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Journal
Physics Letters B, 458

Author
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Publication Date
1998-11-01

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LBNL-42536
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November 1998
Submitted to
Physics Letters B

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LBNL-42536

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This work was supported in part by the Director, Office of Science, Office of High Energy and Nuclear Physics, Division of High Energy Physics, of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098, and by NATO Grant CRG-970214.

LBNL–42536
LANCS–TH/9821
hep-ph/9811375
November 1998

Quintessential difficulties

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Abstract

An alternative to a cosmological constant is quintessence, defined as a slowly-varying scalar field potential $V(\phi)$. If quintessence is observationally significant, an epoch of inflation is beginning at the present epoch, with ϕ the slowly-rolling inflaton field. In contrast with ordinary inflation, quintessence seems to require extreme fine tuning of the potential $V(\phi)$. The degree of fine-tuning is quantified in various cases.

1. In the context of Einstein gravity, a cosmological constant may be regarded as a constant contribution to the energy density of the Universe. One can instead consider a contribution, termed quintessence [1, 2, 3, 4, 5], which is slowly decreasing on the Hubble timescale at the present epoch, and will presumably vanish in the infinite future. The variation in one Hubble time might be negligible, in which case quintessence is observationally the same as a cosmological constant, or it might be significant.

There is evidence, not yet compelling, that a cosmological constant or quintessence gives a significant contribution to the present energy density, with some leaning towards the latter [6]. Such a contribution is of order $\sim (10^{-3}\,\mathrm{eV})^4$, the present value of the critical energy density $3M_\mathrm{P}^2H^2$, and for the sake of simplicity one assumes that the total energy density has the critical value. (As usual $M_\mathrm{P} \equiv (8\pi G)^{-1/2} = 2.4\times10^{18}\,\mathrm{GeV}$ is the reduced Planck scale, and H is the Hubble parameter.)

A cosmological constant may be regarded as a nonzero value of the effective scalar field potential V, at the minimum which corresponds to our vacuum. From a theoretical viewpoint, it is not clear how the required value $V_\mathrm{vac} \sim (10^{-3}\,\mathrm{eV})^4$ would be determined. In units of M_P the required value is

$$\frac{V_\mathrm{vac}}{M_\mathrm{P}^4} \sim 10^{-120}\,. \tag{1}$$

Quintessence corresponds to $V_\mathrm{vac} = 0$, which may be easier to understand. At the present epoch, V is slowly decreasing towards this value. Quintessence, representing a significant fraction of the present energy density, is generated if the present epoch represents the beginning of an era of inflation, with some quintessence field ϕ satisfying the slow-roll approximation $3H\dot\phi = -V'$, and the potential obeying the flatness conditions

$$M_\mathrm{P}|V'/V| \ll 1 \tag{2}$$
$$M_\mathrm{P}^2|V''/V| \ll 1\,. \tag{3}$$

The first condition ensures that V is indeed slowly varying on the Hubble timescale, and the second condition is required for consistency of the slow-roll approximation. Conversely, with both flatness conditions satisfied, slow-roll typically represents an attractor for a wide range of initial conditions.

The flatness requirements Eqs. (2) and (3) are usually considered in the context of the era of inflation that is supposed to set initial conditions for the Hot Big Bang, which we shall call ordinary inflation. Ordinary inflation can be achieved without any significant fine-tuning [8]. One might therefore suppose that quintessence can also be achieved without extreme fine-tuning, and some of the literature seems to support this view.

In particular, there is the proposal [3, 5] that

$$V = \frac{\Lambda^{4+\alpha}}{\phi^\alpha}\,, \tag{4}$$

with $\alpha > 0$ usually of order 1, and Λ some mass scale. This potential satisfies the flatness conditions at $\phi \gtrsim \alpha M_\mathrm{P}$, and it is supposed that our epoch corresponds to the beginning of this regime, $\phi \sim \alpha M_{\rm P}$. For $\alpha = 2$, Eq. (1) is satisfied for $\Lambda \sim 1\,\mathrm{GeV}$, with larger Λ for larger α. Such values can be naturally generated by strong coupling effects and/or dynamical symmetry breaking, which give $\Lambda \sim e^{-8\pi^2/bg^2} M_{\rm P}$, where $g \sim \mathcal{O}(1)$ and the β-function, b, will usually be roughly 1 to 10. One seems indeed to have avoided fine-tuning.

