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Essays On Climate Change and the Macroeconomy

A dissertation submitted in partial satisfaction of the
requirements for the degree
Doctor of Philosophy

in

Economics

by

Stephanie Daniel Fried

Committee in charge:

Professor Mark Jacobsen, Chair
Professor David Lagakos, Co-Chair
Professor Thomas Baranga
Professor James Hamilton
Professor Valerie Ramey
Professor Irina Telyukova
Professor David Victor

2015

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The dissertation of Stephanie Daniel Fried is approved, and it is acceptable in quality and form for publication on microfilm and electronically:

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University of California, San Diego

2015

DEDICATION

for mom

EPIGRAPH

Strive not to be a success, but rather to be of value

—Einstein

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ABSTRACT OF THE DISSERTATION

Essays On Climate Change and the Macroeconomy

by

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Doctor of Philosophy in Economics

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Professor Mark Jacobsen, Chair
Professor David Lagakos, Co-Chair

This dissertation applies quantitative macroeconomic tools to study climate and energy policy in a dynamic general equilibrium framework. The first chapter examines how the cost and effectiveness of different climate policies change as countries develop. The second chapter studies the implications of endogenous innovation for climate policy outcomes over time. Finally, the third chapter develops an overlapping generations model to analyze the distributional and efficiency effects of a carbon tax across both income and age groups.

Chapter 1

Stuck in the Corner? Climate Policy in Developing Countries

1.1 Introduction

Firms in developing countries do not always use capital equipment with energy intensity that is “appropriate” for their resources. Most production technologies make optimal use of the capital, labor, and energy inputs of the richer economies because weak intellectual property rights induce entrepreneurs to target their innovation towards the developed countries’ needs (Acemoglu and Zilibotti (2001), Basu and Weil (1998)). However, cross-country differences in capital-labor ratios lead to different demands for energy intensity. These differences imply that the energy intensity embodied in capital equipment designed for developed countries could be sub-optimal for use in developing countries. This paper explores the effects of this sub-optimality for climate change mitigation in a neoclassical growth model. I find that using capital equipment with inappropriate energy intensity has sizable implications for both the effectiveness and welfare cost of climate policy in developing countries.

I make two adaptations to the standard neoclassical growth model. First, I include fossil energy as a production input. Second, I impose an upper bound on the energy intensity of capital (i.e., the ratio of fossil energy to capital in the production process). This upper bound limits the substitutability between capital and energy. For example, a firm can reduce the energy efficiency components of a car and purchase a car with less capital that uses more energy to produce the same transportation. However, there are limits to this substitution; very low capital cars with very high energy needs (e.g., fuel economy equal to one mile per gallon) are not available.

Firms choose their energy use to maximize profits. Optimal energy intensity (the ratio of energy to capital) is high in countries for which the capital-labor ratio is low. This is because a capital-poor country has the incentive to substitute both labor and energy for capital, since accumulating additional capital takes time. I show analytically that there exists a threshold value of the capital-labor ratio where the upper bound on energy intensity binds for firms below this threshold. When the upper bound binds, a firm’s

energy intensity is inappropriate for its capital-labor ratio, and its optimal energy use is a corner solution. Empirically, the capital-labor ratio rises as countries develop and so I interpret the capital-labor ratio of the representative firm as a measure of the country's economic development. Since each country is inhabited by a representative firm, I use the terms country and firm interchangeably.

There are three key results. First, I compare the effects of a uniform carbon tax in countries with capital-labor ratios above and below the threshold. The percentage emissions reduction from the tax is weakly increasing in the level of development. Since a carbon tax raises the price of energy, it increases firms' incentives to economize on energy use and, thus, can move a firm from the corner where the constraint on energy intensity binds, to the interior. However, if the carbon tax is not large enough to remove the firm from the corner, it will not incentivize any reduction in emissions because the upper bound on energy intensity continues to bind under the tax. In general, the need for the tax remove the firm from the corner in addition to incentivizing the firm to reduce emissions, decreases the effectiveness of the tax for firms below the threshold, resulting in a smaller reduction in emissions. Smaller reductions in emissions reduce the distortions in behavior from the tax, lowering the welfare costs of the policy in developing countries below the threshold.

Second, I compare the effects of a uniform percent reduction in emissions (abatement) in countries above and below the threshold. The carbon tax necessary to achieve this emissions reduction is considerably larger in developing countries below the threshold because the tax must move the firm from the corner to the interior, and then incentivize the reduction in energy use. The bigger taxes cause more deadweight loss and, thus, increase the welfare costs of the uniform abatement target for developing countries.

The notion of fairness across countries is an ongoing issue in international climate negotiations. These results suggest that if policy makers strive to design a climate policy that is "fair" in the sense that it equalizes the welfare cost, it should include larger carbon taxes or smaller abatement targets for developing countries.

Third, I calibrate the model to determine where high emissions countries lie with respect to different possible thresholds. Two developing countries that are key emitters in the current economy are India and China. I find that for most plausible values of the upper bound on energy intensity, India's level of development (measured by its capital-labor ratio) is always below the threshold while China's is just above the threshold. All else constant, these results imply that the welfare cost of a uniform abatement target would be considerably larger in India than in China or the US. Understanding these countries' welfare costs is important for designing a global emissions agreement.

This paper builds on the growing literature which applies macroeconomic models to study climate policy. For examples, see Golosov et al. (2014), Krusell and Smith (2009), and Hassler and Krusell (2012). Like these papers, this paper develops a dynamic model of the macroeconomy, energy, and climate policy. However, this previous work abstracts from the effects of the technology appropriability in developing countries, the focus of the present analysis.

The paper proceeds as follows: Sections 1.2 and 1.3 describe the decentralized model and derive the threshold capital-labor ratio. Section 3.3 calibrates the model and Section 2.5 quantifies the effects of the threshold on the distortionary costs of different climate policy. Section 3.5 concludes.

1.2 Decentralized model

1.2.1 Household

The economy is inhabited by a representative household with inelastic labor supply, L . The household divides its income between consumption, c , and savings in a risk-free asset, a , to maximize lifetime utility

$$U = \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}. \quad (1.1)$$

Fraction, $\frac{1}{\sigma}$, is the intertemporal elasticity of substitution and β is the rate of time preference. The household performs this optimization subject to the per

period budget constraint

$$c_t + a_{t+1} = w_t L + (1 + r_t)a_t + T_t, \quad (1.2)$$

where w and r denote the market wage and risk-free rates, respectively and T is income from government transfers.

