The purpose of this thesis is to further describe the effects of secular resonances between planets at low inclinations and eccentricities with a special focus on the interactions between these resonances and stellar oblateness. To accomplish this, I led four projects. The first project evaluated the stabilizing power of general relativity precession against external perturbation on compact multi-planet systems. I extended existing stability criteria that neglected general relativity, evaluated the proportion of the exoplanet sample for which the updated criteria produces qualitatively different conclusions, and investigated the stability of undetected inner planets within coplanar observed multi-planet systems. My second project further explored the stability of undetected inner planets within observed systems by considering the effect of evolving stellar oblateness on potential secular resonances. I find that, consistent with previous theory, evolving stellar oblateness causes secular resonances between planets to move through parameter space, causing a wide range of potential inner planet orbits to experience resonance. These sweeping resonances systematically transfer angular momentum between planets, permanently changing the mutual inclinations between planets.

In my third project, I consider the three-planet transiting exoplanet sample at large and

investigate how often these sweeping resonances from evolving stellar oblateness occur. Ignoring stellar oblateness, these resonances should be exceptionally rare, requiring almost fine-tuned selections of planetary semimajor axes and masses. However, I demonstrate that the resonance-sweeping vastly increases the regions of parameter space where these resonances may occur. Investigating a clean sample of thirty three-planet systems, I find six (20% of the sample) that have masses and semimajor axes that are susceptible to these resonances if their host stars were born with sufficiently high spins. I argue that this finding of 20%, even if only from a sample of thirty, indicates that such resonances are commonplace among the histories of planetary systems. In my fourth project, I show that sweeping resonances from evolving stellar oblateness resonances have significant impact on the observational outcomes of multi-planet transiting systems as they greatly modify the mutual inclinations between planets. As the outer two planets of three planet systems typically align as a result of these resonances, the probability they are observed transiting together increases. However, the inner planet is misaligned from them as a result, reducing the overall probability that the system is observed as a two- or three-planet transiting system. I discuss the implications these systematic effects on transit probabilities may have on the observed exoplanet sample.

This thesis demonstrates that events early in the life of a planetary system, such as the stellar spin-down, have long-lasting effects on systems' architectures, and provides insight into how these early events can systematically shape what planetary systems are most amenable to observation.

The dissertation of Thea Hendon Faridani is approved.

Erik A. Petigura

Tuan H. Do

Bradley M. Hansen

Smadar Naoz, Committee Chair

University of California, Los Angeles

2025

*To those universal human birthrights,
the sun, moon and stars,
clean water, good food,
quiet peaceful nights,
and the liberty to define “I”*

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The results described in this thesis are derived from published (or in-review) works with co-authors and are reproduced here with permission when required. Chapter 2 is derived from Faridani et al, “Hiding Planets Near and Far: The Parameter Space of Hidden Companions for Known Planetary Systems” *The Astrophysical Journal* (2022), 932, 78. I thank co-authors Smadar Naoz, Lingfeng Wei, and Will Farr.

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VITA

2015–2019	B.S. Physics with honors; <i>aumma cum laude</i> , <i>University of Oregon</i>
2015–2019	B.S. Mathematics with honors; <i>aumma cum laude</i> , <i>University of Oregon</i>
2019 – 2021	M.Sc. in Astronomy & Astrophysics, University of California, Los Angeles.
2019 – 2025	Ph.D. candidate in Astronomy & Astrophysics, University of California, Los Angeles.

CHAPTER 1

Introduction

1.1 A Broad Overview of the State of Exoplanet Science

In this section, I will provide an overview of the milestones and key discoveries that allowed for the starting point of this PhD. This section is intended for the reader who may not be familiar with the state of exoplanet science or its progression over the past fifteen years, and can be safely skipped by a reader with that knowledge. In Section 1.2 of this Introduction, I will discuss the relevant physics and background for this PhD thesis, intended for all readers.

Since the discovery of the first planet orbiting a star that is not our Sun (i.e., an “exoplanet”) in 1992, exoplanet science has been catapulted to one of the forefronts of astronomical and astrophysical research. The first detection of an exoplanet, named PSR B1257+12 b, occurred in 1992 orbiting the Pulsar PSR B1257+12—a stellar remnant formed after its progenitor went supernova [Wolszczan and Frail, 1992]¹. The first planet discovered around a star similar to the Sun was orbiting the star 51 Pegasi, and named 51 Pegasi b following the standard exoplanet naming convention [Mayor and Queloz, 1995]. Since then, almost 6,000 exoplanets have been detected, with over 7,500 more candidate detections awaiting validation and confirmation [NASA Exoplanet Science Institute, 2025].

51 Pegasi b was detected via detailed observations of the spectrum of light emitted from its host star [Mayor and Queloz, 1995]. 51 Pegasi and 51 Pegasi b both orbit a common center of mass. Therefore, the host star 51 Pegasi is moving under the influence of its

¹PSR B1257+12 is presently known to host three planets, PSR B1257+12 b, PSR B1257+12 c, PSR B1257+12 d. In 2015, the International Astronomical Union announced names for the pulsar and its planets following public submissions and voting. The pulsar was dubbed “Lich”, and the three planets, “Draugr”, “Poltergeist”, and “Phobetor” [Ciprari, 2016]

planet’s gravity—approximately 50 m/s. The component of this motion parallel to our line of sight causes the spectrum of light emitted by the star to shift due to the Doppler effect, getting bluer when it is approaching us and redder when it is receding. Remarkably, this small shift in the color of a star—motion relative to us that is slower than a passenger jet—is detectable by modern instruments. This is called the “radial velocity” (RV) method of detecting exoplanets, and has the distinct advantage of being able to constrain the mass of the orbiting planet [Lovis and Fischer, 2010]. A more massive planet shifts the common center of mass further from the center of the star, making the star orbit faster and emit a stronger RV signal. To date, over a thousand planets have been detected via RVs.

Today, exoplanets are predominantly detected via the “transit method.” This is where the brightness of a star appears to dim if one of its planets passes directly between the star and our telescopes. The first planet discovered via transits was OGLE-TR-56 b by the Polish Optical Gravitational Lensing Experiment project in 2002 [Udalski et al., 2002a,b]. However, it was not until 2006 with the French Space Agency’s and European Space Agency’s Convection, Rotation et Transits planétaires (CoRoT) mission in 2006 and the United States’ National Aeronautics and Space Administration’s (NASA) Kepler Space Telescope where transiting planets began to be detected in bulk [see e.g., Barge et al., 2008, Auvergne et al., 2009, Baglin et al., 2006, Léger et al., 2009, Borucki et al., 2010, 2011, Borucki, 2016]. A key advantage of the transit method over the radial velocity method is that it does not require individual spectra to be taken of each star. This enables missions like CoRoT and Kepler to observe many stars at once, vastly increasing their detection output relative to the radial velocity method. However, the transit method does not constrain planetary masses as directly as the radial velocity method—instead yielding the ratio between the host star’s radius and the planet’s [see e.g., Haswell, 2010].

CoRoT detected over 500 transiting planets (though only thirty have been deemed confirmed detections by the mission’s operators) and Kepler, a staggering 2,700 [Moutou et al., 2013, Borucki, 2016]. Both CoRoT and Kepler are now retired, but NASA’s Transiting Exoplanet Survey Satellite (TESS), launched in 2018 has detected over 600 planets with over

7,500 waiting followup confirmation [Ricker et al., 2015]. Moreover, NASA’s Nancy Grace Roman Space Telescope, to be launched in late 2026, is expected to observe over 100,000 transiting planets as part of its Galactic Bulge Time-Domain Survey where it will observe a region of the sky near the center of the Milky Way Galaxy for six approximately 70-day periods over the course of its planned five-year mission [Montet et al., 2017, Wilson et al., 2023].

Critical to understanding the investigations into planetary orbits that I undertake in this thesis is an understanding of how we parametrize planetary orbits—through the orbital elements. Each planet’s orbit is an ellipse defined by five parameters (orbital elements): the semimajor axis (a lengthscale that describes the physical size of the ellipse), the eccentricity (a number between 0 and 1 that describes how stretched and noncircular the ellipse is), and three angles determining the orientation of the ellipse in space and where the short-end of it points (the inclination, longitude of ascending node, and the argument of periapsis). As external factors change planetary orbits, planets will not actually trace out elliptical paths. Fortunately, we can very well approximate the paths that the planets undergo by analyzing the osculating orbital elements—the orbital elements that a planet has at an instant and would retain if all perturbing forces vanished.

1.2 Planetary Demographics and Short-Range Dynamical Forces

With so many planets being detected, planetary demographics has revealed many surprising things. Before exoplanets were detected en masse, the assumption was that exoplanetary systems would broadly look similar to the Solar System. However, this has not borne true. The vast majority of detected exoplanets orbit much closer to their host stars than Mercury, and most detected rocky planets are much larger than Earth [see e.g., Butler et al., 2006, NASA Exoplanet Science Institute, 2025]. Of course, this is influenced by a biased sample: all exoplanet detection methods are more likely to detect bigger planets, and all but direct imaging perform better with closer-orbiting planets. However, even taking these biases into

account, it is clear that the Solar System’s structure is at least somewhat strange, having no planets orbiting within ~ 0.3 au. Additionally, the study of the full exoplanet sample has revealed several oddities. The Kepler mission detected many more systems that contained only one transiting planet than expected from a simple model [see e.g., [Lissauer et al., 2011](#), [Zhu and Dong, 2021](#)]. This discrepancy between high numbers of single-transiting systems compared to multi-planet systems was termed the “Kepler dichotomy.” Over the last 14 years, many explanations for this apparent dichotomy have been proposed [see e.g., [Johansen et al., 2012](#), [Spalding and Batygin, 2016](#), [Ballard and Johnson, 2016](#), [Moriarty and Ballard, 2016](#), [Lai and Pu, 2017](#), [Hansen, 2017](#), [Zhu et al., 2018](#), [Zink et al., 2019](#), [Sandford et al., 2019](#), [Millholland et al., 2021](#)]. At present, the prevailing explanation is that there is not a true dichotomy where singles and multis form from disjoint distributions. Rather, most planetary systems form from the same parent distribution, and observational bias and dynamical effects produce the apparent dichotomy [see e.g., [Hansen, 2017](#), [Romero-Mirza and Kempton, 2018](#), [Millholland et al., 2021](#), [Zhu and Dong, 2021](#)].

Other significant demographic patterns have emerged from the exoplanet sample. The “radius valley” refers to an apparent dearth of planets observed with radii near $1.8 R_{\oplus}$. On either side of $1.8 R_{\oplus}$, there is an occurrence-rate peak [[Fulton et al., 2017](#), [Fulton and Petigura, 2018](#)]. On the lower radius side, planets are typically rocky with thin atmospheres (known as super Earths), whereas above $1.8 R_{\oplus}$, planets are observed to have much more significant atmospheres which increase the total radius [see e.g., [Owen and Wu, 2013, 2017](#)]. These planets are referred to as mini-Neptunes. It is thought that these planets all form as gaseous, but that some fraction of them lose their atmospheres, decreasing in radius and becoming the super-Earths [[Van Eylen et al., 2018](#)]. Two major avenues for this loss of atmosphere have been proposed: core-powered mass loss, where the planet’s internal heat boils off its atmosphere, and photoevaporation, where light from the host star boils off the atmosphere [see e.g., [Owen and Wu, 2016](#), [Gupta and Schlichting, 2019](#), [Ginzburg et al., 2018](#), [Owen and Lai, 2018](#), [Lloyd et al., 2020c](#), [Rogers et al., 2021](#)]. While both avenues are certainly occurring, it is not certain at present which is the dominant channel [[Rogers et al.,](#)

2021]. Another important demographic property important to understanding the transiting exoplanet sample is the “peas in a pod” pattern, where systems of multiple transiting planets are observed to have planets with similar sizes spaced equally apart [Weiss et al., 2018b, Weiss and Petigura, 2020]. This effect implies that planets during formation may be “talking” (i.e., exchanging angular momentum) dynamically between one another, influencing their final spacings and radii [see e.g., Kipping, 2018, Gilbert and Fabrycky, 2020]. Alternatively, it is possible to reproduce the “peas in a pod” pattern without planet-planet interactions and instead having all the planets affected by their host star’s properties but not each other’s [see e.g., Murchikova and Tremaine, 2020b, Zhu and Dong, 2021]. Recent studies evaluate the statistics of planetary eccentricities in the transiting exoplanet sample—specifically, it was suggested that most of the fits of the Kepler sample were not able to extract orbital eccentricity from the transit light curves, or only for ~ 100 s planets [see e.g., Van Eylen and Albrecht, 2015, Van Eylen et al., 2019a]. However, Gilbert et al. [2025] extracted eccentricity information from over 1500 planets to produce statistical information about the eccentricity distributions of these transiting planets. They find that planetary eccentricity increases greatly for planets with radii larger than $4R_{\oplus}$ and that eccentricities are much larger for planets observed transiting alone, consistent with previous studies [see e.g., Xie et al., 2016].

These trends all point towards a key fact of planet formation and evolution: all of these planets being detected are living in a rich dynamical environment. As they orbit their host stars, kept bound by their gravity, they disturb one another through their mutual gravitational interactions. The gravitational perturbations between planets cause the orbital elements of each planet to oscillate or precess. Critically, these oscillations of planetary orbital elements can enter a resonance when conditions are appropriate. In the approximation of low inclinations and eccentricities, the oscillations of the orbital elements due to planet-planet interactions occur with particular frequencies of oscillation, set by the planets’ masses and semimajor axes [see e.g., Murray and Dermott, 2000]. If two or more planets have frequencies of the same rate, they will resonate with one another. This results in enhanced exchanges of angular momentum expressed by larger changes in the oscillation magnitude of

the orbital elements. In certain instances (discussed in detail in Chapter 4), these resonances can cause permanent changes in the status quo—the medians around which orbital elements oscillate.

Of course, interactions between planets are not the only dynamical effects changing the orbits of planets around stars. There is a diversity of short-range forces arising from the host star that can alter its planets. The most commonly known is the tidal force. The differential on either side of a body produces a tidal bulge, and the shifting stresses of this bulge can circularize and shrink the orbit of a planet, among a host of other effects [see e.g., [Hut, 1980](#), [Fabrycky et al., 2007](#), [Wu et al., 2007](#), [Hansen, 2010](#), [Naoz et al., 2011](#), [Hansen, 2012](#), [Naoz et al., 2012](#), [Li et al., 2014a](#), [Hansen and Murray, 2015](#), [Rice, 2015](#), [Petrovich, 2015b](#), [Lai et al., 2018](#), [Stephan et al., 2018](#), [Martin and Triaud, 2015](#)]. While this is a dramatic impact on an orbit, tidal orbital shrinkage, circularization, and precession typically take place over very long timescales. In this thesis, I focus primarily on more rapid-acting effects, so tidal impacts are not explored in detail.

The theory of general relativity [GR, [Einstein, 1915](#)] provides an additional short-range effect: general relativistic precession, which causes precession in the argument of periapsis. Dynamically, precession from GR can act to suppress excitations of the eccentricity of a close-orbiting planet [see e.g., [Naoz et al., 2013a](#), [Liu et al., 2015](#), [Mardling and Lin, 2004](#), [Plavchan et al., 2015](#), [Hansen and Naoz, 2020](#)]. In this way, GR precession acts primarily as a stabilizing force, protecting close-in planets from having their eccentricity excited so much that they fall into their host star or collide with another planet. GR is addressed most directly in Chapter 2, but plays a nontrivial role in the rest of the thesis as well.

Another short range effect—the stellar gravitational quadrupole moment, parametrized by the number J_2 —is particularly central to the work of this thesis. Stars retain angular momentum from formation. A typical sun-like star is born with a rotation period of about 4 days [see e.g., [Gallet and Bouvier, 2013](#)]. This produces a centrifugal bulge at its mid-plane, affecting its gravitational potential. This effect is quantified through the gravitational quadrupole moment, parametrized by the dimensionless number J_2 [see e.g., [Sterne, 1939](#),

[Murray and Dermott, 2000](#)]. The change in potential causes the longitude of the ascending node and the argument of pericenter of any orbiting bodies to precess with a rate proportional to J_2 . Critically, however, because this precession arises from the spin of the star, if that spin changes, so will the speed of precession. And indeed, sun-like stars’ rotation speeds change dramatically over their first billion years of life, transferring angular momentum from their spin to their wind through their magnetic field [e.g., [Parker, 1958](#), [Schatzman, 1962](#), [Kraft, 1967](#), [Weber and Davis, 1967](#), [Mestel, 1968](#), [Skumanich, 1972](#)]. This process, called magnetic braking, will cause dramatic reductions in the J_2 of a star as it spins down, weakening the precessions of the planets caused by its equatorial bulge. This evolution of stellar J_2 with time can produce significant impacts on the planets orbiting a star: inclining them to one another [see e.g., [Ward et al., 1976](#), [Spalding and Batygin, 2016](#), [Li et al., 2020](#), [Becker et al., 2020](#), [Spalding and Millholland, 2020](#), [Schultz et al., 2021](#), [Brefka and Becker, 2021](#), [Chen et al., 2022](#), [Fu and Wang, 2024](#)], destabilizing them [see e.g., [Spalding et al., 2018](#), [Schultz et al., 2021](#)], or hiding them from visibility via transit [see e.g., [Stephan and Gaudi, 2023](#)]. Exploring these effects comprises the bulk of this thesis, especially in chapters 3, 4, and 5.

1.3 Dissertation Summary

The purpose of this thesis is to demonstrate the relevance of planet-planet resonant interactions, particularly in the context of the time-varying potential produced by the spin-down of the host star. This thesis addresses a key gap in the literature, as resonant interactions between planets with the time-varying J_2 had been understudied in systems where the total planets numbered three or more. The results of this work aid the larger goals of the field, including a better understanding of conditions necessary for multi-planet transit and understanding the dynamical environment of planetary systems after the protoplanetary disk has evaporated during their early years of life.

In Chapter 2, I investigate the parameter space of potential undetected planetary compan-

ions in known planetary systems. I first consider the stability impacts of an undetected outer planet to a multi-planet system. Previous work has presented stability criteria that constrain inclined outer planets to inner multi-planet systems by leveraging stabilizing planet-planet interactions between the innermost planets against the destabilizing perturbations of the distant planet. I particularly focus on extending these criteria to evaluate the extent to which GR, which was neglected in the previous criteria, affects where the stability boundary lies in parameter space. Additionally, I explore the possibility of an undetected inner planet to an observed multi-planet system and determine when GR can stabilize such a planet against perturbations from its companions.

In Chapter 3, I further investigate potential undetected inner planets within multi-planet systems. The spin-down of the star through magnetic braking, changes the star’s gravitational quadrupole moment, J_2 , which causes time-varying precessions on all orbiting planets. These precessions may move planet-planet secular resonances through parameter space (“sweeping resonances”), potentially affecting a wider range of planets than previously thought. I specifically focus on the two-planet system HD 106315 as a case-study and determine the conditions under which an undetected planet interior to the observed two planets may exist. I find that such an inner planet is unstable at over 50% of the available semimajor axis range due to sweeping resonances if the host star, HD 106315, was born with sufficient spin.

In Chapter 4, I aim to quantify the prevalence of these sweeping resonances from evolving stellar J_2 among the observed exoplanet sample. To accomplish this, I investigate the observed sample of three-planet systems and determine which of these systems have properties that could have produced impactful sweeping resonances in their pasts. Among a clean sample of thirty transiting systems, I find that six of them (20%) have configurations that indicate they were susceptible to sweeping resonances when their host star was young. This finding indicates that such sweeping resonances may be a common outcome for multi-planet systems.

In Chapter 5, I explore the impacts of sweeping resonances in three-planet systems on

detectability via transit, using the observed three-planet system Kepler-619 as a case-study. I quantify the changes on planetary transit probabilities caused by sweeping resonances from decreasing J_2 . These resonances bifurcate systems, inclining the innermost planet relative to the outermost two. This increases the probability that at least one planet transits, but decreases the probability that at least two planets transit. If these resonances are as common as calculated in Chapter 4, then these resonances constitute a systematic effect frustrating the transit detection of multi-planet systems as multi-planet systems, rather than singles.

CHAPTER 2

The Parameter Space of Hidden Companions for Known Planetary Systems

2.1 Introduction

Recent developments in ground and space based observations have found that multi-planet systems are common in the Galaxy [e.g., [Howard et al., 2010, 2012](#), [Lissauer et al., 2011](#)]. Current leading exoplanet detection methods are more sensitive to larger planets and to planets with shorter periods. For example, the popular transit method is particularly sensitive to close-in planets [[Borucki, 2016](#)].

Because the transit method represents on the order of three quarters of all current exoplanet detections, the biases inherent in the method leave open two classes of planets that are currently difficult to detect: small planets close to their hosts and large planets further out. Radial velocity detection methods, as the second most prolific method, give access to large regimes of wide-orbit parameter space, and have shed light on the life cycle of giant planets living there [[Hatzes et al., 2005](#), [Knutson et al., 2014](#), [Stock et al., 2018](#)].

The observed statistical properties of multi-planet systems have been used to infer the underlying history and evolution of planets [e.g., [Fang and Margot, 2012](#), [Hansen and Murray, 2013](#), [Malhotra, 2015](#), [Pu and Wu, 2015](#), [Steffen and Hwang, 2015](#), [Ballard and Johnson, 2016](#), [Xie et al., 2016](#), [Weiss et al., 2018b](#), [Pu and Lai, 2018](#), [Denham et al., 2019](#), [Tamayo et al., 2020a](#)]. One curious property of the multiplanet systems that have been observed is that the radii of planets in the same system are correlated with one another [[Weiss et al., 2018b,a](#)]. There was debate whether this has been conclusively demonstrated to arise from

formation processes as opposed to observation biases [e.g., [Zhu, 2020](#), [Weiss and Petigura, 2020](#), [Murchikova and Tremaine, 2020a](#)], considering that dynamical evolution may erase the initial conditions of formation, however, correlations not explainable by observation bias have been demonstrated [e.g., [He et al., 2021](#), [Gilbert and Fabrycky, 2020](#)]. Therefore, when we consider hypothetical configurations of planets that are meant to be similar to configurations already observed, we usually assume correlated planet masses.

Observations suggest that far-away companions (giant planets or even stars) are expected to host inner multi-planet systems. This trend was suggested by radial velocity surveys that showed a population of giant planets may exist at large distances from stars that host one or more inner planets [e.g., [Knutson et al., 2014](#), [Konopacky et al., 2016](#), [Zhu and Wu, 2018](#), [Bryan et al., 2016, 2019](#)]. Additionally, several systems have been discovered where the configuration is strictly hierarchical, with a giant planet on a wide orbit outside of one or more very close-in planets [e.g., [Bonomo et al., 2017](#), [Ment et al., 2018](#), [Barbato et al., 2018](#), [Mills et al., 2019](#), [Bryan et al., 2019](#), [Zhu and Wu, 2018](#)]. Furthermore, the most generic example is our solar system, with Jupiter at 5 au. Lastly, since the majority of stars exist in a binary configuration [e.g., [Raghavan et al., 2010](#), [Moe and Di Stefano, 2015](#)], a population of distant stellar companions to multi-planet systems is to be expected.

Companions that may exist far away from their host stars may dynamically affect an inner planetary system. For example, gravitational perturbations from a far-away companion (either a stellar or planetary companion) can excite the inner planets' eccentricity, via the Eccentric Kozai-Lidov (EKL) mechanism [e.g., [Kozai, 1962](#), [Lidov, 1962](#), [Naoz, 2016](#)]. High eccentricity, in combination with tidal dissipation, can lead to shrinking of the semi-major axis of a planet, circularizing it [e.g., [Fabrycky et al., 2007](#), [Wu et al., 2007](#), [Naoz et al., 2011, 2012](#), [Li et al., 2014a](#), [Rice, 2015](#), [Petrovich, 2015b](#), [Lai et al., 2018](#), [Stephan et al., 2018](#), [Martin and Triaud, 2015](#), [Hansen and Murray, 2015](#)]. On the other hand, the eccentricity excitations may plunge the planet into the star [e.g., [Naoz et al., 2012](#), [Petrovich, 2015a](#), [Lai et al., 2018](#), [Stephan et al., 2018, 2020](#)]. In some cases, short-range forces, such as general relativity and tidal precession may suppress these eccentricity excitations [e.g., [Fabrycky](#)

et al., 2007, Naoz et al., 2013a, Liu et al., 2015, Mardling and Lin, 2004, Plavchan et al., 2015]. In general, during the formation of planets, Jupiter-size planets have a large role in the final structure of the system, by driving planetesimals inward [e.g., Morbidelli and Crida, 2007, Raymond and Izidoro, 2017] or by controlling the ability of planetesimals to accrete [e.g., Hansen, 2009, Walsh et al., 2011].

In the case of a multi-planet system, high eccentricity may result in destabilization. However, the gravitational interactions between close-by multi-planetary orbits, and in particular, the angular momentum exchange between different planets, may suppress high eccentricity induced by a far-away companion [e.g., Innanen et al., 1997, Li et al., 2014b, Hansen, 2017, Pu and Lai, 2018, Denham et al., 2019, Wei et al., 2021, Veras and Ford, 2010, Holman and Wiegert, 1999, Naoz et al., 2013b]. Additionally, it may result in synchronized oscillation of the inclinations of all inner planets, resulting in a large obliquity—an observational marker that there may be a hidden far-away companion [e.g., Innanen et al., 1997, Takeda and Rasio, 2005, Takeda et al., 2008, Li et al., 2014b, Denham et al., 2019, Boué and Fabrycky, 2014].

On the other extreme, the lower mass limit of transit surveys today lies on the order of half an Earth radius, for a $1 M_{\odot}$ host star [assuming a favorable noise level Christiansen et al., 2012]. These short-period planets, usually said to have "ultra short periods" (USPs) of less than one day, produce so many transits that they can be detected despite their small size. Already, over a hundred such planets have been detected [e.g., Sanchis-Ojeda et al., 2014a, Xiu-Min and Jiang-Hui, 2020a, Dai et al., 2017a]. One of the key observed qualities of USPs is that they may have radii $\lesssim R_{\oplus}$. Their small radii have been hypothesized to have been linked to the large irradiances they receive [e.g., Frustagli et al., 2020a]. Currently, the properties of the atmospheres of USPs (if they have any atmosphere at all) are relatively unknown [Malavolta et al., 2018a]. Because some models explaining the super earth radius gap rely on strong stellar flux incoming to the star [e.g., Owen and Wu, 2016, 2017, Petigura, 2020a, Loyd et al., 2020a]¹, USPs may be ideal candidates for constraining these models.

¹We note that there are alternatives to the photo-evaporation model [see Gupta and Schlichting, 2020,

Similarly, a range of formation scenarios exist that can explain how these planets came to have such short periods, such as secular chaos and disk migration [e.g., [Pu and Lai, 2019a](#), [Carrera et al., 2019](#), [Petrovich et al., 2019a](#), [Becker et al., 2020](#)]. Though most models predict that they do not form completely *in situ*, at what stage in their evolution they are brought close to the star remains unclear [e.g., [Millholland and Spalding, 2020a](#)].

In this work, we examine the stability of planets in the radius and orbit ranges where their detection via the transit method is currently not possible. Historically, the techniques used in this work have exclusively predicted the permitted locations of large, distant companion planets [e.g., [Denham et al., 2019](#), [Pu and Lai, 2018](#)]. We extend these approaches by incorporating non-Newtonian forces arising from General Relativity (GR), allowing for constraints to be placed on short-period planets as well. In particular, we demonstrate, as a proof of concept, the usage of this analytical stability criterion for several hypothetical and observational examples.

This paper is structured as follows. In Section [2.2](#) we introduce the basic equations used throughout the analysis and show their use in quantifying the stability of systems. We then focus on systems with known two or more planets and put constraints on which of those systems have their stability against perturbation from an outer companion extended by GR precession (see Section [2.3](#)). We then turn our attention to possible ultra-short period planets (USPs) (Section [2.4](#)), and explore the role of GR for these systems (see for the derivation of an analytical stability criterion in Section [2.4.1](#)). GR precession of USPs tends to suppress eccentricity excitations that may be induced due to Laplace-Lagrange resonances (as shown in Section [2.4.2](#)). We then show that our USPs’ stability analytical criterion agrees compared to numerical direct n -body integration (see Section [2.4.3](#)).

In Section [2.5](#), as a case study, we examine the system Kepler 56 and explore the interaction between general relativity and traditional secular methods within it. Finally, in Section [2.6](#) we discuss our findings.

[2021](#)].

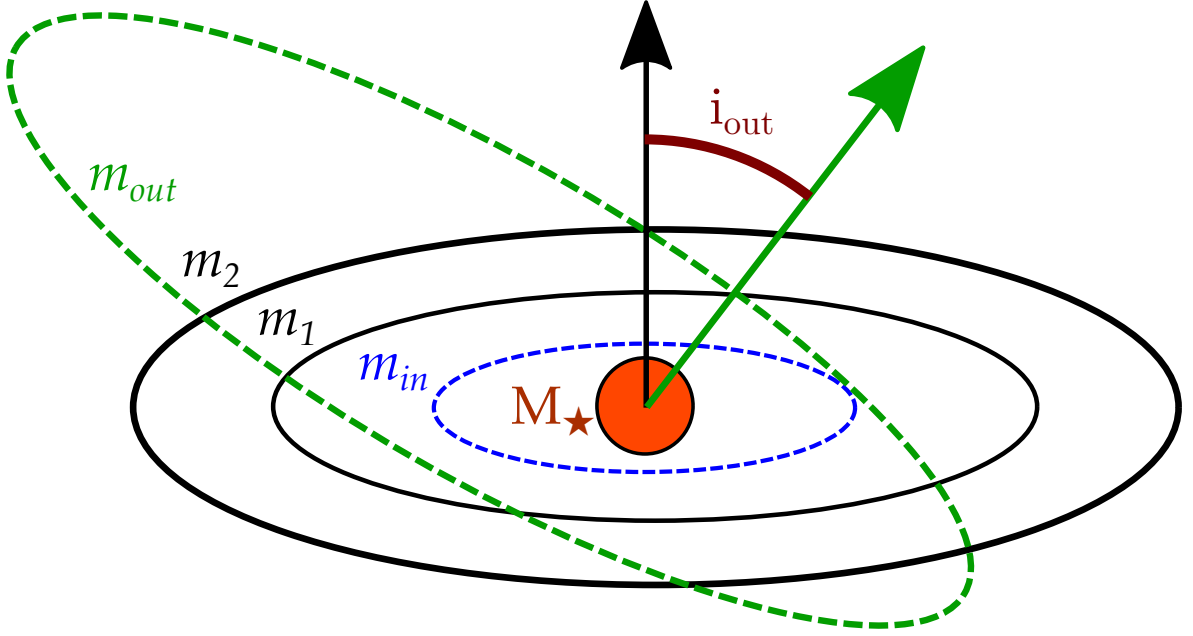


Figure 2.1: Schematic diagram (not to scale) of the systems discussed in this paper. A star of mass $M_\star$ has one or two known planets, m_1 and m_2 . We will consider the stability of this system after the insertion of a hypothetical inner planet m_{in} , or a hypothetical outer planet m_{out} .

2.2 The Two Known Planets - Basic Equations

We consider a fiducial system composed of a star $M_\star$ with two observed planets, m_1 and m_2 , and explore the possibility that a planet may be hiding undetected inward (m_{in}) or outward (m_{out}) of the known planets (see Figure 2.1 for an illustration of the system). The known planets have semi-major axes (eccentricities), denoted by a_1 and a_2 (e_1 and e_2) where $a_1 < a_2$. Angular momentum exchange between the inner planets may stabilize them against gravitational perturbations from the outer companion [e.g., [Denham et al., 2019](#)]. Additionally, general relativity precession of the periapsis may instead further stabilize the system, including a possible inner planet from the perturbations of its neighbors.

Here we describe the basic equations that govern the system. The exchange of angular momentum between planets in the systems is described via the Laplace-Lagrange formalism. Given planets with masses m_j and m_k , the characteristic timescale over which the angular momentum of planet j changes due to planet k , can be written as:

$$\tau_{jk,LL} = \left[A_{jj} + A_{jk} \left(\frac{e_k}{e_j} \right) \cos(\varpi_k - \varpi_j) - B_{jj} - B_{jk} \left(\frac{i_k}{i_j} \right) \cos(\Omega_k - \Omega_j) \right]^{-1}, \quad (2.1)$$

where e_j (e_k) is the eccentricity, and i_j (i_k) is the inclination of planet j (k) and Ω_j (Ω_k) is the longitude of ascending nodes of planet j (k), while ϖ_j (ϖ_k) is the longitude of periapsis. The coefficients A_{jk} and B_{jk} are given by [e.g., [Murray and Dermott, 2000](#)]:

$$A_{jj} = \frac{n_j m_k}{4\pi (M + m_j)} \alpha_{jk} \bar{\alpha}_{jk} f_\psi, \quad (2.2)$$

$$A_{jk} = -\frac{n_j m_k}{4\pi (M + m_j)} \alpha_{jk} \bar{\alpha}_{jk} f_{2\psi}, \quad (2.3)$$

$$B_{jj} = -\frac{n_j m_k}{4\pi (M + m_j)} \alpha_{jk} \bar{\alpha}_{jk} f_\psi, \quad (2.4)$$

$$B_{jk} = \frac{n_j m_k}{4\pi (M + m_j)} \alpha_{jk} \bar{\alpha}_{jk} f_\psi, \quad (2.5)$$

where,

$$\alpha_{jk} = \min \left(\frac{a_j}{a_k}, \frac{a_k}{a_j} \right), \quad (2.6)$$

$$\bar{\alpha}_{jk} = \min \left(\frac{a_j}{a_k}, 1 \right), \quad (2.7)$$

and

$$f_\psi = \int_0^{2\pi} \frac{\cos \psi}{(\alpha_{jk}^2 - 2\alpha_{jk} \cos \psi + 1)^{3/2}} d\psi, \quad (2.8)$$

$$f_{2\psi} = \int_0^{2\pi} \frac{\cos 2\psi}{(\alpha_{jk}^2 - 2\alpha_{jk} \cos \psi + 1)^{3/2}} d\psi. \quad (2.9)$$

We assume that the two known planets were born *in situ*, i.e., their eccentricities and mutual inclinations are small and similar in value. We note that this not a universally advisable assumption. Under this assumption, the precession of the orbit of planet j due to the

aforementioned angular momentum exchange with planet k takes place over a characteristic timescale, estimated as:

$$\tau_{LL,jk} = \left[\frac{n_j m_k}{4\pi(M_\star + m_k)} \alpha_{jk} \bar{\alpha}_{jk} (f_\psi \pm f_{2\psi}) \right]^{-1}, \quad (2.10)$$

where n_j is the mean motion of planet j , and the two cases ($f_\psi \pm f_{2\psi}$) correspond to the minimum (+) and maximum (−) values this timescale can achieve.

Neither f_ψ nor $f_{2\psi}$ have a known closed-form representation to our knowledge, but under the assumption that $\alpha_{jk} \leq 0.9$, we find

$$(f_\psi - f_{2\psi}) \approx 7\alpha_{jk}, \quad (2.11)$$

to within an error of not more than 40%. Because this term has a simple closed-form approximation, when deriving our criterion, we use the ($f_\psi - f_{2\psi}$) case over the ($f_\psi + f_{2\psi}$)

In addition to the Newtonian effect between the two planets, general relativity produces a precession of their orbits. Thus, a far away companion may drive high eccentricity on the inner planets, but in addition to stabilization from Laplace-Lagrange interaction, general relativity precession may suppress this eccentricity excitation [Wei et al., 2021]. The timescale of apsidal precession on planet j caused by general relativity is given by [e.g., Naoz et al., 2013b]

$$\tau_{GR,j} = 2\pi \frac{c^2 a_j^{5/2} (1 - e_j^2)}{3(GM_\star)^{3/2}}, \quad (2.12)$$

where c is the speed of light.

2.3 An Outer Companion: The Role of GR for Known Systems

Consider an outer, inclined companion (m_{out}) to two inner planets (m_1, m_2) that satisfy $a_1 < a_2$. Including this outer companion makes the system hierarchical. This outer companion may excite the eccentricities of the inner planets through the EKL mechanism [e.g., Naoz, 2016]. The associated timescale for this eccentricity excitation on an inner planet j is given

by [e.g., [Antognini, 2015](#)]

$$\tau_{\text{EKL},j} = \frac{16 a_{\text{out}}^3 (1 - e_{\text{out}}^2)^{3/2} \sqrt{(M_\star + m_j)/G}}{15 a_j^{3/2} m_{\text{out}}}, \quad (2.13)$$

where a_{out} is the outer companion semi-major axis. On the other hand, Laplace-Lagrange interactions, which exchange angular momentum between the inner planets, can suppress the outer companion's induced eccentricity excitations [e.g., [Innanen et al., 1997](#), [Takeda and Rasio, 2005](#), [Takeda et al., 2008](#), [Li et al., 2014b](#), [Pu and Lai, 2018](#), [Denham et al., 2019](#)]. For the suppression to occur, the angular momentum exchange due to Laplace-Lagrange interactions, needs to take place on a shorter timescale than the EKL typical timescale. The timescale of the Laplace-Lagrange effect on planet 2's orbit from the gravity of planet 1 is approximately (see Equation (2.1)):

$$\tau_{\text{LL},21,\text{max}} \approx \left[\frac{7n_2 m_1}{4\pi M_\star} \left(\frac{a_1}{a_2} \right)^2 \right]^{-1}. \quad (2.14)$$

[Denham et al. \[2019\]](#) derive an analytical stability criterion indicating the region of parameter space in which a two-planet system is stable against gravitational perturbations from a far away companion. However, short range forces can also tend to stabilize a system [this was noted in the three body systems, e.g., [Ford et al., 2000](#), [Naoz et al., 2013b](#), [Liu et al., 2015](#), [Hansen and Naoz, 2020](#)]. Recently [Wei et al. \[2021\]](#) generalized the aforementioned stability criterion to include general short range forces. In particular, here we consider general relativity precession due to the star².

As was shown in [Wei et al. \[2021\]](#) general relativity can only extend the stability regime beyond what was shown by [Denham et al. \[2019\]](#) if the GR timescale is shorter than the Laplace-Lagrange timescale between the two inner planets. In other words, if the timescale hierarchy is such that $\tau_{\text{GR},2} < \tau_{\text{EKL},2}$, but $\tau_{\text{GR},2} > \tau_{\text{LL},21}$ then the system will be stable against EKL excitation by the outer perturber [as shown by [Denham et al., 2019](#)], but may still be

²Tidal precession is also a relevant short-range force that may work to stabilize the system. General relativity precession is often more important compared to tidal precession [e.g., [Fabrycky and Tremaine, 2007](#), [Liu et al., 2015](#)].

susceptible to detail-dependent excitations by Laplace-Lagrange, as we will discuss in section 2.4.

Considering the population of known two-body systems, we ask in which systems GR effects further help stabilize the system more than the stabilization achieved by Laplace-Lagrange. Observed two-body systems are likely to be stable (or marginally stable) because they have survived long enough for us to observe them. In such a system, if there is an outer perturber, the outermost known planet is affected by the perturber’s EKL excitations more than the inner known planet. Thus, if a hidden companion exists, and GR is assumed not to be relevant, the timescale hierarchy is $\tau_{\text{EKL},2} > \tau_{\text{LL},21}$. Therefore, to examine the importance of GR effects, we must ask when does GR stabilize m_2 against EKL more than Laplace-Lagrange does, i.e., when EKL excitation is involved, GR is necessary to include only when $\tau_{\text{GR},2} < \tau_{\text{LL},21,\text{max}}$, which produces the criterion:

$$\frac{a_1^2}{a_2} \leq \frac{6GM_\star^2}{7c^2(1 - e_{\text{max}}^2)m_1}, \quad (2.15)$$

where we used the approximation in Equation (2.11). Using $\tau_{\text{LL},21,\text{min}}$, we do not have a neat approximation for $(f_\psi + f_{2\psi})$, so the criterion becomes

$$a_1(f_\psi + f_{2\psi}) \leq \frac{6GM_\star^2}{c^2(1 - e_{\text{max}}^2)m_1}. \quad (2.16)$$

In these equations, e_{max} represents the maximum eccentricity induced by the quadrupole-level, EKL oscillation in the presence of general relativity. Aside from e_{max} , none of the terms in these criteria depend on the parameters of the outer perturber inducing the EKL oscillation. In other words, to evaluate whether GR is important when considering EKL oscillation from an outer companion, we only need to consider the eccentricity it induces on m_2 . This is shown in Figure 2.2, where we depict a population of 2-planet systems, with the diagonal lines indicating different choices of e_{max} . Thus, potentially highlighting that for a small but not insignificant fraction of 2-planet population (4-5%), GR plays an important role in stabilizing the system.

Figure 2.2 depicts known two-planet systems, adopted from the **exoplanet archive** [NASA Exoplanet Science Institute, 2025], in the parameter space defined by the plane

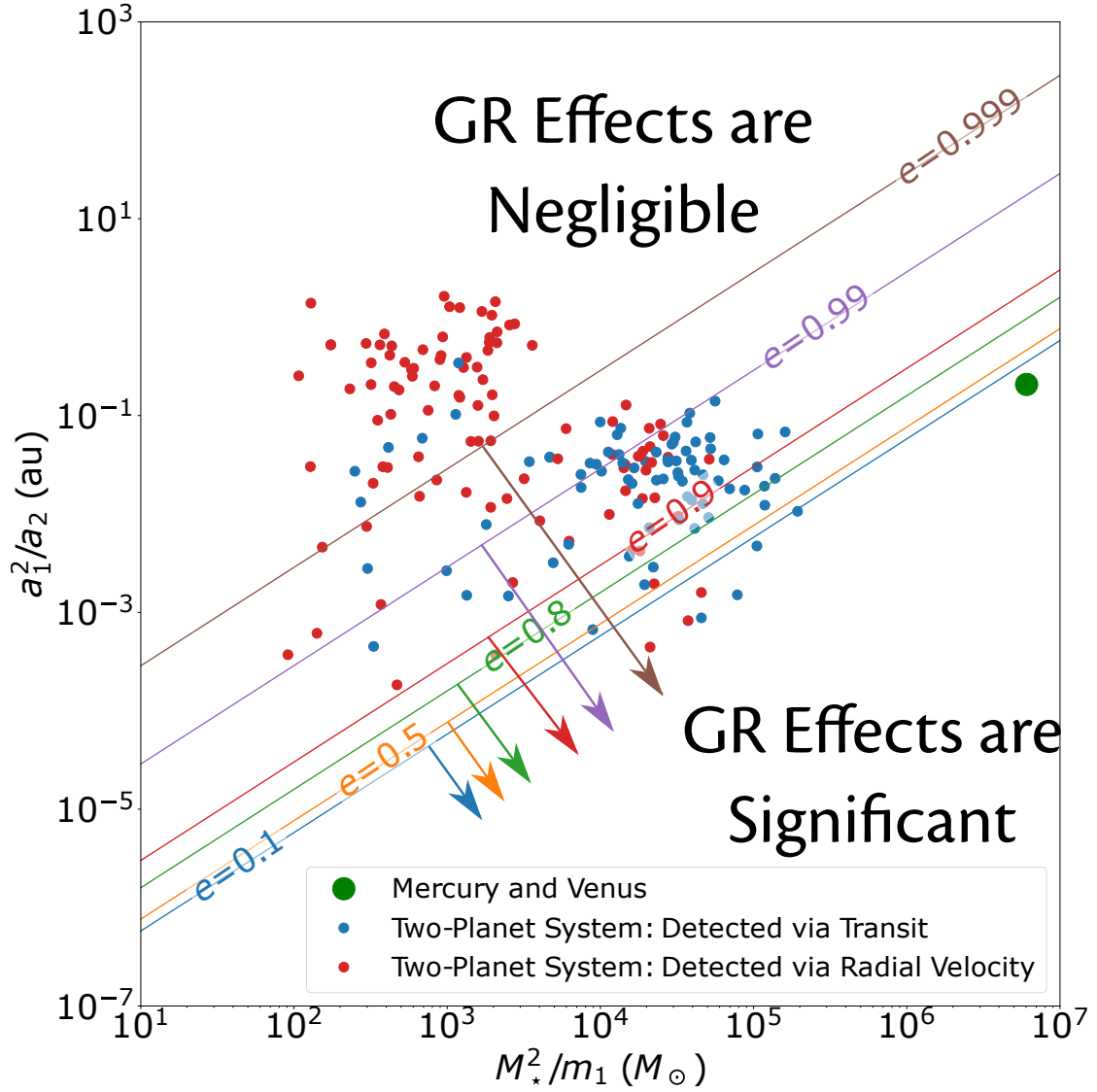


Figure 2.2: Susceptibility of known, two-planet systems to GR necessity. Systems without measured stellar masses are not included. Known two-planet systems (blue and red for planets detected by the Transit Method and the Radial Velocity method, respectively) in the parameter space defined by Equation (2.15), which defines the $a_1^2/a_2 - M^2/m$ plane. Overplotted are lines for different $e_{\max}$ that can be induced by EKL. Below the lines $\tau_{\text{GR}}(m_2) < \tau_{\text{LL}}$. Plotted in green is the Mercury-Venus pair. Systems lying below a given curve have their stability against EKL excitation extended by GR. Notably, At moderate-to-large values $e_{\max}$, the inclusion of GR extends the stability regime of a significant number of systems.

$a_1^2/a_2 - M^2/m$, motivated by Equation (2.15). We have removed systems orbiting binary stars, systems without measurements of stellar mass or where one or both of the two planets lack mass measurements. Overplotted are the boundary lines between GR dominating and Laplace-Lagrange dominating as a function of $e_{\max}$. To first-order, $e_{\max}$ represents the maximum eccentricity that would be obtained within a few EKL or Laplace-Lagrange cycles in the presence of GR precession, which is straightforward to calculate [e.g., [Fabrycky and Tremaine, 2007](#), [Liu et al., 2015](#)]. The value of $e_{\max}$, can be extremely high, even in the presence of GR precession [e.g., [Naoz et al., 2013b](#), [Teyssandier et al., 2013](#), [Li et al., 2014a](#)]. We note that at values of $e_{\max} > 0.8$ much of the parameter space lies below the transition curve, meaning that if $e_{\max}$ is high, GR is making a significant contribution to the stability of a non-negligible portion of the parameter space. It is worthwhile to point out that the Laplace-Lagrange formalism is an expansion in small eccentricities. However, even when higher orders in eccentricity are included, the combination of the GR and Laplace-Lagrange precessions can accurately predicate the stability criterion, as shown in [Wei et al. \[2021\]](#). Note that other processes, especially when high mutual inclinations, and high eccentricities may exist, and higher orders of the approximation can affect the system [e.g., [Naoz, 2016](#)].

2.4 An Inner Companion

Most exoplanet detection methods are sensitive to short orbital periods. For example, the Kepler telescope’s data pipeline could detect planets on a 1.5 year orbit at the longest (because of the pipeline’s requirement of 3 or more transits). Furthermore, Kepler’s methods are insensitive to planets with radii smaller than about $0.5 R_{\oplus}$. For Earth-mass planets it is also insensitive to those with orbital periods longer than about 150 days [see Fig 1 of [Christiansen, 2017](#)].³ Thus, if two planets were detected in a system, an inner planet may still be hidden there with a radius smaller than $R_{\oplus}$. We now investigate the stability of such a planet under perturbations from its companions.

³Stellar and especially stellar activity may push this limit up and down. For example, the $0.354 R_{\oplus}$ planet Kepler 37b was discovered in 2013 orbiting an especially quiet star [e.g., [Barclay et al., 2013](#)].

2.4.1 The Role of GR - Stability Criterion for Inner Companion

We now consider systems where a small (Super-Earth or lighter) planet (m_{in}) orbits within one or more other planets with small mutual inclinations and arbitrary masses (see Figure 2.1). In this scenario, without an outer companion, the EKL mechanism is not pronounced, and the system is non-hierarchical, but significant eccentricities up to ~ 0.4 can still be driven on the super-earth by Laplace-Lagrange effects (see also Section 2.4.2 for example).

Depending on the orbital configuration, this could fatally disrupt this planet. However, if GR precession occurs on a shorter timescale than Laplace-Lagrange excitation, then the eccentricity that can be excited drops significantly. Because we are now concerned with the stability of the innermost planet, we must use the GR precession timescale of the inner planet, $\tau_{\text{GR,in}}$ and require stability against Laplace Lagrange excitations from the further-orbiting m_1 on m_{in} . Note that in Section 2.3, we consider the Laplace-Lagrange interactions between m_1 and m_2 , where the latter is influenced by a distant companion (with an arbitrary geometry). Here we consider a different structure, where m_1 and m_2 are perturbing a hidden planet m_{in} (See Figure 2.1 for an illustration of the system). We wish again to compare the Laplace-Lagrange and GR timescales as we did in Eq (2.15) to investigate stability. However, the new structure changes the semi-major axis dependence of the Laplace-Lagrange timescale (see Eq (2.10) for details), so a new criterion must be derived. The Laplace-Lagrange timescale acting on m_{in} from the perturbations of m_1 is given by Eq. (2.10):

$$\tau_{\text{LL,in1,max}} \approx \left[\frac{7n_{\text{in}}m_1}{4\pi M_\star} \left(\frac{a_{\text{in}}}{a_1} \right)^3 \right]^{-1}, \quad (2.17)$$

$$\tau_{\text{LL,in1,min}} = \left[\frac{n_{\text{in}}m_1}{4\pi M_\star} \left(\frac{a_{\text{in}}}{a_1} \right)^2 (f_\psi + f_{2\psi}) \right]^{-1}. \quad (2.18)$$

Thus, the stability criterion of $\tau_{\text{GR,in}} < \tau_{\text{LL,in1}}$ yields (using $\tau_{\text{LL,in1,max}}$):

$$\frac{a_{\text{in}}^4}{a_1^3} \leq \frac{6GM_\star^2}{7c^2(1 - e_{\text{max}}^2)m_1}, \quad (2.19)$$

and using $\tau_{\text{LL,in1,min}}$)

$$\frac{a_{\text{in}}^3}{a_1^2} (f_\psi + f_{2\psi}) \leq \frac{6GM_\star^2}{c^2(1 - e_{\text{max}}^2)m_1}, \quad (2.20)$$

where $e_{\max}$, is the maximum eccentricity achieved by m_{in} due to Laplace Lagrange resonance.

2.4.2 Example of Resonances in Laplace-Lagrange Secular Evolution

The Laplace-Lagrange secular model has an analytical solution, where the eccentricity and inclination are, as functions of time, algebraic functions of sines/cosines of eigenfrequencies [Murray and Dermott, 2000, p.277-278]. Therefore, there exist some configurations of planets where these eigenfrequencies approach one another in value, driving large eccentricities and inclinations in what we call a "Laplace-Lagrange Resonance". Because the Laplace-Lagrange model assumes eccentricities remain low, and at these resonance points it can predict extreme eccentricity values, the assumptions implicit in Laplace-Lagrange formalism may be violated in these configurations. Thus, n -body simulations may result in higher but not necessarily unity eccentricities. Given two fixed planets, the semi-major axis values where a third planet would be in resonance are not affected by the masses of the two fixed planets. The relative masses of the planets do, however, affect the eccentricity the resonance predicts: if a planet is in resonance and is significantly less massive than its companions, its eccentricity will be excited to larger values.

Angular momentum exchange between planetary orbits via the Laplace-Lagrange perturbations can result in eccentricity excitations even in the absence of an outer companion. In Figure 2.3, bottom panel, we show the maximum eccentricity reached for an Earth-mass planet set at different separations as predicted from Laplace-Lagrange equations. As a case study we consider the HD 15337 system, which is composed of $M_{\star} = 0.9 M_{\odot}$ and has two planets with the following parameters: $a_1 = 0.0522$ au, $m_1 = 7.51 M_{\oplus}$, and $a_2 = 0.1268$ au, $m_2 = 8.11 M_{\oplus}$ [Gandolfi et al., 2019]. In the bottom panel of Figure 2.3, we show the maximum eccentricity predicted by Laplace-Lagrange's aforementioned analytical solution, as a function of a hypothetical inner planet's semi-major axis. This calculation is done in the $m_{\text{in}} \rightarrow 0$ test-particle limit. Outside of this approximation, if m_{in} is set close to m_1 and m_2 , the height of the resonance peaks decrease, but their locations as a function of a_{in} do not change.

We consider a test case of two specific points, one on-resonance, and one off-resonance. Here "on-resonance" refers to a configuration where multiple frequencies in the A matrix (2.2) are close or equal in value, resulting in large predicted eccentricities for one (or more) of the planets in the system. For both of them we use the n -body code **MERCURY**, integrating over 5×10^5 yrs [Chambers, 1999]. This version of **MERCURY**, accounts for general relativity precession from the 1st post-Newtonian term, (M. Payne private communication). Here the Earth-mass planet eccentricity is initially set with $e = 0.1$, and negligible mutual inclination with the known planets in the system. As depicted (top left panel in Figure 2.3), over most of the parameter space, the eccentricity is excited up to about 0.2–0.3, with fluctuation around that eccentricity possible in a configuration of Laplace-Lagrange resonance. On the other hand, (top right panel in Figure 2.3), when the GR precession timescale is shorter than the typical Laplace-Lagrange precession, these eccentricity excitations is suppressed. In general, we find that regardless of whether this system is set up in a resonant configuration or not, when general relativity precession is included in the simulations, its timescale is short enough (by a factor of about 200 in the resonant configuration) to dominate over Laplace-Lagrange and force the eccentricity to remain at its initial value. We also note that general relativity precession is indeed damping out the Laplace-Lagrange resonance rather than simply shifting the eccentricity peak to another location.

