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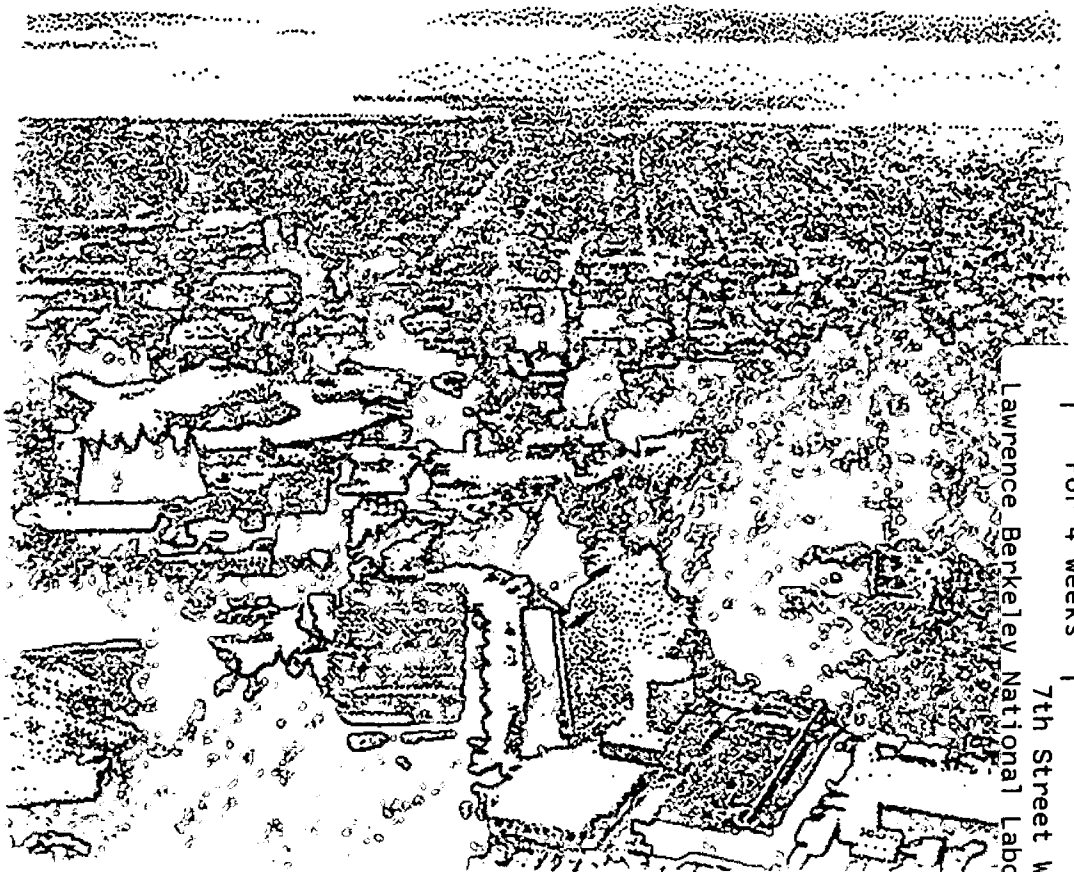
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J. Cline, C. Grojean, and G. Servant

**Physics Division**

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## **Inflating Intersecting Branes and Remarks on the Hierarchy Problem**

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# Inflating Intersecting Branes and Remarks on the Hierarchy Problem

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## Abstract

We generalize solutions of Einstein's equations for intersecting branes in higher dimensional spacetimes to the nonstatic case, modeling an expanding universe. The relation between the Hubble rate, the brane tensions, and the bulk cosmological constant is similar to the case of a single 3-brane in a 5-dimensional spacetime. We discuss how cosmological expansion can be reconciled with the Randall-Sundrum solution of the weak scale hierarchy problem using this class of solutions. We also suggest a new solution of the hierarchy problem in which the Planck scale inflates dynamically.

