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THE CONNECTION BETWEEN ELASTIC AND QUASI-ELASTIC PION SCATTERING FROM NUCLEI

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ABSTRACT

Unitarity relations derived by Tandy *et al.*, are used to reveal the quasi-elastic cross section implicit in several optical potential descriptions of low energy pion-nucleus scattering. An order of magnitude discrepancy between experiment and standard theories is revealed. A qualitative resolution of this disagreement requires the inclusion of elastic unitarity, and the effects of the Pauli exclusion principle. The consequences of this for the pion wave function, and hence for other reactions, are briefly described.

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There has recently been quite a proliferation of theories of low energy pion-nucleus scattering, apparently based on rather different physical assumptions, which nevertheless seem to reproduce the measured elastic differential cross sections.[1-7] Clearly some more stringent test of these theories is required. The purpose of the present work is to suggest that the ability of a theory to predict the quasi-elastic $(\pi,\pi N)$ reaction cross section, can serve as one such test.

The intimate connection between the $(\pi,\pi N)$ reaction, and elastic scattering, has been made most clear in the recent theoretical work of Tandy, Redish and Bollé.[8-10] These authors have spelled out the three-body nature of the first order potential (see also Ref. 11), thereby providing a sound formal basis for a "three body" pion-nucleus optical potential[5,6,12,13]

$$U^{(1)}(\underline{k}',\underline{k}) \sim (A-1)\sum_{a}\int d\underline{p}\;\psi_{a}^{*}(\underline{p}-\underline{q})\;t_{\pi N}(E_{3\text{-body}} = K^{+}_{inc} - E_{B} - (\underline{p}+\underline{k})^{2}/2M)\;\psi_{a}(\underline{p}). \qquad (1)$$

(In Eq. (1) $\{\underline{k},\underline{k}'\}$ are the initial and final pion momenta $(\underline{q} = \underline{k}'-\underline{k})$, "$\underline{p}$" the nucleon fermi momentum, and we have shown explicitly only the sub-energy dependence of the πN t-matrix.)

Most significant for our present considerations, is the unitarity relation proved by Tandy *et al.*[8-10] for a first order optical potential. When applied to pions, their result is that the total cross section for a potential of type (1) (possibly including Pauli effects, but not "true absorption"[1-3,5,6,12,14]) is exactly given by

$$\sigma(\text{Total}) = \sigma(\text{Elastic}) + \sigma(\pi,\pi N). \qquad (2)$$

Furthermore, the quasi-elastic cross section $\sigma(\pi,\pi N)$ is given as

$$\sigma(\pi,\pi N) = \int (\text{Phase space}) \, |\langle \phi_\pi \, \chi_N^{(-)} \, \psi_{(A-1)} | t_{\pi N} | \chi_\pi^{(+)} \, \psi_A \rangle|^2 . \qquad (3)$$

In Eq. (3), $\chi_\pi^{(+)}$ is the initial pion distorted wave and $\{\psi_A, \psi_{A-1}\}$ are the appropriate nuclear wave functions. The deviation from a standard DWIA is that the final pion wave function (ϕ_π) is a plane wave, and the outgoing nucleon wave function ($\chi_N^{(-)}$) is calculated in the (real) single particle (binding) potential. (These last two approximations are a direct consequence of using a first order potential.) The actual form of the "phase space" factor depends critically on the approximation made for the πN interaction energy in Eq. (1). We refer to Ref. 10 for more details, merely noting here that the exact phase space volume is only obtained when the three-body choice of πN sub-energy is made.

We have used results (1) and (2) to calculate the cross section $\sigma(\pi,\pi N)$ implicit in several models of π-nucleus elastic scattering.[15] These calculations are compared in Fig. 1 with the most appropriate available experimental information[16,17] - namely the sum of the total cross sections for π^+ and π^- on ^{12}C leading to particle stable ^{11}C. (It must be realized that this comparison is very approximate, since we actually calculate the cross section for π° to remove either a proton or a neutron from ^{12}C. However, for an order of magnitude comparison, the present approach should be useful.)