1.2.2 Firm

A representative firm combines fossil energy, E , capital, K , and labor, L , to produce output, Y . The production technology is Cobb-Douglas with an upper bound on the energy intensity of capital,

$$Y = K^\alpha E^\theta (AL)^{1-\alpha-\theta} \quad \text{s.t.} \quad \frac{E}{K} \leq X. \quad (1.3)$$

Parameters α , θ , and $1 - \alpha - \theta$ are the factor shares of capital, energy, and labor, respectively. Variable A is exogenous technological progress. The upper bound on the energy intensity of capital, $\frac{E}{K}$, limits the substitutability of these two inputs in the production process.

A frequent critique of the Cobb-Douglas specification is the high substitutability between capital and energy. A piece of capital equipment often has a fixed amount of fossil energy required for its operation. For example, cars require a fixed amount of gas, coal boilers require a fixed amount of coal, etc. These energy requirements suggests that more capital requires more energy, not less, and, thus, are more consistent with a Leontief production technology than with Cobb-Douglas. However, firms can also invest in special types of capital that reduce the fossil energy necessary to operate their machines and buildings. Examples include hybrid breaking technologies in cars, more efficient coal boilers, wind turbines instead of coal boilers, better insulation, etc. These types of capital investments reduce the firm's energy use and suggest that there is some substitutability between capital and energy.

One approach to modeling firm production would be to differentiate between the firm's typical investments in machines and buildings and additional investments it makes to reduce its energy use. I show in Appendix A

that for certain functional forms, such a formulation is equivalent to a Cobb-Douglas production function with an upper bound on the energy intensity of capital, $\frac{E}{K} \leq X$. The upper bound reflects that even if the firm makes no additional investments to reduce its energy use, there is a limit to how much raising energy for a given amount of capital can increase output. For example, a firm could reduce the energy efficiency components of a car and produce a cheaper car (i.e., a car with less capital) that used more energy to produce the same transportation. Similarly, a firm could reduce the energy efficiency components of a coal boiler and produce a cheaper boiler (i.e., a boiler with less capital) that requires more coal to produce the same amount of electricity. However, there is a limit to how much firms can substitute the energy input for capital. Very cheap (low capital) cars with very low fuel economy (say equal to one mile per gallon) are not available. Similarly, very cheap (low capital) boilers with high heat rates (say equal to millions of BTUs per kilowatt hour) are also not available. The upper bound on the energy intensity of capital captures these limits to substitutability.

The firm takes prices as given and chooses the production factors to maximize profits, subject to the upper bound on the energy intensity of capital. The firm's optimization problem is

$$\max_{E,K,L} \{K^\alpha E^\theta (AL)^{1-\alpha-\theta} - RK - P(1+\tau)E\} \quad \text{subject to} \quad \frac{E}{K} \leq X. \quad (1.4)$$

The rental rate of capital, R , is the sum of the risk-free and depreciation rates, $R = r + \delta$.

Parameter P is the exogenous energy price. I assume the country behaves as a small open economy with respect to energy. Energy is imported in exchange for final good with zero trade balance in every period. The small open economy assumption abstracts from the potential general equilibrium effects of climate policy on energy prices. However, global energy supplies and prices are not within any single country's control. Moreover, most countries have very limited market power with respect to energy prices, suggesting that the general equilibrium effects from a unilateral climate policy are likely to be small. Section 1.5.3 discusses the implications of this simplification for the

main results.

Variable, τ , represents a carbon tax. A carbon tax is essentially a tax on energy use, since over eighty percent of all greenhouse gas emissions come from fossil energy combustion. For analytical tractability, I model a carbon tax as a percent of the energy price instead of as an additive tax per unit of energy consumed. This formulation is equivalent to a per unit tax because the energy price is constant. All carbon tax revenues are returned lump-sum to the household through the government transfers, T .

The first order condition for fossil energy yields the optimal level of energy intensity as a function of the capital-labor ratio,

$$\frac{E}{K} = \begin{cases} X & : \lambda > 0 \\ \left(\frac{K}{AL}\right)^{-\left(\frac{1-\alpha-\theta}{1-\theta}\right)} \left(\frac{\theta}{P(1+\tau)}\right)^{\frac{1}{1-\theta}} & : \lambda = 0 \end{cases} \quad (1.5)$$

where λ is the shadow value of the constraint. Higher after-tax energy prices reduce the firm's optimal energy intensity, since they incentivize firms to reduce fossil energy use. Additionally, the firm's optimal energy intensity is decreasing in the ratio of capital to effective labor, $\frac{K}{AL}$. I refer to this value as the capital-labor ratio. The complementarities in production imply that higher labor per capital (i.e., a lower capital-labor ratio) raises the marginal product of fossil energy per capital, increasing optimal energy intensity. Empirically, the average capital-labor ratio rises as countries develop. Therefore, equation (1.5) implies that optimal energy intensity falls with development.

There exists a threshold capital-labor ratio below which the upper bound on energy intensity binds. Equating optimal energy intensity (equation (1.5)) with its upper bound, X , yields this threshold

$$\Omega(\tau) \equiv X^{\frac{1-\theta}{1-\alpha-\theta}} \left(\frac{\theta}{P(1+\tau)}\right)^{\frac{1}{1-\alpha-\theta}}. \quad (1.6)$$

The threshold is decreasing in the tax, $\Omega'(\tau) < 0$, since higher energy prices reduce optimal energy intensity. In particular, the before-tax threshold (i.e., the threshold with a carbon tax of zero, $\Omega(0)$) is bigger than the after-tax threshold, $\Omega(0) > \Omega(\tau > 0)$.

If the capital-labor ratio exceeds the threshold, then the firm's input choices are unconstrained. However, below the threshold, the firm's energy intensity is a corner solution; all else constant, additional fossil energy will not increase firm output. Firms in this region would like to produce with cheaper and less energy efficient capital than what is available. Again, using the average capital-labor ratio to measure economic development, equation (1.6) implies that the constraint on energy intensity binds in countries with development levels below $\Omega(\tau)$. In Section 1.3, I explore the implications of this constraint for climate policy.

Throughout the rest of the paper, I view the capital-labor ratio as a measure of economic development. I consider at the model's implications for climate policies at different values of the capital-labor ratio. I interpret these results as indicative of how the effects of the policy vary with development. I use the terms country and firm interchangeably, since a country is inhabited by a representative firm with the country's capital-labor ratio.

1.2.3 Equilibrium

I define a decentralized competitive equilibrium. The individual state variable is asset holdings, a , and the aggregate state variable is the capital stock, K .