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Although the possibility of extra spatial dimensions is an old idea, it has received more attention lately because of a new twist: perhaps our universe looks 4-dimensional not because of the smallness of the extra dimensions, but because we are trapped on a 3D slice (a 3-brane) [1]. This simple variation has created a wealth of potential new physics signals and hints of solutions to long-standing puzzles.

At first sight, the brane universe scenario poses a new problem: if one embeds a 3-brane with tension (energy density)  $\sigma$  in an empty  $(4 + 1)$ -D spacetime, the space in the 3-brane inflates with a Hubble constant given by [2–5]

$$H^2 = \left( \frac{\kappa^2 \sigma}{6} \right)^2 \quad (1)$$

Here  $\kappa^2$  is the analog of  $8\pi G_N = M_P^{-2}$  (where  $M_P$  is the Planck mass) in  $(4 + 1)$ -D gravity. The linear dependence  $H \sim \sigma$  is contrary to the usual Friedmann equation which gives  $H \sim \sqrt{\sigma}$ . If one tries to model normal cosmology on such a brane universe by replacing the constant tension  $\sigma$  with an energy density  $\rho$  which decreases with the expansion, then  $H$  varies with time and is given by  $1/4t$  instead of  $1/2t$  in a radiation dominated era. This corresponds to a scale factor growing like  $t^{1/4}$  rather than the usual  $t^{1/2}$ . It is likely that such a radical change to the expansion rate can be ruled out using primordial big bang nucleosynthesis [3].

However, one can to a good approximation recover the usual rate of expansion by keeping a nonzero value for the constant part of the brane tension, and canceling its inflationary effect by adding a negative cosmological constant  $\Lambda_b$  in the bulk (the full 5 spatial dimensions) [6,7]. Letting  $\rho$  denote the time-varying part of the energy density on the brane, eq. (1) is modified to read

$$H^2 = \frac{\kappa^4(\sigma + \rho)^2}{36} + \frac{\kappa^2 \Lambda_b}{6}. \quad (2)$$

By tuning the value of  $\Lambda_b$  to

$$\Lambda_b = -\kappa^2 \sigma^2 / 6, \quad (3)$$

the quadratic term in  $\Lambda$  gets canceled, so that the universe is static when  $\rho = 0$ , as expected. Further tuning  $\sigma$  to the value

$$\sigma = \frac{6}{\kappa^4 M_P^2}, \quad (4)$$

one finds that the leading correction to  $H^2$  for  $\rho \ll \sigma$  agrees exactly with the usual Friedmann equation. Only for  $\rho \gtrsim \sigma$  does the unusual  $H \sim \rho$  behavior start to reappear.

In addition to solving the problem of cosmological expansion, the relation (3) has another possible benefit: it might afford a solution to the hierarchy problem, *i.e.*, the question of why the weak scale  $M_W$  is so much smaller than  $M_P$  [8–10]<sup>1</sup>. Randall and Sundrum noticed that the solution for the metric on the 4D space is exponentially suppressed away from the

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<sup>1</sup>The question of the gauge coupling unification has also been addressed in [11]

3-brane, in the direction  $y$  of the 5th dimension:

$$ds^2 = a^2(y) \left( -d\tau^2 + \sum_{i=1}^3 dx_i^2 \right) + b^2 dy^2; \\ a(y) = \exp(-\kappa^2 b \Lambda |y|/6) \quad (5)$$

At a given distance  $y_T$  from the brane at  $y = 0$  (called the ‘‘Planck brane’’),  $a(y_T)$  is exponentially small. If there was another brane located at  $y_T$ , dubbed the ‘‘TeV brane,’’ any particles constrained to exist there would have their masses renormalized by the factor  $a(y_T)$ . Thus even if all mass parameters in the fundamental Lagrangian were of order  $M_P$ , physical masses at position  $y_T$  could easily be of order  $M_W$  or 1 TeV with only a moderately large value of  $y_T$ . The function  $a(y)$  can be interpreted as the wave function of the graviton, showing that gravity is trapped near the brane at  $y = 0$ . Because of this trapping, the usual gravitational force law  $F \sim 1/r^2$  is obeyed at distances  $r \gg 6M^3/b\Lambda$ , even if the extra dimension is infinite in size. It is not obvious whether this happy state of affairs is compatible with getting the correct rate of expansion on the brane at  $y_T$ , but we shall show that it is in fact possible to have both.