The problem, with this or any other model of quintessence, is to prevent additional terms in the potential $V(\phi)$ which would violate the flatness conditions. In this note, we consider both the non-supersymmetric and supersymmetric cases, with emphasis on the latter. We focus mainly on the tree-level contributions to $V(\phi)$, involving the mass and self-couplings of ϕ, and argue that these parameters have to be extremely small compared with their natural values in order to satisfy the flatness conditions. Loop corrections to $V(\phi)$ involve also the couplings of ϕ to other fields, as well as their masses and (at higher order) self-couplings, but in the supersymmetric case we shall not attempt to quantify the degree of suppression of those parameters that is implied by the flatness conditions.

Our work is complementary to that of Carroll [4]. Instead of the flatness conditions, he discussed the constraint implied by the observational limits on a fifth force, in a non-supersymmetric context. The conclusion there is that one requires a moderate suppression of certain non-renormalizable couplings of the quintessence field to other fields.

2. Barring accidental cancellations, the flatness conditions are certainly going to require some internal symmetry. We shall first consider the case that ϕ is the modulus of some complex field, that is charged under a symmetry acting on the phase so that $\phi = 0$ is the fixed point. This eliminates the linear term in $V(\phi)$, and for simplicity we assume that it eliminates the cubic term as well. Then the potential has the form

$$V = V_0 + \frac{1}{2} m^2 \phi^2 + \lambda_4 \phi^4 + \sum_{d=5}^{\infty} \lambda_d M_{\rm P}^{4-d} \phi^d + \cdots . \tag{5}$$

The exhibited terms correspond to the tree-level potential, consisting of the mass term, the renormalizable quartic term, and the non-renormalizable terms with $d \geq 5$. The terms represented by dots represent quantum corrections. The latter include loop corrections, and possible non-perturbative effects giving terms like the one in Eq. (4).

As we are trying to see whether fine-tuning can be avoided, we discount the possibility of accidental cancellations between different contributions to the slope of V. Then the first flatness condition $M_{\rm P}|V'/V| \ll 1$ is implied by the second flatness condition $M_{\rm P}^2 |V''/V| \ll 1$, unless ϕ is far bigger than $M_{\rm P}$. We shall soon see that the latter regime is completely unviable, so we need consider only the second flatness condition which gives

$$m^2 \ll V_{\rm vac}/M_{\rm P}^2 \approx (10^{-42}\,\mathrm{GeV})^2 \tag{6}$$

$$\lambda_d \ll \frac{V_{\rm vac}}{M_{\rm P}^4} \left(\frac{M_{\rm P}}{\phi}\right)^{d-2} \approx 10^{-120} \left(\frac{M_{\rm P}}{\phi}\right)^{d-2} . \tag{7}$$

In the absence of supersymmetry, the mass is unstable against radiative corrections. Suppose ϕ couples to some other field in the theory with (dimensionless) coupling ζ.

Loop corrections to the ϕ-propagator will then shift m^2 by an amount $+\zeta M_{\rm P}^2$ if the field is a boson, and by $-\zeta^2 M_{\rm P}^2$ if it is a fermion. Supersymmetry ensures that the bosonic and fermionic contributions cancel, but without supersymmetry there is no reason for a cancellation. In the absence of a cancellation, the bound Eq. (6) requires $\zeta \ll V/M_{\rm P}^4 \sim 10^{-120}$ for the bosonic couplings, and $\zeta^2 \ll V/M_{\rm P}^4 \sim 10^{-120}$ for the fermionic couplings. This is the same amount of fine-tuning that is needed simply to impose the observational value Eq. (1).

