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### Publication Date

1985-03-01

CUDARE

University of California, Berkeley.  
Dept. of agricultural and  
resource economics  
Working Paper 343

Working Paper No. 343

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March 1985

IMPACT OF EXPECTATION FORMATION ON DYNAMIC INVESTMENT DECISIONS:  
AN APPLICATION TO U. S. AGRICULTURE

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IMPACT OF EXPECTATION FORMATION ON DYNAMIC INVESTMENT DECISIONS:  
AN APPLICATION TO U. S. AGRICULTURE

The application of duality to dynamic optimization problems has led to a means of constructing econometric models which are consistent with dynamic theory. This method, developed by McLaren and Cooper (1980) (hereafter MC) and Epstein (1981), has been applied by Bernstein and Nadiri (1983), Epstein and Denny (1983), and Vasavada and Chambers (1983). One of the limitations of the approach is that it assumes agents have stationary expectations about all variables they do not control. This means, for example, that agents expect current (relative) prices and technology to persist indefinitely. This is more restrictive than the assumptions necessary for a stationary control problem; the latter permits autonomous changes in variables such as price and technology.

The assumption of stationary expectations is inappropriate where the data indicate a trend rather than a fluctuation about a constant mean or where there is evidence of technological or other structural change. Most time series are characterized by at least one of these features. There are several possible resolutions to this conflict between the model and data. One of these is to detrend the data prior to imposing the restrictions implied by dynamic theory. Sargent (1978, p. 1027) argues that this is the correct approach. He claims that the dynamic model should explain the indeterminate component of the series--that portion which is not explained simply by time. However, the restrictions embodied in the investment equations given below involve real economic variables; there is no reason to suppose that the same restrictions hold over detrended or quality-adjusted variables.

Two alternatives are to include an unrestricted time trend while estimating the restricted parameters or to reformulate the original problem to

incorporate the nonstationarity. Both of these alternatives involve a degree of arbitrariness, but they imply different assumptions about the amount of information or rationality of agents. In certain cases the estimates of a subset of parameters is invariant to which assumption is made. This is useful because hypotheses which involve only those parameters can be tested without forcing a choice between two equally probable maintained hypotheses.

The next section reviews MC and Epstein's method of deriving investment functions for a dynamic problem. Section II discusses the complications that arise when nonstationarity is introduced. The section also mentions another issue that arises in the empirical implementation of the duality method: The theory is presented in continuous time, but data sets are available in discrete time, and many dynamic investment problems may be more accurately modeled in discrete time. The discrete time analogs of the continuous time investment rules are given. The continuous time rules can be regarded as a first-order Taylor approximation of the discrete time rules. Section III applies the dynamic model to the estimation of investment rules for U. S. agriculture using aggregate data. These rules indicate the degree of asset fixity and, hence, the adjustment cost in agriculture. It is important to have some idea of the magnitude of this cost because this is a major determinant of the effect of U. S. governmental policy. This section follows the work of Vasavada and Chambers; however, it focuses on the robustness of the parameter estimates and the hypotheses tests with respect to changes in stationarity assumptions. The major points are summarized in a conclusion.

#### I. The Method: A Review

The dynamic adjustment problem assumes that a firm or industry has two types of inputs--variable and quasi-fixed. Variable inputs may be obtained at

a given price; changing the level of quasi-fixed inputs involves an internal, nonlinear adjustment cost as well as the usual cost of renting the additional unit of input. Let  $k$  be a vector of quasi-fixed inputs with rental price  $p$  (normalized by output price), let  $w$  be the vector of normalized prices of the variable inputs  $v$ , and let  $r$  be the discount rate. For economy of notation, suppose that the depreciation rate is 0 so that

$$\frac{dk}{dt} = \dot{k} = i \quad (1)$$

where  $i$  is the vector of investment. Let the maximum instantaneous rate of profits, given  $k$ ,  $i$ ,  $p$ , and  $w$  be  $\pi(k, i, w) - pk$ ;  $\pi(\ )$  is the restricted static profit function and  $pk$  is the rental cost which is fixed in the short run. Hotelling's lemma gives the optimal level of variable inputs as

$$v^* = -\pi_w(\ ), \quad (2)$$

where subscripts indicate partial differentiation.