An obvious question is whether these ideas can be extended to larger numbers ( $N$ ) of extra dimensions, since it is possible that qualitatively new effects might emerge. Thus far no solutions have been constructed for a single 3-brane in  $N > 1$  spatial dimensions. However it is straightforward to do so for a brane with  $(3 + N - 1)$  spatial dimensions (in other words, with codimension 1). Moreover, by taking the intersection of  $N$  such branes, one can single out a 3D region of space which might be identified with a universe like ours, and this kind of solution has also been constructed, in the static case [12, 13]. The static solutions manifest the phenomenon of gravitational trapping and the potential for solving the hierarchy problem analogous to the  $N = 1$  case. Here we wish to consider the generalization to dynamical (expanding or contracting) solutions. We shall see that an expression similar to (2) obtains for the Hubble rate in the intersecting brane model.

To specify the solutions, we consider the case of  $N$  extra dimensions with coordinates  $y_i$ , and  $N$  orthogonally intersecting  $(3 + N)$ -branes located at  $y_i = 0$ , respectively. The action is

$$S = \int d^4x d^N y \sqrt{|g|} \left( \frac{\mathcal{R}}{2\kappa^2} - \Lambda_b - \sum_{i=1}^N \sigma_i \delta(y_i \sqrt{g_{ii}}) \right), \quad (6)$$

where  $\kappa^2$  is related to the  $N$ -dimensional gravity scale  $M$  by  $\kappa^{-2} = M^{N+2}$ . Similarly to ref. [12], we take the conformally flat ansatz

$$ds^2 = a^{-2}(\tau, y_i) \left( -d\tau^2 + \sum_{i=1}^3 dx_i^2 + \sum_{j=1}^N dy_j^2 \right) \quad (7)$$

(notice the negative exponent in our definition) for which the Einstein tensor in  $d = 4 + N$  dimensions has the form

$$G_{\mu\nu} = (N + 2) \left( \frac{\nabla_\mu \nabla_\nu a}{a} - \eta_{\mu\nu} \left( \frac{\nabla^2 a}{a} - \frac{N + 3}{2} \left( \frac{\nabla a}{a} \right)^2 \right) \right). \quad (8)$$

The gradients are simple partial derivatives, using the Minkowski metric  $\eta_{\mu\nu} = \text{diag}(-1, 1, \dots, 1)$ :  $\nabla^2 = \eta^{\mu\nu} \partial_\mu \partial_\nu$ . If the respective branes have tensions  $\sigma_i$ , the stress-energy tensor is given by

$$T_{\mu\nu} = -a^{-2} \left( \Lambda_b \eta_{\mu\nu} + \sum_{i=1}^N (\eta_{\mu\nu} - \delta_{\mu, y_i} \delta_{\nu, y_i}) \sigma_i \delta(y_i/a) \right) \quad (9)$$

Each brane contribution looks like a bulk cosmological term, except in the entry corresponding to  $y_i$  which is zero for the  $i$ th brane.

A solution to the  $(4 + N)$  dimensional Einstein equations,  $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\mathcal{R} = \kappa^2 T_{\mu\nu}$ , is given by

$$a(\tau, y_i) = H\tau + \sum_i k_i |y_i|. \quad (10)$$

This solution was previously found by ref. [14] for the case of  $N = 1$ , that is, a single 3-brane. It is easy to show that the equations are satisfied provided that

$$k_i = \frac{\kappa^2 \sigma_i}{2(N+2)} \quad (11)$$

and the Hubble constant is given by

$$H^2 = \frac{2\kappa^2 \Lambda_b}{(N+2)(N+3)} + \sum_i k_i^2. \quad (12)$$

In particular, the static case where  $H = 0$  is recovered if  $\Lambda_b$  satisfies

$$\Lambda_b = -\frac{(N+3)}{8(N+2)} \kappa^2 \sum_i \sigma_i^2. \quad (13)$$

One can see that this agrees with the previous result (3) in the case of one extra dimension.