On the margins of the Laplace-Lagrange timescale/GR timescale equality, whether a resonance can destabilize a system where the GR timescale and the Laplace-Lagrange timescale differ by only a factor of unity is detail-dependent⁴. However, we find that when the GR timescale is hundreds to thousands times shorter than the Laplace-Lagrange timescale, even resonances tend to be damped out.

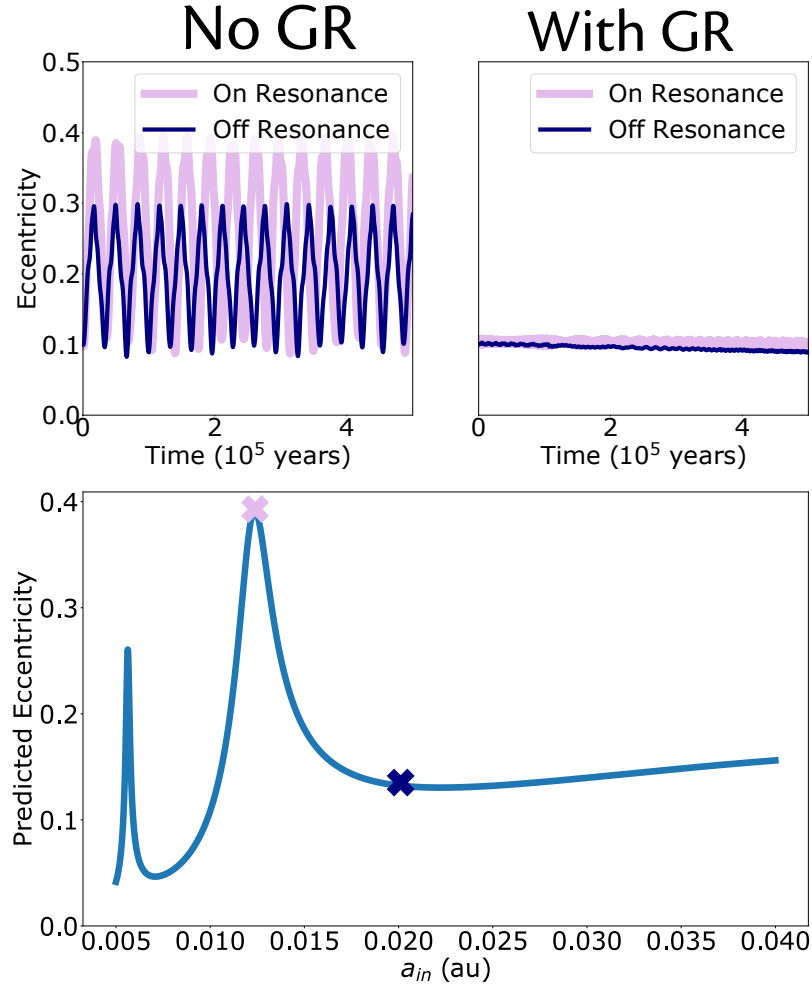


Figure 2.3: An Earth-mass planet was inserted within HD 15337’s two orbits at a semi-major axis of peak resonance, 0.0125 au (marked in light pink), and once off-resonance at 0.02 au (marked in dark blue) both with an initial eccentricity of $e_{\text{in}} = 0.01$ and integrated forward 50,000 years using the *n*-body code **MERCURY**. HD 15337 is a $0.9 M_{\odot}$ star that hosts two known planets with $a_1 = 0.0522$ au, $m_1 = 7.51 M_{\oplus}$, and $a_2 = 0.1268$ au, $m_2 = 8.11 M_{\oplus}$ [Gandolfi et al., 2019]. In the top panels the eccentricities both with (right) and without (left) including GR are plotted. In the bottom panel we show the maximum eccentricity the Laplace Lagrange formalism predicts the planet should achieve over long timescales. The behavior of the eccentricity of the test planet is strongly effected by the predicted resonance, and in the presence of GR, the resonance is completely damped out.

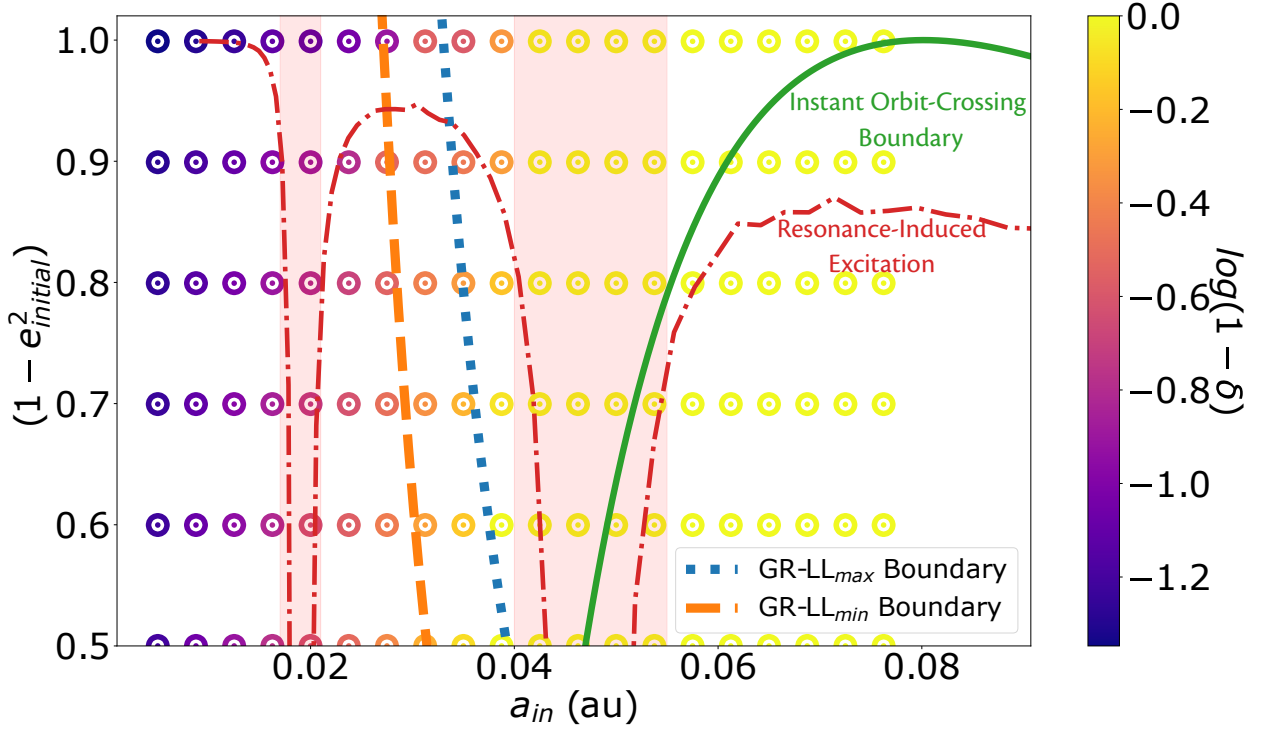


Figure 2.4: Stability map of a hypothetical-inner planet. In particular, we consider the HD 106315, composed out of two planets, HD 106315 b (c) with mass $12.6 M_{\oplus}$ ($15.2 M_{\oplus}$), semi-major axis 0.097 au (0.154 au). We show the inner planet’s initial specific angular momentum, as $1 - e_{\text{initial}}^2$ as a function of it’s semi-major axis a_{in} (here, e_{initial} spans the range from 0.01 at the top, to 0.71 at the bottom). The mass of the inner planet is $0.1 M_{\oplus}$. The points show the numerical integration of the system up to 1 Myr, where the color code represents how close the two orbits came to crossing over the integration according to Eq. (2.21). The dot-dashed red line shows the Laplace-Lagrange prediction of the hidden planet’s eccentricity, and regions where this eccentricity is especially high—the resonances—are shaded red. We over plot the GR-Laplace-Lagrange boundary according to Eqs. (2.19) and (2.20), thick, dashed orange, and dotted blue, respectively. We expect that to the left of these boundaries systems GR precession will suppress eccentricity excitations due to Laplace-Lagrange resonances, and to the right of these boundaries, resonances may lead to orbit crossing. Finally, the green line shows the orbit crossing limit.

2.4.3 Numerical Comparison

Next we test our stability criterion, Equation (2.19) for the inclusion of a hidden inner planet against secular simulations. We utilize the Gauss averaging method to integrate the system, while including 1st Post Newtonian GR precession [c.f. Wei et al., 2021]. In this approximation the line density of an orbit is inversely proportional to the velocity [e.g., Touma et al., 2009]. This type of analysis was shown to be very efficient in calculating long-term evolution [e.g., Nesvold et al., 2016, Denham et al., 2019, Wei et al., 2021, Michtchenko and Malhotra, 2004]. The secular code allows us to study the system for long timescales as we note that the secular code agrees with the n -body code MERCURY, [Chambers, 1999] with general relativity (M. Payne private communication).

As a case study, we consider the HD 106315 system [Zhou et al., 2018b, Livingston et al., 2018], which consist of a $1.024 M_{\odot}$ star and two known planets: HD 106315 b (c) with mass $12.6 M_{\oplus}$ ($15.2 M_{\oplus}$), semi-major axis 0.097 au (0.154 au), and eccentricity 0.093 (0.22) [e.g. Barros et al., 2017, Mayo et al., 2018, Crossfield et al., 2017, Rodriguez et al., 2017]. We consider a possible hypothetical inner planet with $m_{\text{in}} = 0.1 M_{\oplus}$ and explore the stable configuration of the system.

In Figure 2.4, we depict the hidden inner most planet’s initial specific angular momentum ($1 - e_{\text{initial}}^2$), as a function of its semi-major axis a_{in} . Following Denham et al. [2019], we quantify how close the orbits grow over the course of the simulation as:

$$\delta = \frac{a_1(1 - e_1) - a_{\text{in}}(1 + e_{\text{in},\text{max}})}{a_1 - a_{\text{in}}} , \quad (2.21)$$

shown through the coloring of the dots. We highlight the locations of the Laplace-Lagrange resonances (the shaded regions), and the Laplace-Lagrange predicted eccentricity at all a_{in} values (the dot-dashed red line). We overplot our stability criterion, from Equation (2.19) as the dotted thick line as well as a modified version of the criterion where we use the minimum Laplace-Lagrange timescale instead of the maximum (dashed line, see Eq. (2.20)).

⁴Similarly to the resonant-like behaviour that takes place when GR precession on a similar timescale to that of a quadrupole-level hierarchical three body system [e.g. Naoz et al., 2013b].

To the right of this boundary where Laplace-Lagrange dominates, we see that the secular resonance on the inner planet is enough to drive orbit crossing ($\delta \rightarrow 0$), but on the left side of the boundary, where GR dominates, the secular resonance does not induce any additional eccentricity—even when initial eccentricities are set to be as high as $1 - e_{init}^2 = 0.5$. We also see that the difference between using the minimum Laplace-Lagrange timescale instead of the maximum as we do in Equation (2.19) does not make a significant difference in the location of the boundary between Laplace-Lagrange and GR dominance.

To the right of the large resonance we can see in the low initial eccentricity runs that although they are beyond the resonant region, enough eccentricity is excited to achieve orbit crossing. This arises from the structure of the resonances. In systems with exactly two planets, where there are two resonances within the orbit of the closest-in planet, typically the region between the second resonance and a_1 , has a maximum eccentricity higher than the region inward to the first resonance (as also shown in Figure 2.4, dot-dashed line, note that this effect is less pronounced in Figure 2.3, but still present). This behavior can drive orbital crossing even outside the resonant region.

2.4.4 Observational Implications

Figure 2.5 depicts the largest possible semi-major axis (a_{in}) that a hypothetical, earth-analogue inner planet could have while still remaining GR-dominated for an observed system. We show this as a function of the semi-major axis of the closest-in observed planet in the system (a_1). We limit the sample to planets orbiting main-sequence stars with two or fewer known planets. The points' colors represent the mass of the known planet, m_1 . Below the points represents regions of parameter space where an inner planet would be GR dominated, above—Laplace-Lagrange dominated. For each star, a boundary exists where transiting planet can be detected (see Eq. (2.22)).

We use a simplified model of the Kepler mission pipeline's candidate confirmation requirement that the Multiple Event Statistic (MES) must be greater than 7.1 [Christiansen

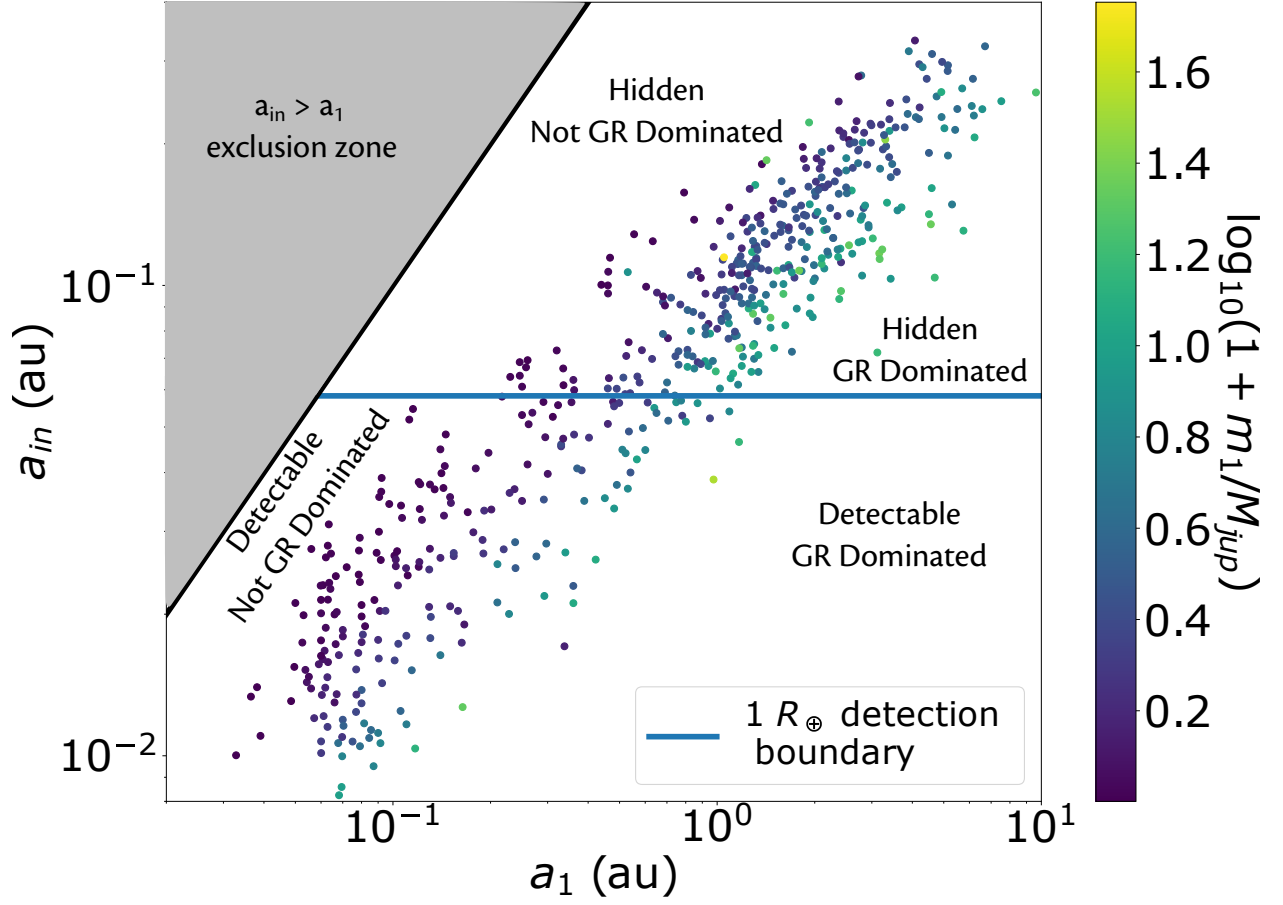


Figure 2.5: The relationship between the semi-major axis of the shortest observed orbit in a system (a_1) and the largest possible semi-major axis a hypothetical inner planet could have while still being GR dominated (see Eq. (2.19)). Each dot represents a known main sequence star with either two or one known planets. The color represents the mass of the known planet m_1 in log. Overplotted is the boundary that separates the region where a $1 R_\oplus$ inner planet would be detectable, via transit, according to Eq. (2.22) if orbiting a sunlike star. Closer orbits (smaller a_{in}) are more likely to be detected via transits and RVs. The region of parameter space where $a_{in} > a_1$ has been grayed out as it is excluded by assumption. The inner planets were assumed to share Earth’s mass and radius.

et al., 2012]. The MES is given approximately by

$$\text{MES} \approx \left(\frac{R_p}{R_\star} \right)^2 \frac{1}{\text{CDPP}_{\text{eff}}} \sqrt{N_t}, \quad (2.22)$$

where $R_\star$ (R_p) is the stellar (planet) radius, CDPP_{eff} is the effective stellar noise in parts per million, and N_t is the number of transits the planet makes over the observing period. While this boundary depends on the stellar radius, as a proof-of-concept, here we over plot the semi-major axis a_{in} beyond which an earth-radius planet would be undetected, for a $1 R_\odot$ ⁵.

The dispersion around the trend reveals that among the set of main sequence stars around which planets have been found, the widest orbit an inner planet can have and remain GR dominated is determined by a_1 to within about an order of magnitude. Importantly, *in this case*, we need only information about one known planet in order to mark the locations in parameter space where a currently undetected planet may be hidden as is done in Figure 2.5. Therefore, by needing very little information as depicted in Figure 2.5, the number of systems to which this criterion is applicable is maximized.

For a particular system, the area above the detection boundary and below that system's associated point, marks the region of parameter space that is both hidden from current observation and is GR dominated. As we show in Figure 2.4, the GR dominated regime stabilizes such planets against Laplace-Lagrange eccentricity excitation. This Hidden and GR Dominated region of parameter space is more stable than non-GR dominated regions and more likely to contain planets than unstable regions. Notably, higher values of the known planets mass m_1 increases the strength of Laplace-Lagrange interactions between it and the inner planet. Thus, the largest allowable a_{in} where the inner planet is GR dominated must shrink as m_1 increases.

Notably, higher values of the known planet's mas m_1 make the largest GR dominated a_{in} in their systems smaller, thus shrinking the Hidden+GR Dominated portion of parameter space. Among systems where m_1 is less than a Jupiter mass, this region is quite a bit larger.

⁵Note that the stellar radii of the observed systems in Figure 2.5, have a typical radius of $1 R_\odot$, up to about a factor of 2.

2.5 Kepler-56: Proof of Concept (Outer and Inner Companions)

Kepler-56 is a $1.32 M_{\odot}$ red giant branch star, hosting three known planets [e.g. Huber et al., 2013, Otor et al., 2016, Lissauer et al., 2011, Hadden and Lithwick, 2014]. These planets feature a significant hierarchical structure—the semi-major axes of the three planets are 0.103, 0.165 and 2.16 au, with masses of $0.07 M_J$ and $0.57 M_J$ for the inner planets, respectively. The inner two planets exhibit large obliquity with respect to the star’s spin axis [Huber et al., 2013]. The outer planet is non-transiting, with an $M \sin i$ value of $5.61 M_J$ [e.g. Otor et al., 2016]. The two inner planets have small eccentricities (< 0.05), indicating that EKL excitations from Kepler-56d are not destabilizing this system, but can result in the large spin-orbit misalignment [Li et al., 2014b]. The outer planet has eccentricity of ~ 0.2 [Otor et al., 2016]. Indeed, applying the criterion defined in Denham et al. [2019], we find that the critical eccentricity of Kepler-56 d above which we expect large eccentricity fluctuation of these planets is ~ 0.83 , much larger than its reported eccentricity.

Using Eq. (2.19), we can constrain a hypothetical Earth-radius planet in Kepler-56 to lie at less than $\lesssim 0.07$ au, depending on this planet’s initial eccentricity. However, we find that a Mercury-radius planet with sufficient eccentricity can remain GR dominated at orbits wider than 0.08 au ⁶ but probably cannot be wider than 0.088 au , which represents about 5 mutual Hill radii from Kepler-56 b [the spacing that can yield stability against planet-planet scattering Chatterjee et al., 2008]. From Eq. (2.22) we see that, such a planet could transit and remain undetected at orbits comparable to its Roche limit with the host star.

Because Kepler-56 contains a known outer planet, we observe the behavior of the system after the insertion of a hypothetical inner planet in Figure 2.6. We simulate the system both with and without GR in four configurations depicted in the middle column of Figure 2.6. The first configuration has no inner planet, and the known outer planet has been removed. The second brings back the outer planet, and its eccentricity has been increased to $e_{\text{out}} = 0.8$ to exaggerate its effect for proof of concept purposes. The third and fourth config-

⁶We note that the stability limits uses Eq. (2.11), which is valid when $a_1/a_2 < 0.9$, which somewhat limits the extent of the use of the equation.

urations introduce a hypothetical inner companion first at an orbit close to its neighboring planets where the GR and Laplace-Lagrange timescales are roughly equal, and second at an orbit close to the host star where the GR timescale dominates. For each configuration we plot the inclination of all the planets and their "orbital range" or the location of each planet's periapsis and apoapsis as functions of time. If one planet's periapsis overlaps with another's apoapsis, we conclude the system will undergo an eventual orbit crossing and is likely unstable. The EKL timescale is smaller than both in all configurations, but because of Laplace-Lagrange and GR having weak influence over inclinations, the outer planet still causes large, synchronized inclination shifts. Broadly, we find that this system is quite stable even with the outer planet's eccentricity artificially increased. However, we find that in the third configuration, the hypothetical planet does undergo orbit crossing after 500,000 years, but only if GR is neglected. Because the outer known planet's eccentricity has been measured to be about $e_{\text{out}} = 0.2$ [Otor et al., 2016], the system is even more stable than these plots show. This modified system is a valuable test case to show how a multiplanet system can be stable in the presence of an extreme companion (where we have adopted a higher eccentricity for the outer planet to more clearly demonstrate its effect) and a possible inner planet. We further demonstrate in Figure 2.6 third row that GR precession extends the stability regime.

2.6 Discussion

Our current exoplanet detection methods are less effective at detecting smaller planets and far-orbiting planets than larger or closer-in planets. Already, for example, it is estimated that about 50% of stars may host Jupiter-like planets at wide separations [Bryan et al., 2016, 5 – 20 au]. In general it was suggested that a population of giant planets often host one or more inner planets [e.g., Knutson et al., 2014, Konopacky et al., 2016, Zhu and Wu, 2018, Bryan et al., 2016, 2019]. Further, some close-in small planets have already been observed [e.g., Sanchis-Ojeda et al., 2014a, Xiu-Min and Jiang-Hui, 2020a, Dai et al., 2017a].

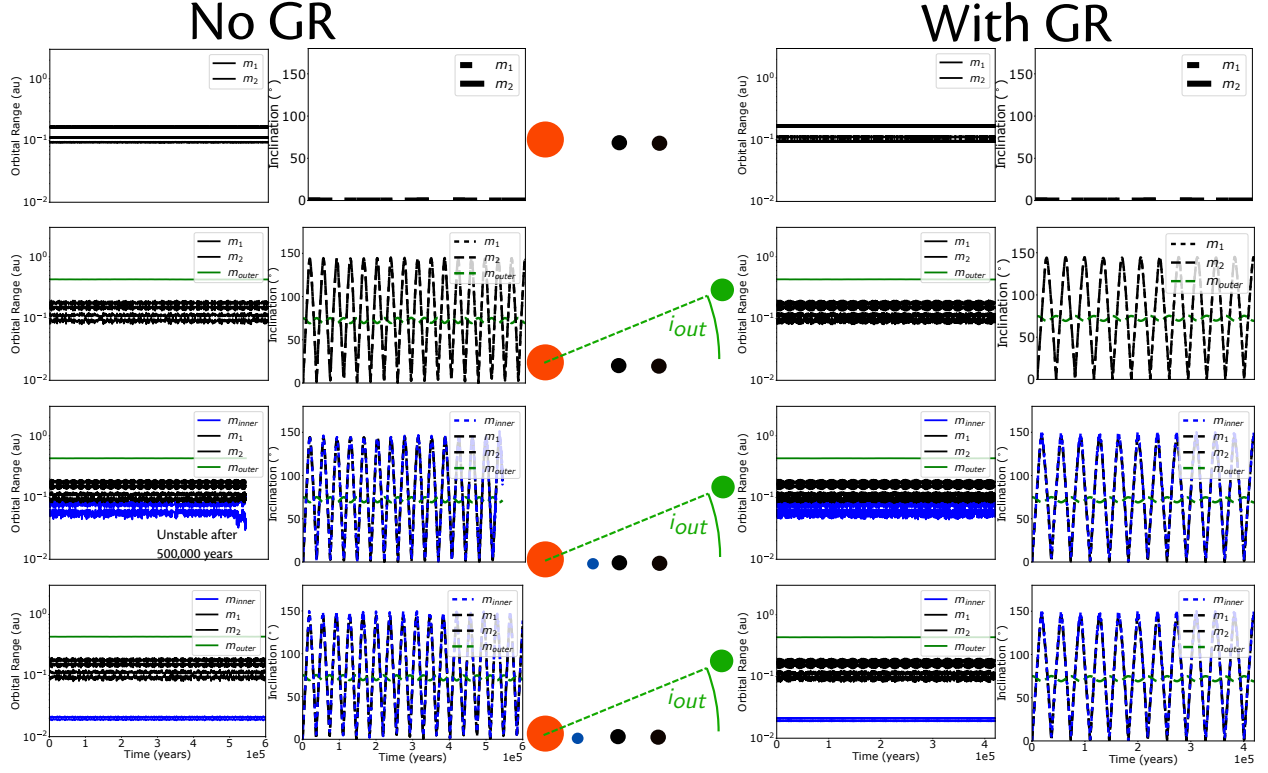


Figure 2.6: Orbital outcomes of our Kepler 56-like system in 4 configurations, with all 4 configurations run once without GR and once with GR. Top to bottom (as indicated by the cartoons in the middle) are the eccentricities and inclinations of the constituent planets when (i) The outermost planet is removed (ii) The outermost planet is included (iii) An Earth-mass planet is inserted into the system close to the innermost planet (iv) An Earth-mass planet is inserted into the system close to the star. Kepler 56 is a $1.32 M_{\odot}$ star that hosts three known planets with $a_1 = 0.1028$ au, $m_1 = 0.58 M_J$, $e_1 = 0.04$, $a_2 = 0.165$ au, $m_2 = 0.57 M_J$, $e_2 = 0.01$, and $a_3 = 2.16$ au, $m_3 = 5.61 M_J$ [Huber et al., 2013, Otor et al., 2016]. We set $e_3 = 0.8$ rather than its measured value of about 0.2 for the purposes of proof-of-concept. The outer planet’s inclination was set to $i_{out} = 75^\circ$. The inner planet is one Earth-mass and starts the integrations with negligible eccentricity

Here we study the stability of known, at most two-planet systems, hosting either a far away companion or a inward small planet.

We have quantified a condition under which the addition of a new planet into a known system does not destabilize the system, considering both inner and outer hypothetical planets. A far away companion, may excite the inner system’s eccentricities, via the EKL mechanism driving them to instability. However, Laplace-Lagrange interactions between neighboring planets can tend to stabilize the system [Pu and Lai, 2018, Denham et al., 2019]. Recently Wei et al. [2021] showed that GR precession expands the stability regime. Here we consider all systems with more than one observed planet, and constrain the regimes where GR precession expands the stability of the system against EKL excitation more than Laplace-Lagrange on its own (depicted in Figure 2.2). This regime depends on the maximum eccentricity excited via EKL from the far-away companion which often can be extremely high [e.g., Li et al., 2014a].

Further, we developed analytic criteria for the stability of an inner planet (see Equations (2.19) and (2.20)). In this case, motivated by Kepler sample of “peas in the pod” [Weiss et al., 2018b], we assume a low-inclination planet inwards of a known planetary system. Laplace-Lagrange resonances can drive this inner planet to high eccentricity values. General relativity precession can suppress these eccentricity excitations, as highlighted in Figure 2.3. In some cases, Laplace-Lagrange resonances may drive the system to instability, i.e. orbital crossing. We show that our analytical criteria (Equations (2.19) and (2.20)) are in good agreement with numerical calculation, as depicted in Figure 2.4.

Using our analytical criterion we examined all systems with either one or two planets and estimated the largest possible, stable semi-major axis of a hypothetical planet (see Figure 2.5). A planet could be hidden in a system that is only Laplace-Lagrange dominated, but if a resonance exists, some locations may be unstable. On the other hand, the GR precession dominated regime will suppress Laplace-Lagrange resonances, making it a stable location for short period planets to exist (right side in Figure 2.5).

We note that stellar oblateness (also known as J2) can induce nodes precession of an

ultra short planet. It was suggested that this precession may even result in inducing small mutual inclination between multi-planet systems [$\lesssim 40^\circ$ e.g., [Becker et al., 2020](#), [Li et al., 2020](#), [Schultz et al., 2021](#)]. As highlighted in [Wei et al. \[2021\]](#), stability analysis can be done for wide variety of short-range forces. Thus, it is straightforward to estimate the induced precession due to the stellar rotation. We find that for a Sun-like star system, assuming a Sun-like rotation rate (~ 25 d period), the oblateness-induced precession timescale is much longer over most part of the parameter space (e.g., in [Figures 2.4 and 2.5](#)). However, we note that G-type stars probably slow their spin during their evolution due to magnetic braking [e.g., [Dobbs-Dixon et al., 2004a](#)]. Thus, it may be that the oblateness-induced precession was more significant compared to GR precession, for young, fast spinning, stars.

Finally, we highlight the relevance of GR precession by considering the system, Kepler-56 [[Huber et al., 2013](#)], and starting with two-planet systems, we systematically add planets (see [Figure 2.6](#) while comparing to evolution with and without GR. We first (second panel) add Kepler-56d, although for a proof-of-concept we adopt a higher eccentricity for it, 0.8. The system indeed is stable to even such high eccentric companion [e.g., [Denham et al., 2019](#)]. We then add a smaller planet inward to Kepler-56b⁷, and show two examples. One of which GR precession suppress eccentricity excitations, due to a resonant location, and another one, off-resonance. As shown in [Figure 2.6](#), a combination of the Laplace-Lagrange and GR precessions stabilize an extreme Kepler-56 like system. In particular, an even more eccentric outer companion than the one observed can still keep the system stable [consistent with [Denham et al., 2019](#)]. Moreover, the system can hide a hypothetical small companion, inward to Kepler 56b, and remain stable.

The aforementioned examples highlight the application of our analytical stability criteria in constraining the possible configurations of hidden planets. In particular, GR precession induced by the star can increase the stabilization of a multi-planet system against perturbation from a far away companion. Further, we showed that GR precession could also

⁷Note that such a planet will not survive Kepler-56 radial expansion as it continues to evolve [[Li et al., 2014b](#)], but it may result in some ejections from the star [e.g., [Stephan et al., 2020](#)].

stabilize an ultra-short period hidden planet, even in the presence of resonances induced by a multi-planet system.

CHAPTER 3

The Effect of Evolving J_2 on the Resonant Structure of a Three-Planet System

3.1 Introduction

It has been shown over the past two decades that systems of tightly packed period planets, inwards to the orbit of Mercury, are common in our galaxy [e.g., [Howard et al., 2010, 2012](#), [Lissauer et al., 2011](#), [Winn and Fabrycky, 2015](#)]. Whether accompanied by far-orbiting companion planets or not, these systems are home to myriad interesting dynamics from planet-planet interactions to star-planet interactions [e.g., [Faridani et al., 2022](#), [Wei et al., 2021](#), [Fabrycky et al., 2007](#), [Naoz et al., 2013a](#), [Liu et al., 2015](#), [Mardling and Lin, 2004](#), [Plavchan et al., 2015](#), [Spalding and Batygin, 2016](#), [Becker and Adams, 2017](#)].

Among these systems exists a population of ultra-short period planets (USPs) which have orbits lasting 24 hours or fewer. Although these planets are intrinsically rare [orbiting only about one star in two hundred [Sanchis-Ojeda et al., 2014b](#)], they are significantly easier to detect than planets with similar mass/size at longer orbital periods because of their frequent transits [[Sanchis-Ojeda et al., 2013](#)], larger radial velocity signals [[Pepe et al., 2013](#)], and high transit probability [[Winn et al., 2011](#)]. As a result, many have been observed and characterized, and they make up a disproportionately high fraction of the known rocky planets with precisely measured densities [e.g., [Jackson et al., 2013](#), [Sanchis-Ojeda et al., 2014b](#), [Adams et al., 2016, 2017](#), [Dai et al., 2017b, 2018](#), [Winn et al., 2018](#), [Malavolta et al., 2018b](#), [Frustagli et al., 2020b](#), [Xiu-Min and Jiang-Hui, 2020b](#)]. USPs are also often found with further-orbiting companion planets [e.g., [Sanchis-Ojeda et al., 2014b](#), [Adams et al.,](#)

2021] Over the past 10 years, already over a hundred of detections of such planets and planet candidates have been made [Sanchis-Ojeda et al., 2014b, Xiu-Min and Jiang-Hui, 2020b, Dai et al., 2017b], with over four hundred TESS Project Objects of Interest with sub-1 day periods [ExoFOP, 2023].

Observations suggest a similar formation pathway for USPs and their longer period multiple planets counterparts [e.g., Mulders et al., 2016, Winn et al., 2017]. It was suggested, for example, that secular chaos or disk migration may be promising processes in producing such short period planets [e.g., Petrovich et al., 2019b, Becker et al., 2021]. Although most models predict that they do not form completely *in situ*, at what stage in their evolution they are brought close to the star remains unclear [e.g., Millholland and Spalding, 2020b]. Shortly after the population of small ultra-short-period planets was revealed by the Kepler mission, it was theorized that they could be the remnants of tidally disrupted giant planets [e.g., Jackson et al., 2013, Armstrong et al., 2020], however the metallicity distribution of the host stars is different from that of Hot Jupiter hosts [Winn et al., 2017]. Furthermore, Lee and Chiang [2017] suggested that planets on the inner edge of the disk could be migrated inward via stellar tides. However, stellar tides alone likely cannot be the cause because the quality factors required are not consistent with observations [e.g., Penev et al., 2012, Pu and Lai, 2019b, Petrovich et al., 2019b]. Planetary tides can produce short enough periods to explain USPs' migration through, for example, following high-eccentricity migration, low-eccentricity migration, and obliquity-driven tidal runaway [e.g., Petrovich et al., 2019b, Pu and Lai, 2019b, Millholland and Spalding, 2020b].

USPs can have significant orbital misalignments in multiplanet systems potentially explicable by the stellar quadrupole moment J_2 [e.g., Becker et al., 2020, Brefka and Becker, 2021, Li et al., 2020, Chen et al., 2022]. The existence of the observed radius gap from 1.5-2.0 Earth radii [e.g., Fulton et al., 2017, Petigura, 2020b, Loyd et al., 2020b] may be connected to USPs via the large stellar flux they receive, driving off their atmospheres and lowering their radii [e.g., Frustagli et al., 2020b, Malavolta et al., 2018b].

Orbiting so close to their host stars, USPs are uniquely affected by short-range forces that

are less significant in spread-out systems like the Solar System. One such force arises from the non-sphericity and non homogeneity of the host star, captured in the multipole expansion of the star’s gravitational potential and represented by the moments J_n . To leading order, the most impactful moment is the gravitational quadrupole moment J_2 (hereafter referred to simply as J_2). A typical Sun-like star’s J_2 is primarily created by centrifugal forces arising from its spin. As a star ages and its spin slows due to magnetic braking, its J_2 decreases as well—all else equal. Importantly, a star’s J_2 can have a significant impact of the structure and evolution of close-orbiting planets [e.g., [Spalding and Batygin, 2016](#), [Becker et al., 2020](#), [Brefka and Becker, 2021](#), [Chen et al., 2022](#)].

There are mainly two approaches adopted in the dynamical analysis of USPs and multi-planet systems. One is exploring the planet-planet interactions in multi-planet system [often including short range forces such as tides or general relativity [Liu et al., 2015](#), [Denham et al., 2019](#), [Wei et al., 2021](#), [Faridani et al., 2022](#)]. The other considers the effect of the aforementioned stellar quadrupole [e.g., [Spalding and Batygin, 2016](#), [Becker and Adams, 2017](#), [Spalding et al., 2018](#), [Becker et al., 2020](#), [Li et al., 2020](#), [Brefka and Becker, 2021](#), [Chen et al., 2022](#)]. Thus, a natural question is how USPs and short-period planets are dynamically affected by these short-range forces when they exist within multi-planet systems.

Here we show that the planet-planet interactions in a multi-planet system combined with evolving stellar spin can affect the stability and detectability of (ultra-)short-period planets. The planet-planet interactions excite high eccentricities and inclinations for specific USPs separations via Laplace-Lagrange resonance. However, stars spin down via magnetic braking as they evolve [e.g., [Dobbs-Dixon et al., 2004b](#)]. As the star spins down, its quadrupole moment, J_2 , decreases, thus, sweeping the resonance locations from high semimajor axes to low. In other words, a short-period planet located in a stable zone, may undergo eccentricity and inclination excitation that may push it beyond transiting limits, or even destabilize the system.

The paper is structured as follows. In section [3.2](#) we outline the mathematical formalism quantifying how J_2 and GR affect the orbital evolution of the system. In section [3.3](#) we

show the impact of the secular resonances that arise in multi-planet configurations and how decreasing J_2 values over time from stellar magnetic braking can shift and strengthen or weaken the effect of secular resonances on close-orbiting planets. In section 3.4 we give a summary of how we treat (in)stability of short-period planets in our simulations. In section 3.5 as a case study, we run a Monte Carlo suite of simulations on a proof-of-concept system based on HD 106315 and show the effects of J_2 on the stability and detectability of a potential planet orbiting inward of the planets observed in that system. In section 3.6 we explore the observed system HD 106315 and mark where in that system an inner, Earth-mass planet could remain stable and transit. Finally, in section 3.7 we summarize our conclusions.

3.2 Key physical processes

Motivated by current observations we adopt a multi-planet system with small mutual inclinations and eccentricities [e.g., Weiss et al., 2018b], orbiting a star with mass M . The time derivatives of e , I , Ω and ϖ on a planet indexed by j from Laplace-Lagrange are given by [e.g., Murray and Dermott, 2000]

$$\begin{aligned} \dot{e}_j &= -\frac{1}{n_j a_j^2 e_j} \frac{\partial \mathcal{R}_j^{(\text{sec})}}{\partial \varpi_j}, & \dot{\varpi}_j &= +\frac{1}{n_j a_j^2 e_j} \frac{\partial \mathcal{R}_j^{(\text{sec})}}{\partial e_j}, \\ \dot{I}_j &= -\frac{1}{n_j a_j^2 I_j} \frac{\partial \mathcal{R}_j^{(\text{sec})}}{\partial \Omega_j}, & \dot{\Omega}_j &= +\frac{1}{n_j a_j^2 I_j} \frac{\partial \mathcal{R}_j^{(\text{sec})}}{\partial I_j}, \end{aligned} \quad (3.1)$$

where a_j , e_j , and I_j represent the semimajor axis, eccentricity, and inclination of planet j respectively. $\mathcal{R}^{(\text{sec})}$ represents the averaged, secular disturbing function, which, in the Laplace-Lagrange, is given by,

$$\begin{aligned} \mathcal{R}_j^{(\text{sec})} &= n_j a_j^2 \left[\frac{1}{2} A_{jj} e_j^2 + \frac{1}{2} B_{jj} I_j^2 \right. \\ &\quad + \sum_{k=1, k \neq j}^N A_{jk} e_j e_k \cos(\varpi_j - \varpi_k) \\ &\quad \left. + \sum_{k=1, k \neq j}^N B_{jk} I_j I_k \cos(\Omega_j - \Omega_k) \right], \end{aligned} \quad (3.2)$$

where

$$A_{jj} = \frac{n_j m_k}{4\pi (M + m_j)} \alpha_{jk} \bar{\alpha}_{jk} f_\psi , \quad (3.3)$$

$$A_{jk} = -\frac{n_j m_k}{4\pi (M + m_j)} \alpha_{jk} \bar{\alpha}_{jk} f_{2\psi} , \quad (3.4)$$

$$B_{jj} = -\frac{n_j m_k}{4\pi (M + m_j)} \alpha_{jk} \bar{\alpha}_{jk} f_\psi , \quad (3.5)$$

$$B_{jk} = \frac{n_j m_k}{4\pi (M + m_j)} \alpha_{jk} \bar{\alpha}_{jk} f_\psi , \quad (3.6)$$

and,

$$\alpha_{jk} = \min \left(\frac{a_j}{a_k}, \frac{a_k}{a_j} \right) , \quad (3.7)$$

$$\bar{\alpha}_{jk} = \min \left(\frac{a_j}{a_k}, 1 \right) , \quad (3.8)$$

and

$$f_\psi = \int_0^{2\pi} \frac{\cos \psi}{\left(1 - 2 \left(\frac{a_1}{a_2} \right) \cos \psi + \left(\frac{a_1}{a_2} \right)^2 \right)^2} d\psi , \quad (3.9)$$

and

$$f_{2\psi} = \int_0^{2\pi} \frac{\cos 2\psi}{\left(1 - 2 \left(\frac{a_1}{a_2} \right) \cos \psi + \left(\frac{a_1}{a_2} \right)^2 \right)^{\frac{3}{2}}} d\psi . \quad (3.10)$$

The effect of General Relativity (GR) precession on the ω of a planet with semimajor axis a and eccentricity e is given by [e.g., [Naoz et al., 2013b](#)]

$$\dot{\omega}_{GR} = \frac{3(GM_\star)^{3/2}}{c^2 a^{5/2} (1 - e^2)} , \quad (3.11)$$

where c is the speed of light.

3.2.1 The Stellar Quadrupole Moment (J_2)

The rotation of a star deforms its surface, causing it to deviate from sphericity with the axis of rotation being an axis of symmetry. This deformation augments its potential, which can

be written as [e.g., [Murray and Dermott, 2000](#)]

$$V_{\text{gravity}}(r, \theta) = -\frac{GM_{\star}}{r} \left[1 - \sum_{n=2}^{\infty} J_n \left(\frac{R_{\star}}{r} \right)^n \mathcal{P}_n(\cos \theta) \right], \quad (3.12)$$

where $M_{\star}$ and $R_{\star}$ are the mass and radius of the star, $\mathcal{P}_n$ is the n th Legendre polynomial, and J_n are dimensionless constants defined by

$$J_n = +\frac{1}{M_{\star} R_{\star}^n} \int_0^{R_{\star}} \int_{-1}^{+1} r^n \mathcal{P}_n(\mu) \rho_{\star}(r, \mu) 2\pi r^2 d\mu dr, \quad (3.13)$$

where $\mu = \cos \theta$ and $\rho_{\star}$ is the star's density. Of particular note is the leading coefficient in this multipole expansion J_2 which is called the (stellar) gravitational quadrupole moment, and can be approximated by [e.g., [Spalding and Batygin, 2016](#)],

$$J_2 = \frac{1}{3} k_2 \left(\frac{\Omega}{\Omega_b} \right)^2, \quad (3.14)$$

where k_2 is the stellar Love number, Ω is the stellar spin, and Ω_b is the stellar breakup spin, $\Omega_b = \sqrt{GM_{\star}/R_{\star}^3}$.

The effect of the stellar gravitational quadrupole moment's (J_2) on ϖ and Ω for a planet with semimajor axis a and mean motion n is given by [e.g., [Murray and Dermott, 2000](#)]

$$\begin{aligned} \dot{\varpi}_{J_2} &= +n \left[\frac{3}{2} J_2 \left(\frac{R_{\star}}{a} \right)^2 \right], \\ \dot{\Omega}_{J_2} &= -n \left[\frac{3}{2} J_2 \left(\frac{R_{\star}}{a} \right)^2 \right]. \end{aligned} \quad (3.15)$$

In this work, we will explore the effect of J_2 evolving as a function of time, and must construct its time evolution. Observed, young solar-type stars have spin periods of approximately 1-10 days [e.g., [Gallet and Bouvier, 2013](#)]. We calculated the J_2 evolution of a $1.1 M_{\odot}$ star with two initial spin periods, 1.5 and 8 days. Using a MESA stellar model to calculate the evolution of the star's radius and Love Number [[Paxton et al., 2011, 2013, 2015, 2018, 2019, Jermyn et al., 2023](#)], the star had J_2 values of $\sim 6 \times 10^{-4}$ and 2×10^{-5} at an age of 10^7 years for the 1.5 and 8 day initial spin period cases respectively. The spin evolution was calculated using the relation [e.g., [Dobbs-Dixon et al., 2004b, Qureshi et al., 2018](#)],

$$\dot{\Omega} = -\alpha \Omega^3, \quad (3.16)$$

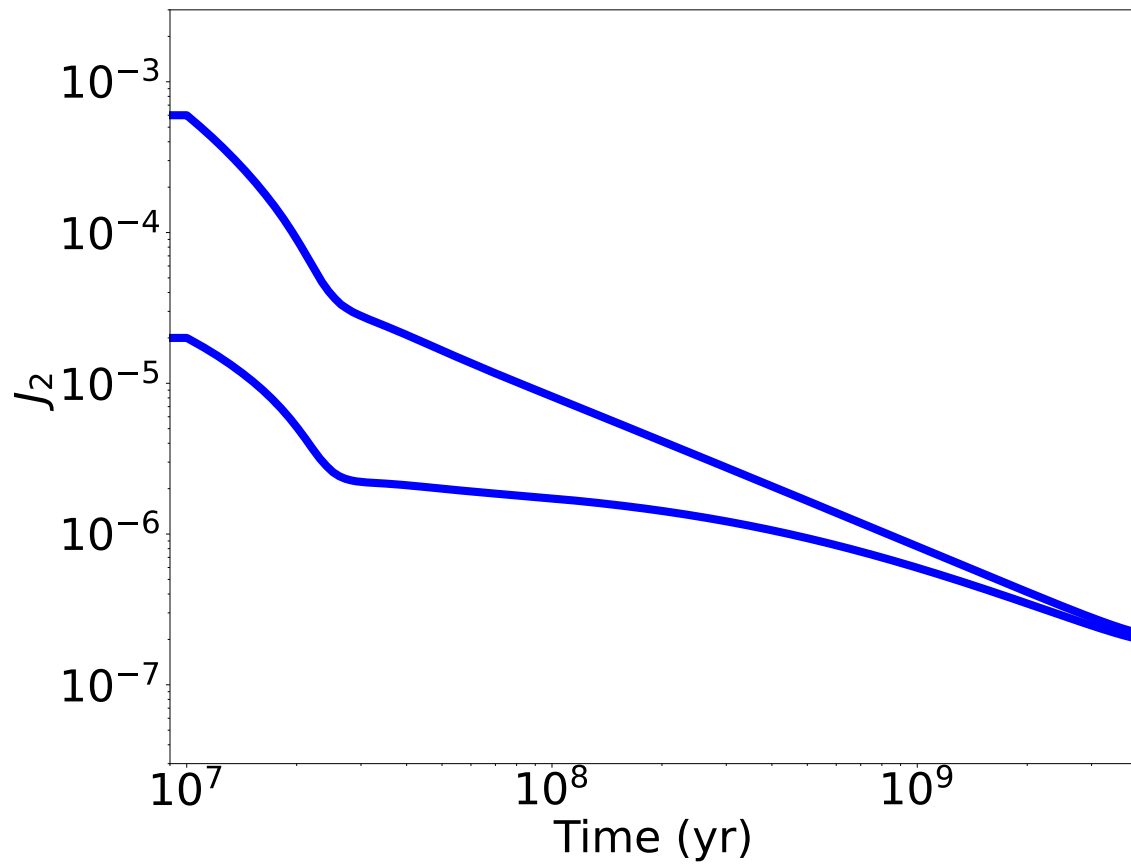


Figure 3.1: A range of possible J_2 evolutions for a $1.1 M_\odot$ star after the evaporation of the disk as a function of time.

with $\alpha = 1.5 \times 10^{-14}$. Figure 3.1 shows the resulting J_2 evolution as a function of time taking into account spin, Love number, and radius evolution.

3.3 Secular Resonances of Eccentricity and Inclination from J_2

When two planets' precessions of ϖ or Ω are similar, they enter a secular resonance in e or I , respectively [e.g., Murray and Dermott, 2000]. This causes them to exchange angular momentum at a significantly higher rate than when they were out of resonance. Two planets can typically only enter secular resonance in systems with 3 or more planets, though Spalding and Batygin [2016] showed that under the influence of J_2 (or, through the same logic, GR), the two planets in a two-planet system can enter secular resonance if the inner planet has more angular momentum than the outer planet.

However, the effect of J_2 on secular resonances is not only significant in two-planet systems. In systems with 3 or more planets, where the innermost planet has the least mass, there are critical semimajor axis where it is in secular resonance with one of its companion planets. The impact of J_2 on this is to shift the values of the critical semimajor axis. Because the host star's spin slows over time due to magnetic braking, this causes the shifts of the critical semimajor axis over time.

This is shown in Figure 3.2, where, in the left column we plot the expected inner planet maximum inclination and maximum eccentricity from secular interactions as functions of inner planet semimajor axis for different values of J_2 for the system described in Table 3.1. For fixed semimajor axes and J_2 , the secular equations defined in Section 3.2 have analytic, periodic solutions [see e.g., the construction in Murray and Dermott, 2000]. To calculate the maximum eccentricities and inclinations shown in Figure 3.2, we evaluate these analytic solutions at sufficiently many times over their periods and retrieve the maximum eccentricity/inclination reported.

In the right column of Figure 3.2 we show the precession of the inner planet's longitude of ascending node (Ω , top) and longitude of periapsis (ϖ , bottom), as well as the eigenfre-

quencies of the secular matrices B (top) and A (bottom) as a function of the inner planet’s semimajor axis. When the inner planet’s precession intersects an eigenfrequency for a particular values of J_2 , this corresponds to a secular resonance, dramatically increasing the eccentricity or inclination excited by the outer planets on the inner planet. In the top row we mark the resonances and intersections for the curves where $J_2 = 10^{-5}$ as an example. Notably, there can only ever (in this system) be two intersections in the Ω precessions, corresponding to there only ever being up to two peaks in the inclination plots. However, the larger, lower eigenfrequency in the ϖ plot means that there can be up to 4 peaks in excited eccentricity for a particular J_2 .

At different values of J_2 the peaks in these plots representing the secular resonances shift dramatically. This implies that if a star’s J_2 significantly evolves over its lifetime, that at some point during that evolution, J_2 will have the appropriate value to send an inner planet into secular resonance for a large range of inner planet semimajor axes.

3.4 Stability of Short-Period Planets

Short-period planets are subject to many dynamical effects, including GR, planet-planet interactions, and precession due to stellar oblateness¹All of these effects can impact the stability of a short-period planet. Secular resonances through planet interactions can cause rapid angular momentum exchange, driving up inner planets’ eccentricities [e.g., [Faridani et al., 2022](#)]. In multi-body systems with far-orbiting outer bodies, angular momentum exchange through the Eccentric Kozai-Lidov-like effect can even more efficiently destabilize an inner planet or body [e.g., [Naoz, 2016](#)]. These eccentricity excitations can be suppressed if GR precession (or other short range forces) takes place faster than the multi-body orbits, and the far away companion, induce precession [e.g., [Denham et al., 2019](#), [Wei et al., 2021](#)]. Sufficiently high values of J_2 can destabilize short-period planets with only one companion

¹Tidal effects such as precession, circularization and shrinking of the planet’s orbit may also contribute to the orbital stability, but they are beyond the scope of this paper. We expect that tidal precession will be of the order of GR precession timescale and thus both will contribute similarly to the stability of the system.

[e.g., [Spalding and Batygin, 2016](#)], and systems with high initial stellar obliquities [e.g., [Spalding et al., 2018](#)], though in this work we will be focusing on short-period planets with 2 (or more) companion planets.

In this work, we characterize the stability of an inner planet over the course of its lifetime with an index, δ ,

$$\delta = \max \left(\frac{r_{\text{peri},1} - r_{\text{apo},\text{in}}}{a_1 - a_{\text{in}}}, 0 \right), \quad (3.17)$$

where a_{in} and $r_{\text{apo},\text{in}}$ represent the semimajor axis and apocenter distance of the inner planet, respectively, and a_1 and $r_{\text{peri},1}$ represent the semimajor axis and the pericenter distance respectively of the closest-orbiting planet to the inner planet. Here, δ represents, over its whole lifetime, the closest an inner planet got to crossing orbits with its closest companion planet.