In the MC and Epstein models, the firm has stationary expectations and seeks to maximize the discounted stream of profits. Its problem is

$$\max_i \int_{t_0}^{\infty} e^{-r(t-t_0)} [\pi(\ ) - pk] dt$$

subject to (1) and  $k(t_0) = k_0$ , given. The maximum value of the discounted stream of profits at  $t_0$ , given  $k_0$  and the parameters  $p$  and  $w$ , is the function  $J(k_0, p, w)$  which satisfies the dynamic programming equation

$$rJ(\ ) = \max_i [\pi(\ ) - pk + J_k(\ )i]. \quad (3)$$

The first-order condition is  $\pi_i(\cdot) = -J_k(\cdot)$  which implicitly defines the control rule

$$i^* = \phi(k, p, w). \quad (4)$$

Instead of specifying the functional form for  $\pi(\cdot)$ , it is generally simpler to specify the form of  $J(\cdot)$ . The system of demand equations for  $v$  and  $i$  can be estimated, and hypotheses on  $\pi(\cdot)$  can be tested via the function  $J(\cdot)$  using duality relations. To determine the demand system, differentiate (3) with respect to  $p$  and  $w$  at the optimal  $i^*$ . The results are

$$rJ_p = -k + J_{kp}\phi \quad (5a)$$

$$rJ_w = \pi_w + J_{kw}\phi. \quad (5b)$$

Equation (5a) gives investment as

$$i^* = \phi(\cdot) = (J_{kp})^{-1} (rJ_p + k). \quad (6)$$

Substituting (6) and (2) in (5b) gives the demand for the variable inputs

$$v^* = -rJ_w + J_{kw}(J_{kp})^{-1} (rJ_p + k). \quad (7)$$

Equations (6) and (7) can be estimated, and the implied cross-equation restrictions can be imposed or tested.

MC and Epstein prove the important duality relations between  $\pi(\cdot)$  and  $J(\cdot)$ . If  $\pi(\cdot)$  is convex in  $w$ , then  $rJ(k, p, w) - J_k(k, p, w)\bar{i}$  is locally convex around  $\bar{p}$  and  $\bar{w}$ , where  $\bar{i}$  is the optimal investment level at  $k$ ,  $\bar{p}$ , and  $\bar{w}$ . In addition,  $J(\cdot)$  is convex in  $w$  and  $p$  (Epstein, lemma 1). If  $\pi$  is concave in  $k$  and  $i$ , then the maximized  $\pi(\cdot)$  is concave in  $k$  and, from (3),  $rJ - J_k i^*$  is

concave in  $k$ . Hence,  $-e^{rt} \frac{d}{dt} J(k)$  and  $-e^{-rt} \frac{d}{dt} J(k)$  are concave in  $k$ . Assume that  $e^{rt} J[k(t)] \rightarrow 0$  or  $t \rightarrow \infty$ . Integrating the last derivative over  $(0, \infty)$  implies that  $J(\cdot)$  is concave in  $k$ . This parallels the proof of Epstein's lemma 1. Since  $i^*$  involves second-order partials of  $J$ , it is necessary to check fourth-order partials of  $J$  to test the concavity of  $\pi$  in  $k$ .

Tests on the parameters of  $J$  indicate whether necessary and sufficient conditions hold to insure the indirect profit function exhibits the expected properties.

## II. Modifications to the Model

For most problems, the stationarity assumption is inappropriate, so equations (6) and (7) cannot be directly applied. Two probable sources of nonstationarity are changes in technology and price levels. This section concentrates on the former; the latter is briefly discussed. There are at least two alternatives to simply detrending the data prior to imposing the restrictions. Both alternatives involve the same ad hoc assumption about the way in which the nonstationarity enters the value function, but they imply different assumptions about the agent's rationality or information set. For an important special case, a subset of parameters is invariant to which assumption is made. Hypotheses involving only those parameters can be tested without explicitly adopting either assumption.

For clarification, suppose that the nonstationarity is caused by exogenous technological change and measured by some function  $f(t)$ . One way to account for the nonstationarity is to include  $f(t)$  as a regressor in equations (6) and (7). This is equivalent to including  $f(t)$  as an argument in the value function  $J(\cdot)$ . However, the dependence of  $J$  on  $f$  is not theoretically



determined. In particular, there is no presumption that  $J$  is either concave or convex in  $f$ . Assumptions about the effect of technological change, e.g., Hick's neutrality, do impose conditions on the way in which  $f$  enters  $\pi(\cdot)$  and, hence,  $J(\cdot)$ ; however, there does not appear to be a simple duality relation between  $\pi$  and  $J$  involving the argument  $f$ . Thus, specification of the dependence of  $J(\cdot)$  on  $f$  is ad hoc.