Without supersymmetry, the same degree of fine-tuning is required by the constraint Eq. (7) on the non-renormalizable couplings, unless ϕ is well below $M_{\rm P}$. Indeed, the expected values of the λ_d are of order 1 since they represent quantum gravity effects at the Planck scale. (At least this should be the case for d not too large; for extremely large d one might reasonably expect [7] a behavior like $\lambda_d \sim 1/d!$).

3. Henceforth, we present our discussion in the context of supersymmetry. This ensures a cancellation between the fermionic and bosonic quantum corrections to m^2, and as we shall see it gives some control over the couplings λ_d. Supersymmetry is treated in several texts, and a summary of the aspects relevant for inflation is given in [8].

Taking the usual chiral formulation, supersymmetry works with complex scalar fields that we shall denote by Φ_n. As a function of these fields, the tree-level potential has a well-known form, consisting of an F-term plus a D-term. The F-term involves the superpotential W, which is holomorphic in the complex fields, and the real Kähler potential K which is taken to be a function of the fields and their complex conjugates. The D-term involves the holomorphic gauge kinetic function f and also K, but it is unlikely to be relevant for quintessence and we ignore it for the moment.

Because W is holomorphic, its form is very strongly constrained by internal symmetries. As a result, one can write down a simple expression corresponding to a model of quintessence (or anything else) and forbid all additional terms. Because of the specific form of V, this gives considerable control over m, λ and the non-renormalizable coefficients λ_d, and in particular allows one to suppress the latter far below the generic value $\lambda_d \sim 1$. However, in contrast with the case for ordinary inflation, this suppression is nowhere near enough to make quintessence viable.

The problem arises because supersymmetry must be broken if it is to realized at all in nature. The scale of supersymmetry breaking M_S is very large compared with $V^{1/4} \sim 10^{-3}\,{\rm eV}$. Indeed, to have a viable low-energy phenomenology one needs $M_S \gtrsim 1\,{\rm TeV}$, and it is usually supposed that $M_S \sim 10^{10}\,{\rm GeV}$. Also, sensible models seem to require at least a significant fraction of M_S to come from the F-term. Assuming for simplicity that M_S comes entirely from the F-term, and involves only say Φ_1, the potential $V(\phi)$ in the presence of supersymmetry breaking is of the form

$$V(\phi) = M_S^4 \left(k(\phi) + \cdots\right) + \cdots , \tag{8}$$

where k is the 1-1 element of the matrix inverse of $\partial^2 K/\partial\Phi_n\partial\Phi_m^*$. (Contributions to V can also come from W, but in specific models holomorphy will often forbid such terms.) Because it is not derived from a holomorphic function, k cannot be controlled by symmetries acting on the phases of the Φ_n. It will therefore have an expansion

$$k = 1 + \sum_{d=2}^{\infty} k_d M_{\mathrm{P}}^{-d} \phi^d \,, \tag{9}$$

with $|k_d| \sim 1$. This will gives contributions to the mass and couplings of order

$$m^2 \;\sim\; M_S^4/M_{\mathrm{P}}^2 \tag{10}$$

$$\lambda_d \;\sim\; M_S^4/M_{\mathrm{P}}^4 \,. \tag{11}$$

The mass-squared is a factor

$$M_S^4/V \sim (1\,\mathrm{TeV}/10^{-3}\,\mathrm{eV})^4 \sim 10^{60} \tag{12}$$

too big, and the same is true of the couplings unless ϕ is far below M_{P}. This represents severe fine-tuning.

Of course, it is always possible that the term in V proportional to M_S^4 might be suppressed because K and W have special forms. This occurs in the form of supersymmetry termed 'no-scale', where the term actually vanishes at tree level. But no-scale supersymmetry does not seem to emerge from string theory.[1] At present, no mechanism is known that would suppress the mass and coefficients below the level of Eqs. (10) and (11).