Our solution differs from previous ones, such as refs. [5, 15], by allowing the extra dimensions to inflate simultaneously with the 3D universe. Without gravitational trapping this would be unacceptable, but in the present context the size of the extra dimensions has no direct effect on 3D gravity, so inflation of the bulk is not necessarily problematic.

To see that  $H$  is indeed the Hubble parameter, one should transform from the conformal time coordinate  $\tau$  to FRW time  $t$ , in which the  $g_{00}$  element of the metric is  $-1$ :  $dt = \pm d\tau/a(\tau, y_i)$ ,  $t = \pm H^{-1} \ln(a)$ . This implies  $a^{-1}(\tau(t), y_i) = \exp(\mp Ht)$ . Choosing the lower sign gives the line element

$$ds^2 = - \left( dt + H^{-1} e^{Ht} \sum_{i=1}^N k_i \text{sign}(y_i) dy_i \right)^2 + e^{2Ht} (d\vec{x}^2 + d\vec{y}^2). \quad (14)$$

The 4D part of the metric has the usual form for an inflationary solution with expansion rate  $H$ . The range of the  $\tau$  coordinate depends on the position in the bulk:  $\tau \in (-\sum_i k_i |y_i|/H, \infty)$ , corresponding to  $t \in (-\infty, \infty)$ .



To construct a realistic inflationary or cosmological scenario, one should replace the constants  $\sigma_i$  with time-varying energy densities:

$$\sigma_i \rightarrow \sigma_i + V_i(t) \quad (15)$$

Here it is envisioned that the  $\sigma_i$  and  $\Lambda_b$  satisfy the condition (13) which ensures that the expansion will stop when the perturbations  $V_i$  settle to their minimum values, presumed to be  $V_i(\infty) = 0$ . The  $V_i$  could be potentials of scalar fields, as in inflation, or the energy density of matter, in the post-inflationary universe. The solution we have obtained is no longer exact, but in the limit where the  $V_i$  are changing adiabatically with time, it is approximately correct. Linearizing in these perturbations gives an expansion rate of

$$H \cong \left( \sum_i \frac{\kappa^4 \sigma_i V_i}{2(N+2)^2} \right)^{1/2} \quad (16)$$

For an observer living on the intersection of all the branes, the apparent 3D energy density due to the  $V_i$  would be zero if the branes were infinitesimally thin, but if they are given a finite thickness  $\Delta$  (so that the delta functions  $\delta(y_i)$  are replaced by top-hat functions of width  $\Delta$ ), then the 3D energy density is

$$\rho = \sum_i V_i \Delta^{N-1}, \quad (17)$$

and the usual rate of expansion,  $H = (\rho/3M_p^2)^{1/2}$ , can be obtained by setting

$$\sigma_i = \frac{2(N+2)^2 \Delta^{N-1}}{3\kappa^4 M_p^2} \quad (18)$$

in agreement with the value (4) in the  $N = 1$  case. In terms of the fundamental gravity scale  $M$ , defined by  $\kappa^2 = M^{-(2+N)}$ , it seems reasonable to imagine a brane thickness on the order of  $\Delta \sim M^{-1}$ , so that  $\sigma_i \sim M^{N+5}/M_p^2$ . This construction leaves unanswered the question of why the matter we see in our universe, if the latter is the intersection of several branes, is constrained to that intersection region, *i.e.*, why we do not feel any interaction with the matter living out of the intersection. This problem does not arise in nonintersecting brane scenarios. For instance, in the case of one extra dimension compactified on  $S^1/\mathbb{Z}_2$ , the two branes localized on the fixed points do not interact provided they correspond to different gauge groups. The standard model resides on the positive-tension brane, whose matter is neutral under the hidden sector of the other, negative tension brane. However in the present proposal, for  $N > 1$  matter lives in higher  $(N+2)$ -dimensional branes and it is not obvious how it can be confined to the 3-dimensional intersection region; nor, if it is so confined, why it does not interact with the matter that exists elsewhere on the branes.