Given an exogenous energy price, P , a carbon tax rate, τ , a level of technology, A , a growth rate of technology, n , and factor prices $\{w, r\}$, a decentralized competitive equilibrium consists of agents' decision rules, $\{c, a'\}$, firms' production plans, $\{E, K, L\}$, and transfers, T , such that the following holds:

1. Given prices, policies, and transfers, the agent maximizes equation (3.3) with respect to equation (1.2) and the non-negativity constraints, $a \geq 0, c \geq 0$.
2. Firm demands for E , K , and L , satisfy:

$$P = \theta K^\alpha E^{\theta-1} (AL)^{1-\alpha-\theta} \quad r = \alpha K^{\alpha-1} E^\theta (AL)^{1-\alpha-\theta} - \delta \quad (1.7)$$

$$w = A(1 - \alpha - \theta)K^\alpha E^\theta (AL)^{-\alpha-\theta}$$

3. Transfers satisfy: $T = \tau PE$

4. Market clearing:

$$a = K \tag{1.8}$$

$$Y + (1 - \delta)K = K' + C + PE \tag{1.9}$$

1.3 Effects of Climate Policy

I explore the implications of the threshold for the effects of climate policy in countries at different stages of development. I consider two climate policies: (1) a uniform carbon tax for countries at all stages of development, and (2) a uniform abatement target for countries at all stages of development. To measure abatement in period t by a firm with capital-labor ratio j , I compare the firm's optimal energy use (and hence emissions) with and without the carbon tax in place. Abatement is then the percent reduction in the firm's emissions as a result of the carbon tax.

1.3.1 Uniform Carbon Tax

I analyze the effects of a uniform carbon tax, τ , on abatement in countries at different levels of development. The capital-labor threshold violates the standard result that a very small carbon tax can induce firms to undertake abatement at little or no distortionary cost to the economy. This result assumes the firm is at an interior optimum, and, thus, holds for capital-labor ratios above $\Omega(\tau)$. However, a very small carbon tax does not incentivize abatement in countries with capital-labor ratios below $\Omega(\tau)$.

Rearranging equation (1.5) and incorporating the upper bound on the energy intensity of capital yields the following condition for firm energy use,

$$E = \begin{cases} \frac{K}{X} & : \frac{K}{AL} \leq \Omega(\tau) \\ \left(\frac{AL}{K}\right)^{\frac{1-\alpha-\theta}{1-\theta}} \left(\frac{\theta}{P(1+\tau)}\right)^{\frac{1}{1-\theta}} K & : \frac{K}{AL} > \Omega(\tau) \end{cases} \tag{1.10}$$

For countries above the before-tax threshold, $\Omega(0)$, the carbon tax, τ , reduces energy use and hence emissions by a factor of $\left(\left(\frac{1}{1+\tau}\right)^{\frac{1}{1-\theta}} - 1\right)$. For countries below the after-tax threshold, $\Omega(\tau > 0)$, the tax leads to zero reduction in emissions. Since the threshold is decreasing in the carbon tax, $\Omega'(\tau) < 0$, there exists a set of countries that are below the before-tax threshold and above the after-tax threshold, $\Omega(\tau > 0) \leq \frac{K}{AL} \leq \Omega(0)$. The tax reduces emissions in this subset of countries by a factor greater than zero, but less than $\left(\left(\frac{1}{1+\tau}\right)^{\frac{1}{1-\theta}} - 1\right)$.

The percentage abatement from a given sized carbon tax is (weakly) increasing in the level of development. This result suggests that the welfare costs of a uniform carbon tax could be larger for developed countries than developing countries because it induces bigger distortions in behavior. In Section 2.5, I numerically solve a calibrated version of the model and quantify the welfare costs of a uniform carbon tax for countries at different stages of development.

1.3.2 Uniform Abatement Targets Across Countries

The previous section showed that the level of abatement from a uniform carbon tax increases with economic development. In this section, I consider the reverse exercise and analyze the size of the carbon tax necessary to achieve a uniform percentage abatement target in countries at different stages of development.

To be concrete, consider a carbon tax designed to achieve an abatement target of $b\%$ in every period. I calculate the size of the tax as function of the capital-labor ratio. In countries with development levels above the before-tax threshold, $\Omega(0)$, a $b\%$ reduction in emissions in period t , requires that the firm decrease its energy intensity of capital by $b\%$ from its optimal value with no carbon tax. Thus the carbon tax in the interior, τ_I , solves: $\left[(1 - b) \frac{E}{K}\right]_{\tau=0} = \left[\frac{E}{K}\right]_{\tau=\tau_I}$. In an interior solution, optimal energy intensity is given by equation (1.5). Therefore, the carbon tax required to attain this

reduction in energy intensity must solve the following equality in every period

$$(1-b) \left(\frac{K}{AL} \right)^{-\left(\frac{1-\alpha-\theta}{1-\theta}\right)} \left(\frac{\theta}{P} \right)^{\frac{1}{1-\theta}} = \left(\frac{K}{AL} \right)^{-\left(\frac{1-\alpha-\theta}{1-\theta}\right)} \left(\frac{\theta}{P(1+\tau_I)} \right)^{\frac{1}{1-\theta}}, \quad (1.11)$$

which yields

$$1 + \tau_I = \left(\frac{1}{1-b} \right)^{1-\theta}. \quad (1.12)$$

The tax is increasing in the stringency of the abatement target and is independent of the capital-labor ratio, provided that the capital-labor ratio is above $\Omega(0)$. This invariance results from the homothetic properties of the Cobb-Douglas production function.

Next, consider the tax necessary to induce firms below $\Omega(0)$ to reduce their emissions by b percent. Below the threshold, the constraint on energy intensity binds and the firm's energy choice is a corner solution. I divide this tax for firms in the corner, τ_C , into two components. Thus the first component of the tax, τ_{C_1} must move the firm from the corner to the interior. Under τ_{C_1} the optimal *unconstrained* energy intensity is X . Thus τ_{C_1} solves the following equality in every period

$$X = \left(\frac{K}{AL} \right)^{-\left(\frac{1-\alpha-\theta}{1-\theta}\right)} \left(\frac{\theta}{P(1+\tau_{C_1})} \right)^{\frac{1}{1-\theta}} \quad (1.13)$$

which yields

$$1 + \tau_{C_1} = X^{-(1-\theta)} \left(\frac{K}{AL} \right)^{-(1-\alpha-\theta)} \left(\frac{\theta}{P} \right). \quad (1.14)$$

Tax, τ_{C_1} , is decreasing in the capital-labor ratio, since optimal energy intensity falls with economic development. However, while this tax moves the firm from the corner to the interior, it does not incentivize emissions reduction. The second component of the tax, τ_{C_2} , must achieve the target abatement. This second component is the same as the tax in countries past $\Omega(0)$, $\tau_{C_2} = \tau_I$. Therefore, the total tax required to attain the abatement target in developing countries below the threshold is $1 + \tau_C = (1 + \tau_{C_1})(1 + \tau_{C_2})$.