Accounting for nonstationary technology in this manner implies that the econometrician has information unavailable to the optimizing agents. To see this, consider the agents' nonstationary problem, where  $f$  is an argument of  $\pi$ . Equations (3), (4), (6), and (7) are replaced by

$$rJ(\cdot) = \max_i [\pi(\cdot) - pk + J_k(\cdot)i + J_f \dot{f}] \quad (3')$$

$$i^* = \phi(k, p, w, r, f). \quad (4')$$

$$i^* = \phi(\cdot) = (J_{kp})^{-1} (rJ_p + k - J_{fp} \dot{f}) \quad (6')$$

$$v^* = -rJ_w + J_{kw} (J_{kp})^{-1} (rJ_p + k - J_{fp} \dot{f}) + J_{fw} \dot{f}. \quad (7')$$

In a certainty equivalent framework, all variables are replaced by their expected values. If the agents expect technology to remain stationary,  $\dot{f}$  is replaced by 0, and (3'), (6'), and (7') reproduce (3), (6), and (7) except that  $f$  is now an argument of  $J$ . Therefore, including  $f$  as an argument of  $J$  and estimating (6) and (7) [rather than (6') and (7')] implies that the econometrician is aware of technical change, but agents expect technology to remain constant.

If agents have the same information set as the econometrician, (6') and (7') replace (6) and (7). If  $J_{kp}$  and  $J_{fp}$  are constant matrices and if  $f$  is a polynomial then (6) and (6') have the same form and impose the same restrictions on  $J_{kp}$ . This means that the estimates and standard errors of  $J_{kp}$  are the same whether (6) or (6') are chosen; other parameter estimates will, in general, differ. This is an important special case since a constant  $J_{kp}$  implies that the investment rule is linear in capital stock  $k$ . Representing  $f$  by a polynomial can be defended as an approximation to a more general form. In this case, tests on the adjustment coefficients, which involve only  $J_{kp}$ , can be made without choosing between the competing hypotheses regarding the agents' view of technological change. Tests which involve other parameters will, in general, require selecting one of the two as maintained hypotheses. However, for the functional form used below, the two specifications result in the same restrictions for all parameters of interest.

The important dual relations between  $\pi$  and  $J$ , which involve the arguments  $p$  and  $w$ , still hold. Convexity of  $J$  in  $p$  and  $w$  can be seen by inspection of MC's proof of theorem 5.1. Inclusion of the argument  $f$  does not alter their demonstration. It is certainly possible to make strong enough assumptions about the regularity of  $\pi$  so that the function  $J$  and its derivatives exist. In that case, Epstein's lemma 1, mentioned above, is altered to state that the function  $rJ(k, p, w, f) - J_k(k, p, w, f)\bar{i} - J_f(k, p, w, f)\dot{f}$  is locally convex around  $\bar{p}$  and  $\bar{w}$ . The assumption that  $\pi(k, i, w, f)$  is concave in  $k$  implies that  $J$  is concave in  $k$  as before. This can be seen by repeating the argument of the previous section, modified to account for the inclusion of  $f$ .

Nonstationary expectations caused by expected changes in price can be handled in a similar way. Suppose that the certainty equivalent price forecast obeys the differential equation  $\dot{w}_t = g(w_t, t)$ , with  $w_0$ , the current price, given. A special case of this is  $\dot{w}_t = g(w_t)$ , an autonomous differential equation. In either case, the solution will be of the form  $w_t = h(w_0, t)$ . Substitute this solution into the function  $\pi(k, i, w_t)$  to obtain  $\pi[k, i, h(w_0, t)] \equiv \tilde{\pi}(k, i, w_0, t)$ . The problem is now of the same form as when nonstationarity is caused by technological change. The function  $\tilde{\pi}$  contains the argument  $w_0$  to indicate that the agent acts as if the boundary condition is fixed;  $w_0$  plays the same role as  $w$  in the previous problem. It is updated at each new observation. There is, however, an additional restriction. The assumption that  $\pi$  is decreasing and convex in  $w$  does not necessarily imply that  $\tilde{\pi}$  is decreasing and convex in  $w_0$ . If  $w$  is a scalar (i.e., there is a single variable input), then a necessary and sufficient condition for  $\partial \tilde{\pi} / \partial w_0 < 0$  is  $\partial h / \partial w_0 > 0$ , and a sufficient condition for  $\partial^2 \tilde{\pi} / \partial w_0^2 > 0$  is  $\partial^2 h / \partial w_0^2 \leq 0$ . If  $g$  is linear in  $w$ , as is often the case in estimation, these conditions are guaranteed to hold. There is, then, the same duality relation between  $\pi$  (or  $\tilde{\pi}$ ) and  $J$  as above.

