

UC Berkeley

UC Berkeley Electronic Theses and Dissertations

Title

Essays in Macroeconomics

Permalink

<https://escholarship.org/uc/item/7r9408x2>

Author

Asriyan, Vladimir A.

Publication Date

2014

Peer reviewed|Thesis/dissertation

Essays in Macroeconomics

by

Vladimir A. Asriyan

A dissertation submitted in partial satisfaction of the

requirements for the degree of

Doctor of Philosophy

in

Economics

in the

Graduate Division

of the

University of California, Berkeley

Committee in charge:

Professor Pierre-Olivier Gourinchas, Co-chair

Assistant Professor William Fuchs, Co-chair

Assistant Professor Demian Pouzo

Associate Professor Yuriy Gorodnichenko

Assistant Professor Brett Green

Spring 2014

Essays in Macroeconomics

Copyright 2014
by
Vladimir A. Asriyan

Abstract

Essays in Macroeconomics

by

Vladimir A. Asriyan

Doctor of Philosophy in Economics

University of California, Berkeley

Professor Pierre-Olivier Gourinchas, Co-chair

Assistant Professor William Fuchs, Co-chair

The research presented in this dissertation has been motivated by the Great Recession that has shown us once again how the financial system can amplify and propagate relatively mild economic shocks into larger scale recessions. In the past decade, we witnessed what many may call the worst crisis since the Great Depression that appears to have resulted from a perverse interaction between real estate markets and borrowers' balance sheets. The three chapters of this dissertation are an attempt to shed light on the mechanisms that may have allowed for such perverse effects to arise. I believe that to understand crisis episodes such as the recent one, it is imperative to study why some economic agents become overly exposed to risk, why financial markets fail to function at times, and what policy makers can do to ameliorate the incidence and repercussions of such adverse events.

In Chapter 1, "*A Theory of Balance Sheet Recessions with Informational and Trading Frictions*," I propose a novel theory to rationalize the limited risk-sharing that drives *balance sheet recessions* as a result of informational and trading frictions in financial markets. I show that borrowers and creditors will find it costly to share macroeconomic risk in environments where creditors value the liquidity of financial claims but where information about the future states of the economy is *dispersed* and the secondary markets for financial claims feature *search* frictions. As a result, borrowers will optimally choose to retain disproportionate exposures to macroeconomic risk on their balance sheets, and adverse shocks will be amplified through the balance sheet channel. I show that the magnitude of this amplification becomes closely linked to the level of information dispersion and the severity of search frictions in financial markets. In this setting, I study the implications of the theory for macro-prudential regulation and find that subsidizing contingent write-downs of borrowers' liabilities can be welfare improving.

In Chapter 2, "*Informed Intermediation over the Cycle*", a joint work with Victoria Vanasco, we construct a dynamic model of financial intermediation in which changes in the information held by financial intermediaries generate asymmetric credit cycles as the ones

documented by Reinhart and Reinhart (2010). We model financial intermediaries as “expert” agents who have a unique ability to acquire information about firm fundamentals. While the level of “expertise” in the economy grows in tandem with information that the “experts” possess, the gains from intermediation are hindered by informational asymmetries. We find the optimal financial contracts and show that the economy inherits not only the dynamic nature of information flow, but also the interaction of information with the contractual setting. We introduce a cyclical component to information by supposing that the fundamentals about which experts acquire information are stochastic. While persistence of fundamentals is essential for information to be valuable, their randomness acts as an opposing force and diminishes the value of expert learning. Our setting then features economic fluctuations due to waves of “confidence” in the intermediaries’ ability to allocate funds profitably.

In Chapter 3, “*Credit Crises, Liquidity Traps, and Demand Externalities*,” I extend the work of Eggertsson and Krugman (2012) to study welfare implications of households’ consumption-saving decisions in New Keynesian economies with incomplete asset markets. My contribution is to show that due to aggregate demand externalities the amount of debt pre-contracted in such economies is generally excessive, and that the amount of “over-borrowing” is increasing in the Central Banker’s inflation-aversion. This externality arises because an individual household does not internalize its contribution to the overall fragility as the latter is only a function of aggregate indebtedness of all borrowers. These findings suggest that macro-prudential policies geared towards limiting household leverage can indeed be welfare improving.

To my grandfather, Karlos A. Parikian

Contents

Contents	ii
List of Figures	iv
List of Tables	v
1 A Theory of Balance Sheet Recessions with Informational and Trading Frictions	1
1.1 Introduction	1
1.2 The Model	6
1.3 Equilibrium	9
1.4 Equilibrium Characterization	14
1.5 Policy Implications	26
1.6 Extensions	30
1.7 Conclusions	33
2 Informed Intermediation Over the Cycle	35
2.1 Introduction	35
2.2 The Model	39
2.3 Optimal Intermediation Contracts	41
2.4 The Dynamic Economy	45
2.5 Conclusions	51
3 Credit Crises, Liquidity Traps, and Demand Externalities	52
3.1 Introduction	52
3.2 The Model	54
3.3 Equilibrium Characterization	57
3.4 Welfare Analysis	63
3.5 Conclusions	69
Bibliography	74
A Appendix for Chapter 1	80

B Appendix for Chapter 2

List of Figures

1.1	Timeline	8
1.2	Equilibrium Fluctuations (a)	23
1.3	Equilibrium Fluctuations (b)	24
1.4	Competitive Equilibrium vs. Social Optimum	28
2.1	Learning by Lending	47
2.2	Structural Shocks	49
2.3	Sectoral Reallocation	50
3.1	Crises and Interest Rates	70
3.2	Crises and Output	70
3.3	Discount Factors	71
3.4	Size of Fluctuations (a)	71
3.5	Size of Fluctuations (b)	72
3.6	Inflation Target (a)	72
3.7	Inflation Target (b)	73

List of Tables

3.1	<i>Baseline Calibration</i>	68
-----	---------------------------------------	----

Acknowledgments

I am highly indebted to the co-chairs of my dissertation, Pierre-Olivier Gourinchas and William Fuchs. Their overarching grasp of the discipline has always been an inspiration to me, and at times when my work has gone astray, their invaluable, timely, and sharp advice has been more than sufficient to put me back on track. I thank Demian Pouzo for the innumerable hours he devoted to aiding me with the many challenges that I encountered throughout my work. Yuriy Gorodnichenko has always been a source of encouragement for me, and I am grateful to him for igniting in me the spark that led me to study Macroeconomics. I thank Brett Green for the many stimulating conversations that were very valuable in the later stages of my dissertation, and my undergraduate advisers Julie Cullen, Navin Kartik, and Michael Bernstein who continued to mentor me throughout my graduate studies.

The feedback I received from other Berkeley faculty members was also instrumental in completing my dissertation. I benefited greatly from the deep insights of Maury Obstfeld and David Romer, and from the numerous discussions with Haluk Ergin, Ben Faber, Nicolae Gârleanu, Amir Kermani, Gustavo Manso, Atif Mian, Marcus Opp, Christine Parlour, Andrés Rodríguez-Clare, and Johan Walden. I want to also thank seminar participants at the UC Berkeley Department of Economics and the Haas School of Business, the Paris School of Economics, Bocconi University, CREI and Universitat Pompeu Fabra, EIEF Rome, the Federal Reserve Board, and the World Bank Research Department for helpful comments and suggestions on the various parts of this dissertation. During my time in Berkeley, I also learned a great deal about economics and many other things from my incredibly talented classmates and colleagues: Miguel Almunia, Liang Bai, Matthew Botsch, Pamela Campa, Sebastian Di Tella, Isaac Hacamo, Emiliano Huet-Vaughn, Claire Lebarz, Matteo Maggiori, Pascal Michailat, John Mondragon, Takeshi Murooka, Aniko Öery, Michaela Pagel, Gautam Rao, Ana Rocca, Peri Saygin, Michel Serafinelli, Slavik Sheremirov, Lucas Siga, Asrat Tesfayesus, and Johannes Wieland.

Throughout these years in Berkeley, I have come to realize how important friends are in one's life. It has been a gift to have had the continued presence and support of my dear friends Santiago Halty, Scott Kostyshak, Joshua Norton, and Stephanie Alfonso, who have remained very close to me since my undergraduate years in San Diego. Meeting my new friends Ana Rocca, Michel Serafinelli, and Asrat Tesfayesus has had a tremendous impact on my life. Ana's energy and passion have always been inspirational to me; Michel has made me realize how impossible it is to fully comprehend the human mind; and the never-ending debates with Asrat have convinced me that 'truth' is a subtle and, perhaps, an illusory concept (one of the few things we have come to agree upon).

Here at Berkeley, I also met the love of my life, Victoria Vanasco. She and I connected at the beginning of our graduate studies and we've been inseparable since. We have supported each other through the ups and downs of the roller-coaster life that a Ph.D student's life can be. Together we went through all the difficult moments of grad school, and together also we enjoyed all the beautiful moments that the past few years have afforded to us. I am

infinitely thankful to Victoria for playing such an important role in this period of my life, and I am looking forward to our promising journey together in the years to come.

Finally, and most importantly, I am grateful to my wonderful family. To my mother, Marina, for being the best mother one can wish for. To my stepfather, Paul, for providing me with all the needed support during these challenging but rewarding years of my life. To my brother, Anthony, whom I love greatly and who has always made me want to improve and become a better role model for him. To my grandfather, Karlos, who passed away last year but who throughout his life has set for me the standard for hard work and honesty that I will strive to adhere to for the rest of my life.

Chapter 1

A Theory of Balance Sheet Recessions with Informational and Trading Frictions

1.1 Introduction

The Great Recession has once again underscored the significant role that financial frictions play in the transmission and propagation of macroeconomic shocks. The macroeconomic literature on credit frictions has long recognized that the conditions of household and firm balance sheets are important drivers of macroeconomic activity. In normal times, a well-functioning financial system allows economic agents to use leverage to undertake productive investments and thus enhance economic growth and prosperity. In bad times, disproportionate exposures to macroeconomic risk by these leveraged agents can lead to perverse feedback effects between asset prices and balance sheets that can turn shocks of modest magnitude into full-blown *balance sheet recessions*.¹ The theory, however, remains incomplete. While the literature has shown how limited sharing of macroeconomic risk between borrowers and creditors can generate balance sheet recessions, we do not have a complete understanding of why risk-sharing become so limited to begin with.² Answering this question is particularly

¹Balance sheet recessions refer to recessions driven by feedback effects between borrowers' balance sheets and general economic activity (e.g. asset prices, aggregate demand). The seminal papers in the literature on balance sheet recessions are Bernanke and Gertler (1989)[9] and Kiyotaki and Moore (1997)[46]. Some of the more recent contributions are He and Krishnamurthy (2011)[43] and Brunnermeier and Sannikov (2012)[13]. Mian et al. (2013)[59] and Adrian et al. (2012)[1] provide empirical support for the importance of balance sheets in the recent recession.

²In standard models of the balance sheet channel, it is typically assumed that agents are unable to write contracts contingent on aggregate states of the economy (e.g. Kiyotaki and Moore (1997)[46], Brunnermeier and Sannikov (2012[13])). The papers by Krishnamurthy (2003)[49] and Di Tella (2013)[25], however, show that amplification effects disappear when contingent contracts are allowed.

important in view of the recent policy discussions on how to regulate systemic risk.³

In this paper, I propose a novel theory that rationalizes limited sharing of macroeconomic risk that drives balance sheet recessions as a result of informational and trading frictions in financial markets. I show that when borrowers issue contracts contingent on the state of the economy, they need to trade off the risk-sharing benefits that contingent contracts provide with the ‘illiquidity’ costs that they must pay creditors for holding such contracts. These costs arise because creditors value claims that are liquid in secondary markets and because *information dispersion* and *search frictions* in these markets prevent contingent contracts from trading at their ‘fair’ value. As a result, borrowers optimally choose to retain disproportionate exposures to macroeconomic risk on their balance sheets, and adverse shocks become amplified through the balance sheet channel. I show that the magnitude of this amplification becomes closely linked to the level of information dispersion and the severity of search frictions in financial markets. In particular, the model predicts that amplification effects should be expected to be large when information dispersion and search frictions are large, and vice versa. In this setting, I study the implications of the theory for macro-prudential regulation and, consistent with some recent policy proposals,⁴ I find that policy measures geared towards subsidizing contingent write-downs of borrowers’ liabilities can be welfare improving.

This paper uses a model of the financial accelerator *à la* Kiyotaki and Moore (1997)[46]. Entrepreneurs (borrowers) issue financial contracts to investors (creditors) to finance long-term projects whose cash-flows are exposed to aggregate risk. If these cash-flows fall short of the promised repayments, then entrepreneurs must meet their liabilities by liquidating projects prematurely. In aggregate, such liquidations lead to ‘fire-sales’ and become amplified through the endogenous interaction of entrepreneurial balance sheets and asset prices, thus generating balance sheet recessions. To understand the risk-sharing benefits of contractual contingencies, I allow economic agents to write financial contracts contingent on the aggregate state of the economy. More specifically, entrepreneurs can contract with investors ex-ante to write-down some of their liabilities in the adverse state of the world. In the absence of secondary market frictions, both entrepreneurs and investors will find such write-downs mutually beneficial; as they are introduced, the aggregate impact of macroeconomic shocks becomes endogenously muted.

To explain why such contingencies in liabilities may be limited, I augment the basic model in three directions. First, I suppose that investors may experience idiosyncratic funding (liquidity) needs prior to the maturity of financial contracts and thus want to sell these contracts in secondary financial markets. These funding needs intend to capture a variety

³See, for example, Calomiris and Herring (2011)[16] and McDonald (2011)[58] for a discussion of Contingent Convertible (CoCo) debt requirements, and Mian (2011)[62] for the introduction of contingent write-downs of households’ liabilities.

⁴For example, Mian (2010)[62] states that contingent write-downs could have considerably ameliorated the negative repercussions of the deleveraging-aggregate demand cycle of the Great Recession. Similarly, Calomiris and Herring (2011)[16] argue that, with such write-downs in place, we would have avoided the financial meltdown of 2008.

of reasons (e.g. intermittent consumption/investment opportunities) for which investors may want to trade financial contracts before they mature. Second, I assume that secondary markets for financial contracts are subject to a form of search friction: an investor who wants to sell his contract can solicit offers only from a finite number of buyers. This assumption is meant to capture the various 'search' frictions that may limit the number of potential counterparties that an investor can trade with on short notice.⁵ These types of frictions are particularly prevalent in over-the-counter type markets where many important assets (corporate bonds, asset-backed securities, a wide range of derivatives, etc.) are traded, as well as in centralized markets that are relatively thin. Finally, I introduce information dispersion by supposing that, prior to trade in financial markets, investors observe noisy private signals about the future state of the economy. This assumption is supported by a growing literature in macroeconomics that documents substantial disagreements among economic agents about a variety of macroeconomic variables; this literature further shows that an important reason for these disagreements is the disparity of information held by economic agents.⁶

These ingredients then deliver the main results of the paper. I show that in equilibrium contracts contingent on the state of the economy are expected to trade at a discount in secondary markets. This discount arises because of informational rents that investors must forgo when selling their contracts in secondary markets, and it is directly linked to the level of informational dispersion and the severity of search frictions in these markets.⁷ As investors rationally anticipate future liquidity needs, they pass these (trading) costs of contractual contingencies onto entrepreneurs, who then face a tradeoff between insurance against macroeconomic fluctuations and the scale of their operations. This gives rise to a 'pecking order' theory for liability design: entrepreneurs prefer to borrow with non-contingent contracts, and write-downs are only introduced when entrepreneurs expect shocks to be sufficiently severe. Finally, as entrepreneurs optimally design their liabilities, the costs of contractual contingency also limit the extent to which macroeconomic fluctuations are endogenously stabilized. The model thus closely links the magnitude of amplification effects that arise in response to economic shocks to the level of information dispersion and the severity of search frictions in financial markets.

The theory suggests that financial market microstructure (i.e. depth) is an important determinant of the extent of risk-sharing and thus of macroeconomic fluctuations. Consistent with this, Shiller (1998)[73] argues that establishing and promoting liquid markets for a

⁵See Duffie (2010)[29] for a discussion of the relevance of search frictions in financial markets. See Zhu (2011)[79] and Duffie and Manso (2007)[31] for related models of financial markets.

⁶Mankiw et al. (2004)[55] find substantial heterogeneity in inflation forecasts among professional forecasters, economists, and consumers, and Dovern et al. (2009)[28] show similar findings for GDP and prices in a cross-country study of surveys of professional forecasters. Coibion and Gorodnichenko (2012)[20] provide extensive evidence that informational disagreements are pervasive across a variety of population groups and macroeconomic variables.

⁷This insight is drawn from the literature on common value auctions (e.g. Milgrom and Weber (1982)[64]), which shows that, in a variety of trading mechanisms, sellers forgo informational rents when faced with buyers who have informational disagreements about the object being offered for sale.

variety of macroeconomic risks would significantly improve households' welfare by allowing them to better manage their risks. Englund et al. (2002)[34] provide an empirical evaluation of these welfare gains in the context of real estate related risks. That liquid secondary markets may further smooth out the business cycle has, for example, been noted by Case et al. (1993)[18]. In line with these claims, my model also predicts that when secondary markets are more liquid (i.e. trading frictions are small), then insuring macroeconomic risks is less costly and fluctuations due to amplification effects are smaller.

The theory also has implications for the optimal design of macro-prudential regulation. I show that a policy of capping (or taxing) entrepreneurial borrowing against adverse states of the world can indeed be welfare improving. This result is due to pecuniary externalities that arise in times of systemic distress: atomistic entrepreneurs undervalue the social benefits of insurance which come from the general equilibrium effects that it has on asset prices. This finding thus suggests that policy makers should actively encourage (subsidize) contingencies in borrowers' liabilities as their stabilizing benefits are likely to remain uninternalized by economic agents. Furthermore, because amplification effects in my framework become closely linked to informational and trading frictions in financial markets, the model also suggests that policy efforts to improve transparency and competition in financial markets may have an extra benefit of stabilizing the macroeconomy.⁸

Related Literature. This paper is related to a large literature on the balance sheet channel. Bernanke and Gertler (1989)[9] and Kiyotaki and Moore (1997)[46] are the seminal contributions to the literature. Some of the more recent works include Bernanke, Gertler, and Gilchrist (1999)[8], He and Krishnamurthy (2012)[43], Brunnermeier and Sannikov (2012)[13]. Much of this literature eliminates risk-sharing considerations by assuming that economic agents are unable to issue contracts contingent on the state of the economy. In contrast, I allow economic agents to write contracts contingent on aggregate states in order to understand why risk-sharing may be limited endogenously. My work is thus closer in spirit to the recent papers by Krishnamurthy (2003)[49], Korinek (2011)[47], Rampini and Viswanathan (2009)[70], and Di Tella (2013)[25] who also study the balance sheet channel when contingent contracts are feasible.

Krishnamurthy (2003)[49] showed that, within the Kiyotaki and Moore (1997)[46] framework, allowing economic agents to write contingent contracts mutes balance sheet amplification and thus has a stabilizing effect on macroeconomic fluctuations. He proposes to explain periods of amplification and limited risk-sharing by problems of commitment on the side of the lenders.⁹ Lenders' commitment, however, is only necessary for equilibrium risk-sharing if debt write-downs are insufficient for borrowers to avoid costly liquidations. In my

⁸For example, the Dodd-Frank Act of 2010 has provisions relating to the transparency and stability of derivatives markets: it requires public disclosure of volumes and price data of standardized derivatives. Furthermore, a wide range of derivatives will be required to be traded on open platforms and cleared centrally. The model suggests that these type of policies may be stabilizing if, of course, they do not introduce other forms of transaction costs.

⁹This insight is related to the literature on asset pricing with market incompleteness arising from lack of commitment (e.g. Alvarez and Jermann (2000)[2], Chien and Lustig (2009)[19]).

framework, debt write-downs are in fact sufficient for risk-sharing but they are endogenously limited (or absent altogether) as they expose lenders to secondary market ‘illiquidity.’

Rampini and Viswanathan (2009)[70] show that, in the presence of collateral constraints, borrowers may optimally choose to forgo risk-sharing opportunities if their funding needs are sufficiently high ex-ante. In particular, they argue that this may potentially explain why poor households and small firms engage insufficiently in hedging macroeconomic risks. While their mechanism arises in my framework as well, I am also able to explain limited sharing of macroeconomic risks when borrowers’ financing needs do not override their risk-sharing concerns.

Korinek (2011)[47] limits equilibrium risk-sharing by supposing that lenders are more risk-averse than borrowers. Di Tella (2013)[25] instead argues that the type of aggregate shock hitting the economy matters. In particular, he shows that in standard models of the balance sheet channel volatility shocks can create balance sheet recessions. My work is complementary to these papers. In my framework, agents have symmetric risk attitudes but creditors have a preference for liquidity; furthermore, the amplification effects in my framework depend on the type of shock that hits the economy through the level of information dispersion and search frictions in markets where contingent contracts are traded.

The limits to risk-sharing in my paper result from discounting of contingent contracts in secondary markets. The secondary market setting in my model, in fact, resembles a collection of first-price common value auctions for financial contracts, but where the seller may also be adversely selected. Thus, the paper is also related to the classic literature on common value auctions (e.g. Wilson (1977)[78] and Milgrom and Weber (1982)[64]). An important insight from this literature is that in a wide variety of trading mechanisms sellers must forgo informational rents when faced with buyers who disagree about the value of the object being offered for sale.¹⁰ Hence, the logic that drives costly contractual contingencies extends well beyond the basic environment presented in this paper.

That liquidity concerns may play an important role in precipitating financial crises through their impact on contractual design has also been emphasized in a recent paper by Dang et al. (2010)[22]. They argue that debt contracts are ‘best’ at providing liquidity to economic agents but that they also expose the financial system to fragility: ‘bad’ news may induce economic agents to acquire private information about contracts being traded, and thus destroy trade. In contrast to their paper, crises (balance sheet recessions) in my paper are driven by limited risk-sharing that arise from such liquidity concerns. Furthermore, because information is dispersed rather than exclusively private, trading/search frictions are essential to generate costs to contractual contingency.¹¹ This is one of the important differences with the traditional literature on security design in the presence of informational asymmetries (e.g. Myers and Majluf (1984)[67], DeMarzo and Duffie (1999)[23]).

¹⁰See, for example, Axelson (2007)[7] for an application to the problem of a firm that designs a security to raise funds from more informed investors.

¹¹While in reality agents may differ in their ability to process information, it is unlikely that any agent has exclusive access to information about aggregate states.

The welfare implications of my paper are related to the recently growing literature on macro-prudential regulation (e.g. Lorenzoni (2008)[54], Korinek (2011)[47], Stein (2010)[42]). As in this literature, my policy results are driven by the pecuniary externalities that arise in periods of systemic distress. One of the interesting differences with the existing analyses is that my framework features the possibility that economic agents endogenously borrow with the non-contingent claims but the planner wants to introduce contingencies into financial contracts. Thus, the model can also rationalize the recent policy proposals to introduce macro-contingencies into firms' and households' liabilities.

Finally, my paper contributes to the literature that investigates the macroeconomic implications of heterogeneity of information. Lucas (1972) is the seminal paper in this literature, and Angeletos and La'O (2009, 2011)[4][5] and Lorenzoni (2006)[53] are some of the recent contributions. These papers also study how information dispersion may help propagate macroeconomic shocks. My work contributes to this literature by showing that informational dispersion influences fluctuations by limiting the extent to which macroeconomic risks can be shared.

Organization. The paper is organized as follows. In Section 1.2, I describe the economic environment. In Section 1.3, I setup the economic agents' problems and I define the equilibrium of the economy. In Section 1.4, I characterize the equilibrium of the economy and I analyze the policy implications of the theory in Section 1.5. In Section 1.6, I consider several extensions, and I conclude in Section 1.7. All proofs are relegated to the Appendix.

1.2 The Model

The model has three periods, $t = 0, 1, 2$. There are two sets of agents, entrepreneurs and investors, each of unit mass. Entrepreneurs have no endowments, are risk-neutral, and consume only in period 2, so their preferences are given by $\mathbb{E}\{\tilde{c}_2\}$. Investors receive endowment e in all periods, are risk-neutral, and are exposed to idiosyncratic liquidity needs $\beta \in \{0, 1\}$. In particular, if an investor is hit with a liquidity shock in $t = 1$, he becomes an "early" type ($\beta = 1$) and does not value consumption in period 2: his preferences are $\mathbb{E}\{c_0 + c_1\}$. Otherwise, an investor becomes a "late" type ($\beta = 0$) and values consumption in all periods: his preferences are $\mathbb{E}\{c_0 + c_1 + c_2\}$. Investors are ex-ante identical and the probability of experiencing an idiosyncratic liquidity need in $t = 1$ is given by λ . Thus, λ also denotes the fraction of investors who become early types in $t = 1$.¹²

Entrepreneurial Technology. Entrepreneurs have access to long-term investment projects. An entrepreneur can install k units of capital in period 0 at a convex cost $\chi(k)$, and each

¹²This modeling device is meant to capture investors' preference for holding liquid claims. For example, an investor may have new investment/consumption opportunity, or an institution may experience a 'run' and need to sell assets to pay its depositors. The main results of the paper do not depend on this particular approach.

unit of capital delivers a cash-flow a at the *beginning* of period 2 and a cash-flow A at the *end* of period 2. The final cash-flow A should be interpreted as the continuation payoff of the project. Entrepreneurial cash-flows are stochastic and vary with the aggregate state of the economy, which is realized in period 2 and is denoted by $s \in \{l, h\}$ with $\Pr(s = l) = \pi(l)$. In particular, the cash-flow a takes values in $\{a(l), a(h)\}$ with $a(l) < a(h)$, while the cash-flow A remains deterministic.¹³

Liquidation and Fire-Sales. In period 2, when in need of funds, an entrepreneur has the option to liquidate her capital in a competitive capital goods market. If an entrepreneur has begun her project at scale k and the price of capital is q , she can liquidate a fraction $z \in [0, 1]$ of her capital and receive qzk units of the consumption good. The remaining $(1 - z)k$ units of capital deliver $A(1 - z)k$ units of the consumption good at the end of the period. Thus, entrepreneurs may face a trade-off: they can liquidate capital pre-maturely to raise funds in the capital goods market vs. they can keep capital intact to receive a per unit return A in the future.

The units of capital that entrepreneurs liquidate are absorbed by a “traditional” sector, that is composed of a mass of competitive firms. Each of these firms can convert capital goods to consumption goods according to an increasing and concave production technology $g(\cdot)$ that satisfies $g'(0) = A$. Thus, the productivity of firms in this sector decreases below that of entrepreneurs as the units of capital employed by these firms increases. This technological assumption is a simple way to introduce amplification effects in the form of ‘fire-sales’.¹⁴

Information. Information about the state of the economy arrives between periods 0 and 1. Investor $i \in [0, 1]$ observes private signal $x_i \in [\underline{x}, \bar{x}]$. The signals $\{x_i\}$ are distributed independently across investors conditional on the state, with a continuously differentiable conditional cdf denoted by $F_s(\cdot)$ for $s \in \{l, h\}$. The conditional distributions are related by the monotone likelihood ratio property; that is, $\frac{f_h(x)}{f_l(x)}$ is increasing in x on $[\underline{x}, \bar{x}]$, where $f_s(\cdot)$ denotes the pdf of signal x_i conditional on state $s \in \{l, h\}$. In addition, I suppose that signals are boundedly informative.¹⁵

Financial Contracts. In period 0, to finance projects, entrepreneurs raise funds from investors by issuing financial contracts to them. An entrepreneur approaches an investor and makes him a take-it-or-leave-it contractual offer that specifies a desired loan amount L and state-contingent repayments $b(s)$ and $B(s)$ made at the beginning and at the end of period 2, respectively.¹⁶ The investor can accept or reject the offer. If he rejects, the entrepreneur

¹³The fluctuations in the intermediate cash-flow capture any fluctuations in the firms’ ability to repay its creditors while in operation. For example, firms may experience demand fluctuations; or financial institutions may experience losses on existing positions (e.g. mortgages).

¹⁴See, for example, Kiyotaki and Moore (1997)[46] for a similar modeling approach. The idea of ‘fire-sales’ in response to common industry shocks goes back to Shleifer and Vishny (1992)[74].

¹⁵There exist positive constants $\underline{\phi}, \bar{\phi}$ such that $\underline{\phi} < \frac{f_h(x)}{f_l(x)} < \frac{f_h(\bar{x})}{f_l(\bar{x})} \leq \bar{\phi}$.

¹⁶As there are no gains from diversification, the assumption of one-to-one matching between entrepreneurs and investors is without loss.

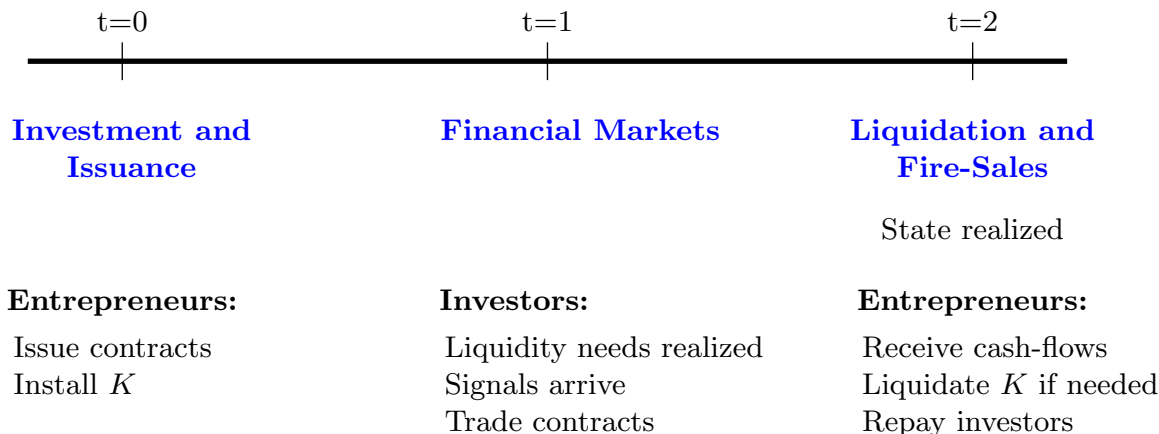


Figure 1.1: **Timeline**

does not invest. Investors' idiosyncratic liquidity needs are assumed to be non-verifiable and thus non-contractable; furthermore, to motivate trade in secondary markets, I assume that there is no centralized mechanism by which investor liquidity needs can be pooled (e.g., Diamond and Dybvig (1983)[27]).

As in Lorenzoni (2008)[54], entrepreneurial promises of repayment must be backed by her capital plus a fraction $\theta \in (0, 1)$ of the project's cash-flows. This friction can be microfounded by assuming that the entrepreneur can always default and walk away with a fraction $1 - \theta$ of her cash-flows; this threat of default and renegotiation then puts a limit on the entrepreneur's borrowing capacity. Finally, I assume that investors do not have claims to back up promises and they are thus unable to borrow. Thus, if the scale and the fraction of the project liquidated are k and $z(s)$ respectively, the financial contracts must satisfy the following no-default conditions for $s \in \{l, h\}$:

$$\begin{aligned} 0 \leq b(s) + B(s) &\leq (\theta a(s) + q(s))k \\ 0 \leq B(s) &\leq \theta A(1 - z(s))k \end{aligned}$$

Secondary Markets. In period 1, *after* liquidity needs and signals are realized, investors participate in secondary markets where they can sell and/or buy financial contracts. If an investor chooses to sell his contract, he solicits *private* price offers from a finite number $n \geq 2$ of buyers and commits to sell to the highest bidder.¹⁷ If an investor chooses to become buyer, then he submits a price offer to a contacting seller and receives the contract if his offer is highest. Contacts are established according to a random process that matches each seller

¹⁷The assumption of commitment to sell is imposed for simplicity. In the baseline case, where investor liquidity needs are observable, this assumption is not binding. However, when liquidity needs are unobservable, this assumption allows me to eliminate the possibility that investors solicit offers, learn other investors' information, but choose not to trade.

with n buyers. A detailed description of investors' sorting strategies and of the matching process is deferred to Section 3.2. Once all trades are executed, secondary markets close, investors consume and holders of financial contracts wait to receive contractual repayments at date 2.

Timeline. The heuristic timeline of the economy is illustrated in Figure 1. To summarize, in period 0, entrepreneurs issue contracts in order to raise funds and invest. In period 1, after liquidity needs and signals are realized, investors are matched to trade in secondary markets. In period 2, the aggregate state is realized, entrepreneurs receive cash-flows, liquidate capital if needed, and repay liabilities.

1.3 Equilibrium

In this section, I setup the problems solved by entrepreneurs and investors, and then I define the general equilibrium of the economy.

Entrepreneurs' Problem

In period 0, an entrepreneur chooses how much to invest and raises funds by proposing contractual terms $\{L, b(s), B(s)\}$ to an investor. An investor accepts the contract if L , the loan amount, does not exceed the expected present value of the repayments to the investor, which in this section I take as given and denote by $\mathcal{L}(\{b(s), B(s)\})$. I refer to the function $\mathcal{L}(\cdot)$ that maps a financial contract to a maximal loan amount as the *contract price schedule*.

Entrepreneur takes the prices $\{q(s)\}$ of capital goods and the schedule $\mathcal{L}(\cdot)$ as given and chooses her investment and the contractual terms in order to maximize her expected period 2 consumption subject to the investor participation constraint, the no-default conditions, and her budget constraints in period 0 and 2. Entrepreneur's problem is thus given by

$$\max_{\{k, L, b(s), B(s), z(s), \tilde{c}_2(s)\}} \mathbb{E}\{\tilde{c}_2(s)\}$$

subject to

$$\tilde{c}_2(s) = a(s)k + q(s)z(s)k - b(s) + A(1 - z(s))k - B(s) \quad (1.1)$$

$$\chi(k) = L \leq \mathcal{L}(\{b(s), B(s)\}) \quad (1.2)$$

$$b(s) \leq a(s)k + q(s)z(s)k \quad (1.3)$$

$$0 \leq b(s) + B(s) \leq (\theta a(s) + q(s))k \quad (1.4)$$

$$0 \leq B(s) \leq \theta A(1 - z(s))k \quad (1.5)$$

$$0 \leq z(s) \leq 1 \quad (1.6)$$

for $s = l, h$.

Constraint (1) is the entrepreneurial budget constraint in period 2, where entrepreneurs consume the returns of the project net of repayments. Constraint (2) is the period 0 budget constraint combined with the investor participation constraint. Because there are no informational asymmetries in period 0, it is without loss to assume that entrepreneurs issue contracts that investors accept. Constraint (3) states that the entrepreneur must be able to cover her repayment $b(s)$ from the intermediate cash-flow $a(s)k$ and capital goods liquidations $q(s)z(s)k$. Constraints (4) and (5) are the no-default conditions and, finally, constraint (6) states that the fraction $z(s)$ of capital goods liquidated must lie between zero and one.

The prices of capital goods in period 2 are determined by the traditional firms' optimal demand for capital and the clearing condition in the capital goods market. The profits (if any) from these firms' operations are rebated lump sum to late investors. Before proceeding further, I make the following parametric assumptions that simplify the subsequent analysis,

Assumption 1 *I assume that (1) $(g'(x) - \theta A)x$ is increasing in x , (2) $\chi'(x) - \frac{\chi(x)}{x}$ is increasing in x , and (3) $a(l) < A \leq a(h)$.*

Assumption 1.1 ensures equilibrium uniqueness in the capital goods market, Assumption 1.2 ensures that entrepreneurs face decreasing returns to scale, and Assumption 1.3 implies that entrepreneurs will not need to liquidate capital in state h .¹⁸ An immediate result that follows from Assumption 1.1 is that, in equilibrium, the prices of capital goods satisfy

$$q(s) = g'(z(s)k) \in (\theta A, A] \tag{1.7}$$

for $s = l, h$.¹⁹ Entrepreneurs thus face a downward sloping inverse demand schedule for capital that is bounded above by the return to capital at its best use, A , and below by the portion of this return that can be credibly promised to investors, θA .