It might at first appear that models of dynamical SUSY breaking or models in which exact superpotentials are calculable, such as those employed in [5], might work as models of quintessence since in the large field limit W is calculable and its flat directions appear to be truly flat. While this it true, such models cannot provide quintessence in a universe that looks like ours. In our universe, SUSY is badly broken and that breaking is (generically) communicated to all fields in the theory. In general, only scalars which are already protected from receiving mass contributions (*e.g.*, Goldstone bosons) remain massless after SUSY-breaking.

Let us comment briefly on the possibility of constructing a quintessence model with $\phi \ll M_{\mathrm{P}}$, which might sufficiently suppress the quartic and non-renormalizable terms. In this case, V needs to be dominated by the constant term V_0, because no single term of the varying part of V will satisfy the flatness conditions on its own and we are trying to avoid delicate cancellations. Given the assumption that V vanishes in the true vacuum (achieved in the far future), V_0 will be a function of the other parameters in the potential,

[1] Let us mention the two popular examples. In weakly coupled heterotic string theory, no-scale supersymmetry corresponds to the case that the superpotential W is independent of the bulk moduli t_I, *i.e.*, $\partial W/\partial t_I = 0$. In the true vacuum, W is non-vanishing, and because of modular invariance one is unlikely to have $\partial W/\partial t_I = 0$. (In contrast, for ordinary inflation a potential of the no-scale form can be obtained [8], since the condition $V = M_S^4$ corresponds to $W = 0$ making it easy to achieve $\partial W/\partial t_I = 0$ without violating modular invariance.) In Horava-Witten M-theory, no-scale supersymmetry does not seem to emerge at all. Finally, one might mention that a recent proposal [9] eliminates the tree-level contribution to m^2, but does not suppress non-renormalizable interactions with the visible sector, so that $m^2 \sim M_W^4/M_{\mathrm{P}}^2$ generically, which is still much too large.

but the problem will be to explain its smallness. This difficulty explains, no doubt, why the literature does not contain any models of quintessence with $\phi \ll M_{\rm P}$.

5. We now turn to models of quintessence [2] in which ϕ is a pseudo-Goldstone boson. This corresponds to an approximate global $U(1)$ symmetry $\phi \to \phi + \text{const}$, and $V(\phi)$ is flat in the limit of exact symmetry. We focus on the usual case, that ϕ corresponds to the phase of a complex field Φ, which is in the bottom of a Mexican Hat potential

$$V = \lambda(|\Phi|^2 - \mu^2/2)^2 + \cdots . \tag{13}$$

At the bottom of the Mexican Hat we write $\Phi = (\mu/\sqrt{2})\exp(i\phi/\mu)$. The dots represent non-renormalizable terms and quantum corrections which may generate a potential for ϕ. For a model of quintessence (or ordinary inflation) it is convenient to set $\phi = 0$ at a maximum of the potential, near which inflation takes place.

In the limit of exact symmetry, $V(\phi)$ is perfectly flat. If the global $U(1)$ is explicitly broken to Z_N, a potential for ϕ is generated of the form

$$V(\phi) = \frac{1}{2}V_0[\cos(N\phi/\mu) + 1] = V_0 - \frac{1}{2}m^2\phi^2 + \cdots \tag{14}$$

where $m^2 = \frac{1}{2}N^2V_0/\mu^2$. Proposals exist [10] for obtaining the required value $V_0 \sim (10^{-3}\,\text{eV})^4$, but we still have to satisfy the flatness conditions, in particular Eq. (6). This requires $\mu \gg M_{\rm P}$, at which point we encounter the problem with using a pseudo-Goldstone boson for quintessence, or ordinary inflation.