As in the case where technology is the source of the nonstationarity, there is, in general, no presumption that  $J$  is either concave or convex in  $t$ . To see this, suppose that the relative price of the single variable input is expected to rise at a constant rate,  $\alpha$ , so that  $w_t = e^{\alpha t} w_0$ . If this is the only source of nonstationarity, then  $\pi_t = \pi_w \alpha w < 0$ , but the sign of  $\pi_{tt} = \alpha w (\pi_{ww} \alpha w + \pi_w \alpha)$  is ambiguous. In this case  $J_t < 0$ , but  $J_{tt}$  is ambiguous. Even in this simple case, specification of the functional dependence of  $J$  on  $t$  is largely ad hoc.

Although it is useful to recognize the similarity of the problems where nonstationary expectations are caused by changing prices or by changing technology, they differ in one respect. If the certainty equivalent forecast for the rate of change of  $w$ ,  $g(w)$ , is autonomous, it is unnecessary to include the argument  $t$  in  $J$ . The last term in (3') is replaced by  $J_w g(w)$  and equations (6') and (7') are replaced by

$$i^* = \phi(\ ) = (J_{kp})^{-1} (rJ_p + k - J_{wp}g) \quad (6'')$$

$$v^* = -rJ_w + J_{kw}(J_{kp})^{-1} (rJ_p + k - J_{wp}g) + J_{ww}g + J_w g_w. \quad (7'')$$

As above, proof of the convexity of  $J$  in  $w$  requires that  $\pi$  be convex in the boundary condition  $w_0$ . The advantage of this formulation is that it is unnecessary to make any ad hoc assumptions about the dependence of  $J$  on  $t$ . If rental prices,  $p$ , are expected to change, the same method can be used to determine the estimation equations.

This is one further point to be made before turning to the empirical model. The theory is typically presented in continuous time, but discrete time optimization is, in many cases, a better description of reality and is more consistent with available data. Both the continuous time equations (6) and (7) and their nonstationary analogs (6') and (7') or (6'') and (7'') can be directly applied to discrete data, but there are two reasons for beginning with a discrete time model. The first is largely pedagogic; the second indicates a need to scale the left side of the variable input equations, (7).

The following assumes stationary price expectations but nonstationary technology. The discount rate is  $\beta = 1/(1 + r)$ . The discrete version of (1) is

$$k_{t+1} - k_t = i_t, k_{t_0} = k_0, \text{ given.} \quad (8)$$

The agent's problem is

$$\max_i \sum_{t=t_0}^{\infty} \beta^{(t-t_0)} [\pi(k_t, i_t, w, f_t) - \beta p k_t]$$

subject to (8). The rent on the quasi-fixed input is paid at the end of the period; this is consistent with the definition of  $p$  used in the previous sections. The optimal value of the problem at  $t = t_0$  is given by the function  $J(k_0, p, w, f_t)$  which satisfies the dynamic programming equation

$$J(k_t, p, w, f_t) = \max_i [\pi(k_t, i, w, f_t)] + \beta [-p k_t + J(k_t + i, p, w, f_{t+1})]. \quad (9)$$

Differentiating (9), evaluated at the optimal  $i^*$  with respect to  $p$  and  $w$  and using (2), gives the demand system

$$J_p(k_t, p, w, f_t) = \beta [J_p(k_{t+1}, p, w, f_{t+1}) - k_t] \quad (10)$$

$$v^* = \beta J_w(k_{t+1}, p, w, f_{t+1}) - J_w(k_t, p, w, f_t). \quad (11)$$

Equation (10) gives investment implicitly. To compare the result with the continuous model, use a Taylor expansion of  $J_p(k_{t+1}, p, w, f_{t+1})$  around  $k_t, f_t$  to obtain

$$0 = r J_p - J_{pk} i_t - J_{pf} \Delta f + k_t + o(\Delta t)$$

where  $\Delta f = f_{t+1} - f_t$  and  $\Delta t$  is the length of a period;  $o(\Delta t)$  contains all higher order terms in the Taylor expansion. Simplifying yields

$$i_t = (J_{pk})^{-1} [rJ_p - J_{pf}\Delta f + k_t + o(\Delta t)],$$

which is identical to (6') except for the term  $o(\Delta t)$ . Where  $J_{pk}$  and  $J_{pf}$  are constants,  $o(\Delta t) \equiv 0$ , and the two equations are identical. As mentioned above, this condition also leads to a linear (in  $k$ ) investment rule and the invariance of the adjustment matrix with respect to the agents' expectations about technology (provided  $f$  is a polynomial). It may appear that, for more general cases, the discrete formulation is superior since the continuous model provides an approximation to it. This apparent advantage is illusory. The problem is that, unless (10) is linear in  $k$ , it does not provide an explicit investment rule. Suppose that (10) is quadratic in  $k$ . It is possible to estimate (10) in its implicit form and test whether the coefficients of the quadratic terms are zero; however, because of the strong correlation between  $k$  and  $k^2$ , it would be very surprising if the tests were not rejected. This cannot be used as evidence of the inadequacy of the linear model. In order to use (10), an explicit solution for investment must be found; this involves a linearization and is equivalent to using (6).