Investors' Problem

In period 0, an investor's problem is to accept or reject the entrepreneur's contractual offer. In period 1, investors participate in secondary markets where they can trade contracts. In period 2, investors consume the payments received on their contractual holdings and the profits from the traditional firms' operations. Since investors' decision whether to accept the contract in period 0 will depend on investors' buying and selling decisions in secondary markets, I solve the investors' problem by backwards induction.

¹⁸The first two assumptions are standard. See, for example, Lorenzoni (2008)[54] and Jeanne and Korinek (2012)[45]. The third assumption is only made for simplicity; the analysis is qualitatively the same if it is relaxed.

¹⁹See Lemma 0 in Appendix.

Investors' Problem at $t=1$: Trading Contracts in Secondary Markets

Let $\mathcal{C} = \{b(s), B(s)\}$ denote the financial contract offered by the entrepreneurs in $t = 0$. After liquidity needs and signals are realized, investors enter secondary markets and decide whether to post their contracts for sale and whether to become potential buyers of contracts from other investors. Let $(x, \beta) \in [\underline{x}, \bar{x}] \times \{0, 1\}$ denote the type of an investor who has received signal x and has a liquidity need β .

Sorting into Buying and Selling. I make the following indifference-breaking assumptions. First, I suppose that an investor becomes a potential buyer only if he is willing to pay a positive price for some contract. Second, I suppose that an investor posts his contract for sale only if he strictly prefers to do so.²⁰ Since only investors of type $\beta = 0$ value consumption in period 2, we immediately have that an investor is a potential buyer if and only if he has $\beta = 0$. On the other hand, an investor's posting decision may depend both on his liquidity need and his signal. Let $\gamma_i \in \{0, 1\}$ denote investor i 's decision whether to sell his contract or not, with $\gamma_i = 1$ iff he decides sell. I consider symmetric posting strategies in which conditional on the contract \mathcal{C} , the strategy γ_i is a measurable function of the investor's type. In particular, for an investor i who has type (x, β) and holds contract \mathcal{C} , $\gamma_i = \gamma(x, \beta, \mathcal{C})$.

Matching Buyers and Sellers. Each seller is matched randomly with n buyers, while each buyer is matched with at most one seller.²¹ In the Appendix, I construct an explicit matching function that has these features, and I provide the conditions on the primitives that ensure that the buyer-to-seller ratio is consistent with such matching.

The secondary market is thus a collection of sub-markets, each with a single seller and n buyers. Given contract \mathcal{C} , each sub-market is fully characterized by the type (x^S, β^S) of the seller, and the signals $\{x_1^B, \dots, x_n^B\}$ of the buyers (recall that all buyers have $\beta = 0$). In the main analysis, I assume that buyers observe the liquidity need of the seller. As I show below, this assumption eliminates the problem of adverse selection on the side of the seller. Because discounting of contingent contracts occurs due to informational rents earned by buyers, the main results of the paper are illustrated best in this baseline case. I relax this assumption in Section 6 and show that the results of the paper remain robust.

Assumption 2 (Baseline) *Buyers observe the liquidity need β^S of the seller.*

Buyer's Payoff. When a buyer is contacted by a seller, he submits a price offer for the seller's contract. I consider symmetric offer strategies, in which conditional on contract \mathcal{C} , the price offer that a buyer submits is a measurable function of his own signal and the liquidity need of the seller. In particular, a buyer who has received signal x and is matched with a seller

²⁰The first assumption eliminates trivial offers from equilibrium, while the second assumption minimizes the ratio of sellers-to-buyers in the economy; it can be rationalized by assuming that there is a small cost to posting contracts.

²¹The assumption that a buyer is contacted by at most one seller is made for simplicity; the results are similar if a buyer is matched with multiple sellers but when trades are executed simultaneously.

with a liquidity need β^S submits offer $p(x, \beta^S, \mathcal{C})$. In what follows, I omit the superscript S on the seller's liquidity need and his signal whenever this dependence is clear. When a buyer is matched with a seller, he (possibly) makes an inference about the seller's signal. The reason why a buyer cares about the seller's signal is that it helps him make an inference about the state of the economy and thus about the payoffs of the entrepreneurial contracts. If other buyers follow offer strategy $p(\cdot, \beta, \mathcal{C})$, the payoff to buyer j who has received signal $x_j^B = x$ and submits offer \hat{p} is given by

$$U^B(\hat{p}|x, \beta, \mathcal{C}, p) = \mathbb{E}\{(b(s) + B(s) - \hat{p}) \phi(\hat{p}, \max_{-j}\{p(x_i, \beta, \mathcal{C})\})|x, \beta, \mathcal{C}\}$$

where

$$\phi(\hat{p}, \max_{-j}\{p(x_i, \beta, \mathcal{C})\}) = \begin{cases} 1 & \text{if } \hat{p} > \max_{-j}\{p(x_i, \beta, \mathcal{C})\} \\ 0 & \text{if } \hat{p} < \max_{-j}\{p(x_i, \beta, \mathcal{C})\} \\ l^{-1} & \text{if } \hat{p} = \max_{-j}\{p(x_i, \beta, \mathcal{C})\} \end{cases}$$

is the allocation rule with $l \in \{1, \dots, n\}$ denoting the number of buyers who have submitted the maximal offer: a buyer gets the contract if his bid is highest and the contract is allocated randomly among highest bidders in case of a tie. By symmetry, the offer strategy $p(\cdot, \beta, \mathcal{C})$ is optimal for buyers if

$$p(x, \beta, \mathcal{C}) \in \operatorname{argmax}_{\hat{p}} U^B(\hat{p}|x, \beta, \mathcal{C}, p)$$

for all $x \in [\underline{x}, \bar{x}]$.²²

Seller's Payoff. A seller commits to sell his contract to the buyer with the highest offer. Let $p^{\max}(\beta, \mathcal{C}) \equiv \max_{j=1, \dots, n}\{p(x_j, \beta, \mathcal{C})\}$ be the maximal offer that the seller receives conditional on buyers having received signals $\{x_j\}_{j=1}^n$, and note that $p^{\max}(\beta, \mathcal{C})$ is random as it depends on the entire vector of realizations of the buyers' signals (I omit this dependence for brevity). The expected payoff to a seller who has received signal x and has a liquidity need β is thus given by

$$U^S(x, \beta, \mathcal{C}) = \mathbb{E}\{p^{\max}(\beta, \mathcal{C})|x\} - \beta \mathbb{E}\{b(s) + B(s)|x\}$$

Investor of type (x, β) will therefore post his contract if only if he is strictly better off selling it than keeping it, i.e. $\gamma(x, \beta, \mathcal{C}) = 1$ if and only if $U^S(x, \beta, \mathcal{C}) > 0$.

Let $\hat{F}(\cdot|x, \beta, \mathcal{C}) : [\underline{x}, \bar{x}] \rightarrow [0, 1]$ denote the belief (cdf) that a buyer with signal x holds over the seller's signal conditional on seller having a liquidity need β and holding contract \mathcal{C} . An equilibrium in secondary markets is then defined as follows,

²²Note that I implicitly assume that investors' endowment e is large enough for their budget constraint not to bind when making offers.

Definition 1 Given financial contract \mathcal{C} , an equilibrium in secondary markets is given by a posting strategy γ , price offer strategy p , and belief function \widehat{F} , such that

1. *Seller Optimality:* $\gamma(x, \beta, \mathcal{C})$ is optimal for an investor of type (x, β) , given that buyers follow strategy p ,
2. *Buyer Optimality:* $p(x, \beta, \mathcal{C})$ is optimal for a buyer with signal x who is matched with a seller with a liquidity need β , given that other buyers follow strategy p and given belief \widehat{F} , and
3. *Belief Consistency:* \widehat{F} is derived from strategy γ using Bayes' rule where possible.

Investors' Problem at $t=0$: Pricing Financial Contracts

An investor decides whether to accept the contract \mathcal{C} offered by the entrepreneur in period 0. He accepts the contract if the loan amount specified by the entrepreneur does not exceed the expected present value of the contract. If the investor accepts the contract, he can keep it to maturity and consume its cash-flows $\{b(s), B(s)\}$ in period 2, or he can sell the contract in secondary markets and consume the proceeds from this sale in period 1. Given the equilibrium strategies γ and p , the expected present value of the contract to the investor is

$$\mathcal{L}(\mathcal{C}) = \mathbb{E}\{\gamma(x, \beta, \mathcal{C})\mathbb{E}\{p^{max}(\beta, \mathcal{C})|x\} + (1 - \gamma(x, \beta, \mathcal{C}))\beta\mathbb{E}\{b(s) + B(s)|x\}\} \quad (1.8)$$

i.e. if the investor is of type (x, β) , then he sells his contract if $\gamma(x, \beta, \mathcal{C}) = 1$, in which case he is expected to receive $\mathbb{E}\{p^{max}(\beta, \mathcal{C})|x\}$; otherwise he keeps the contract and expects to receive $\beta\mathbb{E}\{b(s) + B(s)|x\}$. Since investors are ex-ante symmetric and matching is random, equation (1.8) is computed by taking expectations over x and β . This equation fully specifies the *contract price schedule* $\mathcal{L}(\cdot)$, because it is defined for any contract \mathcal{C} issued by the entrepreneur at the initial date.

General Equilibrium

In the previous sections, I described the entrepreneurial problem for given *contract price schedule* $\mathcal{L}(\cdot)$ and prices of capital $\{q(s)\}$, and the determination of the equilibrium prices of capital. I have also setup the investors' problem for a given financial contract issued by entrepreneurs, and I thus obtained the *contract price schedule* that entrepreneurs face in period 0. A general equilibrium of this economy is then defined as follows,

Definition 2 An equilibrium consists of an allocation $\{k, L, b(s), B(s), z(s), \tilde{c}_2(s)\}$, capital goods prices $\{q(s)\}$, a contract price schedule $\mathcal{L}(\cdot)$, and a triple $\{\gamma, p, \widehat{F}\}$, such that

1. *Entrepreneurs' Optimality: the allocation $\{k, L, b(s), B(s), z(s), \tilde{c}_2(s)\}$ is optimal for entrepreneurs, given the capital goods prices $\{q(s)\}$ and the contract price schedule $\mathcal{L}(\cdot)$,*
2. *Investors' Optimality: the triple $\{\gamma, p, \hat{F}\}$ is an equilibrium in secondary markets, given financial contract $\mathcal{C} = \{b(s), B(s)\}$,*
3. *Market Clearing: the prices $\{q(s)\}$ of capital goods are given by (1.7), and the schedule $\mathcal{L}(\cdot)$ is given by (1.8).*

Note that in defining the equilibrium of the economy, I omitted the allocations of the investors. I have done this only for brevity, as investors' allocations affect the equilibrium quantities of interest only through the prices of capital goods and of the financial contracts in secondary markets. These prices in turn are fully summarized by (i) the contract price schedule implied by optimal posting and offer strategies, and (ii) the traditional sector's optimal demand for capital. I now proceed to characterize the equilibrium of the economy.

1.4 Equilibrium Characterization

In this section, I characterize the equilibrium of the economy and derive the main result of the paper which states that the magnitude of balance sheet amplification is directly linked to the level of information dispersion and the severity of search frictions in financial markets. I solve for the equilibrium in three steps.

First, I show how contractual choices made by entrepreneurs in period 0 determine the magnitude of amplification in period 2, and I show that there are benefits to contingent write-downs of entrepreneurial liabilities in adverse states of the world.

Second, I solve for the equilibrium in secondary markets and determine the costs of contractual contingency that entrepreneurs face when issuing contracts at the initial date.

Finally, I solve for the financial contract issued by entrepreneurs that optimally trades off the costs and benefits of insurance, and then I characterize the equilibrium fluctuations in the economy.

Contracts and Fire-Sales

In period 2, after cash-flows are realized, entrepreneurs decide how much capital to liquidate in order to meet their liabilities. Recall that $\{b(s), B(s)\}$ denote the contractual repayments that entrepreneurs have promised to make to investors in period 2, and let $d(s) \equiv \frac{b(s)+B(s)}{k}$ denote the 'per unit' equivalent of the total period 2 repayments. Since liquidations are costly, it is without loss to assume that entrepreneurs would have chosen to

repay as much as possible at the end of period 2, i.e. $B(s) = \theta A(1 - z(s))k$ for $s \in \{l, h\}$. We can thus re-express entrepreneurial consumption in period 2 as

$$\tilde{c}_2(s) = (a(s) + q(s)z(s) + (1 - z(s))A - d(s))k$$

Entrepreneurs choose $z(s) \in [0, 1]$ to maximize $\tilde{c}_2(s)$ subject to the beginning of period resource constraint given in equation (3), which can be re-written as

$$0 \leq (a(s) + q(s)z(s) - d(s) + \theta A(1 - z(s)))k$$

where I used the fact that $B(s) = \theta A(1 - z(s))k$ for $s \in \{l, h\}$. Since entrepreneurs will only choose to liquidate capital if the resource constraint is violated at $z(s) = 0$, we can immediately make the following conclusion: in equilibrium, entrepreneurs liquidate capital in state s if and only if the cash-flows at hand plus the pledgeable portion of future cash-flows are insufficient to cover debt obligations, i.e. if $d(s) > a(s) + \theta A$. Given this, the following proposition characterizes how entrepreneurs' contractual choices made in period 0 translate into liquidations and equilibrium fluctuations in the prices of capital in period 2,

Proposition 1 (Contracts and Fire-Sales) *In equilibrium, entrepreneurs do not liquidate capital in state h , and they liquidate capital in state l if only if $d(l) > a(l) + \theta A$. The prices of capital goods satisfy*

$$\theta A < q(l) \leq q(h) = A$$

where $q(l) = g'(z(l)k)$ and $z(l) = \max\{0, \frac{d(l) - a(l) - \theta A}{q(l) - \theta A}\} \in [0, 1]$.

Thus, entrepreneurs liquidate capital only in the low state and only if the present value of their liabilities exceeds their cash-flow at hand plus the amount that they can 'roll-over' to the future.²³ In aggregate, as the entrepreneurial sector liquidates, the price of capital in state l also becomes depressed, since $q(l) = g'(z(l)k) < A$.

If contracts are such that $d(l) > a(l) + \theta A$, a negative shock to entrepreneurs' cash-flows will be amplified through the endogenous interaction between the price of capital and the its aggregate liquidations. In particular, note that a decline in the price $q(l)$ of capital forces entrepreneurs to liquidate even more capital to meet the shortfall $d(l) - a(l) - \theta A$, since $z(l) = \frac{d(l) - a(l) - \theta A}{q(l) - \theta A}$ is decreasing in $q(l)$; this further depresses the price of capital, leads to further liquidations, and so on. Feedback effects of this type can potentially explain how small shocks get amplified into larger scale recessions.

The magnitude of such amplification and whether it at all occurs will, however, depend critically on the design of entrepreneurial liabilities at the initial date. When entrepreneurs promise to repay 'too much' in the low state and therefore aggregate liquidations are positive,

²³The result that entrepreneurs do not liquidate capital in state h follows from Assumption 1.3 and the financial constraint $d(h) \leq \theta a(l) + q(h)$.

asset prices become depressed, $q(l) < A$; in this case, entrepreneurs' marginal utility of funds in the low state is $\frac{(1-\theta)A}{q(l)-\theta A} > 1$; this is because entrepreneurs can use each extra 'dollar' to reduce costly liquidations. Entrepreneurs' marginal utility of funds in state h , on the other hand, is equal to 1 as they use each extra 'dollar' for consumption. Therefore, a contract that has lower repayments in state l and higher repayments in state h is potentially beneficial because it allows entrepreneurs to insure against premature liquidations. As I show in the following section, this insurance benefit of contingent contracts will need to be traded off against the 'illiquidity' costs that entrepreneurs will need to pay investors for holding such contracts.

Equilibrium in Secondary Markets

I now solve for the equilibrium in secondary markets where investors trade financial contracts. The main result of this section is that contingent contracts are priced at a discount due to the presence of dispersed information and search frictions in financial markets.

In period 1, an investor who has received signal x and has a liquidity need β decides whether to post his contract for sale. If the investor posts his contract, he is matched with n buyers that submit offers according to strategy $p(\cdot, \beta, \mathcal{C})$, where recall that $\mathcal{C} = \{d(s)k\}$ denotes the financial contract issued by the entrepreneurs in period 0. The first immediate result of this section is that non-contingent contracts are always traded at their 'fair' value,

Lemma 1 *If financial contract $\mathcal{C} = \{d(s)k\}$ satisfies $d(l) = d(h)$, then the offer strategy*

$$p(x, \beta, \mathcal{C}) = \mathbb{E}\{d(s)k\} \quad \text{for } x \in [\underline{x}, \bar{x}] \quad \text{and } \beta \in \{0, 1\}$$

is optimal for buyers for any belief \hat{F} .

This result relies on the fact that when the payoff of the underlying contract does not depend on the state of the economy, informational asymmetries are irrelevant for pricing this contract. Competition then forces buyers to bid the prices of such contracts up to their 'fair' value. As I show below, this logic does not extend to contingent contracts. In this case, information dispersion causes buyers to disagree about expected payoffs of such contracts and, thus, to worry about the winner's curse - the fact that a buyer who gets to buy the contract must be more optimistic about the contract's payoffs than other buyers.

To analyze buyers' offer strategies for contracts contingent on the state, I further restrict my attention to offer strategies p that are monotonic and differentiable in buyers' signals. This restriction is standard in the literature (e.g. Milgrom and Weber (1982)[64]), and it yields the reasonable result that buyers who are most optimistic about contractual payoffs are also the ones who receive the contracts in equilibrium. As I show in the Appendix, because buyers observe the seller's liquidity need β , an investor's optimal posting strategy is always to sell the contract if and only if he has experienced a liquidity need, i.e. $\beta = 1$. The reason is that, because there is common knowledge of gains from trade, non-liquidity hit

investors are unable to earn rents by mimicking investors who have them.²⁴ Hence, to fully characterize the equilibrium pricing of financial contracts, it suffices to compute the offer strategy conditional on buyers' being matched with sellers who have experienced liquidity needs, i.e. $p(\cdot, 1, \mathcal{C})$.

Suppose that a seller of contract \mathcal{C} has been matched with buyers with signals x_1^B, \dots, x_n^B , and that the contract is positively correlated with the state, i.e. $d(h) > d(l)$. Let y_1^+ denote the maximal signal among the signals $\{x_2^B, \dots, x_n^B\}$ received by the opponents of buyer 1. Note that y_1^+ is a random variable from buyer 1's perspective, and that this buyer's valuation of the contract conditional on his signal being $x_1^B = x$ and conditional on receiving the contract is given $\mathbb{E}\{d(s)k|x_1^B = x, y_1^+ < x\}$: buyer 1 conditions on his own signal and the fact that his signal is highest. For a contract that is negatively correlated with the state, buyer 1's valuation of the contract conditional on his signal and conditional on receiving the contract is given by $\mathbb{E}\{d(s)k|x_1^B = x, y_1^- > x\}$, where y_1^- denotes the minimal signal among the signals $\{x_2^B, \dots, x_n^B\}$ received by the opponents of buyer 1. The following lemma states that buyers' offers are strictly below their conditional valuations when contracts are contingent on the state of the economy.

Lemma 2 *If financial contract $\mathcal{C} = \{d(s)k\}$ satisfies $d(l) \neq d(h)$, then the equilibrium offer strategy of the buyers satisfies*

$$p(x, 1, \mathcal{C}) < \begin{cases} \mathbb{E}\{d(s)k|x_1^B = x, y_1^+ < x\} & \text{if } d(h) > d(l) \\ \mathbb{E}\{d(s)k|x_1^B = x, y_1^- > x\} & \text{if } d(h) < d(l) \end{cases}$$

for $x \in [\underline{x}, \bar{x}]$.

The computation of buyers' optimal offer strategies is analogous to Milgrom and Weber (1982)[64], with the exception that off-equilibrium path, buyers' beliefs are also updated to adjust for the adverse selection on the side of the seller.²⁵ Since the expressions for the optimal offer strategies are rather cumbersome, they are relegated to the Appendix.

The intuition for the result in Lemma 2 is the following. When submitting their offers, buyers trade off the probability of having the highest offer and receiving the contract with the profit earned conditional on having the highest offer. If a buyer submits an offer that is equal to his conditional valuation, he expects to earn zero profits; on the other hand, if a buyer shades his offer below, the probability of receiving the contract is reduced²⁶, but he makes a positive rent. Hence, it is always optimal for a buyer to shade his offer below his valuation conditional on receiving the contract.

²⁴As I show in Section 6, this may no longer be the case when liquidity needs are unobservable.

²⁵If a buyer is contacted by a $\beta = 0$ seller, he assigns a belief $\Pr(x^S = \underline{x}) = 1$ for a contract with payoffs $d(h) \geq d(l)$, and he assigns a belief $\Pr(x^S = \bar{x}) = 1$ otherwise.

²⁶Note that probability does not drop to zero because information is dispersed and because there are finitely many buyers in each market.

Because in equilibrium all buyers shade their offers, the unconditional expected resale price of a contingent contract satisfies

$$\mathbb{E}\{p^{\max}(1, \mathcal{C})\} = \mathbb{E}\{\max_{j=1}^n \{p(x_j^B, 1, \mathcal{C})\}\} < \mathbb{E}\{d(s)k\}$$

and is thus below the unconditional expected value of that contract.²⁷ As will be seen below, such discounting of financial contracts in secondary markets is the precise reason why it is costly for entrepreneurs to introduce contingencies into financial contracts when raising funds at the initial date. The model also predicts that there is a dispersion of prices for contingent contracts with similar characteristics. The maximal offer received by a seller of a financial contract is given by $p^{\max}(1, \mathcal{C}) = \max_i^n \{p(x_i, 1, \mathcal{C})\}$, where the signals x_1, \dots, x_n are drawn independently from the distribution $F_s(\cdot)$. The maximal offer received by the seller, therefore, depends on the entire profile of signals received by the buyers within his match.²⁸

The following proposition uses the equilibrium offer strategies to yield the main result of this section: it states that the period 0 price of a generic financial contract is given by its expected value net of a discount that is proportional to the degree of contingency of that contract on the state of the economy,

Proposition 2 (Costs of Contingency) *The price of a generic contract $\mathcal{C} = \{d(s)k\}$ satisfies*

$$\mathcal{L}(\mathcal{C}) = \mathbb{E}\{d(s)k\} - \lambda \zeta_{\mathcal{C}} \cdot |d(h) - d(l)|k \tag{1.9}$$

where $\zeta_{\mathcal{C}} = \zeta^+ \cdot \mathbf{1}\{d(h) \geq d(l)\} + \zeta^- \cdot \mathbf{1}\{d(h) < d(l)\}$ and $\zeta^+, \zeta^- > 0$.

The intuition for this result can be understood as follows. Recall the definition of the *contract price schedule* $\mathcal{L}(\cdot)$ in equation (7) and note that given contract $\mathcal{C} = \{d(s)k\}$ issued by entrepreneurs at the initial date, we have that

$$\begin{aligned} \mathcal{L}(\mathcal{C}) &= \mathbb{E}\{\gamma(x, \beta, \mathcal{C})\mathbb{E}\{p^{\max}(\beta, \mathcal{C})|x\} + (1 - \gamma(x, \beta, \mathcal{C}))\beta\mathbb{E}\{d(s)k|x\}\} \\ &= \lambda\mathbb{E}\{p^{\max}(1, \mathcal{C})\} + (1 - \lambda)\mathbb{E}\{d(s)k\} \\ &= \mathbb{E}\{d(s)k\} - \lambda(\mathbb{E}\{d(s)k\} - \mathbb{E}\{p^{\max}(1, \mathcal{C})\}) \end{aligned}$$

The above expression follows from the fact that an investor sells his contract only when he has $\beta = 1$ (a probability λ event), and the assumption that investor liquidity needs are idiosyncratic and therefore independent of the state of the economy. The precise form of the schedule $\mathcal{L}(\cdot)$ in Proposition 2 then follows from the fact that the offer strategy p is linear in the contractual payoffs $\{d(s)k\}$ and because buyers are able to earn rents only when contracts are contingent on the state of the economy.

²⁷As discounting will be greater when dispersion of information is greater, this is consistent with investors facing larger trading costs in more ‘opaque’ markets. See Bessembinder and Maxwell (2008)[10].

²⁸See Ashcraft and Duffie (2007)[6], Bessembinder and Maxwell (2008), and Ang et al. (2013)[3] for evidence on dispersion in over-the-counter markets.

The schedule $\mathcal{L}(\cdot)$ has the following simple interpretation for the entrepreneurs' cost of funds in period 0. If entrepreneurs want to issue contracts that are non-negatively correlated with the state, i.e. $d(l) \leq d(h)$, then it is as if entrepreneurs face investors who are more pessimistic about the future state of the economy: the implicit probability that they assign to state l is given by $\tilde{\pi}(l) = \pi(l) + \lambda\zeta^+$. As I show in the next section, the magnitude of such implicit pessimism is closely linked to the severity of equilibrium amplification in the adverse state of the world.

Optimal Contracts and Equilibrium Fluctuations

In this section, I characterize the general equilibrium of the economy. In Proposition 3, I derive a pecking order for entrepreneurial liability design. I show that entrepreneurs prefer to raise funds with non-contingent claims unless they expect fluctuations to be sufficiently severe. In Proposition 4, I derive the main result of the paper that relates the magnitude of balance sheet amplification in the economy to the information dispersion and search frictions in financial markets.

In period 0, entrepreneurs choose what financial contract to issue, taking as given the *contract price schedule* $\mathcal{L}(\cdot)$ and the prices $\{q(s)\}$ of capital goods. This choice then fully characterizes the solution to the entrepreneurial problem as well as the equilibrium of the economy: given the contract, investment is pinned down by the period 0 budget constraint, and Proposition 1 fully characterizes entrepreneurial liquidation decisions $\{z(s)\}$ and thus consumptions $\{\tilde{c}_2(s)\}$. Before proceeding further, however, I make a further simplifying assumption that reduces the number of cases to be considered. Let \underline{k} be defined by $\chi(\underline{k}) = (a(l) + \theta A)\underline{k}$ and thus be the largest scale that entrepreneurs can achieve by borrowing with non-contingent claims and avoiding liquidations at the same time (see Proposition 1). Let $k^{fb} \equiv \chi'^{-1}(\mathbb{E}\{a(s) + A\})$ be the scale of investment that would be optimal in a frictionless economy.²⁹ Then I assume that

Assumption 3 $\underline{k} < k^{fb}$

This assumption ensures that in equilibrium entrepreneurs face a meaningful tradeoff between the scale of their investment and the premature liquidations of their projects in the low state. The following proposition provides a pecking order for entrepreneurial liability design. It characterizes the optimal financial contract issued by entrepreneurs as a function of expected fluctuations in the prices of capital goods,

Proposition 3 (Optimal Contract) *Given the contract price schedule $\mathcal{L}(\cdot)$ as in Proposition 2 and prices $\{q(s)\}$ of capital goods that satisfy $q(l) \leq q(h) = A$, there exists a threshold $q \in (\theta A, A)$ such that the optimal financial contract issued by entrepreneurs falls into one of the following categories,*

²⁹In a frictionless economy, projects would be valued at their expected value and the optimal investment scale would satisfy $\chi'(k^{fb}) = \mathbb{E}\{a(s) + A\}$.

- *Type I (Non-Contingency):* If $q(l) > \underline{q}$, then either (i) $a(l) + \theta A \leq d(l) = d(h) \leq \theta a(l) + q(l)$, or (ii) $d(l) = \theta a(l) + q(l) < d(h) \leq \theta a(h) + A$,
- *Type II (Contingency):* If $q(l) < \underline{q}$, then either (i) $a(l) + \theta A = d(l) < d(h) \leq \theta a(h) + A$ or (ii) $a(l) + \theta A < d(l) < d(h) = \theta a(h) + A$,
- *Type III (Indifference):* If $q(l) = \underline{q}$, then $a(l) + \theta A \leq d(l) \leq \theta a(l) + q(l)$ and $d(l) \leq d(h) \leq \theta a(h) + A$.

where the threshold price is given by

$$\underline{q} = \left(\theta + (1 - \theta) \frac{\pi(l)}{1 - \pi(l)} \frac{1 - \pi(l) - \lambda \zeta^+}{\pi(l) + \lambda \zeta^+} \right) \cdot A$$

and is strictly decreasing in $\lambda \zeta^+$.

Proposition 3 states that if the price of capital $q(l)$ in low state is above the threshold \underline{q} , then entrepreneurs prefer to borrow with non-contingent claims: they set their repayments to $d(l) = d(h)$ unless the borrowing capacity in state l is exhausted, i.e. if $d(l) = \theta a(l) + q(l)$. On the other hand, if the price $q(l)$ is below the threshold \underline{q} , then introducing contingency becomes desirable. In fact, entrepreneurs fully insure themselves against fluctuations as long as their borrowing capacity in the high state is not exhausted, i.e. if $d(h) < \theta a(h) + A$. Finally, when the price $q(l)$ is equal to the threshold \underline{q} , entrepreneurs are indifferent between the two types of contracts.

The above result thus shows that because contractual contingencies are costly, entrepreneurs are willing to sacrifice the risk-sharing benefits that contingent contracts provide and liquidate their projects prematurely even if price of capital in low state falls below A . As will be seen next, this behavior is precisely what gives rise to equilibrium fluctuations in asset prices and allows for balance sheet amplification to occur. The final and main result of this section combines the result in Proposition 3 with the results of the previous sections to give a full characterization of the general equilibrium of the economy.

Proposition 4 (Equilibrium Fluctuations) *In equilibrium,*

1. *The optimal financial contract satisfies $d(l) \in (a(l) + \theta A, \theta a(l) + q(l)]$ and $d(h) \in [d(l), \theta a(h) + A]$, and depending on parameters, it may or may not be contingent.*
2. *Entrepreneurs invest at scale $k \in (0, k^{fb})$ and liquidate a fraction $z(l) = \frac{d(l) - a(l) - \theta A}{q(l) - \theta A} > 0$ of capital in state l .*
3. *The prices of capital goods satisfy $q(l) < q(h) = A$. Furthermore, whenever the financing constraint in state h is loose, we have that*

$$0 \leq \frac{q(h) - q(l)}{q(h)} \leq (1 - \theta) \left(1 - \frac{\pi(l)}{1 - \pi(l)} \frac{1 - \pi(l) - \lambda \zeta^+}{\pi(l) + \lambda \zeta^+} \right)$$

and thus the fluctuations in asset prices are bounded by the cost of contractual contingency.

Thus, because contingencies are costly to introduce, in equilibrium entrepreneurs will choose to make 'excessive' repayments in low state, $d(l) > a(l) + \theta A$, and they will thus liquidate capital in that state. As a result, the equilibrium prices of capital will fluctuate, $q(l) < q(h) = A$, and the perverse feedback between prices and liquidations will give rise to a balance sheet amplification: a decline in the price of capital, $q(l)$, will lead to an increase in entrepreneurial liquidations, $z(l) = \frac{d(l) - a(l) - \theta A}{q(l) - \theta A}$, which will lead to a further declines in the prices of capital, $q(l) = g'(z(l)k)$, and so on.

The costs of contractual contingency are essential for this result because in their absence ($\lambda\zeta^+ = 0$) entrepreneurs would begin insuring even the slightest fluctuations in the prices of capital in order to avoid liquidations: note that $\underline{q} < A$ if and only if $\lambda\zeta^+ > 0$. This behavior would then endogenously stabilize the equilibrium prices of capital and potentially eliminate balance sheet amplification altogether. The last part of the proposition, in essence, provides a necessary condition for balance sheet amplification to occur: it states that the fluctuations in the prices of capital and thus the magnitude of amplification will tend to be bounded by the cost of contractual contingency. The only scenario where fluctuations may be larger is when entrepreneurs are so 'desperate' for funds that they still want to raise more funds even after having exhausted their borrowing capacity in state h . In what follows, I will focus on the case where in equilibrium the financial constraint is non-binding in the high state.³⁰ The conditions on the primitives that ensure non-binding constraint in state h are provided in the Appendix. This allows me to illustrate my results most starkly because full-insurance will be obtained in the absence of secondary market frictions.

Comparative Statics

I now provide comparative statics results that relate the magnitude of equilibrium amplification to the primitives of the economy. Because the magnitude of amplification in the model is tightly linked to the fluctuations in the prices of capital, I will measure amplification by the percent fall of the price of capital in the low state below the price in the high state, $V(q) \equiv \frac{q(h) - q(l)}{q(h)}$. By Proposition 4, $V(q)$ is bounded above by a term that is monotonically increasing in the costs of contractual contingency, which are given by $\lambda\zeta^+$ for the equilibrium contracts. These costs are in turn closely linked to the three ingredients introduced in this paper: (i) liquidity needs, (ii) information dispersion, and (iii) search frictions in financial markets. Investor liquidity needs and search frictions in financial markets are captured by an investor's probability λ of being 'early' type and by the number n of buyers that a seller may contact respectively. To measure information dispersion, recall from Section 2 that there

³⁰While asset prices may still fluctuate when the borrowing constraint is exhausted in the high state, amplification will not occur because collateral constraints will play a stabilizing role. See also Krishnamurthy (2003)[49].

exist positive constants $\underline{\phi}, \bar{\phi}$ such that $\underline{\phi} \leq \frac{f_h(x)}{f_l(x)} \leq \bar{\phi}$ for all $x \in [\underline{x}, \bar{x}]$, and define $\psi \equiv \bar{\phi} - \underline{\phi}$ as a measure of signal informativeness. Thus, as ψ decreases to 0, signals become uninformative of the state and information becomes less dispersed.

The following proposition shows that liquidity needs, information dispersion, and search frictions are essential for amplification effects to be present. In particular, it shows that amplification disappears as either liquidity needs, search frictions or information dispersion vanish.

Proposition 5 *$V(q)$ decreases to 0 when either λ or ψ decrease to 0, or when n increases to ∞ .*

The fact that smaller liquidity needs reduce amplification effects is clear. Risk-sharing among entrepreneurs and investors is limited solely because contingent claims trade at a discount in financial markets. In the absence of liquidity needs, there is no trade in these markets, and thus the resale value of financial contracts is irrelevant for pricing them.³¹ On the other hand, as ψ declines to 0, even when financial contracts are traded, the discount that sellers receive on them goes to 0 because the beliefs of all buyers become closer to their priors as their signals become less informative.

The effect of the search friction n is more subtle. While it is difficult to obtain a monotonicity result that relates n to the discount ζ^+ , the asymptotic result in Proposition 5 is derived in Kremer (2002)[48]: he shows that expected prices converge to expected values in first-price common value auctions, which is equivalent to ζ^+ going to 0 in my framework. The intuition is that the larger the number of buyers per seller, valuations within a match become less dispersed, and it becomes more likely that a buyer loses the contract when he sheds his offer below his conditional valuation. Note that I have implicitly decoupled the severity n of search frictions from an investor's probability λ of experiencing liquidity needs. While in the main analysis, I have assumed exclusive matching of buyers to sellers (i.e. that $n\lambda \leq 1 - \lambda$), the result in Proposition 2 is the same in a variant of the model where buyers match with multiple sellers but when all trades are executed simultaneously.

Figure 2 illustrates the upper bound on $V(q)$ derived in Proposition 4 for a particular parameterization of signal distributions: signals are drawn from a truncated normal, $N(\mu(s), \sigma^2)$, on interval $[-0.5, 0.5]$ with state-dependent mean $\mu(s)$ and variance σ^2 . The upper bound on $V(q)$ is depicted on the vertical axis, and the variance of signals σ^2 is depicted on the horizontal axis. I plot this relationship for two different values of the search friction, n . As illustrated in the figure, amplification effects can be large when search frictions are severe and when signals are not too uninformative. The plot, however, also shows that amplification effects are small not only when signals are uninformative (high σ) but also when signals are very informative (low σ). In fact, when information is common across

³¹This result is akin to the insight from Milgrom and Stokey (1982)[63] that non-strategic motives are essential to sustain trade. See Serrano-Padial (2007)[72] for a qualification and refinement of this result to generalized trading mechanisms in a risk-neutral environment.