Larger carbon taxes are required to reduce emissions in developing countries below $\Omega(0)$. Deadweight losses generally increase with the size of

the tax, suggesting that the welfare cost of achieving a given abatement target is substantially larger in these developing countries. In Sections 3.3 and 2.5, I calibrate the model and numerically solve for this welfare cost.

1.4 Calibration

I perform an illustrative calibration to demonstrate the welfare implications of the upper bound on the energy intensity of capital and the accompanying capital-labor threshold for climate policy. The model has eight parameters to be determined: $\{\alpha, \beta, \delta, n, P, \sigma, \theta, X\}$. I use US data on capital, fossil energy consumption and energy prices to calibrate the remaining parameters. Data on fossil energy use and prices are from the Energy Information Administration. All other data are from the Bureau of Economic Analysis. The time period is one year.

1.4.1 Preferences

I determine β to match the US capital-output ratio of 2.7 in an economy at its long-run steady state. This yields $\beta = 0.98$. Following Conesa et al. (2009), I use 0.5 for the inter-temporal elasticity of substitution, $\frac{1}{\sigma} = 0.5$.

1.4.2 Production

Following Golosov et al. (2014), the value of capital's share is $\alpha = 0.3$ and of energy's share is $\theta = 0.04$. I determine the depreciation rate, δ , to match the US investment-output ratio of 0.255 in an economy at its long-run steady state. This yields $\delta = 0.064$. I set the exogenous growth rate of AL equal to the average annual growth rate of US GDP in NIPA from 1949-2011, $n = 0.03$. This choice implies that developed economies grow at 3% per year along their long-run balanced growth path. However, note that developing economies will grow faster than 3% per year as they transition to their long-run balanced growth paths. Finally, I calibrate the relative price of fossil energy to match the energy intensity of US capital over the past ten years (2003-2013),

which equals 1.53 billion BTUs per Million 2014 dollars. This yields $P = 0.01$ million 2014 dollars per billion BTUs. This price calibration is consistent with measures of the composite energy price equal to an average of the prices of coal, natural gas, and petroleum weighted by the relative quantities of each fuel consumed in a given year.

1.4.3 Upper Bound on Energy Intensity

I do not calibrate a value for the upper bound on energy intensity, X . Instead, I consider a range of values equal to 1.5, 2, 2.5, and 3 times the average energy intensity of US capital over the past ten years, 2003-2013. This yields $X = \{2.3, 3.1, 3.8, 4.6\}$ billion BTUs per million dollars of capital. I set $X = 2.3$ in the main specification. Table 1.1 reports the calibrated parameter values.

Table 1.1: Calibrated Parameters

Parameter	Value	Target
Production		
Capital share: α	0.3	Golosov et al. (2014)
Energy share: θ	0.04	Golosov et al. (2014)
Energy price: P	0.01	Data
Growth rate: n	0.03	Data
Depreciation: δ	0.064	$\frac{I}{Y} = 0.25$
Upper bound: X	$\{2.3, 3.1, 3.8, 4.6\}$	Illustration
Preferences		
Discount rate: β	0.98	$\frac{K}{Y} = 2.7$
IES: σ	0.5	Conesa et al. (2009)

1.5 Results

1.5.1 Uniform Carbon Tax and Abatement Target

I perform three simulations: (1) a baseline economy with no carbon tax, (2) a uniform carbon tax, and (3) a uniform abatement target. I set the carbon tax equal to 35 dollars per ton CO₂, approximately 32 percent of the energy

price. In countries beyond the threshold, this size tax attains an annual level of abatement of 25.7 percent. Therefore, I choose the uniform abatement equal to 25.7 percent to facilitate comparison between the two policies. I implement the uniform abatement target by imposing a carbon tax that varies with the capital-labor ratio, and hence changes over time as the country develops. For every value of the capital-labor ratio, j , I choose the size of the tax such that energy use by a firm with capital-labor ratio j under the tax is 25.7 percent below the energy use by a firm with capital-labor ratio j and no carbon tax. In both simulations, I set the upper bound on energy intensity equal to 2.3.

Figure 1.1 plots the welfare and abatement implications from the uniform carbon tax and the uniform abatement target. The left panel of Figure 1.1 plots the consumption equivalent variation (CEV) of each policy. I define the CEV as the uniform percent change in consumption the agent would need in every period of the baseline such that he is indifferent between the baseline and the policy. This measure of CEV only includes the welfare cost of the climate policy because I do not model the climate benefits from the reduced emissions. The x-axis in the right panel of Figure 1.1 is the country's level of development in the year the policy is introduced. For example, the CEV for a capital-labor ratio equal to two measures the effect of the climate policy on a lifetime welfare for a country that implements the policy when its capital-labor ratio equals two.

The right panel of Figure 1.1 plots the annual percent reduction in emissions from the policy as a function of the capital-labor ratio. For example, the present reduction in emissions in the current period for a country whose current-period capital-labor ratio equals two is 25.7 percent. If, five years ago, this country had a capital-labor ratio of one, then its emissions reduction under the carbon tax would have been zero in that period.

The two policies are identical beyond the before-tax threshold, $\Omega(0)$, and, thus, the welfare implications are the same for countries who implement the policies when their development levels exceed the threshold. However, for developing countries who implement the policies before their capital-labor ratios reach the threshold, the welfare cost of the uniform carbon tax is smaller.

Since the upper bound binds in these countries, the carbon tax does not incentivize firms with capital-labor ratios in this portion of the state space to reduce emissions (left panel of Figure 1.1). Yet, the tax still reduces these countries' optimal *unconstrained* level of energy intensity, making it closer to the upper bound. This change makes the upper bound closer to the firm's optimal choice, and, thus reduces the distortions from the upper bound, thereby decreasing the welfare cost of the uniform carbon tax.

In contrast to the uniform carbon tax, the welfare cost of the uniform abatement target is much larger for developing countries whose capital-labor ratios are below the threshold when they implement the policy. As discussed in Section 1.3, much larger carbon taxes are required to attain a given abatement target in countries below the threshold. The tax must first move the firm from the corner to the interior and then incentivize the necessary reduction in emissions. Figure 1.2 plots the carbon tax necessary to attain the uniform abatement target as a function of the capital-labor ratio. The larger carbon taxes increase the deadweight loss from the policy, raising the welfare cost.

The notion of fairness across countries has been a key issue in many recent climate negotiations. These results suggest that neither a uniform carbon tax nor a uniform abatement target are “fair” in the sense that they equalize the welfare costs across countries. Instead, if policy makers strive to equalize welfare costs, then they should either assign developing countries higher carbon-tax rates or lower abatement targets.