A δ of 0 represents a potential-orbit crossing scenario, where the Laplace-Lagrange formalism no longer applies and the system is likely unstable.

Here, orbital crossing is treated as a catastrophic fate for the system, which may lead to the ejection of one or more planets (or changing their orbital energies by a factor of two). However, orbital crossing does not yield an instantaneous destabilization of the system. In fact it was recently shown that orbits within each other’s Roche limit, and even crossing orbits, have a timescale for destabilization, in some cases longer than the system’s lifetime [[Zhang et al., 2023](#)]². However, here for simplicity, when running simulations, we stop the simulation when the orbits cross.

3.5 Proof of Concept System

3.5.1 System Description

Here we consider a constructed system to demonstrate the impact of secular resonances under the influence of evolving J_2 . This system is based on the real system HD 106315

²Non hierarchical systems, but perhaps not orbit crossing, do not instantaneously destabilize [e.g., [Grishin et al., 2018](#), [Mushkin and Katz, 2020](#), [Bhaskar et al., 2021](#)]. Thus, even low values of δ may take time to undergo dramatic changes for their orbits.

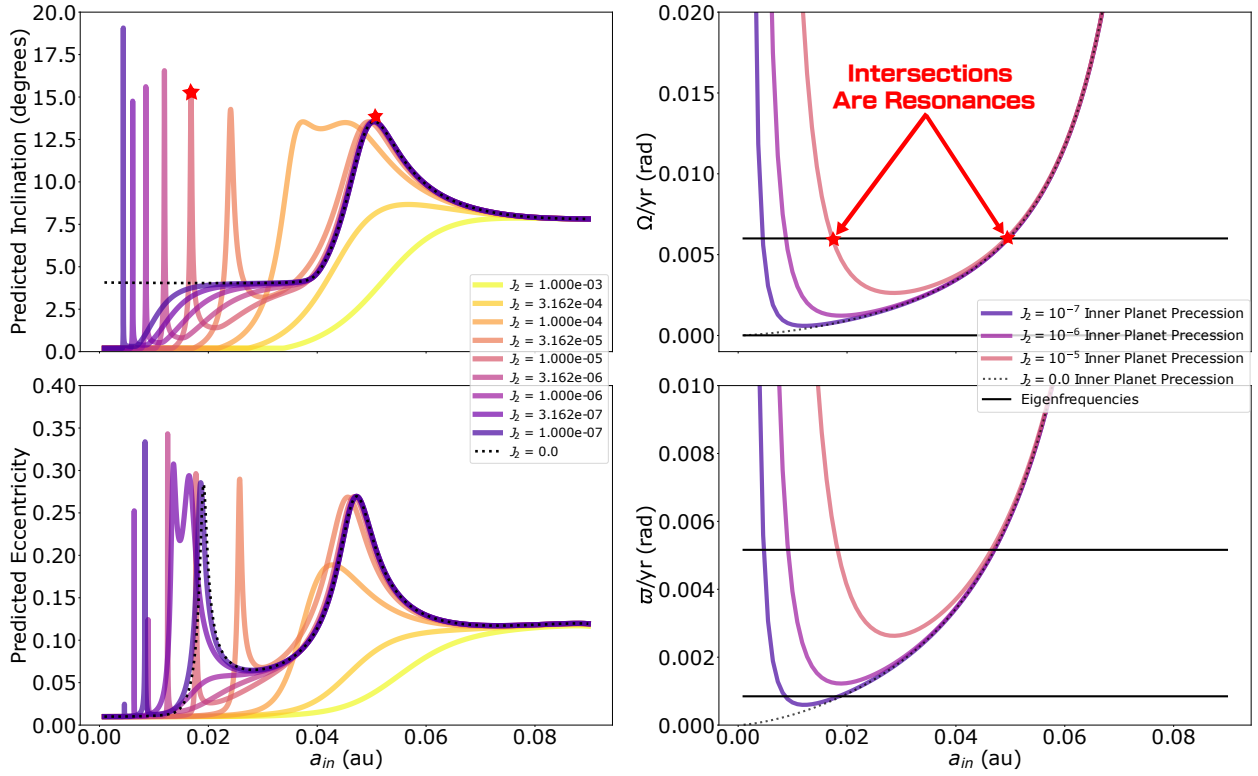


Figure 3.2: **Left Column:** The maximum eccentricity and maximum inclination of a hypothetical planet orbiting inward of our proof-of-concept system's two planets expected by Laplace-Lagrange for various values of J_2 . At small semimajor axes (< 0.04 au) there exists a value of J_2 such that the combined Laplace-Lagrange and J_2 effect produces a resonance at that semimajor axis. The orbital parameters of the system here are shown in Table 3.1, with the hypothetical planet having a mass of $1 M_{\oplus}$, initial eccentricity $e = 0.05$, and initial inclination $i = 1^\circ$.

Right Column: Impact of inner planet semimajor axis on the inner planet's precession of ϖ (top) and Ω (bottom) for several values of J_2 . Also shown are the eigenfrequencies of the secular matrices B (top) and A (bottom). When the inner planet's precession of ϖ or Ω intersects with an eigenfrequency, that corresponds to a secular resonance. The critical a_{in} values that are in resonance for $J_2 = 10^{-5}$ are marked in the top row to demonstrate the relationship between intersections in the right column and resonances in the left column.

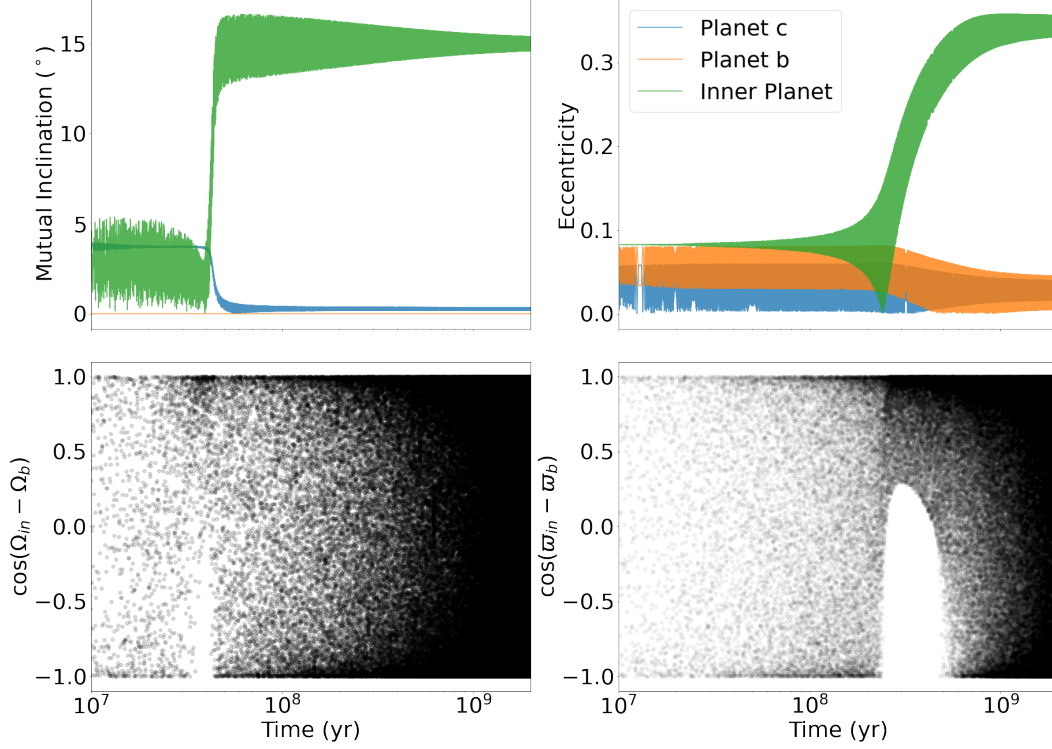


Figure 3.3: Top Row: Eccentricity and mutual inclination with middle planet as a function of time for a run from the Monte Carlo described in Section 3.5.2. An inner planet was inserted into the proof-of-concept system with initial conditions $a_{\text{in}} = 0.02061$, $e_{\text{in}} = 0.08247$, $i_{\text{in}} = 1.07865^\circ$, $\Omega_{\text{in}} = 154.6^\circ$. Representative of the effect of sweeping resonances is the swift transition in the inclination of the inner planet as a resonance passes through it at ~ 40 Myr. The rolling average of the inclination of the middle planet remains at $\sim 2.5^\circ$ over the entire integration, with the magnitude of its inclination oscillations changing from $\pm 4^\circ$ to $\pm 0.5^\circ$ after the inner planet transitions to its higher mutual inclination state.

Bottom Row: Difference of Ω and ϖ angles between the inner and middle planets as a function of time. The alignment of angles during the resonances is shown by large gaps occurring at the same time as the transitions to higher eccentricity and inclination.

Table 3.1: Parameters of Proof-of-Concept System

Object	Mass ($M_{\odot}$)	a (au)	e	i	Ω
Star	1.09				
Inner Planet	$1.0 M_{\oplus}$	$0 - 0.085$	$0 - 0.2$	$0 - 7^{\circ}$	$0 - 360^{\circ}$
Planet b	$12.6 M_{\oplus}$	0.097	0.05	4°	0°
Planet c	$15.2 M_{\oplus}$	0.1536	0.05	1°	0°

[see e.g., [Barros et al., 2017](#)]. HD 106315 has significant eccentricity in both its observed planets, and has an observed stellar obliquity of $\sim 10^{\circ}$ for HD 106315c [though HD 106315 b and c are shown to be coplanar [Zhou et al., 2018a](#)]. These properties make HD 106315 very interesting, though too dynamically hot to act as a representative example of exoplanet systems. To demonstrate the importance of J_2 on secular resonances for a wider variety of systems, we first explore a modified version of HD 106315 with much lower planetary eccentricities and stellar obliquities. HD 106315 was chosen as a base for this work because the spacing and moderate masses of its planets are ideal for showcasing the effect of evolving J_2 on secular resonances.

HD 106315 is a $1.09 M_{\odot}$ star with two observed planets detected via transits. HD 106315 b has semimajor axis, mass and radii 0.097 au, $12.6 M_{\oplus}$, and $2.44 R_{\oplus}$ respectively. HD 106315 c has semimajor axis, mass and radii 0.1536 au, $15.2 M_{\oplus}$, and $4.35 R_{\oplus}$ respectively. The two planets have measured mutual inclinations of $< 5^{\circ}$ [[Zhou et al., 2018a](#)]. As a better proof-of-concept, we prepare both planets with eccentricities of 0.05. The parameters of our proof-of-concept system are summarized in Table 3.1. Note that we have adopted the convention that an inclination of 0° rather than 90° corresponds to a transiting configuration. To avoid confusion arising from this, we have, wherever possible in our figures, shown mutual inclinations between planets rather than the values of the inclination orbital element. To demonstrate the effect of secular resonance evolution, we will introduce a third, Earth-mass planet into the system orbiting within the two observed planets. We will explore the stability and detectability of this introduced inner planet under the influence of evolving resonances.

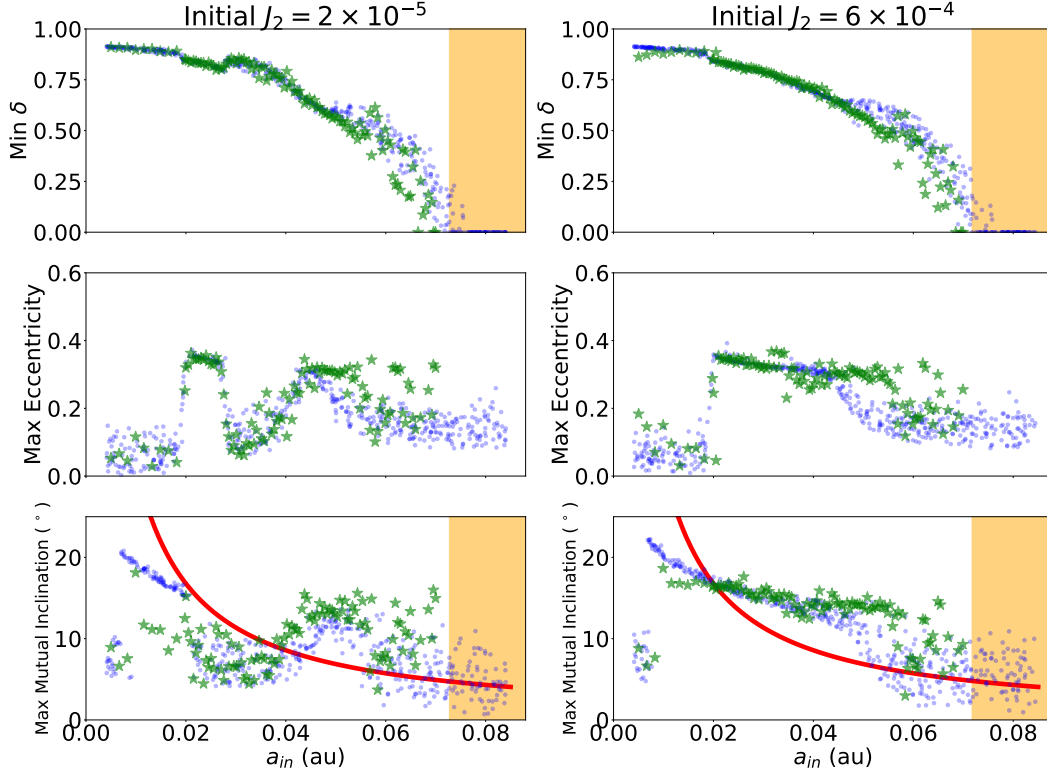


Figure 3.4: **Proof of Concept System's inner planet.** Results of 1000 systems, represented by blue dots, integrated up to 2 Gyr of our proof of concept system with an Earth-mass planet introduced inward of the two observed warm Neptunes, and 220 N-body simulations, represented by green stars. The left column represents initial $J_2 = 2 \times 10^{-5}$ while the right column represents $J_2 = 6 \times 10^{-4}$, these two values represent the bottom and top curves in Figure 3.1. The top row indicates for each run the minimum value of δ (see Eq. 3.17) achieved over the entire run. The shaded regions represent regions of instability either from Roche limit crossing (left) or orbit-crossing indicated by $\delta = 0$ (right). The middle row indicates the maximum eccentricity achieved by the inner planet over each run. The bottom row indicates the maximum mutual inclination between the inner planet and the two warm Neptunes over the course of the run. The red curve indicates the maximum mutual inclination that the inner planet can have with its closest neighbor and still transit.

In Figure 3.3, we show the evolution of the eccentricity and inclination of the planets of our proof-of-concept system with an additional, Earth-mass inner planet included inside Planet b’s orbit. This evolution, and all other evolutions in this work unless stated otherwise are performed by a straightforward integration of the sums of Equations (3.1), (3.11), and (3.15). Allowing the value of J_2 in Equation (3.15) to vary allow us to straightforwardly model the impact of the evolution of J_2 as a function of time. The short-period planet has initial conditions of $a_{\text{in}} = 0.02061$, $e_{\text{in}} = 0.08247$, $i_{\text{in}} = 1.07865^\circ$, $\Omega_{\text{in}} = 154.64^\circ$.

Notable in Figure 3.3 is the sudden transition of the inner planet’s mutual inclination between it and its companions from its starting value to higher values at approximately $t = 40$ Myr. This transition is caused by the evolution of J_2 bringing the inner planet into a secular resonance with its companions. Thus, as J_2 evolves, the inner planet’s inclination can reach its resonant value, depending on its semi-major axis, as shown in Figure 3.2. The relatively fast “jump” in mutual inclination occurs because the width of the resonance is narrow, so small changes in J_2 correspond to large changes in excited inclination. This means that the transition from non-resonant to resonant happens quickly as J_2 continuously evolves. Further, as J_2 continues to evolve, the system leaves the resonance, but the high inclination remains as a signature of past resonance. The growth in inner planet eccentricity shown in Figure 3.3, however, arises because the inner planet’s semimajor axis is very close to a resonance at $J_2 = 0$, this means that as J_2 evolves over the 2 Gyr evolution, the inner planet grows closer and closer to resonance continuously, causing a much slower growth in eccentricity. Additionally, after the transition in inner planet inclination, the mutual inclination between the two outer planets shrinks dramatically, indicating that their co-transiting geometry remains unaffected. The frequencies involved are commensurabilities between the inner planet and planet b’s ϖ and $\dot{\Omega}$ for resonances affecting eccentricity and inclination respectively. This is shown in the bottom row of Figure 3.3, where the differences between the planets’ ϖ and Ω librate (constrained in bounded intervals around 0° and do not extend to 180°), while the transitions from low to high mutual inclination and eccentricity are taking place.

3.5.2 Monte Carlo Exploration

Here we present the results of 1000 simulations of our proof-of-concept system with an inner, earth-mass planet at a semimajor axis drawn from a uniform distribution $a_{\text{in}} \in [0.005, 0.085]$, inward of the two observed planets. . The eccentricity and inclination (in radians) of the inner planet are drawn from Rayleigh distributions with $\sigma = 0.049$, consistent with the observed distribution of small planets in Kepler multiplanet systems [see e.g., [Van Eylen and Albrecht, 2015](#)]. The short-period planet’s longitude of ascending node and argument of periapsis were drawn from uniform distributions in $[0, 2\pi]$. We integrate the Laplace Lagrange equation of motion including an evolving J_2 , due to magnetic braking spindown, see Section 3.2.1. We also include general relativity precession.

All systems are integrated to 2 Gyrs or until orbit crossing occurs. In Figure 3.4 we show the results of these integrations with blue circles. Additionally, we run N-body simulations as verification, represented by colored stars. We show the maximum inclination (bottom row), eccentricity (middle row), and the minimum value of δ , (see Equation (3.17), top row). The latter represents the stability of the system, where zero corresponds to orbital crossing. Because J_2 depends on so many stellar parameters (the Mass, Radius, Spin, and stellar Love Number, see Eq 3.14), we adopt two widely-separated values (2×10^{-5} and 6×10^{-4}) for the initial J_2 in our simulations (see Figure 3.1) to capture the breadth of possible initial conditions.

The evolution of J_2 over the life of the star causes excitations in the eccentricity of the inner planet that change strength and location with time. Compared to a host star with unchanging J_2 , this can reduce the size of the stable regions of parameter space for the inner planet. The critical role that an evolving J_2 plays in the dynamics of the system is highlighted in Figure 3.4. For example, when comparing to Figure 3.2, individual curves in Figure 3.2 yields well defined semimajor axes corresponding to secular resonance for the short-period planet’s eccentricity and inclination. However, as J_2 evolves, these resonances sweep across a large semimajor axis range, yielding a wide plateau in the maximum eccentricity (see middle

row in Figure 3.4).

In the bottom row of Figure 3.4, the mutual inclination between the short-period planet and its two companion planets is plotted. There are two distinct behaviors between the high and low initial J_2 cases, (left and right, column respectively). Namely for the low initial J_2 case there is a clear discontinuity and minimum in mutual inclination between 0.02 – 0.04 au and for the high initial J_2 case, there is a smooth transition in the same region. This behaviour arises because at $J_2 = 10^{-4}$, a secular resonance in inclination for the inner planet exists at $a_{\text{in}} = 0.04$ au (see top panel of Figure 3.2). However, for the lower initial value of J_2 in the left column, those resonances never appear because the initial value of J_2 is lower than the values where these resonances appear. This means that in the left column, inner planets orbiting near 0.04 au will never experience a secular resonance in inclination, and therefore have lower inclinations than their closer-orbiting counterparts.

An immediate consequence from the evolution of the inclination is that the transit probability of a high inclination inner planet is low. In particular, we adopt the simple geometric condition for the critical transiting inclination $i_{\text{mut,tran}} < \tan^{-1}(R_{\star}/a_{\text{in}}$, where $R_{\star}$ is the radius of the star. This boundary is marked with a red curve. As can be seen, Both cases (low and high initial J_2) have the possibility to detect inner planets as close as ~ 0.03 au, with the low initial J_2 allowing for the possibility to detect some further out, thanks to the aforementioned minimum in mutual inclination. The discontinuity at 0.005 au in the inclination arises from J_2 dominates over the dynamics, analogous to the GR dynamics, which result in a discontinuity in eccentricity at 0.02 au.

Significantly, the eccentricity excitation can lead to destabilizing the system up orbital crossing. To estimate this, as mentioned above, we adopt the parameter δ , Equation (3.17), shown in the top panel. Notably, for inner planet semimajor axes larger than ~ 0.07 au, regardless of inner planet initial orbital configuration or initial J_2 , the system reach orbit crossing. However, even many of the inner planets that don't cross orbits have significantly elevated eccentricities, as high as 0.4, which can pose challenges for the long-term stability of the system.

The N-body simulations shown in Figure 3.4, agree well with the secular integrations described above. To run the N-body simulations, we used the N-body code `REBOUND` using the `WHFAST` integrator and the `gr` and `gravitational_harmonics` implemented effects from `REBOUNDx` [Rein and Liu, 2012, Wisdom and Holman, 1991, Rein and Tamayo, 2015, Tamayo et al., 2020b]. However, integrating `REBOUND` for the full 2Gyr for this system is infeasible. To account for this, the N-body integrations shown in Figure 3.4 are run with a J_2 evolution that evolves 100 times faster than shown in Figure 3.1. The high degree of agreement between the secular evolution and the N-body despite orders of magnitude differences in $\dot{J}_2$ prompt the question of the impact of the timescale of J_2 evolution. We have found that, heuristically, the important timescale is the period of secular oscillation produced by planet-planet interactions. If J_2 evolves significantly over timescales less than this period, then the resonant behavior shown in Figure 3.3 does not occur. However, in this system, the period of secular oscillation from planet-planet interactions is approximately 20 kyr, and the transition from low to high mutual inclination mediated by J_2 in Figure 3.3 occurs over ~ 20 Myr. Thus, a 100-fold increase in $\dot{J}_2$ ought not to significantly impact the behavior, which is consistent with the results of the N-body simulations closely matching the secular integrations.

3.5.3 Constraints on the Orbital Configuration of a Possible Inner Planet

The eccentricity and inclination excitation from this channel can constrain the inner planet’s orbital configuration, as well as its stability and detectability. As depicted in the top row of Figure 3.4, for the representative example, any such planets with semimajor axes greater than 0.07 au undergo orbit crossing with their closest neighbor and are likely unstable. Those with semimajor axes between 0.02 – 0.04 au become significantly eccentric ($e > 0.3$) over the evolution of the system, indicating potential instability over the rest of the lifetime of the system from effects not captured by the first-order Laplace-Lagrange.

Detectability via transit is also a concern for these type of planets. For example, in the simulations with high initial J_2 , for planets with semimajor axes higher than 0.02 au,

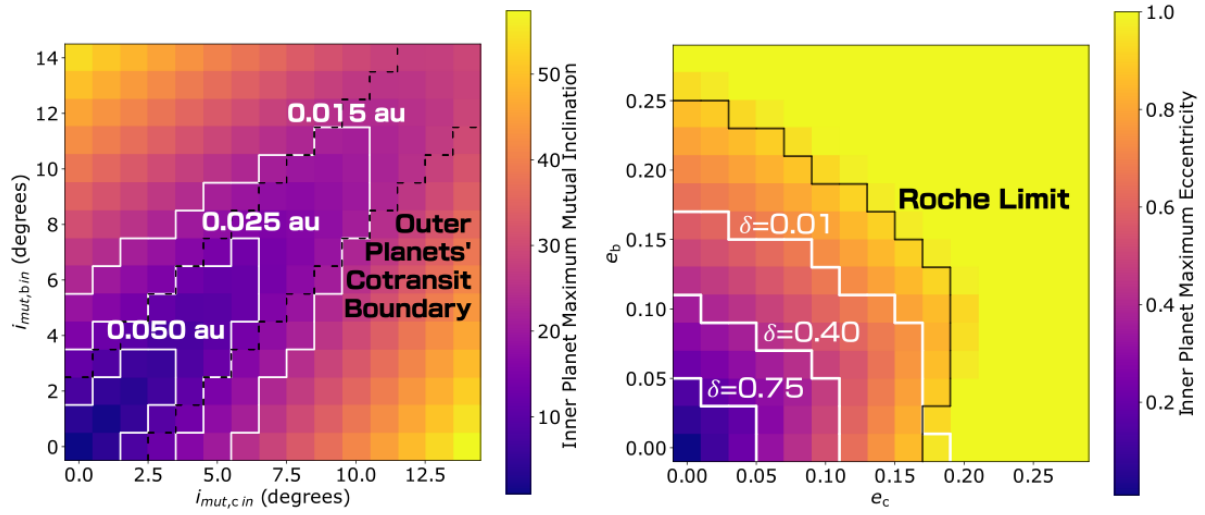


Figure 3.5: The maximum possible eccentricity and mutual inclination between it and its companions achieved by an inner planet as a result of sweeping resonances given the inclinations/eccentricities of the outer planets. The reported maximum eccentricity and inclination values are found by calculating the predicted eccentricity and inclination excitation from a secular resonance. The inner planet’s initial eccentricity and inclination are set to 0.01 and 1° , respectively for these calculations. These maximum values are not significantly affected by J_2 evolution as J_2 serves primarily to move them rather than affect their magnitude (see Figure 3.2). Overplotted in white lines on the inclination panel are the contours within which the inner planet always transits not matter its longitude of ascending node, for the labeled semimajor axis and in black dashed lines is the boundary within which the two companion planets both always transit under the simplifying assumption that the inner of the two companion planets lies approximately on the line-of-sight. On the eccentricity panel are shown the contours within which δ is above the given value, for an inner planet with $a_{in} = 0.046$. Additionally, the contour within which the inner planet does not cross the Roche Limit $R_L = R_p(2M_\star/M_p)^{1/3}$ for $a_{in} = 0.046$.

enough mutual inclination between it and its companions is excited such that these planets may elude transit³, In the case with low initial stellar J_2 , planets with semimajor axes higher than 0.045 au may elude transit, but between 0.03 – 0.045 au, a dip in maximum inclination exists. This dip arises because the inclination resonances only appear in the range 0.03 – 0.045 au for J_2 values higher than 4×10^{-5} (see Figure 3.1 in Appendix 3.2.1).

3.5.4 The Effects of Initial Conditions

As shown in Figure 3.4, the main determinant in outcome for the inner planet with is its semimajor axis, as the spread around the trends in all 6 panels are relatively small indicating that the effect of the sweeping resonances is largely independent of any other initial conditions. The exception to this are semimajor axes greater than 0.05 au, where spreads are significant. The larger variance in maximum eccentricity/inclination at semimajor axis values greater than 0.05 au exists because there are no resonances for any values of J_2 (see Figure 3.2) for those semimajor axes. The random choices of initial conditions drive the increased variance in this range. This indicates that when a planet is affected by a sweeping resonance, that resonance determines its eccentricity and inclination excitation independent of any of its other orbital elements.

However, the effect of the initial eccentricities and inclinations of the outer two planets has great impact. In Figure 3.5, we show the maximum eccentricity and inclination of the inner planet over its evolution predicted by Laplace-Lagrange as functions of the initial inclinations and eccentricities of its two companions. Large eccentricities can be induced on the inner planet from moderate companion eccentricities, greatly impacting inner planet stability. Similarly, an inclination difference between the companions of two degrees can induce tens of degrees of misalignment on the inner planet.

³Note that in Figure 3.4, the transit boundary, depicted as the solid line in the Figure, represents the maximum inclination for which transiting can take place independently of Ω . However, above the line, different values of Ω can yield a transiting system.

Table 3.2: Parameters Used for HD 106315 System

Object	Mass	$a(\text{au})$	e	i	Ω
HD 106315	$1.09 M_{\odot}$				
Inner Planet	$1.0 M_{\oplus}$	$0.005 - 0.085$	$0 - 0.2$	$9 - 16^{\circ}$	0°
HD 106315 b	$12.6 M_{\oplus}$	0.097	0.09	14°	0°
HD 106315 c	$15.2 M_{\oplus}$	0.1536	0.22	11°	0°

Table 3.3: Values aside from inclination drawn from Barros et al. [2017]. Inclination values are consistent with Zhou et al. [2018a], who estimated the planets’ mutual inclinations and measures their obliquity of $\sim 10^{\circ}$.

3.6 HD 106315

Here we repeat the exercise of Section 3.5 except using the measured values of eccentricity and inclination for HD 106315. Zhou et al. [2018a] showed that the two warm Neptunes have low mutual inclinations ($< 5^{\circ}$, consistent with both choices of inclination described in Tables 3.1 and 3.3). Further, Zhou et al. [2018a] also measured the obliquity of HD 106315 c, finding an obliquity of about 10° , and used N-body calculations to show the two planets are likely coplanar. We thus adopt these orbital parameters, and introduce an inner planet and ask what are the consequences of the aforementioned sweeping resonance on the possibility of the existence and orbital configuration of such a hypothetical planet. Particularly, in this exercise we are interested in analyzing the possibility of the system to hide an inner planet.

3.6.1 Constraints on a Possible Inner Companion to HD 106315

Figure 3.6 shows the Minimum δ , and Maximum eccentricities and mutual inclinations for an inner planet introduced inwards of the two warm Neptunes of HD 106315. Notably, HD 106315 c’s eccentricity of 0.22 induces large eccentricities on the inner planet, as predicted by Figure 3.5. inner planets with semimajor axes higher than 0.045 au have minimum $\delta = 0$, indicating that they cross orbits with the innermost warm Neptune. Moreover, even inner planets that don’t cross orbits with the Neptunes have significantly elevated eccentricity unless they orbit at < 0.02 au, where GR dominates, suppressing eccentricity excitation. This is reflected in the N-body simulations, where all systems with inner planet semimajor

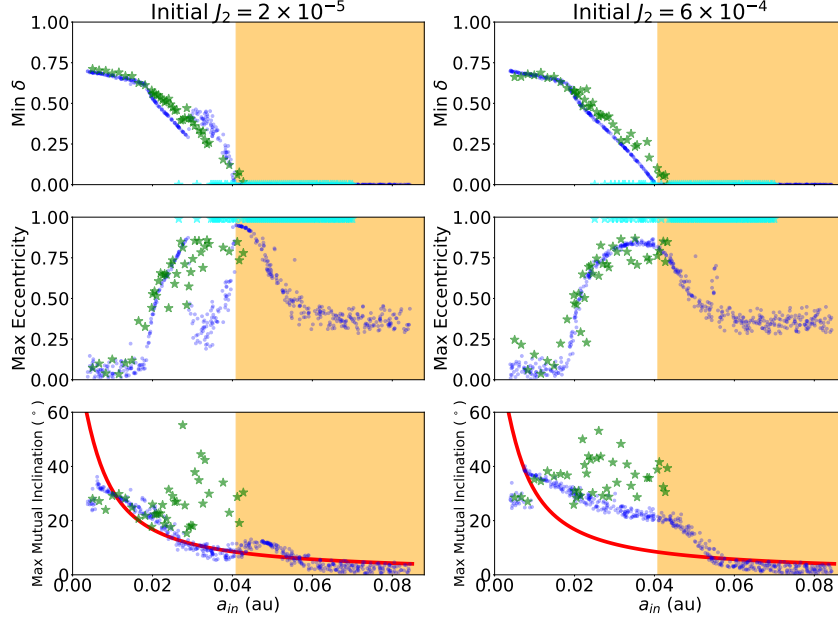


Figure 3.6: **HD 106315’s inner planet.** Results of 1000 systems, represented by blue dots, integrated up to 2 Gyr of HD 106315 with an Earth-mass planet introduced inward of the two observed warm Neptunes, and 220 N-body simulations, represented by green stars if they remained stable, and cyan stars if planet-planet scattering or ejection occurred. Cyan stars do not appear in the bottom row because their maximum mutual inclinations are greater than 60° . The left column represents initial $J_2 = 2 \times 10^{-5}$ while the right column represents $J_2 = 6 \times 10^{-4}$, these two values represent the bottom and top curves in Figure 3.1. The top row indicates for each run the minimum value of δ (see Eq. 3.17) achieved over the entire run. The shaded regions represent regions of instability either from Roche limit crossing (left) or orbit-crossing indicated by $\delta = 0$ (right). The middle row indicates the maximum eccentricity achieved by the inner planet over each run. The bottom row indicates the maximum mutual inclination between the inner planet and the two warm Neptunes over the course of the run. The red curve indicates the maximum mutual inclination that the inner planet can have with its closest neighbor and still transit.

axis > 0.04 au are unstable (cyan stars) and a significant portion of inner planets with semimajor axis > 0.035 au are unstable as well.

In the case of low initial J_2 , there is a region of (relatively) low eccentricity excitation from $0.03 - 0.04$ au, analogous to the same low-eccentricity region in the same place in Figure 3.4. This region also has low mutual inclination with HD106315 b and c, with dots falling below the red curve, indicating that in this more stable region, an inner planet could survive and also transit with HD 106315 b and c. However, as expected in such high eccentricities, the Laplace-Lagrange approximation underestimates the eccentricity and inclination value, and the system retains high eccentricities and mutual inclinations throughout the region. This discrepancy is likely due to the large eccentricities of the companion planets and significant stellar obliquity driving the system outside of the regime where the Laplace-Lagrange approximation is valid. The high number of unstable integrations indicate that only inner planets with semimajor axes $\lesssim 0.03$ au can be reliably stable, despite the secular integration suggesting a low eccentricity region from $0.03 - 0.04$ au.

Overall, the effect of sweeping resonances is to dramatically constrict the regime where a stable, detectable inner planet might live orbiting HD 106315. Inner planet orbits with semimajor axes greater than 0.04 au cross with their closest neighbor, becoming unstable. For those that don't cross orbits, their detectability and stability are heavily affected by the initial J_2 of HD 106315 after the evaporation of the disk, destabilizing them or removing them from the co-transiting regime. There is also the possibility of a stable and detectable orbiting at < 0.015 au, however, at the time of writing, only two planets orbiting that close to a host star of mass $> 1 M_\odot$ have ever been detected (PSR J1719-1438 b, and PSR J2322-2650 b [Bailes et al., 2011, Spiewak et al., 2018]), both orbiting pulsars (though PSR J1719-1438 b may be a carbon white dwarf), indicating that finding a short-period planet orbiting there around HD 106315 (an F star) is unlikely.

3.7 Discussion

Compact multiplanet systems are common in our galaxy [e.g., Howard et al., 2010, 2012, Lissauer et al., 2011, Winn and Fabrycky, 2015]. Further, Weiss et al. [2018a] suggested that planets in a system have similar sizes, called peas-in-a-pods [but see, e.g., Zhu, 2020, Murchikova and Tremaine, 2020b, for possible selection biases]. In fact, it was suggested that USPs are likely an extension of the population of hot Earth-sized planets [Mulders et al., 2016, Winn et al., 2017] to short periods. Thus, to develop a comprehensive understandings of USPs’ dynamics it is imperative to investigate the combined effects of planet-planet interactions with evolving stellar quadrupole.

We demonstrated that the time-evolution of J_2 due to the stellar spin down can affect secular resonances in systems with at least 3 planets⁴. The quadrupole moment J_2 ’s evolution drives planets into and out of secular resonance as it evolves (illustrated in Figure 3.2). In systems like the ones we explored, where the innermost planet is significantly less massive than its companions, the effect of these sweeping resonances is to transition this inner planet to higher eccentricities and mutual inclinations over a relatively short period of time. These effects are most pronounced in systems where, like the ones explored, the period ratio between the inner planet and its next closest companion is large $\gtrsim 2$. This demonstrates the critical interplay between planet-planet interactions and star-planet interactions in determining the lifetime evolution of short-period planets. We suspect that the formalism described here can be used to constrain USP candidates. The applications for these candidate systems are reserved for future work.

We note that tidal effects become important over gigayear timescales. They were not included here as the timescales of tidal precession, circularization, and semimajor axis shrinkage are much slower than the GR, Laplace-Lagrange, and J_2 timescales. This means that tidal effects do not impede the effects of evolving J_2 to transition inner planets to higher

⁴Note, that unlike, Spalding and Batygin [2016], who considered 2 planet systems, here we highlight the importance of the angular momentum exchange in a ≥ 3 planet system, which forms a more complex resonant pattern.

eccentricities and misalign them from their companions. However, when considering the gigayears of evolution after secular resonances have affected the inner planet, tidal effects become significant, bringing the planet to shorter orbits and stabilizing it via circularization.

We showed the impact of sweeping secular resonances arising from evolving J_2 on a 3-body system with initially low mutual inclinations and eccentricities⁵ We began with a pedagogical proof of concept, highlighting the effect of sweeping resonances. For example, for a given semimajor axis for the innermost planet we depicted in Figure 3.3 the sharp eccentricity (inclination) transition as the planet enters its resonance, leaving a long lasting signature on the planet’s eccentricity (inclination).

These sweeping resonances can have large consequences on the planet’s stability and transiting probability. This effect is demonstrated in Figure 3.4, where we varied the innermost planet’s semimajor axis neighbouring two outer planets. The destabilization can even occur when its period ratio with its closest neighbor is low. Moreover, the inner planet can potentially escape transit over a large semimajor axis range (see bottom panels of Figure 3.4). This highlight the impact of sweeping resonances on the transiting probability. In particular, the other two planets orbital architecture does not vary significantly, remaining with their initial low inclinations and eccentricities, but the inner most planet, can evade transit. Using the aforementioned framework we investigated the stability and detectability of the observed system HD 106315. This system has two observed, transiting planets with an $\sim 10^\circ$ stellar obliquity, low mutual inclinations, and its outer planet has a moderate eccentricity of ~ 0.22 [Barros et al., 2017]. While there are no observed transits for an interior planet, we ask if the sweeping resonances can allow for the existence of a interior non-transiting planet.

We find that in HD 106315, a potential Earth-mass inner planet is unstable at any semimajor axes greater than 0.04 au, and has high mutual inclinations with its transiting companions at semimajor axes inward of 0.025 au (see Figure 3.6). This result depends

⁵The impact of J_2 on two-planet systems has been well-demonstrated [e.g., Spalding and Batygin, 2016, Chen et al., 2022, with continuous J_2 evolution in the latter], and has been explored in systems with more planets at several fixed J_2 values [e.g., Li et al., 2020, Becker et al., 2020], however we demonstrate with this system how in systems with 3 or more planets, the continuous evolution of J_2 is critical in understanding their evolution.

on the initial J_2 of the star, which is largely unknown [e.g., Baraffe et al., 2015, Chen et al., 2022]. Specifically, a lower initial J_2 will yield only moderate mutual inclination, but similar stability regimes (see left column in Figure 3.6). This is consistent with findings that transiting USPs and short-period planets often have companions [e.g., Sanchis-Ojeda et al., 2014b, Adams et al., 2021].

Any approximately Earth-radius candidate transit detections in the HD 106315 system orbiting past 0.04 au could be reasonably discarded as unstable, and candidates orbiting at radii less than 0.025 au might be suspect due to their likely high mutual inclination with the two transiting planets. This result depends on the initial J_2 of the host star, with larger initial J_2 values reducing the size of the low-eccentricity region near 0.03 au and increasing the expected mutual inclination between the inner planet and its transiting companions.

3.8 Appendix A: Comparison of Secular Code with N-body

In Figure 3.3, we show the results of our secular evolution. In Figure 3.7, we compare these results with the results from the N-body integrator `HNBody`, as an additional test. The N-body simulations were prepared with initial conditions taken from the secular integration. We find that the N-body agrees well with the secular code’s calculation of the maximum and mutual inclination achieved. However, the N-body shows that the magnitude of oscillations in eccentricity and inclination are significantly larger than predicted by the secular code, ranging from $\sim 10 - 20\%$ error, bringing eccentricity and inclination to lower minima. Additionally, `HNBody` predicts a higher maximum eccentricity of 0.4, a 11% deviation from the secular code.

The impact of the differences between the N-body and secular code to the conclusions of this work is relatively minor. The lower minima in eccentricity oscillations does not significantly stabilize the system, as maximum eccentricity has much more impact on driving close encounters. The enhanced maximum eccentricity, however is not expected to significantly drive more collisions, as the δ values for similar ranges of eccentricity are probed in

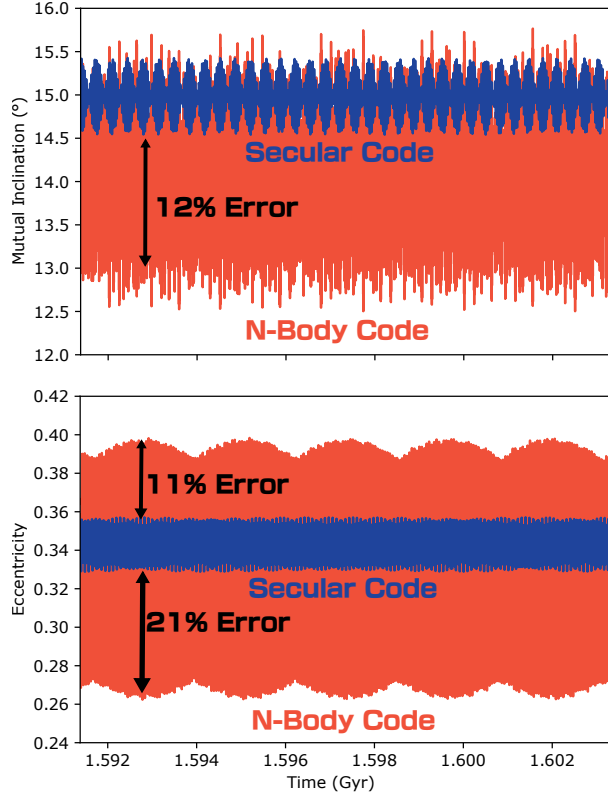


Figure 3.7: Comparison of the inner planet’s eccentricity and mutual inclination with the middle planet when calculated with the secular code used throughout this work (blue, central) and the N-body integrator HNBODY (orange, outer). The green curve is a subsection of the evolution of the inner planet in Figure 3.3, and the red curve was initialized with the orbital elements and J_2 value of all planets 1.591 Gyr into the evolution shown in Figure 3.3.

Figures 3.4 and 3.6, and minimal differences in eccentricity are achieved. For inclination however, the differences might have more impact—affecting whether or not the inner planet transits with its companions. However, from the bottom row of Figure 3.4 and the top panel of Figure 3.5, a mutual inclination of 16° is not enough to remove the inner planet from co-transiting geometry, indicating the the differences between the N-body and secular code are not qualitatively significant in this case.

CHAPTER 4

Inclination-Driving Secular Resonances are Common in Known Exoplanet Systems

4.1 Introduction

The multi-planet Kepler systems, often called “Kepler Multis,” are a class of exoplanet systems uncovered by the *Kepler* mission [Lissauer et al., 2011]. They are characterized by hosting 3 or more planets with even spacing [e.g., Weiss et al., 2018b], low mutual inclinations [e.g., Lissauer et al., 2011, Fang and Margot, 2012, Fabrycky et al., 2014], low eccentricities [e.g., Van Eylen and Albrecht, 2015, Van Eylen et al., 2019b], and a general absence of mean-motion resonant configurations [e.g., Fabrycky et al., 2014]. Moreover, there are suggested correlations in mass and radius of the planets within these systems [i.e., the “Peas in the pod” pattern; Weiss et al., 2018b, Millholland et al., 2017]. While detection bias may be a contributing factor in the detection of this pattern [e.g., Murchikova and Tremaine, 2020b, Zhu, 2020], these biases alone cannot realistically account for all the correlations observed [e.g., Weiss and Petigura, 2020, Weiss et al., 2023]. As these are high-multiplicity exoplanet systems, they experience planet-planet interactions. These interactions have been studied extensively in the literature, and can sculpt the present-day configurations of exoplanet systems both through short-timescale effects like scattering and long-timescale effects like secular evolution [see e.g., Lithwick and Wu, 2011, Van Laerhoven and Greenberg, 2012, Naoz et al., 2013a, Lanza and Shkolnik, 2014, Petrovich et al., 2019b, Denham et al., 2019, Volk and Malhotra, 2020, Wei et al., 2021, Lu et al., 2024].

As an example of the significance of planet-planet interactions, secular resonances can

significantly impact exoplanet systems. Secular resonances can lead to increased rates of angular momentum exchange when certain precessional frequencies align, discussed in detail in section 4.2. These frequencies are much slower than the orbital frequency, hence, “secular”. Secular resonances can interact with tides to circularize or migrate planetary orbits [e.g., [Hansen and Murray, 2015](#)]. In systems with three or more planets, resonances can overlap, causing chaotic evolution of planets’ orbits, known as “secular chaos,” which can destabilize systems, produce ultra short-period planets, and pollute white dwarfs [e.g., [Lithwick and Wu, 2011](#), [Petrovich et al., 2019b](#), [O’Connor et al., 2022](#)].

The secular planet-planet interactions can be quantified to linear order in eccentricity and inclination by the well-studied “Laplace-Lagrange” formalism [see, e.g., Ch. 7 of [Murray and Dermott, 2000](#)]. The Laplace-Lagrange formalism models planets and other bodies orbiting a central body (usually a star) as massive rings, which torque and deform each other, changing their eccentricities and inclinations but conserving energy (semimajor axes). Laplace-Lagrange correctly predicts the initial conditions that produce first-order secular resonances (which depend only on the bodies’ masses and semimajor axes). Laplace-Lagrange itself cannot predict higher-order secular resonances, such as those that create “secular chaos” [see, e.g., [Lithwick and Wu, 2011](#)] that may only appear for particularly excited eccentricity or inclination values. Beyond planet-planet interaction, in recent years, there has been a growing interest in the star-planet interaction, including interactions arising from general relativity, the evaporating protoplanetary disk, and the stellar gravitational quadrupole moment, J_2 [e.g., [Plavchan et al., 2015](#), [Faridani et al., 2022, 2023](#), [Spalding and Batygin, 2016](#), [Becker et al., 2020](#), [Brefka and Becker, 2021](#), [Wei et al., 2021](#), [Chen et al., 2022](#)].

The evolving disk is of particular note because of its time-evolution. Its evolution creates a precession in the planets’ longitudes of ascending node and arguments of periapsis, which can bring them into and out of secular resonance. The time-variation of the precession means that the resonance moves across a (potentially quite large) region of the parameter space rather than staying in place. This is commonly called a “sweeping secular resonance”. If a system is in a region of parameter space where the sweeping resonance occurs, the resonance

can imprint a signature on the orbital elements of the planets post-evaporation. Sweeping resonances have been identified in the evolving disk and have been shown to affect planet formation [e.g., [Best et al., 2024](#)] and excite significant eccentricity and inclination in nearby planets post-formation [e.g., [Heppenheimer, 1980](#), [Ward, 1981](#), [Nagasawa et al., 2003](#), [Li and Lai, 2023](#), [Zanazzi and Chiang, 2024](#), [Petrovich et al., 2020](#)].

The stellar gravitational quadrupole moment, J_2 is a correction to the stellar gravitational potential arising from a star’s deviations from spherical symmetry [e.g., [Murray and Dermott, 2000](#)]. This asymmetry causes nodal recession and periapsis precession for any planets orbiting that star at a rate of

$$\dot{\Omega}_{J_2}, \dot{\varpi}_{J_2} = \mp n \left[\frac{3}{2} J_2 \left(\frac{R_\star}{a} \right)^2 \right] \propto \mp J_2 P^{-7/3}, \quad (4.1)$$

where P is an individual planet’s orbital period [e.g., [Murray and Dermott, 2000](#)]. The quadrupole moment J_2 was shown to misalign tightly packed planets from one another [e.g., [Spalding and Batygin, 2016](#), [Becker et al., 2020](#), [Brefka and Becker, 2021](#)], impact transit probabilities in single-planet systems [e.g., [Stephan and Gaudi, 2023](#)] contribute to tidal migration [e.g., [Pu and Lai, 2019b](#), [Millholland and Spalding, 2020b](#)], and has been used to analyze the orbital inclination of Mercury [e.g., [Ward et al., 1976](#)].

For sun-like, main-sequence stars, the evolution of their spins results in a significant evolution of their J_2 values. During their pre-main-sequence phases, sun-like stars have an observed range of spins, from 10-day periods to as fast as one day [e.g., [Gallet and Bouvier, 2013](#)]. However, it is well-known that main sequence stars lose angular momentum via magnetic braking on their stellar wind [e.g., [Parker, 1958](#), [Schatzman, 1962](#), [Kraft, 1967](#), [Weber and Davis, 1967](#), [Mestel, 1968](#), [Skumanich, 1972](#)]. This spindown can be approximated by a power law for sun-like stars [e.g., [Dobbs-Dixon et al., 2004b](#)], where faster-spinning stars lose more angular momentum per unit time. Over the first gigayear, this dramatically reduces the dispersion in rotational velocities of sun-like stars, and thereafter their spins evolve slower, following the Skumanich relation of $\dot{S} \propto t^{-1/2}$ [[Skumanich, 1972](#)], where S denotes the stellar spin. More massive stars spin down differently: above the “Kraft Break”

at $T_{\text{eff}} > 6250 \text{ K}$ [Kraft, 1967], stars typically experience little angular momentum loss via magnetic braking due to thin convective zones that cannot drive sufficient wind to achieve angular momentum loss.

Most old, sun-like stars have similar spin rates [about a few weeks; Delorme et al., 2011, Meibom et al., 2015, Hartman et al., 2010]. The spin of old ($\sim$ few Gyrs) stars is not correlated with the young stellar spins, and decreases with time [e.g., Barnes, 2003, Skumanich, 1972]. This range of initial conditions and evolution implies that planets are exposed to a wide range of stellar J_2 values during their first gigayear. Therefore, the planets are exposed to a time-varying precession again after the disk evaporates. This can produce a sweeping secular resonance similar to those caused by the evolving disk. This J_2 sweeping resonance can affect three (or more) planet systems, freezing in excited inclinations and eccentricities [Brefka and Becker, 2021, Faridani et al., 2023], and can even excite two-planet systems in a resonance if the inner planet has sufficient angular momentum [e.g., Spalding and Batygin, 2016]. In these resonances, the planets all torque one another, exchanging angular momentum, and the planet with the least angular momentum more easily has its eccentricity and inclination affected by these angular momentum transfers. If planets have similar masses, then the innermost planet will have the least angular momentum and will be the most excited. Angular momentum conservation dictates that the other planets' orientation will have to compensate.

In this work, we focus on observed systems and analyze their secular, planet-planet, and planet-star interaction, with a focus on exploring the potential impact of evolving J_2 on their configuration, and whether evolving J_2 produces sweeping secular resonances in them. We find that about a fifth of all 3-planet observed transiting systems have configurations that indicate they have undergone sweeping resonances due to evolving J_2 in their pasts. This potentially can be used, for example, to constrain the initial conditions of exoplanet systems or to even constrain the initial spin of the host star.

We pedagogically describe the sweeping resonances via the evolving J_2 phenomenon and demonstrate its applicability to observed transiting systems. This paper is structured as

follows: in Section 4.2 we construct the secular model used to explore the secular resonances of interest, in Section 4.3 we demonstrate the impact of system initial conditions on whether sweeping resonances occur and their strengths, and in Section 4.4 we apply these calculations to 31 three-planet transiting systems and show approximately one fifth of them may have experienced sweeping resonances in their past. In Section 4.5 we discuss the implications of these findings and what effects might influence the fraction of systems that encounter inclination-driving secular resonances via evolving J_2 , and in Section 4.6 we conclude.