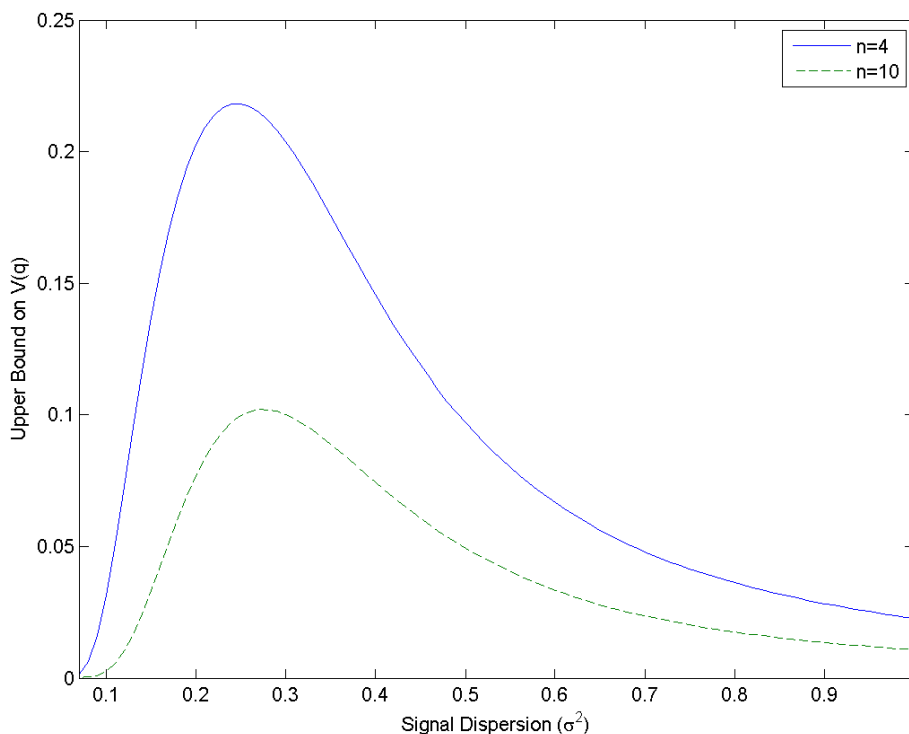


Figure 1.2: **Equilibrium Fluctuations (a)**

Calibration: Signal distribution $N(\mu(s), \sigma^2)$ on $[-0.5, 0.5]$ for $s = l, h$, with $\mu(l) = -0.2$ and $\mu(h) = 0.2$; prior: $\pi(l) = 0.05$; liquidity needs: $\lambda = 0.3$; pledgeability: $\theta = 0.1$.

agents (either uninformative or fully informative signals), then contracts are always traded at their expected value: buyers agree on valuations and compete contract prices to their expected values. Furthermore, one can use the result known in auction theory as the ‘linkage principle’ to show that discounting of contingent claims and thus amplification effects are smaller when public signals are introduced.³² Thus, it is the dispersion of signals across agents rather than the informativeness of these signals that drives the main results of the paper.

While the above results show that amplification effects will be bounded by the magnitudes of three ingredients introduced in the paper, the actual magnitude of amplification will also depend on the severity of the shock to the entrepreneurial sector, i.e. how low is $a(l)$. In Figure 3, I plot the optimal contractual repayments (top diagram), the equilibrium prices of capital goods (middle diagram), and equilibrium liquidations (bottom diagram) as a function

³²Milgrom and Weber (1982)[64] show that a seller’s revenue increases when public signals are introduced. This is equivalent to ζ^+ , and thus the upper bound on $V(q)$, decreasing in my framework.

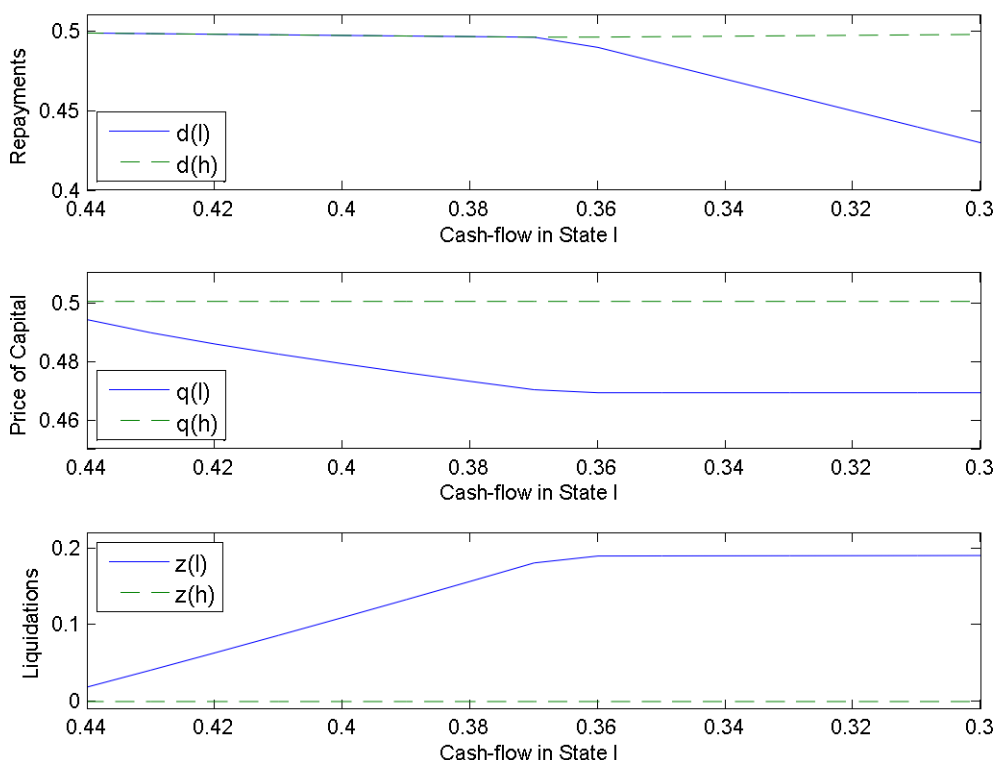


Figure 1.3: **Equilibrium Fluctuations (b)**

Calibration: $\chi(k) = \frac{\chi}{\alpha}k^\alpha$ with $(\chi, \alpha) = (1, 2)$; $g(k) = Ak - \frac{g}{\beta}k^\beta$ with $(g, \beta) = (0.1, 1.7)$; $\theta = 0.1$ and $(a(h), A) = (0.5, 0.5)$; prior: $\pi(l) = 0.05$, signal dispersion: $\sigma = 0.15$; number of buyers: $n = 10$, liquidity needs: $\lambda = 0.3$.

of cash-flows $a(l)$ in the low state. I parameterize signal dispersion to $\sigma^2 = 0.15$, search friction to $n = 10$, and liquidity needs to $\lambda = 0.3$ as before. As can be seen, entrepreneurs borrow with non-contingent claims when the shocks to cash-flows are not severe (Type I). In this region, entrepreneurial liquidations become more severe and asset prices become more depressed the lower their cash-flows are. However, when cash-flows in the low state become sufficiently low ($a(l) = 0.365$), introducing contingencies and bounding liquidations becomes optimal (Type III). In equilibrium, this also puts the lower bound ($q = .469$) on the price of capital in that state, which corresponds precisely to the percent fall in the price of capital for $\sigma^2 = 0.15$ and $n = 10$ depicted in Figure 2.

Discussion

The results presented in the previous section rationalize why macroeconomic risks may not be shared properly and thus why balance sheet recessions may occur even if borrowers and lenders are able to write contracts contingents on the aggregate state of the economy. The theory thus provides a potential rationale for the restriction of contractual incompleteness often exogenously imposed in much of the literature on the balance sheet channel. However, the model also predicts that amplification of aggregate shocks should be expected to be larger for risks about which there are larger informational disagreements and for which the secondary financial markets feature larger search frictions. Hence, while this restriction may be relevant for some risks where hedging markets are not very liquid (e.g. housing), it may be less so for risks where hedging occurs in well-organized competitive exchanges (e.g. energy). Furthermore, as I show in the next section, the model yields policy implications that would not be possible to obtain with exogenous restrictions on the contractual environment.

The model is also able to nest some of the alternative explanations provided in the literature. For example, Rampini and Viswanathan (2009)[70] argue that borrowers may optimally choose to forgo risk-sharing opportunities if their funding needs are sufficiently high ex-ante. In my framework, this would also arise if financing needs are high - Type II contract in Proposition 3. However, the mechanism presented in this paper is applicable independent of the extremity of borrowers' financing needs. Krishnamurthy (2003)[49] and Lorenzoni (2008)[54], on the other hand, derive balance sheet amplification by imposing limited commitment on lenders and supposing that write-downs are insufficient for risk-sharing. More precisely, while in my framework the state l repayment that eliminates fluctuations is given by $d(l) = a(l) + \theta A$, in their framework it is given by $d(l) = a(l) + \theta A - w$, where w is some additional expenditure that the entrepreneur must make in the intermediate period. The assumption of insufficiency of write-downs is then equivalent to $a(l) + \theta A < w$, i.e. cash-flows shocks are sufficiently severe so that even if entrepreneurs wrote down all of their liabilities ($d(l) = 0$) they would still be unable to avoid liquidations. In contrast, my framework can explain periods of balance sheet amplification even when such write-downs suffice for risk-sharing. This result is of further interest in view of the recent policy discussions which stress that write-downs can go a long way in providing insurance to borrowers.

The magnitude of balance sheet amplification in my framework is tightly linked to the information dispersion and search frictions that I have introduced in the paper. While I have analyzed a stylized trading environment, the insight that information disagreements and limited competition allow buyers to earn rents is more general. In particular, the literature on common value auctions has shown that in a variety of trading mechanisms, sellers forgo informational rents when faced with buyers who disagree about the value of the object being offered for sale.³³ While in general the magnitude of rents may depend on the particular

³³See Milgrom and Weber (1982)[64] for this result in alternative trading mechanisms. Duffie and Manso (2009)[30] analyze bilateral trading mechanisms where disagreements lead to probability of no-trade. While Cremer and McLean (1986)[21] and McAfee et al. (1989)[57] show the existence of mechanisms where buyers' rents are zero, such mechanisms are very complex and are unlikely to apply to trade in financial markets.

trading environment, the logic that drives costly contractual contingencies should extend well beyond the basic economic environment presented in this paper.

1.5 Policy Implications

In this section, I consider the policy-implications of the theory. I study the implications of the model for macro-prudential policy, and I discuss the theory's implications for policies geared towards increasing transparency and competition in financial markets.

Macro-Prudential Regulation

I now consider the implications of the theory for macro-prudential regulation. The question I ask is whether atomistic entrepreneurs retain excessive risks in a decentralized economy. In particular, I study the problem of a social planner who can ex-ante coordinate entrepreneurs' contractual choices and whose objective is to maximize entrepreneurial welfare subject to leaving investors as well off as in the competitive equilibrium. The planner must still respect the financial constraints faced by economic agents (i.e. limited commitment constraints). In addition, the planner still allows economic agents to trade in capital goods markets and in secondary markets for financial contracts.

Note that if the planner modifies entrepreneurial contracts, she only affects investors' welfare through the changes in the equilibrium prices of capital $\{q(s)\}$. Thus, to ensure a Pareto improvement, the planner must compensate investors for any losses originating from these changes in prices.³⁴ To simplify the analysis, I suppose that the planner makes compensatory transfers to investors in state h of period 2. Furthermore, I consider the parameterizations of the model for which the financial constraints are loose in both states at the competitive allocation.

Let $T \geq 0$ denote the transfer that the planner makes from entrepreneurs to investors in state h and let $\Pi^{CE}(s)$ ($\Pi(s)$) denote the profits of the traditional sector firms in period 2 and state s at the competitive (at the planner's) allocation. Since investors are ex-ante identical, the transfer T must satisfy

$$(1 - \pi(l))T \geq \sum_s \pi(s) (\Pi^{CE}(s) - \Pi(s))$$

i.e. the expected present value of the transfer to the investors must be at least as great as the expected present value of losses in profits. The transfers and the contract chosen by the

³⁴Transfers are necessary here because the economy is ex-post efficient. In an alternative setting, where increased risk-sharing benefits all agents in the economy ex-post, such transfers may not be needed for Pareto improvement. See, for example, Jeanne and Korinek (2012)[45] for a model with technological externalities.

planner are still required to satisfy the financial constraints

$$\begin{aligned} 0 &\leq b(s) + B(s) \leq (\theta a(s) + q(s))k - T \cdot 1\{s = h\} \\ 0 &\leq B(s) \leq \theta A(1 - z(s))k \end{aligned}$$

and, importantly, the planner internalizes the fact that her contractual choices affect equilibrium prices of capital, i.e. she faces the additional constraint $q(s) = g'(z(s)k)$.

The following proposition provides the main result of this section. It states that the planner chooses to reduce liquidations and repayments in the low state, and it shows that the planner's allocation can be implemented by taxing (or capping) entrepreneurial borrowing against adverse states of the world. Let superscript $\{SP\}$ denote the optimal allocations of the social planner and superscript $\{CE\}$ denote the allocations in the competitive equilibrium, then we have that

Proposition 6 (Prudential Regulation) *The planner chooses to borrow and invest (weakly) less than entrepreneurs, $k^{SP} \leq k^{CE}$, and she strictly reduces repayments and liquidations in the low state, $d^{SP}(l)k^{SP} < d^{CE}(l)k^{CE}$ and $0 < z^{SP}(l)k^{SP} < z^{CE}(l)k^{CE}$. At the planner's allocation, the prices of capital goods satisfy*

$$q^{CE}(l) < q^{SP}(l) < q^{SP}(h) = q^{CE}(h) = A$$

The planner's allocation can be implemented by a compensatory transfer T and a Pigowian tax τ on entrepreneurial repayments against state l .

I provide a numerical illustration of the above result in Figure 5. For the same calibration as in the previous section, I plot the prices and allocations in the two economies (competitive equilibrium vs social optimum) as a function of entrepreneurial cash-flows in the low state (i.e. severity of the shock). As can be seen, when shocks are small, both the contract at CE and SP are non-contingent (top left), and the planner borrows less than entrepreneurs in both states. When shocks become larger, however, the planner makes her contract contingent sooner than the entrepreneurs, and the degree of contingency of the planner's contract is significantly greater. As a result, in the low state, the price of capital goods (bottom left) are lower and liquidations (top right) are smaller. Furthermore, while the planner tends to borrow and invest less overall (bottom right), much of the difference between the competitive and the planner's allocations derives from the degree of contingency of the financial contracts. Thus, the theory also implies that policies of limiting overall indebtedness (leverage) of borrowers may be sub-optimal.

The finding that the planner wants to reduce borrowing against the adverse state of the world is due to pecuniary externalities that arise at times of systemic distress. Here, entrepreneurs do not internalize the positive externality on other entrepreneurs of reducing repayments in the low state. The existence of these externalities in related settings has been pointed out by a number of researchers (e.g. Korinek (2011)[47], Lorenzoni (2008)[54],

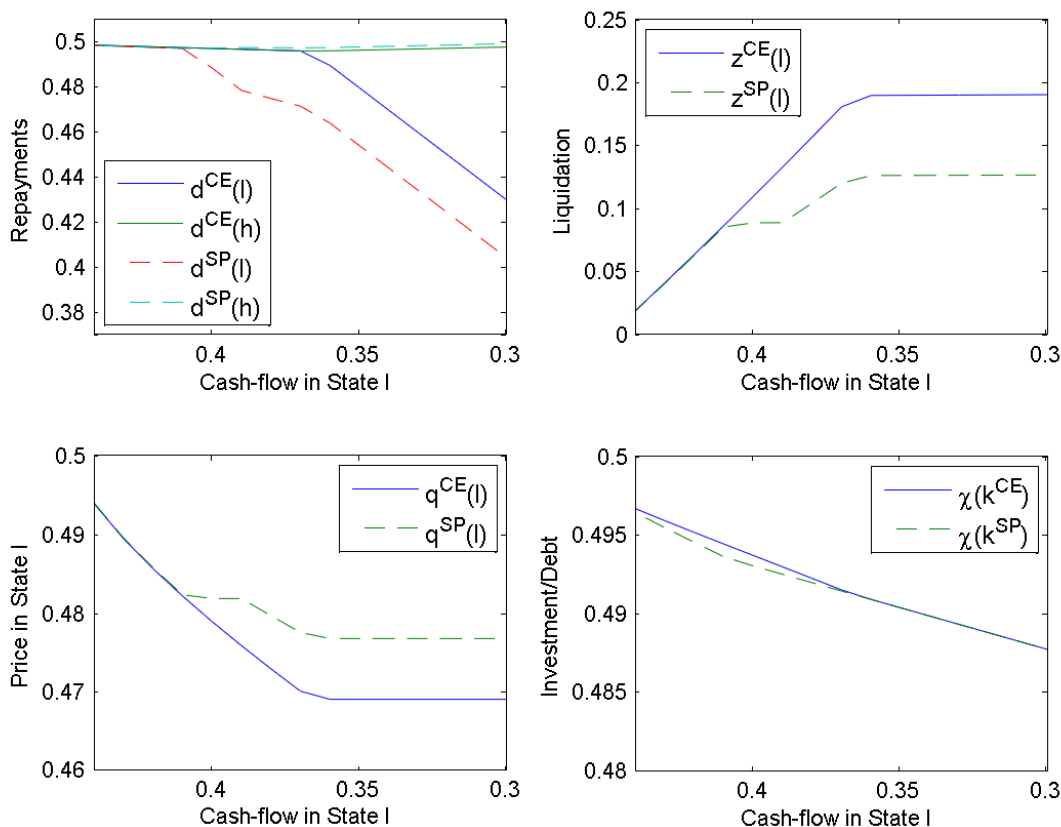


Figure 1.4: **Competitive Equilibrium vs. Social Optimum**

Stein (2010)). One of the interesting differences with the existing analyses is that my framework features the possibility that economic agents endogenously borrow with non-contingent claims but the planner wants to introduce contingencies into financial contracts. Thus, the model can also rationalize the recent policy proposals to introduce macro-contingencies into firms' and households' liabilities. Furthermore, the model suggests that policies targeting borrower leverage alone may be sub-optimal; this is in sharp contrast to implications drawn from models that exogenously restrict contracts to be non-contingent.

These findings thus lend support to some recent regulatory proposals. For example, in a congressional testimony on the role of household debt in the Great Recession, Mian (2011)[62] states that, "mortgage principal can be automatically written down if the local house price index falls beyond a certain threshold. [...] If we had such contingencies present in the current mortgage contracts, we could have avoided the extreme economic pain due to the negative deleveraging aggregate demand cycle." These calls have also been echoed in the discussion of contingent capital requirements (CoCo) for financial institutions. For

instance, Calomiris and Herring (2011)[16] argue that, “If a CoCo requirement had been in place in 2007, the disruptive failures of large financial institutions, and the systemic meltdown after September 2008, could have been avoided.” Thus, in addition to explaining why macroeconomic contingencies may often be absent in borrower liabilities, my theory provides a rationale for why policy makers may want to intervene in order to subsidize such contingencies.

Transparency/Competition Policies

The theory also suggests that the recent policy measures intended to increase transparency and competition in financial markets can have the extra benefit of stabilizing macroeconomic fluctuations. As a result of the Dodd-Frank Act of 2010, many standardized products that were traded over-the-counter will now be required to be traded on regulated exchanges and be cleared centrally.³⁵ According to the Commodity and Futures Trading Commission, “Transparent trading will increase competition and bring better pricing of the marketplace. This will lower costs for businesses and consumers.”

The policy of transparency within the model can be thought of as essentially an introduction of informative public signals: for example, assume that a fraction of trades in the secondary markets is executed before others, and that policy makers mandate public reporting of the terms of trade of each transaction. A statistic of these terms of trade would then be informative about the future state of the economy, and traders in the second set of markets would thus be endowed with an additional public signal. By the ‘linkage principle’, this would lower the buyers’ rents and the costs of contingency to the entrepreneurs. The policy of increasing competition can instead be thought of as increasing the number n of buyers within each market. This would again have the effect of lowering the cost of contingency faced by entrepreneurs. The model thus predicts that these policies would indeed lead to lower costs of funding for borrowers by increasing the extent to which borrowers and lenders can share macroeconomic risks. Furthermore, the model points to the additional effect of these policies on macroeconomic stabilization.

For a complete analysis of the effects of these policies, however, the potential costs associated with such policies must also be incorporated: presumably, there are reasons why trade in many financial assets occurs over-the-counter to begin with. For example, among other reasons, a typical rationale for trading OTC is that these markets allow economic agents to tailor financial contracts to their specific ‘tastes’ and that they are associated with lower transaction costs for traders. Biais et al. (2012)[11] argue that the lower degree of anonymity in OTC markets allows economic agents to better search, screen, and monitor their counterparties. Some of these costs can be incorporated in the model simply by introducing a problem of adverse selection: a fraction of entrepreneurial collateral backing financial contracts is bad (never pays), but the quality of collateral can be screened only in

³⁵Similar policies have been put in place in 2007 through Markets in Financial Instruments Directive in the European Union.

OTC markets. The model would then predict that centralized trading would be dominated if this fraction is sufficiently large.

Furthermore, as shown in Dang et al. (2010)[22], transparency may not always be beneficial. In my setting, public signals are always beneficial because signals about the aggregate state are always received by traders. However, this may not be the case if such information is costly to acquire: it may not pay agents to obtain costly signals when this state is believed unlikely to occur. Public signals that increase the public's perception of the likelihood of the low state may then incentivize information acquisition, which will then introduce costs to contractual contingency and thus limit risk-sharing. While the study of endogenous market formation is beyond the scope of this paper, the theory does suggest an additional social benefit to transparent and competitive markets – macroeconomic stabilization – that should be considered when evaluating policies related to market design.

1.6 Extensions

In this section, I consider several extensions of the basic economic environment. First, I extend the model to the case where investor liquidity needs are unobservable and show that the main results of the paper remain robust to this case. Second, I decouple the magnitude of investor liquidity needs from their frequency and show that the magnitude of implied balance sheet amplification will be increasing in the importance that investors attach to liquidity of financial claims. Finally, I discuss other directions in which the model can be fruitfully extended.

Unobservable Liquidity Needs

In this section, I generalize the main result of the paper to the case when investor liquidity needs are unobservable. In particular, I show that contingent contracts are as before discounted due to informational rents earned by buyers in secondary financial markets.

Consider financial contracts $\mathcal{C} = \{d(s)k\}$ that are positively correlated with the state of the economy, i.e. $d(h) > d(l)$. The case of negatively correlated contract is similar, and non-contingent contracts are again priced at their 'fair' value as in Lemma 2. When liquidity needs are unobservable, pessimistic investors who have not experienced liquidity needs may also decide to sell their contracts. To this end, I restrict my attention to threshold posting strategies: while investors with liquidity needs, $\beta = 1$, will as before sell their contracts independently of their signals, investors without liquidity needs, $\beta = 0$, will sell if they receive signals below some threshold $\hat{x} \in [\underline{x}, \bar{x}]$, where $\hat{x} = \underline{x}$ if these investors never sell.³⁶ An equilibrium in secondary markets is defined as before, except that now buyers cannot condition their offers on the liquidity need of the seller: a buyer with signal x submits an

³⁶I still maintain that there are sufficiently many buyers to have an exclusive matching of n buyers per seller; a sufficient condition for this is that λ is not too large.

offer $p(x, \mathcal{C}, \hat{x})$, where \hat{x} appears in the argument because buyers will know the equilibrium threshold strategy.

Let \hat{x} denote the equilibrium posting strategy. Suppose as before that a seller of contract \mathcal{C} has been matched with buyers with signals x_1^B, \dots, x_n^B , and let y_1^+ denote the maximal signal among the signals $\{x_2^B, \dots, x_n^B\}$ received by the opponents of buyer 1. This buyer's valuation of the contract conditional on his signal being $x_B^1 = x$ and conditional on receiving the contract is given $\mathbb{E}\{d(s)k | x_1^B = x, y_1^+ < x, H(\hat{x})\}$, where now buyer 1 also has to condition on the event that the investor has chosen to sell the contract rather than keep it, $H(\hat{x}) \equiv (\{x^S < \hat{x}\} \cap \{\beta^S = 0\}) \cup \{\beta^S = 1\}$. As I show in the Appendix, the derivation of buyers' offers p for a given threshold \hat{x} follows analogously to that in Proposition 2. Buyers first update their beliefs to adjust for the possible adverse selection on the side of the seller and then they submit offers just as before. The threshold \hat{x} is then determined by the marginal investor's indifference condition: investor with signal \hat{x} who does not experience a liquidity need is indifferent to whether to post his contract for sale.

The following proposition yields the main result of this section. It shows that the contract price schedule that results from the equilibrium in secondary markets takes a form analogous to that in Proposition 2,

Proposition 7 *The price of a generic contract $\mathcal{C} = \{d(s)k\}$ is given by*

$$\mathcal{L}(\mathcal{C}) = \mathbb{E}\{d(s)k\} - \hat{\zeta} \cdot |d(h) - d(l)|k$$

where $\hat{\zeta} = \hat{\zeta}^+ \cdot 1\{d(h) \geq d(l)\} + \hat{\zeta}^- \cdot 1\{d(h) < d(l)\}$ and $\hat{\zeta}^+, \hat{\zeta}^- > 0$.

The reasoning behind why contingent contracts are discounted is the same as before: buyers first update their beliefs to take into account that the seller may be adversely selected, and then shade their offers below their conditional valuations in order to earn informational rents. The schedule $\mathcal{L}(\mathcal{C})$ takes the same form as before again due to linearity of buyers' offer strategies. Thus, this result generalizes Proposition 2 to the case of unobservable liquidity needs and shows that the results of the paper also remain robust to this case.

Magnitude of Liquidity Needs

In the main analysis, I assumed that investors who are 'distressed' have the same marginal valuation of funds in $t = 1$ as investors who are not 'distressed'. As a result, the value of liquidity to investors was captured by the probability λ of experiencing a liquidity need. In practice, however, the marginal valuation of funds is likely to be greater for 'distressed' investors. To incorporate this effect, consider modifying investor preferences to $\mathbb{E}\{c_0 + c_1 + c_2\}$ if the investor is a 'late' type and to $\mathbb{E}\{c_0 + \delta c_1\}$ if the investors is an 'early' types, where $\delta > 1$ is the marginal valuation of funds to 'distressed' investors in $t = 1$. Thus, in period 1, a 'dollar' is valued at $\delta > 1$ by early investors and only at 1 by the late investors. With these preferences, the ex-ante price of a generic contract $\mathcal{C} = \{d(s)k\}$ is now given by

$$\mathcal{L}(\mathcal{C}) = (1 + \lambda(\delta - 1))\mathbb{E}\{d(s)k\} - \lambda\delta\zeta_c |d(h) - d(l)|k$$

where ζ_C is given as in Proposition 2. Thus, the increased value of liquidity has two effects. First, the average cost of funds to the entrepreneurs is now lower because entrepreneurial claims enable investors to make welfare improving transfers at $t = 1$. Second, as resale value becomes a more important determinant of financial contract pricing, the cost to contractual contingency plays a more prominent role. In particular, the threshold price of capital given in Proposition 3 that determines entrepreneurs' incentives to insure cash-flow fluctuations is now given by

$$\underline{q}(\delta) \equiv \left(\theta + (1 - \theta) \frac{\pi(l)}{1 - \pi(l)} \frac{1 - \pi(l) - \frac{\delta}{1 + \lambda(\delta - 1)} \lambda \zeta^+}{\pi(l) + \frac{\delta}{1 + \lambda(\delta - 1)} \lambda \zeta^+} \right) A$$

and is therefore decreasing in δ . As a result, the upper bound on price fluctuations and balance sheet amplification given in Proposition 4 is also increasing in δ . Thus, even if the probability λ of experiencing liquidity needs is small, their effect on risk-sharing may still be large if the magnitude δ of these liquidity needs is large.

In the above example, I considered the case when investor liquidity needs are observable. With unobservable liquidity needs, when $\delta > 1$, we would also have additional discounting of claims due to informational rents earned by non-liquidity hit sellers. Because I have modeled the seller side of the economy monopolistically, however, discounting here would be indistinguishable from the standard models of adverse selection where private information is about idiosyncratic states. Incorporating competition on the seller side of the economy would be more realistic, and it would also allow to distinguish frictions that arise from *dispersed* information rather than information held exclusively by one party.

Other Extensions

Informational Spillovers. Information about the future state was assumed to arrive only once. In reality, however, investors may receive information repeatedly, perhaps through market based interactions with other investors. An interesting way to introduce information arrival is by supposing that not all trades are executed simulatenously and that buyers travel through markets sequentially and privately learn new information from each trade. This approach has been taken up in Duffie and Manso (2007)[31]. In a similar market setting, they study how information that is common across markets accumulates over time. Buyers infer each others' signals over time by trading and eventually dispersed information becomes aggregated. However, recall from Figure 2 that the costs of contingency are largest when signals are of intermediate informativeness (intermediate σ). Hence, even if each investor's information set is initially uninformative, there would be periods in which disagreements among investors become large; as a result, the discount on financial contracts in these periods would also be large. Of course, eventually, information would be fully aggregated and the discount on contingent contracts would converge to zero.

Heterogeneity. The model can also incorporate ex-ante heterogeneity among investors: some investors may face lower search frictions or may be better informed than others. Introducing

such heterogeneity can have interesting implications for the allocation of macroeconomic risk in the economy. For example, the model would predict that risks should be concentrated more heavily in portfolios of investors who face lower search frictions and who have higher ability to process information. In this case, it would also be interesting to incorporate potential ‘capacity’ constraints (risk-aversion, financing constraints, etc.) that may endogenously limit the extent to which these risks can be concentrated in the portfolios of the more connected/informed investors.

Insuring Liquidity Needs. I have precluded contracting on investor liquidity needs by assuming that these are non-verifiable. If these liquidity needs were instead verifiable, then investors may be able to avoid the secondary market by contracting to exchange financial contracts for consumption goods at pre-agreed terms. Furthermore, there may be gains from re-trading between investors and entrepreneurs in period 1. For example, an investor who experiences a liquidity need may be willing to exchange his risky contract for a safer one at a cost in order to minimize trading losses in secondary markets. As a result, entrepreneurs may benefit from this exchange by reducing their overall repayments, but their exposures to cash-flow fluctuations may now increase when such an exchange occurs. This may then introduce interesting interactions between secondary market liquidity and entrepreneurial balance sheets; especially, if aggregate liquidity needs also fluctuate. Such an extension is left for future research because the (informational) heterogeneity among entrepreneurs and the multi-dimensional (liquidity and information) heterogeneity among investors make the problem considerably more complex.

Cyclicalities of the Discount. Incorporating fluctuations in secondary market ‘liquidity’ may also yield interesting implications. In particular, the three components (liquidity needs, info dispersion, and search frictions) that give rise to discounting of contingent contracts are likely to be subject to cyclical fluctuations as well. Furthermore, because it is the interaction of these three ingredients that gives rise to discounting of financial contracts, their co-variance will also be an important determinant of contract pricing. Incorporating such fluctuations is not only interesting theoretically, but it is also likely to yield cyclical predictions with regards to the extent of macroeconomic risk-sharing.

1.7 Conclusions

In this paper, I argued that balance sheet recessions can be rationalized as a result of informational and trading frictions in financial markets. I showed that information dispersion about the future states of the macroeconomy and search frictions in financial markets make issuance and trade of contracts contingent on the state of the economy costly. As a result, it was optimal for borrowers to sacrifice the risk-sharing benefits that such contracts provide and, in aggregate, such behavior allows for aggregate shocks to be amplified into balance sheet recessions. The magnitude of this amplification was shown to be closely linked to the level of informational and trading frictions introduced in this paper. I also studied

the policy-implications of the model and found that active policy measures geared towards subsidizing contingent write-downs of borrowers' liabilities can be welfare improving. The theory also suggests that enhancing transparency and competition in financial markets may have an additional benefit of stabilizing macroeconomic fluctuations. While the economic environment presented in this paper is stylized, I argued that the basic mechanism that limits risk-sharing and leads to periods of balance sheet amplification should apply more generally. Further exploration of these ideas and their quantitative evaluation are left for future research.

Chapter 2

Informed Intermediation Over the Cycle

Joint with Victoria Vanasco

2.1 Introduction

Credit cycles are a pervasive feature of all modern economies. As documented by Reinhart and Reinhart (2010)[71], financial crises are preceded by long periods of credit expansion and rising leverage, and followed by slow recoveries initiated by flight to quality episodes and strong cuts on new lending that can take up to a decade to recover. Another well documented feature of these crises is the asymmetric behavior of credit and investment: strong booms are followed by sharp declines and gradual rebounds. Since the financial sector plays a prominent role in the intermediation of funds between savers and borrowers, understanding the behavior of financial intermediaries should shed some light on the forces that underpin these credit cycles. Financial intermediaries such as banks, hedge funds, and pension funds manage the major bulk of household financial holdings and are the dominant source of external finance for non-financial businesses in all economies. As Gorton and Winton (2002)[39] state, “the savings-investment process, the workings of capital markets, corporate finance decisions, and consumer portfolio choices cannot be understood without studying financial intermediation.” We adopt this view here and present a simple framework that puts a central emphasis on financial intermediaries.

We construct a dynamic model of financial intermediation in which changes in the information held by financial intermediaries allow us to rationalize key features of the documented credit cycles, but also of the financial contracts observed in practice. We suppose that some agents in the economy, whom we call “experts”, have a unique ability to acquire information about firm and sector fundamentals. Better information allows for better allocation of resources, and this informational advantage makes these experts the natural contenders to intermediate funds between households and businesses. The level of “expertize” in the

economy and the potential gains from intermediation grow in tandem with the information that these experts possess; these gains, however, are hindered since experts' information is inherently private. Financial contracts must be strained to balance allocational efficiency with the provision of appropriate incentives for these experts. The economy therefore inherits not only the dynamic nature of information flow, but also the interaction of information with the contractual setting. We introduce a cyclical component to information by supposing that the fundamentals about which experts acquire information are stochastic. While persistence of fundamentals is essential for information to be valuable, their randomness acts as an opposing force and diminishes the value of expert learning.

The combination of a model of financial intermediation and a dynamic model of private information not only allows us to study credit cycles from a new perspective, but it also provides new testable predictions about the connection between confidence in the financial sector, intermediation fees, and financial players' portfolios. We provide a novel mechanism that connects the severity of credit contractions with structural changes in an economy, understood as changes in the underlying productivities of different economic sectors. In our model, the financial system amplifies and propagates real shocks to fundamentals by contracting credit to productive sectors, and by slowing down the access of these sectors to credit in the years to follow. The intuition of the mechanism is as follows. During stable times, financial intermediaries raise funds, lend, and acquire information. Over time their expertise increases, their perceived uncertainty about the investment set is reduced, and this is reflected in higher credit to risky sectors. On top of this, households' confidence in experts increases, and so do intermediation fees in response. Given the nature of our learning process, unexpected changes in underlying fundamentals act as a volatility shock for financial intermediaries, since their accumulated expertise becomes obsolete. This generates a loss of confidence in financial intermediaries, a reduction in their fees, and a contraction of credit to risky sectors. As time passes, intermediaries accumulate information again, and credit slowly recovers, together with the confidence in the financial sector and its fees.

We find that economic fluctuations can be rationalized by waves of "confidence" in the experts' ability to allocate funds profitably. The asymmetry of credit cycles arises from the asymmetric nature of information acquisition: even though it takes time to acquire information through a learning-by-lending process, expertise can be lost the moment a shock to fundamentals is perceived. Credit to risky sectors responds one to one to these changes in expertise, and so do intermediation fees. These results arise in a framework in which optimal intermediation contracts match those observed in reality: a portfolio manager receives an intermediation fee that is proportional to assets under management, and a percentage of the total portfolio. In the literature on intermediation fees the fraction of the portfolio is referred to as an incentive fee, since in most of the literature moral hazard is the prevalent friction. In our paper, as there is no moral hazard, sharing portfolio returns is the result of risk-sharing between households and experts rather than incentives. We believe both arguments are reasonable, reality is probably in between. Given the optimal contracts that we find, we analyze how allocations evolve as information, and thus financial expertise, is

accumulated.