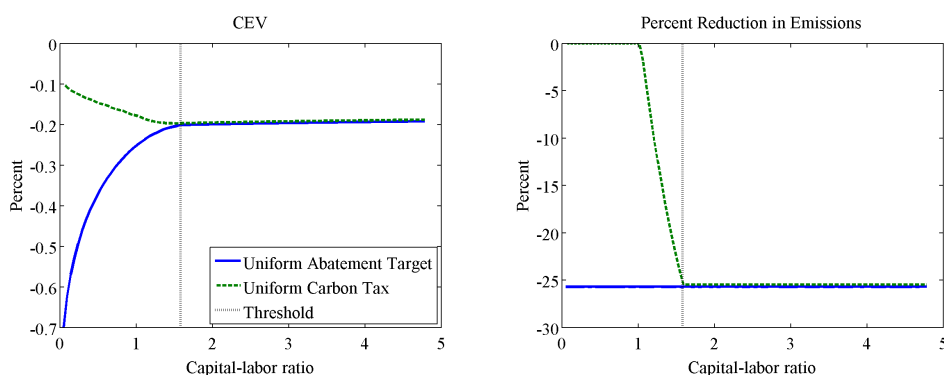


Figure 1.1: Welfare and Abatement Effects of the Tax and Target

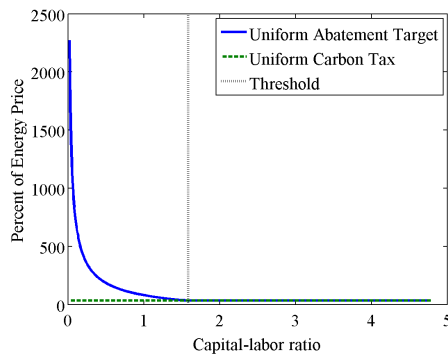


Figure 1.2: Size of the Carbon Tax

1.5.2 Developing Countries and the Threshold

In this section, I analyze the implications of the above results for some of the world’s largest, developing-country, carbon emitters: Brazil, China, India, Indonesia, and Mexico. Using 2011 data on capital, labor, and total factor productivity from the Penn World Tables, I calculate the capital-labor ratio in each country.

As discussed in the previous sections, the upper bound on energy intensity leads to a threshold capital-labor ratio. The effects of climate policy are different for countries above and below this threshold. Table 1.2 reports these thresholds for different values of the upper bound on energy intensity. I consider upper bounds equal to 1.5, 2, 2.5, and 3 times the average US energy intensity of capital from 2003-2013. Table 1.3 compares each country’s capital-labor ratio with the thresholds.

Table 1.2: Capital-Labor Thresholds for Different Upper Bounds

X	$1.5 \times \left(\frac{E}{K}\right)_{US}$	$2 \times \left(\frac{E}{K}\right)_{US}$	$2.5 \times \left(\frac{E}{K}\right)_{US}$	$3 \times \left(\frac{E}{K}\right)_{US}$
$\Omega(0)$	2.4	1.6	1.1	0.9

Table 1.3: Capital-Labor Threshold Comparison

Country	$\frac{K}{AL}$	Exceeds?			
		$\Omega = 2.4$	$\Omega = 1.6$	$\Omega = 1.1$	$\Omega = 0.9$
Brazil	2.22	No	Yes	Yes	Yes
China	2.32	No	Yes	Yes	Yes
India	0.68	No	No	No	No
Indonesia	1.16	No	No	Yes	Yes
Mexico	1.77	No	Yes	Yes	Yes
US	4.34	Yes	Yes	Yes	Yes

If the upper bound is relatively tight, equal to 1.5 times the US energy intensity of capital, then all the countries in Table 1.3 except the US are below the threshold. This result suggests that uniform abatement targets would be considerably more costly in these countries than in developed countries like the US and the EU. Moreover, a uniform carbon tax would be considerably less costly in these countries, but it would also attain a smaller percentage reduction in emissions.

If the upper bound is less tight, greater than two times the US energy intensity of capital, then Brazil, China, and Mexico are all above the threshold. In contrast, India's capital-labor ratio is so small that the upper bound would have to be greater than three times the US energy intensity of capital for India to be above the threshold.

1.5.3 Generalization: Exogenous Technology and Fossil Energy Price

Factor augmenting technical progress is exogenous. However, if technology is endogenous and innovation can be directed towards reductions in energy intensity, then a similar analysis could imply that there exists a technology threshold below which the firm does not invest innovation resources in technology that reduces energy intensity. This additional threshold would further support the result that uniform abatement targets are more distortionary in developing countries with capital-labor ratios below a threshold value.

The price of fossil energy is exogenous and, thus, I abstract from the general equilibrium effects of a carbon tax on energy prices. If energy prices are endogenous, then a carbon tax should reduce the before tax energy price since it reduces the demand for fossil energy. This effect implies a smaller after-tax energy price. The threshold is inversely proportional to the after-tax price of fossil energy and, thus, these general equilibrium effects should increase the threshold. Therefore, I view the threshold with exogenous energy prices as a lower bound.

1.6 Conclusion

This paper develops a macroeconomic model in which firms choose capital, labor, and fossil energy to maximize profits subject to an upper bound on the energy intensity of capital. There are three key results. First, a firm's incentives to economize on energy use increase with its capital-labor ratio, implying that the upper bound on the energy intensity of capital is more likely to bind in developing countries.

Second, much larger carbon taxes are necessary to achieve a given abatement target when this upper bound binds than when it does not. The larger taxes create more deadweight loss and, thus, the welfare cost of a uniform abatement target is larger in developing countries for which the upper bound on energy intensity binds. Similarly, a uniform carbon tax results in a smaller reduction in emissions when the upper bound binds. Thus, both the level of abatement and welfare cost of a uniform carbon tax are smaller in developing countries in which the upper bound on energy intensity binds.

Third, the calibration implies that for most plausible values of the upper bound, the constraints on energy intensity bind in India but not in China. Therefore, the model suggests that the welfare cost of a uniform abatement target would be considerably larger in India, while the welfare cost of uniform carbon tax would be considerably smaller.

1.7 Appendix

A frequent critique of the Cobb-Douglas specification is the substitutability between capital and energy. After all, more capital in the form of buildings and machines requires more energy, not less. In this section, I show that the Cobb-Douglas production structure with an upper bound on energy intensity is equivalent to a production function that directly models the substitutability between capital and fossil energy, through improved fossil efficiency.

Consider the following production structure. The representative firm combines capital, labor, and fossil energy to maximize profits. In each period, the firm divides its total capital stock, K , between a productive and a green use. Productive capital is used to generate output. Green capital reduces the firm's fossil energy expenses by increasing its fossil efficiency. Increases in fossil efficiency include both improving the energy efficiency of machines (i.e., cars with better fuel economy) and fuel switching (i.e., using wind turbines instead of coal to produce electricity). Let g be the fraction of total capital that is green.