4.2 Secular Resonances and the Relation to J_2

Consider a star with a mass $M_\star$ and a radius $R_\star$, orbited by a three-planet ($j = 0, 1, 2$) system in which the j th planet has a mass m_j , semi-major axis a_j , period P_j (orbital frequency $n_j = 2\pi/P_j$), eccentricity e_j , longitude of ascending node Ω_j , longitude of periapsis ϖ_j , and inclination with respect to the star's spin axis I_j . A schematic of the system is depicted in Figure 4.1. For completeness, here, we specify the Laplace-Lagrange equations. Given the secular disturbing function $\mathcal{R}^{(\text{sec})}$ [e.g., [Murray and Dermott, 2000](#)]

$$\begin{aligned} \mathcal{R}_j^{(\text{sec})} = & n_j a_j^2 \left[\frac{1}{2} A_{jj} e_j^2 + \frac{1}{2} B_{jj} I_j^2 \right. \\ & + \sum_{k=1, k \neq j}^N A_{jk} e_j e_k \cos(\varpi_j - \varpi_k) \\ & \left. + \sum_{k=1, k \neq j}^N B_{jk} I_j I_k \cos(\Omega_j - \Omega_k) \right], \end{aligned} \quad (4.2)$$

where

$$\begin{aligned} A_{jj} = & n_j \left[\frac{3}{2} J_2 \left(\frac{R_\star}{a_j} \right)^2 \right. \\ & \left. + \sum_{k=1, k \neq j}^N \frac{m_k}{4\pi (M_\star + m_j)} \alpha_{jk} \bar{\alpha}_{jk} f_\psi(\alpha_{jk}) \right], \end{aligned} \quad (4.3)$$

where J_2 is the quadrupole moment of the star. Additionally,

$$A_{jk} = -\frac{n_j m_k}{4\pi (M_\star + m_j)} \alpha_{jk} \bar{\alpha}_{jk} f_{2\psi}(\alpha_{jk}), \quad (4.4)$$

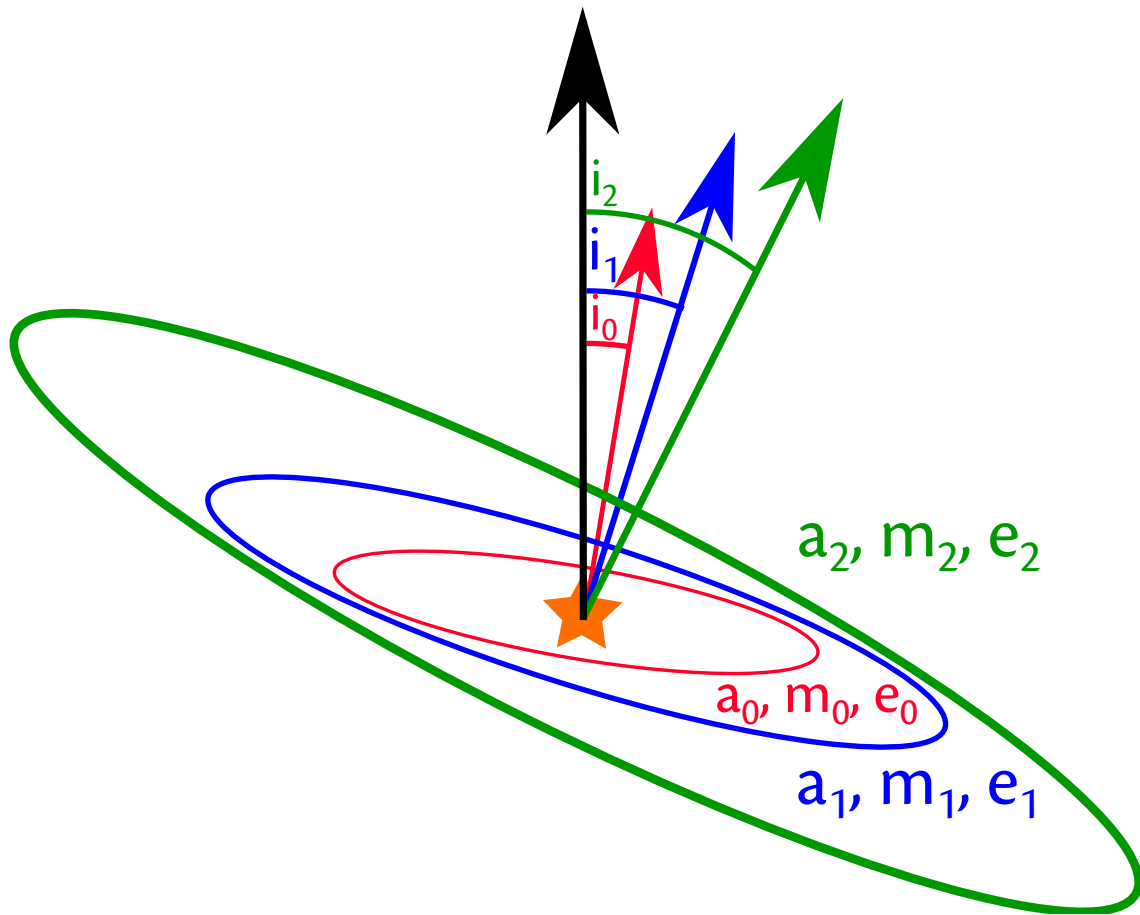


Figure 4.1: A simplified depiction of the three-planet systems considered in this work (not to scale), showing the labeling convention of the planets adopted.

$$B_{jj} = -n_j \left[\frac{3}{2} J_2 \left(\frac{R_\star}{a_j} \right)^2 + \sum_{k=1, k \neq j} \frac{m_k}{4\pi (M_\star + m_j)} \alpha_{jk} \bar{\alpha}_{jk} f_\psi(\alpha_{jk}) \right], \quad (4.5)$$

$$B_{jk} = \frac{n_j m_k}{4\pi (M_\star + m_j)} \alpha_{jk} \bar{\alpha}_{jk} f_\psi(\alpha_{jk}), \quad (4.6)$$

where

$$\alpha_{jk} = \min \left(\frac{a_j}{a_k}, \frac{a_k}{a_j} \right), \quad (4.7)$$

$$\bar{\alpha}_{jk} = \min \left(\frac{a_j}{a_k}, 1 \right), \quad (4.8)$$

$$f_\psi(\alpha_{jk}) = \int_0^{2\pi} \frac{\cos \psi}{(1 - 2\alpha_{jk} \cos \psi + \alpha_{jk}^2)^2} d\psi, \quad (4.9)$$

and

$$f_{2\psi}(\alpha_{jk}) = \int_0^{2\pi} \frac{\cos 2\psi}{(1 - 2\alpha_{jk} \cos \psi + \alpha_{jk}^2)^{\frac{3}{2}}} d\psi. \quad (4.10)$$

Using the disturbing function, Eq. 4.2, we can find the time evolution of the system's orbital parameters as follows

$$\begin{aligned} \dot{e}_j &= -\frac{1}{n_j a_j^2 e_j} \frac{\partial \mathcal{R}_j^{(\text{sec})}}{\partial \varpi_j}, & \dot{\varpi}_j &= +\frac{1}{n_j a_j^2 e_j} \frac{\partial \mathcal{R}_j^{(\text{sec})}}{\partial e_j}, \\ \dot{I}_j &= -\frac{1}{n_j a_j^2 I_j} \frac{\partial \mathcal{R}_j^{(\text{sec})}}{\partial \Omega_j}, & \dot{\Omega}_j &= +\frac{1}{n_j a_j^2 I_j} \frac{\partial \mathcal{R}_j^{(\text{sec})}}{\partial I_j}, \end{aligned} \quad (4.11)$$

For all planets, a small correction to their ϖ precessional frequency can be made to account for (to first-order) general relativity,

$$\dot{\omega}_{j,GR} = \frac{3(GM_\star)^{3/2}}{c^2 a_j^{5/2} (1 - e_j^2)}. \quad (4.12)$$

We include this term for all simulations in this work. However, for all the systems of interest, the planets orbit too far out for this term to ever dominate over the planet-planet interactions or the precession caused by J_2 . In other words, we see that the GR precession term has very little impact in the simulations. Therefore, for our analytic calculations, we omit this term, as it is negligible. Moreover, as GR precession primarily impacts eccentricity evolution (as

eccentricity affects the conjugate momentum associated with the ϖ angle), and in this work we are primarily concerned with inclination evolution, the omission of GR is not expected to have significant impact. When eccentricity evolution is of primary concern, GR can have significant impacts [see e.g., [Naoz et al., 2013b](#), [Pu and Wu, 2015](#), [Denham et al., 2019](#), [Wei et al., 2021](#), [Faridani et al., 2022](#), [Hansen and Naoz, 2020](#)], and can be incorporated into the Laplace-Lagrange A matrix if desired [see e.g., [Marzari and Nagasawa, 2020](#)].

Here, we allow the stellar quadrupole moment to evolve as a function of time, adopting the following relation [e.g., [Spalding and Batygin, 2016](#)]:

$$J_2 = \frac{1}{3}k_2 \left(\frac{S}{S_b} \right)^2, \quad (4.13)$$

where k_2 is the Love number of the star, S represents the star’s spin, and S_b is the breakup spin denoted by $S_b = \sqrt{GM_\star/R_\star^3}$. During the main sequence stage, the stellar spin slows down as it loses angular momentum due to magnetic stellar winds – a process known as magnetic braking [e.g., [Parker, 1958](#), [Schatzman, 1962](#), [Weber and Davis, 1967](#), [Mestel, 1968](#)]. We adopt the spindown rate following [Dobbs-Dixon et al. \[2004b\]](#),

$$\dot{S} = -\alpha S^3, \quad (4.14)$$

with $\alpha = 1.5 \times 10^{-14}$. The effect on J_2 is shown in [Faridani et al. \[2023, Figure 1\]](#), duplicated here in [Figure 4.2](#). Briefly, the ranges shown in [Figure 4.2](#) correspond to the possible range of initial spins for young sun-like stars, and the evolution is derived from [Equation 4.14](#) and from internal parameters derived from a MESA model [[Paxton et al., 2011, 2013, 2015, 2018, 2019](#), [Jermyn et al., 2023](#)]. The power-law spin-down relation is chosen for simplicity. The exact functional form of J_2 evolution has little effect as long as $\dot{J}_2$ is slower than other relevant timescales in the system [see e.g., [Becker et al., 2020](#), [Faridani et al., 2023](#)].

In [Faridani et al. \[2023\]](#), we noted that as J_2 evolves, an exoplanetary system may be swept into a resonance that leaves an imprint on the planet’s eccentricity and inclination. In preparation to map the resonances for a wide range of the planets’ parameter space, we first depict an example. Specifically, consider the system Kepler-9 with parameters given in

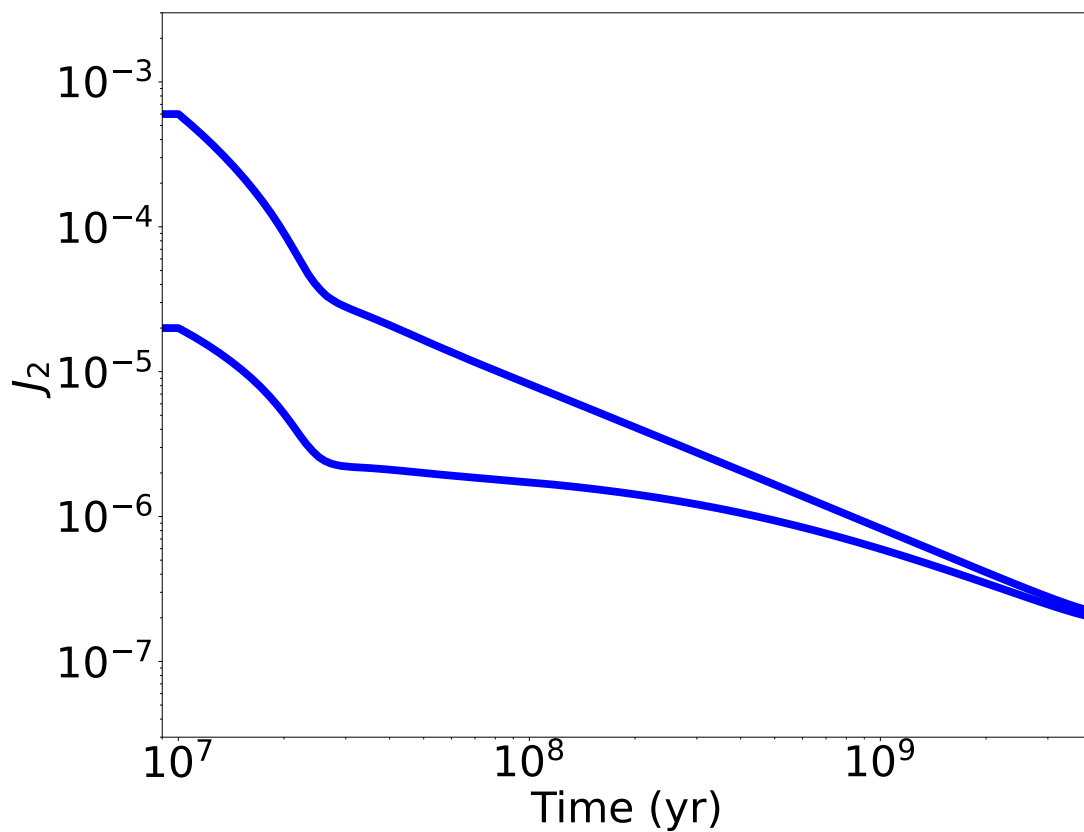


Figure 4.2: Depiction of a range of possible J_2 evolutions for sun-like stars. Figure used with permission from [Faridani et al. \[2023\]](#), and that work describes the models used to generate it.

Object	Mass	Radius	$a(\text{au})$	$e^\dagger$	$i^\dagger$	$\Omega^\dagger$	$\omega^\dagger$	Reference
Star	$1.09 M_\odot$	$1.02 R_\odot$						Torres et al. [2011]
Kepler-9 d	$3.342 M_\oplus$	$1.64 R_\oplus$	0.0273	0.021	0.57°	1.61°	89.5°	Torres et al. [2011]
Kepler-9 b	$43.4 M_\oplus$	$8.29 R_\oplus$	0.1437	0.026	1.72°	1.58°	123.36°	Borsato et al. [2019]
Kepler-9 c	$29.9 M_\oplus$	$8.08 R_\oplus$	0.23015	0.045	4.21°	0.29°	38.47°	Borsato et al. [2019]

Table 4.1: Initial conditions of Kepler-9 used in Figure 4.3. Bolded masses indicate masses calculated using the mass-radius relation shown in Müller et al. [2023]. Semimajor axes are derived from periods rather than the associated references’ reported value to ensure the observed period ratios are maintained. † : parameters randomized according to the scheme described in Table 4.2.

Table 4.1. The inner planet mass was determined using the mass-radius relation described in Müller et al. [2023].

In Figure 4.3, we show the time evolution of Kepler-9’s planetary system. Specifically, we show the mutual inclination between the planets (left top panel) and eccentricity (left bottom panel) as well as the alignment or misalignment of the outer two planets’ longitude of ascending nodes (top right panel) and longitudes of periapsis (bottom right panel). Overplotted on all panels are the eigenfrequencies of the system (blue and red). Mutual inclinations were obtained by combining I_j and Ω_j to get the 3D planetary angular momentum vector for each planet and the mutual inclination was calculated using a dot product. The initial mutual inclination between the outer planets is chosen to be 2.5° . For simplicity, we adopt that the initial longitudes of ascending node of the planets are set to (approximately) zero. This evolution was obtained by integrating Equations 4.11 (which include J_2 precession) and adding GR precession. As Equations 4.11 are secularly averaged, the evolution in Figure 4.3 is therefore purely secular.

Initially, J_2 is high, and the eccentricities and mutual inclinations oscillate about their initial value and do not evolve significantly for the first few Myr. This is because the J_2

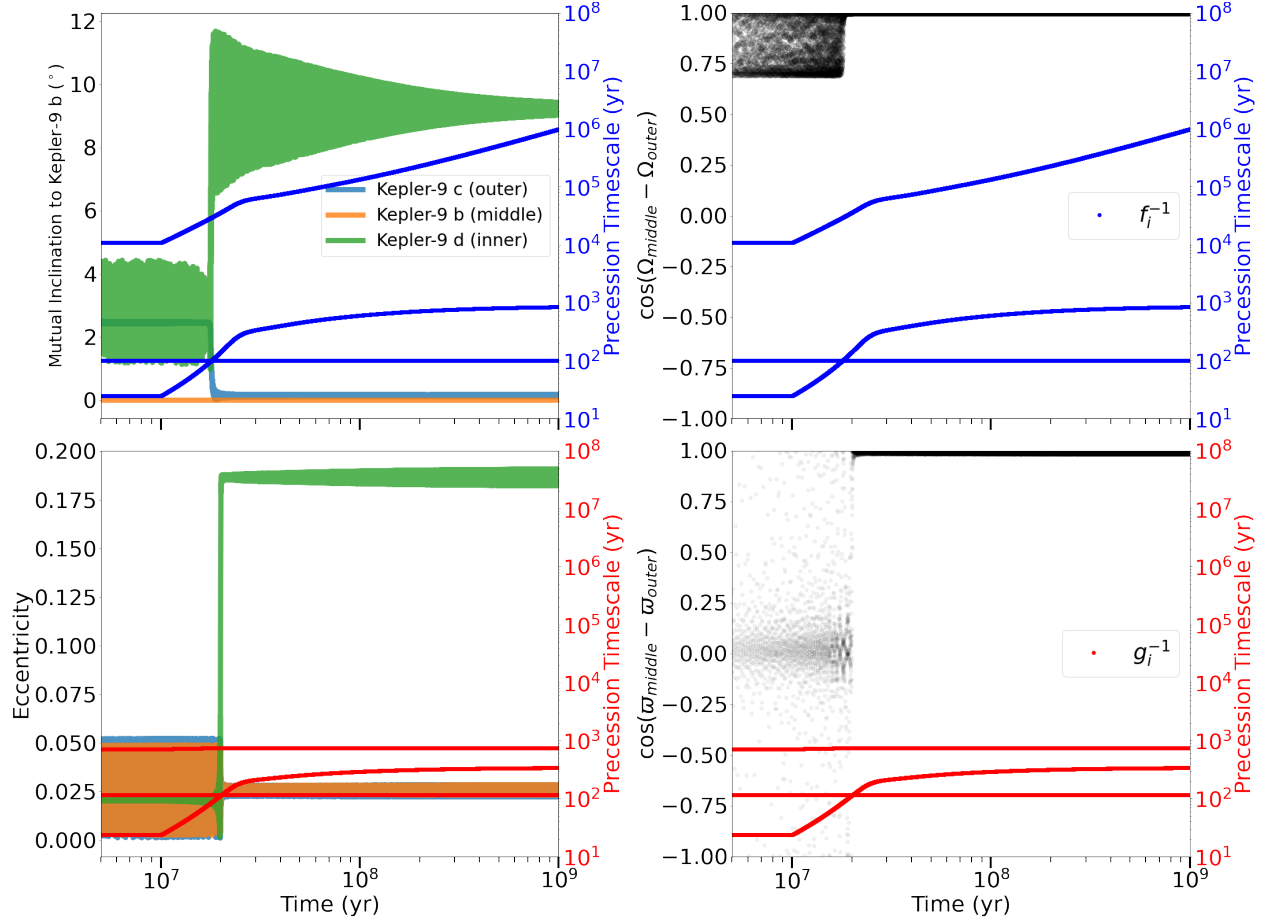


Figure 4.3: Time evolution of sun-like star Kepler-9's planetary system. Upper Left: Mutual inclination between planets b, c, and d and Kepler-9 b (the middle planet) as a function of time. Lower Left: Eccentricity of Kepler 9 b, c, and d as a function of time. Upper Right: Evolution of the difference in longitudes of ascending node (Ω) between Kepler-9 b and c (the middle and outer planets) as a function of time. The alignment near 2×10^7 years marks the resonance that misaligns Kepler-9 d from the rest of the planets, and simultaneously locks Kepler-9 b and c into coplanarity. Lower Right: Evolution of the difference in longitudes of periaapsis (ϖ) between Kepler-9 b and c as a function of time.

Overplotted on all figures are the Laplace-Lagrange eigenfrequencies f_i and g_i .

evolution in Figure 4.2 remains constant for the first 10 Myr.¹ The inner planet initially has the majority of its eccentricity and inclination oscillation amplitude in the shortest-period eigenmode. The outer two planets have vanishingly small amplitude in this mode. However, as J_2 evolves, for both inclination and eccentricity, this mode eventually has the same period as one of the other two modes—bringing the inner planet into resonance with its companions. Kepler-9 is swept into a resonance, exciting Kepler-9d’s eccentricity and inclination, and the outer two planets’ longitudes of ascending nodes lock together ($\Omega_b = \Omega_c$). A similar effect occurs in the eccentricities and longitudes of pericenter ($\varpi_b = \varpi_c$), though at a later time in the evolution. The inclination excitation occurs at $t \approx 17$ Myr and the eccentricity excitation at ≈ 22 Myr. Note that despite appearing instantaneous, the exchange of angular momentum that excites the inner planet takes place over $\sim 2 \times 10^5$ years. However, despite the exchange occurring over such a long timescale, the exchange is not an adiabatic resonant capture—the inner planet does not remain in resonance with the outer two planets as the colliding eigenvalues diverge. The amplitude of the largest-amplitude eigenmode of the inner planet (i.e., $\max(e_{00}, e_{01}, e_{02})$ and $\max(I_{00}, I_{01}, I_{02})$ in Eq 4.16) is an adiabatic invariant before and after the resonant passage [see e.g., Ward et al., 1976, for a complete treatment], but transitions to a new value during the resonant passage. Moreover, the resonance is only ever achieved by the inner planet’s highest-amplitude eigenfrequency approaching another eigenfrequency from above (see the overplotted eigenmode periods in Figure 4.3). To approach from below would require J_2 to increase with time.

Clearly, the effects of resonances mediated by J_2 can be drastic. Thus, exploring the maximum inclination (and eccentricity) achieved for a range of J_2 values is a promising avenue for evaluating the impact of evolving J_2 in different systems. For a fixed value of J_2 and neglecting General Relativity precession, the maximum value of the planets’ inclinations over all time can be calculated as follows². Let g_i and f_i be the eigenvalues of the matrices

¹see Appendix 4.8 for a discussion

²Including GR does not change the calculation when finding maximum inclinations because GR precesses the argument of pericenter instead of the longitude of ascending node. The maximum eccentricity, however, is affected by GR, and when a planet orbits close enough to its host that it has a period shorter than a few days, this calculation may not be accurate.

A and B and $\bar{e}_{ij}$ and $\bar{I}_{ij}$ the matrix of normalized eigenvectors of A and B . It can be shown that for h_j , k_j , p_j , and q_j defined as

$$\begin{aligned} h_j &= e_j \sin \varpi_j, & k_j &= e_j \cos \varpi_j, \\ p_j &= I_j \sin \Omega_j, & q_j &= I_j \cos \Omega_j, \end{aligned} \quad (4.15)$$

their time evolution, for a fixed value of J_2 , is

$$\begin{aligned} h_j &= \sum_{i=1}^N e_{ji} \sin(g_i t + \beta_i), & k_j &= \sum_{i=1}^N e_{ji} \cos(g_i t + \beta_i), \\ p_j &= \sum_{i=1}^N I_{ji} \sin(f_i t + \gamma_i), & q_j &= \sum_{i=1}^N I_{ji} \cos(f_i t + \gamma_i), \end{aligned} \quad (4.16)$$

for scaled e_{ji} and I_{ji} defined as, $S_i \bar{e}_{ji} = e_{ji}$ and $T_i \bar{I}_{ji} = I_{ji}$, and phases β_i and γ_i . This time evolution is only true for a fixed value of J_2 because changing J_2 changes the eigenvalues g_i and f_i , as A_{jj} and B_{jj} are functions of J_2 .

Therefore, for a fixed J_2 , the maximum eccentricity and inclination of planet j can be calculated by finding the values when all the oscillations are in phase: $e_{j,\max} = \sum_{i=1}^N |e_{ji}|$, and $I_{j,\max} = \sum_{i=1}^N |I_{ji}|$. To find, e_{ji} and I_{ji} the values of the scaling constants S_i , T_i are required. S_i , T_i and the phases β_i and γ_i can be found by solving the linear equations,

$$\begin{aligned} e_{ji} S_i \sin \beta_i &= h_j(t=0), & e_{ji} S_i \cos \beta_i &= k_j(t=0), \\ I_{ji} T_i \sin \gamma_i &= p_j(t=0), & I_{ji} T_i \cos \gamma_i &= q_j(t=0). \end{aligned} \quad (4.17)$$

These calculations enable analytical prediction of the inclination induced by J_2 if they are repeated for a wide variety of J_2 values.

4.3 The Effect of Initial Conditions on Secular Resonances

In Figure 4.4, we show, for a synthetic three-planet system, the relationship between the semimajor axis ratios between the planets and the subsequent inclination induced on the innermost planet by interactions with the outer planets. Planet radii for the inner, middle, and outer planets, were selected as median radii in that orbital position across the 31 three-planet systems studied in Section 4.4. Planet masses were calculated using the Mass-Radius relation of Müller et al. [2023]. The analytic calculations described in the previous section

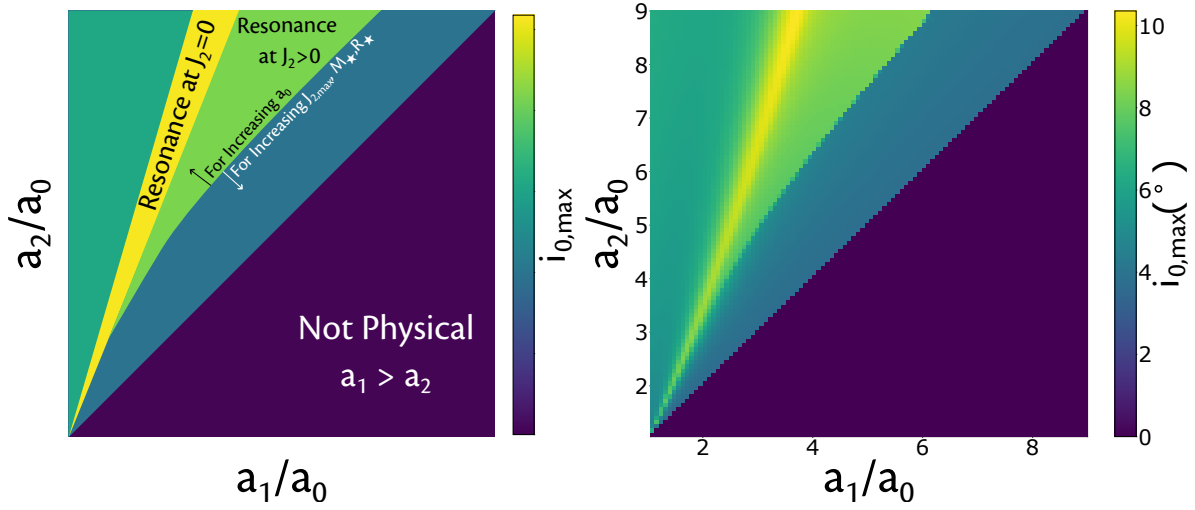


Figure 4.4: Left: Heuristic relationships between companion planet semimajor axis ratios (axes) and maximum inclination induced on the inner planet of a three planet system via planet-planet interactions (color bar) as J_2 decreases over time. Annotations describe how variations of system parameters affect the structure. The boundaries between regions were drawn by eye as a representation of the right panel.

Right: Analytical calculation of relation between semimajor axis ratios and inner planet maximum inclination over all values of J_2 explored by the star as it spins down.. The calculations were performed following the prescription described at the end of Section 4.2. Planets 1 and 2 were initialized with a 3° mutual inclination. Planet radii were selected as median radii in that orbital position across the 31 three-planet systems studied in Section 4.4 (i.e., the inner planet has the median radius of all inner planets in our sample of 3-planet systems (see Table 4.5), and so on for middle and outer planet). The inner planet was set at a semimajor axis of $a_0 = 0.1$ au, J_2 spanned from $10^{-3} - 10^{-8}$, and the inner, middle, and outer planet masses were set to $3.2, 9.2, 12.3 M_\oplus$ respectively using the mass-radius relation of Müller et al. [2023].

describe how to calculate the maximum inclination of the inner planet due to planet-planet interactions and J_2 precession for a fixed value of J_2 . To produce the right panel of Figure 4.4, these calculations were repeated for many J_2 values between 10^{-3} and 10^{-8} , and the maximum inner planet inclination across all J_2 values is shown for each semimajor axis ratio in the grid. This calculation produces five visually distinct regions, which we show heuristically in the left panel. The region boundaries in the left panel were drawn by hand. The notations on the left panel describe how these regions change when parameters are adjusted.

In the leftmost region, $a_2 \gg a_1$, there is no resonance, and inclinations induced on the inner planet are small. To the right of this region is a thin strip where inner planet inclination is maximized. This occurs for only a narrow subset of choices of a_1 and a_2 . This region is caused by secular resonances when $J_2 = 0$ or when J_2 is sufficiently small to be negligible. These $J_2 = 0$ resonances occur for the inner planet when the inner planet’s oscillation has majority amplitude in the same eigenmode as the outer planets at $J_2 = 0$, causing it to precess together with one or more of the outer planets. This increases the amount of eccentricity and inclination oscillation of the inner planet relative to the amount of oscillation if the semimajor axis of the inner planet was perturbed slightly [see e.g., [Faridani et al., 2022](#)]. To the right of the $J_2 = 0$ resonant region is a much larger region of enhanced inclination. This is caused by secular resonances for a range of nonzero, nontrivial J_2 values. This region is not in resonance for $J_2 = 0$, but for every point in the region, there exists some $J_2 \neq 0$ which precesses the planets just enough to bring the planets into secular resonance. We call this the “Sweeping Resonance” region because for a particular value of J_2 , only a small portion of the region is in resonance, and it is only as J_2 continuously evolves that that resonant portion “sweeps” across the parameter space. To the right of this region is a nonresonant region where $a_1 \approx a_2$. To the right of that region, indicated by the dark lower triangle, is a forbidden region by the restriction that $a_2 > a_1$. Written in the left panel are descriptions of how these regions’ boundaries change for different parameter values. For increasing a_0 , the sweeping resonant region for $J_2 > 0$ decreases in size, but the $J_2 = 0$ resonance stays

in place. For increasing maximum J_2 , $M_\star$, and $R_\star$, the sweeping resonant region for $J_2 > 0$ increases in size.

The general shape shown in the right panel of Figure 4.4 is conserved across many choices of system parameters. Indeed, if $J_2 = 0$, there are no terms in Eqs (4.3)-(4.10) that depend on the individual semimajor axes of the planets—only the ratios between planets. Moreover, when $J_2 = 0$, assuming that $m_j \ll M_\star$ for all planets j , then scaling all the planet masses by the same amount will not affect the resulting inclination of the inner planet, as all elements of the matrices will be scaled by the same amount—keeping the eigenvectors constant. However, relaxing $J_2 = 0$ can turn on the “Sweeping Resonance” region. In this case, scaling up all the planet masses reduces the magnitude of the precession caused by J_2 compared to planet-planet interactions. This weakening pushes the region where the inner planet experiences “Sweeping Resonances” to larger values of a_1 and a_2 , where the mass-boosted planet-planet interactions no longer dominate the J_2 precession.

The effect of the planet masses can also be mapped out like we have for the semimajor axes. In Figure 4.5, we show the impact of varying the ratios between the outer planets’ masses and the inner planet’s mass for four different sets of semimajor axes. As a base system, we use the characteristic parameters shown in the right panel of Figure 4.4. The four choices of semimajor axis ratios are labeled on the left panel of Figure 4.5, and feature fixed outer and inner planet semimajor axes, with a varying middle planet semimajor axis between them. The middle planet’s semimajor axis is varied such that the system explores the four regimes: nonresonant with a small middle planet semimajor axis (labeled ‘A’), static resonance (labeled ‘B’), sweeping resonance (labeled ‘C’), and nonresonant with a large middle planet semimajor axis (labeled ‘D’). The green stars indicate the masses that were used when calculating the left side of Figure 4.5. In region ‘A,’ the nonresonant region, the relationships between mass and inclination induced are smooth, with a slightly larger inclination achieved when m_2 increases and m_1 stays the same. In region ‘B,’ the effect of resonance is made clear by the vastly enhanced inclination when both masses m_1 and m_2 increase relative to m_{in} . In region ‘C,’ the sweeping resonance region, the figure shows

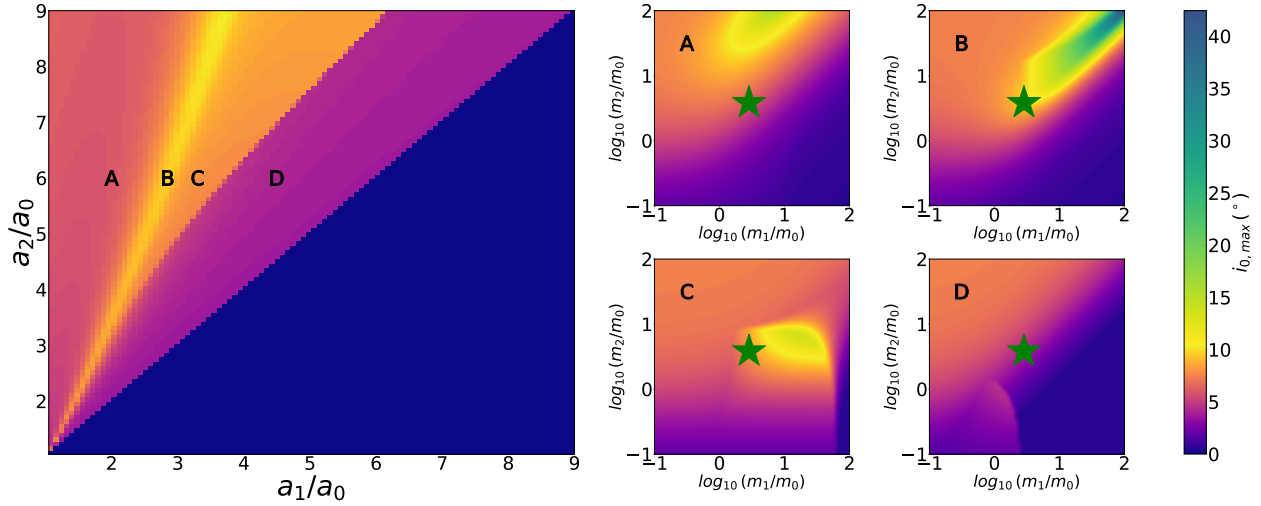


Figure 4.5: Left: Relationship between companion planet semimajor axis ratios (axes) and maximum inclination induced on candidate inner planet via resonance (color bar). Planets 1 and 2 were initialized with a 4° mutual inclination

Right: Effects of the variation of the masses of the outer two planets at four different sets of semimajor axis ratios shown on the leftmost panel corresponding to the four different qualitative regions shown: A) a_{in} and a_1 are too close and a_2 too far away for the system to enter resonance; B) a_1 and a_2 are such that the inner planet is in resonance with the outer two at $J_2 = 0$; C) a_1 and a_2 are such that the inner planet is in resonance with the outer two for some particular $J_2 \neq 0$; D) a_1 and a_2 are such that the inner planet will never be in resonance with the outer two for any of the J_2 values the star will explore. The green stars mark the mass ratio assumed in the left panel.

the range of masses that allow for sweeping resonance. For $0 < \log_{10}(m_1/m_{in}) < 1.25$ and $-1 < \log_{10}(m_2/m_{in}) < 1$, there is a wedge of parameter space with enhanced inclination. Outside this wedge, the induced inclination is similar to region ‘A,’ the nonresonant region, indicating that for those masses, no sweeping resonance exists. Region ‘D’ appears similar to region ‘A,’ which is fitting as they are both nonresonant; however, at smaller values of m_1 and m_2 , a similar wedge shape to region ‘C’ appears. This indicates that for choices of masses that lie within that wedge, there would be a sweeping resonance at the semimajor axis ratios denoted by region ‘D.’

Notable in Figure 4.5 is the effect of increasing companion planet masses. In the resonant regions (panels ‘B’ and ‘C’), increasing the masses of the outer planets relative to the innermost planet (i.e., moving the green star upward and rightward) tends to increase the amount of misalignment produced by the resonances. However, especially in panel ‘C,’ this is not guaranteed. If the masses of the outer two planets are increased past a certain point, then the strength of J_2 precession relative to the precession caused by planet-planet interactions decreases, and the requisite J_2 to push the inner planet into resonance is higher than the maximum J_2 allowed. This defines the top edge of the wedge shape in panel ‘C.’ Unlike region ‘C,’ in region ‘B’ where $J_2 = 0$, there is no maximum outer planet masses beyond which no resonance occurs. The green strip extending to high mass ratios indicates that for a large, given outer planet mass, a middle planet mass exists that brings the inner planet into resonance. A similar phenomenon is also shown in panel ‘A’. If, from the mass ratios of the green star, m_2/m_0 is increased and m_1/m_0 is held constant, then the system can enter the resonant region visible above the green star. In this sense, region ‘A’ is not truly non-resonant for all planet masses.

4.4 Observed Systems

Clearly, sweeping secular resonances can have significant impact on exoplanet systems. This naturally poses the question: to what extent are these effects reflected in the known exoplanet

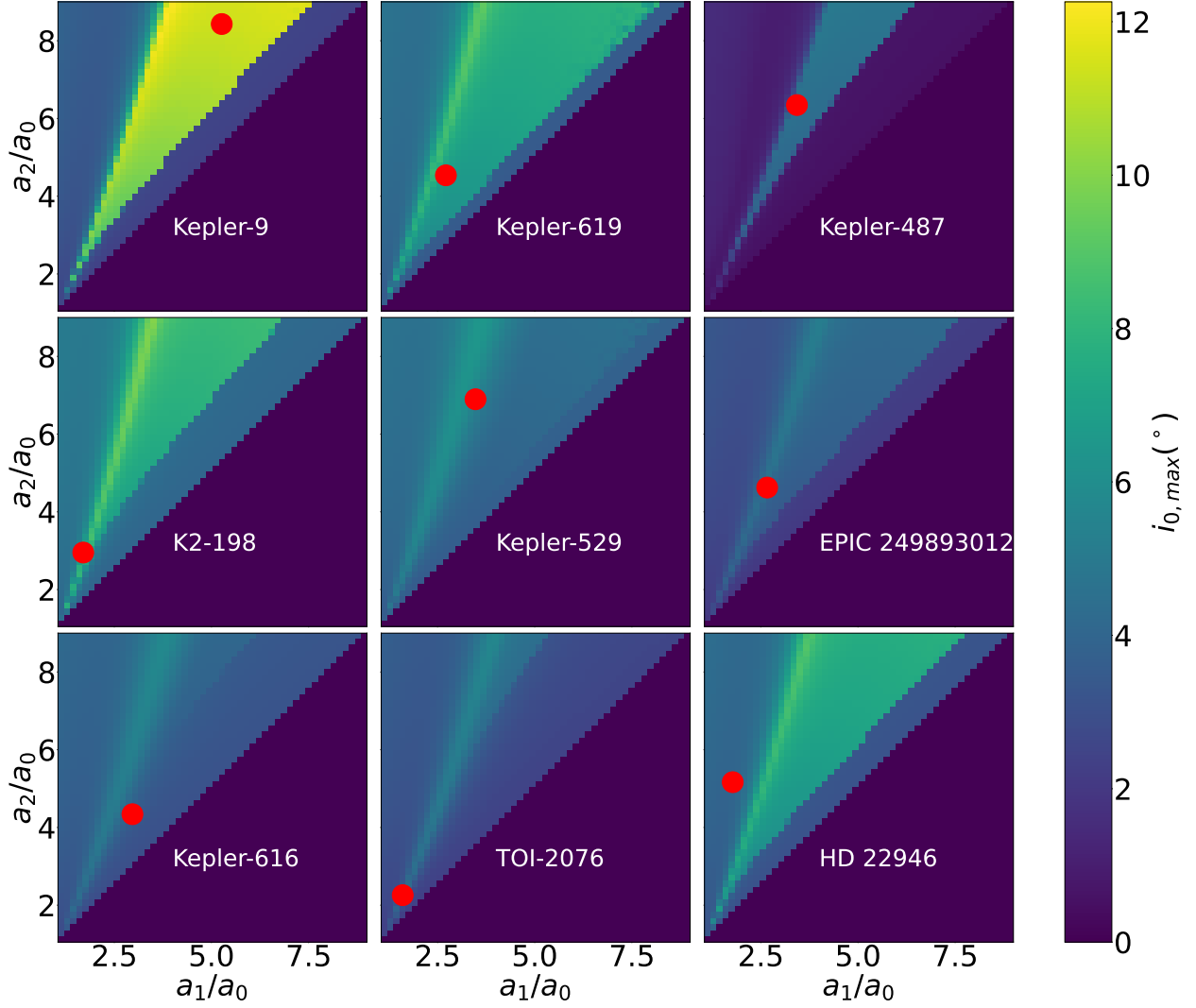


Figure 4.6: Relation between companion semimajor axes and inner planet maximum inclination for nine of the 31 identified three-planet systems in our sample. The initial mutual inclination between the outer planets for each panel was set to 3° . The top two rows contain only systems that undergo sweeping resonances, and the bottom row contains only systems that do not.

Object	Mass, Stellar Radius	a (au)	e	i	Ω	ω
Star	From Reference					
Inner Planet	From Reference	From Reference	$\mathcal{R}(0.049)$	1°	$\mathcal{U}(2\pi)$	$\mathcal{U}(2\pi)$
Middle Planet	From Reference	From Reference	$\mathcal{R}(0.049)$	$\mathcal{R}(0.049 \text{ rad})$	$\mathcal{U}(2\pi)$	$\mathcal{U}(2\pi)$
Outer Planet	From Reference	From Reference	$\mathcal{R}(0.049)$	$\mathcal{R}(0.049 \text{ rad})$	$\mathcal{U}(2\pi)$	$\mathcal{U}(2\pi)$

Table 4.2: Randomization Scheme for Simulations. Simulations were integrated for 2 Gyr. $\mathcal{R}(\sigma)$ represents a random variable drawn from a Rayleigh distribution with scale σ . $\mathcal{U}(X)$ represents a random variable drawn from a uniform distribution between 0 and X .

sample? To investigate this, we consider 31 known exoplanet systems. These systems were selected from the NASA Exoplanet Archive [NASA Exoplanet Science Institute, 2025] with the following criteria: **1)** The host star has its temperature in the range for FGK-type stars below the Kraft break; **2)** The host star has its mass and radius measured and is not significantly evolved; **3)** The planetary system has exactly three observed planets; **4)** Each planet in the system transits its host star; and **5)** Each planet in the system has its mass or its radius measured. After all five criteria have been applied, only 31 systems remain. In cases where a planet only had its radius measured, its mass was assigned by the Mass-Radius relation given in Müller et al. [2023]. The parameters adopted are listed in Table 4.5.

To quantify which systems undergo systematic misalignments similar to those depicted in Figure 4.3, for each of these 31 systems, we run 100 integrations of the secular equations described in Eqs 4.11. In these integrations, we include the effects of precession due to J_2 and GR, and we let J_2 evolve according to the upper curve of Figure 4.2. Each simulation was allowed to run for 2 Gyr. In each simulation the initial conditions of the planets were randomized according to the prescription given in Table 4.2. We find that six of these 31 systems exhibit permanent misalignments between planets mediated by J_2 , corresponding to about one fifth of the sample. Similar to the evolution depicted in Figure 4.3, when J_2 passes a critical value, the outer two planets in these systems align their orbits. Simultaneously, the innermost planet becomes misaligned from the outer two. The values of J_2 and the corresponding times when the misalignment occurs are listed in Table 4.3.

In Figure 4.6, we show the structure of increases of inner-planet inclination in a_1/a_0 – a_2/a_0 space of nine systems. Each panel of Figure 4.6 represents a different observed system,

System	Resonant J_2	t_{res} (yr)
Kepler-9	1.3×10^{-4}	1.7×10^7
Kepler-619	7×10^{-5}	2×10^7
Kepler-487	1.1×10^{-4}	1.85×10^7
K2-198	8.3×10^{-5}	2×10^7
Kepler-529	1.8×10^{-5}	4.6×10^7
EPIC 249893012	2.1×10^{-4}	1.5×10^7

Table 4.3: Values of J_2 for system inclination resonance for the six sweeping-resonant systems identified in Section 4.4. Additionally, the corresponding times for this J_2 value are listed (following the top curve of Figure 4.2).

with the observed values of the semimajor axis ratios (derived from the extremely accurately measured period ratios) marked with a red dot. The top two rows of Figure 4.6 are the six systems from our 31-system sample which were found in the Monte Carlo simulations to undergo permanent misalignments. The bottom row of Figure 4.6 contains three of the 25 systems whose simulations did not undergo permanent misalignments. The bottom row systems are chosen from various regions of non-sweeping parameter space: high a_1/a_0 (Kepler-616; see Table 4.5), “static” resonance that does not sweep, and therefore does not produce misalignments (TOI-2076; discussed in detail in Section 4.4.2), and low a_1/a_0 (HD 22946; see Table 4.5). The values of $i_{0,max}$ shown in Figure 4.6 are calculated using the method described at the end of Section 4.2 for initial mutual inclinations between middle and outer planets of 3° . The predictions of inner-planet inclination of Figure 4.6 and the simulations shown later in the red circles of Figure 4.7 demonstrate good agreement between the analytic prediction and the simulated result.

To quantify the misalignment present in a system caused by changing J_2 , we define the parameter

$$\Delta i_{jk}^{mut} = \langle i_{jk}^{mut} \rangle_{late} - \langle i_{jk}^{mut} \rangle_{early} , \quad (4.18)$$

where $\langle i_{jk}^{mut} \rangle_{early}$ is the time-averaged mutual inclination between planets j and k before

$t = 1$ Myr (this is before J_2 evolution is turned on at $t = 10$ Myr) and $\langle i_{jk}^{mut} \rangle_{late}$ is the average mutual inclination between planets j and k during the last 40 Myr of the simulation. A positive value of Δi_{jk}^{mut} implies that planets j and k became more misaligned over the course of the simulation, and a negative value implies that planets j and k aligned themselves. We choose to define Δi_{jk}^{mut} in terms of time-averages at early and late times to remove dependence on the initial phase of inclination oscillation.

In each panel of Figure 4.7, we show the relationship between the initial mutual inclination between the outer two planets (x-axis) and the change in planets' relative alignment (y-axes, Δi_{01}^{mut} in red and Δi_{12}^{mut} in blue). The top two rows are systems whose positions in a_1/a_0 – a_2/a_0 space indicate they undergo sweeping resonances. For these systems, there is an approximately linear relationship between initial mutual inclination between outer planets, and the (mis-)alignments between the planets after J_2 evolution has had its effect. In each panel, the dotted red line represents the minimum mutual inclination between the middle and inner planets such that the inner planet may not co-transit with the middle planet under the simplifying assumption that the middle planet's transit impact parameter $b_1 = 0$. Many of the sweeping resonant systems would require very large initial mutual inclinations between the outer two planets to make the inner planet no longer transit.

However, that is not the only constraint we can place on the initial conditions. If the outer two planets are prepared with high initial mutual inclinations, under what conditions do they align enough over the course of the simulation such that at the end, they both transit? This is shown in the blue dotted lines. For blue triangles above the blue dotted line, the initial mutual inclination between the outer two planets is high enough that they may not transit together (assuming the middle planet's impact parameter $b_1 = 0$), even after the sweeping resonance from evolving J_2 partially aligns their orbits. Of course, if the longitudes of ascending nodes are favorable, even two greatly mutually inclined planets may still co-transit. Therefore, even systems with red dots above the red dotted line or blue triangles above the blue dotted line may have all three planets transit depending on nodal positioning. To account for these two constraints on $i_{mut,12,init}$, regions on Figure 4.7 are

shaded. The shaded regions correspond to the ranges of $i_{mut,12,init}$ such that all three planets are guaranteed to co-transit under the $b_1 = 0$ assumption. The color of the shaded region indicates which planet escapes transit first as $i_{mut,12,init}$ increases—red for the inner planet and blue for the outer planet. Therefore, to be guaranteed consistency with observations, $i_{mut,12,init}$ must be smaller than the rightmost edge of the shaded region. For example, Kepler-487 (which experiences sweeping resonance), has only a small range of $i_{mut,12,init}$ values where co-transit is guaranteed. The outer planets of Kepler-487 do not align sufficiently during resonance to overcome large initial mutual inclinations, and at $i_{mut,12,init} \sim 2^\circ$ the outer two planets no longer always co-transit. Systems that do not undergo sweeping resonance—such as Kepler-616, for example—have far stricter restraints on $i_{mut,12,init}$, as they are subject to the geometric requirements for co-transit without a sweeping resonance to align the outer planets. Additionally, Kepler-487, as well as K2-198 and EPIC 249893012, also has an interesting property where if $i_{mut,12,init}$ is near zero, then the inner and middle planets will be the ones aligning, and the outer planet misaligns from the rest. This is represented by negative (positive) values of Δi_{01}^{mut} (Δi_{12}^{mut}).

With these constraints on initial inclinations in mind, it may be worthwhile to consider whether they are consistent with known constraints on the exoplanet population. For example, [He et al. \[2020\]](#) and [Millholland et al. \[2021\]](#) find that the *Kepler* sample’s properties are consistent with each system having the maximum stable angular momentum deficit [AMD; see e.g., [Laskar and Petit, 2017](#)] distributed randomly, weighted by mass, between the planets. The sweeping resonances described here conserve angular momentum (and therefore AMD), meaning that these resonances do not produce a systematic signal contrary to the maximum-AMD model. However, as can be seen in Figures 4.3 and 4.7, these resonances cause the outer two planets to align and circularize, dumping much of their AMD into the (usually less-massive) inner planet to make its orbit misaligned and eccentric. If the initial distribution of AMD between planets is as assumed by [He et al. \[2020\]](#), where the more massive planets hold more of the system’s AMD, then observations of lower-mass inner planets holding outsized proportions of AMD could be an indication that these sweeping resonances

have occurred.

4.4.1 Sweeping Resonant Systems

In Figure 4.6 we showed six systems that lie in the range of parameter space where sweeping resonances via J_2 evolution can occur. Parameters adopted for all 31 systems in the sample are shown in Table 4.5. Below we comment on Kepler-9 and EPIC 249893012, as their properties are of note.

4.4.1.1 Kepler-9

Kepler-9 is a well-studied system, discovered by [Holman et al. \[2010\]](#). As it was one of the first-validated Kepler systems, its parameters have been studied and adjusted many times through transits, transit timing variations, and radial velocity measurements [e.g., [Borsato et al., 2019](#), [Albrecht et al., 2021](#), [Berger et al., 2018](#), [Wang et al., 2018](#), [Hadden and Lithwick, 2017](#), [Holczer et al., 2016](#), [Morton et al., 2016](#), [Borsato et al., 2014](#), [Hadden and Lithwick, 2014](#), [Torres et al., 2011](#), [Weiss et al., 2024](#)]. The outer two planets are near, but outside, a 2 : 1 mean-motion resonance that integrations of the most recent fits indicate is an unstable resonance [e.g., [Antoniadou and Libert, 2020](#)]. The literature suggests that the outer two planets have aligned orbital axes [e.g., [Freudenthal et al., 2018](#)], which we note is consistent with the damping out of any primordial orbital misalignments between them as shown in Figures 4.3 and 4.7.

4.4.1.2 EPIC 249893012

EPIC 249893012 is a sun-like star just beginning its evolution off the main sequence [[Hidalgo et al., 2020](#)]. As it is no longer a main-sequence star, it would normally have been excluded from this sample. However, as there is substantial evidence this star is just leaving the main sequence, its mass while its J_2 was spinning down would have been approximately the same as its current mass and therefore the planets should not have experienced stellar evolution-

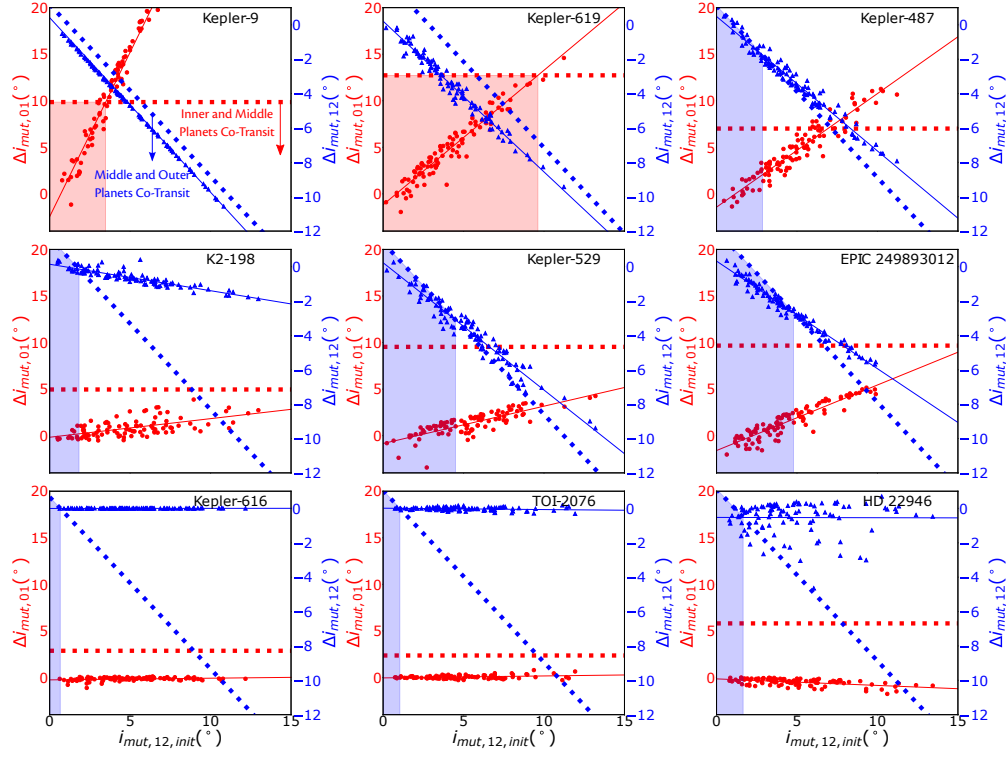


Figure 4.7: Relation between initial mutual inclination between the two outermost planets and changes in alignment between planets over the course of the simulation for nine systems. The top six systems undergo resonance-mediated misalignments. In red circles are the change in alignment between the inner and middle planets, $(\Delta i_{mut,01})$ —the average mutual inclination at late times between the inner and middle planet minus the initial mutual inclination between the inner and middle planet (see Eq 4.18). The red dotted line represents the minimum $\Delta i_{mut,01}$ such that the inner planet fails to transit. In blue triangles are $(\Delta i_{mut,12})$. The blue dotted line represents the maximum value of $\Delta i_{mut,12}$ as a function of $i_{mut,12,init}$ such that the outer two planets are guaranteed to co-transit. Lines of best fit are plotted for both the red circles and blue triangles. The shaded regions represent the range of $i_{mut,12,init}$ such that all three planets are guaranteed to transit. The color of the shaded region indicates which planet escapes transit first as $i_{mut,12,init}$ increases—red for the inner planet and blue for the outer planet. The scheme by which orbital parameters were varied between simulations is described in Table 4.2.

based migration. Therefore, it is a valid inclusion, and for the purposes of this work, we have set its radius to $1.1 R_{\odot}$.