We consider an overlapping generations model with heterogeneous agents. Each generation is exogenously divided between households and experts. There is a storage technology and risky projects available to all agents. We assume agents do not know the true underlying distribution of project returns, but they are born with a prior about it. After investing in a particular project, however, an agent receives a signal about the mean of the distribution that can be used to compute a more precise posterior of project returns. We assume experts have the ability to process these signals in a more sophisticated way, and are thus able to get more precise information than households. This can be rationalized by thinking that experts have access to “soft” information that only they are able to interpret. In this context, we think of financial intermediaries as experts that intermediate funds between households and projects, i.e. they manage households’ portfolios. We model the dynamics of information by assuming that information can be freely transmitted to those that can interpret it, i.e. from old to young.

In this context, experts have private information about investment opportunities, and this is an obstacle when they need to raise funds from uninformed households. We solve a signaling model in which experts with superior, private, information are able to signal uninformed households their private information through their own investment choices. The idea that investment choices can signal private information was first introduced by Leland and Pyle (1977)[52]. As in their setting, the presence of asymmetric information introduces an inefficiency. Experts over-invest in risky assets in an attempt to make households overly optimistic and extract higher intermediation fees. In equilibrium households understand this and information is perfectly transmitted, but over-investment in risky assets cannot be avoided.

There is a large strand of literature that tries to rationalize the role of financial intermediaries. Diamond and Dybvig (1983)[27] argue that intermediaries allow households to smooth their uncertain consumption needs, while Holmström and Tirole (1997)[44] show that intermediaries can help firms to pool their liquidity more efficiently. Another view posits that the role of intermediaries is to reduce monitoring costs in the presence of agency problems (Diamond (1984)[26], Williamson (1987)[77]). In this paper, however, we model financial intermediaries as information producers. The main proponents of this theory are Leland and Pyle (1977), Campbell and Kracaw (1980)[17], and Boyd and Prescott (1986)[12]. These papers focus on the ability of an intermediary to solve the classic “reliability” and “appropriability” problems that arise when there are costs to acquiring information. In these models, financial intermediaries are coalitions of agents that acquire information on behalf of others and their existence is endogenous. In our paper, however, the presence of agents with the ability to learn more than households is exogenously given. We view this as a simplifying assumption that can be micro-funded by the previously mentioned papers.

In particular, a recent paper about trust in financial advisers by Gennaioli, Vishny, and Shleifer (2012)[36], connects very closely to our work. They construct a model of money man-

agement in which investors delegate their portfolio decision to managers based on “trust,” and not only on performance. Their main idea is that a good manager is able to reduce an investor’s uncertainty exogenously, by reducing their anxiety about taking risks. Mullainathan et al. (2010)[66] conduct an audit of financial advisers and find evidence to support the fact that investors choose their financial advisers based on factors other than past performance. In particular, they argue that many financial advisers do not advertise based on past performance, but rather on experience and dependability. This is a key feature of our paper, where households choose to delegate their portfolio decisions to those agents with better quality information that they have acquired with experience. In contrast to Gennaioli, Vishny, and Shleifer (2012), the trust households put on our intermediaries is rational, since households understand that experts have more precise information. In both papers, the decision to delegate does not depend on past performance, but on “confidence” in the knowledge of the portfolio manager.

A number of studies have found evidence supporting the theory that intermediaries do possess superior information about borrowers with whom they have established a relationship. The explanation often given is that in the process of establishing a relation with a firm, the lender obtains “soft” information about the firm. Slovin, Sushka, and Polonchek (1993) examine the stock price of bank borrowers after the announcement of the failure of their main bank, Continental Illinois. They find that Continental borrowers incurred negative abnormal returns of 4.2% on average after the failure announcement. If bank loans were indistinguishable from corporate bonds, borrowers could borrow directly from the market when their bank disappeared; this, however, was not the case, leading the authors to the conclusion that the intermediary had some information about the borrowers that the market did not. Gibson (1995) reaches a similar conclusion by studying the effect of Japanese banks’ health on borrowing firms. Petersen and Rajan (1994) and Berger and Udell (1995) both show in independent studies that a longer bank relationship (controlling for firms’ age), implies better access to credit in the form of lower interest rates or less collateral requirement.

Finally, recent works by Veldkamp (2005)[75], Ordoñez (2009)[69], and Kurlat (2013) [51] emphasize the role of information over the cycle. These papers point to cyclical asymmetries that arise from the naturally asymmetric flow of information over the cycle. In contrast, we study the role of financial intermediaries in generating and amplifying these informational cycles.

The paper is organized as follows. In Section 2.2, the model setup is described. In section 2.3, we solve the static problem and characterize optimal intermediation contracts. In section 2.4, we introduce dynamics and characterize how intermediation activity evolves over time; we also incorporate aggregate shocks and study how intermediaries propagate and amplify shocks to the real economy. We discuss two interesting extensions in Section 2.4. Section 2.5 concludes.

2.2 The Model

We construct an overlapping generations model (OLG) with two-period lived agents. Each generation is of unit mass and is exogenously divided between an equal number of experts (e) and households (h). There is a single consumption good that can be stored at an exogenous gross risk-free rate $R_f \geq 1$.

Preferences and Endowments. Agents are born with an endowment w^j ($j = e, h$) units of the consumption good. They consume only when old and have preferences $u(c) = -e^{-\gamma^j c}$ with $\gamma^j > 0$ for $j = e, h$. The objective of agent j born at date t is to maximize expected utility $U_t^j = E_t^j \{u(c_{t+1}^j)\}$.

Technology. There are N risky projects and investment in these projects is costly: an agent who makes investments in risky projects experiences a non-pecuniary cost $\chi > 0$. The payoff structure of these projects is summarized by the vector of project returns $R_{t+1} \equiv [R_{1,t+1}, \dots, R_{N,t+1}]$ which follows the stochastic process given by

$$\begin{aligned} R_{t+1} &= \theta_t + \varepsilon_{t+1} \\ \theta_t &= (1 - X_t) \theta_{t-1} + X_t \tilde{\theta}_t \quad \forall t > 0 \\ \theta_0 &= \bar{\theta}_0 \end{aligned}$$

where we suppose that $\varepsilon_{t+1} \sim^{iid} N(0, \Sigma_\varepsilon)$ and $\tilde{\theta}_t \sim^{iid} N(\bar{\theta}, \Sigma_\theta)$, and where $X_t \sim \text{Bernoulli}(p)$ for all $t \geq 0$. The project returns at date $t + 1$ are thus decomposed into a transitory component given by ε_{t+1} and a persistent component given by θ_t . The parameter p captures the persistence of returns and thus the degree to which historic data is useful to understanding future investment returns.

Information and Expertize. Experts and households understand the model of the economy but do not know the realization of θ_t . They are born with prior beliefs $\theta_t \sim N(\bar{\theta}, \Sigma_\theta^e)$ and $\theta_t \sim N(\bar{\theta}, \Sigma_\theta^h)$ respectively, where $\Sigma_\theta^e = \Sigma_\theta$ and $\Sigma_\theta^h = \Sigma_\theta + \Sigma_N$, i.e. we assume experts know the underlying distribution of θ_t , while households have a more dispersed prior.

Learning. Agents are born with a prior $\theta \sim N(\bar{\theta}, \Sigma_\theta)$ and after receiving a signal $s \sim N(\theta, \Sigma_s)$, they update their beliefs using Bayes' rule to $\theta \sim N(\hat{\theta}, \hat{\Sigma}_\theta)$ where

$$\begin{aligned} \hat{\theta} &= E[\theta|s] = [\Sigma_\theta^{-1} + \Sigma_s^{-1}]^{-1} [\Sigma_\theta^{-1} \bar{\theta} + \Sigma_s^{-1} s] \\ \hat{\Sigma}_\theta &= V[\theta|\Sigma_s^{-1}] = [\Sigma_\theta^{-1} + \Sigma_s^{-1}]^{-1} \end{aligned}$$

The arrival of signal s reduces the perceived volatility of mean project returns and, in the absence of intermediation, this reduction in volatility is larger for experts than for households. This asymmetry alone is sufficient for the households to want to delegate their investment decisions to the experts, since we will assume that the experts' informational advantage is common knowledge.

Intermediation. Experts have a comparative advantage over households in investment activity since they face a more precise distribution of project returns than households. Therefore, households may want to delegate their portfolio decisions to the experts. We define intermediation as the investment activity that the experts conducts on behalf of the households; that is, experts intermediate funds between the households and the investment projects. In our setting, there are three potential sources of gains from such intermediation. Firstly, there is a fixed non-pecuniary cost of investing in risky assets, and it can be split among agents if investment occurs jointly. Second, the risks from investment activity can be spread more widely across agents. And finally, and more importantly, the funds in the economy can be allocated more efficiently due to the presence of expert information. This last channel is the focus of our paper; the two other motives severely simplify the problem by fixing the outside option of households. To be consistent with Leland and Pyle (1977), we make the following two assumptions to motivate the contractual setting:

Assumption 4 [*Complex Information*]

- *Experts' posterior beliefs are not observable by households, and*
- *Households know the distribution of expert's private signals.*

Assumption 5 [*Contractable Information*]

- *Portfolios chosen by experts are ex-post verifiable by the participating households,*
- *Contractual terms between households and experts are not publicly observable.*

By ex-post verifiable, we mean that portfolio weights can only be verified after contracts have been accepted, i.e. portfolio weights can only be observed when the investment of funds is actually made in a given portfolio. This ex-post assumption is not only realistic, but desirable, since it allows the experts to exploit their informational advantage when offering the contract.

Assumption 6 [*Costly Investment*] *The non-pecuniary cost of investment $\chi > 0$ satisfies:*

$$\frac{1}{2} [\mu_0^h - R_f 1_N]' \Sigma_0^{h-1} [\mu_0^h - R_f 1_N] < \chi < \frac{1}{2} [\mu_0^e - R_f 1_N]' \Sigma_0^{e-1} [\mu_0^e - R_f 1_N]$$

We now discuss the implications of the above assumptions for the contractual setting. First, note that Assumption (4) implies that the only information that cannot be communicated between experts and households is the mean of the experts' posterior distribution $\hat{\theta}_t$. The reason for this is that the experts' posterior can be fully characterized by its mean and variance, and that the variance of the experts' posterior is common knowledge by Assumption (4). Second, since project returns are public information, Assumption (5) implies that portfolio returns are verifiable. Therefore, Assumption (4) and (5) imply that experts and

households can contract upon the precision of the expert information, the expert portfolio choice, and the realized portfolio returns. Finally, assumption (6) ensures that households are not willing to invest in risky assets on their own, but would do so through an expert.

2.3 Optimal Intermediation Contracts

At each date t , an expert and a household get randomly matched. After the match is realized, the expert offers the household a take it or leave it *intermediation contract* that the household can accept or reject.¹ We define an *intermediation contract* as a contract in which an expert asks the household to deposit its funds in return for payoffs contingent on verifiable outcomes. If the household rejects the contract, then both the expert and the household invest on their own (these are their outside options). If the household accepts the contract, contractual terms are executed.

As Leland and Pyle (1977) have shown, total funds that are invested in the risky asset by the expert are a signal about the expert's private information. We solve a signaling problem, in which experts offer contracts taking into account that their portfolio weights signal to households their private information. Given Assumption (6), contracts can (and will) be contingent on portfolio weights. The timing of the per period problem is as follows. First, the expert offers the household to pull their funds together in exchange for a payoff contingent on the constructed portfolio and on the realized return of the chosen portfolio. To ensure that the household participates, the expert chooses the payoff functions so that the household gets its outside option.² Second, once the funds have been raised, the expert chooses her preferred portfolio and commits to the per-specified return-contingent payoff function for that particular portfolio choice.

The presence of overlapping generations of experts and households that are randomly matched to enter an intermediation contract allows us to isolate the per period problem, given the state variables: i) the state of the economy $\{X_t, \theta_t, \tilde{\theta}_t\}$, and ii) the private information of the experts $\{R^t\}$ summarized in their posterior $\{\mu_t, \Sigma_t\}$. First, we focus on the problem of an expert that enters period t with a posterior distribution $\theta \sim N(\mu_t, \Sigma_t)$, and we solve for the optimal intermediation contract, and optimal consumption and investment allocations. Second, we introduce dynamics to characterize the evolution of key variables over time.

For the analyzes of the per period problem, we drop the t subscripts when characterizing the stage problem. Let $\mu = \hat{\theta}_t$ and $\Sigma = \hat{\Sigma}_t$ denote the mean and precision of the expert's

¹Our qualitative results do not depend on the distribution of bargaining power.

²WLOG, this can be modeled as the expert offering a menu to the households, given by $\{c^e(R, \alpha(\mu)), c^h(R, \alpha(\mu)), \alpha(\mu)\}_{\forall \mu}$ where $c^e(R, \alpha(\mu)), c^h(R, \alpha(\mu))$ are the consumption allocations of the expert and the household, contingent on realized returns and on portfolio weights, and μ is the expert's private information (mean of its posterior distribution). In equilibrium, after the contract is accepted, the expert has to choose from that menu the triple consistent with its real $\mu = \hat{\theta}_t$. We show in the Appendix that these two problems are equivalent, and that the mechanism is optimal.

information, c^e, c^h denote the consumption allocations of experts and households determined by the contract, and α denote the expert's chosen portfolio. To solve the expert's problem, we formulate the following conjecture.

Conjecture 1 *Portfolio weights chosen by the expert (fully) reveal his private information.*

Let $\tilde{\mu}(\alpha)$ denote the signal about underlying beliefs embedded in the portfolio choice, an expert with posterior beliefs characterized by μ solves the following problem:

$$\begin{aligned} \max_{c^e, c^h, \alpha} E[u^e(c^e) | \mu] & \quad (2.1) \\ E[u^h(c^h) | \tilde{\mu}(\alpha)] & \geq \bar{U}^h \quad (\lambda_{pc}) \\ c^e(R) + c^h(R) & \leq [\alpha'(R - R_f 1_N) + R_f] w \quad (\lambda_{fc1}(R)) \end{aligned}$$

where w denotes the total funds of an intermediary, and it is given by the sum of experts and households initial endowments, $w = w^h + w^e$; and $E[x|\mu]$ denotes the expected value of x conditional on beliefs characterized by μ (experts), and $\tilde{\mu}(\alpha)$ (households). The experts problem is to maximize its expected utility subject to the participation constraint of the household (multiplier λ_{pc} , and the feasibility constraint (multiplier λ_{fc}).)³ Since we focus on separating equilibria, we impose the following ‘‘truth revelation’’ condition: $\tilde{\mu}(\alpha(\mu)) = \mu$.

Proposition 8 *Under the optimal intermediation contract the expert receives a fixed payment and a fraction of the returns of the portfolio where all funds are invested. Consumption allocations under the optimal contract are given by*

$$\begin{aligned} c^e(R, \alpha) &= \frac{\gamma^h}{\gamma^h + \gamma^e} R_p w + Z(\alpha) \\ c^h(R, \alpha) &= \frac{\gamma^h}{\gamma^h + \gamma^e} R_p w - Z(\alpha) \end{aligned}$$

where $R_p \equiv R_f + [R - R_f 1_N]'\alpha$ are the total portfolio returns and $Z(\alpha)$ is a transfer contingent on portfolio weights.

The optimal contract presented in Proposition 8 has a straight-forward interpretation. The first term of the contract is a variable payoff and it is given by the corresponding fraction of the total portfolio returns that each agent receives ($\frac{\gamma^e}{\gamma^h + \gamma^e} R_p$ for households and $\frac{\gamma^h}{\gamma^h + \gamma^e} R_p$ for experts). This fraction is chosen to smooth marginal utilities across states between households and experts, to attain full-risk sharing. To see this, note that for a given portfolio α , consumption allocations presented in Proposition 8 guarantee that:

³Households outside option \bar{U}^h is given by the utility households derive from investing their endowments in the risk-free firms, i.e. $\bar{U}^h \equiv u^h(w^h R_f)$.

$$u^{e'}(c^e(R, \alpha)) = \lambda u^{h'}(c^h(R, \alpha)) \quad \forall R \quad (2.2)$$

for $u^h(c) = -\exp[-\gamma^h c]$ and $u^e(c) = -\exp[-\gamma^e c]$.

The last term of the contract is a transfer made from the household to the expert. For exposition purposes, we decompose the transfer into a fixed payment plus a payment contingent on portfolio weights: $Z(\alpha) = \bar{f} + f(\alpha)$. The first term of this decomposition, \bar{f} , is a constant transfer that ensures that the participation constraint of the household binds. The second term, $f(\alpha)$, is what we refer to as an intermediation fee, since it reflects the value of acquiring the expert's intermediation services. One interesting result is that this fee is contingent on the portfolio weights chosen by the expert. This is because, from Conjecture 1, portfolio weights affect households' beliefs (and thus the value of intermediation) by signaling the expert's private information.

Proposition 9 *For a given portfolio α chosen by the expert, the optimal transfer that the expert receives from the household is given by $Z(\alpha) = \bar{f} + f(\alpha)$ where*

$$\bar{f} = R_f \left[\frac{\gamma^e}{\gamma^h + \gamma^e} w^e - \frac{\gamma^h}{\gamma^h + \gamma^e} w^h \right]$$

$$f(\alpha) = \frac{1}{\gamma^h} \left[\bar{\gamma} [\mu(\alpha) - R_f 1_N]' \alpha w - \frac{\bar{\gamma}^2}{2} (\alpha w)' \Sigma (\alpha w) \right]$$

Proof. Using Conjecture 1, we take the optimal consumption allocations and make the participation constraint of the households bind. ■ The previous results shows that by choosing portfolio weights, α , experts do not only affect the expected returns that arise from the portfolio, but also the fee charged to the households by manipulating their beliefs. This result relies on the fact that the funds invested in risky assets signal the expert's private information about the return of these assets (see also Leland and Pyle (1977)). Using the consumption allocations presented in Proposition 9, we solve for the expert's optimal portfolio choice. We proceed as follows. First, we conjecture the functional relationship between α and μ . Second, given our conjecture, we find the expert's optimal portfolio choice α when both the portfolio and the manipulation of beliefs effects are considered. Finally, we verify our conjecture.

Conjecture 2 *Given expert's private information μ , portfolio weights are given by $\alpha(\mu) = \kappa [w\bar{\gamma}\Sigma]^{-1} [\mu - R_f 1_N]$ with $\kappa > 1$.*

In Conjecture 2 we claim that when portfolio weights signal the expert's private information, there is a multiplicative distortion from optimal portfolios. In the absence of private information, optimal portfolios are given by the standard Sharpe ratio $[w\bar{\gamma}\Sigma]^{-1} [\mu - R_f 1_N]$. When portfolios signal private information, by investing more in the risky assets the experts can make households more optimistic about portfolio returns and thus increase the fee charged

for intermediation. The experts distort their portfolio choice towards riskier positions to increase intermediation fees.

Proposition 10 *The total funds invested in risky projects are given by*

$$\alpha w = \kappa (\bar{\gamma} \Sigma)^{-1} [\mu - R_f 1_N]$$

where $\kappa = \frac{\gamma^h + 2\gamma^e}{\gamma^h + \gamma^e}$ and $\bar{\gamma} = \frac{\gamma^h \gamma^e}{\gamma^h + \gamma^e}$.

Proof. Using the FOC with respect to α of the expert's problem (2.1), plugging in the conjecture, and using the method of undetermined coefficients yields result. (See Appendix for details). ■

Corollary 1 *The expert with posterior beliefs $\{\mu^e, \Sigma^e\}$ charges the following portfolio-contingent intermediation fee:*

$$f = \frac{1}{2} \frac{\gamma^h + 2\gamma^e}{(\gamma^h + \gamma^e)^2} [\mu^e - R_f 1_N]' (\Sigma^e)^{-1} [\mu^e - R_f 1_N]$$

When the investment in risky assets signals private information about the underlying quality of these assets, experts overinvest. The distortion in portfolios is generated by the expert's incentives to inflate the household's beliefs about portfolios returns, and thus raise a higher fee from intermediation. This overinvestment occurs despite the fact that in equilibrium the expert's strategy is inferred by households. The distortion to the expert's portfolio choice, $\kappa - 1$, is equal to the percentage of risk that the household is exposed to, $\gamma^e / (\gamma^e + \gamma^h)$, and is thus increasing (decreasing) in expert's (household's) risk aversion. The more the household is exposed to risk, the larger the expert's gains from convincing the household that risky returns are favorable.

Optimal contracts in our model match qualitatively the contracts offered by many hedge funds and portfolio managers. This is popularly referred to as the 2/20 fee structure, where managers charge a fee of 2% of assets under management, and receive 20% of the returns of the chosen portfolio. As Deuskar et al. (2011)[24] show, however, this type of contracts are a generalization, since when looking at the data on hedge fund fees, there are significant cross-section and time series variations in these amounts. What this means is that in reality, even though the contracts do look like a manager's fee and a fraction of the portfolio, the level of these two components is variable. This is consistent with our model, where fees can vary with the level of expertise of financial intermediaries, and with the set of investment opportunities experts can offer to households. In most of the literature on portfolio managers' fees, the variable component is referred to as the incentive fee. In our model, there is no moral hazard and thus the reasons why experts hold a fraction of the portfolio are: i) that they are actually investing their own funds in these portfolio, and (most importantly), ii) they share risks optimally with households. If moral hazard was introduced into the model, the variable component would not only be a function of the risk aversions, but some distortion might

arise to provide incentives to experts. We choose to avoid adding the moral hazard friction to be able to fully focus on the asymmetric information problem that arises when portfolio managers possess superior information about the quality of assets they invest households funds in, since we believe this is an interesting problem on its own.

Finally, we would like to end this section with an interesting fact documented in Mulainathan et al. (2008) in their audit to financial advisers. They find that “some advisers refused to offer any specific advice as long as the potential client has not transferred the account to the company of the adviser,” and they argue that this happens because no useful information wants to be revealed before the contract is accepted. This supports the view that there is not only a moral hazard problem present in intermediation, but an asymmetric information problem at the moment of contracting as well. They say: “it makes sense that advisers want to protect their time and insights so that clients do not replicate the advice for free.” Even though it makes sense, they find this result puzzling, since investors need to make decisions without knowing what the adviser knows. In our model this is exactly the case. We also present a solution to this puzzle: investors (households in our model) are conceptually being offered a contract that is contingent on portfolio weights, and these weights signal the expert’s private information ex-post. As we have shown, the problem of appropriability and reliability of private information is solved when fees are made contingent on the investment choices the advisers make after the contract is accepted.

In the following section we introduce dynamics to understand how private information, and thus portfolios and intermediation fees, evolve over time. The dynamic model provides interesting testable predictions for the correlation between the riskiness of portfolios and intermediation fees, and for the evolution of the overall income of financial intermediaries as a function of confidence in their expertise.

2.4 The Dynamic Economy

The dynamic economy is a straight-forward extension of the static problem presented in the previous section. Due to their short horizon, the contracts between experts and households are still short-term, but the information that the expert possess evolves over time. Before setting up the dynamic problem, we discuss the evolution of learning and how it responds to structural shocks. We have assumed that the mean of project returns follows the process given by:

$$\theta_t = (1 - X_t) \theta_{t-1} + X_t \tilde{\theta}_t \quad \forall t > 0$$

where $X_t \sim \text{Binomial}\{1, p\} \quad \forall t > 0$ and $X_0 = 1$. Thus, the mean θ_t remains unchanged as long as $X_t = 0$, but is redrawn anew from distribution $N(\bar{\theta}, \Sigma_\theta)$ whenever $X_t = 1$. This process allows to add a cyclical component to information by supposing that the fundamentals about which experts acquire information are stochastic. In particular, while persistence of fundamentals is essential for information to be valuable ($X_t = 0$), their randomness acts as an opposing force and diminishes the value of expert learning (when $X_t = 1$). Since X_t

is public information for all t , when computing posterior distributions for θ_t , agents only incorporate signals received after the change of state, i.e. signals received after date T given by $T = \sup\{\tau < t : X_\tau = 1\}$. The experts' posterior mean and variance are therefore given by

$$\mu_t = \left[\Sigma_\theta^{-1} + \Sigma_{s_t^R}^{-1} \right]^{-1} \left[\Sigma_\theta^{-1} \bar{\theta} + \Sigma_{s_t^R}^{-1} s_t^R \right] \quad (2.3)$$

$$\hat{\Sigma}_{\theta,t} = \left[\Sigma_\theta^{-1} + \Sigma_{s_t^R}^{-1} \right]^{-1} \quad (2.4)$$

where the updating is conditional on signals $s_t^R = (t - T)^{-1} \sum_{\tau=T}^t R_\tau$ with precision $\Sigma_{s_t^R}^{-1} = (t - T) \Sigma_\epsilon^{-1}$. The expert's problem at time $t \geq 0$, with public information $\{\hat{\Sigma}_{\theta,t}, X^t\}$, is given by

$$V^e \left(\mu_t, \hat{\Sigma}_{\theta,t}, X_t \right) = \max_{c_{t+1}^e, c_{t+1}^h, \alpha_t} E \left[u^e (c_{t+1}^e) \mid \mu_t, \hat{\Sigma}_{\theta,t} \right] \quad (2.5)$$

$$E \left[u^h (c_{t+1}^h) \mid \tilde{\mu}_t (\alpha_t), \hat{\Sigma}_{\theta,t} \right] \geq \bar{U}_t^h \quad (\lambda_{pc})$$

$$c_{t+1}^e (R_{t+1}) + c_{t+1}^h (R_{t+1}) \leq ([R_{t+1} - R_f 1_N]' \alpha_t + R_f) w \quad (\lambda_{fc1} (R_{t+1})) \quad \forall R_{t+1}$$

$$R_{t+1} = \theta_t + \varepsilon_{t+1} \quad \varepsilon_{t+1} \sim^{iid} N(0, \Sigma_\varepsilon)$$

$$\theta_t = (1 - X_t) \theta_{t-1} + X_t \tilde{\theta}_t \quad \tilde{\theta}_t \sim N(\bar{\theta}, \Sigma_\theta)$$

where μ_t and $\hat{\Sigma}_{\theta,t}$ are given by equations (2.3) and (2.4) respectively and θ_t is not observed by agents.

It is straightforward to verify that the solution to the dynamic problem matches the solution to the static model, given the prevalent state variables. This is because we have chosen an overlapping generations framework, where agents have short-horizons. The qualitative results of the model would not change if both agents had an infinite horizon, as long as some relevant level of informational asymmetry persists. We chose to avoid long horizons to avoid situations in which after long periods of stability households learn through their own experience with the expert, and thus intermediation is no longer motivated by expertise (information asymmetries become irrelevant when both agents have precise posteriors). We believe that the assumption that experts hold consistently more precise information than households is a realistic one, and this is why we have chosen a simple framework where the entrance of new uninformed households allows the informational asymmetry to persist over time. Finally, note that in this model there would be no gains from allowing agents to write long-term contracts, or contracts contingent on past information, as portfolio weights are a sufficient statistic for the expert's private information. Using results from the previous section, we characterize the equilibrium of the dynamic economy.

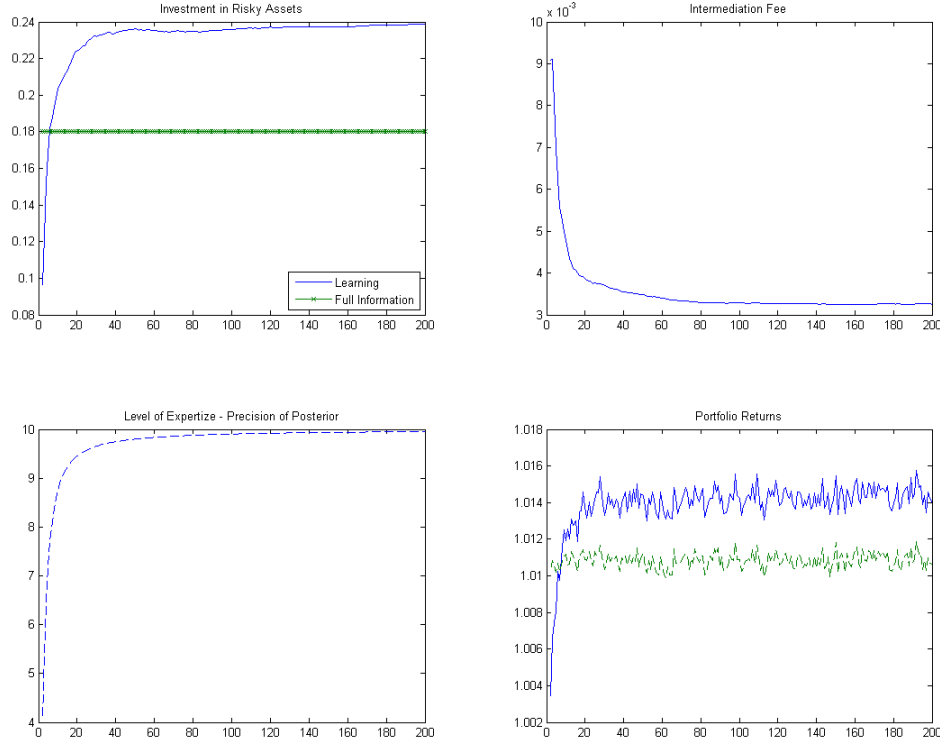


Figure 2.1: Learning by Lending

Number of simulations is 1000. Economy consists of one safe asset and two risky assets with time horizon $T = 200$. Parameter values are: $\gamma^e = 5, \gamma^h = 10, \omega^e = \omega^h = 1, R_f = 1, p = 0.01, \sigma_0 = 0.1, \sigma^e = 0.3, \sigma^\eta = 1, \theta = \theta_0 = 1.03$.

Proposition 11 *In the dynamic economy, with public information $\{\hat{\Sigma}_{\theta,t}, X^t\}$, and state variables $\{\theta_t, \tilde{\theta}_t, X_t\}$, the consumption allocations are given by*

$$c^e(R_{t+1}, \alpha_t) = \frac{\gamma^h}{\gamma^h + \gamma^e} ([R_{t+1} - R_f 1_N]' \alpha_t + R_f) w + \bar{f} + f(\alpha_t)$$

$$c^h(R_{t+1}, \alpha_t) = \frac{\gamma^e}{\gamma^h + \gamma^e} ([R_{t+1} - R_f 1_N]' \alpha_t + R_f) w - \bar{f} - f(\alpha_t)$$

$\forall t > 0$; the expert's portfolio choice is given by

$$\alpha_t = \kappa [w \bar{\gamma} \hat{\Sigma}_{\theta,t}]^{-1} [\mu_t - R_f 1_N]$$

where $\kappa = \frac{\gamma^h + 2\gamma^e}{\gamma^h + \gamma^e}$, and $(\mu_t, \hat{\Sigma}_{\theta,t})$ are given by equations (2.3) and (2.4). Finally, the inter-mediation fee charged to households is:

$$f(\mu_t) = (\gamma^h)^{-1} \left(\kappa - \frac{1}{2}\kappa^2 \right) [\mu_t - R_f 1_N]' \hat{\Sigma}_{\theta,t}^{-1} [\mu_t - R_f 1_N]$$

Proof. See Propositions 1-3. ■

We now illustrate the dynamics of our economy graphically. First, in Figure 2.1, we use a simple parametrization of our model to simulate a period of economic stability following an initial draw of the fundamental state θ . Then, in Figure 2.2, we show the effects of a shock to mean of project returns θ . We use the economy with full information, where all agents know the true value of θ , as a useful benchmark against which to compare our results. Figure 2.1 shows that the economy with learning is more volatile relative to the economy with full information, but that volatility diminishes over time as the level of expertise (precision of posterior beliefs) in the economy grows. Thus, periods of continued economic stability are associated with periods of gradually declining volatility. Volatility of investment between the two economies is a good case in point. In the full information economy, risky investment is fixed by a sharp ratio that is constant - all agents know the true mean of project returns and the returns are iid. When there is learning, however, both the perceived mean of project returns and their perceived variance change over time.

The mechanism we aim to highlight is the detrimental effect that the loss of “inside” information has when the economy experiences changes in fundamentals. A shock to fundamentals (positive or negative) generates an endogenous volatility shock for experts due to the loss of their expertise, reflected in a decrease in the precision of their private information. As a response, experts contract credit to risky sectors, while they start learning about the new economy. This amplifies and propagates shocks to the real economy, that in a standard model with full information would only generate a once and for all change in allocations at the moment of impact, and no change at all in uncertainty. We do not interpret these type of shocks as drivers of business cycles, these are permanent shocks that alter the distribution of productivities across firms or sectors in an economy. They should be interpreted as technological shocks that change the relative productivity of sectors within an economy. For example, the introduction of the internet affected some sectors positively and other negatively, this is one example of a shock that has altered relative productivities. The dot.com boom is a good example to describe the mechanism we have in mind: while at the beginning it was difficult to raise funding for internet related activities, once credit started going into this sector, optimism increased over time, possibly generating the so called dot.com bubble. Finally, it became clear that the sector was not as profitable as expected, credit to the private sector contracted, the U.S. had a recession, but eventually funds were allocated to new sectors (real state related investment, for example).

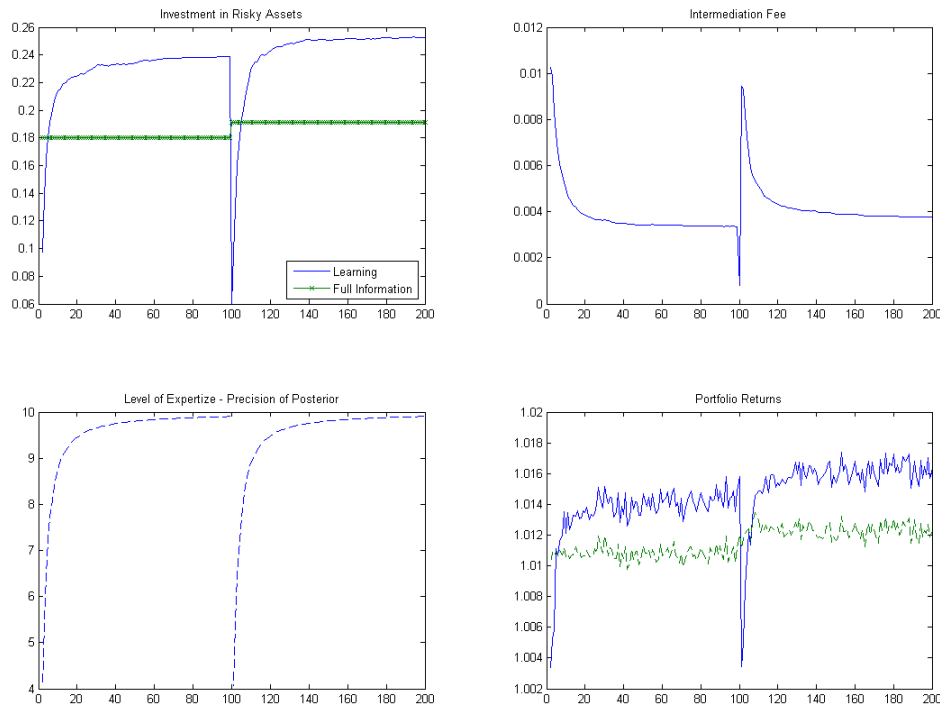


Figure 2.2: Structural Shocks

Same calibration as before. At $T = 100$, there is a structural shock, i.e. $X_T = 1$.