For a given amount of green capital, firm production is Leontief in a productive-capital-labor composite and fossil energy. There are two exogenous types of technological progress: factor augmenting technological progress, A , and a fossil energy saving or "green" technological progress, A_g . This production function is analogous to the one estimated in Hassler et al. (2012) with the addition of green capital. The functional form is

$$Y = \min \left[A((1-g)K)^\alpha L^{1-\alpha}, \left(\frac{A((1-g)K)^\alpha L^{1-\alpha}}{\Theta(g, K)} \right) A_g E \right].$$

Equating the two arguments of the min function implies that the total fossil energy required to produce $A((1-g)K)^\alpha L^{1-\alpha}$ units of output is $\frac{\Theta(\cdot)}{A_g}$. This fossil energy requirement is endogenously determined by the firm's investments in green and productive capital, and exogenously determined by green technological progress. I refer to $\Theta(g, K)$ as the energy requirement function.

The functional form for the energy requirement should satisfy three conditions to capture the key features of energy use. First, the energy re-

quirement should be increasing in productive capital and decreasing in green capital. Second, there should be diminishing returns to green capital, implying increasing marginal costs of emissions abatement. Third, the energy requirement should be homogenous of degree one in K . This homogeneity implies that if the total capital stock doubles and the fractions of productive and green capital are unchanged, then fossil energy use will double. A simple function form that satisfies these three conditions is $\Theta(g, K) = K(1 - g)^{1+\phi}$.

Given this functional form for the energy requirement, the energy intensity of productive capital is

$$\frac{E}{(1 - g)K} = \frac{(1 - g)^\phi}{A_g}. \quad (1.15)$$

Higher levels of green capital reduce the energy intensity of capital, reducing the energy requirement. The endogenous energy requirement explicitly models the substitutability between productive capital and energy through investment in green capital.

The optimization problem for the firm with this production structure is

$$\max_{K, g, L} A((1 - g)K)^\alpha L^{1-\alpha} - P(1 + \tau)E - RK - wL \quad (1.16)$$

subject to the energy requirement

$$E = \frac{K(1 - g)^{1+\phi}}{A_g}, \quad (1.17)$$

and the constraint $0 \leq g \leq 1$. This formulation is equivalent to the Cobb-Douglas production function with an upper bound on energy intensity. Solving the fossil energy requirement (equation (1.17)) for $(1 - g)$ and substituting this expression into the firm maximization problem (equation (1.16)) and constraints yields

$$\max_{E, K, L} AA_g^{\frac{\alpha}{1+\phi}} E^{\frac{\alpha}{1+\phi}} K^{\alpha - \frac{\alpha}{1+\phi}} L^{1-\alpha} - P(1 + \tau)E - RK - wL$$

subject to $\frac{E}{K} \leq A_g$. This optimization problem is isomorphic to the production function and upper bound introduced in Section 1.2.

1.8 Acknowledgement

Chapter 1, in full, has been submitted for publication of the material as it may appear in the Journal of Development Economics. Fried, Stephanie.

Chapter 2

Climate Policy and Innovation: A Macroeconomic Analysis

2.1 Introduction

A carbon tax can induce innovation in green technologies. Over time, these technological advances lower the cost of reducing carbon emissions. However, how much innovation responds and the magnitudes of the accompanying effects on energy prices, production, and carbon emissions remain open questions. Much of the macroeconomic literature studying climate policy assumes that innovation is exogenous (e.g., Golosov et al. (2014); Krusell and Smith (2009); Hassler and Krusell (2012); and Nordhaus (2008)) while much of the environmental literature has concentrated on endogenous innovation in partial equilibrium (see Popp et al. (2009) for an overview). This paper combines these two approaches by studying a carbon tax in a general equilibrium model with endogenous innovation. I use this model to analyze the dynamic effects of a carbon tax and to quantify the importance of endogenous innovation for climate policy outcomes. I find that the tax induces large movements in innovation that have considerable effects on energy prices, production, and other macroeconomic aggregates. Moreover, abstracting from endogenous innovation –and modeling technological progress as exogenous– results in a substantial overestimation of the carbon tax necessary to attain a given reduction in emissions.

The central contribution of this paper is to quantify the interaction between endogenous innovation and climate policy in a dynamic, general equilibrium framework. The model builds on the literature on directed technical change and climate (e.g., Acemoglu, Aghion, Burzysten, and Hemous (2012) (AABH), Acemoglu, Akcigit, Hanley, and Kerr (2014); Hart (2012); Hassler, Krusell and Olovsson (2012); Hemous (2014); Smulders and de Nooij (2003)). This earlier work is mainly theoretical, and the models are generally not designed for quantitative analysis.¹ I complement this earlier work by quantifying the importance of endogenous innovation for climate policy outcomes.

¹For example, AABH state that their “objective is not to provide a comprehensive quantitative evaluation” (AABH, p. 154). One exception is Acemoglu et al. (2014), which is a quantitative paper focused on the relative roles of carbon taxes and subsidies to green energy research in the structure of optimal climate policy.

In many of the existing models, such as AABH, innovation occurs in only one energy sector (i.e., fossil or green) on the long-run balanced growth path. However, US data on fossil and green innovation show positive and substantial amounts of innovation in both of these sectors since the 1970s. To match this empirical fact, I incorporate technology spillovers across the different sectors. The spillovers imply that technology developed for one sector increases the productivity of innovation in the other sectors. If the spillovers are sufficiently strong, then the unique balanced growth path is an interior solution in which innovation occurs in both the fossil and green energy sectors. This result is similar to results in Acemoglu (2002) and Hart (2012), which show the importance of knowledge spillovers for the stability of interior long-run balanced growth paths in models of directed technical change.

I develop a novel calibration strategy using the energy price increases triggered by oil shocks and the accompanying changes in energy production and innovation. It is important for the model to capture the empirical relationships among energy prices, production, and innovation. These are key links because many climate policies, including a carbon tax and a cap and trade system, create incentives to reduce fossil energy consumption through changes in energy prices. The oil shocks provide empirical evidence of the response of energy innovation and production to an aggregate increase in the energy price. This variation is particularly useful for disciplining the parameter values since economy-wide historical examples of climate policies are scarce.

I perform two exercises to fully explore the interactions between endogenous innovation and climate policy. First, to evaluate the dynamic effects of climate policy with endogenous innovation, I introduce a constant carbon tax into my benchmark model with endogenous innovation. I compare the movements in technology, relative prices, and other macroeconomic aggregates in the model with the tax to their values in the model without the tax. Next, to quantify the importance of endogenous innovation for climate policy evaluation, I introduce a carbon tax into an alternative model with the endogenous innovation channel shut down. I refer to this model as the exogenous-innovation model because innovation cannot respond to the tax.