Although the discrete time formulation does not extend the available range of models, it does indicate how nonstationarity is dealt with in the certainty equivalent framework. Arguments of the function  $J(\ )$  on the right side of (9) and (10) are the expectations at  $t$  of the future values of prices and technology. Determining how the model performs under different assumptions about expectations requires estimating the investment equations using

different explanatory variables. For example, static expectations imply that the period  $t + 1$  prices and technology variable equal the period  $t$  values; perfect foresight is modeled by using the realized values at  $t + 1$ ; a form of adaptive expectations requires using the period  $t$  predictions of the  $t + 1$  variables. This is done in the next section.

The equations for variable inputs differ slightly in the discrete and continuous models. Expanding (11) gives

$$(1 + r) v = -rJ_w + J_{kw}i + J_{fw}\Delta f + o(\Delta t).$$

This reproduces the continuous time equation except that the left side is scaled by  $1 + r$ . In the continuous time model, there is no distinction between this period and the next, so the scaling factor goes to one. This correction suggests that parameter estimates of variable input equations obtained from continuous time models should be deflated if the decision problem actually occurs in discrete time.

### III. The Empirical Model

The methods discussed above are used to study the importance of asset fixity in U. S. agriculture and to determine the robustness of the results to different specifications of expectations. The hypothesis that producers regard certain assets as (quasi) fixed has often been advanced to explain their apparent unwillingness to reduce production in the face of continued low prices. The source of asset fixity or "adjustment costs" is usually left vague. Presumably, it is caused by some technical or deep structural reason such as imperfections in an input market.

An evaluation of U. S. agricultural policy depends, in large part, on the magnitude of the adjustment cost facing producers. Major importers of U. S. agricultural products (e.g., the European Community and Japan) impose substantial import restrictions. The purpose of these is probably to protect their domestic producers, but they also depress world price which permits importers to capture oligopsony rents. If asset fixity causes long-run U. S. supply to be very inelastic, then the effect of U. S. policies (such as deficiency payments and the loan rate) which maintain high producer prices is chiefly to transfer income to producers. If, on the other hand, asset fixity is not an important determinant of supply (adjustment costs are small), then U. S. supply in the absence of government price supports would be quite elastic; and the cost to importers of import restrictions would be large. If this is the case, government price supports are a principal cause of the inelasticity of supply and, thus, government policies serve chiefly to subsidize importers. That is, U. S. policies are an obstacle to the reduction of import restrictions because they increase the cost to producers in the importing countries of a reduction in restrictions and because they increase the importer's ability to capture rents by restricting trade.

Whether U. S. policies should be viewed primarily as a transfer to U. S. producers or as a transfer to importers and an obstacle to liberalized trade depends, in large part, on the magnitude of adjustment costs. This is an empirical question. The results reported below are not conclusive, but they offer some support for the second view.

#### Data and Specification

The data set, obtained from Ball (1984), consists of 32 observations over the period 1948-1979 for six outputs and seven inputs. These data are



comprehensive in coverage of output and input items of the agricultural production sector. They reflect an intensive examination of basic data underlying published U. S. Department of Agriculture series. Input data were aggregated using a tornqvist aggregation process into two quasi-fixed inputs (durable equipment and real estate) and two variable inputs (labor and miscellaneous inputs). Quantities are measured in constant 1977 dollars with corresponding price indices which equal 1.0 in 1977. A real discount rate ( $r$ ) equal to .065 and a capital depreciation rate of ( $\delta$ ) equal to .05 was used in estimation.

The value function  $J(k, p, w, t)$  was specified as a modified Generalized Leontief form:

$$J(\ ) = p'Ek + w'Fk + (w \cdot 5)' A p \cdot 5 + (p \cdot 5)' B p \cdot 5 + (w \cdot 5)' (w \cdot 5) \\ + t(g_1'p + h_1'w) + t^2(g_2'p + h_2'w)$$

where  $p$ ,  $w$ , and  $k$  are two dimensional;  $k_1$  is the stock of real estate (land and buildings);  $k_2$  is the stock of durable capital (durable equipment and farm-produced durables);  $p_1$  and  $p_2$  are the rental prices of real estate and durable capital, respectively;  $w_1$  and  $w_2$  are the prices of labor and miscellaneous inputs (materials, energy, and other nondurable inputs), respectively;  $t$  is time which acts as a proxy for technological change, and the notation  $z \cdot 5$  means a vector whose  $i$ th element is  $z_i \cdot 5$ .