Discussion

Re-Allocation of Funds

One interesting feature of our model is that it can generate contractions as a response to a sectoral reallocation of productivities. In the previous simulations, the shock that hits the economy has an impact on aggregate productivity, since it changes the mean of project returns. In response to this, total investment to the risky sectors changes not only as response to the loss of expertise, but also as a response to the changes in fundamentals. In this section, we analyze a shock that shuffles productivities across sectors, but has no impact on aggregate returns if portfolios are re-adjusted accordingly. The response to a shock of this nature in a model with full information is to simply reallocate capital across sectors according to the new distribution of productivities. In our model, experts understand that there is a shock but they do not know how productivities have been re-shuffled. The lack of knowledge about the nature of the shock acts as a volatility shock, generating a fall in credit, and a slow recovery. As experts learn the new distribution of productivities and reallocate funds accordingly, the economy converges to its previous levels. Figure 3 shows the results

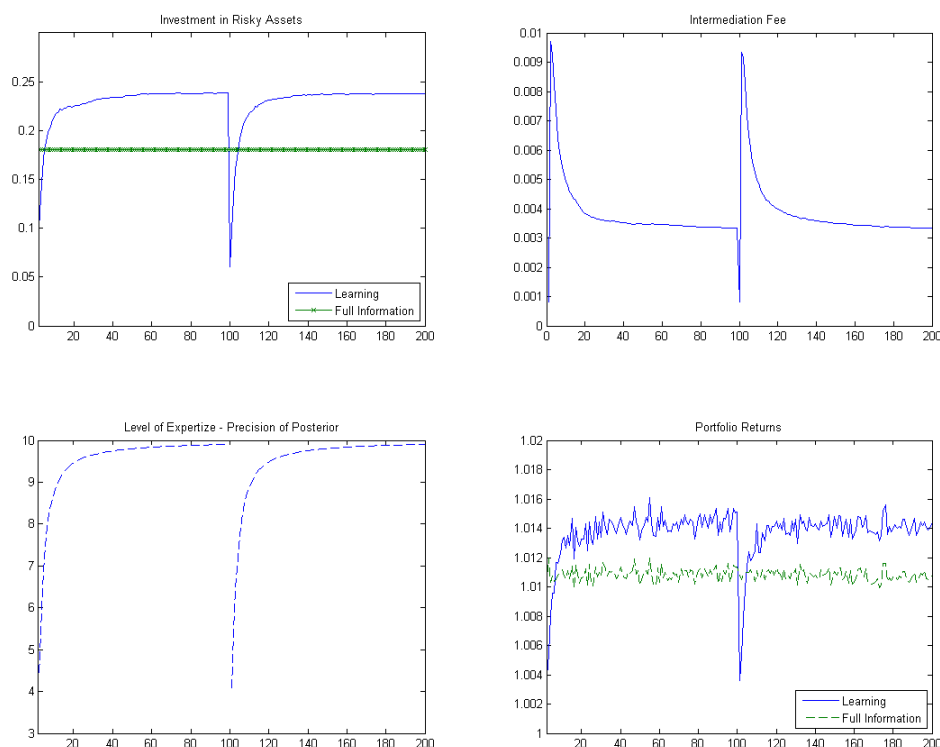


Figure 2.3: Sectoral Reallocation

Same calibration as before. At $T = 100$, there is a structural shock that has no aggregate effect, $\theta'_1 = \theta_2$ and $\theta'_2 = \theta_1$.

for a simulation in which the mean return of sectors one and three is interchanged. Notice that in this example, the benchmark economy of full information experiences no change in the levels of investment to the risky sectors.

Unobservable Aggregate States

A natural extension to this model is to make the aggregate state, X_t unobservable. In this scenario, agents should infer the change of state from observed returns. In a model with Bayesian learning, it is very hard to generate strong reactions to bad signals after long periods of stability. In the presence of a negative shock to fundamentals, experts would take a long time to realize that the state has changed, and credit cycles would not be strongly asymmetric as observed in the data. An alternative learning mechanism that allows to generate highly asymmetric responses was introduced by Marcet and Nicolini (2003)[56]. Their paper presents a boundedly rational learning model, where agents re-adjust their beliefs

very strongly once they observe realizations that are highly unlikely under their prevalent beliefs. The drawback of this learning mechanism is that it is not fully rational. However, it is extremely intuitive and a good candidate to generate abrupt changes in beliefs. If we were to make the aggregate state X_t unobservable, we would model expert's beliefs as follows: when realizations of returns are very far on the tails of the posterior distribution (a threshold is imposed), agents understand there has been a new draw of fundamentals, and the economy behaves as if the shock had been public. There are two new implications of using this learning mechanism: first, after a negative shock to fundamentals, it might take time for experts to realize this, and thus for some periods returns are going to be low on average. Second, cycles could be generated without having shocks to fundamentals at all. If an outlier is drawn, experts would interpret this as an aggregate shock and start reacting accordingly by putting more weight on recent observations, and disregarding past data. We believe that the results that could be obtained from an alternative learning as the one described here are very interesting, but we postpone that analyzes to future research.

2.5 Conclusions

We presented a dynamic model of financial intermediation in which changes in the information held by financial intermediaries generate asymmetric credit cycles as the ones documented by Reinhart and Reinhart (2010). Our model is able to generate long periods of credit expansion, followed by sharp contractions in lending and slow recoveries. We model financial intermediaries as information producers, we assume they are “expert” agents that have a unique ability to acquire information about firm/sector fundamentals. Better information allows for better allocation of resources, and this informational advantage makes these actors be the natural contenders to intermediate funds between households and businesses. The level of “expertize” in the economy and the potential gains from intermediation grow in tandem with the information that these experts possess; these gains, however, are hindered since experts' information is inherently private. We find the optimal financial contracts that balance allocational efficiency with the provision of appropriate incentives. The economy therefore inherits not only the dynamic nature of information flow, but also the interaction of information with the contractual setting. To generate contractions in credit we introduce a cyclical component to information by supposing that the fundamentals about which experts acquire information are stochastic. While persistence of fundamentals is essential for information to be valuable, their randomness acts as an opposing force and diminishes the value of expert learning. Our setting then features economic fluctuations due to waves of “confidence” in the experts' ability to allocate funds profitably.

Chapter 3

Credit Crises, Liquidity Traps, and Demand Externalities

3.1 Introduction

From 2000 to 2006, household indebtedness in the US grew from about 90 to 130 percent of disposable income. Similar trends were observed in a number of Eurozone countries. The catastrophic events for the world economy beginning with the housing slowdown in 2006 are well known by now. There is mounting evidence connecting the rise in household debt to the severity of the post-2006 economic slowdown. On a macro level, economies that had seen the fastest rise in household debt appear also to be the ones experiencing the sharpest declines in economic activity (Glick and Lansing (2010, 2011)[38][37]). On a micro level, the US counties with largest growth in household debt in the pre-slowdown period have also seen the largest drops in consumption and increases in unemployment during the recession (Mian and Sufi (2010, 2011)[60][61]). Finally, Mishkin (1978)[65] and Olney (1999)[68] argue that household balance sheets were important in propagating the initial downturn during the Great Depression. As the link from household balance sheets to the severity of economic slowdowns seems strong, a policy-maker may wonder if the benefits associated with high levels of household debt are worth the costs that these entail. From an economic theorists' point of view, however, the inefficiency of these boom-bust cycles is not evident. Perhaps, households do weigh their costs and benefits rationally and the consumption-savings decisions they make are optimal. In this paper, I present a simple theoretical framework within which I ask whether this is indeed the case.

I construct a stylized New Keynesian model with heterogeneous agents and incomplete asset markets to investigate whether the choice of debt in such economies is efficient. I build on the framework introduced by Eggertsson and Krugman (2012)[32]. I assume that the economy is composed of households with heterogeneous temporal preferences for consumption. Households with stronger preferences for early consumption desire to issue debt at early dates and roll it over at intermediate dates. I introduce credit crises by assuming that

in some states of the world borrowers are unable to roll over a portion of their debts into future periods. When a credit crisis occurs, borrowers are forced to delever by cutting their consumptions collectively, and unless monetary authorities are able to fully stabilize the aggregate demand, the aggregate income in the economy contracts. Eggertsson and Krugman (2012)[32] consider unexpected credit crises and focus on ex-post stabilization policies. I depart from their framework in two respects. First, I allow households to expect credit crises to occur with positive probability. This departure allows me to investigate the welfare implications of households' ex-ante consumption-savings decisions. Second, I assume that asset markets are incomplete by restricting debt contracts in the economy to be short-term and non-contingent¹. With this market incompleteness, the remaining margin through which households can potentially reduce the impact of credit crises is by reducing the amount of debt that they issue ex-ante. I find however that in a decentralized equilibrium households collectively undervalue the benefit that lower aggregate debt provides and borrow excessively. Intuitively, while the costs of debt reduction are borne by the debt reducing households, the resulting benefits from lower demand fluctuations accrue to all the households in the economy. Due to this negative externality, household debt tends to be excessive in equilibrium and policies limiting household indebtedness may be Pareto improving. The crucial ingredients for these results are the incompleteness of asset markets and nominal rigidities, both of which are plausible departures from a frictionless economic environment.

The framework I adopt is closely related to the one in Eggertsson and Krugman (2012)[32] who build on the "liquidity trap" literature that has been revived by Krugman (1998)[50] and has since grown substantially (e.g. Eggertsson and Woodford[33] (2003), Guerrieri and Lorenzoni (2011)[41], Werning (2011)[76]). My work is also related to the literature on equilibrium pecuniary externalities. Greenwald and Stiglitz (1986)[40] and Geanakoplos and Polemarchakis (1986)[35] provide seminal contributions. They show that the equilibria in economies with incomplete markets are in general constrained inefficient due to equilibrium price movements that are not internalized by atomistic economic agents. This is also the case in the present model. My contribution is rather to show that in economies with incomplete markets and nominal rigidities, the equilibrium inefficiencies that arise tend to result in excessive household debt. In this respect, the works of Caballero and Krishnamurthy (2001, 2004)[14][15], Lorenzoni (2008)[54], and Korinek (2011)[47] are closer in spirit. They show that in economies with collateral constraints a la Kiyotaki and Moore (1997)[46], firms or financial institutions may contract excessive debt due to fire-sale externalities that arise during crises. My findings are complimentary to this literature but more suitable to the analysis of households' consumption-savings decisions. Finally, the empirical support for the importance of household balance sheets is provided by Mishkin (1978), Olney (1999), Mian and Sufi (2010, 2011), Glick and Lansing (2010, 2011).

I organize the paper as follows. In Section 3.2, I describe the setup of the model. In Section 3.3, I characterize the model's equilibria. In Section 3.4, I conduct the welfare

¹Allowing for some default does not change the qualitative results. On the other hand, making debt fully contingent completes the asset market.

analysis. In Section 3.5, I conclude by summarizing my findings and giving suggestions for further research.

3.2 The Model

The economy lasts for three dates, $t = 0, 1, 2$, and consists of households, firms, and policy-makers who are in turn divided into monetary and fiscal authorities. I first describe the problems solved by the households and firms and then lay out the policies available to monetary and fiscal authorities.

Households The economy is composed of two types of households of measure $\frac{1}{2}$ each, indexed by $j = a, b$. Households derive utility from consumption and leisure. Heterogeneity between the two types of households arises from heterogeneity in their temporal preferences. In particular, households of type b are assumed to have a relatively stronger preference for early consumption. Households' period utilities from consumption and leisure are identical and assumed to be $u(c, l) = \log(c) + \chi \log(1 - l)$ and their lifetime welfare is given by

$$U_j = E_0 \sum_{t=0}^2 \beta_{j,t} \{ \log(c_{j,t}) + \chi \log(1 - l_{j,t}) \}, \quad j = a, b$$

$$\beta_{a,1}/\beta_{a,0} > \beta_{b,1}/\beta_{b,0}$$

$$\beta_{a,2}/\beta_{a,1} \geq \beta_{b,2}/\beta_{b,1}$$

where $\beta_{j,t}/\beta_{j,t-1}$ is the discount factor between dates $t - 1$ and t ². I assume that $\beta_{a,1}/\beta_{a,0} > \beta_{b,1}/\beta_{b,0}$ and $\beta_{a,2}/\beta_{a,1} \geq \beta_{b,2}/\beta_{b,1}$ to ensure that households of type b want to issue debt at date 0 in order to finance early consumption and that some of this debt is rolled over at date 1.

Consumption $c_{j,t}$ is assumed to be a CES basket of differentiated varieties indexed by $i \in [0, 1]$. This assumption is only needed for the economy with sticky prices where imperfect competition is an essential modeling ingredient. More specifically, $c_{j,t} = \left(\int c_{i,j,t}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}$ and θ is the elasticity of substitution between the varieties. The price index associated with the basket $c_{j,t}$ is denoted by $P_t = \left(\int p_{i,t}^{1-\theta} dj \right)^{\frac{1}{1-\theta}}$.

Households derive income from employment ($\frac{w}{P}l$), fiscal transfers (T), and ownership of firms ($\nu\Pi$). Households are endowed with shares of firm equity - ν , and may participate in a competitive financial market where they can trade both firm equity and short-term non-contingent real debt (d). I assume that the trades in firm equity are subject to short-sale

²Preferences $u(c, l) = \log(c) + \chi \log(1 - l)$ (or $u(c, l) = (c)^\gamma + \chi(1 - l)^\gamma$ more generally) ensure that when prices are fully flexible, aggregate income in this economy is independent of the cross-sectional wealth distribution. As the focus of the paper is on externalities that arise due to nominal rigidities, this is a convenient simplification.

constraints, i.e. $\nu_{j,t} \geq 0$ for $j = a, b$ and $t = 1, 2^3$. Households' flow budget constraint is given by

$$c_{j,t} \leq \frac{w_{j,t}}{P_t} l_{j,t} + T_{j,t} + \nu_{j,t} \Pi_t + (\nu_{j,t} - \nu_{j,t+1}) Q_t + d_{j,t} - (1 + r_{t-1}) d_{j,t-1}$$

$$d_{-1} = r_{-1} = 0$$

where r_t denotes the real interest rate on debt and Q_t denotes the price of firms' equity.

Debt Limit: At date 1, households may be faced with an upper limit ϕ on the stock of debt carried to date 2, i.e. $d_{j,1} \leq \phi$. This debt limit is stochastic. There are two states of nature s_L and s_H possible at date 1. The states are realized prior to any date 1 decision that households make. The probabilities of the two states are denoted by $p(s_L)$, $1 - p(s_L)$ respectively. I set $\phi(s_H)$ to be the "natural" debt limit, i.e. the maximal liability that households can repay at date 2. The debt limit in state s_L is assumed to be strictly lower, $\phi(s_L) < \phi(s_H)$.

Households' Problem: Household j takes prices $(w_t(s), \{p_{i,t}(s)\}_i, r_t(s), Q_t(s))_t$ as given and solves

$$V_j = \max_{c_{j,i,t}, l_{j,i,t}, d_{j,t}} E_0 \sum_{t=0}^2 \beta_{j,t} \{ \log(c_{j,t}) + \chi \log(1 - l_{j,t}) \}, \quad j = a, b$$

subject to

$$c_{j,t} \leq \frac{w_{j,t}}{P_t} l_{j,t} + T_{j,t} + \nu_{j,t} \Pi_t + (\nu_{j,t} - \nu_{j,t+1}) Q_t + d_{j,t} - (1 + r_{t-1}) d_{j,t-1}$$

$$d_{j,-1} = r_{-1} = 0$$

$$d_{j,1} \leq \phi$$

$$-\nu_{j,t} \leq 0$$

$$c_{j,t} = \left(\int c_{j,i,t}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}$$

$$P_t = \left(\int p_{i,t}^{1-\theta} dj \right)^{\frac{1}{1-\theta}}$$

Firms There is a continuum of firms indexed by $i \in [0, 1]$, each firm producing a single differentiated variety. Firms produce goods using labor only,

$$y_{i,t} = L_{i,t}^\alpha$$

³The assumption is in line with the assumption that household debts are non-contingent.

$$L_{i,t} = \sum_j \frac{1}{2} l_{i,j,t}$$

Firms are monopolistically competitive in the production of their variety and set their prices internalizing the effect they have on the demand for their good. Let $\Gamma_{i,t}$ denote the date t discount factor of firm i 's owners. Firms' discounted profits are given by

$$\begin{aligned} \Lambda_i &= \sum_{t=0}^2 E_0 \{ \Gamma_{i,t} \Pi_{i,t} \} \\ &= \sum_{t=0}^2 E_0 \left\{ \Gamma_{i,t} \left(\frac{p_{i,t}}{P_t} y_{i,t} - \frac{w_t}{P_t} L_{i,t}^\alpha \right) \right\} \end{aligned}$$

Nominal Rigidities: I introduce nominal rigidities by assuming that firms need to decide their non-contingent price $p_{i,1}$ before realization of $s \in \{s_L, s_H\}$. As is standard in the New Keynesian literature, when the state s is realized and firms are unable to reset their prices, firms adjust production to satisfy the demand for their goods. Firms do so as long as profits are non-negative. Firms ration production otherwise.

Firms' Problem: Firm i takes prices $\left(w_t(s), \{p_{k,t}(s)\}_{k \neq i}, r_t(s) \right)_t$ as given and solves

$$L = \max_{p_{i,t}, y_{i,t}, L_{i,t}, l_{j,i,t}} \sum_{t=0}^2 E_0 \left\{ \Gamma_{i,t} \left(\frac{p_{i,t}}{P_t} y_{i,t} - \frac{w_t}{P_t} L_{i,t}^\alpha \right) \right\}$$

subject to

$$y_{i,t} \leq L_{i,t}^\alpha = \left(\sum_j \frac{1}{2} l_{i,j,t} \right)^\alpha$$

$$y_{i,t} \leq \sum_j \frac{1}{2} c_{j,i,t}$$

$$P_t = \left(\int p_{i,t}^{1-\theta} dj \right)^{\frac{1}{1-\theta}}$$

where when prices are sticky, the price $p_{i,1}$ is non-contingent and preset at date 0; and as long as profits are non-negative, production $y_{i,1}$ adjusts to satisfy demand $c_{i,1} = \sum_j \frac{1}{2} c_{j,i,1}$.

Monetary Authorities When prices are flexible, the real side of the economy is independent of monetary policy and, in that case, I normalize the price level to unity and ignore monetary policy altogether. With nominal rigidities, monetary policy plays a central role. I analyze monetary policy under full commitment and assume that monetary authorities set the short term nominal rate - i and have full control of the economy-wide price level - P in periods when prices are flexible. Since prices are assumed to be fully flexible at date 0, the

date 0 price level - P_0 and nominal rate - i_0 are inessential in determining the allocations of the sticky price economy. I therefore specify monetary policy as the set of instruments $\{i_1(s), P_2(s)\}_s$. Finally, I do not allow monetary authorities to overcome the debt limit in a credit crisis⁴. The interest rate $i_1(s_L)$ is thus the rate at which households can save or borrow up to the debt limit $\phi(s_L)$. I specify the objective of monetary authorities in the next section. As I show, monetary authorities have the ability to replicate the allocations of the flexible price economy provided that they are not averse to setting high enough inflation targets. I do not model inflation costs explicitly but instead assume that these costs may exist in the background. If, due to such costs, monetary authorities are unwilling to set relatively high inflation targets, they may not be able to fully stabilize output and credit crises will lead to output fluctuations.

Fiscal Authorities I only introduce fiscal authorities in order to analyze the efficiency of the households' ex-ante consumption-savings decisions. I restrict fiscal authorities to a simple set of policy tools. I assume that at date 0, the fiscal authorities set a debt tax τ_0 on the borrowers and specify lump sum transfers $\{T_{j,0}\}_j$. Fiscal authorities do not intervene at dates 1 or 2.

Credit Crises I refer to the situation where the debt limit at date 1 - state s_L binds, i.e. $d_1(s_H) > \phi(s_L)$, as a credit crisis. When the debt limit binds, borrowers are effectively rationed away from credit markets and the implicit rate at which they can borrow is set "abnormally" high. Credit crises are exogenous in this model as in Eggertsson and Krugman (2012)[32], but I assume that the economic agents have a full knowledge of their possibility. One may think of the debt limit fluctuations as representing sharp declines in collateral values or shocks to the intermediating sector, both of which can hinder the supply of funds to the borrowers.

3.3 Equilibrium Characterization

I now define the equilibria in the flexible and the sticky price economies. I then proceed to the characterization of each equilibrium in turn. The main results of this section are Propositions 12 and 16. In Proposition 12, I show that credit crises do occur in equilibrium if the debt limit is tight enough in state s_L ; with full price flexibility, aggregate income is always fixed in the economy and credit crises lead to fluctuations in interest rates. In Proposition 16, I show that in addition, when prices are sticky, credit crises may lead to output fluctuations. This latter conclusion will depend crucially on the willingness of monetary authorities to increase their inflation targets.

The equilibrium in an economy with flexible prices is defined as follows:

⁴If monetary authorities can lend to borrowers in a credit crisis, then the complete markets allocation can be trivially achieved.

Definition 3 *An equilibrium in the flexible price economy is a collection of prices*

$$\{w_t(s), \{p_{i,t}(s)\}_i, r_t(s), Q_t(s)\}_t$$

allocations,

$$\left\{ \{c_{j,i,t}(s)\}_{j,i}, \{d_{j,t}(s)\}_j, \{l_{j,i,t}(s)\}_{j,i}, \{y_{i,t}(s)\}_i, \{\nu_{j,t}(s), \Pi_t(s)\} \right\}_t$$

and given fiscal policy $\{\tau_0, \{T_{j,0}\}_j\}$ such that:

- a) $\left\{ \{c_{j,i,t}(s)\}_{j,i}, \{d_{j,t}(s)\}_j, \{l_{j,i,t}(s)\}_{j,i} \right\}$ *solve households' problem where households take prices and fiscal policy as given,*
- b) $\left\{ p_{i,t}(s), y_{i,t}(s), \{l_{j,i,t}(s)\}_{j,i} \right\}_t$ *solve firms' problem where firms take prices (except their own) and fiscal policy as given, and*
- c) *prices $\{w_t(s), \{p_{i,t}(s)\}_i, r_t(s), Q_t(s)\}_t$ are such that goods, labor, and asset markets clear.*

On the other hand, the equilibrium in an economy with nominal rigidities is defined as follows:

Definition 4 *An equilibrium in the sticky price economy is a collection of prices*

$$\{w_t(s), \{p_{i,t}(s)\}_i, r_t(s), Q_t(s)\}_t$$

allocations

$$\left\{ \{c_{j,i,t}(s)\}_{j,i}, \{d_{j,t}(s)\}_j, \{l_{j,i,t}(s)\}_{j,i}, \{y_{i,t}(s)\}_i, \{\nu_{j,t}(s), \Pi_t(s)\} \right\}_t$$

and fiscal and monetary policies $\{\tau_0, \{T_{j,0}\}_j\}$ and $\{i_1(s), P_2(s)\}$ such that:

- a) $\left\{ \{c_{j,i,t}(s)\}_{j,i}, \{d_{j,t}(s)\}_j, \{l_{j,i,t}(s)\}_{j,i} \right\}$ *solve households' problem where households take prices, fiscal and monetary policies as given,*
- b) $\left\{ p_{i,t}(s), y_{i,t}(s), \{l_{j,i,t}(s)\}_{j,i} \right\}_t$ *solve firms' problem where firms take prices (except their own), fiscal and monetary policies as given, and are restricted to setting non-contingent price $p_{i,1}(s) = p_{i,1}$ for date 1, and*
- c) *prices $\{w_t(s), \{p_{i,t}(s)\}_i, r_t(s), Q_t(s)\}_t$ are such that goods, labor and asset markets clear.*

In Proposition 12, I characterize the behavior of the flexible price economy. Aggregate income is fixed in this economy at all dates and states, and in the event of a credit crisis, when impatient households are prevented from rolling over their debts, real interest rates adjust downwards to induce the patient households to increase their consumption. Let $\epsilon = \frac{\beta_{a,1}}{\beta_{a,0}} - \frac{\beta_{b,1}}{\beta_{b,0}}$, then

Proposition 12 (*Flexible Price Economy*) In the flexible price economy with no fiscal intervention, 1) the aggregate income, consumption, and employment are given by $\bar{Y} = \left(\frac{1}{1+\frac{\mu}{\alpha}}\right)^\alpha$ at all dates and states; 2a) there exists a $\hat{\phi}_f > 0$ such that for $\epsilon > 0$ and $\phi(s_L) \leq \hat{\phi}_f$ there is an open set of crisis probabilities $[0, k(\phi(s_L))]$ for which the debt limit binds strictly for impatient households in state s_L ; and 2b) when $\phi(s_L) < \hat{\phi}_f$ and $p(s_L) \in [0, k(\phi(s_L))]$ the market clearing real interest rate is lower in state s_L than in state s_H , i.e. $r(s_L) < r(s_H)$.

Proof. (Sketch) 1) The expression for aggregate income can be derived by aggregate the households' consumption leisure choice $\chi c = \frac{w}{P}(1-l)$ across households and combining it with firm pricing decision $\frac{p}{w} = \frac{\mu}{\alpha} Y^{\frac{1-\alpha}{\alpha}}$. 2a) The model's equilibrium allocations and prices are continuous and differentiable in $p(s_L)$, except at the point where the debt limit just binds. Hence, if the debt limit $\phi(s_L)$ binds for $p(s_L) = 0$ (which can be ensured by reducing $\hat{\phi}_f$ and/or increasing ϵ), then there is an open set $[0, k(\phi(s_L))]$ such that the debt limit $\phi(s_L)$ binds for all $p(s_L) \in [0, k(\phi(s_L))]$. 2b) Inequality $r(s_L) < r(s_H)$ is derived from the patient households' Euler equation. ■

The behavior of the sticky price economy differs from that of the flexible price economy in one crucial respect. When the debt limit fluctuates and prices are rigid, some of the interest rate adjustment may translate into fluctuations in the aggregate income. Whether this is the case will crucially depend on the behavior of the monetary authorities. To this end, I argue next that the set of monetary rules $\{i_1(s), P_2(s)\}$ can be restricted considerably. In Proposition 13, I show that monetary authorities can potentially replicate the allocations of the flexible price economy. In Proposition 14, I argue that it is sufficient to consider monetary policies that feature non-contingent inflation targetting.

Proposition 13 There exist monetary rules $\{i_1(s), P_2(s)\}$ such that the flexible and sticky price economies result in identical allocations. The family F of monetary rules that replicate the flexible price economy is given by

$$F = \left\{ \{i_1(s), P_2(s)\} : \exists P_1 \text{ s.t. } i_1(s) = \left(1 + r_1^f(s)\right) \frac{P_2(s)}{P_1} - 1, i_1(s) \geq 0 \text{ for } s \in \{s_L, s_H\} \right\}$$

where $r_1^f(s)$ denotes the market clearing rate of interest in state s when prices are flexible. The set of rules F is non-empty.

Proof. Let $\bar{P} > 0$ and let $\{r_1^f(s)\}_s$ be the real interest rates that would prevail if prices were fully flexible. If monetary authorities set interest rates $\{i_1(s)\}$ so that

$$(1 + i_1(s_H)) = \frac{1 + r_1^f(s_H)}{1 + r_1^f(s_L)} (1 + i_1(s_L))$$

with

$$i_1(s_L) \geq 0$$

then the real interest rates $\{r_1(s)\}_s$ in the sticky price economy are identical with $\{r_1^f(s)\}_s$. The allocations in the two economies are thus identical and the inflation rate between dates 1 and 2 can be computed as

$$\frac{1 + i_1(s_L)}{1 + r_1^f(s_L)} - 1 = \frac{\bar{P}}{P_1} - 1 = \pi_1$$

Since aggregate income at date 1 is equal to \bar{Y} , firms' ex-ante prices p_1 are also ex-post optimal and π_1 is the equilibrium rate of inflation. ■

Note that the monetary rules $\{i_1(s), P_2(s)\}$ constructed in the proof of Proposition 13 specify a non-contingent price level $P_2(s) = \bar{P}$. Hence, I conclude that

Proposition 14 (*Inflation Targetting*) For each monetary rule $\{i_1(s), P_2(s)\}$ and fixed \bar{P} , there exist a rule $\{\tilde{i}_1(s), \bar{P}\}$ such that the equilibrium allocations under rules $\{i_1(s), P_2(s)\}$ and $\{\tilde{i}_1(s), \bar{P}\}$ are identical.

Proof. The result follows from proof of Proposition 13. ■

Let $\pi_1 = \frac{\bar{P}}{P_1} - 1$ be the equilibrium inflation rate between dates 1 and 2. Under policy rules $\{i_1(s), \bar{P}\}$ and assumption of full price rigidity, π_1 is non-contingent. The result in Proposition 14 therefore states that it is without loss of generality to consider monetary rules that specify non-contingent inflation targets⁵. It may at first appear that monetary authorities can achieve any inflation target π_1 at will. In Proposition 15, I argue however that due to the lower bound on the nominal rate, this is indeed the case for inflation targets that are high enough. I first make the following definition,

Definition 5 *Inflation target π is attainable if there exist monetary rules $\{i_1(s), \bar{P}\}$ such that π is the realized equilibrium inflation rate between dates 1 and 2. Given an attainable inflation target π , a monetary rule $\{i_1(s), \bar{P}\}$ is π -consistent if the equilibrium inflation rate under rule $\{i_1(s), \bar{P}\}$ is at most π .*

Let $F(\pi)$ denote the set of π -consistent monetary rules, then have that

Proposition 15 If $F(\pi^0)$ is non-empty, then $F(\pi^1)$ is non-empty for all $\pi^1 \geq \pi^0$. In addition, there exists a $\tilde{\pi}$ such that for all $\pi \geq \tilde{\pi}$, the allocations under rules in F are attainable under some rules in $F(\pi)$.

⁵To be precise, if one allows for prices to be partially rigid, then inflation rate π_1 will be state-contingent. Full price rigidity is a convenient simplification. In fact, the allocations of the partially rigid economy are subsumed in the fully rigid economy: since allocations of the sticky price economy are pinned down by the date 1 interest rates, the interest rates of the partially rigid economy can always be achieved by a policy of the form $\{i_1(s), \bar{P}\}$ as in Proposition 13.

Proof. Let π^0 be an attainable inflation rate and $\{i_1(s), \bar{P}\}_s$ be monetary rule associated with π^0 . Fix $\pi^1 > \pi^0$ and consider the rule $\{\tilde{i}_1(s), \bar{P}\}_s$ where

$$\tilde{i}_1(s) = \frac{1 + \pi^1}{1 + \pi^0} (1 + i_1(s)) - 1, \quad s = s_L, s_H$$

Then we have that $\tilde{i}_1(s) \geq 0$ for $s = s_L, s_H$ and the inflation rate delivered under rule $\{\tilde{i}_1(s), \bar{P}\}_s$ is π^1 . The real interest rates and thus the allocations under the two rules are the same. ■

To summarize the results in Propositions 13, 14, and 15, I have shown that it suffices to consider inflation targetting monetary rules, that inflation targets can be made non-contingent, that high enough inflation targets are always attainable, and that there exists a threshold inflation rate such that the allocations of the flexible price economy are attainable under targets above that threshold.

Prior to characterizing the behavior of the sticky price economy, it is natural to restrict the set of policy rules $F(\pi)$ a bit further. I assume that monetary authorities in addition want to target output stabilization at date 1. To that end, let

$$Stab(\pi) = \left\{ \{i_1(s), \bar{P}\} : \{i_1(s), \bar{P}\} \in F(\pi), \text{ and } i_1(s) \in \operatorname{argmin}_{\{i_1(s) \geq 0\}} (Y_1(s_L) - \bar{Y})^2 \right\}$$

be the subset of monetary rules in $F(\pi)$ that target maximum output stabilization⁶. Thus, I implicitly restrict my attention to monetary rules that put inflation targetting ahead of output stabilization as a primary policy objective. The results however would remain qualitatively the same if instead a loss function with quadratic costs from inflation and output gaps were specified. The behavior of the sticky price economy is summarized next.

Proposition 16 (*Sticky Price Economy*) In the sticky price economy with no fiscal intervention, with an attainable inflation rate π and a monetary rule $\{i_1(s), \bar{P}\} \in Stab(\pi)$, 1) the aggregate income, consumption, and employment are given by $\bar{Y} = \bar{Y} = \left(\frac{1}{1+\frac{\pi\epsilon}{\alpha}}\right)^\alpha$ at dates 0 and 2; 2a) there exists a $\hat{\phi}_s > 0$ such that for $\epsilon > 0$ and $\phi(s_L) \leq \hat{\phi}_s$ there is an open set of crisis probabilities $[0, k(\phi(s_L))]$ for which the debt limit binds strictly for impatient households in state s_L , 2b) when $\phi(s_L) < \hat{\phi}_s$ and $p(s_L) \in [0, k(\phi(s_L))]$ then either the asset market clearing interest rate is lower in state s_L than in state s_H , or aggregate income is lower in state s_L than in state s_H , or both, and 2c) if aggregate income falls in state s_L , then $Y_1(s_L) < \bar{Y} < Y_1(s_H)$ for $p(s_L) > 0$.

Proof. (Sketch) 1) The expression for aggregate income at dates 0 and 2 is derived similarly as in the flexible price economy. At date 1, aggregate income is determined by equilibrium

⁶Notice that $Stab(\pi)$ only includes monetary rules that minimize recessions. This in fact is sufficient for full stabilization as monetary authorities are constrained only by the lower bound on the nominal rate.

price setting⁷, monetary policy, and labor market clearing. 2) The equilibrium prices and allocations are continuous and differentiable in $p(s_L)$ except at the point where the debt limit just binds and where the nominal rate just hits the zero lower bound. Hence, if the debt limit $\phi(s_L)$ binds for $p(s_L) = 0$ (which can be ensured by reducing $\hat{\phi}_s$ and/or increasing ϵ), and the inflation target is low enough, there is an open set $[0, k(\phi(s_L))]$ such that the debt limit $\phi(s_L)$ strictly binds and the nominal rate is at 0 for all $p(s_L) \in [0, k(\phi(s_L))]$. In that case, only $Y_1(s_L) \leq \bar{Y} \leq Y(s_H)$ is consistent with rational price setting, and inequalities must be strict when $0 < p(s_L) < 1$. ■

Discussion

Proposition 16 shows the key distinction between the flexible and the sticky price economies. In the flexible price economy, when the debt limit binds at date 1, real interest rates fall in order to induce the patient households to reduce saving and increase consumption. This mechanism is available in the sticky price economy but only to a limited extent. From the Fisher identity

$$1 + r_1 = (1 + i_1) \frac{P_1}{\bar{P}}$$

the adjustment in r_1 can either be achieved through the price level P_1 or through the nominal rate i_1 . The former channel is absent when prices are sticky and the latter is limited due to the zero bound on the nominal rate. Hence, when patient households cannot be induced to consume, aggregate demand and income fall to equilibrate the markets. In Figures 1-3, I illustrate the behavior of the sticky price economy by varying the debt limit (severity of the credit crisis) in state s_L .

Figure 3.1 shows the equilibrium “excess rollover”, $d_1(s_H) - \phi(s_L)$, and the behavior of equilibrium interest rates at date 1. Notice that the real rate is lower in state s_L than in state s_H as long as the excess rollover is positive. In the sticky price economy, however, the lowest the real interest rate can be set is the negative of the inflation rate, which here is calibrated to 2%. When the debt limit reaches about 0.1, an interest rate of -2% coincides with the flexible price interest rate. The real interest rate in state s_H is lower with sticky prices than with flexible prices and this is due to firms’ desire to set prices so as to hedge the aggregate demand fluctuations at date 1. As the debt limit increases and credit crises become smaller, the interest rates in the two states come closer together.

Figure 3.2 shows the behavior of aggregate income across the two states. Again, for values of the debt limit below 0.1, the zero lower bound on the nominal rate is binding and recessions cannot be avoided. Notice that rational price setters ensure that the fluctuations in income are not one-sided and that booms occur in state s_H . When the debt limit is higher than 0.1, monetary authorities are able to use their interest rate policy to fully stabilize the date 1 aggregate income.

⁷Symmetric price setters set $1 = \frac{p_i}{P} = \frac{E\{\Gamma_i MC\}}{E\{\Gamma_i Y\}}$ where $MC = \frac{1}{\alpha} \frac{w}{P} Y^{\frac{1-\alpha}{\alpha}}$ and $\frac{w}{P} = \frac{\chi Y}{1-Y^{\frac{1}{\alpha}}}$.

Finally, Figure 3.3 shows that crises are worse for the borrowers than for the savers. The volatility of consumptions and discount factors in general increases with the severity of crises. This volatility however increases more rapidly for the borrowers than for savers. The means of the discount factors are equalized across households, as households are unconstrained ex-ante, and increase with the severity of credit crises due to households' precautionary motives.