Comparing the effects of the tax in the endogenous and exogenous-innovation models allows me to quantify the interaction between endogenous innovation and climate policy. In both models, I choose the size of the carbon tax to achieve a 30-percent reduction in emissions by 2030, one version of the emissions target that the US government set with the announcement of the Clean Power Plan in June 2014.

There are two main findings. First, comparing the endogenous-innovation model with and without the tax, I find that the tax induces substantial movements in innovation, energy prices, and other macroeconomic aggregates. For example, by 2030, the tax causes green innovation to be 50 percent higher and fossil innovation to be 60 percent lower than what they would have been without the tax. These movements in innovation are accompanied by substantial changes in relative prices. In the model with the tax, the relative price of green compared to fossil energy is 7 percent lower in 2030 and 17 percent lower on the new balanced growth path than in the model without the tax.

Second, comparing the results from the tax in the exogenous- and endogenous-innovation models, I find that endogenous innovation has substantial implications for the effectiveness of the carbon tax and for the relative price of green energy. The carbon taxes required to achieve the Power Plan target in the exogenous- and endogenous-innovation models are \$30 and \$24 per ton of CO₂, respectively. Endogenous innovation reduces the carbon tax by close to 20 percent because it increases incentives for carbon abatement. The intuition for this result is that regardless of whether innovation is endogenous, the carbon tax operates through prices to shift demand from fossil to green energy, reducing emissions. However, when innovation is endogenous, this shift in demand spurs green innovation. Over time, the increase in green innovation reduces the marginal cost of producing green energy, lowering its equilibrium price and creating stronger incentives for agents to switch from fossil to green. Thus, endogenous innovation amplifies the price incentives created by the carbon tax, implying that the emissions target can be achieved with a 20 percent smaller tax.

The standard equivalence between carbon taxes and carbon permits

holds in this model. An alternative interpretation of these results is that endogenous innovation reduces the predicted carbon permit price from an equivalent cap and trade system by close to 20 percent. This interpretation is consistent with the United States' experience using tradable permits to reduce acid rain in the 1990s. Initial forecasts of the permit price were orders of magnitude higher than the realized prices, partly because of technological advances in low-sulfur coal mining, fuel mixing, and scrubber installation and performance (Sandor et al. (2014)).

Additionally, I find that endogenous innovation has offsetting effects on the welfare costs of attaining a given abatement target. The carbon tax is smaller when innovation is endogenous, and, hence, the accompanying distortionary cost is smaller. However, the shift in innovation from fossil to green energy in response to the tax reduces the aggregate growth rate along the transition path to a new long-run equilibrium, raising the welfare cost of the policy. As a result, the net effect of endogenous innovation on the welfare costs of the carbon tax is small. In particular, the consumption equivalent variation (CEV) of the tax is -0.5 percent in the endogenous-innovation model and -0.6 percent in the exogenous-innovation model.

The paper proceeds as follows: Sections 2 and 3 describe the model. Section 4 discusses the oil shocks and the calibration strategy. Section 5 presents the results and Section 6 concludes.

2.2 Model

I adapt the standard directed technical change framework to a setting with fossil, green, and non-energy intermediate inputs and oil shocks. Fossil energy refers to energy derived from coal, oil, or natural gas. Green energy refers to energy derived from any non-carbon energy source. This category includes renewable energy, such as wind and solar, as well as nuclear energy and energy savings from improved fossil energy efficiency, such as better insulation, higher fuel economy, etc.

2.2.1 Final good

The unique final consumption good, Y , is produced competitively from energy, E , and non-energy inputs, N , according to the CES production function

$$Y_t = \left(\delta_y E_t^{\frac{\varepsilon_y - 1}{\varepsilon_y}} + (1 - \delta_y) N_t^{\frac{\varepsilon_y - 1}{\varepsilon_y}} \right)^{\frac{\varepsilon_y}{\varepsilon_y - 1}}, \quad (2.1)$$

where $\varepsilon_y < 1$ is the elasticity of substitution between the energy and non-energy inputs. Energy is a nested CES composite of fossil energy, green energy, and foreign oil,

$$E_t = \left(\tilde{F}_t^{\frac{\varepsilon_e - 1}{\varepsilon_e}} + G_t^{\frac{\varepsilon_e - 1}{\varepsilon_e}} \right)^{\frac{\varepsilon_e}{\varepsilon_e - 1}} \quad \text{and} \quad \tilde{F}_t = \left(\delta_{\tilde{F}} F_t^{\frac{\varepsilon_f - 1}{\varepsilon_f}} + (1 - \delta_{\tilde{F}}) (O_t^*)^{\frac{\varepsilon_f - 1}{\varepsilon_f}} \right)^{\frac{\varepsilon_f}{\varepsilon_f - 1}}. \quad (2.2)$$

Parameter $1 < \varepsilon_f < \infty$ denotes the elasticity of substitution between fossil energy (produced domestically), F , and foreign oil imports, O^* . Since fossil energy is a mixture of coal, oil, and natural gas, foreign oil and fossil energy are not perfect substitutes. Parameter $\varepsilon_e > 1$ is the elasticity of substitution between green energy and the CES composite of domestic fossil energy and foreign oil.

Following AABH and Hemous (2014), I do not include a distribution parameter between green energy and the CES composite of fossil energy and foreign oil. Differences in the quantities of fossil and green energy result exclusively from differences in their relative prices and not from an underlying asymmetry in the production function. Both fossil and green energy contribute equally at the margin to the energy composite, E , when relative prices are the same. For example, a boiler that uses one less ton of coal (higher G) is equivalent to additional coal (higher \tilde{F}). However, the finite elasticity of substitution implies that there is some heterogeneity in the production process, so agents do not substitute indefinitely into either \tilde{F} or G . The final good is the numeraire.

2.2.2 Intermediate inputs

Fossil energy, green energy, and non-energy intermediate inputs are produced competitively and sold at market prices to the final good producer. The production functions are constant returns to scale in labor, L_j , and a unit mass of sector-specific machines, each indexed by i , X_{ji} where $j \in \{f, g, n\}$,

$$\begin{aligned} F_t &= L_{ft}^{1-\alpha_f} \int_0^1 X_{fit}^{\alpha_f} A_{fit}^{1-\alpha_f} di, & G_t &= L_{gt}^{1-\alpha_g} \int_0^1 X_{git}^{\alpha_g} A_{git}^{1-\alpha_g} di \\ N_t &= L_{nt}^{1-\alpha_n} \int_0^1 X_{nit}^{\alpha_n} A_{nit}^{1-\alpha_n} di. \end{aligned} \quad (2.3)$$

Variable A_{ji} denotes the factor-augmenting technology embodied in machine X_{ji} , and α_j is the factor share of machines in sector J . A representative intermediate-goods producer chooses machines and labor to maximize profits, taking prices as given. Labor market clearing requires that $L_{ft} + L_{gt} + L_{nt} \leq L$, where L is the fixed exogenous supply of workers in the economy.