[Gupta and Schlichting \[2021\]](#) calculates that the inner planet, EPIC 249893012 b, is losing mass at a rate of approximately $10^8 g/s$. However, if this mass loss rate has been sustained for the entire age of the system ~ 9 Gyr, the mass of EPIC 249893012 b would have changed by only $0.005 M_{\oplus}$ since its formation. Therefore, we adopted EPIC 249893012 b’s currently measured mass for use in our simulations, as its atmospheric mass loss does not alter significantly its total mass.

4.4.2 The Impact of Static vs Sweeping Resonances

As depicted in [Figure 4.4](#), we identify a region in resonance associated with $J_2 = 0$ (the yellow strip, labeled “Resonance at $J_2 = 0$ ”). Not all of this $J_2 = 0$ resonant region produces systematic misalignments similar to those demonstrated in [Figure 4.3](#). At small values of a_1/a_0 and a_2/a_0 or when a_0 is large, or both, the precession rate caused by J_2 is slow (even at $J_2 = 10^{-3}$) compared to the precession caused by planet-planet interactions. In this case, J_2 precession is not fast enough to shift the location of the secular resonance for any value of J_2 the star can be reasonably expected to achieve. We call these unmoving resonances “static”.

Static resonances notably do not produce the misalignment from evolving J_2 that sweeping resonances do. For example, the system TOI-2076 [observed by e.g., [Osborn et al., 2022](#), [Hedges et al., 2021](#)] lies on such a static resonance as depicted in the bottom middle panel of [Figure 4.6](#). In [Figure 4.9](#), we show the semimajor axis parameter space of TOI-2076 for $J_2 = 10^{-3}$ (left panel) and $J_2 = 0$ (right panel). The location of TOI-2076’s observed semimajor axis ratios is marked with a red dot. In both panels, TOI-2076 lies on a secular resonance with the same expected inner planet maximum inclination. This indicates that there is no expected change to the system as J_2 evolves from $\sim 10^{-3}$ to 0. Therefore, we do not expect the inner planet to systematically and/or permanently misalign from its

companions any more than is provided in its initial conditions. Performing the eigendecomposition described in Section 4.2, calculating the amplitudes of each of the planets in each of the eigenfrequencies of the B matrix, one finds that all three planets have their maximum amplitude in the same mode for any of the values of J_2 taken on in Figure 4.2. This mode’s period can be seen to vary from 10^4 years to 10^7 years in the top row of Figure 4.8. Therefore, the three planets have their Ω values precess together forever, with all the angles $\Omega_1 - \Omega_0$, $\Omega_2 - \Omega_0$, and $\Omega_2 - \Omega_1$ librating.

Indeed, in Figure 4.8, we show the time evolution of one of the simulations of TOI-2076 from the Monte Carlo. In the upper right panel, we can see that the longitudes of ascending nodes between the outer two planets are locked together; however, the upper left panel shows that mutual inclinations do not significantly change. The planets do not misalign. In the bottom middle panel of Figure 4.7, we can see that even for large initial values of $i_{mut,12}$, no permanent misalignment occurs between the planets in this system—they retain whatever mutual inclinations they are initialized with.

Therefore, the misalignments shown in the sweeping resonant systems depicted in the top two rows of Figure 4.7 arise because these systems are initially not in resonance for high J_2 values, then enter resonance as J_2 decreases, increasing the inner planet’s inclination predicted from Laplace-Lagrange. Depending on their locations in a_1/a_0 – a_2/a_0 space, the planets may later leave resonance, but the mark of having passed through resonance is left on their mutual inclinations. It is the change/increase in predicted inclinations from Laplace-Lagrange due to evolving J_2 that produces the misalignments.

4.5 Discussion

Out of our clean sample of 31 three-planet transiting systems, we found six systems whose parameters indicate they may have undergone sweeping resonance during their first Gyr due to J_2 evolution. This corresponds to one system in five, or $\sim 20\%$, indicating that the architectures that produce these misalignments may be common. While performing a full

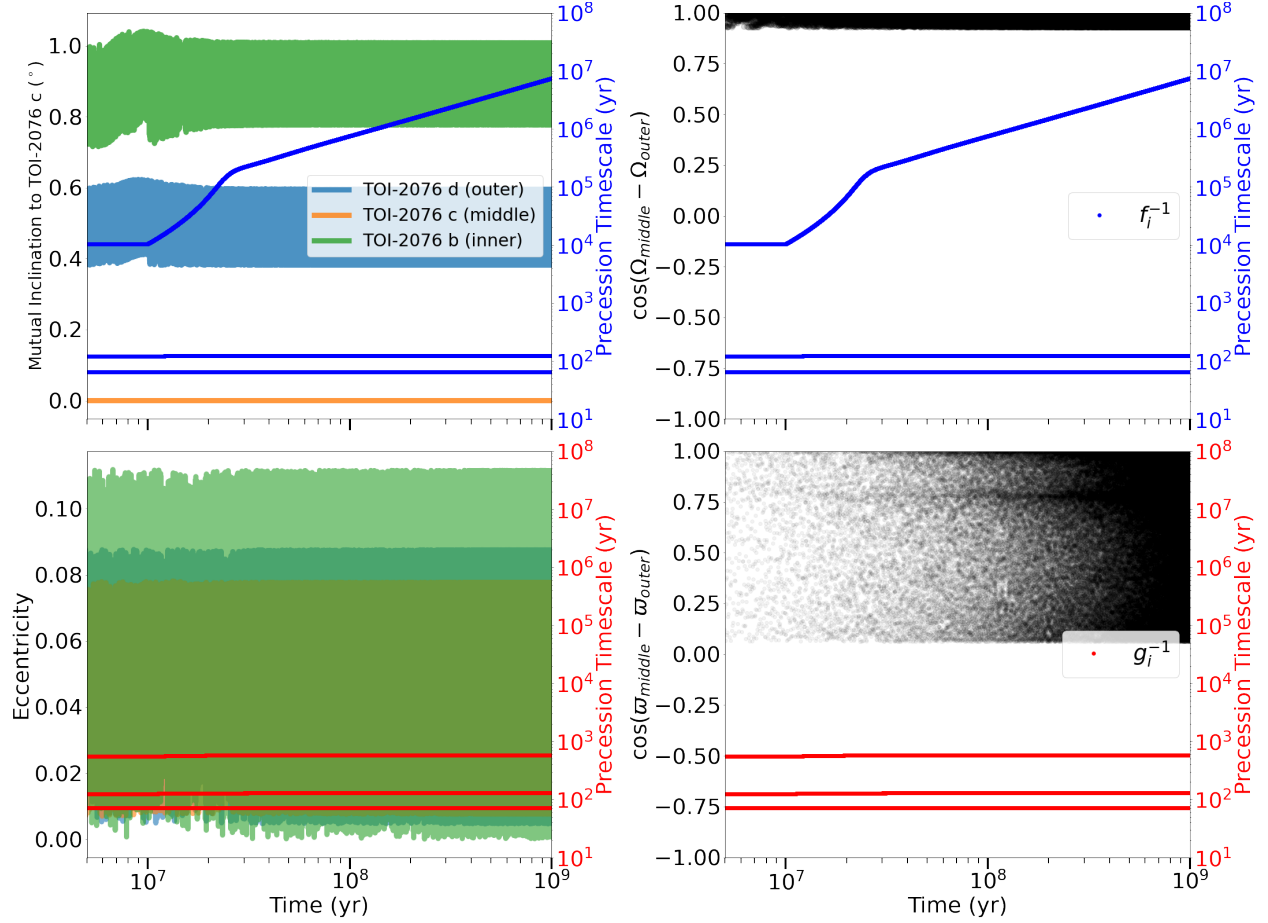


Figure 4.8: Time evolution of K-star TOI-2076's planetary system. Upper Left: Mutual inclination between planets b, c, and d and TOI-2076 c as a function of time. Lower Left: Eccentricity of TOI-2076 b, c, and d as a function of time. Upper Right: Evolution of the difference in longitudes of ascending node (Ω) between TOI-2076 c and d (the middle and outer planets) as a function of time. Lower Right: Evolution of the difference in longitudes of periapsis (ϖ) between TOI-2076 c and d as a function of time.

Overplotted on all figures are the Laplace-Lagrange eigenfrequencies f_i and g_i .

Object	Mass	Radius	a (au)	$e^\dagger$	$i^\dagger$	$\Omega^\dagger$	$\omega^\dagger$	Reference
TOI-2076	$0.82 M_\odot$	$0.77 R_\odot$						Osborn et al. [2022]
TOI-2076 b	$10.4 M_\oplus$	$2.51 R_\oplus$	0.087	0.032	0.57°	1.25°	107.5°	Osborn et al. [2022]
TOI-2076 c	$17.2 M_\oplus$	$3.50 R_\oplus$	0.139	0.066	1.61°	1.24°	281.4°	Osborn et al. [2022]
TOI-2076 d	$15.2 M_\oplus$	$3.23 R_\oplus$	0.196	0.057	2.03°	1.7°	318.2°	Osborn et al. [2022]

Table 4.4: Initial conditions of TOI-2076 used in Figure 4.8. Bolded masses indicate masses calculated using the mass-radius relation shown in Müller et al. [2023]. Bolded semimajor axes indicate semimajor axes calculated from stellar mass and planetary orbital period. † : parameters randomized according to the scheme described in Table 4.2.

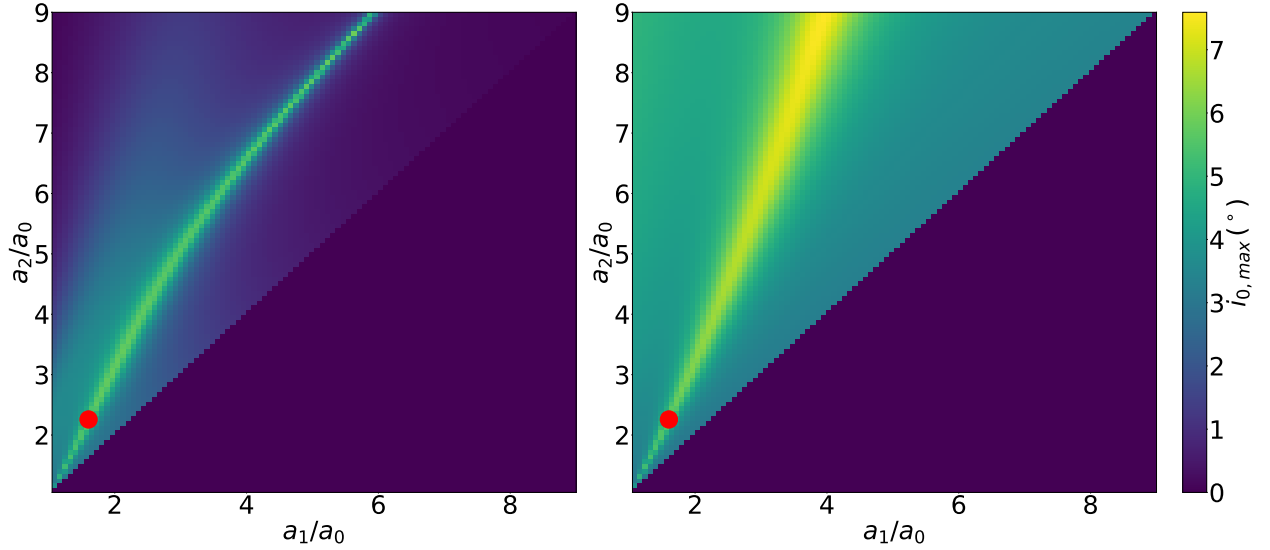


Figure 4.9: Parameter space of TOI-2076 with J_2 fixed at 10^{-3} (left panel) and 0 (right panel). The observed locations of the outer planets are shown by a red dot. The mutual inclination between the outer two planets of TOI-2076 was set to 3° .

completeness analysis to estimate the true number of systems with these architectures is beyond the scope of this work, below, we identify some considerations that may affect the true value of the 1 in 5 number if accounted for.

1. Closer-orbiting planets are more sensitive to J_2 sweeping resonances than further-orbiting planets. Because closer-orbiting planets are easier to detect via transit, systems with closer-orbiting planets will be overrepresented in the transit catalogue compared to the true, underlying distribution. Accounting for this would decrease the 1 in 5 rate.
2. Three-planet systems seem ubiquitous in our Galaxy, with an estimated 30% of sun-like stars hosting a system of ~ 3 planets with sub-400 day periods [Zhu et al., 2018]. As this is an expected value, higher multiplicity systems are also likely and have been detected (for example, the 7-planet TRAPPIST-1 system [Gillon et al., 2016] or the 8-planet Kepler-90 system [e.g., Cabrera et al., 2014, Shallue and Vanderburg, 2018]). A larger number of planets in a system increases the possibility of a resonance, and finding previously undetected planets in observed systems is a common occurrence. Therefore, some three-planet systems may be four-planet systems or higher. Accounting for this would increase the 1 in 5 rate.
3. Our sample of 31 systems only includes systems where all three planets transit. This biases our sample towards systems that have low present-day mutual inclinations. Any system that was sufficiently misaligned by evolving J_2 from a transiting configuration such that it does not transit today could never be included in our sample. In other words, our selection criteria can only produce systems where the induced misalignments are low enough for the planets to remain transiting. Accounting for this would increase the 1 in 5 rate.
4. In our calculations we adopted the upper estimate of the J_2 evolution depicted in Figure 4.2. Not all stars will be born on the upper end of the initial spin distribution, and thus some systems may not undergo sweeping resonance that they might otherwise

experience if the host star was born spinning faster. Accounting for this would reduce the 1 in 5 rate.

The third consideration naturally prompts the following questions. Shouldn't the sweeping resonant systems we found have misaligned themselves out of their co-transiting configurations? Is it surprising that as many systems as we found lie within the sweeping resonance region that misaligns orbits? As an example, consider the Kepler-9 system. Kepler-9 d, the inner planet, orbits with $a/R_\star = 5.54$. Assuming for simplicity that the outer two planets have transit impact parameters of 0, then Kepler-9 d would need a mutual inclination of over $\arctan(R_\star/a) \approx 10^\circ$ with the outer planets in order to escape transit. Figure 4.7 indicates that to become that inclined, the initial mutual inclination between the outer planets would have to be $\sim 4^\circ$ or greater. The distribution of mutual inclinations for observed co-transiting planets is well-modeled by a Rayleigh Distribution with $\sigma = 1^\circ - 2^\circ$ [see e.g., Fang and Margot, 2012, Fabrycky et al., 2014]. If the true distribution of initial mutual inclinations is similar to this model, it makes sense that Kepler-9 d still transits today. Even though it has likely been misaligned from its companions via J_2 sweeping resonances, it would require an initial mutual inclination between outer planets of $\sim 4^\circ$ to prevent the inner planet from transiting (see Figure 4.7). Of the systems we found, Kepler-9 is the most easily inclined, so for none of the systems identified is it particularly surprising that all three planets transit.

This naturally brings to mind the possibility of systems where the inner planet is so greatly misaligned from its companions that it no longer transits. These systems would likely present as two-planet transiting systems, as the outer planets align as a consequence of misaligning the inner (see the negative slopes of the blue curves in Figure 4.7). However, two effects add together to potentially limit the occurrence rate of such systems: 1) high required mass ratios and 2) configurational restrictions. The first effect is relatively simple: the higher the mass ratio between the inner and outer planets, the more misalignment is produced. For example, the mass ratio between Kepler-9's inner and middle planets is ~ 5 , which enables the steep slope of Δi_{01}^{mut} with respect to $i_{mut,12,init}$. While a mass-ratio of 5 is relatively small compared to the ~ 300 between Earth and Jupiter, mass ratios in observed,

compact, multi-transiting systems tend to be relatively small [e.g., [Weiss et al., 2018b](#)].

The second effect limiting how often systems can be misaligned out of transit is configurational. As mentioned, further-orbiting inner planets require less misalignment in order to escape transit. But the minimum semimajor axis ratios for sweeping resonance to occur increase as the inner planet is further from the host star. This is because sweeping resonance requires commensurability in magnitude between planet-planet precession and J_2 precession, which can't happen if all planets are far from the star, but close to one another. This means that the further the inner planet orbits from the star, the larger the semimajor axis ratios between the middle and outer planets and the inner planet need to be (see for example the difference in sweeping resonance locations for Kepler-619 compared to Kepler-616 in [Figure 4.6](#)). The further the inner planet orbits, the more physically isolated it must be from its companions in order to experience sweeping resonance, and such systems don't appear to be common.

The impact of sweeping resonances via J_2 also poses an interesting possibility. The size of the sweeping region in semimajor axis ratio space (see [Figure 4.6](#)) depends on the maximum value of J_2 that the star experiences. This means that the present-day mutual inclinations between planets depend on the maximum J_2 achieved, meaning that the maximum spin of the star (and therefore the maximum J_2) could be constrained by present-day planetary configurations. Because stellar spins as functions of time tend to be relatively indistinguishable after ~ 1 Gyr regardless of initial spin [see, e.g., [Barnes, 2003](#), [Hartman et al., 2010](#), [Delorme et al., 2011](#), [Meibom et al., 2015](#)], this dynamical channel could provide a way to probe the initial conditions of billions of years-old stars. Simultaneously, the trends in [Figure 4.7](#) show a remarkable dependence between mean mutual inclination at late times and initial mutual inclination. This relationship may allow some constraints to also be put on the maximum mutual inclination between outer planets during the early years of the system's life—when J_2 was moving through its high values that drive the resonances. A straightforward implementation of this is present in [Figure 4.7](#), represented by the blue dotted line: blue dots above this line should not present as co-transiting systems and are thus forbidden.

We note that if a system has parameters today that lie within the sweeping resonance region, that is not a guarantee that it actually underwent sweeping resonance in its past. One potential concern is whether the true stellar mass or radius deviates significantly from the values in the literature—it is noted in Figure 4.1 that both $M_\star$ and $R_\star$ affect where the sweeping resonance lies. Moreover, if J_2 decrease begins earlier than we assume, the disk may still be present and the planets may still be migrating, meaning that the starting J_2 of the post-migration epoch would be lower. We discuss this possibility in detail in Appendix 4.8. We explore observed systems to indicate how common sweeping-resonant architectures may be rather than attempting to claim with certainty which of these 31 systems did or did not undergo sweeping resonance via J_2 . Making such claims is possible; however, we do not do so here.

4.6 Conclusion

In this work, we explored the implications of the evolution of stellar oblateness (J_2) on systems of multiple planets during the first Gyr, showing that J_2 evolution can bring non-resonant systems into and out of secular resonance (“sweeping resonance”). While sweeping resonances were pointed out before in the context of J_2 in multi-planet systems [e.g., [Brekka and Becker, 2021](#), [Faridani et al., 2023](#), [Spalding et al., 2018](#)], here we focus on the implications for observed systems.

As a case study, we focus on three-planet systems. As time goes by, magnetic braking slows the spin of the star, reducing J_2 . A three-planet system, initially not in resonance, can be captured in a Laplace-Lagrange resonance at a certain J_2 . If the inner planet is as-massive or less-massive than its companions, the mutual inclination between the outer two planets is damped out as the system enters resonance. In that case, the mutual inclination between the outer pair and the inner planet increases to conserve angular momentum. This can produce large mutual inclinations between the inner planet and the outer pair.

Whether or not sweeping resonances from J_2 occur at all depends on the semimajor axes

and masses of the planets. In Figure 4.4, we show the relationship between the semimajor axis ratios in the system and the subsequent inclination induced on the inner planet by sweeping resonances from evolving J_2 . A large portion of the parameter space exhibits sweeping resonances when a_1/a_0 and a_2/a_0 are both large—a much larger portion of the space than is produced without evolving J_2 . Similarly, when a_0 decreases, the rate of J_2 precession increases relative to planet-planet precession, so more of the parameter space exhibits sweeping resonances when a_0 is small. The mass ratios between the planets also affect where the resonances are located, but the sensitivity is small, with mass changes up to a factor of 2 or 3 not necessarily affecting whether a system enters resonance.

To determine how common sweeping resonances from evolving J_2 are, we explored the population of observed, three-planet systems. We found that $\sim 20\%$ of a sample of three-planet all-transiting systems have mass and semimajor axis ratios that indicate they underwent sweeping resonances early in their lives. In Figure 4.7, we show that the ability of sweeping resonances to misalign the inner planet from its companions is dependent on the initial mutual inclination between the outer two planets. Moreover, for all six sweeping-resonant systems we identified, they would require very large initial outer planet mutual inclinations to make the inner, transiting planet no longer transit. As such high initial mutual inclinations are unlikely, that these systems are observed in co-transiting configurations is unsurprising. Our findings have significant implications for the connection between exoplanet systems’ initial conditions and their present configurations. A fraction of systems experiencing sweeping resonances of near 20% constitutes a generic, ubiquitous phenomenon.

4.7 Appendix B: The Impact of Lying Near a Mean-Motion Resonance

Of the six sweeping-resonant systems shown in Figure 4.6, two of them have planets that lie near mean-motion resonances (MMRs). The two outer planets of Kepler-9 are approximately 1% wide of a 2 : 1 commensurability, and the planets of Kepler-487 lie near a 32 : 5 : 2

commensurability. The outer planet’s orbit is much closer to the 16 : 1 commensurability with the inner planet than the 5 : 2 with the middle planet. Mean-motion resonances occur on orbital timescales, and are therefore not accounted for in the secular model used heretofore, requiring further investigation.

To verify that MMRs are not at play in these systems, we integrate Kepler-9 and Kepler-487 using different values of the outer planet semimajor axis and randomized inner planet initial inclination. This allows us to quantify how close these systems are to actually being in an MMR and to what extent that changes their behavior. We integrate using the N-body code REBOUND and use the add-on pack REBOUNDx with its implemented gravitational harmonics to incorporate a changing values of J_2 using the WHFast integrator [Rein and Liu, 2012, Tamayo et al., 2020b, Rein and Tamayo, 2015]. The timestep for the WHFast integrator was set to be 3% of the shortest orbital period in the system. For Kepler-9 (Kepler-487), the outer planet’s semimajor axis was varied between a period ratio with the middle (inner) planet of $1.97 - 2.03$ ($1.59 - 16.1$); the initial mutual inclination between outer planets was fixed to 2.5° (4.5°) for all integrations; the inner planet’s initial inclination was randomized uniformly between $0 - 3^\circ$ ($0 - 3^\circ$); and all other orbital elements were held constant between integrations. The integrations were run for 5×10^6 years with a J_2 evolution rate of 10 times what is shown in Figure 4.2. Faridani et al. [2023] found that such increased J_2 evolution does not qualitatively affect the results and only minorly affects them quantitatively. Even at the increased evolution rate, the timescale of $\dot{J}_2$ is still much slower than the Laplace-Lagrange timescales between the planets.

We show the results of these integrations in Figure 4.10, where we plot the period ratio between the outer two planets versus Δi_{mut} over the integrations. Both $\Delta i_{mut,01}$ (red) and $\Delta i_{mut,12}$ (blue) are shown. Both systems display no effect from the MMRs at their observed period ratios. For Kepler-9, wide of resonance (where the planets are observed to lie today) and short of the 2 : 1 resonance, the systems display similar behavior, producing similar mutual inclination excitation between the inner planet and the outer pair. However, on-resonance, the system exhibits chaotic behavior, where the inclinations and eccentricities of

the planets vary within a broader range (consistent with observations; see e.g. [Rice et al. \[2023\]](#)). For Kepler-487, the outer and inner planets lie very close to a 16 : 1 ratio. However, the effect of the commensurability is negligible, with changes to the outer planet period not having an effect on the system’s evolution. This non-effect persists even when the 16 : 1 ratio is near-exact. Moreover, the outer two planets of Kepler-487 are too distant from the 5 : 2 commensurability for it to have an effect.

While neither Kepler-9 nor Kepler-487 lie close enough to their respective commensurabilities to impact the conclusions drawn from the secular calculations, this is not the case for all systems. As can be seen from the left panel of Figure 4.10, a 2 : 1 commensurability between outer planets can have significant effect on the change on $\Delta i_{mut,01}$ caused by evolving J_2 . For the integrations near the commensurability, the evolution became chaotic. However, that an orbit is chaotic does not mean that secular resonances via evolving J_2 have no effect. Even in the chaotic regime, the mutual inclination between inner and middle planets almost always increases dramatically as J_2 passes through the secular resonant value (though the chaos occasionally drives the inner and middle two planets to temporarily align again, reducing the value of $\Delta i_{mut,01}$). Because these MMR configurations are chaotic, this may also drive the inner planet’s inclination even higher, potentially systematically removing such systems from transiting configurations, and therefore removing them from our sample.

4.8 Appendix C: Sweeping Resonance from J_2 , Planetary Migration, and the Disk

The times we find these resonances occurring are relatively early in the system’s life (< 50 Myr). During this same period or earlier, the protoplanetary disk is evaporating and the planets may be migrating. The protoplanetary disk introduces its own potential, and its evaporation may prompt resonances of its own [e.g., [Heppenheimer, 1980](#), [Ward, 1981](#), [Nagasawa et al., 2003](#), [Petrovich et al., 2020](#), [Li and Lai, 2023](#), [Zanazzi and Chiang, 2024](#)]. Planetary migration would cause a system’s location in a_1/a_0 – a_2/a_0 space to vary, potentially

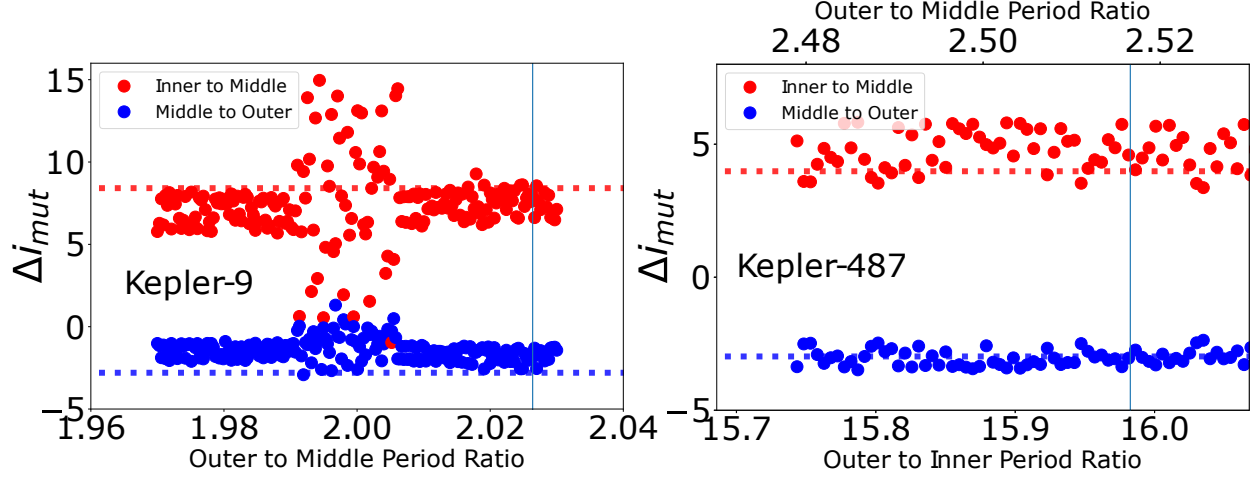


Figure 4.10: Relation between period ratio of outer two planets and Δi_{mut} for Kepler-9 (left) and relation between period ratio of outer two planets and outer/inner for Kepler-487. The changing of the period ratio was performed by changing the outer planet’s semimajor axis. The solid light blue lines represents the observed period ratios. The dotted red and blue lines represent the values of Δi_{mut} predicted by the fits for Kepler-9 and Kepler-487 shown in Figure 4.7.

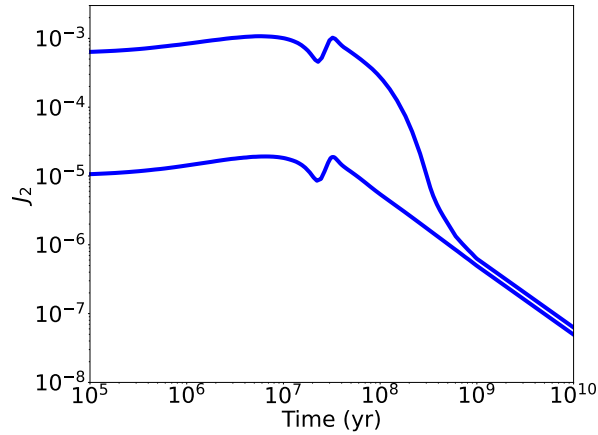


Figure 4.11: Model of J_2 evolution for a $1.1 M_\odot$ star. Stellar parameters were calculated by a MESA model (the same model as Figure 4.2), but spin evolution was calculated by the wind model given in Matt et al. [2015].

interfering with the sweeping secular resonances from evolving J_2 , and any mean-motion resonances the system passes through could have a significant impact as well.

Neither of these possibilities are taken into account in our model, which assumes that the system initial conditions may be sculpted by these processes, but that they are finished making significant impact by the time J_2 begins evolving in our simulations at $t = 10$ Myr. While protoplanetary disks typically evaporate in less than 10 Myr, older disks have been discovered, indicating the disk lifetime distribution may be wider than previously thought [see e.g. [Silverberg et al., 2020](#), [Pfalzner and Dincer, 2024](#)]. Therefore, it is necessary to consider when J_2 begins to evolve and whether that happens while the disk is still present.

The 10 Myr delay shown in Figure 4.2 is of somewhat arbitrary duration. However, the existence of a period where J_2 is roughly constant before it starts decreasing due to stellar spin-down is not arbitrary. For example, [Becker et al. \[2020\]](#) show in their Figure 5 the J_2 evolution of a $0.686 M_\odot$ star where J_2 remains constant for the first 80 Myr of the star’s life. They calculate this using [Baraffe et al. \[2015\]](#)’s stellar models and [Matt et al. \[2015\]](#)’s wind model. Using that wind model but using a MESA model for a $1.1 M_\odot$ star produces a ~ 60 Myr lag before the J_2 drops below its initial value [[Paxton et al., 2011, 2013, 2015, 2018, 2019](#), [Jermyn et al., 2023](#)]. This alternative J_2 evolution is shown in Figure 4.11. After 60 Myr, the disk is almost certainly no longer causing significant precession and migration will have stopped. Therefore, we assume disk effects and migration need not be included in our simulations. If J_2 begins to decrease early, then the initial J_2 of the post-migration epoch will be lower than assumed here.

4.9 Appendix D: The Sample of 31 three-planet all-transiting systems

Name	M_* ($M_\odot$)	R_* ($R_\odot$)	Star Reference	a_0 (au)	a_1 (au)	a_2 (au)	m_0 ($M_\oplus$)	m_1 ($M_\oplus$)	m_2 ($M_\oplus$)	Planet Reference(s)
Kepler-9	1.07	1.02	Torres et al. [2011]	0.027 [†]	0.143 [*]	0.230 [*]	3.342 [†]	43.4[*]	29.9[*]	[†] Torres et al. [2011]; [*] Borsato et al. [2019]
Kepler-619	1.09	1.11	Mo16	0.022 [†]	0.062 [†]	0.103 [*]	3.596 [†]	14.72 [†]	22.56 [*]	[†] Mo16; [*] Stassun et al. [2019]
Kepler-487	0.91	0.88	Mo16	0.033 [†]	0.115 [*]	0.213 [*]	12.06 [†]	106.3 [*]	11.43 [*]	[†] V22; [*] Mo16
K2-198	0.8	0.76	Hedges et al. [2019]	0.040	0.069	0.120	2.364	9.882	22.72	Hedges et al. [2019]
Kepler-529	1.07	1.14	Mo16	0.031 [†]	0.109 [†]	0.217 [*]	4.543 [†]	5.422 [†]	12.19 [*]	[†] Mo16; [*] V22
EPIC 249893012	1.05	1.70 ⁺	Hidalgo et al. [2020]	0.046	0.124	0.215	8.75	14.67	10.18	Hidalgo et al. [2020]
HD 22946	1.1	1.12	Garai et al. [2023]	0.051	0.091	0.264	2.124	9.205	10.95	Garai et al. [2023]
HD 28109	1.26	1.45	Dransfield et al. [2022]	0.170	0.309	0.406	18.49	7.943	5.681	Dransfield et al. [2022]
K2-19	0.88	0.82	Petigura [2020c]	0.034	0.074	0.097	1.289	32.4	10.8	Petigura [2020c]
K2-219	1.02	1.19	Ma18	0.048	0.069	0.098	2.071	2.425	10.75	Ma18
K2-352	0.98	0.95	de Leon et al. [2021]	0.046	0.079	0.117	2.155	4.908	8.615	de Leon et al. [2021]
K2-37	0.9	0.85	Simukoff et al. [2016]	0.051	0.065	0.110	3.194	11.89	11.76	Simukoff et al. [2016]
K2-58	0.89	0.85	Crossfield et al. [2016]	0.035	0.069	0.151	3.243	11.43	3.700	Crossfield et al. [2016]
K2-80	0.9	0.86	Crossfield et al. [2016]	0.059 [†]	0.134 [†]	0.177 [*]	2.559 [†]	5.489 [†]	10.60 [*]	[†] Crossfield et al. [2016]; [*] Ma18
Kepler-1073	0.92	1.12	Thompson et al. [2018a]	0.035	0.048	0.080	2.636	3.342	9.035	Thompson et al. [2018a]
Kepler-1530	0.92	0.88	Mo16	0.035 [†]	0.058 [†]	0.083 [*]	3.342 [†]	3.391 [†]	15.59 [*]	[†] Mo16; [*] V22
Kepler-18	0.97	1.11	Cochran et al. [2011]	0.044	0.075	0.117	6.9	17.3	16.4	Cochran et al. [2011]
Kepler-27	0.95	0.85	Mo16	0.066 [†]	0.117 [*]	0.189 [*]	11.60 [†]	21.16 [*]	28.92 [*]	[†] V22; [*] Steffen et al. [2012]
Kepler-271	0.9	0.87	Mo16	0.057 [†]	0.071 [*]	0.090 [*]	0.362 [†]	0.882 [*]	1.656 [*]	[†] Mo16; [*] Rowe et al. [2014]
Kepler-289	1.08	1.0	Schmitt et al. [2014]	0.212	0.328	0.504	7.3	4.0	132.0	Schmitt et al. [2014]
Kepler-30	0.99	0.95	Sanchis-Ojeda et al. [2012]	0.185	0.300	0.534	11.3	640.0	23.09	Sanchis-Ojeda et al. [2012]
Kepler-31	1.21	1.22	Fabrycky et al. [2012]	0.158	0.254	0.411	34.55	32.63	20.35	Fabrycky et al. [2012]
Kepler-431	1.07	1.09	Everett et al. [2015]	0.071	0.084	0.104	0.518	0.373	1.289	Everett et al. [2015]
Kepler-450	1.19	1.64	Stassun et al. [2019]	0.079	0.128	0.193	0.647	11.04	40.92	Van Eylen and Albrecht [2015]
Kepler-51	1.04	0.93	Masuda [2014]	0.251	0.384	0.509	2.1	4.0	7.6	Masuda [2014]
Kepler-549	0.9	0.85	Thompson et al. [2018b]	0.157	0.228	0.445	22.81	10.71	13.11	Thompson et al. [2018b]
Kepler-616	0.98	0.97	Mo16	0.090 [†]	0.267 [*]	0.391 [†]	9.832 [†]	11.25 [†]	16.56 [*]	[†] Mo16; [*] V22
Kepler-770	0.94	0.92	Mo16	0.024	0.049	0.136	1.560	2.272	9.035	Mo16
KOI-7892	1.15	1.24	Stassun et al. [2019]	0.099 [†]	0.205 [†]	0.365 [*]	13.49 [†]	21.98 [*]	9.646 [*]	[†] V22; [*] Wang et al. [2015]
TOI-125	0.86	0.85	Nielsen et al. [2020]	0.051	0.081	0.137	9.5	6.629	13.6	Nielsen et al. [2020]
TOI-2076	0.82	0.77	Osborn et al. [2022]	0.087	0.139	0.196	10.38	17.21	15.24	Osborn et al. [2022]

Table 4.5: Parameters adopted for the 31-system sample. All semimajor axes were derived from orbital periods rather than reported semimajor axes in listed references, to ensure that period ratios are maintained. Bolded masses were adopted from references, and unbolded masses were derived from the mass-radius relation derived in Müller et al. [2023]. ⁺: See discussion of EPIC 249893012’s radius in Section 4.4.1. Morton et al. [2016], Valizadegan et al. [2022], and Mayo et al. [2018] are abbreviated as Mo16, Va22, and Ma18 respectively.

CHAPTER 5

Secular Resonances from Evolving Stellar Oblateness Impede Transit Detections

5.1 Introduction

After contracting from their pre-main sequence phase, sun-like stars about 100 Myr of age have spin periods ranging from about $\sim 0.5 - 10$ days [see e.g., [Gallet and Bouvier, 2013](#)]. At this age, these stars experience a rapid deceleration of their spins due to magnetic braking — the torquing of the wind by the star’s magnetic field transferring angular momentum away from the star [e.g., [Parker, 1958](#), [Schatzman, 1962](#), [Kraft, 1967](#), [Weber and Davis, 1967](#), [Mestel, 1968](#), [Skumanich, 1972](#)]. When sun-like stars age to 600 – 1000 Myr, the correlation between the strength of magnetic braking and the stellar spin rate brings all sun-like stars to similar spins.

For planetary systems around sun-like stars, the host stars’ spin rates at early times have the potential to significantly alter their planets’ orbital angles. Stars’ spins rotationally flatten them at their poles, altering their gravitational potential. This deviation from a point-mass potential is, to leading order, parametrized by the gravitational quadrupole moment, J_2 . The effect of J_2 is to induce a precession in the arguments of periapsis and longitudes of ascending node of all orbiting bodies [see e.g., [Murray and Dermott, 2000](#)]. These precessions can have a wide range of effects, such as misaligning tightly-packed systems [e.g., [Spalding and Batygin, 2016](#), [Becker et al., 2020](#), [Brefka and Becker, 2021](#), [Chen et al., 2022](#), [Faridani et al., 2023, 2025](#)] or aiding tidal migration [e.g., [Pu and Lai, 2019b](#), [Millholland and Spalding, 2020b](#)]. As spin rates decrease over time, J_2 does as well, reducing the rates

of these precessions.

Mutual gravitational interactions between the planets also cause precessions in these angles at a wide range of frequencies. If there are first-order commensurabilities between the slower of these frequencies (i.e., frequencies with periods much longer than the orbital periods in the system), the orbiting bodies enter what is known as a “secular resonance.” In such a resonance, the involved bodies rapidly exchange angular momentum [see e.g., [Murray and Dermott, 2000](#)]. The results of this exchange can range from planetary interactions with tides [see e.g., [Hansen and Murray, 2015](#)], to circularize and migrate [potentially causing chaotic evolution through overlap of many higher-order resonances, see e.g., [Lithwick and Wu, 2011](#), [Petrovich et al., 2019b](#), [O’Connor et al., 2022](#)], to coupling with planetary spin and tilting planets’ spin axes [see e.g., [Millholland and Laughlin, 2019](#)].

The evolving stellar J_2 early in the star’s life can significantly affect when and where secular resonances occur in an exoplanetary system [e.g., [Brefka and Becker, 2021](#), [Faridani et al., 2023, 2025](#)]. The decreasing J_2 can move the locations (in semimajor-axis ratio space) of secular resonances, dramatically increasing the range of planetary system architectures where resonances can occur [[Faridani et al., 2025](#)]. Specifically, [Faridani et al. \[2025\]](#) showed that out of a sample of 30 observed, three-planet systems orbiting stars below 6200 K¹, that 6 of them (approximately 20%) exist in configurations where a sufficiently high initial stellar J_2 would cause the planets, at some point during the star’s spin-down, to enter secular resonance and exchange angular momentum. This exchange typically misaligns the inner planet from the outer two, which align with each other in the process [e.g., [Faridani et al., 2025](#)]. Therefore, systems susceptible to resonances mediated by evolving J_2 may experience significant orbital changes depending on whether or not the initial stellar spin was large enough to reach the critical J_2 required for resonance. In this work, we calculate how this

¹6200 K denotes the Kraft break, above which stars are not expected to form the convective envelopes necessary to produce a strong magnetic field and efficiently undergo magnetic braking. Stars above 6200 K therefore retain high spins for much longer [see e.g., [Kraft, 1967](#)]. Note, however, that [Beyer and White \[2024\]](#) find that the Kraft Break — defined specifically as a rotational discontinuity, rather than a dividing line of stellar structure — is centered at ~ 6550 K. In this work, we do not discuss any star hotter than 6200 K, but note that stars between 6200–6550 K may also spin down and cause resonance-induced misalignments in their planets.

(mis-) aligned configuration affects transit probability.

As a test case, we consider the Kepler-619 system. Kepler-619 is a sun-like star with mass $1.09 M_\odot$ and radius $1.11 R_\odot$, hosting three planets (‘c’, ‘b’, and ‘d’) with periods of $1.21 d$, $5.4 d$, $11.68 d$ and radii of $1.69 R_\oplus$, $3.16 R_\oplus$, and $4.38 R_\oplus$, respectively [Morton et al., 2016, Valizadegan et al., 2023].

This paper is structured as follows: in Section 5.2, we introduce the mathematical model. In Section 5.3 we explore the effects of sweeping J_2 resonances on the transit probabilities for observed three-planet system Kepler-619. In Section 5.4 we discuss and conclude.

5.2 Sweeping Secular Resonances and the Relation to J_2

The gravitational influence of each planet in a multi-planet system on its neighbors over secular timescales is described by the Laplace-Lagrange secular model [see e.g., Murray and Dermott, 2000]. The precessions of inclination and longitude of ascending node (eccentricity and longitude of periapsis) are determined by the Laplace-Lagrange formalism, defined, for a host star of mass $M_\star$, radius $R_\star$, and quadrupole moment J_2 , by

$$\begin{aligned} \mathcal{R}_j^{(\text{sec})} = & n_j a_j^2 \left[\frac{1}{2} A_{jj} e_j^2 + \frac{1}{2} B_{jj} I_j^2 \right. \\ & + \sum_{k=1, k \neq j}^N A_{jk} e_j e_k \cos(\varpi_j - \varpi_k) \\ & \left. + \sum_{k=1, k \neq j}^N B_{jk} I_j I_k \cos(\Omega_j - \Omega_k) \right], \end{aligned} \quad (5.1)$$

$$\begin{aligned} A_{jj} = & n_j \left[\frac{3}{2} J_2 \left(\frac{R_\star}{a_j} \right)^2 \right. \\ & \left. + \sum_{k=1, k \neq j} \frac{m_k}{4\pi (M_\star + m_j)} \alpha_{jk} \bar{\alpha}_{jk} f_\psi(\alpha_{jk}) \right], \end{aligned} \quad (5.2)$$

$$A_{jk} = - \frac{n_j m_k}{4\pi (M_\star + m_j)} \alpha_{jk} \bar{\alpha}_{jk} f_{2\psi}(\alpha_{jk}), \quad (5.3)$$

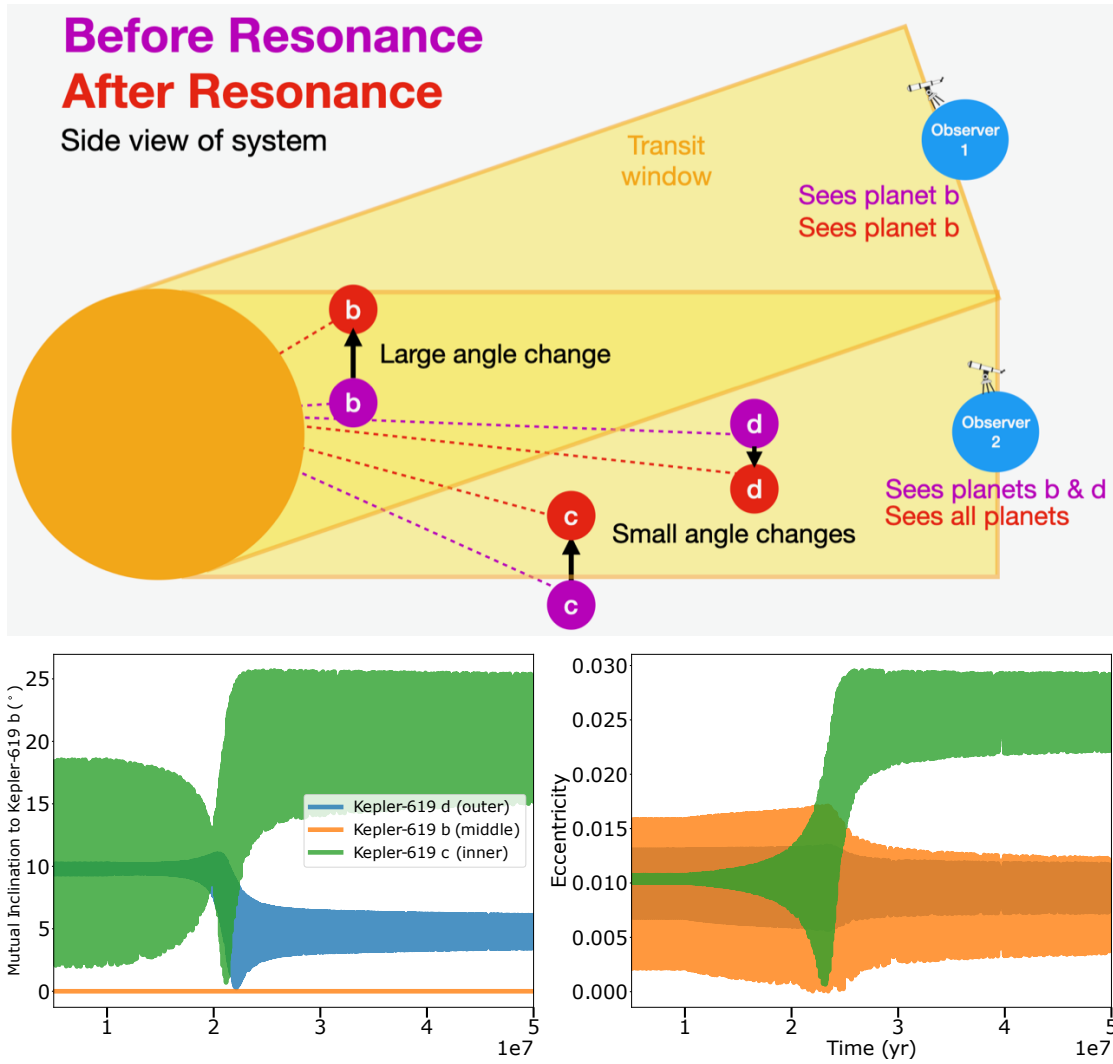


Figure 5.1: (Top) Qualitative schematic of impact-parameter geometry before and after sweeping resonance. Angles, distances and radii are not to scale.

(Bottom) Time evolution of the inclinations of the Kepler-619 system with high initial J_2 . The inner, middle, and outer planets were initialized with inclinations of 10.3° , 6.1° , 4.5° , eccentricities of 0.010, 0.006, and 0.0125, and longitudes of ascending node of 1.8, 3.1, 4.9 radians, respectively. These initial conditions were chosen from the Monte Carlo described in Section 5.3, and therefore, were chosen arbitrarily to demonstrate clearly what resonance-induced misalignments look like as functions of time.

$$B_{jj} = -n_j \left[\frac{3}{2} J_2 \left(\frac{R_\star}{a_j} \right)^2 + \sum_{k=1, k \neq j} \frac{m_k}{4\pi (M_\star + m_j)} \alpha_{jk} \bar{\alpha}_{jk} f_\psi(\alpha_{jk}) \right], \quad (5.4)$$

$$B_{jk} = \frac{n_j m_k}{4\pi (M_\star + m_j)} \alpha_{jk} \bar{\alpha}_{jk} f_\psi(\alpha_{jk}), \quad (5.5)$$

where

$$\alpha_{jk} = \min \left(\frac{a_j}{a_k}, \frac{a_k}{a_j} \right), \quad (5.6)$$

$$\bar{\alpha}_{jk} = \min \left(\frac{a_j}{a_k}, 1 \right), \quad (5.7)$$

$$f_\psi(\alpha_{jk}) = \int_0^{2\pi} \frac{\cos \psi}{(1 - 2\alpha_{jk} \cos \psi + \alpha_{jk}^2)^2} d\psi, \quad (5.8)$$

and

$$f_{2\psi}(\alpha_{jk}) = \int_0^{2\pi} \frac{\cos 2\psi}{(1 - 2\alpha_{jk} \cos \psi + \alpha_{jk}^2)^{\frac{3}{2}}} d\psi, \quad (5.9)$$

$$J_2 = \frac{1}{3} k_2 \left(\frac{S}{S_b} \right)^2, \quad (5.10)$$

$$\begin{aligned} \dot{e}_j &= -\frac{1}{n_j a_j^2 e_j} \frac{\partial \mathcal{R}_j^{(\text{sec})}}{\partial \varpi_j}, \quad \dot{\varpi}_j = +\frac{1}{n_j a_j^2 e_j} \frac{\partial \mathcal{R}_j^{(\text{sec})}}{\partial e_j}, \\ \dot{I}_j &= -\frac{1}{n_j a_j^2 I_j} \frac{\partial \mathcal{R}_j^{(\text{sec})}}{\partial \Omega_j}, \quad \dot{\Omega}_j = +\frac{1}{n_j a_j^2 I_j} \frac{\partial \mathcal{R}_j^{(\text{sec})}}{\partial I_j}, \end{aligned} \quad (5.11)$$

$$\dot{\omega}_{j,GR} = \frac{3(GM_\star)^{3/2}}{c^2 a_j^{5/2} (1 - e_j^2)}. \quad (5.12)$$

where the j th planet has a mass m_j , semi-major axis a_j , mean motion n_j , eccentricity e_j , longitude of ascending node Ω_j , longitude of periapsis ϖ_j , and inclination with respect to the star's spin axis I_j . S corresponds to the stellar spin, and S_b to the breakup spin defined by $S_b = \sqrt{GM_\star/R_\star^3}$. Integrating these equations provides the evolution of all the orbits in the system. Unless specified otherwise, all simulations in this work are calculated using numerical integrations of Equations 5.11 and 5.12.

Note that here we present Equation (5.12) as a correction to the precessions arising from the disturbing function (in other words, the contribution of the precession from Equation

(5.12) is added to the $\dot{\varpi}$ term in Equation (5.11) at each timestep) rather than incorporating it into the disturbing function (as e.g. Marzari and Nagasawa [2020] do). Without GR, the elements of the secular A and B matrices change slowly relative to secular timescales, and they can therefore be recalculated less often than every timestep while maintaining sufficient accuracy. GR precession includes eccentricity-dependent terms, which necessitate recalculating the A and B matrices at least every secular timescale (if not every timestep). By isolating the rapidly changing, eccentricity-dependent terms into a correction, we achieve a substantial speedup.

If the proper precession of an additional orbiting body’s longitude of ascending node or longitude of periapsis matches an eigenfrequency of the B or A matrix, respectively, then that body has entered a linear secular resonance. In such a resonance, angular momentum is exchanged between bodies at an enhanced rate, potentially destabilizing the system or significantly altering its architecture. As can be seen by Equations (5.2) and (5.5), a change in the host star’s J_2 as it spins down will cause a change in the eigenfrequencies of the system. This results in a change over time of the range of orbits that are in secular resonance. This is generically called a “sweeping resonance”, and is discussed thoroughly in the context of J_2 evolution in Faridani et al. [2025].

We illustrate the impact of sweeping resonances on Kepler-619, caused by evolving J_2 , in the bottom panels of Figure 5.1. Stellar J_2 is initialized to a high value of 6×10^{-4} . This corresponds to a spin period of 1.6 days, approximately at the 75th percentile for young sun-like stars [see e.g., Gallet and Bouvier, 2013]. This is higher than the critical value of $J_2 \sim 10^{-4}$ (corresponding to a 4 day spin period, approximately 40th percentile for initial spin) where the system enters the secular resonance. At $t \sim 2 \times 10^7$ years into the simulation, J_2 hits the critical value, which brings the dominant eigenmode of the inner planet’s longitude of ascending node precession equal to the dominant mode of the outer planets’ precession, entering the resonance. The mutual inclination of the inner planet with respect to the outer two is then transitioned to a higher value, and, to conserve angular momentum, the mutual inclination of the outer two planets is reduced significantly. This

resonance-induced alignment occurs over approximately 5×10^6 years, many times longer than the period of the inner planet’s dominant eigenmode. In order for these changes in alignment to occur, the outer planets must have some initial mutual inclination; otherwise, if all planets are aligned, they cannot torque one another out of alignment.