In the original incarnation of large extra dimensions, inflation of the latter was associated with time variation of Newton's constant, since the largeness of the Planck mass was linked to the size of the extra dimensions. Since there are strong constraints on time variations in

the strength of gravity, we should verify that it is not a problem for our solution. In fact one might be concerned about this because ref. [12] showed that the relationship between the fundamental gravity scale and the observed Planck mass is

$$M_P^2 = M^{N+2} \int d^N y a^{-2-N}(y_i) \quad (19)$$

in the static case. This comes from integrating  $\sqrt{g}\mathcal{R}$  over the extra dimensions to find the effective 4D action, and using the scaling property of the Ricci scalar under conformal transformations of the metric. Naively applying the same formula using our dynamical solution gives

$$M_P^2 = \frac{2^N M^{N+2}}{(N+1)! \prod_i k_i} e^{2Ht}, \quad (20)$$

using the fact that  $(H\tau)^{-1} = \exp(Ht)$ . However this apparent time dependence of the Planck mass is not correct; the above formula would predict the same thing even in the case of no extra dimensions, and we know that inflation in 4D does not cause  $M_P$  to increase. One can always rescale the 4D coordinates  $x^\mu \rightarrow e^{Ht} x^\mu$  to get rid of the factor  $e^{2Ht}$  which comes into the effective 4D Lagrangian  $\int d^N y \sqrt{g}\mathcal{R}^{(4)}$ , showing that the strength of gravity is unaffected by the inflation, regardless of the number of extra dimensions.

Another way to see that the strength of gravity is not time-dependent in the present solution is to consider the ratio of  $M_P$  to physical masses of particles  $m_{\text{phys}}$  on the brane. These too grow like  $e^{2Ht}$  in the coordinate system  $x^\mu$ , for exactly the same reason that particles at a distance  $y$  from the fiducial brane have their masses suppressed by  $a^{-1}(y)$  in the original Randall-Sundrum proposal. For inflationary solutions we have  $a^{-1}(t, y) = e^{Ht} a^{-1}(0, y)$ . Thus  $M_P/m_{\text{phys}}$  remains constant. Alternatively,  $m_{\text{phys}}$  is a constant in the rescaled coordinates,  $x^\mu \rightarrow e^{Ht} x^\mu$ , which is the same coordinate system where  $M_P$  is unchanging.

We have argued that an observer at the intersection of  $N$  branes in  $4+N$  dimensions will experience the correct rate of cosmological expansion, provided that the conditions (18), (13) are satisfied, and that the apparent 3D energy density is small compared to that coming from the brane tensions,  $\rho \ll \sigma_i \Delta^{N-1}$ . However for this observer there is no immediate solution to the weak scale hierarchy problem. Only for a 3-brane which is located some distance away from the intersection region are masses suppressed by the geometrical factor  $a(y_i)$ . A potential problem is whether the Hubble rate will be correct when measured on this ‘‘TeV brane,’’ which presumably should have a smaller tension than the ‘‘Planck brane’’ intersection region, so as not to significantly perturb the geometry induced by the Planck brane. One might expect the expansion of the universe to be controlled by the large energy density on the Planck brane, rather than the small one on the TeV brane. An observer on the latter might find his universe expanding at a rate that was not directly correlated with the local energy density.