In the next section, I proceed to the welfare analysis, which is the main contribution of this paper. The welfare analysis is motivated by the following observation. When prices are flexible, the marginal rate of substitution between consumption and leisure is equated, modulus the monopolistic distortion, to the marginal rate of transformation in the economy. When prices are sticky and credit crises are large, then aggregate output fluctuates at date 1 and the rate of substitution diverges from the rate of transformation. The level of production is either too low (state s_L) or too high (state s_H) in this economy at date 1. As fluctuations in consumption interact with fluctuations in income, higher output fluctuations in addition translate into larger volatilities in discount factors in a way that risk sharing across households is reduced (Figure 3.3). Notice that households borrowing decisions are one for one tied with the magnitude of date 1 income fluctuations through the equilibrium excess rollover, $d_1 - \phi$. This is an equilibrium relationship, however, and while households do not internalize the ex-post social costs of their borrowing decisions, fiscal authorities can. There may thus be room for policy-makers to improve welfare by regulating household debt ex-ante. As I show next, this line of reasoning is correct. When credit crises are large so as to lead to output fluctuations, reducing household debt ex-ante does indeed lead to Pareto improvement.

3.4 Welfare Analysis

In this section, I analytically investigate the welfare properties of the decentralized equilibrium in the sticky price economy. To this end, I define the concept of constrained efficiency against which I compare the decentralized equilibrium as follows:

Definition 6 *The equilibrium of the sticky price economy with monetary rule $\{i_1(s), \bar{P}\} \in \text{Stab}(\pi)$ is constrained efficient if there does not exist a set of fiscal instruments $\{\tau_0, \{T_{j,0}\}_j\}$ with $\tau_0 \neq 0$ that achieve strict Pareto improvement.*

Since I am interested in whether the aggregate debt in this economy is excessive, I define equilibria with excessive household debt as follows:

Definition 7 *The equilibrium of the sticky price economy with monetary rule $\{i_1(s), \bar{P}\} \in \text{Stab}(\pi)$ features excessive household debt if there exists fiscal instruments $\{\tau_0, \{T_{j,0}\}_j\}$ with $\tau_0 > 0$ that achieve strict Pareto improvement.*

Notice that the concept of constrained efficiency in Definition 7 is subject to a particular ex-post reaction function of the authorities (here only monetary). In particular, it may be the case that the equilibrium is constrained inefficient under monetary rules $\{i_1(s), \bar{P}\} \in \text{Stab}(\pi^0)$ but not under rules $\{i_1(s), \bar{P}\} \in \text{Stab}(\pi^1)$. The same of course is true for the classification of equilibria according to whether household debt is excessive or not.

In what follows, I first conduct a welfare analysis for a simplified version of the sticky price economy where I do not allow households to trade firm equity. I then show that the economy with endogenous trade in firm equity is similar to a special case of no-equity trade economy. To preview the results, I show analytically that the decentralized equilibrium in these economies is generally not *constrained efficient*. In the case when credit crises are large enough to lead to output fluctuations, I am able to show that household debt is *excessive*. I do this analytically for a particular parameterization of the no-equity trade economy and verify that this is the case for other parameterizations numerically. In the numerical exercise, I also compute the size of these inefficiencies.

Exogenous Firm Ownership

In Proposition 17, I derive the effect on households' welfare of a marginal debt tax imposed on borrowers at date 0. The transfers $\{T_{j,0}\}$ are chosen so as to leave the savers' indifferent.

Proposition 17 (*Externalities I*) In the sticky price economy with a monetary rule $\{i(s), \bar{P}\} \in \text{Stab}(\pi)$, the change in impatient households' welfare from a marginal increase in debt tax evaluated at $\tau = 0$ that leaves patient households' indifferent is given by

$$\begin{aligned} \frac{dV_b}{d\tau} \Big|_{\tau=0} &= -\beta_{b,0} (c_{b,0})^{-1} E_0 \left\{ (\Gamma_{b,2} - \Gamma_{a,2}) (d_1) \frac{dr_1}{d\tau} \right\} + \\ &+ \beta_{b,0} (c_{b,0})^{-1} E_0 \left\{ (\nu_b \Gamma_{b,1} + (2 - \nu_b) \Gamma_{a,1}) \left(1 - \frac{w_1}{P_1} \right) \left(\frac{dY_1}{d\tau} \right) \right\} + \\ &+ \beta_{b,0} (c_{b,0})^{-1} \times \frac{1}{2} \times E_0 \left\{ (\Gamma_{b,1} - \Gamma_{a,1}) (l_{b,1} - l_{a,1}) \frac{d\left(\frac{w_1}{P_1}\right)}{d\tau} \right\} \\ &+ \beta_{b,0} (c_{b,0})^{-1} \left(\frac{1}{2} \times E_0 \left\{ (\Gamma_{b,1} - \Gamma_{a,1}) (1 - \nu_b) (Y_1) \frac{d\left(\frac{w_1}{P_1}\right)}{d\tau} \right\} \right) \end{aligned}$$

where ν_b denotes the impatient households' endowment of firm equity and $\Gamma_{j,t}(s) = \frac{\beta_{j,t}}{\beta_{j,0}} \left(\frac{c_{j,t}(s)}{c_{j,0}} \right)^{-1}$ is the stochastic discount factor with which household j discounts consumption from date t and state s to date 0. In addition, $\Gamma_{b,1}(s_L) > \Gamma_{a,1}(s_L)$, $\Gamma_{b,1}(s_H) < \Gamma_{a,1}(s_H)$, and $\Gamma_{b,2}(s_H) < \Gamma_{a,2}(s_H)$.

Proof. (Sketch) Apply Envelope Theorem to the consumption-leisure choices for each date, state, and each type of household, and to the Euler conditions for each date and state for household a , and for date 0 and date 1 - state s_H for household b . Use the fact that intervention is marginal and that b households are still constrained in state s_L after the intervention, i.e. $\frac{dd_{b,1}(s_L)}{d\tau} = 0$. Finally, transfer the present value of the change in patient households' welfare to the impatient household. ■

Whether the decentralized equilibrium is *constrained efficient* depends on whether $\frac{dV_b}{d\tau}|_{\tau=0}$ is 0 or not. It is clear from the expression for $\frac{dV_b}{d\tau}|_{\tau=0}$ that the equilibrium will in general not be *constrained efficient*. The question then is about the direction of this inefficiency, i.e. whether household debt is too high or too low. To this end, I decompose the welfare effect of debt taxation into three components. First, debt taxation results in a deadweight loss the ratio of date 1 to date 0 marginal utilities is no longer equalized across households. At $\tau = 0$, this distortion is of second order and thus does not appear in the expression for $\frac{dV_b}{d\tau}|_{\tau=0}$. Second, debt choices affect equilibrium interest rates at date 1 (first term), and due to asset market incompleteness this effect of debt taxation is of first order. Finally, debt choice at date 0 affects the choice of rollover at date 1 and thus aggregate demand at date 1. This effect is present solely due to nominal rigidities and is also of first order due to asset market incompleteness.

I consider each of the terms in the expression for $\frac{dV_b}{d\tau}|_{\tau=0}$ in turn. The term

$$-\beta_{b,0} (c_{b,0})^{-1} \left(E_0 \left\{ (\Gamma_{b,2} - \Gamma_{a,2}) \left(\frac{dr_1}{d\tau} \right) (d_1) \right\} \right)$$

resulting from date 1 interest rate movements is present even if prices are fully flexible prices. The welfare effect of this term is in general ambiguous. However, when the nominal rate is at the zero lower bound, as is the case when credit crises lead to output fluctuations (see Prop 8), this term is positive. At the zero bound, we have that $\frac{dr_1}{d\tau}(s_L) = \frac{di_1}{d\tau}(s_L) = 0$. As debt taxation reduces debt roll-over and thus leads to smaller aggregate income fluctuations at date 1, any monetary rule $\{i(s), \bar{P}\} \in \text{Stab}(\pi)$ implies that $\frac{dr_1}{d\tau}(s_H) = \frac{di_1}{d\tau}(s_H) > 0$. Hence, we have that

$$-\beta_{b,0} (c_{b,0})^{-1} \left(E_0 \left\{ (\Gamma_{b,2} - \Gamma_{a,2}) (d_1) \left(\frac{dr_1}{d\tau} \right) \right\} \right) > 0$$

The term

$$\beta_{b,0} (c_{b,0})^{-1} \left(E_0 \left\{ (\nu_b \Gamma_{b,1} + (2 - \nu_b) \Gamma_{a,1}) \left(1 - \frac{w_1}{P_1} \right) \left(\frac{dY_1}{d\tau} \right) \right\} \right)$$

reflects the change in firm profitability that results from a change in aggregate demand at date 1, holding wages fixed. In particular, the first best level of production with employment subsidy $\omega = 1 - \mu^{-1}$ requires that $\frac{w_1}{P_1} = 1$. With nominal rigidities, however, real wages are depressed in recessions ($\frac{w_1}{P_1} < 1$) and, if subsidies are present, are excessive in booms ($\frac{w_1}{P_1} > 1$).

Since debt taxation reduces output fluctuations, when employment subsidy $\omega = 1 - \mu^{-1}$ is in place, then

$$\beta_{b,0} (c_{b,0})^{-1} \left(E_0 \left\{ (\nu_b \Gamma_{b,1} + (2 - \nu_b) \Gamma_{a,1}) \left(1 - \frac{w_1}{P_1} \right) \left(\frac{dY_1}{d\tau} \right) \right\} \right) > 0$$

Numerically, I find that this is the case even when employment subsidies are absent.

Debt taxation also provides a form of insurance to the borrowers through the labor market. Using household budget constraints and labor market clearing condition, I can re-write the term

$$\beta_{b,0} (c_{b,0})^{-1} \left(\frac{1}{2} \times E_0 \left\{ (\Gamma_{b,1} - \Gamma_{a,1}) (l_{b,1} - l_{a,1}) \frac{d \left(\frac{w_1}{P_1} \right)}{d\tau} \right\} \right)$$

as

$$Cov \left\{ (\Gamma_{b,1} - \Gamma_{a,1}), \Delta \times \chi \frac{d \left(\frac{w_1}{P_1} \right) / d\tau}{\left(\frac{w_1}{P_1} \right)} \right\}$$

where Δ denotes the wealth transfer from the impatient to the patient households at date 1. Since these wealth transfers are larger in credit crises, they covary negatively with aggregate income (and wages) at date 1, i.e. $\Delta(s_L) > \Delta(s_H)$. As debt taxation stabilizes wages, $\frac{d \left(\frac{w_1}{P_1} \right)}{d\tau}(s_L) > 0 > \frac{d \left(\frac{w_1}{P_1} \right)}{d\tau}(s_H)$, and the marginal value of a unit of consumption is greater to the borrowers in crises, i.e. $(\Gamma_{b,1} - \Gamma_{a,1})(s_L) > (\Gamma_{b,1} - \Gamma_{a,1})(s_H)$, we have that

$$\begin{aligned} \beta_{b,0} (c_{b,0})^{-1} \left(\frac{1}{2} \times E_0 \left\{ (\Gamma_{b,1} - \Gamma_{a,1}) (l_{b,1} - l_{a,1}) \frac{d \left(\frac{w_1}{P_1} \right)}{d\tau} \right\} \right) &= \\ &= Cov \left\{ (\Gamma_{b,1} - \Gamma_{a,1}), \Delta \times \chi \frac{d \left(\frac{w_1}{P_1} \right) / d\tau}{\left(\frac{w_1}{P_1} \right)} \right\} > 0 \end{aligned}$$

The final term in the expression for $\frac{dV_b}{d\tau}|_{\tau=0}$ is

$$\beta_{b,0} (c_{b,0})^{-1} \left(\frac{1}{2} \times E_0 \left\{ (\Gamma_{b,1} - \Gamma_{a,1}) (1 - \nu_b) (Y_1) \frac{d \left(\frac{w_1}{P_1} \right)}{d\tau} \right\} \right)$$

This term reflects the effect of wage changes on the income of firm owners. This term is positive if $\nu_b < 1$ and negative otherwise.

Thus, in the special case where $\nu_b = 1$, optimal fiscal interventions necessarily feature positive debt taxes, and I conclude that household debt at date 0 is excessive. By continuity, one can also argue that there is an upper bound $\bar{\nu}$ on ν_b such that the debt taxes τ are positive for all $\nu_b < \bar{\nu}$. As I show in the numerical section, even if impatient households own all the shares of the monopolistic firms, optimal fiscal interventions still feature positive debt taxes.

Endogenous Firm Ownership

I now show that when firm ownership is endogenous, the expression for the welfare impact of the date 0 debt tax is similar to a special case of the no-equity trade economy. To this end, I first characterize the equilibrium firm ownership in this economy in Lemma 3.4:

Lemma 3 *The impatient households' portfolios are either $\{\nu_{b,1} = 2, \nu_{b,2} = 0\}$ or $\{\nu_{b,1} = 0, \nu_{b,2} = 0\}$*

Proof. (Sketch) In a credit crisis, the impatient households will sell all firm equity, i.e. they set $\nu_{b,2}(s_L) = 0$. In state s_H on the other hand, portfolios are indeterminate (date 2 profits are constant) and setting $\nu_{b,2}(s_H) = 0$ is without loss of generality. If firm ownership provides the impatient households with some insurance against credit crises, the impatient households will purchase all firm equity at date 0, i.e. $\nu_{b,1} = 2$. Otherwise, the impatient households will sell all firm equity at date 0, i.e. $\nu_{b,1} = 0$. Since by definition of credit crises, households' discount factors are not equalized at date 1, $\nu_{b,1}$ must either be 0 or 2. The portfolio with $\nu_{b,1} = 1$ can only arise in equilibrium only in a knife-edge scenario where firm profits at date 1 are constant. In that case, however, portfolios are again indeterminate and fixing $\nu_{b,1} = 0$ or 2 is without loss. ■

Using this result, I now derive the effect on households' welfare of a marginal debt tax imposed on borrowers at date 0. The transfers $\{T_{j,0}\}$ are again chosen so as to leave the savers' indifferent.

Proposition 18 (*Externalities II*) In the sticky price economy with a monetary rule $\{i(s), \bar{P}\} \in \text{Stab}(\pi)$, the change in the impatient households' welfare from a marginal increase in debt tax evaluated at $\tau = 0$ that leaves the patient households' welfare unaffected is given by

$$\begin{aligned} \frac{dV_b}{d\tau}|_{\tau=0} = & -\beta_{b,0} (c_{b,0})^{-1} \left(E_0 \left\{ (\Gamma_{b,2} - \Gamma_{a,2}) \left(\frac{dr_1}{d\tau} \right) (d_1) \right\} \right) - \\ & - \left(\nu_{b,1} \times E_0 \left\{ (\Gamma_{b,1} - \Gamma_{a,1}) \left(\frac{dr_1}{d\tau} \right) \left(\frac{dQ_1}{dr_1} \right) \right\} \right) + \\ & \beta_{b,0} (c_{b,0})^{-1} \times 2 \times E_0 \left\{ (\nu_{b,1} \Gamma_{b,1} + (2 - \nu_{b,1}) \Gamma_{a,1}) \left(\frac{dY_1}{d\tau} \right) \left(1 - \frac{w_1}{P_1} \right) \right\} + \\ & + \beta_{b,0} (c_{b,0})^{-1} \times \frac{1}{2} \times E_0 \left\{ (\Gamma_{b,1} - \Gamma_{a,1}) (l_{b,1} - l_{a,1}) \frac{d\left(\frac{w_1}{P_1}\right)}{d\tau} \right\} + \\ & + \beta_{b,0} (c_{b,0})^{-1} \left(\frac{1}{2} \times E_0 \left\{ (\Gamma_{b,1} - \Gamma_{a,1}) (1 - \nu_{b,1}) Y_1 \frac{d\left(\frac{w_1}{P_1}\right)}{d\tau} \right\} \right) \end{aligned}$$

Table 3.1: *Baseline Calibration*

$\frac{\beta_{a,1}}{\beta_{a,0}}$	$\frac{\beta_{b,1}}{\beta_{b,0}}$	$\frac{\beta_{a,1}}{\beta_{a,0}}$	$\frac{\beta_{a,1}}{\beta_{a,0}}$	χ	μ	α	$\nu_{b,0}$	π	ϕ	p
1.40	0.60	0.97	0.97	1	1.1	0.8	0	0.02	0.03	0.2

where $\Gamma_{j,t}(s) = \frac{\beta_{j,t}}{\beta_{j,0}} \left(\frac{c_{j,t}(s)}{c_{j,0}} \right)^{-1}$ is the stochastic discount factor with which household j discounts consumption from date t and state s to date 0. In addition, $\Gamma_{b,1}(s_L) > \Gamma_{a,1}(s_L)$, $\Gamma_{b,1}(s_H) < \Gamma_{a,1}(s_H)$, and $\Gamma_{b,2}(s_H) < \Gamma_{a,2}(s_H)$.

Proof. Analogous to the Proof of Proposition 17. ■

I again conclude that the decentralized equilibrium is in general not *constrained efficient*. The expression for $\frac{dV_b}{d\tau}|_{\tau=0}$ in Proposition 18 is similar to the one in Proposition 17 if we set $\nu_b = \nu_{b,1}$. The only additional term is

$$\beta_{b,0} (c_{b,0})^{-1} \left(\nu_{b,1} \times E_0 \left\{ (\Gamma_{b,1} - \Gamma_{a,1}) \left(\frac{dr_1}{d\tau} \right) \left(\frac{dQ_1}{dr_1} \right) \right\} \right)$$

This term arises with endogenous firm ownership because the impatient households sell all their holdings of firm equity at date 1. As debt taxation affects date 1 equilibrium interest rates, it also affects the equity prices that are the present value of (constant) date 2 profits. This term is non-negative for the reason that $\frac{dr_1}{d\tau}(s_L) = 0 < \frac{dr_1}{d\tau}(s_H)$ and $\frac{dQ_1}{dr_1} = \left(\frac{d}{dr_1} \right) \left(\frac{\Pi_2}{1+r_1} \right)$. Hence, if the optimal debt tax τ_0 were positive with no equity trade and $\nu_b = \nu_{b,1}$, it would also be positive with endogenous equity trade. In what follows, I conduct the welfare analysis numerically. I show that household debt is indeed *excessive* when credit crises are large enough to lead to output fluctuations. I compute and characterize these inefficiencies for various parameterizations of the model.

Numerical Analysis

In this section, I compute the magnitude of the inefficiencies identified above. To preview the results, I find that these equilibrium inefficiencies robustly increase with the severity and likelihood of credit crises. The welfare costs of these inefficiencies are roughly proportional to the total costs of output fluctuations. In Table 3.1, I show the baseline calibration for the model.

I choose the distance between date 1 discount factors to generate substantial rollover at date 1 (Figures 3.1, and 3.2). The date 2 discount factors are chosen so as to set the date 1 equilibrium interest rate in the absence of crises to 3%. With an inflation target of 2%, this gives the Central Bank some room for its stabilization policy. I calibrate the markups μ , “disutility” from labor χ , and “returns to scale” α to be 1.1, 1, and 0.8 respectively. The qualitative results do not change as I vary these parameters within reasonable range. The

debt limit and the likelihood of crises are calibrated to 0.03 and 0.2 respectively. In Figures 3.4 through 3.7, I show the welfare results by varying the debt limit and the inflation targets.

Figure 3.4 shows the magnitude of output fluctuations as I vary the debt limit from 0 to 0.1 (flex price output is 0.5). The more severe the crises are, the larger the output fluctuations associated with them. In Figure 3.5, I display the characteristics of the optimal interventions for each value of the debt limit. The marginal debt tax is positive as long as output fluctuates, and the tax is larger the tighter the debt limit. Larger optimal debt taxes coincide with larger reductions in date 1 rollover, smaller fluctuations in aggregate output, and larger welfare gains for the households.

In line with the findings on the severity of crises, the higher the monetary authorities set their inflation targets, the smaller the size of inefficiencies (Figures 3.6 and 3.7). I have not modelled the costs of inflation explicitly here. For a given cost of inflation, however, the desirability of adjusting inflation targets upwards is larger the more severe or likely credit crises are.

To summarize, the inefficiencies due to the aggregate demand externalities identified in this paper are increasing in the likelihood and severity of credit crises. Monetary authorities may be able to remedy these inefficiencies by adjusting their inflation targets upwards, but this will generally depend on the social costs of inflation.

3.5 Conclusions

In this paper I investigated the optimality of decentralized equilibria in a stylized New Keynesian economy with incomplete markets. I asked whether households' borrowing decisions are socially optimal when credit crises are expected to occur with positive probability. I found that when credit crises are expected to be large, household indebtedness is excessive. This result is due to the aggregate demand externalities that arise when highly indebted households cut their consumption in order to delever during credit crises. The main ingredients that contribute to this finding are the incompleteness of asset markets and the presence of nominal rigidities.

In the model, regulations that constrain household debt are desirable. This conclusion however will in general depend on whether there are pre-existing distortions in asset markets or if there are distortionary costs to such regulations. If, for example, the fixed costs associated with regulation are large, then the desirability of interventions will depend on the severity and the likelihood of credit crises. Similarly, if there are pre-existing distortions, the initial losses associated with regulations are no longer second order. Finally, as the model is highly stylized, the results I obtain are mainly qualitative. A more elaborate model would need to be constructed to evaluate these conclusions quantitatively. It would be interesting to also investigate the interactions between the aggregate demand externalities identified in this paper with other pecuniary externalities identified in the recent literature on fire-sales in economies with collateral constraints.

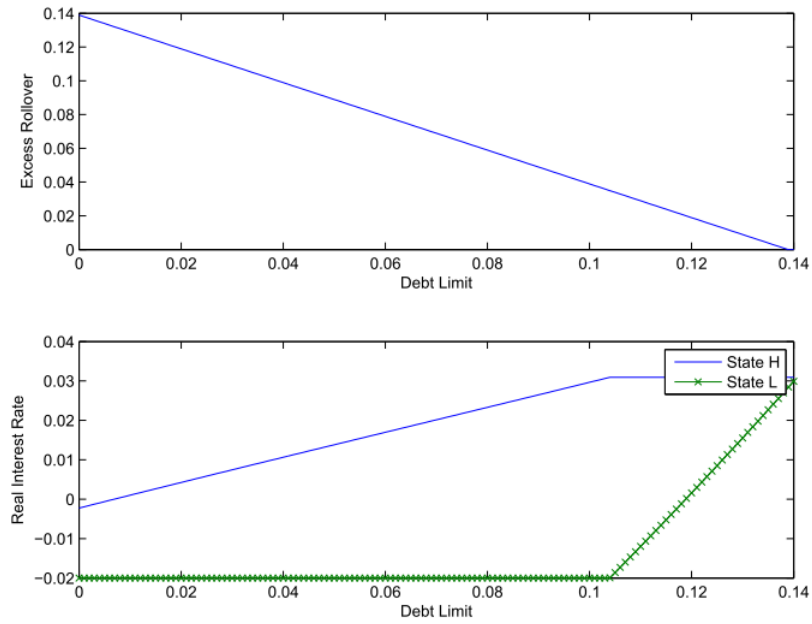


Figure 3.1: Crises and Interest Rates

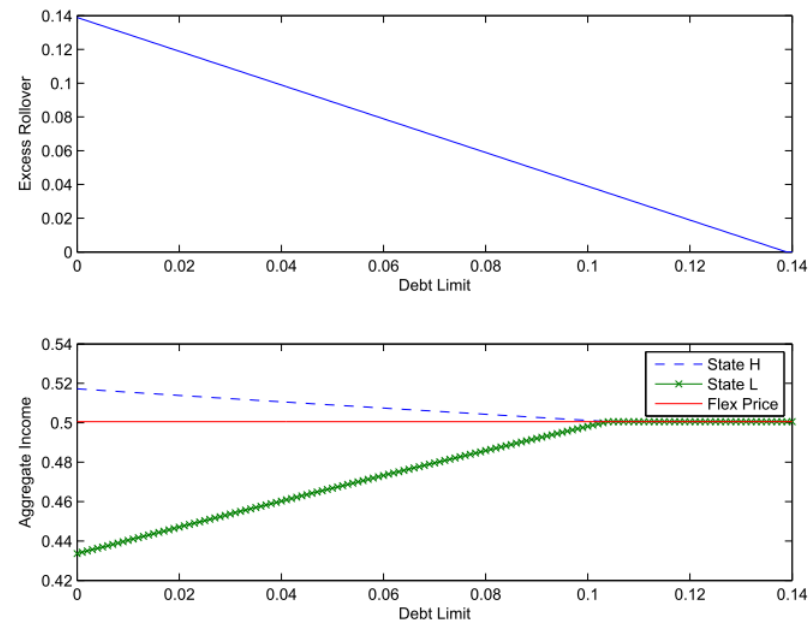


Figure 3.2: Crises and Output

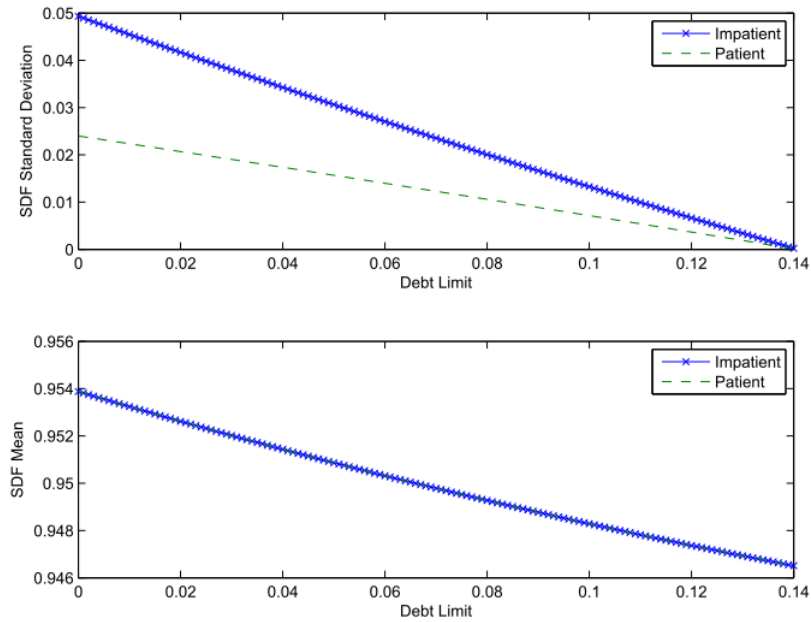


Figure 3.3: Discount Factors

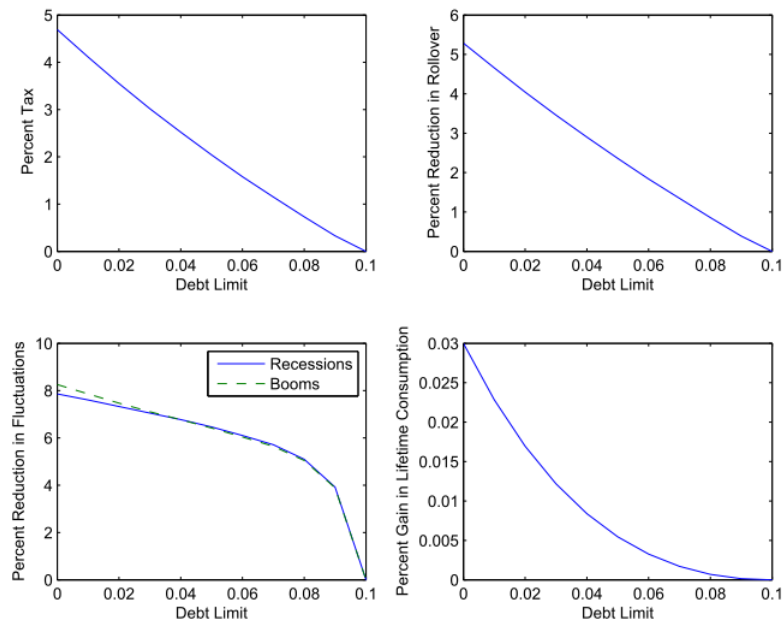


Figure 3.4: Size of Fluctuations (a)

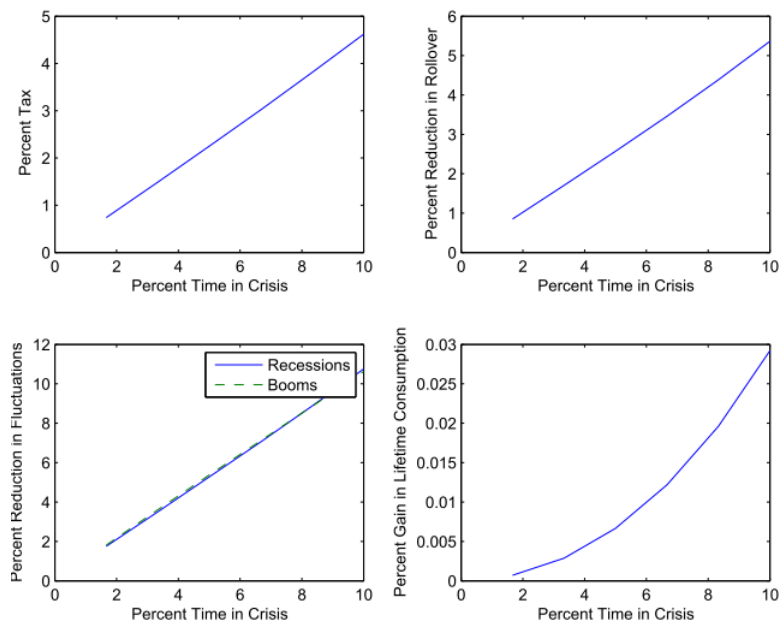


Figure 3.5: Size of Fluctuations (b)

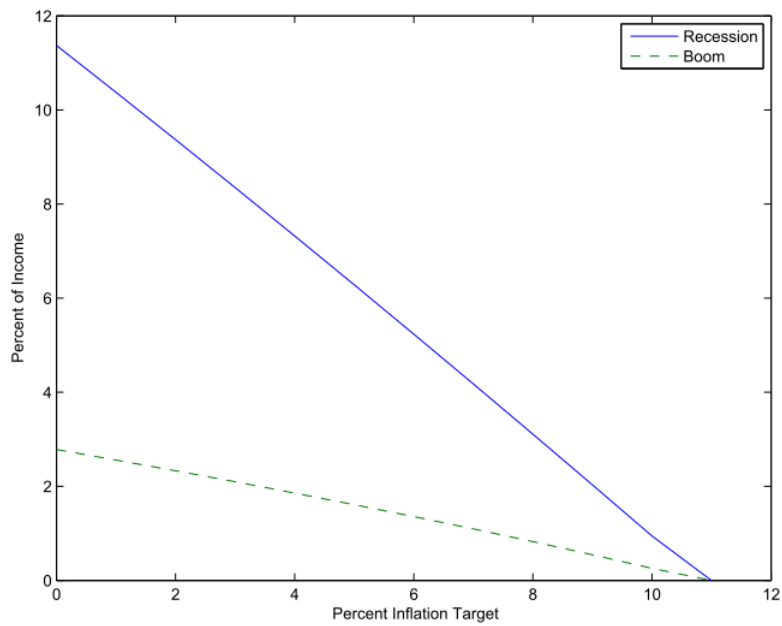


Figure 3.6: Inflation Target (a)

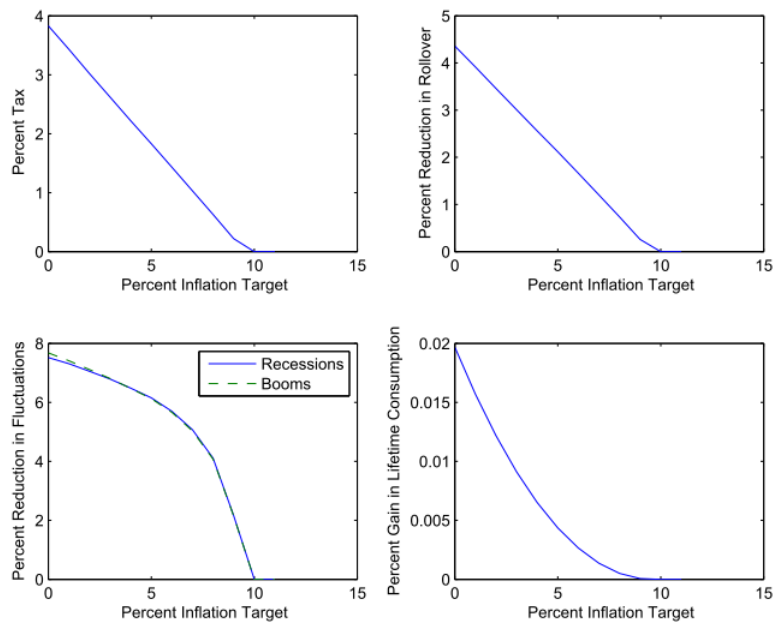


Figure 3.7: Inflation Target (b)

Bibliography

- [1] Tobias Adrian and Hyun Song Shin. “Liquidity and leverage”. In: *Journal of financial intermediation* 19.3 (2010), pp. 418–437.
- [2] Fernando Alvarez and Urban J Jermann. “Efficiency, Equilibrium, and Asset Pricing with Risk of Default”. In: *Econometrica* 68.4 (2000), pp. 775–797.
- [3] Andrew Ang, Assaf A Shtauber, and Paul C Tetlock. “Asset Pricing in the Dark: The Cross Section of OTC Stocks”. In: *Work. Pap.* (2013).
- [4] George-Marios Angeletos and Jennifer La’O. “Decentralization, Communications, and the Origins of Fluctuations”. In: (2011).
- [5] George-Marios Angeletos and Jennifer La’O. “Noisy Business Cycles”. In: *NBER Macroeconomics Annual 2009, Volume 24*. University of Chicago Press, 2010, pp. 319–378.
- [6] Adam B Ashcraft and Darrell Duffie. “Systemic Illiquidity in the Federal Funds Market”. In: *The American Economic Review* 97.2 (2007), pp. 221–225.
- [7] Ulf Axelson. “Security Design with Investor Private Information”. In: *The Journal of Finance* 62.6 (2007), pp. 2587–2632.
- [8] Ben Bernanke, Mark Gertler, and Simon Gilchrist. *The Financial Accelerator in a Quantitative Business Cycle Framework*. NBER Working Papers 6455. National Bureau of Economic Research, Inc, Mar. 1998. URL: <http://ideas.repec.org/p/nbr/nberwo/6455.html>.
- [9] Ben S. Bernanke and Mark Gertler. “Agency costs, Net worth, and Business Fluctuations”. In: *The American Economic Review* 79.1 (Mar. 1989), pp. 14–31. ISSN: 0036-8075. DOI: 10.1126/science.151.3712.867-a.
- [10] Hendrik Bessembinder and William Maxwell. “Markets: Transparency and the Corporate Bond Market”. In: *The Journal of Economic Perspectives* 22.2 (2008), pp. 217–234.
- [11] Bruno Biais and Richard C Green. “The Microstructure of the Bond Market in the 20th Century”. In: *Tepper School of Business* (2007), p. 134.
- [12] John Boyd and Edward C Prescott. “Financial Intermediary-Coalitions”. In: Research Department Staff Report (1985).