From Fig. 1 we see that both simple optical potentials, namely the Kisslinger[2,18,19] model, and the separable potential model with the non-three-body interaction energy[20,21] (labelled $E_{2\text{-body}}$), lie over an order of magnitude above the data. Even the very crude modification of the "πN" input to take account of Pauli effects in Ref. 11 (labelled "modified Kisslinger"), lies an order of magnitude too high. When the three-body sub-energy is used (curve $E_{3\text{-body}}$), the cross sections decrease significantly. This underlines one crucial aspect of the three-body energy in Eq. (1). That is, when there is not sufficient energy for nucleon knock-out (i.e. $K_{inc} < E_B$) the πN t-matrix, and therefore the optical potential, is real. There is therefore no cross section for (π,πN), and elastic unitarity is satisfied. Thus even as crude a solution to the three-body problem as Eq.(1) contains an extra, important physical constraint.

Although the use of $E_{3\text{-body}}$ lessens the discrepancy between theory and experiment, something more is needed. In Fig. 1 we also show the (π,πN) cross section implicit in the optical model calculations when the πN t-matrix is modified, as described in Ref. 12 to take into account the restriction of the intermediate nucleon states because of the Pauli exclusion principle (curves labelled "--+ Pauli").[12,22-28] Clearly this effect is capable of eliminating the factor (6-10) discrepancy in the region of 40 MeV.

The question of Pauli effects in elastic scattering has been investigated many times,[6,12,23-26] unfortunately with little evidence for their importance. For example, from the field theoretic viewpoint, it has been shown that the suppression of the nucleon pole graph is more complicated than suggested by Bethe.[23] The improved treatment for the pionic atom case by Barshay *et al.*,[24] found only small (essentially undetectable) effects. Similarly, Dover *et al.*,[26] found effects of order 15% or less in pionic atoms using separable potentials for the πN interaction. Finally, our inclusion of Pauli blocking in a study of low energy pion elastic scattering[6,12], also produced relatively small changes in the elastic differential cross section. The meagre evidence in these studies of the elastic channel, contrasts sharply with the order of magnitude correction in the $(\pi,\pi N)$ reaction channel!

At this point it is important to comment on the quantitative aspects of this calculation vis-a-vis the experimental data. While there has been a qualitative improvement by an order of magnitude, particularly in the region below 60 MeV, it is apparent that the quantitative agreement is far from perfect. At the present stage of development this is only to be expected. First, as we explained above, the experimental data is only approximately comparable to what we calculate. As the pion energy rises there is more likelihood for an inner core nucleon to be removed, leaving an unstable final nucleus which is not measured. Second, the fact that the $(\pi,\pi N)$ cross section implicit in our optical model is given by the DWIA (3) with no distortion for the final pion, means that for the higher pion energies we expect to lie above the data (since this pion can itself eject another nucleon). It is relevant to note in this regard, that the π - ^{12}C <u>total cross sections</u> calculated with potentials like Eq. (1), tend to give too high a peak in the (3,3) resonance region. This is to be expected in a theory which greatly overestimates the $(\pi,\pi N)$ cross section.

In all of this we have not discussed the effect of the extra term ($U^{(abs)}$) in the potential describing the effects of real pion absorption. Within a quasi-deuteron model, this has been shown to play an important role in pionic atoms,[14,29] and very recently in low energy elastic scattering.[6] With our much smaller cross section for quasi-elastic scattering, this term is also necessary to reproduce the total cross section measurements! The inclusion of such a term will of course complicate the unitarity relation (2), and tend to make the $(\pi,\pi N)$ cross section slightly smaller. The omission of this effect is another reason why our cross section tends to rise above experiment at higher pion energy. (It is certainly not the reason for the original order of magnitude discrepancy in Fig. 1 however.)

In conclusion, we repeat that an important criterion in assessing the validity of any description of π-nucleus elastic scattering is its implicit prediction for the quasi-elastic $(\pi,\pi N)$ reaction. The order of magnitude overestimate of this reaction cross section in standard models may be a serious drawback in calculations of other reactions. For example, the p-wave co-ordinate space wave function at 50 MeV in the "$E_{2\text{-body}}$" case, is a factor of two smaller inside the nucleus than that calculated with "$E_{3\text{-body}}$ + Pauli" and a term representing true absorption. This is simply a reflection of the excessive and unphysical loss of flux from the elastic channel in the former case.

We believe that the effects of elastic unitarity and the Pauli principle which we have demonstrated, deserve further detailed theoretical study. In addition, we repeat the call[30] for careful experimental measurements of the reactions through which pions are lost from the elastic channel. In particular, in view of the present work, we stress the importance of a thorough study of the $(\pi,\pi N)$ reaction at low and intermediate energies.