To investigate this question we will consider the simplest case, that of  $N = 1$ . The extension of our previous solution to incorporate a Planck brane and a TeV brane, having respective positions  $y = y_P$ ,  $y = y_T$  and tensions  $\sigma_P$ ,  $\sigma_T$ , is

$$a(\tau, y) = \begin{cases} H\tau + k_P |y - y_P|, & y < y_T \\ H\tau + k_T |y - y_T| + k_P |y_P - y_T|, & y \geq y_T \end{cases} \quad (21)$$

This is a generalization of the static solution found in ref. [15]. By computing the  $G_{\mu\nu}$  for this metric one finds that it solves the Einstein equations if

$$k_P = \frac{\kappa^2}{6}\sigma_P; \quad k_T = \frac{\kappa^2}{6}(\sigma_P + 2\sigma_T), \quad (22)$$

and if  $\Lambda_b$  changes discontinuously at the interface provided by the TeV brane,

$$\Lambda_T - \Lambda_P \equiv \Lambda_b \Big|_{y=y_T-\epsilon}^{y=y_T+\epsilon} = -\frac{2\kappa^2}{3}\sigma_T(\sigma_T + \sigma_P) \quad (23)$$

The Hubble rate is given by

$$\begin{aligned} H^2 &= \frac{\kappa^4\sigma_P^2}{36} + \frac{\kappa^2\Lambda_P}{6} \\ &= \frac{\kappa^4(\sigma_P + 2\sigma_T)^2}{36} + \frac{\kappa^2\Lambda_T}{6} \end{aligned} \quad (24)$$

Let us first construct the static configuration where  $H = 0$ . The term  $H\tau$  can be replaced by a constant in eq. (21) to maintain the regularity of the solutions. The bulk cosmological constants in the two regions  $y < y_T$  and  $y > y_T$  are related to the brane tensions by

$$\begin{aligned} \Lambda_P &= -\frac{\kappa^2}{6}\sigma_P^2; \\ \Lambda_T &= -\frac{\kappa^2}{6}(\sigma_P + 2\sigma_T)^2. \end{aligned} \quad (25)$$

To get expanding solutions, we now perturb around the static case by adding small energy densities  $V_P$  and  $V_T$  to the branes, and linearizing. Eq. (24) becomes

$$H^2 \cong \frac{\kappa^4}{18}\sigma_P V_P \quad (26)$$

$$\cong \frac{\kappa^4}{18}(\sigma_P + 2\sigma_T)(V_P + 2V_T) \quad (27)$$

The fact that eqs. (26) and (27) must agree implies that the perturbations on the two branes are proportional,

$$V_T = -\frac{\sigma_T}{2\sigma_T + \sigma_P}V_P \cong -\frac{\sigma_T}{\sigma_P}V_P, \quad (28)$$

so that an observer on the TeV brane would relate the expansion rate to his local energy density by

$$H^2 = -\frac{\kappa^4}{18}\frac{\sigma_P}{\sigma_T}(\sigma_T + \sigma_P)V_T \cong -\frac{\kappa^4}{18}\frac{\sigma_P^2}{\sigma_T}V_T \quad (29)$$

Now the Planck brane tension  $\sigma_P$  must be positive to insure that  $a^{-1}$  is decreasing away from  $y = y_P$ , as is needed to solve the hierarchy problem; then eq. (26) implies  $V_P > 0$  as well. From (29) it follows that  $\sigma_T$  and  $V_T$  must have the opposite sign. This is an improvement

over the original Randall-Sundrum proposal, where the extra dimension being compactified on a circle led to the topological restriction that  $\sigma_T = -\sigma_P$ , hence the conclusion that  $V_T$  had to be negative. In the present realization we can take  $\sigma_T$  negative but smaller in magnitude than  $\sigma_P$ , leading to the conclusion that  $V_T > 0$ . Since we would like  $V_T$  to represent the energy driving cosmological expansion as seen on the TeV brane, this is encouraging.