The effect of the resonance-induced alignments on the detectability of a three-planet system via transit is stark, as will be quantified in the following sections. Any reduction in the mutual inclination between the outer planets will increase the probability that they can be observed transiting together, and the misalignment of the inner planet will reduce the probability that it transits together with one of its companions. These contrasting effects mean that the change in probability of detecting all three planets transiting together is not a priori clear. However, what is certain is that the alignment of the outer planets will increase their pairwise co-transit probability, and the misalignment of the inner planet to its companions will decrease its pairwise co-transit probabilities.

5.3 Test Case of Observed Transiting Systems

5.3.1 Monte Carlo Design

To investigate the impact of the misalignments caused by sweeping secular resonances from evolving J_2 , we explore how ranges of initial conditions impact the system. For each selection of initial conditions, we integrate numerically Equations (5.11) with the addition of a GR correction to $\dot{\varpi}$ described in Equation 5.12. Inclinations were drawn from Rayleigh distributions with various σ_i values, ranging linearly from $0.01 \text{ rad} \approx 0.6^\circ$ to $0.15 \text{ rad} \approx 8.6^\circ$. The end of this range was chosen to produce mutual inclinations much larger than the observed median mutual inclination distribution between transiting planets in three-planet transiting systems, so as to cover a large range of potential distributions [see [Zhu et al., 2018](#), who estimate the median mutual inclination between planets in a three-planet transiting system

is $\sim 4.8^\circ$]. The Rayleigh distribution is defined by

$$p(x) = \frac{x}{\sigma^2} e^{-x^2/(2\sigma^2)}, \quad (5.13)$$

and is commonly used to model exoplanet inclinations [see e.g., [Fabrycky et al., 2014](#)]. Planetary eccentricities were drawn from radius-dependent beta distributions, whose α and β parameters were taken from Table S2 of [Gilbert et al. \[2025\]](#), where we adopt $R_{\text{gap}} = 1.8R_\oplus$. Initial J_2 values were drawn uniformly from 10 bins spaced equally in log between 2×10^{-5} and 6×10^{-4} inclusive. In total, we run 22,000 runs up to 5×10^7 yrs, which is well beyond the time the resonance takes place [[Faridani et al., 2025](#)].

To calculate the transit probabilities of the planets, we use the open-source CORBITS package, which calculates single- and multiple-transit probabilities given a system’s orbital elements [[Brakensiek and Ragozzine, 2016](#)]². We calculate the mean probabilities at the end of the simulation using the orbital elements at 1000 times spaced evenly over the last 1% of the simulation.

5.3.2 Transit Probabilities

As shown in Figure 5.1, for Kepler-619 (and more generally), the resonance-crossing aligns the outer two planets of the three-planet system and misaligns the inner relative to them. As discussed, this will have significant effects on the co-transit probabilities. As an example, in Figure 5.2 we show the results of 2000 Monte-Carlo simulations of Kepler-619 with Rayleigh $\sigma_i = 0.08 \approx 4.5^\circ$. This value of σ_i was chosen as it produces median mutual inclinations — namely $\sim 7.4^\circ$, assuming uniformly random longitudes of ascending node for the planets — between planets greater than the observed mutual inclinations between transiting planets. [Zhu et al. \[2018\]](#) fit a multiplicity-dependent mutual inclination distribution to observed transiting planets and find a median mutual inclination of $\sim 4.8^\circ$.³ Each simulation’s probability of the outer two planets of Kepler-619 transiting together is shown relative to that

²We used a fork that was wrapped in Python by Samuel Yee <https://github.com/samuelyeew1/CORBITS>

³The mutual inclination distribution for observed transiting planets is somewhat uncertain. For example, [Fabrycky et al. \[2014\]](#) find that the mode of the mutual inclination distribution of 365 multi-planet systems has a mode between 1° - 2.2° . [He et al. \[2020\]](#), construct a model that asserts exoplanet systems are as close

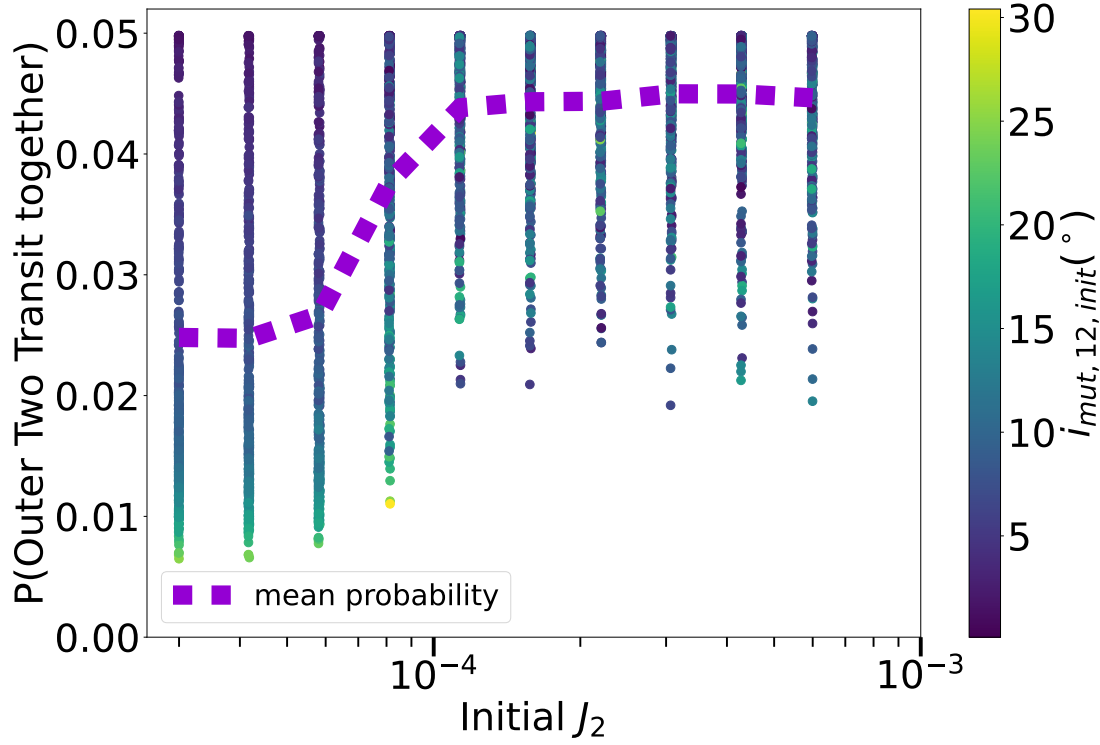


Figure 5.2: Overview of the effects of J_2 on transit probability for the outer planets in Kepler-619. The probability that the outer two observed planets transit together when viewed from a random angle (irrespective of whether the inner planet transits at this angle) is shown for 2000 integrations of orbital parameters as a function of initial stellar J_2 . The inclination distribution for these integrations used a Rayleigh-sigma of $\sigma_i = 0.08 \text{ rad} \approx 4.8^\circ$. The purple curve represents the mean probability as a function of J_2 .

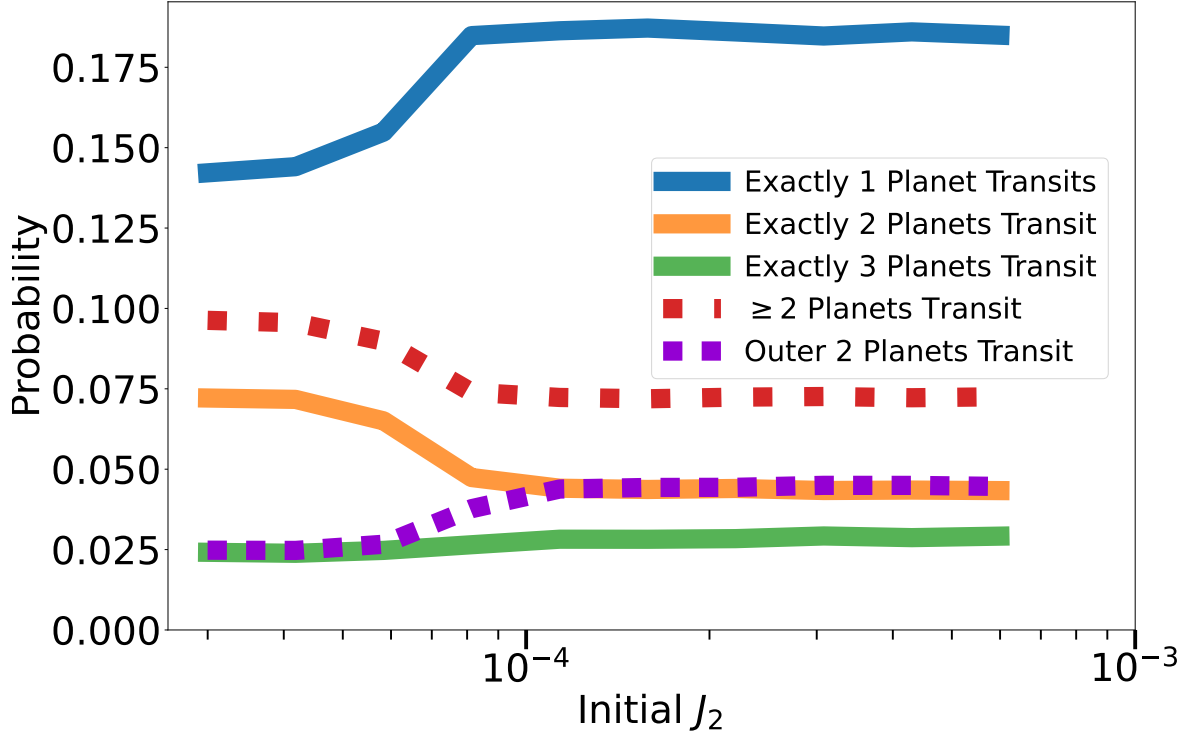


Figure 5.3: Overview of the effects of J_2 on average transit probability for the planets in Kepler-619 over a Monte Carlo of 2000. Longitudes of ascending node were assigned uniformly and inclinations were drawn from a Rayleigh distribution with $\sigma_i = 0.08 \approx 4.5^\circ$. The blue, orange, and green lines represent the probabilities that exactly one, two, or three planets transit, respectively. The red dotted line represents the probability that two or more planets transit, and the purple dotted line represents the probability that the outer two planets transit. Overall, at high J_2 values, the system is more likely to be seen as a single-transiting system and less likely with two or more transiting planets, despite the outer two planets co-transiting more commonly at high J_2 values.

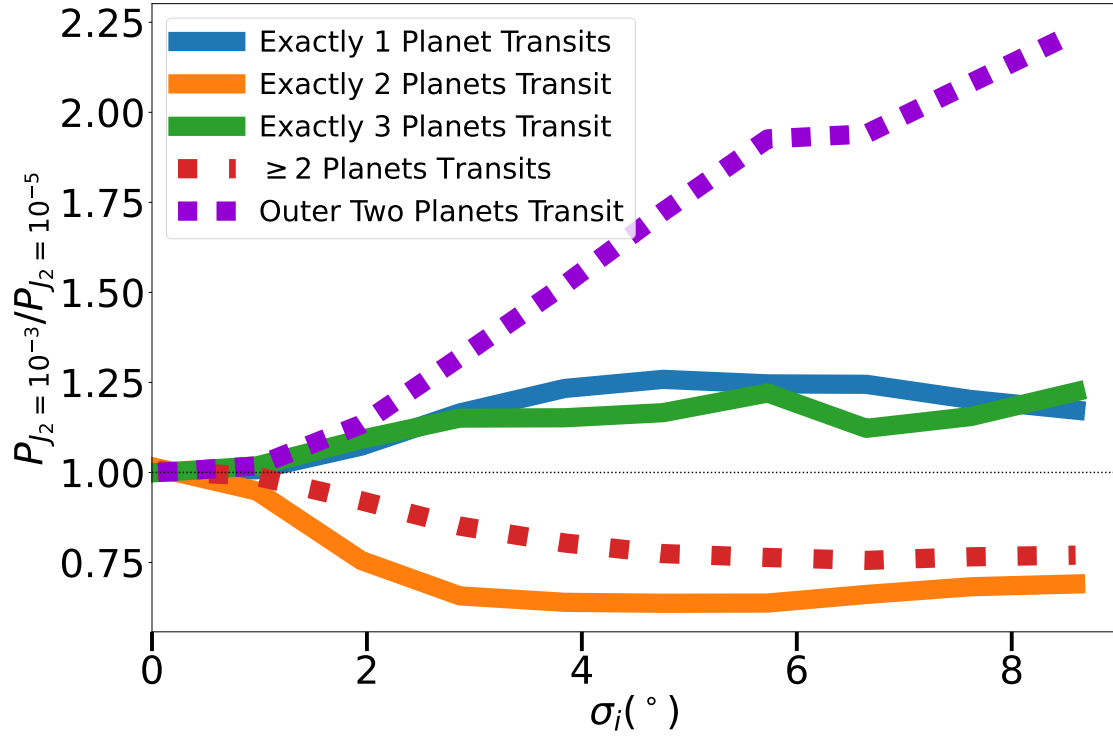


Figure 5.4: Ratios of transit probabilities between initial J_2 values of 10^{-3} and 10^{-5} as a function of the Rayleigh distribution σ_i for the initial inclination distribution for Kepler-619. The blue, orange, and green lines represent the probabilities that exactly one, two, or three planets transit, respectively. The red dotted line represents the probability that two or more planets transit, and the purple dotted line represents the probability that the outer two planets transit (irrespective of the inner). To account for the breadth of potential models describing the intrinsic inclination distribution of exoplanet systems, we display each case out to large Rayleigh distribution widths σ_i (see Section 5.4 for more details)..

simulation’s initial J_2 . The dashed purple line corresponds to the mean probability over all initial inclinations at a given J_2 . When the initial J_2 is above the resonance-crossing threshold of $\sim 10^{-4}$, the outer planets align and their co-transit probability is increased by almost a factor of 2. We note that the transition between low and high probability in Figure 5.2 spans a range, resulting in a gradual rise between $J_2 \sim 6 \times 10^{-5} - 10^{-4}$ rather than an abrupt increase when initial J_2 lies above the resonant threshold. This arises because the region of elevated inclinations has a finite width in J_2 [see e.g., Figure 2 of [Brefka and Becker, 2021](#)], so a system initialized with J_2 just below the exact resonance may still be excited.

During the resonance-induced alignment of the outer two planets, the inner planet misaligns from its companions and remains so. Thanks to its proximity to the star, it is the highest-probability planet to observe in transit (see Figure 5.1). Therefore, the probability of seeing all three planets transit together slightly increases due to the sweeping resonance, as seen in Figure 5.3. However, the large misalignment from its companions reduces the probability of detecting more than one planet in transit, as depicted by the dashed red line.

Figure 5.3 also shows, as functions of initial J_2 , the mean probabilities that exactly 1, 2, 3, ≥ 2 , and the outer two planets are observed transiting when Kepler-619 is viewed from a random angle for the same 2000 integrations used in Figure 5.2. Counterintuitively, the probability that two planets are observed transiting decreases drastically when the initial J_2 is high, despite the higher probability that the outer two planets transit together. This is because there are three ways to observe two planets transiting in a three-planet system: inner/middle, inner/outer, and middle/outer. When the outer two align, their combined transit shadow covers a smaller area, and the misalignment of the inner planet reduces its shadow’s overlap with that of the outer two. This means that the inner/middle and inner/outer probabilities plummet more than the boost to middle/outer can supplement. Moreover, the probability that any number of planets transit (the 1 planet transits curve plus the ≥ 2 planets transit curve) changes relatively little compared to the fractional change

to instability as possible without tipping into it and find that the median mutual inclination between planets in three-planet systems is 2.7° .

in the 1- or 2-planet curves. This suggests that these resonances do not significantly alter the probability that the system can be detected in the first place, but rather the multiplicity of that detection.

The change in mean probability of co-transit during a resonance-induced alignment is strongly dependent on the assumed underlying distribution of initial orbital inclinations. Consider a case where every planetary system is born with no mutual inclinations and perfectly circular orbits whose normals are parallel to the stellar spin-axis. Then, as each planet is at its maximum angular momentum for its mass and semimajor axis, the system is dynamically cold and will not be affected if stellar J_2 passes through a resonance. Angular momentum would not be transferred between planets, as they each hold the maximum angular momentum permitted by their mass and semimajor axis. Therefore, the underlying inclination distribution is an important factor to consider when evaluating the effects of sweeping J_2 resonances on exoplanetary systems⁴. In Figure 5.4, we show the results of 20,000 integrations of Kepler-619’s planetary system with varying values of the inclination distribution’s Rayleigh σ_i (2,000 each for 10 different values of σ_i). We plot the ratio of several transit probabilities of initial high J_2 and initial low J_2 as functions of σ_i . This represents the change in transit probability between the cases in which Kepler-619 was born with a high versus a low J_2 . As expected, when $\sigma_i \approx 0$ rad, there is no difference — the transit probabilities are the same regardless of whether the system had high initial J_2 or not. However, as the inclination distribution gets warmer, the differences between high and low initial J_2 become clear. At higher values of σ_i , it is expected that the probability that the outer two planets transit together decreases — as they are born with higher mutual inclinations. If a J_2 resonance occurs, the outer planets will align such that the warmer the underlying inclination distribution is, the greater the proportional impact of that resonance. Interestingly, the proportional impact on the probabilities that exactly 1, 2, and all 3 planets transit level off after $\sigma_i \sim 3^\circ$. In other words, hot and very hot inclination distributions have

⁴The case in which all planets are strictly coplanar is strongly disfavored by the evidence. Transiting systems are broadly consistent with having mutual inclinations of a few degrees [see e.g., [Fabrycky et al., 2014](#), [Chen et al., 2022](#)]. If the inclination distribution varies with planetary multiplicity, then larger dispersions are consistent with the observed population [see e.g., [Zhu et al., 2018](#), [He et al., 2020](#), [Zhu and Dong, 2021](#)]

the same proportional change in detection probability at high initial J_2 compared to low initial J_2 .

5.4 Discussion and Conclusion

We have demonstrated that sweeping resonances, thanks to stellar magnetic braking in a multi-planet system, can significantly affect the transit probability of the system. In initially misaligned systems, angular momentum is transferred from the innermost planet to one or more of the outer companions. This process causes three-planet systems that experience resonance-induced misalignments to split, resulting in an aligned outer pair and a misaligned inner planet.

A natural question is how these resonances affect the observed transiting exoplanet population. Broadly speaking, resonance-induced alignment can increase the transit probability, as illustrated in Figure 5.1. For example, in the three-planet system Kepler-619, the resonance-induced alignment increases the transit probability of the outer two planets for initially high J_2 , i.e., high initial spin.

Interestingly, through a brief calculation, we can estimate that Kepler-619 had a 50-50 chance of undergoing a sweeping resonance from evolving J_2 . This estimate is based on Kepler-619’s planets’ masses and separations, which determine the critical $J_2 \sim 10^{-4}$ for the system to experience resonance [e.g., Faridani et al., 2025, see also Figure 5.2]. For a stellar spin distribution adopted from Gallet and Bouvier [2013], a sun-like star born rotating at a median period of 3.5 d will have a J_2 at age 10^7 years of $\sim 10^{-4}$ from Equation (5.10) for a star of stellar mass and radius, and $k_2 = 0.28$. This shows that Kepler-619 had an approximately 50% probability of being born spinning fast enough to cause a sweeping resonance in its planetary system. If, like Kepler-619, the typical system has a 50% of being born spinning fast enough, and assuming 20% of systems are susceptible to sweeping resonance [see Faridani et al., 2025], then $\sim 10\%$ of the transiting exoplanet population will have undergone these resonances.

Out of the six observed systems found to be resonant-susceptible in [Faridani et al. \[2025\]](#) (which does include Kepler-619), five had resonance-threshold J_2 values within a factor of ~ 2 of 10^{-4} . All six systems had innermost planets orbiting within 0.05 au. These systems also have a diverse range of semimajor axis ratios, with the outermost planets’ semimajor axes ranging from 2-8 times those of the innermost planets. Therefore, we suggest that Kepler-619, with its threshold J_2 of 10^{-4} , serves as a good test case for the implications of the sweeping resonance on the configuration and the observational properties.

However, this means we expect our prediction of 10% of systems to be maximally valid for Kepler-like systems: somewhat compact systems of planets with low (< 10) mass ratios. For these systems, a threshold of $J_2 \sim 10^{-4}$ is a reasonable estimate, as it matches the systems identified that are susceptible to resonance-sweeps. To consider more general system architectures, which may be underrepresented in the data relative to their true frequency, many factors can affect where the threshold J_2 lies. We have found that the dominant factor is the innermost planet’s semimajor axis, because the precession frequency from J_2 decreases with semimajor axis squared. [Faridani et al. \[2025\]](#) found that mass ratios between planets greatly impact the magnitude of the angular momentum transfer, but somewhat weakly affect the system’s threshold: the size of the resonant region in mass space is large. As such, we expect our 10% estimate to be most valid for systems with close-orbiting inner planets, but with wide applicability to other systems with different parameters.

Specifically, we identify two key factors that affect the resonance behavior and thus the transit probability. These are the initial stellar spin (which determines J_2) and the initial misalignment between the planets. Therefore, heuristically, transiting systems fall into one of two outcomes. If the initial misalignment between planets is small [on the order of $< 1^\circ$, disfavored by observations, see e.g., [Fabrycky et al., 2014](#), [Zhu et al., 2018](#)], then the resonance will have little effect regardless of the initial stellar spin — as if no resonance occurred at all. Similarly, if the spin is below the critical threshold ($\gtrsim 4$ d spin period), no resonance will occur regardless of the misalignments. The second outcome occurs with significant misalignment and high spin. Here, the resonance can occur, and the planets exchange angular momentum

(mis)aligning relative to one another.

For example, as described in Figure 5.2, assuming a Rayleigh distribution for the mutual inclinations, with $\sigma = 4.8^\circ$, an initial high J_2 (above 2×10^{-4} for Kepler-619) has a 75% higher transit probability for the outer two planets compared to initially low J_2 . In this case, as expected, the probability that only one planet is transiting is also increasing (by $\sim 30\%$). Because this gain is mediated by the inner planet misaligning from the outer two, there is a corresponding decrease in the probability that exactly two planets transit (by $\sim 85\%$). The probability that all three planets transit together increases substantially — by $\sim 10\%$ — when the resonance occurs. Overall, this would produce an enhancement in the detection of single-planet transiting systems and a decrease in the number of systems detected as multi-planet. Thus, as highlighted, observing a multi-transiting system in this case is challenging. As closer-orbiting inner planets are more susceptible to these sweeping resonances [i.e., there are more configurations of the outer planets that produce resonance, see Faridani et al., 2025], the misalignment of the inner planet may enhance the mutual inclinations of short- and ultra-short period planets with their companions [as is seen in, e.g., Dai et al., 2018].

As shown in Figure 5.4, the underlying inclination distribution of exoplanets has a major impact on the relative change in transit probability induced by the resonance. For dynamically hotter distributions, the relative impact of the resonance on the transit probability of the outer two planets in a three-planet system is greater. However, for other probabilities (e.g., exactly 1, 2, or 3 planets transiting), the relative impact of the resonance saturates, resulting in warm and hot inclination distributions having the same relative change in detection probabilities.

The functional form and width of the intrinsic inclination distribution of exoplanetary systems is uncertain; however, a wide range of intrinsic distributions is also consistent with the data. For example, Bovaird and Lineweaver [2017] find that a disk model of constant height (approximately $2 R_\star$), rather than one increasing with semimajor axis, can reproduce the observed populations of single- and multi-transits. Such a model produces very large mutual inclinations for planets at small semimajor axes. Meanwhile, He et al. [2020] find that

a model which assumes inner planetary systems are as close to instability as possible while still remaining stable can also reproduce the statistics of the observed Kepler sample.⁵ This model predicts a wide spread in mutual inclinations between systems. External companions may also significantly increase the inclination dispersion among transiting planets [see e.g., [Denham et al., 2019](#), [Rodet and Lai, 2021](#)]. Of course, neither of these example warmer models has an inclination distribution reducible to a simple Rayleigh distribution. However, the general trend – that higher average inclinations imply resonances have a greater impact on outer-planet transits compared to other detection outcomes — is independent of the functional form.

Overall, these misalignments should produce an observable signal in the exoplanet statistics. This signal would be characterized by reduced numbers of two-planet transiting systems overall. However, we suggest a higher outer two-planet detection rate compared to an estimated intrinsic distribution that does not take resonance-induced (mis-)alignment into account. Specifically, these two transiting planets orbit far enough out from their host star that an undetected inner planet could stably exist. Overrepresented detections over predictions of such two-planet systems would indicate that these resonances are more common, even if the inner planets are not detected. As just one example, Kepler-204 hosts two transiting planets with periods of ~ 14 and ~ 26 days, leaving plenty of space to host an additional planet interior to both that was misaligned from the observed planets by resonance. Moreover, the relative numbers of distant pairs of transiting planets compared to a short-period planet and a nearby companion could constrain the dynamical heat of the underlying inclination distribution. The resonances’ relative effect on transit probabilities is dependent on the spread of the inclination distribution. Therefore, the strength of this signal depends on the intrinsic spread of the exoplanet inclination distribution and could constrain it. With NASA’s Roman set to observe 100,000 transiting exoplanets [[Montet et al., 2017](#), [Wilson](#)

⁵They measure stability by the minimum necessary angular momentum deficit (AMD) to produce orbit-crossing. AMD is defined as the difference between the total angular momentum of a system as it is and if all its orbits were circular and coplanar [[Laskar and Petit, 2017](#), [Petit et al., 2017](#)]. This “maximum-AMD” model takes advantage of the fact that collisions between bodies always decrease total AMD [[Petit et al., 2017](#)], pushing the system closer to stability. Therefore, one might expect that inner planetary systems may have total AMD just below the threshold to bring them into instability.

et al., 2023], the sample size for observing this effect will greatly increase.

The reduction in probability for detecting multiple planets caused by the resonance certainly contributes to the effect known as the ‘Kepler Dichotomy,’ where more single-transiting systems were observed by Kepler than predicted [see e.g., [Lissauer et al., 2011](#), [Johansen et al., 2012](#), [Hansen and Murray, 2013](#), [Ballard and Johnson, 2016](#), [Millholland et al., 2021](#)]. Misalignments from sweeping J_2 resonances represent a way to reproduce some of the dichotomous statistics without requiring separate, distinct distributions to produce the discrepancy. However, as we estimate these resonances occur in $\sim 10\%$ of systems, other channels may also contribute [see, e.g., [Johansen et al., 2012](#), [Spalding and Batygin, 2016](#), [He et al., 2019](#), [MacDonald et al., 2020](#), [Millholland et al., 2021](#)].

In higher-multiplicity systems, the picture becomes more complicated. With more planets, there are more opportunities for resonances to occur, and it is not always the innermost planet having its angular momentum transferred away, as a middle planet can be brought into resonance with far-orbiting companions. In this case, any far-orbiting companions, by virtue of their distance from the host star, would very rarely transit compared to the closer planets. By misaligning one of the inner planets relative to the rarely-transiting outer planets, these resonances may also reduce the probability of multiple planets transiting together, similar to what is shown in the three-planet case in Figure 5.3. Moreover, any significant misalignment of one planet would increase the probability that the system is observed with a single transiting planet, as by definition, misaligning that planet’s orbit moves its transit shadow into regions with no overlap with its companions.

We have found that these resonances should make an imprint on the statistics of transiting planets. By taking into account the required spin for resonance-induced misalignments to occur, we estimated that 10% of three-planet transiting exoplanet systems will have undergone these resonances — a substantial fraction. These misalignments frustrate multiplanet transit detection overall, but enhance the probability of specifically the outermost planets transiting together. This effect could produce a subclass of transiting systems observed as two-planet systems that maintain a hidden, misaligned inner planet within them. Our find-

ings demonstrate a significant, post-migration effect that dramatically impacts the kinds of planetary systems that transit, and how they present themselves.

CHAPTER 6

Conclusions

This thesis targets the dynamical environment of gravitational interactions between planets particularly when subject to time-varying precessions caused by the time-varying stellar oblateness caused by the spin-down of the host, sun-like star. I extended models and conclusions for two-planet systems to generalizable conclusions for systems with three or more planets—a key gap in the literature. The results contained herein provide key insights into the dynamical lives and detection realities of multi-planet transiting systems.

In Chapter 2, I extended non-relativistic models of exoplanet stability in the face of external perturbers to include stabilizing effects from general relativistic precession. I found that these non-relativistic methods are insufficient to determine stability against external perturbation for approximately 5% of detected two-planet systems. I also derived an analytic criteria for the stability through GR of hidden planets interior to observed planets against perturbations by those planets and identified the orbital ratios where these hidden, GR-stabilized planets may live.

In Chapter 3 I showed how the time-varying precessions from evolving stellar oblateness can shift the locations of secular resonances within exoplanetary systems. I focused on the observed two-planet system HD 106315 as a case-study, inserted a hypothetical inner planet interior to the observed two and showed that this inner planet becomes destabilized for a majority of its allowed semimajor axes if the host star HD 106315 was born spinning sufficiently fast. Critically, if one neglects J_2 's time evolution, one would miss the instability of this inner planet, as secular resonances would not move through parameter space.

In Chapter 4, I constructed a parameter space for these secular resonances sweeping due

to time-varying stellar J_2 . I found that when secular resonances sweep, the total portion of parameter space they occupy at some point during their evolution is greatly enhanced compared to if time-varying stellar J_2 is neglected. Looking at a sample of observed three-planet transiting systems, I found that the expansion of resonant parameter space from evolving J_2 indicates that 20% of the three-planet transiting exoplanet sample exists in configurations where these resonances can occur. This result is much larger than the common wisdom that secular resonances like these are rare (and indeed they are if one neglects varying J_2).

In Chapter 5, I explored the effects on transit detectability that these sweeping resonances from evolving J_2 induce on planetary systems. I found that they are efficient in splitting up three-planet systems by misaligning the innermost planet greatly from its outer two companions. Overall this increases the total probability that the system is observed transiting, as misaligned planets produce more total transit shadow. However, as tradeoff, the probability that such a three-planet system is seen having two or more planets transit is significantly reduced, as the planets are less coplanar. Taking into account the occurrence estimate found in Chapter 4, I found that potentially up to 10% of three-planet systems will undergo these bifurcations. This would produce a significant impact on the observed statistics of the multi-planet transiting exoplanet sample.

BIBLIOGRAPHY

- E. R. Adams, B. Jackson, and M. Endl. Ultra-short-period Planets in K2 SuPerPiG Results for Campaigns 0-5. *AJ*, 152(2):47, Aug. 2016. doi: 10.3847/0004-6256/152/2/47.
- E. R. Adams, B. Jackson, M. Endl, W. D. Cochran, P. J. MacQueen, D. A. Duev, R. Jensen-Clem, M. Salama, C. Ziegler, C. Baranec, S. Kulkarni, N. M. Law, and R. Riddle. Ultra-short-period Planets in K2 with Companions: A Double Transiting System for EPIC 220674823. *AJ*, 153(2):82, Feb. 2017. doi: 10.3847/1538-3881/153/2/82.
- E. R. Adams, B. Jackson, S. Johnson, D. R. Ciardi, W. D. Cochran, M. Endl, M. E. Everett, E. Furlan, S. B. Howell, P. Jayanthi, P. J. MacQueen, R. A. Matson, C. Partyka-Worley, J. Schlieder, N. J. Scott, S. M. Stanton, and C. Ziegler. Ultra-short-period Planets in K2. III. Neighbors are Common with 13 New Multiplanet Systems and 10 Newly Validated Planets in Campaigns 0-8 and 10. *The Planetary Science Journal*, 2(4):152, Aug. 2021. doi: 10.3847/PSJ/ac0ea0.
- S. H. Albrecht, M. L. Marcussen, J. N. Winn, R. I. Dawson, and E. Knudstrup. A Preponderance of Perpendicular Planets. *ApJ*, 916(1):L1, July 2021. doi: 10.3847/2041-8213/ac0f03.
- J. M. O. Antognini. Timescales of Kozai-Lidov oscillations at quadrupole and octupole order in the test particle limit. *MNRAS*, 452:3610–3619, Oct. 2015. doi: 10.1093/mnras/stv1552.
- K. I. Antoniadou and A.-S. Libert. Exploiting periodic orbits as dynamical clues for Kepler and K2 systems. *A&A*, 640:A55, Aug. 2020. doi: 10.1051/0004-6361/202037779.
- D. J. Armstrong, T. A. Lopez, V. Adibekyan, R. A. Booth, E. M. Bryant, K. A. Collins, M. Deleuil, A. Emsenhuber, C. X. Huang, G. W. King, J. Lillo-Box, J. J. Lissauer, E. Matthews, O. Mousis, L. D. Nielsen, H. Osborn, J. Otegi, N. C. Santos, S. G. Sousa, K. G. Stassun, D. Veras, C. Ziegler, J. S. Acton, J. M. Almenara, D. R. Anderson, D. Barrado, S. C. C. Barros, D. Bayliss, C. Belardi, F. Bouchy, C. Briceño, M. Brogi, D. J. A. Brown, M. R. Burleigh, S. L. Casewell, A. Chaushev, D. R. Ciardi, K. I. Collins,

- K. D. Colón, B. F. Cooke, I. J. M. Crossfield, R. F. Díaz, E. Delgado Mena, O. D. S. Demangeon, C. Dorn, X. Dumusque, P. Eigmüller, M. Fausnaugh, P. Figueira, T. Gan, S. Gandhi, S. Gill, E. J. Gonzales, M. R. Goad, M. N. Günther, R. Helled, S. Hojjatpanah, S. B. Howell, J. Jackman, J. S. Jenkins, J. M. Jenkins, E. L. N. Jensen, G. M. Kennedy, D. W. Latham, N. Law, M. Lendl, M. Lozovsky, A. W. Mann, M. Moyano, J. McCormac, F. Meru, C. Mordasini, A. Osborn, D. Pollacco, D. Queloz, L. Raynard, G. R. Ricker, P. Rowden, A. Santerne, J. E. Schlieder, S. Seager, L. Sha, T.-G. Tan, R. H. Tilbrook, E. Ting, S. Udry, R. Vanderspek, C. A. Watson, R. G. West, P. A. Wilson, J. N. Winn, P. Wheatley, J. N. Villasenor, J. I. Vines, and Z. Zhan. A remnant planetary core in the hot-Neptune desert. *Nature*, 583(7814):39–42, July 2020. doi: 10.1038/s41586-020-2421-7.
- M. Auvergne, P. Bodin, L. Boissard, J. T. Buey, S. Chaintreuil, G. Epstein, M. Jouret, T. Lam-Trong, P. Levacher, A. Magnan, R. Perez, P. Plasson, J. Plessier, G. Peter, M. Steller, D. Tiphène, A. Baglin, P. Agogué, T. Appourchaux, D. Barbet, T. Beaufort, R. Bellenger, R. Berlin, P. Bernardi, D. Blouin, P. Boumier, F. Bonneau, R. Briet, B. Butler, R. Cautain, F. Chiavassa, V. Costes, J. Cuvillo, V. Cunha-Parro, F. de Oliveira Fialho, M. Decaudin, J. M. Defise, S. Djalal, A. Docclo, R. Drummond, O. Dupuis, G. Exil, C. Fauré, A. Gaboriaud, P. Gamet, P. Gavalda, E. Grolleau, L. Gueguen, V. Guivarc’h, P. Guterman, J. Hasiba, G. Huntzinger, H. Hustaix, C. Imbert, G. Jeanville, B. Johlander, L. Jorda, P. Journoud, F. Karioty, L. Kerjean, L. Lafond, V. Lapeyrere, P. Landiech, T. Larqué, P. Laudet, J. Le Merrer, L. Leporati, B. Leruyet, B. Levieuge, A. Llebaria, L. Martin, E. Mazy, J. M. Mesnager, J. P. Michel, J. P. Moalic, W. Monjoin, D. Naudet, S. Neukirchner, K. Nguyen-Kim, M. Ollivier, J. L. Orcesi, H. Ottacher, A. Oulali, J. Parisot, S. Perruchot, A. Piacentino, L. Pinheiro da Silva, J. Platzner, B. Pontet, A. Pradines, C. Quentin, U. Rohbeck, G. Rolland, F. Rollenhagen, R. Romagnan, N. Russ, R. Samadi, R. Schmidt, N. Schwartz, I. Sebbag, H. Smit, W. Sunter, M. Tello, P. Toulouse, B. Ulmer, O. Vandermarcq, E. Vergnault, R. Wallner, G. Wautier, and P. Zanatta. The CoRoT satellite in flight: description and performance. *A&A*, 506(1): 411–424, Oct. 2009. doi: 10.1051/0004-6361/200810860.

- A. Baglin, M. Auvergne, P. Barge, M. Deleuil, C. Catala, E. Michel, W. Weiss, and COROT Team. Scientific Objectives for a Minisat: CoRoT. In M. Fridlund, A. Baglin, J. Lochard, and L. Conroy, editors, *The CoRoT Mission Pre-Launch Status - Stellar Seismology and Planet Finding*, volume 1306 of *ESA Special Publication*, page 33, Nov. 2006.
- M. Bailes, S. D. Bates, V. Bhalerao, N. D. R. Bhat, M. Burgay, S. Burke-Spolaor, N. D’Amico, S. Johnston, M. J. Keith, M. Kramer, S. R. Kulkarni, L. Levin, A. G. Lyne, S. Milia, A. Possenti, L. Spitler, B. Stappers, and W. van Straten. Transformation of a Star into a Planet in a Millisecond Pulsar Binary. *Science*, 333(6050):1717, Sept. 2011. doi: 10.1126/science.1208890.
- S. Ballard and J. A. Johnson. The Kepler Dichotomy among the M Dwarfs: Half of Systems Contain Five or More Coplanar Planets. *ApJ*, 816(2):66, Jan. 2016. doi: 10.3847/0004-637X/816/2/66.
- I. Baraffe, D. Homeier, F. Allard, and G. Chabrier. New evolutionary models for pre-main sequence and main sequence low-mass stars down to the hydrogen-burning limit. *A&A*, 577:A42, May 2015. doi: 10.1051/0004-6361/201425481.
- D. Barbato, A. Sozzetti, S. Desidera, M. Damasso, A. S. Bonomo, P. Giacobbe, L. S. Colombo, C. Lazzoni, R. Claudi, R. Gratton, G. LoCurto, F. Marzari, and C. Mordasini. Exploring the realm of scaled solar system analogues with HARPS. *A&A*, 615:A175, Aug. 2018. doi: 10.1051/0004-6361/201832791.
- T. Barclay, J. F. Rowe, J. J. Lissauer, D. Huber, F. Fressin, S. B. Howell, S. T. Bryson, W. J. Chaplin, J.-M. Désert, E. D. Lopez, G. W. Marcy, F. Mullally, D. Ragozzine, G. Torres, E. R. Adams, E. Agol, D. Barrado, S. Basu, T. R. Bedding, L. A. Buchhave, D. Charbonneau, J. L. Christiansen, J. Christensen-Dalsgaard, D. Ciardi, W. D. Cochran, A. K. Dupree, Y. Elsworth, M. Everett, D. A. Fischer, E. B. Ford, J. J. Fortney, J. C. Geary, M. R. Haas, R. Handberg, S. Hekker, C. E. Henze, E. Horch, A. W. Howard, R. C. Hunter, H. Isaacson, J. M. Jenkins, C. Karoff, S. D. Kawaler, H. Kjeldsen, T. C.

- Klaus, D. W. Latham, J. Li, J. Lillo-Box, M. N. Lund, M. Lundkvist, T. S. Metcalfe, A. Miglio, R. L. Morris, E. V. Quintana, D. Stello, J. C. Smith, M. Still, and S. E. Thompson. A sub-Mercury-sized exoplanet. *Nature*, 494(7438):452–454, Feb. 2013. doi: 10.1038/nature11914.
- P. Barge, A. Baglin, M. Auvergne, H. Rauer, A. Léger, J. Schneider, F. Pont, S. Aigrain, J. M. Almenara, R. Alonso, M. Barbieri, P. Bordé, F. Bouchy, H. J. Deeg, D. La Reza, M. Deleuil, R. Dvorak, A. Erikson, M. Fridlund, M. Gillon, P. Gondoin, T. Guillot, A. Hatzes, G. Hebrard, L. Jorda, P. Kabath, H. Lammer, A. Llebaria, B. Loeillet, P. Magain, T. Mazeh, C. Moutou, M. Ollivier, M. Pätzold, D. Queloz, D. Rouan, A. Shporer, and G. Wuchterl. Transiting exoplanets from the CoRoT space mission. I. CoRoT-Exo-1b: a low-density short-period planet around a G0V star. *A&A*, 482(3):L17–L20, May 2008. doi: 10.1051/0004-6361:200809353.
- S. A. Barnes. On the Rotational Evolution of Solar- and Late-Type Stars, Its Magnetic Origins, and the Possibility of Stellar Gyrochronology. *ApJ*, 586(1):464–479, Mar. 2003. doi: 10.1086/367639.
- S. C. C. Barros, H. Gosselin, J. Lillo-Box, D. Bayliss, E. Delgado Mena, B. Brugger, A. Santerne, D. J. Armstrong, V. Adibekyan, J. D. Armstrong, D. Barrado, J. Bento, I. Boisse, A. S. Bonomo, F. Bouchy, D. J. A. Brown, W. D. Cochran, A. Collier Cameron, M. Deleuil, O. Demangeon, R. F. Díaz, A. Doyle, X. Dumusque, D. Ehrenreich, N. Espinoza, F. Faedi, J. P. Faria, P. Figueira, E. Foxell, G. Hébrard, S. Hojjatpanah, J. Jackman, M. Lendl, R. Ligi, C. Lovis, C. Melo, O. Mousis, J. J. Neal, H. P. Osborn, D. Pollacco, N. C. Santos, R. Sefako, A. Shporer, S. G. Sousa, A. H. M. J. Triaud, S. Udry, A. Vigan, and A. Wyttenbach. Precise masses for the transiting planetary system HD 106315 with HARPS. *A&A*, 608:A25, Dec. 2017. doi: 10.1051/0004-6361/201731276.
- J. Becker, K. Batygin, D. Fabrycky, F. C. Adams, G. Li, A. Vanderburg, and J. E. Rodriguez. The Origin of Systems of Tightly Packed Inner Planets with Misaligned, Ultra-short-period Companions. *AJ*, 160(6):254, Dec. 2020. doi: 10.3847/1538-3881/abbad3.

- J. C. Becker and F. C. Adams. Effects of unseen additional planetary perturbers on compact extrasolar planetary systems. *MNRAS*, 468:549–563, June 2017. doi: 10.1093/mnras/stx461.
- J. C. Becker, K. Batygin, and F. C. Adams. Migrating Planets into Ultra-short-period Orbits during Episodic Accretion Events. *ApJ*, 919(2):76, Oct. 2021. doi: 10.3847/1538-4357/ac111e.
- T. A. Berger, D. Huber, E. Gaidos, and J. L. van Saders. Revised Radii of Kepler Stars and Planets Using Gaia Data Release 2. *ApJ*, 866(2):99, Oct. 2018. doi: 10.3847/1538-4357/aada83.
- M. Best, A. A. Sefilian, and C. Petrovich. The Influence of Cold Jupiters in the Formation of Close-in Planets. I. Planetesimal Transport. *ApJ*, 960(1):89, Jan. 2024. doi: 10.3847/1538-4357/ad0965.
- A. C. Beyer and R. J. White. The Kraft Break Sharply Divides Low-mass and Intermediate-mass Stars. *ApJ*, 973(1):28, Sept. 2024. doi: 10.3847/1538-4357/ad6b0d.
- H. Bhaskar, G. Li, S. Hadden, M. J. Payne, and M. J. Holman. Mildly Hierarchical Triple Dynamics and Applications to the Outer Solar System. *AJ*, 161(1):48, Jan. 2021. doi: 10.3847/1538-3881/abcbfc.
- A. S. Bonomo, S. Desidera, S. Benatti, F. Borsa, S. Crespi, M. Damasso, A. F. Lanza, A. Sozzetti, G. Lodato, F. Marzari, C. Boccato, R. U. Claudi, R. Cosentino, E. Covino, R. Gratton, A. Maggio, G. Micela, E. Molinari, I. Pagano, G. Piotto, E. Poretti, R. Smareglia, L. Affer, K. Biazzo, A. Bignamini, M. Esposito, P. Giacobbe, G. Hébrard, L. Malavolta, J. Maldonado, L. Mancini, A. Martinez Fiorenzano, S. Masiero, V. Nascimbene, M. Pedani, M. Rainer, and G. Scandariato. The GAPS Programme with HARPS-N at TNG . XIV. Investigating giant planet migration history via improved eccentricity and mass determination for 231 transiting planets. *A&A*, 602:A107, June 2017. doi: 10.1051/0004-6361/201629882.

- L. Borsato, F. Marzari, V. Nascimbeni, G. Piotto, V. Granata, L. R. Bedin, and L. Malavolta. TRADES: A new software to derive orbital parameters from observed transit times and radial velocities. Revisiting Kepler-11 and Kepler-9. *A&A*, 571:A38, Nov. 2014. doi: 10.1051/0004-6361/201424080.
- L. Borsato, L. Malavolta, G. Piotto, L. A. Buchhave, A. Mortier, K. Rice, A. Collier Cameron, A. Coffinet, A. Sozzetti, D. Charbonneau, R. Cosentino, X. Dumusque, P. Figueira, D. W. Latham, M. Lopez-Morales, M. Mayor, G. Micela, E. Molinari, F. Pepe, D. Phillips, E. Poretti, S. Udry, and C. Watson. HARPS-N radial velocities confirm the low densities of the Kepler-9 planets. *MNRAS*, 484(3):3233–3243, Apr. 2019. doi: 10.1093/mnras/stz181.
- W. J. Borucki. KEPLER Mission: development and overview. *Reports on Progress in Physics*, 79(3):036901, Mar. 2016. doi: 10.1088/0034-4885/79/3/036901.
- W. J. Borucki, D. Koch, G. Basri, N. Batalha, T. Brown, D. Caldwell, J. Caldwell, J. Christensen-Dalsgaard, W. D. Cochran, E. DeVore, E. W. Dunham, A. K. Dupree, T. N. Gautier, J. C. Geary, R. Gilliland, A. Gould, S. B. Howell, J. M. Jenkins, Y. Kondo, D. W. Latham, G. W. Marcy, S. Meibom, H. Kjeldsen, J. J. Lissauer, D. G. Monet, D. Morrison, D. Sasselov, J. Tarter, A. Boss, D. Brownlee, T. Owen, D. Buzasi, D. Charbonneau, L. Doyle, J. Fortney, E. B. Ford, M. J. Holman, S. Seager, J. H. Steffen, W. F. Welsh, J. Rowe, H. Anderson, L. Buchhave, D. Ciardi, L. Walkowicz, W. Sherry, E. Horch, H. Isaacson, M. E. Everett, D. Fischer, G. Torres, J. A. Johnson, M. Endl, P. MacQueen, S. T. Bryson, J. Dotson, M. Haas, J. Kolodziejczak, J. Van Cleve, H. Chandrasekaran, J. D. Twicken, E. V. Quintana, B. D. Clarke, C. Allen, J. Li, H. Wu, P. Tenenbaum, E. Verner, F. Bruhweiler, J. Barnes, and A. Prsa. Kepler Planet-Detection Mission: Introduction and First Results. *Science*, 327(5968):977, Feb. 2010. doi: 10.1126/science.1185402.
- W. J. Borucki, D. G. Koch, G. Basri, N. Batalha, T. M. Brown, S. T. Bryson, D. Caldwell, J. Christensen-Dalsgaard, W. D. Cochran, E. DeVore, E. W. Dunham, T. N. Gautier, III, J. C. Geary, R. Gilliland, A. Gould, S. B. Howell, J. M. Jenkins, D. W. Latham,

- J. J. Lissauer, G. W. Marcy, J. Rowe, D. Sasselov, A. Boss, D. Charbonneau, D. Ciardi, L. Doyle, A. K. Dupree, E. B. Ford, J. Fortney, M. J. Holman, S. Seager, J. H. Steffen, J. Tarter, W. F. Welsh, C. Allen, L. A. Buchhave, J. L. Christiansen, B. D. Clarke, S. Das, J.-M. Désert, M. Endl, D. Fabrycky, F. Fressin, M. Haas, E. Horch, A. Howard, H. Isaacson, H. Kjeldsen, J. Kolodziejczak, C. Kulesa, J. Li, P. W. Lucas, P. Machalek, D. McCarthy, P. MacQueen, S. Meibom, T. Miquel, A. Prsa, S. N. Quinn, E. V. Quintana, D. Ragozzine, W. Sherry, A. Shporer, P. Tenenbaum, G. Torres, J. D. Twicken, J. Van Cleve, L. Walkowicz, F. C. Witteborn, and M. Still. Characteristics of Planetary Candidates Observed by Kepler. II. Analysis of the First Four Months of Data. *ApJ*, 736:19, July 2011. doi: 10.1088/0004-637X/736/1/19.
- G. Boué and D. C. Fabrycky. Compact Planetary Systems Perturbed by an Inclined Companion. II. Stellar Spin-Orbit Evolution. *ApJ*, 789(2):111, July 2014. doi: 10.1088/0004-637X/789/2/111.
- T. Bovaird and C. H. Lineweaver. A flat inner disc model as an alternative to the Kepler dichotomy in the Q1-Q16 planet population. *MNRAS*, 468(2):1493–1504, June 2017. doi: 10.1093/mnras/stx414.
- J. Brakensiek and D. Ragozzine. Efficient Geometric Probabilities of Multi-Transiting Exoplanetary Systems from CORBITS. *ApJ*, 821(1):47, Apr. 2016. doi: 10.3847/0004-637X/821/1/47.
- L. Brefká and J. C. Becker. A General Origin for Multiplanetary Systems With Significantly Misaligned USP Planets. *AJ*, 162(6):242, Dec. 2021. doi: 10.3847/1538-3881/ac2a32.
- M. L. Bryan, H. A. Knutson, A. W. Howard, H. Ngo, K. Batygin, J. R. Crepp, B. J. Fulton, S. Hinkley, H. Isaacson, J. A. Johnson, G. W. Marcy, and J. T. Wright. Statistics of Long Period Gas Giant Planets in Known Planetary Systems. *ApJ*, 821:89, Apr. 2016. doi: 10.3847/0004-637X/821/2/89.
- M. L. Bryan, H. A. Knutson, E. J. Lee, B. J. Fulton, K. Batygin, H. Ngo, and T. Meshkat.

- An Excess of Jupiter Analogs in Super-Earth Systems. *AJ*, 157(2):52, Feb. 2019. doi: 10.3847/1538-3881/aaf57f.
- R. P. Butler, J. T. Wright, G. W. Marcy, D. A. Fischer, S. S. Vogt, C. G. Tinney, H. R. A. Jones, B. D. Carter, J. A. Johnson, C. McCarthy, and A. J. Penny. Catalog of Nearby Exoplanets. *ApJ*, 646(1):505–522, July 2006. doi: 10.1086/504701.
- J. Cabrera, S. Csizmadia, H. Lehmann, R. Dvorak, D. Gandolfi, H. Rauer, A. Erikson, C. Dreyer, P. Eigmüller, and A. Hatzes. The Planetary System to KIC 11442793: A Compact Analogue to the Solar System. *ApJ*, 781(1):18, Jan. 2014. doi: 10.1088/0004-637X/781/1/18.
- D. Carrera, E. B. Ford, and A. Izidoro. Formation of short-period planets by disc migration. *MNRAS*, 486(3):3874–3885, July 2019. doi: 10.1093/mnras/stz974.
- J. E. Chambers. A hybrid symplectic integrator that permits close encounters between massive bodies. *MNRAS*, 304(4):793–799, Apr. 1999. doi: 10.1046/j.1365-8711.1999.02379.x.
- S. Chatterjee, E. B. Ford, S. Matsumura, and F. A. Rasio. Dynamical Outcomes of Planet-Planet Scattering. *ApJ*, 686(1):580–602, Oct. 2008. doi: 10.1086/590227.
- C. Chen, G. Li, and C. Petrovich. Mutual Inclination of Ultra-short-period Planets with Time-varying Stellar J_2 Moments. *ApJ*, 930(1):58, May 2022. doi: 10.3847/1538-4357/ac6024.
- J. L. Christiansen. Planet Detection Metrics: Pixel-Level Transit Injection Tests of Pipeline Detection Efficiency for Data Release 25. Kepler Science Document KSCI-19110-001, June 2017.
- J. L. Christiansen, J. M. Jenkins, D. A. Caldwell, C. J. Burke, P. Tenenbaum, S. Seader, S. E. Thompson, T. S. Barclay, B. D. Clarke, J. Li, J. C. Smith, M. C. Stumpe, J. D.