- [13] Markus K Brunnermeier and Yuliy Sannikov. “A Macroeconomic Model with a Financial Sector”. In: (2013).
- [14] Ricardo J Caballero and Arvind Krishnamurthy. “International and domestic collateral constraints in a model of emerging market crises”. In: *Journal of Monetary Economics* 48.3 (2001), pp. 513–548.
- [15] Ricardo J Caballero and Arvind Krishnamurthy. “Smoothing sudden stops”. In: *Journal of Economic Theory* 119.1 (2004), pp. 104–127.
- [16] Charles W Calomiris and Richard J Herring. “Why and How to Design a Contingent Convertible Debt Requirement”. In: *Available at SSRN 1815406* (2011).
- [17] Tim S Campbell and William A Kracaw. “Information production, market signalling, and the theory of financial intermediation”. In: *The Journal of Finance* 35.4 (1980), pp. 863–882.
- [18] Karl E Case, Robert J Shiller, and Allan N Weiss. “Index-Based Futures and Options Markets in Real Estate”. In: *The Journal of Portfolio Management* 19.2 (1993), pp. 83–92.
- [19] YiLi Chien and Hanno Lustig. “The Market Price of Aggregate Risk and the Wealth Distribution”. In: *Review of Financial Studies* 23.4 (2010), pp. 1596–1650.
- [20] Olivier Coibion and Yuriy Gorodnichenko. “Information Rigidity and the Expectations Formation Process : A Simple Framework and New Facts”. In: *Working Paper* (2012).
- [21] Jacques Cremer and Richard McLean. “Optimal Selling Strategies under Uncertainty for a Discriminating Monopolist when Demands are Interdependent”. In: *Econometrica* 53.2 (1985), pp. 345–361.
- [22] Tri Vi Dang, Gary Gorton, and Bengt Holmström. “Financial Crises and the Optimality of Debt for Liquidity Provision”. In: *Working Paper* (2010).
- [23] Peter DeMarzo and Darrell Duffie. “A liquidity-based model of security design”. In: *Econometrica* 67.1 (1999), pp. 65–99.
- [24] Prachi Deuskar et al. “The dynamics of hedge fund fees”. In: *Available at SSRN* (2011).
- [25] Sebastian Di Tella. “Uncertainty Shocks and Balance Sheet Recessions”. In: *Working Paper* (2012), pp. 1–59.
- [26] Douglas W. Diamond. “Financial Intermediation and Delegated Monitoring”. In: *Economic Analysis* 51.3 (1984), pp. 393–414.
- [27] Douglas W. Diamond and Philip H. Dybvig. “Bank Runs, Deposit Insurance, and Liquidity”. In: *Journal of Political Economy* 91.3 (Jan. 1983), p. 401.
- [28] Jonas Dovern, Ulrich Fritsche, and Jiri Slacalek. “Disagreement among forecasters in G7 countries”. In: *Review of Economics and Statistics* 94.4 (2012), pp. 1081–1096.
- [29] Darrell Duffie. *Dark markets: Asset pricing and information transmission in over-the-counter markets*. Princeton University Press, 2011.

- [30] Darrell Duffie, Semyon Malamud, and Gustavo Manso. “Information Percolation in Segmented Markets”. In: *Econometrica* 77.5 (2009), pp. 1513–1574.
- [31] Darrell Duffie and Gustavo Manso. “Information Percolation in Large Markets”. In: *The American Economic Review* 97.2 (2007), pp. 203–209.
- [32] Gauti B Eggertsson and Paul Krugman. “Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo Approach*”. In: *The Quarterly Journal of Economics* 127.3 (2012), pp. 1469–1513.
- [33] Gauti B Eggertsson and Michael Woodford. *Optimal monetary policy in a liquidity trap*. Tech. rep. National bureau of economic research, 2003.
- [34] Peter Englund, Min Hwang, and John M Quigley. “Hedging Housing Risk*”. In: *The Journal of Real Estate Finance and Economics* 24.1-2 (2002), pp. 167–200.
- [35] John Geanakoplos and Heraklis M Polemarchakis. “Existence, regularity and constrained suboptimality of competitive allocations when the asset market is incomplete”. In: *Uncertainty, information and communication: essays in honor of KJ Arrow* 3 (1986), pp. 65–96.
- [36] Nicola Gennaioli, Andrei Schleifer, and Robert W Vishny. “Money Doctors”. In: NBER Working Paper (2012).
- [37] Reuven Glick and Kevin J Lansing. “Consumers and the economy, part I: Household credit and personal saving”. In: *FRBSF Economic Letter* 1 (2011).
- [38] Reuven Glick and Kevin J Lansing. “Global household leverage, house prices, and consumption”. In: *FRBSF Economic Letter* 1 (2010).
- [39] Gary Gorton and Andrew Winton. “Financial intermediation”. In: *Handbook of the Economics of Finance* 1 (2003), pp. 431–552.
- [40] Bruce C Greenwald and Joseph E Stiglitz. “Externalities in economies with imperfect information and incomplete markets”. In: *The quarterly journal of economics* 101.2 (1986), pp. 229–264.
- [41] Veronica Guerrieri and Guido Lorenzoni. *Credit crises, precautionary savings, and the liquidity trap*. Tech. rep. National Bureau of Economic Research, 2011.
- [42] Samuel Hanson, Anil Kashyap, and Jeremy Stein. “A Macroprudential Approach to Financial Regulation”. In: *Chicago Booth Research Paper* (2010).
- [43] Zhiguo He and Arvind Krishnamurthy. “A Model of Capital and Crises”. In: *The Review of Economic Studies* 79.2 (2012), pp. 735–777.
- [44] Bengt Holmstrom and Jean Tirole. “Financial Intermediation, Loanable Funds, and The Real Sector”. In: *Quarterly Journal of Economics* 112.3 (Aug. 1997), pp. 663–691.
- [45] Olivier Jeanne and Anton Korinek. “Macroprudential Regulation Versus Mopping Up After the Crash”. In: *Working Paper* (2012).

- [46] Nobuhiro Kiyotaki and John Moore. “Credit Chains”. In: *The Journal of Political Economy* 105 (1997), pp. 211–248.
- [47] Anton Korinek. *Systemic risk-taking: amplification effects, externalities, and regulatory responses*. Tech. rep. European Central Bank, 2011.
- [48] Illan Kremer. “Information Aggregation in Common Value Auctions”. In: *Econometrica* 70.4 (2002), pp. 1675–1682.
- [49] Arvind Krishnamurthy. “Collateral Constraints and The Amplification Mechanism”. In: *Journal of Economic Theory* 111.2 (Aug. 2003), pp. 277–292. ISSN: 00220531. DOI: 10.1016/S0022-0531(03)00098-X. URL: <http://linkinghub.elsevier.com/retrieve/pii/S002205310300098X>.
- [50] Paul R Krugman, Kathryn M Dominquez, and Kenneth Rogoff. “It’s baaack: Japan’s slump and the return of the liquidity trap”. In: *Brookings Papers on Economic Activity* (1998), pp. 137–205.
- [51] Pablo Kurlat. “Lemons markets and the transmission of aggregate shocks”. In: *The American Economic Review* 103.4 (2013), pp. 1463–1489.
- [52] Hayne E Leland and David H Pyle. “Informational Asymmetries, Financial Structure, and Financial Intermediation”. In: *The Journal of Finance* 32.2 (1977).
- [53] Guido Lorenzoni. *A theory of demand shocks*. Tech. rep. National Bureau of Economic Research, 2006.
- [54] Guido Lorenzoni. “Inefficient Credit Booms”. In: *The Review of Economic Studies* 75.3 (July 2008), pp. 809–833. ISSN: 0034-6527. DOI: 10.1111/j.1467-937X.2008.00494.x. URL: <http://restud.oxfordjournals.org/lookup/doi/10.1111/j.1467-937X.2008.00494.x>.
- [55] N Gregory Mankiw, Ricardo Reis, and Justin Wolfers. *Disagreement About Inflation Expectations*. Vol. 18. July. 2004. ISBN: 026207253X.
- [56] Albert Marcet and Juan Pablo Nicolini. “Recurrent hyperinflations and learning”. In: *The American Economic Review* 93.5 (2003), pp. 1476–1498.
- [57] R Preston McAfee, John McMillan, and Philip J Reny. “Extracting the Surplus in the Common-Value Auction”. In: *Econometrica: Journal of the Econometric Society* (1989), pp. 1451–1459.
- [58] Robert L McDonald. “Contingent Capital with a Dual Price Trigger”. In: *Journal of Financial Stability* (2011).
- [59] Atif Mian, Kamalesh Rao, and Amir Sufi. “Household Balance Sheets , Consumption , and the Economic Slump”. In: *Working Paper* (2013).
- [60] Atif Mian and Amir Sufi. “Household Leverage and the Recession of 2007–09”. In: *IMF Economic Review* 58.1 (2010), pp. 74–117.

- [61] Atif Mian and Amir Sufi. “The consequences of mortgage credit expansion: Evidence from the US mortgage default crisis”. In: *The Quarterly Journal of Economics* 124.4 (2009), pp. 1449–1496.
- [62] Atif Mian and Amir Sufi. “The Great Recession: Lessons from Microeconomic Data”. In: *The American Economic Review: Papers and Proceedings* 100.2 (May 2010), pp. 1–10. ISSN: 0002-8282. DOI: 10.1257/aer.100.2.51. URL: <http://pubs.aeaweb.org/doi/abs/10.1257/aer.100.2.51>.
- [63] Paul Milgrom and Nancy Stokey. “Information, trade and common knowledge”. In: *Journal of Economic Theory* 26.1 (1982), pp. 17–27.
- [64] Paul R Milgrom and Robert J Weber. “A Theory of Auctions and Competitive Bidding”. In: *Econometrica* 50.5 (1982), pp. 1089–1122.
- [65] Frederic S Mishkin. “The household balance sheet and the Great Depression”. In: *The Journal of Economic History* 38.04 (1978), pp. 918–937.
- [66] Sendhil Mullainathan, Markus Noth, and Antoinette Schoar. “The Market for Financial Advice : An Audit Study”. In: (2010).
- [67] Stewart C Myers and Nicholas S Majluf. “Corporate Financing and Investment Decisions When Firms Have Information That Investors Do Not Have”. In: *Journal of financial economics* 13.2 (1984), pp. 187–221.
- [68] Martha Olney. “Avoiding Default: the Role of Credit in the Consumption Collapse of 1930”. In: *The Quarterly Journal of Economics* February (1999).
- [69] Guillermo Luis Ordoñez. *Larger crises, slower recoveries: the asymmetric effects of financial frictions*. Federal Reserve Bank of Minneapolis, Research Department, 2009.
- [70] Adriano A Rampini and Viswanathan Vish. “Household Risk Management ”. In: (2009).
- [71] Carmen M Reinhart and Vincent R Reinhart. “After the fall”. In: (2010).
- [72] Ricardo Serrano-Padial. “On the Possibility of Trade with Pure Common Values under Risk Neutrality”. In: *Working Paper* (2007).
- [73] Robert J Shiller. “Macro Markets: Creating Institutions for Managing Society’s Largest Economic Risks”. In: *OUP Catalogue* (1998).
- [74] Andrei Shleifer and Robert W Vishny. “Liquidation Values and Debt Capacity: A Market Equilibrium Approach”. In: *The Journal of Finance* 47.4 (1992), pp. 1343–1366.
- [75] Laura L Veldkamp. “Slow boom, sudden crash”. In: *Journal of Economic Theory* 124.2 (2005), pp. 230–257.
- [76] Ivan Werning. *Managing a Liquidity Trap: Monetary and Fiscal Policy*. Tech. rep. National Bureau of Economic Research, 2011.

- [77] Stephen Williamson. “Costly Monitoring , Loan Contracts , and Equilibrium Credit Rationing”. In: *The Quarterly Journal of Economics* 102.1 (1987), pp. 135–146.
- [78] Robert Wilson. “A Bidding Model of Perfect Competition”. In: *The Review of Economic Studies* 44.3 (1977), pp. 511–518.
- [79] Haoxiang Zhu. “Finding a Good Price in Opaque Over-the-Counter Markets”. In: *The Review of Financial Studies* 25.4 (Dec. 2011), pp. 1255–1285. ISSN: 0893-9454. DOI: 10.1093/rfs/hhr140. URL: <http://rfs.oxfordjournals.org/cgi/doi/10.1093/rfs/hhr140>.

Appendix A

Appendix for Chapter 1

Proof of Lemma 0. Traditional sector firms set $k^d(s) = 0$ if $q(s) > A$ and $k^d(s) = g'^{-1}(q(s))$ if $q(s) \leq A$. Since entrepreneurs will liquidate all of their capital if $q(s) > A$ and entrepreneurs choose $k > 0$, $q(s) > A$ cannot be an equilibrium; hence, $q(s) \leq A$. Capital goods market clearing then implies that $q(s) = g'(z(s)k)$ for some $z(s) \in [0, 1]$ and $k > 0$. Finally, since $(g'(x) - \theta A)x$ is increasing in x , we have $(p(s) - \theta A)z(s)k = (g'(z(s)k) - \theta A)z(s)k > (g'(0) - \theta A)0 = 0$ if $z(s)k > 0$, and as $g'(0) = A$, we have that $\theta A < q(s) \leq A$.

Proof of Proposition 1. Entrepreneurial budget constraint and the consumption non-negativity constraints in period 2 are

$$\begin{aligned} c(s) &= (a(s) - d(s)k + z(s)q(s) + (1 - z(s))A)k \\ d(s) &\leq a(s) + \widehat{d}(s) + z(s)q(s) \end{aligned}$$

Since by Lemma 0, in equilibrium $q(s) \in (\theta A, A]$ with $q(s) < A$ if and only if $z(s) > 0$, in equilibrium entrepreneurs set $z(s) > 0$ if and only if the consumption non-negativity constraint binds with $z(s) = 0$. Thus, $z(s) > 0$ if and only if $d(s) > a(s) + \widehat{d}(s)$, for $s \in \{l, h\}$. From the consumption non-negativity constraint, entrepreneurial liquidations in state s are decreasing in $\widehat{d}(s)$. Suppose that entrepreneurs set $\widehat{d}(h) = \theta A$, then we have

$$\begin{aligned} a(h) - d(h) + \widehat{d}(h) &= a(h) - d(h) + \theta A \\ &\geq a(h) - (\theta a(h) + p(h)) + \theta A \\ &\geq a(h) - (\theta a(h) + A) + \theta A \\ &= (1 - \theta)(a(h) - A) \\ &\geq 0 \end{aligned}$$

where the first inequality follows from the financial constraint, the second inequality follows from Lemma 1, and the last inequality holds by Assumption 1.3. Thus, Lemma 2 implies that $z(h) = 0$. On the other hand, Lemma 4 also implies that entrepreneurs will liquidate capital in state l if $d(l) > a(l) + \theta A$. If the latter inequality holds, then entrepreneurs will

liquidate the minimum possible units, i.e. set $z(l) = \max\{0, \frac{d(l)-a(l)-\theta A}{q(l)-\theta A}\}$, in which case the equilibrium price of capital in state l is $q(l) = g'(z(l)k)$ by Lemma 1. Note that $z(l) < 1$ because $d(l) - a(l) - \theta A \leq \theta a(l) + p(l) - a(l) - \theta A < p(l) - \theta A$.

Proof of Lemma 1. The result follows directly from the fact that buyers agree about the contractual payoffs and because $n \geq 2$ buyers compete a la Bertrand.

Proof of Lemma 2. Suppose that buyers have matched with an investor with $\beta = 0$ and believe that the investor has received signal $x^S \in A$ where A is some measurable subset of $[\underline{x}, \bar{x}]$. Suppose also that the contract is positively correlated with the state of the economy. The payoff to buyer 1 who receives signal x but bids as if he has received signal z is given by

$$\Pi(z, x) = \int_{\underline{x}}^x (v^+(x, y, A) - p(z)) f_{y_1^+}(y|x, A) dy$$

where for $x, y \in [\underline{x}, \bar{x}]$, $v^+(x, y, A) \equiv \mathbb{E}\{d(s)k|x_1^B = x, y_1^+ = y, x^S \in A\}$ and $F_{y_1^+}(\cdot|x, A)$ is the conditional distribution of the second highest signal among signals $\{x_2^B, \dots, x_n^B\}$, conditional on $x_1^B = x$ and $x^S \in A$ (I omit indexation by contract and liquidity needs for brevity). In particular, we have that

$$F_{y_1^+}(y|x, A) = \frac{\sum_s F_s^{n-1}(y) f_s(x) \pi_A(s)}{\sum_s f_s(x) \pi_A(s)}$$

$$f_{y_1^+}(y|x, A) = \frac{\sum_s (n-1) f_s^{n-2}(y) f_s(x) \pi_A(s)}{\sum_s f_s(x) \pi_A(s)}$$

where $\pi_A(s) \equiv \Pr(s|x^S \in A)$, i.e. all buyers bid as in a standard first-price common value auction only after adjusting their priors from $\pi(l)$ to $\pi_A(l)$. Differentiation with respect to z and evaluation at $x = z$ implies that the equilibrium strategy satisfies the ODE

$$p'(x) = (v^+(x, x, A) - p(x)) \frac{f_{y_1^+}(x|x, A)}{F_{y_1^+}(x|x, A)}$$

With the boundary condition $p(\underline{x}) = v(\underline{x}, \underline{x}, A)$, the solution to the above ODE is given by

$$p(x) = \int_{\underline{x}}^x v^+(z, z, A) dG^+(z|x, A) \tag{A.1}$$

for $x \in [\underline{x}, \bar{x}]$, and where for $z \leq x$, $G^+(z|x, A) = \exp\left(-\int_z^x \frac{f_{y_1^+}(t|x, A)}{F_{y_1^+}(t|x, A)} dt\right)$. The boundary condition holds since a buyer with signal \underline{x} earns a negative payoff if $p(\underline{x}) > v^+(\underline{x}, \underline{x}, A)$, and if $p(\underline{x}) < v^+(\underline{x}, \underline{x}, A)$ then he can deviate and earn a positive payoff. The function $v^+(x, x, A)$ is increasing in x and $G^+(z|x') \geq G^+(z|x)$ for $z \leq x$ and $x' > x$; thus the strategy $p(\cdot)$ is

indeed increasing and differentiable. To show that p is a maximum, note that MLRP implies that

$$\frac{d\Pi}{dz} = F_{y_1^+}(z|x) \left[(v^+(x, z, A) - p(z)) \frac{f_{y_1^+}(z|x, A)}{F_{y_1^+}(z|x, A)} - p'(z) \right]$$

is positive for $z < x$, negative for $z > x$, and it is zero at $z = x$ by construction. This establishes that the strategy p is optimal for buyers. The derivation of buyers' offers for the case of a negatively correlated contract is analogous. The equilibrium offer strategy in that case is decreasing and is given by

$$p(x) = \int_x^{\bar{x}} v^-(z, z, A) dG^-(z|x, A) \quad (\text{A.2})$$

for $x \in [\underline{x}, \bar{x}]$, where $v^-(y, x, A) = \mathbb{E}\{d(s)k|x_1^B = x, y_1^- = y, x^S \in A\}$, and where for $z \geq x$, $G^-(z|x, A) = \exp\left(-\int_x^z \frac{f_{y_1^-}(t|t, A)}{1-F_{y_1^-}(t|t, A)} dt\right)$. To compute the off-equilibrium strategies, set $A = \{\bar{x}\}$ for negatively correlated contracts, and $A = \{\underline{x}\}$ otherwise.

That in equilibrium investors with $\beta = 1$ sell their contracts is clear: contractual prices are always positive and these investors do not value consumption in period 2. Suppose now that investors with $\beta = 0$ and signals in some measurable set $A \subset [\underline{x}, \bar{x}]$ also sell their contracts in equilibrium, and consider buyers who have matched with a seller with $\beta = 0$ and contract \mathcal{C} that is contingent on the state. Note that seller with $\beta = 0$ is indifferent between selling a non-contingent contract and keeping it, and thus he will not sell it by the indifference-breaking assumption. Let $p(\cdot, 0, \mathcal{C})$ denote an equilibrium offer strategy followed by buyers, and let $p^{\max}(0, \mathcal{C})$ denote the maximal offer implied by this strategy (as defined in Section 3.2). Then since buyers observe the seller's liquidity need, then for non-negatively correlated contracts we have that

$$\begin{aligned} p(x) &= \int_x^{\bar{x}} v^+(z, z, A) dG^+(z|x, A) \\ &\leq \int_x^{\bar{x}} v^+(x, z, A) dG^+(z|x, A) \\ &\leq \int_x^{\bar{x}} v^+(x, z, A) dF^+(z|x, A) \\ &= \mathbb{E}\{d(s)k|x_1^B = x, y_1^+ < x, x^S \in A\} \end{aligned}$$

The same reasoning shows that $p(x) \leq \mathbb{E}\{d(s)k|x_1^B = x, y_1^- > x, x^S \in A\}$ for negatively correlated contract. Using symmetry and integrating over buyers' signals, we have that

$$\mathbb{E}\{p^{\max}(0, \mathcal{C})|x^S \in A\} \leq \mathbb{E}\{d(s)k|x^S \in A\}$$

But the payoff to a seller with $\beta = 0$ and with signal $x \in A$ is $U^S(x, 1, \mathcal{C}) = \mathbb{E}\{d(s)k|x^S = x\} - \mathbb{E}\{p^{max}(0, \mathcal{C})|x^S = x\}$ and must be positive for all $x \in A$ by the indifference-breaking assumption; integration over $x^S \in A$ then implies that

$$\mathbb{E}\{p^{max}(0, \mathcal{C})|x^S \in A\} > \mathbb{E}\{d(s)k|x^S \in A\}$$

Hence, we have a contradiction, i.e. there is no equilibrium in which investors with $\beta = 0$ post their contracts for sale.

Proof of Proposition 2. Since investors sell their contracts if and only if they experience $\beta = 1$, the expected present value of contract $\mathcal{C} = \{d(s)k\}$ to an investor is given by

$$\begin{aligned} \mathcal{L}(\{d(s)k\}) &= \mathbb{E}\{d(s)k\} + \lambda \mathbb{E}\{p^{max}(1, \mathcal{C})\} \\ &= \mathbb{E}\{d(s)k\} + \lambda (\mathbb{E}\{p^{max}(1, \mathcal{C})\} - \mathbb{E}\{d(s)k\}) \end{aligned}$$

where $\mathbb{E}\{p^{max}(1, \mathcal{C})\}$ denotes the expected price at which an investor with $\beta = 1$ sells his contract. From Proposition 2, we have that

$$\mathbb{E}\{p^{max}(1, \mathcal{C})\} = \begin{cases} \mathbb{E}\left\{\int_{\underline{x}}^{x_1^B} v^+(z, z, A) dG^+(z|x_1^B)|\{x_1^B > y_1^+\}\right\} & \text{if } d(h) \geq d(l) \\ \mathbb{E}\left\{\int_{x_1^B}^{\bar{x}} v^-(z, z, A) dG^-(z|x_1^B)|\{x_1^B < y_1^-\}\right\} & \text{otherwise} \end{cases}$$

where $v^+(x, y) = \mathbb{E}\{d(s)k|x_1^B = x, y_1^+ = y\}$ and $v^-(x, y) = \mathbb{E}\{d(s)k|x_1^B = x, y_1^- = y\}$ because investors with $\beta = 1$ sell their contracts irrespective of signals received. Using the forms of the functions $v^+(\cdot, \cdot)$ and $v^-(\cdot, \cdot)$, we have that

$$\mathcal{L}(\{d(s)k\}) = \mathbb{E}\{d(s)k\} - \lambda \zeta |d(h) - d(l)|k$$

where $\zeta = \zeta^+ \cdot 1\{d(h) \geq d(l)\} + \zeta^- \cdot 1\{d(h) < d(l)\}$ with ζ^+ and ζ^- given by

$$\begin{aligned} \zeta^+ &= \mathbb{E} \left\{ \int_{\underline{x}}^{x_1^B} \Pr(s = l | x_1^B = z, y_1^+ = z) dG^+(z|x_1^B) | \{x_1^B > y_1^+\} \right\} - \pi \\ \zeta^- &= \pi - \mathbb{E} \left\{ \int_{x_1^B}^{\bar{x}} \Pr(s = l | x_1^B = z, y_1^- = z) dG^-(z|x_1^B) | \{x_1^B < y_1^-\} \right\} \end{aligned}$$

That $\zeta^+ > 0$ follows from the fact that $\Pr(s = l | x_1^B = z, y_1^+ = z) > \Pr(s = l | x_1^B = x, y_1^+ = z)$ for all $x > z$, and from the fact that $G^+(\cdot|x)$ is fbsd dominated by $F_{y_1^+}(\cdot|x)$ for all x ,

$$\begin{aligned} \pi &= \mathbb{E} \left\{ \int_{\underline{x}}^{x_1^B} \Pr(s = l | x_1^B > y_1^+ = z) dF_{y_1^+}(z|x_1^B) | \{x_1^B > y_1^+\} \right\} \\ &< \mathbb{E} \left\{ \int_{\underline{x}}^{x_1^B} \Pr(s = l | x_1^B = y_1^+ = z) dF_{y_1^+}(z|x_1^B) | \{x_1^B > y_1^+\} \right\} \\ &\leq \mathbb{E} \left\{ \int_{\underline{x}}^{x_1^B} \Pr(s = l | x_1^B = y_1^+ = z) dG_{y_1^+}(z|x_1^B) | \{x_1^B > y_1^+\} \right\} \\ &= \zeta^+ + \pi \end{aligned}$$

The proof that $\zeta^- > 0$ is analogous.

Proof of Proposition 3. That it suffices to consider contracts with $d(l) \leq d(h)$ follows because contingent contracts are costly to issue regardless of the contract's correlation with the state and because entrepreneurs value insurance against state l . I derive the results in Proposition 3 in several steps.

First I show that Assumption 3 implies that $a(l) + \theta A \leq d(l)$. Suppose to the contrary that at the optimum $d(l) < a(l) + \theta A$. Then entrepreneur's marginal utility of wealth in state l is also 1 and thus $d(h) = d(l) = d$ because $\zeta > 0$. Now, consider increasing d by a small amount ϵk . For ϵ small, the marginal cost of this increase is ϵk , while the marginal benefit is given by $\frac{\sum_s (a(s) + A) - d}{\chi'(k) - d} \cdot \epsilon k$ where k satisfies $\chi(k) = (d + \epsilon)k < (a(l) + \theta A)k$. Assumption 3 then implies that $\chi'(k) < \sum_s (a(s) + A)$ and that thus such an increase is optimal for the entrepreneur. Since this argument applies for any $d(l) < a(l) + \theta A$, the result follows. Using the previous result, the entrepreneurial objective can be re-written as

$$\max_{\{k, d(l), d(h)\}} \left[\pi(l)(a(l) + \theta A - d(l)) \frac{(1 - \theta)A}{q(l) - \theta A} + (1 - \pi(l))(a(h) + \theta A - d(h)) + (1 - \theta)A \right] k$$

subject to

$$\begin{aligned} \chi(k) &= \left(\sum_s \xi(s) d(s) \right) k \\ d(l) &\leq \theta a(l) + q(l) \\ d(h) &\leq \theta a(h) + A \\ a(l) + \theta A &\leq d(l) \\ d(l) &\leq d(h) \end{aligned}$$

where $\xi(l) \equiv \pi(l) + \lambda \zeta^+$ and $\xi(h) \equiv 1 - \xi(l)$. Let $\nu, \mu(l), \mu(h), \omega(l), \omega(h) \geq 0$ denote the multipliers on the above constraints in order as they appear. The entrepreneur's first order conditions with respect to $k, d(l), d(h)$ are then given by

$$\begin{aligned} \nu &= \frac{\pi(l)(a(l) + \theta A - d(l)) \frac{(1 - \theta)A}{q(l) - \theta A} + (1 - \pi(l))(a(h) + \theta A - d(h)) + (1 - \theta)A}{\chi'(k) - \sum_s \xi(s) d(s)} \\ \nu(1 - \xi(l)) &= 1 - \pi(l) + (\mu(h) - \omega(h))k^{-1} \\ \nu \xi(l) &= \pi(l) \frac{(1 - \theta)A}{q(l) - \theta A} + (\mu(l) - \omega(l) + \omega(h))k^{-1} \end{aligned}$$

The complementary slackness conditions are given by

$$\begin{aligned} \mu(h)(\theta a(h) + A - d(h)) &= 0 \\ \mu(h)(\theta a(l) + q(l) - d(l)) &= 0 \\ \omega(l)(d(l) - a(l) - \theta A) &= 0 \\ \omega(h)(d(h) - d(l)) &= 0 \end{aligned}$$

These conditions together with the period 0 budget constraint and the inequality constraints for the repayments fully characterize the solution to the entrepreneurial problem.

- Type I: Note that $q(l) > \underline{q} \implies \frac{\pi(l)(1-\theta)A}{\xi(l)q(l)-\theta A} < \frac{1-\pi(l)}{1-\xi(l)}$. Therefore, from the foc's we have that

$$\frac{(\mu(l) - \omega(l) + \omega(h))k^{-1}}{\xi(l)} > \frac{(\mu(h) - \omega(h))k^{-1}}{1 - \xi(l)}$$

If the contract satisfies $d(l) = d(h)$, then we are done (part (i)). If, on the other hand, $d(l) < d(h)$, then we have $\omega(h) = 0$, and thus $\mu(l) > 0$; otherwise, we have that $\mu(h) < 0$ since $\omega(l) \geq 0$, a contradiction. Thus, $d(l) < d(h) \implies d(l) = \theta a(l) + p(l)$.

- Type II: Note that $q(l) < \underline{q} \implies \frac{\pi(l)(1-\theta)A}{\xi(l)q(l)-\theta A} > \frac{1-\pi(l)}{1-\xi(l)}$. Therefore, from the foc's we have that

$$\frac{(\mu(l) - \omega(l) + \omega(h))k^{-1}}{\xi(l)} < \frac{(\mu(h) - \omega(h))k^{-1}}{1 - \xi(l)}$$

If the contract satisfies $a(l) + \theta A = d(l)$, then we are done (part (i)). On the other hand, $a(l) + \theta A < d(l) \implies \omega(l) = 0 \leq \mu(l)$ and thus $\mu(h) > 0$; otherwise, $\omega(h) < 0$ because $\mu(l) - \omega(l) + \omega(h) \geq 0$. Thus, $d(l) < d(h)$ implies $d(h) = \theta a(h) + A$.

- Type III: Note that $q(l) = \underline{q} \implies \frac{\pi(l)(1-\theta)A}{\xi(l)q(l)-\theta A} = \frac{1-\pi(l)}{1-\xi(l)}$ and thus the bank is indifferent about what contract to issue.

Proof of Proposition 4. Part 1) To show that $d(l) > a(l) + \theta A$ and $q(l) < A$, suppose to the contrary that at the optimum $d(l) = a(l) + \theta A$ and thus $q(l) = A$; note that foc's w.r.t. $d(h)$ and $d(l)$ then imply that $d(h) = d(l) = d = a(l) + \theta A$. Then we must have that

$$\begin{aligned} \nu &= \frac{\pi(l)(a(l) + A - d) + (1 - \pi(l))(a(h) + A - d)}{\chi'(\underline{k}) - d} \\ &= \frac{\mathbb{E}\{a(s) + A\} - d}{\chi'(\underline{k}) - d} > 1 \end{aligned}$$

where the last inequality follows from Assumption 3. Adding the foc's with respect to $d(l)$ and $d(h)$, we have

$$\nu = \pi(l) \frac{(1 - \theta)A}{q(l) - \theta A} + 1 - \pi(l) - \omega(l)k^{-1} < 1$$

since $\mu(l) = 0 < \omega(l)$ and $q(l) = A$, a contradiction. Thus, since the unique $q(l)$ that solves $q(l) = g' \left(\frac{d(l) - a(l) - \theta A}{q(l) - \theta A} \right)$ is continuous in $d(l)$, and ν is continuous in $d(l)$ and $q(l)$, we must have that $d(l) > a(l) + \theta A$ and thus $q(l) < A$. The remaining characterization of the financial

contract follows from the financial constraints and Proposition 3. Part 2) That investment is below first-best follows from the result in Part 1) which implies that $\nu > 1$ and thus

$$\begin{aligned}
\left[\chi'(k) - \sum_s \xi(s)d(s) \right] \nu &= \pi(l)(a(l) + q(l) - d(l)) \frac{(1-\theta)A}{q(l) - \theta A} + (1 - \pi(l))(a(h) + A - d(h)) \\
&\implies \\
\chi'(k) - \sum_s \xi(s)d(s) &< \pi(l)(a(l) + q(l) - d(l)) \frac{(1-\theta)A}{q(l) - \theta A} + (1 - \pi(l))(a(h) + A - d(h)) \\
&\implies \\
\chi'(k) &< \pi(l)(a(l) + A) + (1 - \pi(l))(a(h) + A) + \sum_s (\xi(s) - \pi(s)) d(s) \\
&\leq \pi(l)(a(l) + A) + (1 - \pi(l))(a(h) + A) = \chi'(k^{fb})
\end{aligned}$$

and the convexity of schedule $\chi(\cdot)$. The expression for liquidations follows from the beginning of period 2 budget constraint, and liquidations are positive by Part 1). Part 3) That the price of capital in state l is below A follows from Part 2) and Proposition 1. The bounds on asset price fluctuations follow from combining Proposition 1 with the foc's w.r.t. $d(l)$ and $d(h)$ by setting $\mu(h) = 0$.

Lemma (non-binding constraint in state h). Let \bar{k} be implicitly defined by

$$\chi(\bar{k}) = \xi(l)(a(l) + \theta A)\bar{k} + (1 - \xi(l))(\theta a(h) + A)\bar{k}$$

Then a sufficient condition for the collateral constraint to be loose in state h is that $\chi'(\bar{k}) \geq \sum_s \xi(s)(a(s) + A)$. To show this, suppose to the contrary that in equilibrium $d(h) = \theta a(h) + A$ and recall that in equilibrium $d(l) > a(l) + \theta A$ and $q(l) < A$. Hence, the entrepreneur's foc's yield

$$\begin{aligned}
\nu(1 - \xi(l)) &\geq 1 - \pi(l) \\
\nu\xi(l) &\geq \pi(l)\nu(l) \\
\nu &= \frac{\pi(l)\nu(l)(a(l) + q(l) - d(l)) + (1 - \pi(l))(a(h) + A - d(h))}{\chi'(k) - \sum_s \xi(s)d(s)} \\
&< \frac{\xi(l)\nu(a(l) + A - d(l)) + (1 - \xi(l))\nu(a(h) + A - d(h))}{\chi'(k) - \sum_s \xi(s)d(s)}
\end{aligned}$$

Now, note that $d(l) > a(l) + \theta A$ implies that $k > \bar{k}$ and thus

$$\begin{aligned}
1 &< \frac{\xi(l)\nu(a(l) + A - d(l)) + (1 - \xi(l))\nu(a(h) + A - d(h))}{\chi'(k) - \sum_s \xi(s)d(s)} \\
&< \frac{\sum_s \xi(s)(a(l) + A) - \sum_s \xi(s)d(s)}{\chi'(k) - \sum_s \xi(s)d(s)} < 1
\end{aligned}$$

where the last inequality follows from $\chi'(k) > \chi'(\bar{k}) \geq \sum \xi(s)(a(s) + A)$, a contradiction. Note that this assumption holds if the cost schedule is not too convex. Alternatively, in place of a technological assumption, if entrepreneurs had an endowment w in period 0, then non-binding constraint in state h can be ensured by assuming that w is not too low.

Proof of Proposition 5. It suffices to show that $\lambda\zeta^+$ goes to 0. That $\lambda\zeta^+$ goes to 0 as λ goes to 0 is clear. The proof for when n goes ∞ is contained in Kremer (2002), Theorem 3. To show the result as ψ goes to 0, note that the resale value of financial contracts is always bounded below by the offer of the buyer who is most pessimistic about contract's payoffs, i.e. buyer with signal \underline{x} when $d(h) \geq d(l)$ and with signal \bar{x} when $d(h) < d(l)$. Thus, it suffices to show that as $\psi \downarrow 0$, we have

$$\begin{aligned} \Pr(s = l | x_1^B = \underline{x}, y_1^+ = \underline{x}) &\uparrow \Pr(s = l) \\ \Pr(s = l | x_1^B = \bar{x}, y_1^- = \bar{x}) &\downarrow \Pr(s = l) \end{aligned}$$

But note that

$$\begin{aligned} \Pr(s = l | x_1^B = \underline{x}, y_1^+ = \underline{x}) &= \frac{1}{\Pr(s = l) + (1 - \Pr(s = l)) \left(\frac{f_h(\underline{x})}{f_l(\underline{x})}\right)^n} \cdot \Pr(s = l) \\ \Pr(s = l | x_1^B = \bar{x}, y_1^- = \bar{x}) &= \frac{1}{\Pr(s = l) + (1 - \Pr(s = l)) \left(\frac{f_h(\bar{x})}{f_l(\bar{x})}\right)^n} \cdot \Pr(s = l) \end{aligned}$$

and since $\psi \downarrow 0$ implies that $\frac{f_h(\underline{x})}{f_l(\underline{x})} \rightarrow 1$ and $\frac{f_h(\bar{x})}{f_l(\bar{x})} \rightarrow 1$, the result follows.