In the above construction, we observe that it is not after all necessary to assume that  $\sigma_T \ll \sigma_P$ , as is suggested by the names ‘‘TeV’’ and ‘‘Planck’’ for the two branes. All that is really needed is to have  $-\sigma_P/2 < \sigma_T < 0$ . As long as this is true, all the quantities  $\sigma_P$ ,  $\sigma_T$ ,  $\Lambda_P$  and  $\Lambda_T$  can be of order  $M_P$  to the appropriate power. Then the expansion rate goes like  $H \sim \sqrt{V_i}/M_P$  in terms of the excess energy density on either brane, as desired. Moreover it looks straightforward to generalize this construction to higher dimensions. Then different  $N$ -dimensional hypercubic regions would have different values of the bulk cosmological constant, changing discontinuously at the interfaces where the analogues of the TeV brane are located.

One annoying feature of our construction is the negative tension attributed to the TeV brane. Such a brane might be unstable to crinkling up into something with an infinite volume since, in the absence of some stabilizing mechanism, this would minimize the energy. However in a superstring context such a negative brane could be seen as an orientifold and supersymmetric properties could stabilize it [16]. It is also possible to arrange for positive tension TeV branes; as pointed out in ref. [17], in solutions with several parallel branes in  $N = 1$ , the signs of the tensions alternate. Thus one could create a positive tension TeV brane if desired. However a negative tension brane between the TeV and Planck branes is still required, so it is not clear whether this would be an improvement.

To demonstrate this is a straightforward generalization of our previous solution. If we add a third brane at  $y = y_{T'} > y_T$  with tension  $\sigma_{T'}$ , then we have the previous relations, and in addition

$$\begin{aligned} k_{T'} &= \frac{\kappa^2}{6}(\sigma_P + 2(\sigma_T + \sigma_{T'})), \\ \Lambda_{T'} &= -\frac{\kappa^2}{6}(\sigma_P + 2(\sigma_T + \sigma_{T'}))^2, \end{aligned} \quad (30)$$

in order to maintain the static condition. If we now add excess energies  $V_P$ ,  $V_T$  and  $V_{T'}$  to the respective branes, it is straightforward to show that in terms of  $V_{T'}$  the expansion rate is given by

$$H^2 = -\frac{\kappa^4}{18} \frac{1}{\sigma_{T'}} (2\sigma_T + \sigma_P) (2(\sigma_T + \sigma_{T'}) + \sigma_P) V_{T'} \quad (31)$$

If  $\sigma_{T'} > 0$  and  $V_{T'} > 0$ , then  $\sigma_T$  must be in the range  $-\sigma_P/2 - \sigma_{T'} < \sigma_T < -\sigma_P/2$ , showing that the middle brane has negative tension, although the outer ones have positive tension.

A final mystery is the question of why the time-dependent parts of the energy densities of the two branes should be proportional to each other. A priori one would think that they are parallel universes which could have arbitrarily different sources of stress-energy. Whether this is an artifact of having a simple ansatz for the solutions, or there is some deeper reason, is not obvious.

In summary, we have found expanding global solutions for a  $(4+N)$  dimensional universe, in which the intersection of  $N$  orthogonal branes of codimension one plays the role of a 4D universe. The rate of expansion can be made to agree with the Friedmann equation if the brane tensions  $\sigma$  are balanced against a negative bulk cosmological constant  $\Lambda_b$  in a particular way. Both quantities can be of order  $M_P$  to the appropriate power, so only a tuning of their relative values is necessary; the magnitudes of  $\Lambda_b$  and  $\sigma$  are natural. An energy density  $\rho$  which is in excess of these particular values for the brane tensions is what appears to drive the expansion of the universe, at the expected rate  $H \sim \sqrt{\rho}/M_P$ . Furthermore it is possible to introduce extra branes whose physical masses are exponentially suppressed by the distance from the primary brane, thereby possibly solving the weak scale hierarchy problem on the extra branes, while maintaining the correct rate of cosmological expansion. However, this combination of two virtues seems to come always at the expense of introducing some negative tension branes.

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