The system of investment equations for quasi-fixed inputs is

$$i_t = k_{t+1} - (1 - \delta)k_t = Mk_t + Gf(t) + Hh(w, p) + d \quad (12)$$

where  $M$ ,  $G$ , and  $H$  are parameter matrices (vectors), and  $f$  and  $h$  are, respectively, vector functions of  $t$  and  $(w, p)$ . An advantage of the functional form used for  $J$  is that the estimates of a subset of parameters are the same whether it is assumed that agents take technology as static [(6) and (7) are used] or recognize that technology changes [(6') and (7')] are used. Only the parameters in  $G f(t) + d$  depend on which assumption is adopted; this means that hypotheses about asset fixity and convexity can be tested without specifying agents' expectations about technology.

Three variations of the basic model were estimated. These are (i) static expectations ( $E_t p_{t+1} = p_t$ ,  $E_t w_{t+1} = w_t$ ), (ii) perfect foresight ( $E_t p_{t+1} = p_{t+1}$ ,  $E_t w_{t+1} = w_{t+1}$ ), and (iii) adaptive expectations<sup>1</sup> ( $E_t p_{t+1} = \alpha + \beta p_t$ ,  $E_t w_{t+1} = \gamma + \rho w_t$ ) with  $\beta$  and  $\rho$  diagonal matrices. For (iii), the parameters  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\rho$  were estimated and the resulting values were then used in the estimation of (12). For both (i) and (ii), the matrix  $M$  is given by  $M = (r + \delta)I + E^{-1}$ . For (iii), the adaptive expectations variation,  $M = E^{-1}[(1 + r)\beta E + \beta] - (1 - \delta)I$ . In general,  $\beta$  is not a scalar matrix; in this case,  $M$  is a fairly complicated function of  $E$ . For (i) and (ii), (12) involves  $E^{-1}$  but not  $E$  which simplifies estimation and hypothesis testing.

The matrix of parameters  $H$  is the same for all three variations and is given by

$$H = E^{-1} \begin{bmatrix} a_{11} & a_{12} & b_{12} + b_{21} & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{21} & a_{22} & b_{12} + b_{21} \end{bmatrix}.$$

The regressors  $h$  depend on the assumption about price expectations. For (i),

$$h' = \left(\frac{r}{2}\right) \left[ \left(\frac{w_1}{p_1}\right)^{1/2}, \left(\frac{w_2}{p_1}\right)^{1/2}, \left(\frac{p_2}{p_1}\right)^{1/2}, \left(\frac{w_1}{p_2}\right)^{1/2}, \left(\frac{w_2}{p_2}\right)^{1/2}, \left(\frac{p_1}{p_2}\right)^{1/2} \right]_{it}.$$

For (ii), the jth element of h is

$$\frac{1}{2} \left[ (1+r) \left(\frac{w_j}{p_j}\right)_{it}^{1/2} - \left(\frac{w_j}{p_j}\right)_{it+1}^{1/2} \right].$$

For (iii), the jth element of h is

$$\frac{1}{2} \left[ \left(\frac{1+r}{\beta_{jj}}\right) \left(\frac{w_j}{p_j}\right)_{it}^{1/2} - \left(\frac{w_j^*}{p_j^*}\right)^{1/2} \right];$$

where  $p_j^* = \alpha_j + \beta_{jj}p_{j,t}$ ,  $w_j^* = \gamma_j + \rho_{jj}w_{j,t}$ .

The elements of G, f, and d can be determined using (10) and the specification of J. For variations (i) and (ii), the restrictions embodied in the parameter matrices are the same; only the regressors h are different. For variation (iii), there is, in addition to the change in M mentioned above, a slight modification of the terms f and d.

The results for the three variations constitute a sensitivity analysis of an intermediate model since each of the variations embodies an extreme assumption about what expectations are used in decision making. In principle, it is possible to nest (i) and (iii) in a more general model by estimating  $\alpha_i$ ,  $\gamma_i$ ,  $\beta_{ii}$ , and  $\rho_{ii}$  in (12) (these appear in M, f, h, and d) jointly with the other parameters of (12). The restriction that the estimates  $\hat{\alpha}_i$ ,  $\hat{\gamma}_i$ ,  $\hat{\beta}_{ii}$ , and  $\hat{\rho}_{ii}$  equal, respectively, 0, 0, 1, and 1, tests whether variation (i) is

the correct model. The restriction that the estimates equal the corresponding parameter estimates of the price forecast equations tests whether variation (iii) is the correct model. This procedure requires estimating eight additional parameters with a considerable increase in the degree of nonlinearity. It also creates an identification problem when the Generalized Leontief is used. The objective of this paper is to determine the sensitivity of estimation results to different assumptions about expectations rather than to distinguish which of the assumptions about expectations is more consistent with the data. For these reasons, the three variations are compared only informally.