As discussed in [8] for the latter case, a non-renormalizable term like $\lambda_d^{(\Phi)} M_{\rm P}^{4-d}\Phi^d + h.c.$ will have the generic magnitude $|\lambda_d^{(\Phi)}| \sim M_S^4/M_{\rm P}^4$ that we discussed before. A Z_N symmetry can eliminate many such terms, but at some order a term $\lambda_d^{(\Phi)} M_{\rm P}^{4-N}\Phi^N + h.c.$ will eventually lift the potential for ϕ. As long as $\mu \sim M_{\rm P}$, all such terms at any order may be regarded as equally dangerous. Alternatively, in the spirit of [7], we may suppose that $\lambda_d \propto 1/d! \sim e^{-d}$ for very large d, and ask to what order λ_d must then be eliminated. The answer is $d \sim \ln(M_S^4/V_0) \gtrsim 60\ln 10 \sim 240$, which seems quite unreasonable.

Dual to our discussion for the modulus of Φ, a possibility [11] which has not yet been explored (for either quintessence or ordinary inflation) is to suppose that one has a hybrid inflation model, where some field other than Φ is displaced from the minimum of the potential and gives a constant term V_0 which dominates. This would again allow $\mu \ll M_{\rm P}$, placing the non-renormalizable terms under control, but as before the problem would be to explain the tiny magnitude of V_0.

We have yet to consider the moduli fields emerging from string theory, which are not charged under symmetries acting on their phases. At present it does not seem that any of them will give quintessence. However, the dilaton field s does look hopeful at first sight. At $s \gg M_{\rm P}$ its potential is supposed to be of the form $V \propto e^{-cs}$ with $c \sim 1/M_{\rm P}$ and no corrections. This satisfies the flatness conditions, but does not lead to viable quintessence because the unified gauge coupling is proportional to $1/s$ and one cannot tolerate significant time-dependence for that coupling [5]. (In this article we are not considering another requirement often imposed on quintessence models, which is that in the early Universe quintessence should scale with the radiation/matter energy density. The dilaton violates that requirement too [5].)

Finally, it does not help to make ϕ a condensate rather than an elementary field. There are actually two possibilities here. One is that ϕ exists only *below* some mass scale $\Lambda \ll M_{\rm P}$, analogous to the situation for the Higgs in Technicolor extensions of the Standard Model. This makes things much worse, because the effective field theory now has an ultraviolet cutoff Λ and the natural value of the non-renormalizable coefficients λ_d defined in Eq. (5) is $\lambda_d \sim (M_{\rm P}/\Lambda)^{d-4} \gg 1$. (Equivalently, the coefficients are of order 1 if we replace $M_{\rm P}$ by Λ).

The opposite possibility, that ϕ exists only *above* some scale, is the one invoke [5] for the model of Eq. (4). Such a behavior would be expected, for example, if ϕ parameterizes a flat direction in a supersymmetric theory. However, this makes no difference at all to our discussion, because at large values of ϕ (which are required by slow roll), the theory is weakly-coupled and ϕ can be treated as a fundamental field with canonical normalization.

6. In contrast with the above situation, ordinary inflation need not involve fine-tuning. The basic reason is that V during ordinary inflation need not be small compared with the scale of supersymmetry breaking. Indeed, the only theoretical constraint is $V \leq M_S^4$, and in fact one has $V = M_S^4$ in most models of inflation[2]. The value Eq. (11) of the couplings λ_d, that can be achieved with supersymmetry, is then sufficient to satisfy the flatness condition Eq. (3) for $d > 2$, provided that the model is constructed so that $\phi \ll M_{\rm P}$. Finally, the mass term ($d = 2$) corresponding to Eq. (10) only marginally violates Eq. (3) ($m^2 \sim V$ instead of $m^2 \ll V$), and ways are known that will achieve the necessary marginal reduction without fine-tuning.

Acknowledgements

We would like to thank C. Csaki, A. D. Linde and J. Terning for helpful discussions. This work was supported in part by the Director, Office of Science, Office of High Energy and Nuclear Physics, Division of High Energy Physics, of the U.S. Department of Energy under Contract DE–AC03–76SF00098 and by NATO grant CRG–970214.

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Prepared for the U.S. Department of Energy under Contract No. DE-AC03-76SF00098