- Twicken, and J. Van Cleve. The Derivation, Properties, and Value of Kepler’s Combined Differential Photometric Precision. *PASP*, 124(922):1279, Dec. 2012. doi: 10.1086/668847.
- L. Ciprari. Battezzati 3 esopianeti attorno alla pulsar PSR1257+12Battezzati 3 esopianeti attorno alla pulsar PSR1257+12Haunted exoplanets around the pulsar PSR1257+12 received scary names. *Astronomia. La rivista dell’ Unione Astrofili Italiani*, 5:24–28, May 2016.
- W. D. Cochran, D. C. Fabrycky, G. Torres, F. Fressin, J.-M. Désert, D. Ragozzine, D. Sasselov, J. J. Fortney, J. F. Rowe, E. J. Brugamyer, S. T. Bryson, J. A. Carter, D. R. Ciardi, S. B. Howell, J. H. Steffen, W. J. Borucki, D. G. Koch, J. N. Winn, W. F. Welsh, K. Uddin, P. Tenenbaum, M. Still, S. Seager, S. N. Quinn, F. Mullally, N. Miller, G. W. Marcy, P. J. MacQueen, P. Lucas, J. J. Lissauer, D. W. Latham, H. Knutson, K. Kinemuchi, J. A. Johnson, J. M. Jenkins, H. Isaacson, A. Howard, E. Horch, M. J. Holman, C. E. Henze, M. R. Haas, R. L. Gilliland, I. Gautier, Thomas N., E. B. Ford, D. A. Fischer, M. Everett, M. Endl, B.-O. Demory, D. Deming, D. Charbonneau, D. Caldwell, L. Buchhave, T. M. Brown, and N. Batalha. Kepler-18b, c, and d: A System of Three Planets Confirmed by Transit Timing Variations, Light Curve Validation, Warm-Spitzer Photometry, and Radial Velocity Measurements. *ApJS*, 197(1):7, Nov. 2011. doi: 10.1088/0067-0049/197/1/7.
- I. J. M. Crossfield, D. R. Ciardi, E. A. Petigura, E. Sinukoff, J. E. Schlieder, A. W. Howard, C. A. Beichman, H. Isaacson, C. D. Dressing, J. L. Christiansen, B. J. Fulton, S. Lépine, L. Weiss, L. Hirsch, J. Livingston, C. Baranec, N. M. Law, R. Riddle, C. Ziegler, S. B. Howell, E. Horch, M. Everett, J. Teske, A. O. Martinez, C. Obermeier, B. Benneke, N. Scott, N. Deacon, K. M. Aller, B. M. S. Hansen, L. Mancini, S. Ciceri, R. Brahm, A. Jordán, H. A. Knutson, T. Henning, M. Bonnefoy, M. C. Liu, J. R. Crepp, J. Lothringer, P. Hinz, V. Bailey, A. Skemer, and D. Defrere. 197 Candidates and 104 Validated Planets in K2’s First Five Fields. *ApJS*, 226(1):7, Sept. 2016. doi: 10.3847/0067-0049/226/1/7.
- I. J. M. Crossfield, D. R. Ciardi, H. Isaacson, A. W. Howard, E. A. Petigura, L. M. Weiss, B. J. Fulton, E. Sinukoff, J. E. Schlieder, D. Mawet, G. Ruane, I. de Pater, K. de Kleer,

- A. G. Davies, J. L. Christiansen, C. D. Dressing, L. Hirsch, B. Benneke, J. R. Crepp, M. Kosiarek, J. Livingston, E. Gonzales, C. A. Beichman, and H. A. Knutson. Two Small Transiting Planets and a Possible Third Body Orbiting HD 106315. *AJ*, 153(6):255, June 2017. doi: 10.3847/1538-3881/aa6e01.
- F. Dai, J. N. Winn, D. Gandolfi, S. X. Wang, J. K. Teske, J. Burt, S. Albrecht, O. Barragán, W. D. Cochran, M. Endl, M. Fridlund, A. P. Hatzes, T. Hirano, L. A. Hirsch, M. C. Johnson, A. Bo Justesen, J. Livingston, C. M. Persson, J. Prieto-Arranz, A. Vanderburg, R. Alonso, G. Antoniciello, P. Arriagada, R. P. Butler, J. Cabrera, J. D. Crane, F. Cusano, S. Csizmadia, H. Deeg, S. B. Dieterich, P. Eigmüller, A. Erikson, M. E. Everett, A. Fukui, S. Grziwa, E. W. Guenther, G. W. Henry, S. B. Howell, J. A. Johnson, J. Korth, M. Kuzuhara, N. Narita, D. Nespral, G. Nowak, E. Palles, M. Pätzold, H. Rauer, P. M. Rodríguez, S. A. Shectman, A. M. S. Smith, I. B. Thompson, V. V. Eylen, M. W. Williamson, and R. A. Wittenmyer. The Discovery and Mass Measurement of a New Ultra-short-period Planet: K2-131b. *AJ*, 154(6):226, Dec. 2017a. doi: 10.3847/1538-3881/aa9065.
- F. Dai, J. N. Winn, D. Gandolfi, S. X. Wang, J. K. Teske, J. Burt, S. Albrecht, O. Barragán, W. D. Cochran, M. Endl, M. Fridlund, A. P. Hatzes, T. Hirano, L. A. Hirsch, M. C. Johnson, A. Bo Justesen, J. Livingston, C. M. Persson, J. Prieto-Arranz, A. Vanderburg, R. Alonso, G. Antoniciello, P. Arriagada, R. P. Butler, J. Cabrera, J. D. Crane, F. Cusano, S. Csizmadia, H. Deeg, S. B. Dieterich, P. Eigmüller, A. Erikson, M. E. Everett, A. Fukui, S. Grziwa, E. W. Guenther, G. W. Henry, S. B. Howell, J. A. Johnson, J. Korth, M. Kuzuhara, N. Narita, D. Nespral, G. Nowak, E. Palles, M. Pätzold, H. Rauer, P. M. Rodríguez, S. A. Shectman, A. M. S. Smith, I. B. Thompson, V. V. Eylen, M. W. Williamson, and R. A. Wittenmyer. The Discovery and Mass Measurement of a New Ultra-short-period Planet: K2-131b. *AJ*, 154(6):226, Dec. 2017b. doi: 10.3847/1538-3881/aa9065.
- F. Dai, K. Masuda, and J. N. Winn. Larger Mutual Inclinations for the Shortest-period

- Planets. *ApJ*, 864(2):L38, Sept. 2018. doi: 10.3847/2041-8213/aadd4f.
- J. P. de Leon, J. Livingston, M. Endl, W. D. Cochran, T. Hirano, R. A. García, S. Mathur, K. W. F. Lam, J. Korth, A. A. Trani, F. Dai, E. Díez Alonso, A. Castro-González, M. Fridlund, A. Fukui, D. Gandolfi, P. Kabath, M. Kuzuhara, R. Luque, A. B. Savel, H. Gill, C. Dressing, S. Giacalone, N. Narita, E. Palte, V. Van Eylen, and M. Tamura. 37 new validated planets in overlapping K2 campaigns. *MNRAS*, 508(1):195–218, Nov. 2021. doi: 10.1093/mnras/stab2305.
- P. Delorme, A. C. Cameron, L. Hebb, J. Rostron, T. A. Lister, A. J. Norton, D. Pollacco, and R. G. West. Stellar Rotation in the Hyades and Praesepe: Gyrochronology and Braking Timescale. In C. Johns-Krull, M. K. Browning, and A. A. West, editors, *16th Cambridge Workshop on Cool Stars, Stellar Systems, and the Sun*, volume 448 of *Astronomical Society of the Pacific Conference Series*, page 841, Dec. 2011.
- P. Denham, S. Naoz, B.-M. Hoang, A. P. Stephan, and W. M. Farr. Hidden planetary friends: on the stability of two-planet systems in the presence of a distant, inclined companion. *MNRAS*, 482(3):4146–4154, Jan. 2019. doi: 10.1093/mnras/sty2830.
- I. Dobbs-Dixon, D. N. C. Lin, and R. A. Mardling. Spin-Orbit Evolution of Short-Period Planets. *ApJ*, 610(1):464–476, July 2004a. doi: 10.1086/421510.
- I. Dobbs-Dixon, D. N. C. Lin, and R. A. Mardling. Spin-Orbit Evolution of Short-Period Planets. *ApJ*, 610(1):464–476, July 2004b. doi: 10.1086/421510.
- G. Dransfield, A. H. M. J. Triaud, T. Guillot, D. Mekarnia, D. Nesvorný, N. Crouzet, L. Abe, K. Agabi, M. Buttu, J. Cabrera, D. Gandolfi, M. N. Günther, F. Rodler, F.-X. Schmider, P. Stee, O. Suarez, K. A. Collins, M. Dévora-Pajares, S. B. Howell, E. C. Matthews, M. R. Standing, K. G. Stassun, C. Stockdale, S. N. Quinn, C. Ziegler, I. J. M. Crossfield, J. J. Lissauer, A. W. Mann, R. Matson, J. Schlieder, and G. Zhou. HD 28109 hosts a trio of transiting Neptunian planets including a near-resonant pair, confirmed by ASTEP from Antarctica. *MNRAS*, 515(1):1328–1345, Sept. 2022. doi: 10.1093/mnras/stac1383.

- A. Einstein. The field equations of gravitation. *Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.)*, 1915:844–847, 1915.
- M. E. Everett, T. Barclay, D. R. Ciardi, E. P. Horch, S. B. Howell, J. R. Crepp, and D. R. Silva. High-Resolution Multi-Band Imaging for Validation and Characterization of Small Kepler Planets. *AJ*, 149(2):55, Feb. 2015. doi: 10.1088/0004-6256/149/2/55.
- ExoFOP. Exoplanet follow-up observing program - tess, 2023. URL <https://catcopy.ipac.caltech.edu/doi/doi.php?id=10.26134/EXOFOP3>.
- D. Fabrycky and S. Tremaine. Shrinking Binary and Planetary Orbits by Kozai Cycles with Tidal Friction. *ApJ*, 669:1298–1315, Nov. 2007. doi: 10.1086/521702.
- D. C. Fabrycky, E. T. Johnson, and J. Goodman. Cassini States with Dissipation: Why Obliquity Tides Cannot Inflate Hot Jupiters. *ApJ*, 665:754–766, Aug. 2007. doi: 10.1086/519075.
- D. C. Fabrycky, E. B. Ford, J. H. Steffen, J. F. Rowe, J. A. Carter, A. V. Moorhead, N. M. Batalha, W. J. Borucki, S. Bryson, L. A. Buchhave, J. L. Christiansen, D. R. Ciardi, W. D. Cochran, M. Endl, M. N. Fanelli, D. Fischer, F. Fressin, J. Geary, M. R. Haas, J. R. Hall, M. J. Holman, J. M. Jenkins, D. G. Koch, D. W. Latham, J. Li, J. J. Lissauer, P. Lucas, G. W. Marcy, T. Mazeh, S. McCauliff, S. Quinn, D. Ragozzine, D. Sasselov, and A. Shporer. Transit Timing Observations from Kepler. IV. Confirmation of Four Multiple-planet Systems by Simple Physical Models. *ApJ*, 750(2):114, May 2012. doi: 10.1088/0004-637X/750/2/114.
- D. C. Fabrycky, J. J. Lissauer, D. Ragozzine, J. F. Rowe, J. H. Steffen, E. Agol, T. Barclay, N. Batalha, W. Borucki, D. R. Ciardi, E. B. Ford, T. N. Gautier, J. C. Geary, M. J. Holman, J. M. Jenkins, J. Li, R. C. Morehead, R. L. Morris, A. Shporer, J. C. Smith, M. Still, and J. Van Cleve. Architecture of Kepler’s Multi-transiting Systems. II. New Investigations with Twice as Many Candidates. *ApJ*, 790(2):146, Aug. 2014. doi: 10.1088/0004-637X/790/2/146.

- J. Fang and J.-L. Margot. Architecture of Planetary Systems Based on Kepler Data: Number of Planets and Coplanarity. *ApJ*, 761(2):92, Dec. 2012. doi: 10.1088/0004-637X/761/2/92.
- T. H. Faridani, S. Naoz, L. Wei, and W. M. Farr. Hiding Planets Near and Far: The Parameter Space of Hidden Companions for Known Planetary Systems. *ApJ*, 932(2):78, June 2022. doi: 10.3847/1538-4357/ac6e38.
- T. H. Faridani, S. Naoz, G. Li, and N. Inzunza. Let’s Sweep: The Effect of Evolving J_2 on the Resonant Structure of a Three-planet System. *ApJ*, 956(2):90, Oct. 2023. doi: 10.3847/1538-4357/acf378.
- T. H. Faridani, S. Naoz, G. Li, M. Rice, and N. Inzunza. More Likely Than You Think: Inclination-driving Secular Resonances Are Common in Known Exoplanet Systems. *ApJ*, 978(1):18, Jan. 2025. doi: 10.3847/1538-4357/ad8ebf.
- E. B. Ford, K. J. Joshi, F. A. Rasio, and B. Zbarsky. Theoretical Implications of the PSR B1620-26 Triple System and Its Planet. *ApJ*, 528:336–350, Jan. 2000. doi: 10.1086/308167.
- J. Freudenthal, C. von Essen, S. Dreizler, S. Wedemeyer, E. Agol, B. M. Morris, A. C. Becker, M. Mallonn, S. Hoyer, A. Ofir, L. Tal-Or, H. J. Deeg, E. Herrero, I. Ribas, S. Khalafinejad, J. Hernández, and M. M. Rodríguez S. Kepler Object of Interest Network. II. Photodynamical modelling of Kepler-9 over 8 years of transit observations. *A&A*, 618: A41, Oct. 2018. doi: 10.1051/0004-6361/201833436.
- G. Frustagli, E. Poretti, T. Milbourne, L. Malavolta, A. Mortier, V. Singh, A. S. Bonomo, L. A. Buchhave, L. Zeng, A. Vanderburg, S. Udry, G. Andreuzzi, A. Collier-Cameron, R. Cosentino, M. Damasso, A. Ghedina, A. Harutyunyan, R. D. Haywood, D. W. Latham, M. López-Morales, V. Lorenzi, A. F. Martinez Fiorenzano, M. Mayor, G. Micela, E. Molinari, F. Pepe, D. Phillips, K. Rice, and A. Sozzetti. An ultra-short period rocky super-Earth orbiting the G2-star HD 80653. *A&A*, 633:A133, Jan. 2020a. doi: 10.1051/0004-6361/201936689.

- G. Frustagli, E. Poretti, T. Milbourne, L. Malavolta, A. Mortier, V. Singh, A. S. Bonomo, L. A. Buchhave, L. Zeng, A. Vanderburg, S. Udry, G. Andreuzzi, A. Collier-Cameron, R. Cosentino, M. Damasso, A. Ghedina, A. Harutyunyan, R. D. Haywood, D. W. Latham, M. López-Morales, V. Lorenzi, A. F. Martinez Fiorenzano, M. Mayor, G. Micela, E. Molinari, F. Pepe, D. Phillips, K. Rice, and A. Sozzetti. An ultra-short period rocky super-Earth orbiting the G2-star HD 80653. *A&A*, 633:A133, Jan. 2020b. doi: 10.1051/0004-6361/201936689.
- T. Fu and Y. Wang. Sculpting of Exoplanetary Systems Driven by a Misaligned Disk and Stellar Oblateness: Origin of Perpendicular Orbits in HD 3167. *ApJ*, 973(2):L43, Oct. 2024. doi: 10.3847/2041-8213/ad77d6.
- B. J. Fulton and E. A. Petigura. The California-Kepler Survey. VII. Precise Planet Radii Leveraging Gaia DR2 Reveal the Stellar Mass Dependence of the Planet Radius Gap. *AJ*, 156(6):264, Dec. 2018. doi: 10.3847/1538-3881/aae828.
- B. J. Fulton, E. A. Petigura, A. W. Howard, H. Isaacson, G. W. Marcy, P. A. Cargile, L. Hebb, L. M. Weiss, J. A. Johnson, T. D. Morton, E. Sinukoff, I. J. M. Crossfield, and L. A. Hirsch. The California-Kepler Survey. III. A Gap in the Radius Distribution of Small Planets. *AJ*, 154(3):109, Sept. 2017. doi: 10.3847/1538-3881/aa80eb.
- F. Gallet and J. Bouvier. Improved angular momentum evolution model for solar-like stars. *A&A*, 556:A36, Aug. 2013. doi: 10.1051/0004-6361/201321302.
- D. Gandolfi, L. Fossati, J. H. Livingston, K. G. Stassun, S. Grziwa, O. Barragán, M. Fridlund, D. Kubyskhina, C. M. Persson, F. Dai, K. W. F. Lam, S. Albrecht, N. Batalha, P. G. Beck, A. B. Justesen, J. Cabrera, S. Cartwright, W. D. Cochran, S. Csizmadia, M. D. Davies, H. J. Deeg, P. Eigmüller, M. Endl, A. Erikson, M. Esposito, R. A. García, R. Goeke, L. González-Cuesta, E. W. Guenther, A. P. Hatzes, D. Hidalgo, T. Hirano, M. Hjorth, P. Kabath, E. Knudstrup, J. Korth, J. Li, R. Luque, S. Mathur, P. Montañes Rodríguez, N. Narita, D. Nespral, P. Niraula, G. Nowak, E. Palles, M. Pätzold, J. Prieto-Arranz,

- H. Rauer, S. Redfield, I. Ribas, M. Skarka, A. M. S. Smith, P. Rowden, G. Torres, V. Van Eylen, and M. L. Vezie. The Transiting Multi-planet System HD15337: Two Nearly Equal-mass Planets Straddling the Radius Gap. *ApJ*, 876(2):L24, May 2019. doi: 10.3847/2041-8213/ab17d9.
- Z. Garai, H. P. Osborn, D. Gandolfi, A. Brandeker, S. G. Sousa, M. Lendl, A. Bekkelien, C. Broeg, A. C. Cameron, J. A. Egger, M. J. Hooton, Y. Alibert, L. Delrez, L. Fossati, S. Salmon, T. G. Wilson, A. Bonfanti, A. Tuson, S. Ulmer-Moll, L. M. Serrano, L. Borsato, R. Alonso, G. Anglada, J. Asquier, D. Barrado y Navascues, S. C. C. Barros, T. Bárczy, W. Baumjohann, M. Beck, T. Beck, W. Benz, N. Billot, F. Biondi, X. Bonfils, M. Buder, J. Cabrera, V. Cessa, S. Charnoz, S. Csizmadia, P. E. Cubillos, M. B. Davies, M. Deleuil, O. D. S. Demangeon, B. O. Demory, D. Ehrenreich, A. Erikson, V. Van Eylen, A. Fortier, M. Fridlund, M. Gillon, V. Van Grootel, M. Güdel, M. N. Günther, S. Hoyer, K. G. Isaak, L. L. Kiss, M. H. Kristiansen, J. Laskar, A. L. des Etangs, C. Lovis, A. Luntzer, D. Magrin, P. F. L. Maxted, C. Mordasini, V. Nascimbeni, G. Olofsson, R. Ottensamer, I. Pagano, E. Pallé, G. Peter, G. Piotto, D. Pollacco, D. Queloz, R. Ragazzoni, N. Rando, H. Rauer, I. Ribas, N. C. Santos, G. Scandariato, D. Ségransan, A. E. Simon, A. M. S. Smith, M. Steller, G. M. Szabó, N. Thomas, S. Udry, J. Venturini, and N. Walton. Refined parameters of the HD 22946 planetary system and the true orbital period of planet d. *A&A*, 674:A44, June 2023. doi: 10.1051/0004-6361/202345943.
- G. J. Gilbert and D. C. Fabrycky. An Information Theoretic Framework for Classifying Exoplanetary System Architectures. *AJ*, 159(6):281, June 2020. doi: 10.3847/1538-3881/ab8e3c.
- G. J. Gilbert, E. A. Petigura, and P. M. Entrican. Planets larger than neptune have elevated eccentricities. *Proceedings of the National Academy of Sciences*, 122(11):e2405295122, 2025. doi: 10.1073/pnas.2405295122. URL <https://www.pnas.org/doi/abs/10.1073/pnas.2405295122>.
- M. Gillon, E. Jehin, S. M. Lederer, L. Delrez, J. de Wit, A. Burdanov, V. Van Grootel,

- A. J. Burgasser, A. H. M. J. Triaud, C. Opitom, B.-O. Demory, D. K. Sahu, D. Bardalez Gagliuffi, P. Magain, and D. Queloz. Temperate Earth-sized planets transiting a nearby ultracool dwarf star. *Nature*, 533(7602):221–224, May 2016. doi: 10.1038/nature17448.
- S. Ginzburg, H. E. Schlichting, and R. Sari. Core-powered mass-loss and the radius distribution of small exoplanets. *MNRAS*, 476(1):759–765, May 2018. doi: 10.1093/mnras/sty290.
- E. Grishin, H. B. Perets, and G. Fragione. Quasi-secular evolution of mildly hierarchical triple systems: analytics and applications for GW sources and hot Jupiters. *MNRAS*, 481(4):4907–4923, Dec. 2018. doi: 10.1093/mnras/sty2477.
- A. Gupta and H. E. Schlichting. Sculpting the valley in the radius distribution of small exoplanets as a by-product of planet formation: the core-powered mass-loss mechanism. *MNRAS*, 487(1):24–33, July 2019. doi: 10.1093/mnras/stz1230.
- A. Gupta and H. E. Schlichting. Signatures of the core-powered mass-loss mechanism in the exoplanet population: dependence on stellar properties and observational predictions. *MNRAS*, 493(1):792–806, Mar. 2020. doi: 10.1093/mnras/staa315.
- A. Gupta and H. E. Schlichting. Caught in the act: core-powered mass-loss predictions for observing atmospheric escape. *MNRAS*, 504(3):4634–4648, July 2021. doi: 10.1093/mnras/stab1128.
- S. Hadden and Y. Lithwick. Densities and Eccentricities of 139 Kepler Planets from Transit Time Variations. *ApJ*, 787(1):80, May 2014. doi: 10.1088/0004-637X/787/1/80.
- S. Hadden and Y. Lithwick. Kepler Planet Masses and Eccentricities from TTV Analysis. *AJ*, 154(1):5, July 2017. doi: 10.3847/1538-3881/aa71ef.
- B. M. S. Hansen. Formation of the Terrestrial Planets from a Narrow Annulus. *ApJ*, 703(1):1131–1140, Sept. 2009. doi: 10.1088/0004-637X/703/1/1131.
- B. M. S. Hansen. Calibration of Equilibrium Tide Theory for Extrasolar Planet Systems. *ApJ*, 723:285–299, Nov. 2010. doi: 10.1088/0004-637X/723/1/285.

- B. M. S. Hansen. Calibration of Equilibrium Tide Theory for Extrasolar Planet Systems. II. *ApJ*, 757(1):6, Sept. 2012. doi: 10.1088/0004-637X/757/1/6.
- B. M. S. Hansen. Perturbation of Compact Planetary Systems by Distant Giant Planets. *MNRAS*, 467(2):1531–1560, May 2017. doi: 10.1093/mnras/stx182.
- B. M. S. Hansen and N. Murray. Testing in Situ Assembly with the Kepler Planet Candidate Sample. *ApJ*, 775(1):53, Sept. 2013. doi: 10.1088/0004-637X/775/1/53.
- B. M. S. Hansen and N. Murray. Secular effects of tidal damping in compact planetary systems. *MNRAS*, 448(2):1044–1059, Apr. 2015. doi: 10.1093/mnras/stv049.
- B. M. S. Hansen and S. Naoz. The stationary points of the hierarchical three-body problem. *MNRAS*, 499(2):1682–1700, Dec. 2020. doi: 10.1093/mnras/staa2602.
- J. D. Hartman, G. Á. Bakos, G. Kovács, and R. W. Noyes. A large sample of photometric rotation periods for FGK Pleiades stars. *MNRAS*, 408(1):475–489, Oct. 2010. doi: 10.1111/j.1365-2966.2010.17147.x.
- C. A. Haswell. *Transiting Exoplanets*. 2010.
- A. P. Hatzes, E. W. Guenther, M. Endl, W. D. Cochran, M. P. Döllinger, and A. Bedalov. A giant planet around the massive giant star HD 13189. *A&A*, 437(2):743–751, July 2005. doi: 10.1051/0004-6361:20052850.
- M. Y. He, E. B. Ford, and D. Ragozzine. Architectures of exoplanetary systems - I. A clustered forward model for exoplanetary systems around Kepler’s FGK stars. *MNRAS*, 490(4):4575–4605, Dec. 2019. doi: 10.1093/mnras/stz2869.
- M. Y. He, E. B. Ford, D. Ragozzine, and D. Carrera. Architectures of Exoplanetary Systems. III. Eccentricity and Mutual Inclination Distributions of AMD-stable Planetary Systems. *AJ*, 160(6):276, Dec. 2020. doi: 10.3847/1538-3881/abba18.

- M. Y. He, E. B. Ford, and D. Ragozzine. Friends and Foes: Conditional Occurrence Rates of Exoplanet Companions and Their Impact on Radial Velocity Follow-up Surveys. *AJ*, 162(5):216, Nov. 2021. doi: 10.3847/1538-3881/ac1db8.
- C. Hedges, N. Saunders, G. Barentsen, J. L. Coughlin, J. V. d. M. Cardoso, V. B. Kostov, J. Dotson, and A. M. Cody. Four Small Planets Buried in K2 Systems: What Can We Learn for TESS? *ApJ*, 880(1):L5, July 2019. doi: 10.3847/2041-8213/ab2a74.
- C. Hedges, A. Hughes, G. Zhou, T. J. David, J. Becker, S. Giacalone, A. Vanderburg, J. E. Rodriguez, A. Bieryla, C. Wirth, S. Atherton, T. Fetherolf, K. A. Collins, A. M. Price-Whelan, M. Bedell, S. N. Quinn, T. Gan, G. R. Ricker, D. W. Latham, R. K. Vanderspek, S. Seager, J. N. Winn, J. M. Jenkins, J. F. Kielkopf, R. P. Schwarz, C. D. Dressing, E. J. Gonzales, I. J. M. Crossfield, E. C. Matthews, E. L. N. Jensen, E. Furlan, C. L. Gnilka, S. B. Howell, K. V. Lester, N. J. Scott, D. L. Feliz, M. B. Lund, R. J. Siverd, D. J. Stevens, N. Narita, A. Fukui, F. Murgas, E. Palle, P. J. Sutton, K. G. Stassun, L. G. Bouma, M. Vezie, J. N. Villaseñor, E. V. Quintana, and J. C. Smith. TOI-2076 and TOI-1807: Two Young, Comoving Planetary Systems within 50 pc Identified by TESS that are Ideal Candidates for Further Follow Up. *AJ*, 162(2):54, Aug. 2021. doi: 10.3847/1538-3881/ac06cd.
- T. A. Heppenheimer. Secular resonances and the origin of eccentricities of Mars and the asteroids. *Icarus*, 41(1):76–88, Jan. 1980. doi: 10.1016/0019-1035(80)90160-8.
- D. Hidalgo, E. Pallé, R. Alonso, D. Gandolfi, M. Fridlund, G. Nowak, R. Luque, T. Hirano, A. B. Justesen, W. D. Cochran, O. Barragán, L. Spina, F. Rodler, S. Albrecht, D. Anderson, P. Amado, E. Bryant, J. A. Caballero, J. Cabrera, S. Csizmadia, F. Dai, J. De Leon, H. J. Deeg, P. Eigmüller, M. Endl, A. Erikson, M. Esposito, P. Figueira, I. Georgieva, S. Grziwa, E. Guenther, A. P. Hatzes, M. Hjorth, H. J. Hoeijmakers, P. Kabath, J. Korth, M. Kuzuhara, M. Lafarga, M. Lampon, I. C. Leão, J. Livingston, S. Mathur, P. Montañes-Rodriguez, J. C. Morales, F. Murgas, E. Nagel, N. Narita, L. D. Nielsen, M. Patzold, C. M. Persson, J. Prieto-Arranz, A. Quirrenbach, H. Rauer, S. Redfield, A. Reiners, I. Ribas,

- A. M. S. Smith, J. Šubjak, V. Van Eylen, and P. A. Wilson. Three planets transiting the evolved star EPIC 249893012: a hot 8.8- $M_{\oplus}$ super-Earth and two warm 14.7 and 10.2- $M_{\oplus}$ sub-Neptunes. *A&A*, 636:A89, Apr. 2020. doi: 10.1051/0004-6361/201937080.
- T. Holzer, T. Mazeh, G. Nachmani, D. Jontof-Hutter, E. B. Ford, D. Fabrycky, D. Ragozzine, M. Kane, and J. H. Steffen. Transit Timing Observations from Kepler. IX. Catalog of the Full Long-cadence Data Set. *ApJS*, 225(1):9, July 2016. doi: 10.3847/0067-0049/225/1/9.
- M. J. Holman and P. A. Wiegert. Long-Term Stability of Planets in Binary Systems. *AJ*, 117(1):621–628, Jan. 1999. doi: 10.1086/300695.
- M. J. Holman, D. C. Fabrycky, D. Ragozzine, E. B. Ford, J. H. Steffen, W. F. Welsh, J. J. Lissauer, D. W. Latham, G. W. Marcy, L. M. Walkowicz, N. M. Batalha, J. M. Jenkins, J. F. Rowe, W. D. Cochran, F. Fressin, G. Torres, L. A. Buchhave, D. D. Sasselov, W. J. Borucki, D. G. Koch, G. Basri, T. M. Brown, D. A. Caldwell, D. Charbonneau, E. W. Dunham, T. N. Gautier, J. C. Geary, R. L. Gilliland, M. R. Haas, S. B. Howell, D. R. Ciardi, M. Endl, D. Fischer, G. Fürész, J. D. Hartman, H. Isaacson, J. A. Johnson, P. J. MacQueen, A. V. Moorhead, R. C. Morehead, and J. A. Orosz. Kepler-9: A System of Multiple Planets Transiting a Sun-Like Star, Confirmed by Timing Variations. *Science*, 330(6000):51, Oct. 2010. doi: 10.1126/science.1195778.
- A. W. Howard, G. W. Marcy, J. A. Johnson, D. A. Fischer, J. T. Wright, H. Isaacson, J. A. Valenti, J. Anderson, D. N. C. Lin, and S. Ida. The Occurrence and Mass Distribution of Close-in Super-Earths, Neptunes, and Jupiters. *Science*, 330:653, Oct. 2010. doi: 10.1126/science.1194854.
- A. W. Howard, G. W. Marcy, S. T. Bryson, J. M. Jenkins, J. F. Rowe, N. M. Batalha, W. J. Borucki, D. G. Koch, E. W. Dunham, T. N. Gautier, III, J. Van Cleve, W. D. Cochran, D. W. Latham, J. J. Lissauer, G. Torres, T. M. Brown, R. L. Gilliland, L. A. Buchhave, D. A. Caldwell, J. Christensen-Dalsgaard, D. Ciardi, F. Fressin, M. R. Haas,

- S. B. Howell, H. Kjeldsen, S. Seager, L. Rogers, D. D. Sasselov, J. H. Steffen, G. S. Basri, D. Charbonneau, J. Christiansen, B. Clarke, A. Dupree, D. C. Fabrycky, D. A. Fischer, E. B. Ford, J. J. Fortney, J. Tarter, F. R. Girouard, M. J. Holman, J. A. Johnson, T. C. Klaus, P. Machalek, A. V. Moorhead, R. C. Morehead, D. Ragozzine, P. Tenenbaum, J. D. Twicken, S. N. Quinn, H. Isaacson, A. Shporer, P. W. Lucas, L. M. Walkowicz, W. F. Welsh, A. Boss, E. Devore, A. Gould, J. C. Smith, R. L. Morris, A. Prsa, T. D. Morton, M. Still, S. E. Thompson, F. Mullally, M. Endl, and P. J. MacQueen. Planet Occurrence within 0.25 AU of Solar-type Stars from Kepler. *ApJS*, 201:15, Aug. 2012. doi: 10.1088/0067-0049/201/2/15.
- D. Huber, J. A. Carter, M. Barbieri, A. Miglio, K. M. Deck, D. C. Fabrycky, B. T. Montet, L. A. Buchhave, W. J. Chaplin, S. Hekker, J. Montalbán, R. Sanchis-Ojeda, S. Basu, T. R. Bedding, T. L. Campante, J. Christensen-Dalsgaard, Y. P. Elsworth, D. Stello, T. Arentoft, E. B. Ford, R. L. Gilliland, R. Handberg, A. W. Howard, H. Isaacson, J. A. Johnson, C. Karoff, S. D. Kawaler, H. Kjeldsen, D. W. Latham, M. N. Lund, M. Lundkvist, G. W. Marcy, T. S. Metcalfe, V. Silva Aguirre, and J. N. Winn. Stellar Spin-Orbit Misalignment in a Multiplanet System. *Science*, 342:331–334, Oct. 2013. doi: 10.1126/science.1242066.
- P. Hut. Stability of tidal equilibrium. *A&A*, 92:167–170, Dec. 1980.
- K. A. Innanen, J. Q. Zheng, S. Mikkola, and M. J. Valtonen. The Kozai Mechanism and the Stability of Planetary Orbits in Binary Star Systems. *AJ*, 113:1915–+, May 1997. doi: 10.1086/118405.
- B. Jackson, C. C. Stark, E. R. Adams, J. Chambers, and D. Deming. A Survey for Very Short-period Planets in the Kepler Data. *ApJ*, 779(2):165, Dec. 2013. doi: 10.1088/0004-637X/779/2/165.
- A. S. Jermyn, E. B. Bauer, J. Schwab, R. Farmer, W. H. Ball, E. P. Bellinger, A. Dotter, M. Joyce, P. Marchant, J. S. G. Mombarg, W. M. Wolf, T. L. Sunny Wong, G. C. Cinquegrana, E. Farrell, R. Smolec, A. Thoul, M. Cantiello, F. Herwig, O. Toloza, L. Bildsten,

- R. H. D. Townsend, and F. X. Timmes. Modules for Experiments in Stellar Astrophysics (MESA): Time-dependent Convection, Energy Conservation, Automatic Differentiation, and Infrastructure. *ApJS*, 265(1):15, Mar. 2023. doi: 10.3847/1538-4365/acae8d.
- A. Johansen, M. B. Davies, R. P. Church, and V. Holmelin. Can Planetary Instability Explain the Kepler Dichotomy? *ApJ*, 758:39, Oct. 2012. doi: 10.1088/0004-637X/758/1/39.
- D. Kipping. Do planets remember how they formed? *MNRAS*, 473(1):784–795, Jan. 2018. doi: 10.1093/mnras/stx2383.
- H. A. Knutson, B. J. Fulton, B. T. Montet, M. Kao, H. Ngo, A. W. Howard, J. R. Crepp, S. Hinkley, G. Á. Bakos, K. Batygin, J. A. Johnson, T. D. Morton, and P. S. Muirhead. Friends of Hot Jupiters. I. A Radial Velocity Search for Massive, Long-period Companions to Close-in Gas Giant Planets. *ApJ*, 785:126, Apr. 2014. doi: 10.1088/0004-637X/785/2/126.
- Q. M. Konopacky, J. Rameau, G. Duchêne, J. C. Filippazzo, P. A. G. Godfrey, C. Marois, E. L. Nielsen, L. Pueyo, R. R. Rafikov, E. L. Rice, J. J. Wang, S. M. Ammons, V. P. Bailey, T. S. Barman, J. Bulger, S. Bruzzone, J. K. Chilcote, T. Cotten, R. I. Dawson, R. J. D. Rosa, R. Doyon, T. M. Esposito, M. P. Fitzgerald, K. B. Follette, S. Goodsell, J. R. Graham, A. Z. Greenbaum, P. Higon, L.-W. Hung, P. Ingraham, P. Kalas, D. Lafrenière, J. E. Larkin, B. A. Macintosh, J. Maire, F. Marchis, M. S. Marley, B. C. Matthews, S. Metchev, M. A. Millar-Blanchaer, R. Oppenheimer, D. W. Palmer, J. Patience, M. D. Perrin, L. A. Poyneer, A. Rajan, F. T. Rantakyö, D. Savransky, A. C. Schneider, A. Sivaramakrishnan, I. Song, R. Soummer, S. Thomas, J. K. Wallace, K. Ward-Duong, S. J. Wiktorowicz, and S. G. Wolff. DISCOVERY OF a SUB-STELLAR COMPANION TO THE NEARBY DEBRIS DISK HOST HR 2562. *The Astrophysical Journal*, 829(1):L4, sep 2016. doi: 10.3847/2041-8205/829/1/L4. URL <https://doi.org/10.3847/2041-8205/829/1/L4>.
- Y. Kozai. Secular perturbations of asteroids with high inclination and eccentricity. *AJ*, 67: 591–+, Nov. 1962. doi: 10.1086/108790.

- R. P. Kraft. Studies of Stellar Rotation. V. The Dependence of Rotation on Age among Solar-Type Stars. *ApJ*, 150:551, Nov. 1967. doi: 10.1086/149359.
- D. Lai and B. Pu. Hiding Planets behind a Big Friend: Mutual Inclinations of Multi-planet Systems with External Companions. *AJ*, 153(1):42, Jan. 2017. doi: 10.3847/1538-3881/153/1/42.
- D. Lai, K. R. Anderson, and B. Pu. How do external companions affect spin-orbit misalignment of hot Jupiters? *MNRAS*, 475(4):5231–5236, Apr. 2018. doi: 10.1093/mnras/sty133.
- A. F. Lanza and E. L. Shkolnik. Secular orbital evolution of planetary systems and the dearth of close-in planets around fast rotators. *MNRAS*, 443(2):1451–1462, Sept. 2014. doi: 10.1093/mnras/stu1206.
- J. Laskar and A. C. Petit. AMD-stability and the classification of planetary systems. *A&A*, 605:A72, Sept. 2017. doi: 10.1051/0004-6361/201630022.
- E. J. Lee and E. Chiang. Magnetospheric Truncation, Tidal Inspiral, and the Creation of Short-period and Ultra-short-period Planets. *ApJ*, 842(1):40, June 2017. doi: 10.3847/1538-4357/aa6fb3.
- A. Léger, D. Rouan, J. Schneider, P. Barge, M. Fridlund, B. Samuel, M. Ollivier, E. Guenther, M. Deleuil, H. J. Deeg, M. Auvergne, R. Alonso, S. Aigrain, A. Alapini, J. M. Almenara, A. Baglin, M. Barbieri, H. Bruntt, P. Bordé, F. Bouchy, J. Cabrera, C. Catala, L. Carone, S. Carpano, S. Csizmadia, R. Dvorak, A. Erikson, S. Ferraz-Mello, B. Foring, F. Fressin, D. Gandolfi, M. Gillon, P. Gondoin, O. Grasset, T. Guillot, A. Hatzes, G. Hébrard, L. Jorda, H. Lammer, A. Llebaria, B. Loeillet, M. Mayor, T. Mazeh, C. Moutou, M. Pätzold, F. Pont, D. Queloz, H. Rauer, S. Renner, R. Samadi, A. Shporer, C. Sotin, B. Tingley, G. Wuchterl, M. Adda, P. Agogu, T. Appourchaux, H. Balans, P. Baron, T. Beaufort, R. Bellenger, R. Berlin, P. Bernardi, D. Blouin, F. Baudin, P. Bodin, L. Boisdard, L. Boit, F. Bonneau, S. Borzeix, R. Briet, J. T. Buey, B. Butler, D. Cailleau, R. Cautain, P. Y. Chabaud, S. Chaintreuil, F. Chiavassa, V. Costes, V. Cuna

- Parrho, F. de Oliveira Fialho, M. Decaudin, J. M. Defise, S. Djalal, G. Epstein, G. E. Exil, C. Fauré, T. Fenouillet, A. Gaboriaud, A. Gallic, P. Gamet, P. Gavalda, E. Grolleau, R. Gruneisen, L. Gueguen, V. Guis, V. Guivarc'h, P. Guterman, D. Hallouard, J. Hasiba, F. Heuripeau, G. Huntzinger, H. Hustaix, C. Imad, C. Imbert, B. Johlander, M. Jouret, P. Journoud, F. Karioty, L. Kerjean, V. Lafaille, L. Lafond, T. Lam-Trong, P. Landiech, V. Lapeyrere, T. Larqué, P. Laudet, N. Lautier, H. Lecann, L. Lefevre, B. Leruyet, P. Levacher, A. Magnan, E. Mazy, F. Mertens, J. M. Mesnager, J. C. Meunier, J. P. Michel, W. Monjoin, D. Naudet, K. Nguyen-Kim, J. L. Orcesi, H. Ottacher, R. Perez, G. Peter, P. Plasson, J. Y. Plessier, B. Pontet, A. Pradines, C. Quentin, J. L. Reynaud, G. Rolland, F. Rollenhagen, R. Romagnan, N. Russ, R. Schmidt, N. Schwartz, I. Sebbag, G. Sedes, H. Smit, M. B. Steller, W. Sunter, C. Surace, M. Tello, D. Tiphène, P. Toulouse, B. Ulmer, O. Vandermarcq, E. Vergnault, A. Vuillemin, and P. Zanatta. Transiting exoplanets from the CoRoT space mission. VIII. CoRoT-7b: the first super-Earth with measured radius. *A&A*, 506(1):287–302, Oct. 2009. doi: 10.1051/0004-6361/200911933.
- G. Li, S. Naoz, B. Kocsis, and A. Loeb. Eccentricity Growth and Orbit Flip in Near-coplanar Hierarchical Three-body Systems. *ApJ*, 785(2):116, Apr. 2014a. doi: 10.1088/0004-637X/785/2/116.
- G. Li, S. Naoz, F. Valsecchi, J. A. Johnson, and F. A. Rasio. The Dynamics of the Multi-planet System Orbiting Kepler-56. *ApJ*, 794:131, Oct. 2014b. doi: 10.1088/0004-637X/794/2/131.
- G. Li, F. Dai, and J. Becker. Mutual Inclination Excitation by Stellar Oblateness. *ApJ*, 890(2):L31, Feb. 2020. doi: 10.3847/2041-8213/ab72f4.
- J. Li and D. Lai. Resonant Excitation of Planetary Eccentricity due to a Dispersing Eccentric Protoplanetary Disk: A New Mechanism of Generating Large Planetary Eccentricities. *ApJ*, 956(1):17, Oct. 2023. doi: 10.3847/1538-4357/aced89.
- M. L. Lidov. The evolution of orbits of artificial satellites of planets under the action of

- gravitational perturbations of external bodies. *planss*, 9:719–759, Oct. 1962. doi: 10.1016/0032-0633(62)90129-0.
- J. J. Lissauer, D. Ragozzine, D. C. Fabrycky, J. H. Steffen, E. B. Ford, J. M. Jenkins, A. Shporer, M. J. Holman, J. F. Rowe, E. V. Quintana, N. M. Batalha, W. J. Borucki, S. T. Bryson, D. A. Caldwell, J. A. Carter, D. Ciardi, E. W. Dunham, J. J. Fortney, T. N. Gautier, III, S. B. Howell, D. G. Koch, D. W. Latham, G. W. Marcy, R. C. Morehead, and D. Sasselov. Architecture and Dynamics of Kepler’s Candidate Multiple Transiting Planet Systems. *ApJS*, 197:8, Nov. 2011. doi: 10.1088/0067-0049/197/1/8.
- Y. Lithwick and Y. Wu. Theory of Secular Chaos and Mercury’s Orbit. *ApJ*, 739(1):31, Sept. 2011. doi: 10.1088/0004-637X/739/1/31.
- B. Liu, D. J. Muñoz, and D. Lai. Suppression of extreme orbital evolution in triple systems with short-range forces. *MNRAS*, 447:747–764, Feb. 2015. doi: 10.1093/mnras/stu2396.
- J. H. Livingston, M. Endl, F. Dai, W. D. Cochran, O. Barragan, D. Gandolfi, T. Hirano, S. Grziwa, A. M. S. Smith, S. Albrecht, J. Cabrera, S. Csizmadia, J. P. de Leon, H. Deeg, P. Eigmüller, A. Erikson, M. Everett, M. Fridlund, A. Fukui, E. W. Guenther, A. P. Hatzes, S. Howell, J. Korth, N. Narita, D. Nespral, G. Nowak, E. Palles, M. Pätzold, C. M. Persson, J. Prieto-Arranz, H. Rauer, M. Tamura, V. Van Eylen, and J. N. Winn. 44 Validated Planets from K2 Campaign 10. *AJ*, 156(2):78, Aug. 2018. doi: 10.3847/1538-3881/aaccde.
- C. Lovis and D. Fischer. Radial Velocity Techniques for Exoplanets. In S. Seager, editor, *Exoplanets*, pages 27–53. 2010.
- R. O. P. Loyd, E. L. Shkolnik, A. C. Schneider, T. Richey-Yowell, T. S. Barman, S. Peacock, and I. Pagano. Current Population Statistics Do Not Favor Photoevaporation over Core-powered Mass Loss as the Dominant Cause of the Exoplanet Radius Gap. *ApJ*, 890(1):23, Feb. 2020a. doi: 10.3847/1538-4357/ab6605.
- R. O. P. Loyd, E. L. Shkolnik, A. C. Schneider, T. Richey-Yowell, T. S. Barman, S. Peacock, and I. Pagano. Current Population Statistics Do Not Favor Photoevaporation over Core-

- powered Mass Loss as the Dominant Cause of the Exoplanet Radius Gap. *ApJ*, 890(1): 23, Feb. 2020b. doi: 10.3847/1538-4357/ab6605.
- R. O. P. Loyd, E. L. Shkolnik, A. C. Schneider, T. Richey-Yowell, T. S. Barman, S. Peacock, and I. Pagano. Current Population Statistics Do Not Favor Photoevaporation over Core-powered Mass Loss as the Dominant Cause of the Exoplanet Radius Gap. *ApJ*, 890(1): 23, Feb. 2020c. doi: 10.3847/1538-4357/ab6605.
- T. Lu, Q. An, G. Li, S. C. Millholland, G. M. Brandt, and T. D. Brandt. Planet-Planet Scattering and ZLK Migration – The Dynamical History of HAT-P-11. *arXiv e-prints*, art. arXiv:2405.19511, May 2024. doi: 10.48550/arXiv.2405.19511.
- M. G. MacDonald, R. I. Dawson, S. J. Morrison, E. J. Lee, and A. Khandelwal. Forming Diverse Super-Earth Systems In Situ. *ApJ*, 891(1):20, Mar. 2020. doi: 10.3847/1538-4357/ab6f04.
- L. Malavolta, A. W. Mayo, T. Loudén, V. M. Rajpaul, A. S. Bonomo, L. A. Buchhave, L. Kreidberg, M. H. Kristiansen, M. Lopez-Morales, A. Mortier, A. Vand erburg, A. Coffinet, D. Ehrenreich, C. Lovis, F. Bouchy, D. Charbonneau, D. R. Ciardi, A. Collier Cameron, R. Cosentino, I. J. M. Crossfield, M. Damasso, C. D. Dressing, X. Dumusque, M. E. Everett, P. Figueira, A. F. M. Fiorenzano, E. J. Gonzales, R. D. Haywood, A. Harutyunyan, L. Hirsch, S. B. Howell, J. A. Johnson, D. W. Latham, E. Lopez, M. Mayor, G. Micela, E. Molinari, V. Nascimbeni, F. Pepe, D. F. Phillips, G. Piotto, K. Rice, D. Sasselov, D. Ségransan, A. Sozzetti, S. Udry, and C. Watson. An Ultra-short Period Rocky Super-Earth with a Secondary Eclipse and a Neptune-like Companion around K2-141. *AJ*, 155(3):107, Mar. 2018a. doi: 10.3847/1538-3881/aaa5b5.
- L. Malavolta, A. W. Mayo, T. Loudén, V. M. Rajpaul, A. S. Bonomo, L. A. Buchhave, L. Kreidberg, M. H. Kristiansen, M. Lopez-Morales, A. Mortier, A. Vand erburg, A. Coffinet, D. Ehrenreich, C. Lovis, F. Bouchy, D. Charbonneau, D. R. Ciardi, A. Collier Cameron, R. Cosentino, I. J. M. Crossfield, M. Damasso, C. D. Dressing, X. Dumusque,

- M. E. Everett, P. Figueira, A. F. M. Fiorenzano, E. J. Gonzales, R. D. Haywood, A. Harutyunyan, L. Hirsch, S. B. Howell, J. A. Johnson, D. W. Latham, E. Lopez, M. Mayor, G. Micela, E. Molinari, V. Nascimbeni, F. Pepe, D. F. Phillips, G. Piotto, K. Rice, D. Sasselov, D. Ségransan, A. Sozzetti, S. Udry, and C. Watson. An Ultra-short Period Rocky Super-Earth with a Secondary Eclipse and a Neptune-like Companion around K2-141. *AJ*, 155(3):107, Mar. 2018b. doi: 10.3847/1538-3881/aaa5b5.
- R. Malhotra. The Mass Distribution Function of Planets. *ApJ*, 808(1):71, July 2015. doi: 10.1088/0004-637X/808/1/71.
- R. A. Mardling and D. N. C. Lin. On the Survival of Short-Period Terrestrial Planets. *ApJ*, 614(2):955–959, Oct. 2004. doi: 10.1086/423794.
- D. V. Martin and A. H. M. J. Triaud. Circumbinary planets - why they are so likely to transit. *MNRAS*, 449(1):781–793, May 2015. doi: 10.1093/mnras/stv121.
- F. Marzari and M. Nagasawa. Secular evolution of close-in planets: the effects of general relativity. *MNRAS*, 493(1):427–436, Mar. 2020. doi: 10.1093/mnras/staa271.
- K. Masuda. Very Low Density Planets around Kepler-51 Revealed with Transit Timing Variations and an Anomaly Similar to a Planet-Planet Eclipse Event. *ApJ*, 783(1):53, Mar. 2014. doi: 10.1088/0004-637X/783/1/53.
- S. P. Matt, A. S. Brun, I. Baraffe, J. Bouvier, and G. Chabrier. The Mass-dependence of Angular Momentum Evolution in Sun-like Stars. *ApJ*, 799(2):L23, Jan. 2015. doi: 10.1088/2041-8205/799/2/L23.
- A. W. Mayo, A. Vanderburg, D. W. Latham, A. Bieryla, T. D. Morton, L. A. Buchhave, C. D. Dressing, C. Beichman, P. Berlind, M. L. Calkins, D. R. Ciardi, I. J. M. Crossfield, G. A. Esquerdo, M. E. Everett, E. J. Gonzales, L. A. Hirsch, E. P. Horch, A. W. Howard, S. B. Howell, J. Livingston, R. Patel, E. A. Petigura, J. E. Schlieder, N. J. Scott, C. F. Schumer, E. Sinukoff, J. Teske, and J. G. Winters. 275 Candidates and 149 Validated

- Planets Orbiting Bright Stars in K2 Campaigns 0-10. *AJ*, 155(3):136, Mar. 2018. doi: 10.3847/1538-3881/aaadff.
- M. Mayor and D. Queloz. A Jupiter-mass companion to a solar-type star. *Nature*, 378(6555): 355–359, Nov. 1995. doi: 10.1038/378355a0.
- S. Meibom, S. A. Barnes, I. Platais, R. L. Gilliland, D. W. Latham, and R. D. Mathieu. A spin-down clock for cool stars from observations of a 2.5-billion-year-old cluster. *Nature*, 517(7536):589–591, Jan. 2015. doi: 10.1038/nature14118.
- K. Ment, D. A. Fischer, G. Bakos, A. W. Howard, and H. Isaacson. Radial Velocities from the N2K Project: Six New Cold Gas Giant Planets Orbiting HD 55696, HD 98736, HD 148164, HD 203473, and HD 211810. *AJ*, 156(5):213, Nov. 2018. doi: 10.3847/1538-3881/aaelf5.
- L. Mestel. Magnetic braking by a stellar wind-I. *MNRAS*, 138:359, Jan. 1968. doi: 10.1093/mnras/138.3.359.
- T. A. Michtchenko and R. Malhotra. Secular dynamics of the three-body problem: application to the ν Andromedae planetary system. *Icarus*, 168(2):237–248, Apr. 2004. doi: 10.1016/j.icarus.2003.12.010.
- S. Millholland and G. Laughlin. Obliquity-driven sculpting of exoplanetary systems. *Nature Astronomy*, 3:424–433, Mar. 2019. doi: 10.1038/s41550-019-0701-7.
- S. Millholland, S. Wang, and G. Laughlin. Kepler Multi-planet Systems Exhibit Unexpected Intra-system Uniformity in Mass and Radius. *ApJ*, 849(2):L33, Nov. 2017. doi: 10.3847/2041-8213/aa9714.
- S. C. Millholland and C. Spalding. Formation of Ultra-short-period Planets by Obliquity-driven Tidal Runaway. *ApJ*, 905(1):71, Dec. 2020a. doi: 10.3847/1538-4357/abc4e5.
- S. C. Millholland and C. Spalding. Formation of Ultra-short-period Planets by Obliquity-driven Tidal Runaway. *ApJ*, 905(1):71, Dec. 2020b. doi: 10.3847/1538-4357/abc4e5.