The systems for variable inputs for the three variations can be obtained using (11) and the specification of J. Due to the small sample size, the system for the quasi-fixed inputs was estimated first; the results were used in the estimation of the demand for variable inputs. For present purposes, the system of quasi-fixed inputs is of greatest interest.

### Estimation Results

The demand system (12) is nonlinear in parameters. It was estimated using a Newton algorithm (SYSNLIN, the nonlinear estimation program in SAS) which minimizes a generalized sum of squares; the resulting estimates are asymptotically efficient. Different starting values produced estimates which were identical to the third decimal; this is evidence that a global optimum was achieved.

Define  $\tilde{M} = M - \delta I$  so that the difference equation for  $k$  may be written  $k_{t+1} = (I + \tilde{M})k_t + \theta(p, w, t)$ . The estimated adjustment matrices for the three variations are, respectively

$$\tilde{M}(i) = \begin{bmatrix} -.948 & -.523 \\ -.058 & -.731 \end{bmatrix}$$

$$\tilde{M}(ii) = \begin{bmatrix} -1.048 & -.803 \\ .01 & -.548 \end{bmatrix}$$

$$\tilde{M}(iii) = \begin{bmatrix} -1.016 & -.612 \\ .027 & -.736 \end{bmatrix} .$$

In all three variations,  $I + \tilde{M}$  is a stable matrix. In variations (i) and (ii), there is one positive and one negative root so  $k$  may fluctuate around its long-run (nonstationary) equilibrium. In (iii) both roots are positive so  $k$  approaches its long-run equilibrium monotonically.

The following hypotheses were tested using a likelihood ratio test (Gallant and Jorgenson, 1979):

1. Symmetry of the adjustment matrix ( $m_{12} = m_{21}$ ).
2.  $k_1$  is perfectly variable ( $m_{11} = -1, m_{21} = 0$ ).
3.  $k_2$  is perfectly variable ( $m_{22} = -1, m_{12} = 0$ ),

where  $m_{ij}$  is the  $i, j$  element of  $\tilde{M}$ . The results are given in Table 1. For all three variations, hypotheses 1 and 3 are rejected, and the tests fail to reject hypothesis 2. The conclusions regarding hypotheses 2 and 3 are consistent with Vasavada and Chamber's results. This provides evidence that real estate ( $k_1$ ) can be treated as a variable input and that durable capital ( $k_2$ ) should be regarded as quasi-fixed. This result is robust to widely different assumptions regarding producers' expectations about future prices. It suggests that incorrectly assuming static expectations may not affect important conclusions. Although durable goods are apparently not perfectly variable, the estimates of  $m_{22}$  indicate that a large percentage (between

Table 1  
Testing Hypotheses About Adjustment Matrix

Hypothesis	Critical $\chi^2$	$\chi^2$ statistics for		
		Static expecta- tions	Perfect foresight	Adaptive expecta- tions
Symmetry ( $m_{12} = m_{21}$ )	3.84	7.43	20.46	18.73
Real estate is variable ( $m_{11} = -1, m_{21} = 0$ )	5.99	0.64	0.14	0.16
Durable goods are variable ( $m_{21} = -1, m_{12} = 0$ )	5.99	20.34	42.41	31.03

one-half and three-quarters) of the deviation between actual and equilibrium stock is eliminated in a given year. This is not significantly changed when hypothesis 2 is maintained. These results suggest that asset fixity is unlikely to be a significant explanation of overproduction in agriculture. For a different view, see Vasavada and Chambers, who estimate that only about 20% of the deviation between the actual and equilibrium stock of durable goods is eliminated in a year. The rejection of hypothesis 1 implies rejection of the hypothesis of independent adjustment. The three variations reached identical conclusions, but those conclusions are most decisive in variation (ii) (perfect foresight) and least decisive in variation (i) (static expectations). Hypothesis 3 was also tested while maintaining hypothesis 2 with no change in the conclusion.

A sufficient condition for convexity of  $J$  in  $p$  and  $w$  is the convexity of  $\pi$  in  $w$ . The convexity of  $J$  was informally tested by calculating whether  $J$  was convex over the sample using estimated parameter values. This procedure was used by Epstein and Denny. A formal test, which requires imposing convexity, is extremely difficult and may not be desirable. The choice of a simple functional form for  $J$  may lead to the failure of convexity to hold over some range of  $p$  and  $w$ , when the relevant consideration is convexity over the sample.