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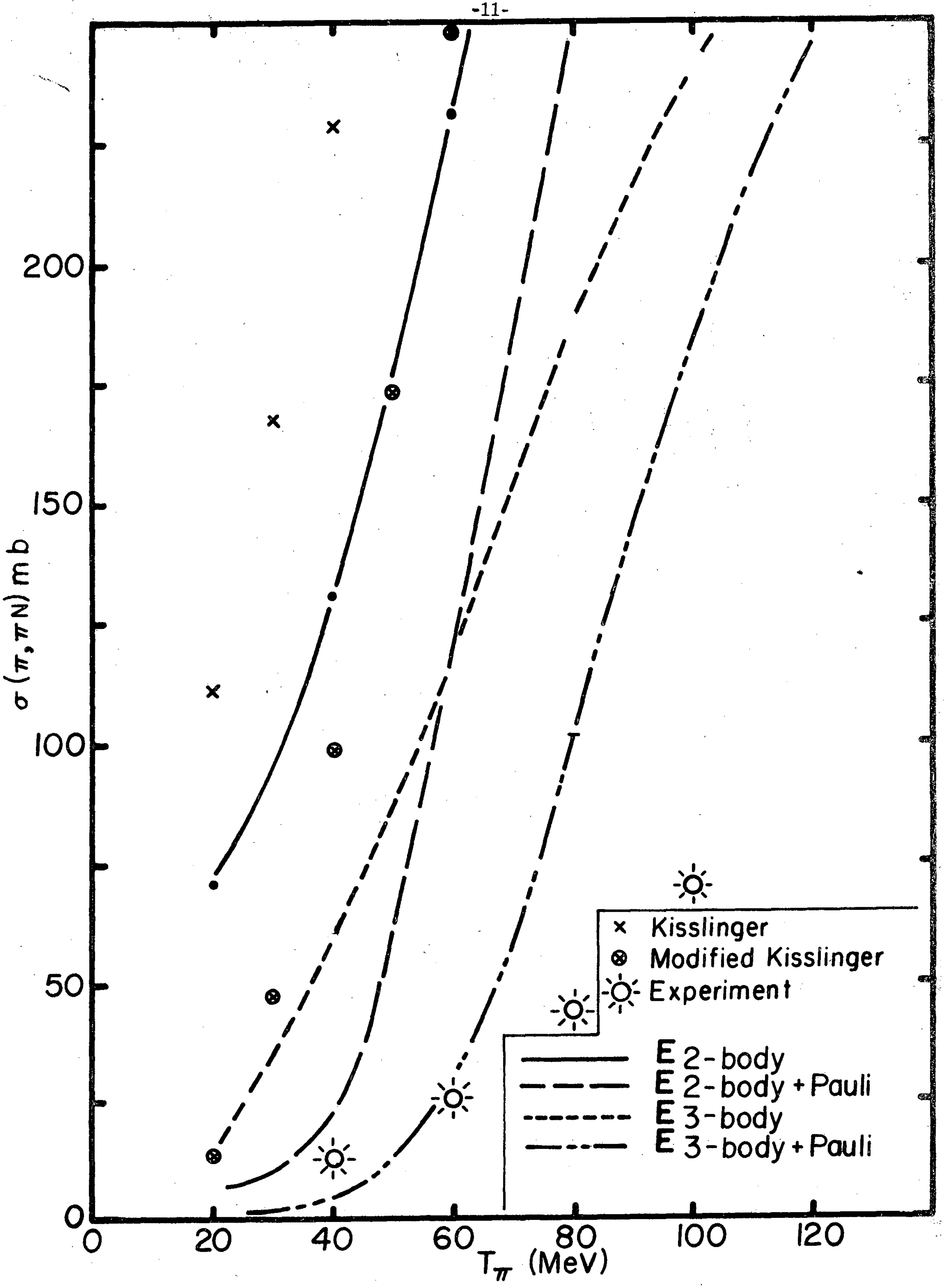
Figure 1: Comparison of the total cross-section for the $(\pi,\pi N)$ reaction implicit in several theories of π-nucleus elastic scattering, with the available data (see text for full details).


σ (π, πN) mb
200
150
100
50
0
0
20
40
60
80
100
120
T_π (MeV)
× Kisslinger
⊗ Modified Kisslinger
Experiment
E 2-body
E 2-body + Pauli
E 3-body
E 3-body + Pauli

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