- S. C. Millholland, M. Y. He, E. B. Ford, D. Ragozzine, D. Fabrycky, and J. N. Winn. Evidence for a Nondichotomous Solution to the Kepler Dichotomy: Mutual Inclinations of Kepler Planetary Systems from Transit Duration Variations. *AJ*, 162(4):166, Oct. 2021. doi: 10.3847/1538-3881/ac0f7a.
- S. M. Mills, A. W. Howard, L. M. Weiss, J. H. Steffen, H. Isaacson, B. J. Fulton, E. A. Petigura, M. R. Kosiarek, L. A. Hirsch, and J. H. Boisvert. Long-period Giant Companions to Three Compact, Multiplanet Systems. *AJ*, 157(4):145, Apr. 2019. doi: 10.3847/1538-3881/ab0899.
- M. Moe and R. Di Stefano. A New Class of Nascent Eclipsing Binaries with Extreme Mass Ratios. *ApJ*, 801(2):113, Mar. 2015. doi: 10.1088/0004-637X/801/2/113.
- B. T. Montet, J. C. Yee, and M. T. Penny. Measuring the Galactic Distribution of Transiting Planets with WFIRST. *PASP*, 129(974):044401, Apr. 2017. doi: 10.1088/1538-3873/aa57fb.
- A. Morbidelli and A. Crida. The dynamics of Jupiter and Saturn in the gaseous protoplanetary disk. *Icarus*, 191(1):158–171, Nov. 2007. doi: 10.1016/j.icarus.2007.04.001.
- J. Moriarty and S. Ballard. The Kepler Dichotomy in Planetary Disks: Linking Kepler Observables to Simulations of Late-stage Planet Formation. *ApJ*, 832(1):34, Nov. 2016. doi: 10.3847/0004-637X/832/1/34.
- T. D. Morton, S. T. Bryson, J. L. Coughlin, J. F. Rowe, G. Ravichandran, E. A. Petigura, M. R. Haas, and N. M. Batalha. False Positive Probabilities for all Kepler Objects of Interest: 1284 Newly Validated Planets and 428 Likely False Positives. *ApJ*, 822:86, May 2016. doi: 10.3847/0004-637X/822/2/86.
- C. Moutou, M. Deleuil, T. Guillot, A. Baglin, P. Bordé, F. Bouchy, J. Cabrera, S. Csizmadia, and H. J. Deeg. CoRoT: Harvest of the exoplanet program. *Icarus*, 226(2):1625–1634, Nov. 2013. doi: 10.1016/j.icarus.2013.03.022.

- G. D. Mulders, I. Pascucci, D. Apai, A. Frasca, and J. Molenda-Żakowicz. A Super-solar Metallicity for Stars with Hot Rocky Exoplanets. *AJ*, 152(6):187, Dec. 2016. doi: 10.3847/0004-6256/152/6/187.
- S. Müller, J. Baron, R. Helled, F. Bouchy, and L. Parc. The mass-radius relation of exoplanets, revisited. *arXiv e-prints*, art. arXiv:2311.12593, Nov. 2023. doi: 10.48550/arXiv.2311.12593.
- L. Murchikova and S. Tremaine. Peas in a Pod? Radius Correlations in Kepler Multiplanet Systems. *AJ*, 160(4):160, Oct. 2020a. doi: 10.3847/1538-3881/abab9e.
- L. Murchikova and S. Tremaine. Peas in a Pod? Radius Correlations in Kepler Multiplanet Systems. *AJ*, 160(4):160, Oct. 2020b. doi: 10.3847/1538-3881/abab9e.
- C. D. Murray and S. F. Dermott. *Solar System Dynamics*. 2000.
- J. Mushkin and B. Katz. A simple random walk model explains the disruption process of hierarchical, Eccentric three-body systems. *MNRAS*, 498(1):665–673, Oct. 2020. doi: 10.1093/mnras/staa2492.
- M. Nagasawa, D. N. C. Lin, and S. Ida. Eccentricity Evolution of Extrasolar Multiple Planetary Systems Due to the Depletion of Nascent Protostellar Disks. *ApJ*, 586(2):1374–1393, Apr. 2003. doi: 10.1086/367884.
- S. Naoz. The Eccentric Kozai-Lidov Effect and Its Applications. *ARA&A*, 54:441–489, Sept. 2016. doi: 10.1146/annurev-astro-081915-023315.
- S. Naoz, W. M. Farr, Y. Lithwick, F. A. Rasio, and J. Teyssandier. Hot Jupiters from secular planet-planet interactions. *Nature*, 473:187–189, May 2011. doi: 10.1038/nature10076.
- S. Naoz, W. M. Farr, and F. A. Rasio. On the Formation of Hot Jupiters in Stellar Binaries. *ApJ*, 754:L36, Aug. 2012. doi: 10.1088/2041-8205/754/2/L36.

- S. Naoz, W. M. Farr, Y. Lithwick, F. A. Rasio, and J. Teyssandier. Secular dynamics in hierarchical three-body systems. *MNRAS*, 431:2155–2171, May 2013a. doi: 10.1093/mnras/stt302.
- S. Naoz, B. Kocsis, A. Loeb, and N. Yunes. Resonant Post-Newtonian Eccentricity Excitation in Hierarchical Three-body Systems. *ApJ*, 773:187, Aug. 2013b. doi: 10.1088/0004-637X/773/2/187.
- NASA Exoplanet Science Institute. Planetary systems table, 2025. URL <https://catcopy.ipac.caltech.edu/doi/doi.php?id=10.26133/NEA12>.
- E. R. Nesvold, S. Naoz, L. Vican, and W. M. Farr. Circumstellar Debris Disks: Diagnosing the Unseen Perturber. *ApJ*, 826:19, July 2016. doi: 10.3847/0004-637X/826/1/19.
- L. D. Nielsen, D. Gandolfi, D. J. Armstrong, J. S. Jenkins, M. Fridlund, N. C. Santos, F. Dai, V. Adibekyan, R. Luque, J. H. Steffen, M. Esposito, F. Meru, S. Sabotta, E. Bolmont, D. Kossakowski, J. F. Otegi, F. Murgas, M. Stalport, F. Rodler, M. R. Díaz, N. T. Kurtovic, G. Ricker, R. Vanderspek, D. W. Latham, S. Seager, J. N. Winn, J. M. Jenkins, R. Allart, J. M. Almenara, D. Barrado, S. C. C. Barros, D. Bayliss, Z. M. Berdiñas, I. Boisse, F. Bouchy, P. Boyd, D. J. A. Brown, E. M. Bryant, C. Burke, W. D. Cochran, B. F. Cooke, O. D. S. Demangeon, R. F. Díaz, J. Dittman, C. Dorn, X. Dumusque, R. A. García, L. González-Cuesta, S. Grziwa, I. Georgieva, N. Guerrero, A. P. Hatzes, R. Helled, C. E. Henze, S. Hojjatpanah, J. Korth, K. W. F. Lam, J. Lillo-Box, T. A. Lopez, J. Livingston, S. Mathur, O. Mousis, N. Narita, H. P. Osborn, E. Palte, P. A. P. Rojas, C. M. Persson, S. N. Quinn, H. Rauer, S. Redfield, A. Santerne, L. A. dos Santos, J. V. Seidel, S. G. Sousa, E. B. Ting, M. Turbet, S. Udry, A. Vanderburg, V. Van Eylen, J. I. Vines, P. J. Wheatley, and P. A. Wilson. Mass determinations of the three mini-Neptunes transiting TOI-125. *MNRAS*, 492(4):5399–5412, Mar. 2020. doi: 10.1093/mnras/staa197.
- C. E. O’Connor, J. Teyssandier, and D. Lai. Secular chaos in white dwarf planetary systems: origins of metal pollution and short-period planetary companions. *MNRAS*, 513(3):4178–4195, July 2022. doi: 10.1093/mnras/stac1189.

- H. P. Osborn, A. Bonfanti, D. Gandolfi, C. Hedges, A. Leleu, A. Fortier, D. Futyan, P. Gutermann, P. F. L. Maxted, L. Borsato, K. A. Collins, J. Gomes da Silva, Y. Gómez Maqueo Chew, M. J. Hooton, M. Lendl, H. Parviainen, S. Salmon, N. Schanche, L. M. Serrano, S. G. Sousa, A. Tuson, S. Ulmer-Moll, V. Van Grootel, R. D. Wells, T. G. Wilson, Y. Alibert, R. Alonso, G. Anglada, J. Asquier, D. Barrado y Navascues, W. Baumjohann, T. Beck, W. Benz, F. Biondi, X. Bonfils, F. Bouchy, A. Brandeker, C. Broeg, T. Bárczy, S. C. C. Barros, J. Cabrera, S. Charnoz, A. Collier Cameron, S. Csizmadia, M. B. Davies, M. Deleuil, L. Delrez, B. O. Demory, D. Ehrenreich, A. Erikson, L. Fossati, M. Fridlund, M. Gillon, M. A. Gómez-Munoz, M. Güdel, K. Heng, S. Hoyer, K. G. Isaak, L. Kiss, J. Laskar, A. Lecavelier des Etangs, C. Lovis, D. Magrin, L. Malavolta, J. McCormac, V. Nascimbeni, G. Olofsson, R. Ottensamer, I. Pagano, E. Pallé, G. Peter, D. Piazza, G. Piotto, D. Pollacco, D. Queloz, R. Ragazzoni, N. Rando, H. Rauer, C. Reimers, I. Ribas, O. D. S. Demangeon, A. M. S. Smith, L. Sabin, N. Santos, G. Scandariato, U. Schroffenegger, R. P. Schwarz, A. Shporer, A. E. Simon, M. Steller, G. M. Szabó, D. Ségransan, N. Thomas, S. Udry, I. Walter, and N. Walton. Uncovering the true periods of the young sub-Neptunes orbiting TOI-2076. *A&A*, 664:A156, Aug. 2022. doi: 10.1051/0004-6361/202243065.
- O. J. Otor, B. T. Montet, J. A. Johnson, D. Charbonneau, A. Collier-Cameron, A. W. Howard, H. Isaacson, D. W. Latham, M. Lopez-Morales, C. Lovis, M. Mayor, G. Micela, E. Molinari, F. Pepe, G. Piotto, D. F. Phillips, D. Queloz, K. Rice, D. Sasselov, D. Ségransan, A. Sozzetti, S. Udry, and C. Watson. The Orbit and Mass of the Third Planet in the Kepler-56 System. *AJ*, 152(6):165, Dec. 2016. doi: 10.3847/0004-6256/152/6/165.
- J. E. Owen and D. Lai. Photoevaporation and high-eccentricity migration created the sub-Jovian desert. *MNRAS*, 479(4):5012–5021, Oct. 2018. doi: 10.1093/mnras/sty1760.
- J. E. Owen and Y. Wu. Kepler Planets: A Tale of Evaporation. *ApJ*, 775(2):105, Oct. 2013. doi: 10.1088/0004-637X/775/2/105.

- J. E. Owen and Y. Wu. Atmospheres of Low-mass Planets: The “Boil-off”. *ApJ*, 817(2):107, Feb. 2016. doi: 10.3847/0004-637X/817/2/107.
- J. E. Owen and Y. Wu. The Evaporation Valley in the Kepler Planets. *ApJ*, 847(1):29, Sept. 2017. doi: 10.3847/1538-4357/aa890a.
- E. N. Parker. Interaction of the Solar Wind with the Geomagnetic Field. *Physics of Fluids*, 1:171–187, May 1958. doi: 10.1063/1.1724339.
- B. Paxton, L. Bildsten, A. Dotter, F. Herwig, P. Lesaffre, and F. Timmes. Modules for Experiments in Stellar Astrophysics (MESA). *ApJS*, 192:3, jan 2011. doi: 10.1088/0067-0049/192/1/3.
- B. Paxton, M. Cantiello, P. Arras, L. Bildsten, E. F. Brown, A. Dotter, C. Mankovich, M. H. Montgomery, D. Stello, F. X. Timmes, and R. Townsend. Modules for Experiments in Stellar Astrophysics (MESA): Planets, Oscillations, Rotation, and Massive Stars. *ApJS*, 208:4, sep 2013. doi: 10.1088/0067-0049/208/1/4.
- B. Paxton, P. Marchant, J. Schwab, E. B. Bauer, L. Bildsten, M. Cantiello, L. Dessart, R. Farmer, H. Hu, N. Langer, R. H. D. Townsend, D. M. Townsley, and F. X. Timmes. Modules for Experiments in Stellar Astrophysics (MESA): Binaries, Pulsations, and Explosions. *ApJS*, 220:15, sep 2015. doi: 10.1088/0067-0049/220/1/15.
- B. Paxton, J. Schwab, E. B. Bauer, L. Bildsten, S. Blinnikov, P. Duffell, R. Farmer, J. A. Goldberg, P. Marchant, E. Sorokina, A. Thoul, R. H. D. Townsend, and F. X. Timmes. Modules for Experiments in Stellar Astrophysics (MESA): Convective Boundaries, Element Diffusion, and Massive Star Explosions. *ApJS*, 234:34, feb 2018. doi: 10.3847/1538-4365/aaa5a8.
- B. Paxton, R. Smolec, J. Schwab, A. Gaudy, L. Bildsten, M. Cantiello, A. Dotter, R. Farmer, J. A. Goldberg, A. S. Jermyn, S. M. Kanbur, P. Marchant, A. Thoul, R. H. D. Townsend, W. M. Wolf, M. Zhang, and F. X. Timmes. Modules for Experiments in Stel-

- lar Astrophysics (MESA): Pulsating Variable Stars, Rotation, Convective Boundaries, and Energy Conservation. *ApJS*, 243(1):10, Jul 2019. doi: 10.3847/1538-4365/ab2241.
- K. Penev, B. Jackson, F. Spada, and N. Thom. Constraining Tidal Dissipation in Stars from the Destruction Rates of Exoplanets. *ApJ*, 751(2):96, June 2012. doi: 10.1088/0004-637X/751/2/96.
- F. Pepe, A. C. Cameron, D. W. Latham, E. Molinari, S. Udry, A. S. Bonomo, L. A. Buchhave, D. Charbonneau, R. Cosentino, C. D. Dressing, X. Dumusque, P. Figueira, A. F. M. Fiorenzano, S. Gettel, A. Harutyunyan, R. D. Haywood, K. Horne, M. Lopez-Morales, C. Lovis, L. Malavolta, M. Mayor, G. Micela, F. Motalebi, V. Nascimbeni, D. Phillips, G. Piotto, D. Pollacco, D. Queloz, K. Rice, D. Sasselov, D. Ségransan, A. Sozzetti, A. Szentgyorgyi, and C. A. Watson. An Earth-sized planet with an Earth-like density. *Nature*, 503(7476):377–380, Nov. 2013. doi: 10.1038/nature12768.
- E. A. Petigura. Two Views of the Radius Gap and the Role of Light Curve Fitting. *AJ*, 160(2):89, Aug. 2020a. doi: 10.3847/1538-3881/ab9fff.
- E. A. Petigura. Two Views of the Radius Gap and the Role of Light Curve Fitting. *AJ*, 160(2):89, Aug. 2020b. doi: 10.3847/1538-3881/ab9fff.
- E. A. Petigura. Two Views of the Radius Gap and the Role of Light Curve Fitting. *AJ*, 160(2):89, Aug. 2020c. doi: 10.3847/1538-3881/ab9fff.
- A. C. Petit, J. Laskar, and G. Boué. AMD-stability in the presence of first-order mean motion resonances. *A&A*, 607:A35, Nov. 2017. doi: 10.1051/0004-6361/201731196.
- C. Petrovich. Steady-state Planet Migration by the Kozai-Lidov Mechanism in Stellar Binaries. *ApJ*, 799(1):27, Jan. 2015a. doi: 10.1088/0004-637X/799/1/27.
- C. Petrovich. Hot Jupiters from Coplanar High-eccentricity Migration. *ApJ*, 805(1):75, May 2015b. doi: 10.1088/0004-637X/805/1/75.

- C. Petrovich, E. Deibert, and Y. Wu. Ultra-short-period Planets from Secular Chaos. *AJ*, 157(5):180, May 2019a. doi: 10.3847/1538-3881/ab0e0a.
- C. Petrovich, E. Deibert, and Y. Wu. Ultra-short-period Planets from Secular Chaos. *AJ*, 157(5):180, May 2019b. doi: 10.3847/1538-3881/ab0e0a.
- C. Petrovich, D. J. Muñoz, K. M. Kratter, and R. Malhotra. A Disk-driven Resonance as the Origin of High Inclinations of Close-in Planets. *ApJ*, 902(1):L5, Oct. 2020. doi: 10.3847/2041-8213/abb952.
- S. Pfalzner and F. Dincer. Low-mass Stars: Their Protoplanetary Disk Lifetime Distribution. *ApJ*, 963(2):122, Mar. 2024. doi: 10.3847/1538-4357/ad1bef.
- P. Plavchan, X. Chen, and G. Pohl. What is the Mass of α Cen B b? *ApJ*, 805(2):174, June 2015. doi: 10.1088/0004-637X/805/2/174.
- B. Pu and D. Lai. Eccentricities and inclinations of multiplanet systems with external perturbers. *MNRAS*, 478(1):197–217, July 2018. doi: 10.1093/mnras/sty1098.
- B. Pu and D. Lai. Low-eccentricity migration of ultra-short-period planets in multiplanet systems. *MNRAS*, 488(3):3568–3587, Sept. 2019a. doi: 10.1093/mnras/stz1817.
- B. Pu and D. Lai. Low-eccentricity migration of ultra-short-period planets in multiplanet systems. *MNRAS*, 488(3):3568–3587, Sept. 2019b. doi: 10.1093/mnras/stz1817.
- B. Pu and Y. Wu. Spacing of Kepler Planets: Sculpting by Dynamical Instability. *ApJ*, 807: 44, July 2015. doi: 10.1088/0004-637X/807/1/44.
- A. Qureshi, S. Naoz, and E. L. Shkolnik. Signature of Planetary Mergers on Stellar Spins. *ApJ*, 864(1):65, Sept. 2018. doi: 10.3847/1538-4357/aad562.
- D. Raghavan, H. A. McAlister, T. J. Henry, D. W. Latham, G. W. Marcy, B. D. Mason, D. R. Gies, R. J. White, and T. A. ten Brummelaar. A Survey of Stellar Families: Multiplicity of Solar-type Stars. *ApJS*, 190:1–42, Sept. 2010. doi: 10.1088/0067-0049/190/1/1.

- S. N. Raymond and A. Izidoro. Origin of water in the inner Solar System: Planetesimals scattered inward during Jupiter and Saturn’s rapid gas accretion. *Icarus*, 297:134–148, Nov. 2017. doi: 10.1016/j.icarus.2017.06.030.
- H. Rein and S. F. Liu. REBOUND: an open-source multi-purpose N-body code for collisional dynamics. *A&A*, 537:A128, Jan. 2012. doi: 10.1051/0004-6361/201118085.
- H. Rein and D. Tamayo. WHFAST: a fast and unbiased implementation of a symplectic Wisdom-Holman integrator for long-term gravitational simulations. *MNRAS*, 452(1):376–388, Sept. 2015. doi: 10.1093/mnras/stv1257.
- K. Rice. Can Kozai-Lidov cycles explain Kepler-78b? *MNRAS*, 448(2):1729–1737, Apr. 2015. doi: 10.1093/mnras/stv073.
- M. Rice, X.-Y. Wang, S. Wang, A. Shporer, K. Barkaoui, R. Brahm, K. A. Collins, A. Jordán, N. Lowson, R. P. Butler, J. D. Crane, S. Shectman, J. K. Teske, D. Osip, K. I. Collins, F. Murgas, G. Boyle, F. J. Pozuelos, M. Timmermans, E. Jehin, and M. Gillon. Evidence for Low-level Dynamical Excitation in Near-resonant Exoplanet Systems. *AJ*, 166(6):266, Dec. 2023. doi: 10.3847/1538-3881/ad09de.
- G. R. Ricker, J. N. Winn, R. Vanderspek, D. W. Latham, G. Á. Bakos, J. L. Bean, Z. K. Berta-Thompson, T. M. Brown, L. Buchhave, N. R. Butler, R. P. Butler, W. J. Chaplin, D. Charbonneau, J. Christensen-Dalsgaard, M. Clampin, D. Deming, J. Doty, N. De Lee, C. Dressing, E. W. Dunham, M. Endl, F. Fressin, J. Ge, T. Henning, M. J. Holman, A. W. Howard, S. Ida, J. M. Jenkins, G. Jernigan, J. A. Johnson, L. Kaltenegger, N. Kawai, H. Kjeldsen, G. Laughlin, A. M. Levine, D. Lin, J. J. Lissauer, P. MacQueen, G. Marcy, P. R. McCullough, T. D. Morton, N. Narita, M. Paegert, E. Palle, F. Pepe, J. Pepper, A. Quirrenbach, S. A. Rinehart, D. Sasselov, B. Sato, S. Seager, A. Sozzetti, K. G. Stassun, P. Sullivan, A. Szentgyorgyi, G. Torres, S. Udry, and J. Villaseñor. Transiting Exoplanet Survey Satellite (TESS). *Journal of Astronomical Telescopes, Instruments, and Systems*, 1:014003, Jan. 2015. doi: 10.1117/1.JATIS.1.1.014003.

- L. Rodet and D. Lai. Inclination dynamics of resonant planets under the influence of an inclined external companion. *MNRAS*, 502(3):3746–3760, Apr. 2021. doi: 10.1093/mnras/stab094.
- J. E. Rodriguez, G. Zhou, A. Vanderburg, J. D. Eastman, L. Kreidberg, P. A. Cargile, A. Bieryla, D. W. Latham, J. Irwin, A. W. Mayo, M. L. Calkins, G. A. Esquerdo, and J. Mink. A Multi-planet System Transiting the $V = 9$ Rapidly Rotating F-Star HD 106315. *AJ*, 153(6):256, June 2017. doi: 10.3847/1538-3881/aa6dfb.
- J. G. Rogers, A. Gupta, J. E. Owen, and H. E. Schlichting. Photoevaporation versus core-powered mass-loss: model comparison with the 3D radius gap. *MNRAS*, 508(4):5886–5902, Dec. 2021. doi: 10.1093/mnras/stab2897.
- C. E. Romero-Mirza and E. M. R. Kempton. No Metallicity Correlation Associated with the Kepler Dichotomy. *AJ*, 155(3):134, Mar. 2018. doi: 10.3847/1538-3881/aaab5e.
- J. F. Rowe, S. T. Bryson, G. W. Marcy, J. J. Lissauer, D. Jontof-Hutter, F. Mullally, R. L. Gilliland, H. Issacson, E. Ford, S. B. Howell, W. J. Borucki, M. Haas, D. Huber, J. H. Steffen, S. E. Thompson, E. Quintana, T. Barclay, M. Still, J. Fortney, T. N. Gautier, R. Hunter, D. A. Caldwell, D. R. Ciardi, E. Devore, W. Cochran, J. Jenkins, E. Agol, J. A. Carter, and J. Geary. VALIDATION OF KEPLER'S MULTIPLE PLANET CANDIDATES. III. LIGHT CURVE ANALYSIS AND ANNOUNCEMENT OF HUNDREDS OF NEW MULTI-PLANET SYSTEMS. *The Astrophysical Journal*, 784(1):45, mar 2014. doi: 10.1088/0004-637x/784/1/45. URL <https://doi.org/10.1088%2F0004-637x%2F784%2F1%2F45>.
- R. Sanchis-Ojeda, D. C. Fabrycky, J. N. Winn, T. Barclay, B. D. Clarke, E. B. Ford, J. J. Fortney, J. C. Geary, M. J. Holman, A. W. Howard, J. M. Jenkins, D. Koch, J. J. Lissauer, G. W. Marcy, F. Mullally, D. Ragozzine, S. E. Seader, M. Still, and S. E. Thompson. Alignment of the stellar spin with the orbits of a three-planet system. *Nature*, 487(7408):449–453, July 2012. doi: 10.1038/nature11301.

- R. Sanchis-Ojeda, S. Rappaport, J. N. Winn, A. Levine, M. C. Kotson, D. W. Latham, and L. A. Buchhave. Transits and Occultations of an Earth-sized Planet in an 8.5 hr Orbit. *ApJ*, 774(1):54, Sept. 2013. doi: 10.1088/0004-637X/774/1/54.
- R. Sanchis-Ojeda, S. Rappaport, J. N. Winn, M. C. Kotson, A. Levine, and I. El Mellah. A Study of the Shortest-period Planets Found with Kepler. *ApJ*, 787(1):47, May 2014a. doi: 10.1088/0004-637X/787/1/47.
- R. Sanchis-Ojeda, S. Rappaport, J. N. Winn, M. C. Kotson, A. Levine, and I. El Mellah. A Study of the Shortest-period Planets Found with Kepler. *ApJ*, 787(1):47, May 2014b. doi: 10.1088/0004-637X/787/1/47.
- E. Sandford, D. Kipping, and M. Collins. The multiplicity distribution of Kepler’s exoplanets. *MNRAS*, 489(3):3162–3173, Nov. 2019. doi: 10.1093/mnras/stz2350.
- E. Schatzman. A theory of the role of magnetic activity during star formation. *Annales d’Astrophysique*, 25:18, Feb. 1962.
- J. R. Schmitt, E. Agol, K. M. Deck, L. A. Rogers, J. Z. Gazak, D. A. Fischer, J. Wang, M. J. Holman, K. J. Jek, C. Margossian, M. R. Omohundro, T. Winarski, J. M. Brewer, M. J. Giguere, C. Lintott, S. Lynn, M. Parrish, K. Schawinski, M. E. Schwamb, R. Simpson, and A. M. Smith. Planet Hunters. VII. Discovery of a New Low-mass, Low-density Planet (PH3 C) Orbiting Kepler-289 with Mass Measurements of Two Additional Planets (PH3 B and D). *ApJ*, 795(2):167, Nov. 2014. doi: 10.1088/0004-637X/795/2/167.
- K. Schultz, C. Spalding, and K. Batygin. The distribution of mutual inclinations arising from the stellar quadrupole moment. *MNRAS*, 506(2):2999–3009, Sept. 2021. doi: 10.1093/mnras/stab1899.
- C. J. Shallue and A. Vanderburg. Identifying Exoplanets with Deep Learning: A Five-planet Resonant Chain around Kepler-80 and an Eighth Planet around Kepler-90. *AJ*, 155(2): 94, Feb. 2018. doi: 10.3847/1538-3881/aa9e09.

- S. M. Silverberg, J. P. Wisniewski, M. J. Kuchner, K. D. Lawson, A. S. Bans, J. H. Debes, J. R. Biggs, M. K. D. Bosch, K. Doll, H. A. D. Luca, A. Enachioaie, J. Hamilton, J. Holden, and M. Hyogo. Peter Pan Disks: Long-lived Accretion Disks Around Young M Stars. *ApJ*, 890(2):106, Feb. 2020. doi: 10.3847/1538-4357/ab68e6.
- E. Sinukoff, A. W. Howard, E. A. Petigura, J. E. Schlieder, I. J. M. Crossfield, D. R. Ciardi, B. J. Fulton, H. Isaacson, K. M. Aller, C. Baranec, C. A. Beichman, B. M. S. Hansen, H. A. Knutson, N. M. Law, M. C. Liu, R. Riddle, and C. D. Dressing. Eleven Multiplanet Systems from K2 Campaigns 1 and 2 and the Masses of Two Hot Super-Earths. *ApJ*, 827(1):78, Aug. 2016. doi: 10.3847/0004-637X/827/1/78.
- A. Skumanich. Time Scales for Ca II Emission Decay, Rotational Braking, and Lithium Depletion. *ApJ*, 171:565, Feb. 1972. doi: 10.1086/151310.
- C. Spalding and K. Batygin. Spin-Orbit Misalignment as a Driver of the Kepler Dichotomy. *ApJ*, 830(1):5, Oct. 2016. doi: 10.3847/0004-637X/830/1/5.
- C. Spalding and S. C. Millholland. Stellar Oblateness versus Distant Giants in Exciting Kepler Planet Mutual Inclinations. *AJ*, 160(3):105, Sept. 2020. doi: 10.3847/1538-3881/aba629.
- C. Spalding, N. W. Marx, and K. Batygin. The Resilience of Kepler Systems to Stellar Obliquity. *AJ*, 155(4):167, Apr. 2018. doi: 10.3847/1538-3881/aab43a.
- R. Spiewak, M. Bailes, E. D. Barr, N. D. R. Bhat, M. Burgay, A. D. Cameron, D. J. Champion, C. M. L. Flynn, A. Jameson, S. Johnston, M. J. Keith, M. Kramer, S. R. Kulkarni, L. Levin, A. G. Lyne, V. Morello, C. Ng, A. Possenti, V. Ravi, B. W. Stappers, W. van Straten, and C. Tiburzi. PSR J2322-2650 - a low-luminosity millisecond pulsar with a planetary-mass companion. *MNRAS*, 475(1):469–477, Mar. 2018. doi: 10.1093/mnras/stx3157.
- K. G. Stassun, R. J. Oelkers, M. Paegert, G. Torres, J. Pepper, N. De Lee, K. Collins, D. W. Latham, P. S. Muirhead, J. Chittidi, B. Rojas-Ayala, S. W. Fleming, M. E. Rose, P. Tenen-

- baum, E. B. Ting, S. R. Kane, T. Barclay, J. L. Bean, C. E. Brassuer, D. Charbonneau, J. Ge, J. J. Lissauer, A. W. Mann, B. McLean, S. Mullally, N. Narita, P. Plavchan, G. R. Ricker, D. Sasselov, S. Seager, S. Sharma, B. Shiao, A. Sozzetti, D. Stello, R. Vanderspek, G. Wallace, and J. N. Winn. The Revised TESS Input Catalog and Candidate Target List. *AJ*, 158(4):138, Oct. 2019. doi: 10.3847/1538-3881/ab3467.
- J. H. Steffen and J. A. Hwang. The period ratio distribution of Kepler’s candidate multiplanet systems. *MNRAS*, 448(2):1956–1972, Apr. 2015. doi: 10.1093/mnras/stv104.
- J. H. Steffen, D. C. Fabrycky, E. B. Ford, J. A. Carter, J.-M. Désert, F. Fressin, M. J. Holman, J. J. Lissauer, A. V. Moorhead, J. F. Rowe, D. Ragozzine, W. F. Welsh, N. M. Batalha, W. J. Borucki, L. A. Buchhave, S. Bryson, D. A. Caldwell, D. Charbonneau, D. R. Ciardi, W. D. Cochran, M. Endl, M. E. Everett, T. N. Gautier, R. L. Gilliland, F. R. Girouard, J. M. Jenkins, E. Horch, S. B. Howell, H. Isaacson, T. C. Klaus, D. G. Koch, D. W. Latham, J. Li, P. Lucas, P. J. MacQueen, G. W. Marcy, S. McCauliff, C. K. Middelour, R. L. Morris, F. R. Mullally, S. N. Quinn, E. V. Quintana, A. Shporer, M. Still, P. Tenenbaum, S. E. Thompson, J. D. Twicken, and J. Van Cleve. Transit timing observations from Kepler - III. Confirmation of four multiple planet systems by a Fourier-domain study of anticorrelated transit timing variations. *MNRAS*, 421(3):2342–2354, Apr. 2012. doi: 10.1111/j.1365-2966.2012.20467.x.
- A. P. Stephan and B. S. Gaudi. Exoplanet Nodal Precession Induced by Rapidly Rotating Stars: Impacts on Transit Probabilities and Biases. *ApJ*, 950(1):32, June 2023. doi: 10.3847/1538-4357/accd6b.
- A. P. Stephan, S. Naoz, and B. S. Gaudi. A-type Stars, the Destroyers of Worlds: The Lives and Deaths of Jupiters in Evolving Stellar Binaries. *AJ*, 156:128, Sept. 2018. doi: 10.3847/1538-3881/aad6e5.
- A. P. Stephan, S. Naoz, B. S. Gaudi, and J. M. Salas. Eating Planets for Lunch and Dinner: Signatures of Planet Consumption by Evolving Stars. *ApJ*, 889(1):45, Jan. 2020. doi: 10.3847/1538-4357/ab5b00.

- T. E. Sterne. Apsidal motion in binary stars. *MNRAS*, 99:451–462, Mar. 1939. doi: 10.1093/mnras/99.5.451.
- S. Stock, S. Reffert, and A. Quirrenbach. Precise radial velocities of giant stars. X. Bayesian stellar parameters and evolutionary stages for 372 giant stars from the Lick planet search. *A&A*, 616:A33, Aug. 2018. doi: 10.1051/0004-6361/201833111.
- G. Takeda and F. A. Rasio. High Orbital Eccentricities of Extrasolar Planets Induced by the Kozai Mechanism. *ApJ*, 627:1001–1010, July 2005. doi: 10.1086/430467.
- G. Takeda, R. Kita, and F. A. Rasio. Planetary Systems in Binaries. I. Dynamical Classification. *ApJ*, 683:1063–1075, Aug. 2008. doi: 10.1086/589852.
- D. Tamayo, H. Rein, P. Shi, and D. M. Hernandez. REBOUNDx: a library for adding conservative and dissipative forces to otherwise symplectic N-body integrations. *MNRAS*, 491(2):2885–2901, Jan. 2020a. doi: 10.1093/mnras/stz2870.
- D. Tamayo, H. Rein, P. Shi, and D. M. Hernandez. REBOUNDx: a library for adding conservative and dissipative forces to otherwise symplectic N-body integrations. *MNRAS*, 491(2):2885–2901, Jan. 2020b. doi: 10.1093/mnras/stz2870.
- J. Teyssandier, S. Naoz, I. Lizarraga, and F. A. Rasio. Extreme Orbital Evolution from Hierarchical Secular Coupling of Two Giant Planets. *ApJ*, 779:166, Dec. 2013. doi: 10.1088/0004-637X/779/2/166.
- S. E. Thompson, J. L. Coughlin, K. Hoffman, F. Mullally, J. L. Christiansen, C. J. Burke, S. Bryson, N. Batalha, M. R. Haas, J. Catanzarite, J. F. Rowe, G. Barentsen, D. A. Caldwell, B. D. Clarke, J. M. Jenkins, J. Li, D. W. Latham, J. J. Lissauer, S. Mathur, R. L. Morris, S. E. Seader, J. C. Smith, T. C. Klaus, J. D. Twicken, J. E. Van Cleve, B. Wöhler, R. Akeson, D. R. Ciardi, W. D. Cochran, C. E. Henze, S. B. Howell, D. Huber, A. Prša, S. V. Ramírez, T. D. Morton, T. Barclay, J. R. Campbell, W. J. Chaplin, D. Charbonneau, J. Christensen-Dalsgaard, J. L. Dotson, L. Doyle, E. W. Dunham, A. K.

- Dupree, E. B. Ford, J. C. Geary, F. R. Girouard, H. Isaacson, H. Kjeldsen, E. V. Quintana, D. Ragozzine, M. Shabram, A. Shporer, V. Silva Aguirre, J. H. Steffen, M. Still, P. Tenenbaum, W. F. Welsh, A. Wolfgang, K. A. Zamudio, D. G. Koch, and W. J. Borucki. Planetary Candidates Observed by Kepler. VIII. A Fully Automated Catalog with Measured Completeness and Reliability Based on Data Release 25. *ApJS*, 235(2):38, Apr. 2018a. doi: 10.3847/1538-4365/aab4f9.
- S. E. Thompson, J. L. Coughlin, K. Hoffman, F. Mullally, J. L. Christiansen, C. J. Burke, S. Bryson, N. Batalha, M. R. Haas, J. Catanzarite, J. F. Rowe, G. Barentsen, D. A. Caldwell, B. D. Clarke, J. M. Jenkins, J. Li, D. W. Latham, J. J. Lissauer, S. Mathur, R. L. Morris, S. E. Seader, J. C. Smith, T. C. Klaus, J. D. Twicken, J. E. Van Cleve, B. Wohler, R. Akeson, D. R. Ciardi, W. D. Cochran, C. E. Henze, S. B. Howell, D. Huber, A. Prša, S. V. Ramírez, T. D. Morton, T. Barclay, J. R. Campbell, W. J. Chaplin, D. Charbonneau, J. Christensen-Dalsgaard, J. L. Dotson, L. Doyle, E. W. Dunham, A. K. Dupree, E. B. Ford, J. C. Geary, F. R. Girouard, H. Isaacson, H. Kjeldsen, E. V. Quintana, D. Ragozzine, M. Shabram, A. Shporer, V. Silva Aguirre, J. H. Steffen, M. Still, P. Tenenbaum, W. F. Welsh, A. Wolfgang, K. A. Zamudio, D. G. Koch, and W. J. Borucki. Planetary Candidates Observed by Kepler. VIII. A Fully Automated Catalog with Measured Completeness and Reliability Based on Data Release 25. *ApJS*, 235(2):38, Apr. 2018b. doi: 10.3847/1538-4365/aab4f9.
- G. Torres, F. Fressin, N. M. Batalha, W. J. Borucki, T. M. Brown, S. T. Bryson, L. A. Buchhave, D. Charbonneau, D. R. Ciardi, E. W. Dunham, D. C. Fabrycky, E. B. Ford, I. Gautier, Thomas N., R. L. Gilliland, M. J. Holman, S. B. Howell, H. Isaacson, J. M. Jenkins, D. G. Koch, D. W. Latham, J. J. Lissauer, G. W. Marcy, D. G. Monet, A. Prsa, S. N. Quinn, D. Ragozzine, J. F. Rowe, D. D. Sasselov, J. H. Steffen, and W. F. Welsh. Modeling Kepler Transit Light Curves as False Positives: Rejection of Blend Scenarios for Kepler-9, and Validation of Kepler-9 d, A Super-earth-size Planet in a Multiple System. *ApJ*, 727(1):24, Jan. 2011. doi: 10.1088/0004-637X/727/1/24.

- J. R. Touma, S. Tremaine, and M. V. Kazandjian. Gauss’s method for secular dynamics, softened. *MNRAS*, 394:1085–1108, Apr. 2009. doi: 10.1111/j.1365-2966.2009.14409.x.
- A. Udalski, B. Paczynski, K. Zebrun, M. Szymanski, M. Kubiak, I. Soszynski, O. Szewczyk, L. Wyrzykowski, and G. Pietrzynski. The Optical Gravitational Lensing Experiment. Search for Planetary and Low-Luminosity Object Transits in the Galactic Disk. Results of 2001 Campaign. *Acta Astron.*, 52:1–37, Mar. 2002a. doi: 10.48550/arXiv.astro-ph/0202320.
- A. Udalski, K. Zebrun, M. Szymanski, M. Kubiak, I. Soszynski, O. Szewczyk, L. Wyrzykowski, and G. Pietrzynski. The Optical Gravitational Lensing Experiment. Search for Planetary and Low- Luminosity Object Transits in the Galactic Disk. Results of 2001 Campaign – Supplement. *Acta Astron.*, 52:115–128, June 2002b. doi: 10.48550/arXiv.astro-ph/0207133.
- H. Valizadegan, M. J. S. Martinho, L. S. Wilkens, J. M. Jenkins, J. C. Smith, D. A. Caldwell, J. D. Twicken, P. C. L. Gerum, N. Walia, K. Hausknecht, N. Y. Lubin, S. T. Bryson, and N. C. Oza. ExoMiner: A Highly Accurate and Explainable Deep Learning Classifier That Validates 301 New Exoplanets. *ApJ*, 926(2):120, Feb. 2022. doi: 10.3847/1538-4357/ac4399.
- H. Valizadegan, M. J. S. Martinho, J. M. Jenkins, D. A. Caldwell, J. D. Twicken, and S. T. Bryson. Multiplicity Boost of Transit Signal Classifiers: Validation of 69 New Exoplanets using the Multiplicity Boost of ExoMiner. *AJ*, 166(1):28, July 2023. doi: 10.3847/1538-3881/acd344.
- V. Van Eylen and S. Albrecht. Eccentricity from Transit Photometry: Small Planets in Kepler Multi-planet Systems Have Low Eccentricities. *ApJ*, 808(2):126, Aug. 2015. doi: 10.1088/0004-637X/808/2/126.
- V. Van Eylen, C. Agentoft, M. S. Lundkvist, H. Kjeldsen, J. E. Owen, B. J. Fulton, E. Pe-

- figura, and I. Snellen. An asteroseismic view of the radius valley: stripped cores, not born rocky. *MNRAS*, 479(4):4786–4795, Oct. 2018. doi: 10.1093/mnras/sty1783.
- V. Van Eylen, S. Albrecht, X. Huang, M. G. MacDonald, R. I. Dawson, M. X. Cai, D. Foreman-Mackey, M. S. Lundkvist, V. Silva Aguirre, I. Snellen, and J. N. Winn. The Orbital Eccentricity of Small Planet Systems. *AJ*, 157(2):61, Feb. 2019a. doi: 10.3847/1538-3881/aaf22f.
- V. Van Eylen, S. Albrecht, X. Huang, M. G. MacDonald, R. I. Dawson, M. X. Cai, D. Foreman-Mackey, M. S. Lundkvist, V. Silva Aguirre, I. Snellen, and J. N. Winn. The Orbital Eccentricity of Small Planet Systems. *AJ*, 157(2):61, Feb. 2019b. doi: 10.3847/1538-3881/aaf22f.
- C. Van Laerhoven and R. Greenberg. Characterizing multi-planet systems with classical secular theory. *Celestial Mechanics and Dynamical Astronomy*, 113(2):215–234, June 2012. doi: 10.1007/s10569-012-9410-6.
- D. Veras and E. B. Ford. Secular Orbital Dynamics of Hierarchical Two-planet Systems. *ApJ*, 715(2):803–822, June 2010. doi: 10.1088/0004-637X/715/2/803.
- K. Volk and R. Malhotra. Dynamical Instabilities in Systems of Multiple Short-period Planets Are Likely Driven by Secular Chaos: A Case Study of Kepler-102. *AJ*, 160(3):98, Sept. 2020. doi: 10.3847/1538-3881/aba0b0.
- K. J. Walsh, A. Morbidelli, S. N. Raymond, D. P. O’Brien, and A. M. Mandell. A low mass for Mars from Jupiter’s early gas-driven migration. *Nature*, 475(7355):206–209, July 2011. doi: 10.1038/nature10201.
- J. Wang, D. A. Fischer, T. Barclay, A. Picard, B. Ma, B. P. Bowler, J. R. Schmitt, T. S. Boyajian, K. J. Jek, D. LaCourse, C. Baranec, R. Riddle, N. M. Law, C. Lintott, K. Schawinski, D. J. Simister, B. Grégoire, S. P. Babin, T. Poile, T. L. Jacobs, T. Jebson, M. R. Omo-hundro, H. M. Schwengeler, J. Sejkpa, I. A. Terentev, R. Gagliano, J.-P. Paakkonen, H. K.

- Otnes Berge, T. Winarski, G. R. Green, A. R. Schmitt, M. H. Kristiansen, and A. Hoekstra. Planet Hunters. VIII. Characterization of 41 Long-period Exoplanet Candidates from Kepler Archival Data. *ApJ*, 815(2):127, Dec. 2015. doi: 10.1088/0004-637X/815/2/127.
- S. Wang, M. Jones, A. Shporer, B. J. Fulton, L. A. Paredes, T. Trifonov, D. Kossakowski, J. Eastman, M. N. Gunther, C. X. Huang, S. Millholland, D. Seligman, D. Fischer, R. Brahm, X.-Y. Wang, B. Cruz, H.-S. James, B. Addison, T. Henry, E.-S. Liang, A. B. Davis, R. Tronsgaard, K. Worku, J. Brewer, M. Kurster, C. A. Beichman, A. Bieryla, T. M. Brown, J. L. Christiansen, D. R. Ciardi, K. A. Collins, G. A. Esquerdo, A. W. Howard, H. Isaacson, D. W. Latham, T. Mazeh, E. A. Petigura, S. N. Quinn, S. Shahaf, R. J. Siverd, G. R. Ricker, R. Vanderspek, S. Seager, J. N. Winn, J. M. Jenkins, P. T. Boyd, G. Furesz, C. Henze, A. M. Levine, R. Morris, M. Paegert, K. G. Stassun, E. B. Ting, M. Vezie, and G. Laughlin. HD 202772A B: A Transiting Hot Jupiter Around A Bright, Mildly Evolved Star In A Visual Binary Discovered By Tess. *ArXiv e-prints*, Oct. 2018.
- W. R. Ward. Solar nebula dispersal and the stability of the planetary system I. Scanning secular resonance theory. *Icarus*, 47(2):234–264, Aug. 1981. doi: 10.1016/0019-1035(81)90169-X.
- W. R. Ward, G. Colombo, and F. A. Franklin. Secular Resonance, Solar Spin Down, and the Orbit of Mercury. *Icarus*, 28(4):441–452, Aug. 1976. doi: 10.1016/0019-1035(76)90117-2.
- E. J. Weber and J. Davis, Leverett. The Angular Momentum of the Solar Wind. *ApJ*, 148: 217–227, Apr. 1967. doi: 10.1086/149138.
- L. Wei, S. Naoz, T. Faridani, and W. M. Farr. Relativistic Dynamical Stability Criterion of Multiplanet Systems with a Distant Companion. *ApJ*, 923(1):118, Dec. 2021. doi: 10.3847/1538-4357/ac2c70.
- L. M. Weiss and E. A. Petigura. The kepler peas in a pod pattern is astrophysical. *The*

- Astrophysical Journal*, 893(1):L1, apr 2020. doi: 10.3847/2041-8213/ab7c69. URL <https://doi.org/10.3847/2041-8213/ab7c69>.
- L. M. Weiss, H. T. Isaacson, G. W. Marcy, A. W. Howard, E. A. Petigura, B. J. Fulton, J. N. Winn, L. Hirsch, E. Sinukoff, J. F. Rowe, and T. California Kepler Survey. The California-Kepler Survey. VI. Kepler Multis and Singles Have Similar Planet and Stellar Properties Indicating a Common Origin. *AJ*, 156(6):254, Dec. 2018a. doi: 10.3847/1538-3881/aae70a.
- L. M. Weiss, G. W. Marcy, E. A. Petigura, B. J. Fulton, A. W. Howard, J. N. Winn, H. T. Isaacson, T. D. Morton, L. A. Hirsch, E. J. Sinukoff, A. Cumming, L. Hebb, and P. A. Cargile. The California-Kepler Survey. V. Peas in a Pod: Planets in a Kepler Multi-planet System Are Similar in Size and Regularly Spaced. *AJ*, 155(1):48, Jan. 2018b. doi: 10.3847/1538-3881/aa9ff6.
- L. M. Weiss, S. C. Millholland, E. A. Petigura, F. C. Adams, K. Batygin, A. M. Block, and C. Mordasini. Architectures of Compact Multi-Planet Systems: Diversity and Uniformity. In S. Inutsuka, Y. Aikawa, T. Muto, K. Tomida, and M. Tamura, editors, *Protostars and Planets VII*, volume 534 of *Astronomical Society of the Pacific Conference Series*, page 863, July 2023.
- L. M. Weiss, H. Isaacson, A. W. Howard, B. J. Fulton, E. A. Petigura, D. Fabrycky, D. Jontof-Hutter, J. H. Steffen, H. E. Schlichting, J. T. Wright, C. Beard, C. L. Brinkman, A. Chontos, S. Giacalone, M. L. Hill, M. R. Kosiarek, M. G. MacDougall, T. Močnik, A. S. Polanski, E. V. Turtelboom, D. Tyler, and J. Van Zandt. The Kepler Giant Planet Search. I. A Decade of Kepler Planet-host Radial Velocities from W. M. Keck Observatory. *ApJS*, 270(1):8, Jan. 2024. doi: 10.3847/1538-4365/ad0cab.
- R. F. Wilson, T. Barclay, B. P. Powell, J. Schlieder, C. Hedges, B. T. Montet, E. Quintana, I. McDonald, M. T. Penny, N. Espinoza, and E. Kerins. Transiting Exoplanet Yields for the Roman Galactic Bulge Time Domain Survey Predicted from Pixel-level Simulations. *ApJS*, 269(1):5, Nov. 2023. doi: 10.3847/1538-4365/acf3df.

- J. N. Winn and D. C. Fabrycky. The Occurrence and Architecture of Exoplanetary Systems. *ARA&A*, 53:409–447, Aug. 2015. doi: 10.1146/annurev-astro-082214-122246.
- J. N. Winn, J. M. Matthews, R. I. Dawson, D. Fabrycky, M. J. Holman, T. Kallinger, R. Kuschnig, D. Sasselov, D. Dragomir, D. B. Guenther, A. F. J. Moffat, J. F. Rowe, S. Rucinski, and W. W. Weiss. A Super-Earth Transiting a Naked-eye Star. *ApJ*, 737(1): L18, Aug. 2011. doi: 10.1088/2041-8205/737/1/L18.
- J. N. Winn, R. Sanchis-Ojeda, L. Rogers, E. A. Petigura, A. W. Howard, H. Isaacson, G. W. Marcy, K. C. Schlaufman, P. Cargile, and L. Hebb. Absence of a Metallicity Effect for Ultra-short-period Planets. *AJ*, 154(2):60, Aug. 2017. doi: 10.3847/1538-3881/aa7b7c.
- J. N. Winn, R. Sanchis-Ojeda, and S. Rappaport. Kepler-78 and the Ultra-Short-Period planets. *New Astronomy Reviews*, 83:37–48, Nov. 2018. doi: 10.1016/j.newar.2019.03.006.
- J. Wisdom and M. Holman. Symplectic maps for the N-body problem. *J*, 102:1528–1538, Oct. 1991. doi: 10.1086/115978.
- A. Wolszczan and D. A. Frail. A planetary system around the millisecond pulsar PSR1257 + 12. *Nature*, 355(6356):145–147, Jan. 1992. doi: 10.1038/355145a0.
- Y. Wu, N. W. Murray, and J. M. Ramsahai. Hot Jupiters in Binary Star Systems. *ApJ*, 670:820–825, Nov. 2007. doi: 10.1086/521996.
- J.-W. Xie, S. Dong, Z. Zhu, D. Huber, Z. Zheng, P. De Cat, J. Fu, H.-G. Liu, A. Luo, Y. Wu, H. Zhang, H. Zhang, J.-L. Zhou, Z. Cao, Y. Hou, Y. Wang, and Y. Zhang. Exoplanet orbital eccentricities derived from LAMOST-Kepler analysis. *Proceedings of the National Academy of Science*, 113(41):11431–11435, Oct. 2016. doi: 10.1073/pnas.1604692113.
- H. Xiu-Min and J. Jiang-Hui. The Recent Research Progress of Exoplanets of Ultra-Short Periods. *Chinese Astron. Astrophys.*, 44(3):283–312, July 2020a. doi: 10.1016/j.chinastron.2020.08.001.

- H. Xiu-Min and J. Jiang-Hui. The Recent Research Progress of Exoplanets of Ultra-Short Periods. *Chinese Astronomy and Astrophysics*, 44(3):283–312, July 2020b. doi: 10.1016/j.chinastron.2020.08.001.
- J. J. Zanazzi and E. Chiang. Sweeping secular resonances and giant planet inclinations in transition discs. *MNRAS*, 527(3):7203–7216, Jan. 2024. doi: 10.1093/mnras/stad3066.
- E. Zhang, S. Naoz, and C. M. Will. A Stability Timescale for Non-Hierarchical Three-Body Systems. *arXiv e-prints*, art. arXiv:2301.08271, Jan. 2023. doi: 10.48550/arXiv.2301.08271.
- G. Zhou, J. E. Rodriguez, A. Vanderburg, S. N. Quinn, J. Irwin, C. X. Huang, D. W. Latham, A. Bieryla, G. A. Esquerdo, P. Berlind, and M. L. Calkins. The Warm Neptunes around HD 106315 Have Low Stellar Obliquities. *AJ*, 156(3):93, Sept. 2018a. doi: 10.3847/1538-3881/aad085.
- G. Zhou, J. E. Rodriguez, A. Vanderburg, S. N. Quinn, J. Irwin, C. X. Huang, D. W. Latham, A. Bieryla, G. A. Esquerdo, P. Berlind, and M. L. Calkins. The Warm Neptunes around HD 106315 Have Low Stellar Obliquities. *AJ*, 156(3):93, Sept. 2018b. doi: 10.3847/1538-3881/aad085.
- W. Zhu. On the Patterns Observed in Kepler Multi-planet Systems. *AJ*, 159(5):188, May 2020. doi: 10.3847/1538-3881/ab7814.
- W. Zhu and S. Dong. Exoplanet Statistics and Theoretical Implications. *ARA&A*, 59: 291–336, Sept. 2021. doi: 10.1146/annurev-astro-112420-020055.
- W. Zhu and Y. Wu. The Super Earth-Cold Jupiter Relations. *AJ*, 156(3):92, Sept. 2018. doi: 10.3847/1538-3881/aad22a.
- W. Zhu, C. Petrovich, Y. Wu, S. Dong, and J. Xie. About 30% of Sun-like Stars Have Kepler-like Planetary Systems: A Study of Their Intrinsic Architecture. *ApJ*, 860(2):101, June 2018. doi: 10.3847/1538-4357/aac6d5.

J. K. Zink, J. L. Christiansen, and B. M. S. Hansen. Accounting for incompleteness due to transit multiplicity in Kepler planet occurrence rates. *MNRAS*, 483(4):4479–4494, Mar. 2019. doi: 10.1093/mnras/sty3463.