For all three variations,  $J$  failed to be convex in either  $p$  or  $w$  at even the majority of sample points. When the parameter estimates were adjusted by one standard deviation,  $J$  was convex in  $p$  and  $w$  at all sample points in variation (ii) (perfect foresight); convex in  $w$ , but not  $p$  in variation (iii) (adaptive expectations); and not convex in  $p$  or  $w$  in variation (i) (static expectations). Thus, convexity cannot be rejected for variation (ii). Recall

that this is a necessary, not a sufficient, condition for the convexity of  $\pi$  in  $w$ . Thus, the conclusion is fairly weak: Where producers have perfect foresight, the investment rules meet a necessary condition for consistency with a well-behaved static profit function. Since the conclusions regarding hypotheses 1-3 were robust to the specification of the role of expectations, it seems likely that imposing convexity over the sample (if that were possible) would not alter these conclusions.

Input demand elasticities are calculated for each observation in the data set using estimated parameter values. Table 2 gives mean values of long- and short-run elasticities for the three hypotheses. A summary of cross-price elasticities is available upon request.

Results from expectation hypothesis (i) are generally inconsistent with static theory (i.e., nonpositive own-price elasticities and long-run elasticities which are greater in absolute value than short-run elasticities) while results from expectation hypotheses (ii) and (iii) tend to be consistent with static theory. Treadway (1970) and Mortensen (1973) have shown that positive own-price effects and less elastic long-run effects are not necessarily inconsistent with dynamic theory. In a dynamic model positive own-price elasticities may be an indication that inputs not only contribute to production activities but also to growth activities of the firm or industry. Short-run effects which exhibit greater elasticity than long-run effects can be interpreted in the following manner: Some inputs may facilitate the dynamic adjustment process thereby lowering the marginal adjustment cost of the quasi-fixed factors; therefore, the firm may use more of the input in the short-run than in the long-run in response to a given price change.



Table 2  
Summary of Elasticities

	(i) Static expectations	(ii) Perfect foresight	(iii) Adaptive expectations
Real estate	1.423 <sup>a</sup> (-.785) <sup>b</sup>	.891 (-.047)	-.102 (-.040)
Capital	.512 (-.33)	-.196 (-.011)	-2.675 (-2.146)
Labor	.24 (1.31)	.043 (-.065)	.372 (-.816)
Miscellaneous	.584 (3.994)	.004 (-.015)	-1.873 (-3.989)

<sup>a</sup>Mean values of long-run, own-price elasticities.

<sup>b</sup>Values in parentheses give short-run elasticities.

The elasticity results obtained under hypothesis (i) exhibit this growth-augmenting property in the majority of input groupings. Hypothesis (ii) exhibits the growth-augmenting property in real estate and both long-run variable input elasticities. Hypothesis (iii) exhibits the growth-augmenting property only in long-run labor demand. The elasticities do not exhibit the robustness over the different expectation schemes which was evident for hypotheses tests on properties of the adjustment matrix.

Comparison of the dynamic elasticities to a sampling of static elasticities [Ray (1982), Antle (1984), and Lopez (1984)] show elasticities to be similar for some input groupings. In contrast to static elasticities, dynamic elasticities exhibit a stronger tendency toward complementarity. This tendency toward complementarity serves to strengthen the growth-augmenting hypothesis which was outlined above.

#### IV. Conclusion

If the firm's dynamic optimization problem is nonstationary due, for example, to technological change, specification of the dependence of the value function on time is ad hoc. This is the case whether agents are assumed to regard technology or static or to possess the same information as the econometrician. Either assumption seems preferable to detrending variables prior to imposing the restrictions from the dynamic model. The two assumptions result in different investment rules but, for the model used here, the parameter estimates of the adjustment matrix are invariant to the two assumptions.

Different assumptions about producers' expectations were maintained in the estimation of dynamic investment rules. The conclusions concerning which inputs are quasi-fixed and which are variable were insensitive to these assumptions. This information is important since it is difficult to determine how

agents form expectations of future prices and how they use their expectations in making decisions. In each case it was concluded that real estate was approximately in continuous equilibrium. This finding provides further evidence that land rental markets adjust very quickly to changes in agricultural profitability. Durable goods appear to be a quasi-fixed input, but the adjustment to equilibrium is quite fast which suggests that adjustment costs are not large.

The degree of asset fixity in agriculture is of considerable practical importance since government policies sustain high producer prices. If asset fixity would cause supply to remain high even in the absence of these policies, the chief effect of the policies is to support producer income. If, on the other hand, the policies are responsible for inflexible supply, then the policies are a subsidy to importers and an obstacle to trade liberalization. This paper offers some evidence for the second view.

FOOTNOTES

<sup>1</sup>Although this is not the standard meaning of "adaptive expectations," no confusion should result in its use.

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