Proof of Proposition 6. I prove this proposition in several steps.

Claim 1. $a(l) + \theta A < d^*(l) \leq d^*(h)$ Since increasing borrowing also reduces the transfer that the planner must make to the investors, it suffices to show that the planner will choose to liquidate capital even in the absence of transfers. Suppose that $d(l) = d(h) = d = a(l) + \theta A$ and consider increasing d by ϵ . The entrepreneurial consumption in period 2 is now given by

$$\left[\pi(l)(a(l) + \theta A - d - \epsilon) \frac{(1 - \theta)A}{p_\epsilon(l) - \theta A} + (1 - \pi(l))(a(h) + \theta A - d - \epsilon) + (1 - \theta)A \right] k_\epsilon$$

where k_ϵ denotes the new scale of investment which satisfies $\chi(k_\epsilon) = (d + \epsilon)k_\epsilon$. Let dk and $dq(l)$ denote the change in the investment and price of capital resulting from this increase in borrowing. Then, for ϵ near 0, the change in consumption is given by

$$(\mathbb{E}\{a(s) + A\} - d) \frac{dk_\epsilon}{d\epsilon} - k$$

where I use the fact that $q(l)$ is close to A and k_ϵ is close to k for ϵ small. Note that the change in price does not enter the above expression because its effect on consumption is

proportional to liquidations which are close to 0 for ϵ small. Similarly, using the budget constraint, we get that

$$\frac{dk}{d\epsilon} = \frac{k}{\chi'(k) - d}$$

Hence, the total change in consumption is given by

$$\left((\mathbb{E}\{a(s) + A\} - d) \frac{1}{\chi'(k) - d} - 1 \right) k$$

Assumption 3 then implies that at k satisfying $\chi(k) = (a(l) + \theta A)k$, the above expression is strictly positive. Hence, increasing $d(l), d(h)$ above $a(l) + \theta A$ is welfare improving. Since $\zeta^+ > 0$, a similar argument shows that increasing $d(l)$ above $a(l) + \theta A$ is welfare improving.

Planner's Problem. Using the previous result, the planner's problem is given by

$$\max_{\{k, d(l), d(h), \tau(h)\}} \left[\pi(l) (a(l) + \theta A - d(l)) \frac{(1 - \theta)A}{q(l) - \theta A} \right] + (1 - \pi(l)) [(a(h) + \theta A - d(h) - \tau(h)) + (1 - \theta)A] k$$

subject to

$$\begin{aligned} \chi(k) &= \left(\sum_s \xi(s) d(s) \right) k \\ d(l) &\leq \theta a(l) + q(l) \\ d(h) &\leq \theta a(h) + A - \tau(h) \\ d(l) &\leq d(h) \\ 0 &\leq \tau(h) \end{aligned}$$

$$\pi(l) (\Pi^{CE}(l) - g(z(l)k) + q(l)z(l)k) \leq (1 - \pi(l))\tau(h)k$$

where $q(l) = g'(z(l)k)$, $z(l) = \frac{d(l) - a(l) - \theta A}{q(l) - \theta A}$, and $\tau(h) \equiv \frac{T(h)}{k}$. Let $\hat{\nu}, \mu(l), \mu(h), \omega(l), \omega(h), \gamma(h), \lambda$ denote the multipliers on the constraints of the planner's problem in the order as they appear. The planner's first order condition w.r.t. k is given by

$$\begin{aligned} \hat{\nu} \left[\sum_s \xi(s) d(s) - \chi'(k) \right] + \left[\sum_s \pi(s) \nu(s) (a(s) + \theta A - d(s) - \tau(s)) + (1 - \theta)A \right] \\ + \lambda [(1 - \pi(l))\tau(h) - \pi(l)(z(l)k)p_k(l)] + \mu(l)p_k(l) + \pi(l)\nu(l)(z(l)k)p_k(l) = 0 \end{aligned}$$

where $\nu(h) = 1 < \frac{(1 - \theta)A}{q(l) - \theta A} = \nu(l)$, and where $p_k(l)$ denotes the derivative of the equilibrium price of capital w.r.t. k . The planner's first order conditions with respect to $d(l)$ and $d(h)$ yield

$$\begin{aligned} \hat{\nu} \xi(l)k - \pi(l)\nu(l)k - \omega(h) - \mu(l) + \pi(l)\nu(l)(z(l)k)p_d(l) + \mu(l)p_d(l) - \lambda\pi(l)(z(l)k)p_d(l) &= 0 \\ \hat{\nu}(1 - \xi(l))k - (1 - \pi(l))k + \omega(h) - \mu(h) &= 0 \end{aligned}$$

where $p_d(l)$ denotes the derivative of the equilibrium price of capital w.r.t. $d(l)$ and thus satisfies $kp_k(l) = (d(l) - a(l) - \theta A)p_d(l)$. Her first-order conditions w.r.t. $\tau(h)$ is

$$-(1 - \pi(l))k - \mu(h) + \gamma(h) + (1 - \pi(l))\lambda k = 0$$

Finally, we have the complementary slackness conditions

$$\begin{aligned} \mu(h)(\theta a(h) + A + \tau(h) - d(h)) &= 0 \\ \mu(h)(\theta a(l) + q(l) - d(l)) &= 0 \\ \omega(h)(d(h) - d(l)) &= 0 \\ \gamma(h)\tau(h) &= 0 \\ \lambda [(1 - \pi(l))\tau(h)k - \pi(l) (\Pi^{CE}(l) - g(z(l)k) + q(l)z(l)k)] &= 0 \end{aligned}$$

These conditions fully characterize the solution to the planner's problem. For $s \in \{l, h\}$, define $\widehat{v}(s)$ by

$$\begin{aligned} \widehat{v}(l) &= \nu(l) + (\lambda - \nu(l)) z(l)p_d(l) - \mu(l)p_d \\ \widehat{v}(h) &= \nu(h) = 1 \end{aligned}$$

and note that then the foc w.r.t. k becomes

$$\widehat{v} \left(\chi'(k) - \sum_s \xi(s)d(s) \right) = \left(\sum_s \pi(s)\widehat{v}(s)(a(s) + \theta A - d(s)) + (1 - \theta)A \right) + (1 - \pi(l))(\lambda - \nu(h))\tau(h)$$

The foc's w.r.t. $d(l)$ and $d(h)$ then become

$$\begin{aligned} \widehat{v}\xi(l) &= \pi(l)\widehat{v}(l) + (\mu(l) + \omega(h))k^{-1} \\ \widehat{v}(1 - \xi(l)) &= 1 - \pi(l) + (\mu(h) - \omega(h))k^{-1} \end{aligned}$$

Claim 2. The planner sets $k^* \leq k^{CE}$ and $d^*(l)k^* < d^{CE}(l)k^{CE}$. Note that the foc's w.r.t. $d(l)$ and $d(h)$ become

$$\pi(l)\widehat{v}(l) = \xi(l)\widehat{v} - (\omega(h) + \mu(l))k^{-1}$$

$$1 - \pi(l) = (1 - \xi(l))\widehat{v} + (\omega(h) - \mu(h))k^{-1}$$

and plugging these into the foc w.r.t. k , we get

$$\begin{aligned} \widehat{v}\chi'(k) &= \sum_s \xi(s)\widehat{v}(a(s) + \theta A) + (1 - \theta)A - (\omega(h) + \mu(l))k^{-1}(a(l) + \theta A - d(l)) + \\ &\quad (\omega(h) - \mu(h))k^{-1}(a(h) + \theta A - d(h)) + \mu(h)\tau(h)k^{-1} \end{aligned}$$

Conjecture that $\mu(l) = 0$ at the planner's allocation. Thus

$$\widehat{\nu}\chi'(k) = \sum_s \xi(s) \widehat{\nu}(a(s) + \theta A) + (1 - \theta) A + [1 - \pi(l) - (1 - \xi(l)) \widehat{\nu}] (1 \{\omega(h) > 0\} (a(h) - a(l)) + 1 \{\mu(h) > 0\} (1 - \theta) (a(h) - A))$$

which defines k as a decreasing function of $\widehat{\nu}$. Note that an identical expression holds in the competitive equilibrium, with ν^{CE} , k^{CE} , $\omega^{CE}(h)$, and $\mu^{CE}(h)$ in place of those corresponding to the planner's solution. An immediate conclusion is that if the entrepreneur issues a contingent contract, then the planner does as well. Suppose that $d^{CE}(l) < d^{CE}(h)$ and thus

$$\nu^{CE} = \pi(l) \nu^{CE}(l) + 1 - \pi(l)$$

and note that at the competitive allocation, we have $\widehat{\nu}^{CE} = \nu^{CE}$ and $\widehat{\nu}^{CE} > \nu^{CE}(l)$. If the planner issues a non-contingent contract, then it must be the case that

$$\widehat{\nu} = \pi(l) \widehat{\nu}(l) + 1 - \pi(l) \leq \frac{1 - \pi(l)}{1 - \xi(l)} = \pi(l) \nu^{CE}(l) + 1 - \pi(l) = \nu^{CE}$$

But then it follows that the planner borrows liquidates strictly less in state l . Hence, we have that $k^* \geq k^{CE}$ and $d^*(l) = d^*(h) < d^{CE}(l) < d^{CE}(h)$, a contradiction.

Thus we are left to consider the following cases:

- Case 1: Both contracts are non-contingent. This case follows immediately since at the competitive allocation, we have

$$\widehat{\nu}^{CE} = \nu^{CE} = \pi(l) \nu^{CE}(l) + 1 - \pi(l) < \pi(l) \widehat{\nu}^{CE}(l) + 1 - \pi(l)$$

and therefore $k^* < k^{CE}$ and $d^*(l) = d^*(h) < d^{CE}(l) = d^{CE}(h)$.

- Case 2: Both contracts are contingent. Suppose that $\widehat{\nu} = \frac{1 - \pi(l)}{1 - \xi(l)}$, then $k^* = k^{CE}$ and we must have that $\widehat{\nu}(l) = \nu^{CE}(l)$, which implies that $d^*(l) < d^{CE}(l)$. On the other hand, if $\widehat{\nu} > \frac{1 - \pi(l)}{1 - \xi(l)}$ then $k^* < k^{CE}$. Now suppose that $d^*(l) = d^{CE}(l)$ and $d^*(h) < d^{CE}(h)$, and note that since $\widehat{\nu} = \pi(l) \widehat{\nu}(l) + 1 - \pi(l)$, the planner must strictly prefer to decrease $d^*(l)$ and increase $d^*(h)$. Hence, $d^*(l) < d^{CE}(l)$.

- Case 3: If the contract is non-contingent in the competitive equilibrium, then we have that at the competitive allocation

$$\widehat{\nu}^{CE} = \nu^{CE} = \pi(l) \nu^{CE}(l) + 1 - \pi(l) < \pi(l) \widehat{\nu}^{CE}(l) + 1 - \pi(l)$$

Suppose that $k^* \geq k^{CE}$, then we have that $\widehat{\nu} \leq \widehat{\nu}^{CE}$ and the planner's contract is therefore also non-contingent. Thus, also $d^*(l) \geq d^{CE}(l)$ and $\widehat{\nu}^{CE}(l) \leq \widehat{\nu}(l)$, a contradiction. But if the planner issues a contingent contract, then it is also clear that $d^*(l) < d^{CE}(l)$, since otherwise $d^{CE}(l) = d^{CE}(h) \leq d^*(l) < d^*(h)$ implies that $k^* > k^{CE}$, a contradiction. Thus, we conclude that $k^* < k^{CE}$ and $d^*(l) < d^{CE}(l)$.

Note that the conjecture that $\mu(l) = 0$ at the planner's allocation is verified. Thus, I proved that the planner (weakly) reduces borrowing and investment, and that she reduces repayments and liquidations in state l . Since, thus, we have that $\widehat{\nu}(l) > \nu^{CE}(l)$, we also conclude that the threshold price $\underline{q}^*(\zeta)$ at which the planner decides to issue a contingent contract is strictly higher.

Implementation. To implement the planner's allocation set the transfer to satisfy $(1 - \pi(l))T = \pi(l) (\Pi^{CE}(l) - g(z^*(l)k^*(l)) + p^*(l)z^*(l)k^*(l)) > 0$. Let τ denote the tax on entrepreneurial borrowing against state l and T_0 denote the lump sum transfers of these taxes back to entrepreneurs, so that the entrepreneur's period 0 budget constraint is given by

$$\chi(k) = \left(\sum_s \xi(s)d(s) \right) k - \tau \cdot d(l)k + T_0$$

If at the planner's allocation $\mu(h) = 0 < \omega(h)$, then set

$$\tau^* = 1 - \frac{\pi(l)\nu^*(l) + 1 - \pi(l)}{\pi(l)\widehat{\nu}^*(l) + 1 - \pi(l)}$$

where ν^* denotes the entrepreneur's marginal utility of wealth in state l at the planner's allocation. On the other hand, if $\mu(h) \geq 0 = \omega(h)$, then set the tax to

$$\tau^* = \xi(l) \left(1 - \frac{\nu^*(l)}{\widehat{\nu}^*(l)} \right)$$

We only need to show that the entrepreneur's optimality conditions hold at the planner's allocation. Now, note that at the planner's allocation, the entrepreneur's marginal utility of wealth ν^* is equal to that of planner, $\widehat{\nu}^*$. If at the planner's allocation, we have $\mu(h) = 0 < \omega(h)$, then the entrepreneur's foc's at this allocation must satisfy

$$(1 - \tau^*)\nu^* = \pi(l)\nu^*(l) + 1 - \pi(l)$$

and this is verified by plugging the corresponding expression for τ^* and using the fact the planner sets $\widehat{\nu}^* = \pi(l)\widehat{\nu}^*(l) + 1 - \pi(l)$. On the other hand, when $\mu(h) \geq 0 = \omega(h)$, the entrepreneur's foc's at the planner's allocation must satisfy

$$(\xi(l) - \tau^*)\nu^* = \pi(l)\nu^*(l)$$

which is also verified by plugging the corresponding expression for τ^* and using the fact that the planner sets $\xi(l)\widehat{\nu}^* = \pi(l)\widehat{\nu}^*(l)$. Finally, note that since $\nu^*(l) < \widehat{\nu}^*(l)$, we have that $\tau^* > 0$ and $T^* > 0$.

Proof of Lemma 3. The derivation of buyers' optimal offer strategies is analogous to the proof of Lemma 2, except that buyers' beliefs are augmented with the belief that the seller is in the set $H^S = \{\beta = 1\} \cup (\{\beta = 0\} \cap \{x^S < \widehat{x}\})$, i.e. buyers' prior is updated to

$$\widehat{\pi}(l) \equiv \Pr(s = l | H^S) = \frac{(\lambda + (1 - \lambda)F_l(\widehat{x}))\pi_l}{(\lambda + (1 - \lambda)F_l(\widehat{x}))\pi_l + (\lambda + (1 - \lambda)F_h(\widehat{x}))(1 - \pi_l)}$$

and then buyers submit offers as if in a standard first-price common value auction. It suffices to show that there is a threshold such that non-liquidity hit investors with signals below that threshold post their contracts for sale. Let \hat{x} be the selling threshold, and let y^{max} be the maximal order statistic among signals x_1, \dots, x_n drawn independently from distribution $F_s(\cdot)$. Then for an investor with signal x , the buyers' optimal offer strategy (given in Lemma 7) implies a payoff from selling of

$$\begin{aligned} \mathbb{E} \{p^{max}(\mathcal{C}, \hat{x}) | x\} &= \int_{\underline{x}}^{\hat{x}} p(y, \hat{x}) f_{y^{max}}(y|x) dy \\ &= \int_{\underline{x}}^{\hat{x}} p(y, \hat{x}) \frac{\sum \pi(s) f_s^n(y) f_s(x)}{f(x)} dy \\ &= \int_{\underline{x}}^{\hat{x}} p(y, \hat{x}) \sum f_s^n(y) \frac{\pi(s) f_s(x)}{f(x)} dy \\ &= \int_{\underline{x}}^{\hat{x}} p(y, \hat{x}) \sum_s f_s^n(y) \Pr(s|x) dy \end{aligned}$$

First, I show that $U(x, \hat{x}) = \mathbb{E} \{d(s) k|x\} - \mathbb{E} \{p^{max}(y, \hat{x}) | x\}$ is increasing in x . Let $\hat{\pi}_l(y, \hat{x})$ denote the implied state l probability corresponding to the offer $p(y, \hat{x})$, then we need to show that

$$\int_{\underline{x}}^{\hat{x}} \hat{\pi}_l(y, \hat{x}) \sum_s f_s^n(y) \Pr(s|x) dy - \Pr(l|x)$$

is decreasing in $\Pr(l|x)$. Differentiation w.r.t. $\Pr(l|x)$ yields

$$\int_{\underline{x}}^{\hat{x}} \hat{\pi}_l(y, \hat{x}) (f_l^n(y) - f_h^n(y)) dy - 1 = \int_{\underline{x}}^{\hat{x}} \hat{\pi}_l(y, \hat{x}) f_l^n(y) \left(1 - \frac{f_h^n(y)}{f_l^n(y)}\right) dy - 1$$

Now recall that signals are boundedly informative: $\frac{f_h^n(y)}{f_l^n(y)} \geq \underline{\phi} > 0$ for all $y \in [\underline{x}, \bar{x}]$. There thus exists a $\underline{\phi} \in (0, 1)$ such that the above expression is negative for all $\hat{x} \in [\underline{x}, \bar{x}]$. To show that there exists a threshold \hat{x} , consider the following two cases:

- Case 1: If $U(\underline{x}, \underline{x}) \geq 0$, then we have that $\hat{x} = \underline{x}$ is an equilibrium because even the most pessimistic investor with $\beta = 0$ does not want to sell when the threshold is \underline{x} . Since $U(\cdot, \underline{x}) \geq 0$ is increasing, it follows that more optimistic investors will also not want to sell. Note that this equilibrium coincides with the equilibrium in the economy with observable liquidity needs.
- Case 2: If $U(\underline{x}, \underline{x}) < 0$, then threshold \underline{x} cannot be an equilibrium because otherwise, by continuity of $U(\cdot, \underline{x})$, there would be a set of signals above \underline{x} for which investors with $\beta = 0$ would want to sell. Consider the equation $U(x^*, \hat{x}) = 0$, which defines a map $x^* : [\underline{x}, \bar{x}] \rightarrow [\underline{x}, \bar{x}]$ that, for a given threshold \hat{x} , gives the signal $x^*(\hat{x})$ of the

non-liquidity hit investor who is indifferent between selling his contract and keeping it. Note that $x^*(\underline{x}) > \underline{x}$ by assumption that $U(\underline{x}, \underline{x}) < 0$ and $x^*(\bar{x}) < \bar{x}$ since the expected resale price is always strictly lower than the most optimistic valuation ($x^*(\hat{x})$ is a singleton because $U(\cdot, \hat{x})$ is increasing). In addition, since $U(x, \hat{x})$ is continuous in both x and \hat{x} , we have that the map $x^*(\cdot)$ is continuous on $[\underline{x}, \bar{x}]$. There thus exists an \hat{x} such that $x^*(\hat{x}) = \hat{x}$.

Proof of Proposition 7. Let \hat{x} be an equilibrium threshold and let $H(\hat{x})$ denote the event that an investor sells contract \mathcal{C} , and note that

$$\Pr(H(\hat{x})) = \lambda + (1 - \lambda)F(\hat{x})$$

The contract price schedule is given by

$$\begin{aligned} \mathcal{L}(\{d(s)k\}) &= \mathbb{E}\{\gamma(x, \beta, \mathcal{C})\mathbb{E}\{p^{\max}(\mathcal{C}, \hat{x})|x\} + (1 - \gamma(x, \beta, \mathcal{C}))\beta\mathbb{E}\{d(s)k|x\}\} \\ &= \Pr(H(\hat{x}))\mathbb{E}\{p^{\max}(\mathcal{C}, \hat{x})|H(\hat{x})\} + (1 - \Pr(H(\hat{x})))\mathbb{E}\{d(s)k|H(\hat{x})^c\} \\ &= \mathbb{E}\{d(s)k\} + \Pr(H(\hat{x}))\mathbb{E}\{p^{\max}(\mathcal{C}, \hat{x}) - d(s)k|H(\hat{x})\} \end{aligned}$$

Finally, that $\Pr(\hat{x})\mathbb{E}\{d(s) - p^{\max}(\mathcal{C}, \hat{x})|\hat{x}\} = \hat{\zeta} \cdot |d(h) - d(l)|k$ for some $\hat{\zeta} > 0$ follows from the linearity of offer strategies and the fact that buyers earn informational rents. The proof that informational rents are positive is analogous to that of Proposition 2.

Matching Function. I construct a random matching function that maps each seller of a contract to n buyers randomly selected from the set of potential buyers, i.e. non-liquidity hit investors. Recall that investors' decision whether to post their contracts for sale is given by a measurable map $\gamma : [\underline{x}, \bar{x}] \times \{0, 1\} \rightarrow \{0, 1\}$ that maps signals $x \in [\underline{x}, \bar{x}]$ and liquidity needs $\beta \in \{0, 1\}$ into sale or no sale decision $\{0, 1\}$. Investors' decision of whether to become buyers is simply given by their liquidity needs: an investor is a buyer if and only if he has $\beta = 0$. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a canonical probability space. Fix a measure space (G, \mathcal{G}, μ) of investors, where $G = [0, 1]$, \mathcal{G} denotes the Lebesgue measurable subsets of $[0, 1]$, and μ is the Lebesgue measure on $[0, 1]$. The random state of nature is given by $s \in \{l, h\}$ and is assumed to satisfy $\Pr(s = l) = \pi(l)$. The stochastic processes for signals $\{x_i\}$ and for liquidity needs $\{\beta_i\}$ in turn satisfy

- Process $\{\beta_i\}$ satisfies the following properties
 - For all $(i, \omega) \in G \times \Omega$, $\beta_i(\omega) \in \{0, 1\}$ and $\mathbb{P}(\omega \in \Omega : \beta_i = 1) = \lambda$, and
 - For any distinct i_1, \dots, i_k with $k < \infty$ and $s \in \{l, h\}$, $\mathbb{P}(\omega \in \Omega : \beta_{i_1} = 1, \dots, \beta_{i_k} = 1) = \lambda^k$ (independence).
- Process $\{x_i\}$ satisfies the following properties

- For all $(i, \omega) \in G \times \Omega$, $x_i(\omega) \in [\underline{x}, \bar{x}]$ and $\mathbb{P}(\omega \in \Omega : x_i \leq x) = F(x) = \pi(l) F_l(x) + (1 - \pi_l) F_h(x)$, and
- For any distinct i_1, \dots, i_k and any $x_{i_1}, \dots, x_{i_k} \in [\underline{x}, \bar{x}]$ with $k < \infty$, $\mathbb{P}(\omega \in \Omega : x_{i_1} \leq x_1, \dots, x_{i_k} \leq x_k | s) = \prod_{l=1}^k F_s(x_l)$ for $s \in \{l, h\}$ (independence).
- The processes $\{\beta_i\}$ and $\{x_i\}$ satisfy : for all $i \in [0, 1]$, $\mathbb{P}(\omega \in \Omega : x_i \leq x, \beta_i = 1) = F(x) \lambda$ for any $x \in [\underline{x}, \bar{x}]$ (independence).

Given $\omega \in \Omega$, define the set of sellers by $S(\omega) \equiv \{(i, \omega) \in [0, 1] \times \Omega : \gamma(x_i, \beta_i) = 1\}$, and the set of buyers by $B(\omega) \equiv \{(i, \omega \in [0, 1] : \beta_i(\omega) = 0\}$. These are measurable subsets of G , with the property that $n \cdot \mu(S(\omega)) \leq \mu(B(\omega))$ holds ω -almost surely. A sufficient condition for this inequality to hold is that $n\lambda \leq (1 - \lambda)$ in the case with observable liquidity needs. In the case of unobservable liquidity needs, λ must satisfy $n[\lambda + (1 - \lambda) F_l(\hat{x})] \leq (1 - \lambda)$, and this is feasible because \hat{x} is decreasing with λ and is close to \underline{x} when λ becomes small. Let $(\Omega', \mathcal{F}', \mathbb{P}')$ be an alternative probability space, and consider a map $\pi : G \times \Omega \times \Omega' \rightarrow G^n \cup \{\emptyset\}$ that satisfies the following properties

- For $i \notin S(\omega)$ and $(\omega, \omega') \in \Omega \times \Omega'$, $\pi(i, \omega, \omega') = \{\emptyset\}$,
- For $i \in S(\omega)$ and $(\omega, \omega') \in \Omega \times \Omega'$, $\pi(i, \omega, \omega') = \{j_1, \dots, j_n\}$ where $j_1 < \dots < j_n$, $i \neq j_k$ for any $k \in \{1, \dots, n\}$,
- For $(\omega, \omega') \in \Omega \times \Omega'$, the map $\pi(\cdot, \omega, \omega')$ is injective when restricted to the set $S(\omega)$, and

The process $\{\pi(i, \cdot, \cdot)\}_i$ satisfies

- For $\omega \in \Omega'$ and for any distinct $i_1, \dots, i_k \in S(\omega)$ and measurable sets $A_1, \dots, A_{i_k} \in G^n$ with $k < \infty$,

$$\Pr(\omega' \in \Omega' : \pi(i_1, \omega, \omega') \in A_1, \dots, \pi(i_k, \omega, \omega') \in A_k) = \prod_{l=1}^k \Pr(\omega' \in \Omega' : \pi(i_l, \omega, \omega') \in A_{i_k})$$

Finally, the sigma algebras \mathcal{F} and \mathcal{F}' are assumed to be independent and the space $\Omega \times \Omega'$ is endowed with the corresponding product sigma algebra $\mathcal{F} \times \mathcal{F}'$ and product measure $\mathbb{P} \times \mathbb{P}'$. Thus, in state (ω, ω') , investor $i \in S(\omega)$ is matched with n distinct investors $j_1, \dots, j_n \in \pi(i, \omega, \omega')$ and investor $i \notin S(\omega)$ remains unmatched; buyers are matched with at most one seller by the definition of the map $\pi(\cdot, \omega, \omega')$ and no two sellers are matched with the same buyers by injectivity of the map $\pi(\cdot, \omega, \omega')$. Conditional on state ω , a seller is matched with a random selection of buyers due to the independence assumption on the process $\{\pi(i, \cdot, \cdot)\}_i$. Finally, the independence of the two sigma algebras ensures that conditional on being a seller and being a buyer, no additional information is revealed by the match itself.

Matching with Multiple Sellers. Each seller can contact n buyers, and each buyer who is matched is assumed to be able to trade with m sellers, where $m(1 - \lambda) \geq n$. To ensure

consistency, matching occurs by adjusting the fraction of buyers who are matched with sellers. Thus, if the measure of sellers is given by μ , a fraction $\omega = \frac{n}{m} \frac{\mu}{1-\lambda}$ of buyers is matched with m sellers and the remaining buyers are matched with 0. This is a simple way to introduce noise into the matching process and ensure that the number of matches that a buyer gets does not fully reveal the state of the economy. Once buyers and sellers have matched, trades are assumed to be executed simultaneously. This latter assumption eliminates informational spillovers across markets; however, see Section 6. As in Section 4, consider the case of observable liquidity needs and conjecture that in equilibrium an investor is a seller if and only if he has a liquidity need, i.e. type $\beta = 1$. Then the fraction of buyers who are matched in state s is given by $\omega_s = \frac{n}{m} \frac{\lambda}{1-\lambda}$ for $s \in \{l, h\}$ and is thus independent of the state. Thus, on equilibrium path, buyers do not make inferences about the state from the match and their offer strategies are the same as in Section 4. For deviating investors of type $\beta = 0$, again suppose that buyers assign the most pessimistic belief. These off-equilibrium beliefs can then be shown to support the equilibrium with sorting solely on liquidity needs.

Appendix B

Appendix for Chapter 2

Expert's Problem

Let c^e, c^h be the payoff of experts and households respectively, let μ be the mean of the experts' posterior distribution, and let $\tilde{\mu}$ be the households beliefs about the expert's private information (beliefs updated after observing portfolio allocations: α). The expert's problem is given by:

$$\begin{aligned} & \max_{c^e, c^h, \alpha} E [u^e (c^e) | \mu] \\ & E [u^h (c^h) | \tilde{\mu}] \geq \bar{U}^h \quad (\lambda_{pc} (\tilde{\mu})) \\ & c^e (R, \alpha) + c^h (R, \alpha) \leq [\alpha' (R - R_f) + R_f] w \quad (\lambda_{fc1} (R, \alpha)) \\ & w \leq w^e + w^h \quad (\lambda_{fc2}) \\ & \tilde{\mu} (\alpha (\mu)) = \mu \end{aligned}$$

Proof of Proposition 1. Combining the constraints, the problem of expert with private information μ , can be re-written as follows:

$$\begin{aligned} & \max_{c^e(R, \alpha), \alpha} E [u^e ([\alpha' (R - R_f) + R_f] w - c^h (R)) | \mu] \\ & E [u^h (c^h (R, \alpha)) | \tilde{\mu}] \geq \bar{U}^h \quad (\lambda_{pc} (\tilde{\mu})) \\ & \tilde{\mu} (\alpha (\mu)) = \mu \end{aligned}$$

Given that $u'(c) = \gamma \exp[-\gamma c]$, the FOC with respect to consumptions evaluated at $\tilde{\mu} (\alpha (\mu)) = \mu$ yield:

$$c^e (R, \alpha) = \frac{\gamma^h}{\gamma^h + \gamma^e} [\alpha' (R - R_f) + R_f] w + Z (\alpha)$$

$$c^h(R, \alpha) = \frac{\gamma^e}{\gamma^h + \gamma^e} [\alpha' (R - R_f) + R_f] w - Z(\alpha)$$

where $Z(\alpha) = \frac{1}{\gamma^h + \gamma^e} \log \left(\frac{\lambda_{pc} \gamma^e}{\gamma^h} \right)$.

Proof of Proposition 2. To solve for \bar{f} and $f(\alpha)$, we find the fee that makes the participation constraint of the households, for given beliefs $\mu(\alpha)$, bind:

$$E \left[u^h \left(\frac{\gamma^e}{\gamma^h + \gamma^e} [\alpha' (R - R_f) + R_f] w - Z(\alpha) \right) \mid \mu(\alpha) \right] = \bar{U}^h$$

and using that $u^h(x) = e^{-\gamma^h x}$ and $E \left\{ e^{-\gamma^h x} \right\} = e^{-\gamma^h E\{x\} + \frac{1}{2} \gamma^{h2} V(x)}$ for x that is normally distributed, we have

$$\begin{aligned} -\gamma^h \left(\frac{\gamma^e}{\gamma^h + \gamma^e} [R_f + [\mu(\alpha) - R_f 1_N]' \alpha] w - Z(\alpha) \right) + \frac{1}{2} \gamma^{h2} \left(\frac{\gamma^e}{\gamma^h + \gamma^e} \right)^2 (\alpha w)' \Sigma (\alpha w) = \\ = -\gamma^h R_f w^h \end{aligned}$$

\Leftrightarrow

$$\begin{aligned} Z(\alpha) = \frac{\gamma^e}{\gamma^h + \gamma^e} [\mu(\alpha) - R_f 1_N]' \alpha w - \frac{1}{2} \gamma^h \left(\frac{\gamma^e}{\gamma^h + \gamma^e} \right)^2 (\alpha w)' \Sigma (\alpha w) + \\ + R_f \left(\frac{\gamma^e}{\gamma^h + \gamma^e} w^e - \frac{\gamma^e}{\gamma^h + \gamma^e} w^h \right) \end{aligned}$$

Decomposing the fee as in the text, $Z(\alpha) = \bar{f} + f(\alpha)$, we have that

$$\bar{f} = R_f \left(\frac{\gamma^e}{\gamma^h + \gamma^e} w^e - \frac{\gamma^e}{\gamma^h + \gamma^e} w^h \right)$$

$$f(\alpha) = \frac{\gamma^e}{\gamma^h + \gamma^e} [\mu(\alpha) - R_f 1_N]' \alpha w - \frac{1}{2} \gamma^h \left(\frac{\gamma^e}{\gamma^h + \gamma^e} \right)^2 (\alpha w)' \Sigma (\alpha w)$$

Proof of Proposition 3. Using the results of Proposition 2, the expert's problem can be expressed as

$$\max_{c^e(R, \alpha), \alpha} \left\{ -\gamma^e \left[\frac{\gamma^h}{\gamma^h + \gamma^e} [\alpha (\mu - R_f) + R_f] w + \bar{f} + f(\alpha) \right] + \frac{1}{2} \gamma^{e2} \left(\frac{\gamma^h}{\gamma^h + \gamma^e} \right)^2 (\alpha w)' \Sigma (\alpha w) \right\}$$

s.t.

$$\bar{f} = R_f \left(\frac{\gamma^e}{\gamma^h + \gamma^e} w^e - \frac{\gamma^e}{\gamma^h + \gamma^e} w^h \right)$$

$$f(\alpha) = \frac{\gamma^e}{\gamma^h + \gamma^e} [\mu(\alpha) - R_f \mathbf{1}_N]' \alpha w - \frac{1}{2} \gamma^h \left(\frac{\gamma^e}{\gamma^h + \gamma^e} \right)^2 (\alpha w)' \Sigma (\alpha w)$$

where as before we use that that $u^h(x) = e^{-\gamma^h x}$ and $\mathbb{E} \left\{ e^{-\gamma^h x} \right\} = e^{-\gamma^h \mathbb{E}\{x\} + \frac{1}{2} \gamma^{h2} \mathbb{V}(x)}$ for x that is normally distributed. The first order condition with respect to α yields

$$-\gamma^e \left[\frac{\gamma^h}{\gamma^h + \gamma^e} (\mu - R_f) w + f'(\alpha) \right] + \gamma^{e2} \left(\frac{\gamma^h}{\gamma^h + \gamma^e} \right)^2 w \Sigma \alpha w = 0$$

where

$$f'(\alpha) = \frac{\gamma^e}{\gamma^h + \gamma^e} [\mu(\alpha) - R_f \mathbf{1}_N] w + \frac{\gamma^e}{\gamma^h + \gamma^e} \left[\frac{d}{d\alpha} \mu(\alpha) \right]' \alpha w - \gamma^h \left(\frac{\gamma^e}{\gamma^h + \gamma^e} \right)^2 w \Sigma \alpha w$$

Using Conjecture 1, we have that

$$\frac{d}{d\alpha} \mu(\alpha) = \kappa^{-1} \Sigma$$

and combining these expressions and solving for α yields

$$\alpha = \kappa \Sigma^{-1} [\mu - R_f \mathbf{1}_N]$$

with $\kappa = \frac{\gamma^h + 2\gamma^e}{\gamma^h + \gamma^e}$.

Proof of Corollary 1. Plugging the optimal choice of α from Proposition 3 into the expression for $f(\alpha)$ yields

$$f(\alpha) = \frac{1}{2} \frac{\gamma^h + 2\gamma^e}{(\gamma^h + \gamma^e)^2} [\mu - R_f \mathbf{1}_N]' \Sigma^{-1} [\mu - R_f \mathbf{1}_N]$$

Optimality of Delegation (implication of Assumption 4). The household's welfare within the contract is given by

$$U^h = \gamma^h R_f w^h$$

If the household was able to invest in the portfolio with prior beliefs (μ_0, Σ_0) , then its welfare would be

$$U^h = \frac{1}{2} [\mu_0 - R_f \mathbf{1}_N]' \Sigma_0^{-1} [\mu_0 - R_f \mathbf{1}_N] + \gamma^h R_f w^h$$

and so if the non-pecuniary cost of investment satisfies $\chi > \frac{1}{2} [\mu_0 - R_f \mathbf{1}_N]' \Sigma_0^{-1} [\mu_0 - R_f \mathbf{1}_N]$, the household will only invest through the expert.