2.2.3 Machine producers

There is a unit mass of machine producers in each of the three sectors. The machine producers sell their machines to the intermediate-goods producers in their specific sectors. Each machine embodies technology. A machine producer can hire scientists to innovate on the embodied technology. A machine costs one unit of the final good to produce, regardless of the sector or the level of technology embodied in the machine. The market for scientists is competitive, and the machine producer must pay the scientists he hires the market wage, w_{sj} , where $j \in \{f, g, n\}$. However, the market for machines is monopolistically competitive, and the machine producers earn positive profits from the sale of their machines.

The evolution of technology for machine producer i in each of the sectors

F , G , and N is

$$\begin{aligned} A_{fit} &= A_{ft-1} \left(1 + \gamma \left(\frac{S_{fit}}{\rho_f} \right)^\eta \left(\frac{A_{t-1}}{A_{ft-1}} \right)^\phi \right), \\ A_{git} &= A_{gt-1} \left(1 + \gamma \left(\frac{S_{git}}{\rho_g} \right)^\eta \left(\frac{A_{t-1}}{A_{gt-1}} \right)^\phi \right), \\ A_{nit} &= A_{nt-1} \left(1 + \gamma \left(\frac{S_{nit}}{\rho_n} \right)^\eta \left(\frac{A_{t-1}}{A_{nt-1}} \right)^\phi \right). \end{aligned} \quad (2.4)$$

Scientists affect the growth rate of the machine producer's technology. Hence, there is path dependence in innovation; higher existing technology in a sector increases the marginal product of research in that sector.

Parameter $\eta \in (0, 1)$ implies that there are diminishing returns to scientific research within a given period. This modeling choice captures the “stepping on toes” effect discussed in the endogenous growth literature, where scientists are more likely to duplicate discoveries within a given period (Jones and Williams (1998)). Parameter γ measures the efficiency with which scientists produce new ideas.

Parameters (ρ_f, ρ_g, ρ_n) adjust for differences in sector diversity. Specifically, ρ_f is the number of processes on which a scientist can innovate in fossil energy. Fossil energy scientists divide their time equally among all available processes (and likewise for green and non-energy scientists). Accounting for differences in sector diversity is particularly important because there are diminishing returns to innovation in each sector. Without a diversity adjustment, the marginal product of a non-energy scientist is much lower than that of an energy scientist simply because there are more non-energy scientists.

Variable A_j , $j \in \{f, g, n\}$ denotes the aggregate (average) level of technology in sector J :

$$A_f = \int_0^1 A_{fi} di, \quad A_g = \int_0^1 A_{gi} di, \quad A_n = \int_0^1 A_{ni} di. \quad (2.5)$$

Variable A denotes the aggregate level of technology in the economy. I define aggregate technology as the average of the technologies in each sector weighted

by the number of processes:

$$A = \frac{\rho_f A_f + \rho_g A_g + \rho_n A_n}{\rho_f + \rho_g + \rho_n}. \quad (2.6)$$

The TFP catchup ratios $\left(\frac{A_{t-1}}{A_{ft-1}}\right)^\phi$, $\left(\frac{A_{t-1}}{A_{gt-1}}\right)^\phi$, and $\left(\frac{A_{t-1}}{A_{nt-1}}\right)^\phi$ incorporate technology spillovers across the different sectors.² Parameter $\phi \in (0, 1)$ determines the strength of these spillovers. The intuition for the spillovers and their implications for the long-run behavior of the model are discussed in Section 2.3.

In addition to the across-sector technology spillovers, the technology accumulation process also incorporates technology spillovers within a sector after one period. The technology of machine producer i in sector J tomorrow depends on the level of knowledge in sector J today and on any new ideas that machine producer i accumulates. Hence, a given machine producer's discoveries are secret for one period. After the period is over, other machine producers in his sector observe his discoveries and can incorporate them into their own innovation processes. This modeling choice is empirically reasonable, provided that the period is sufficiently long and is in line with similar assumptions made in the literature. I discuss evidence of these within-sector spillovers in fossil and green energy and an appropriate period length in Section 3.3.

Each machine producer chooses the quantity of machines, the machine price, and the number of scientists that will maximize his profits. He takes existing levels of technology as given. Scientist market clearing requires that $S_{ft} + S_{gt} + S_{nt} \leq S$, where S is the fixed exogenous supply of scientists in the economy and S_{jt} is the number of scientists in sector J .

2.2.4 A carbon tax and an oil shock

Carbon emissions, \mathcal{E} , accumulate from the use of fossil energy and foreign oil,

$$\mathcal{E}_t = \omega_f F_t + \omega_o O_t^*.$$

²To account for differences in sector diversity, the TFP catchup terms also incorporate spillovers across processes within a sector.

Parameter ω_f and ω_o convert fossil energy and foreign oil into carbon emissions.

A carbon tax places a price on the externality, carbon. Thus, the tax, τ , is a tax per unit of carbon consumed, which is independent of the price. The tax increases the price of fossil energy from P_{ft} to $P_{ft} + \tau_f$ and the price of foreign oil from P_{ot}^* to $P_{ot}^* + \tau_o$, where $\tau_f = \tau \times (\text{carbon content of fossil energy})$ and $\tau_o = \tau \times (\text{carbon content of foreign oil})$. An oil shock is an exogenous increase in the price of foreign oil, P_{ot}^* .

2.2.5 Household

The representative household is inhabited by a unit mass of machine producers in each sector, L workers, and S scientists. The utility function is $U(C) = \frac{C^{1-\theta}}{1-\theta}$, where $\frac{1}{\theta}$ is the intertemporal elasticity of substitution. There is no mechanism through which the household can save, and, thus, it consumes its income. The budget constraint is

$$C_t = w_{lft}L_{ft} + w_{lgt}L_{gt} + w_{lnt}L_{nt} + w_{sft}S_{ft} + w_{sgt}S_{gt} + w_{snt}S_{nt} + \int_0^1 (\pi_{fit} + \pi_{git} + \pi_{nit}) di + T_t. \quad (2.7)$$

Variable π_{ji} denotes profits to machine producer i in sector $J \in \{F, G, N\}$, and T_t is lump sum transfers from a carbon tax.

The aggregate resource constraint implies that the final good can be consumed, converted to machines, or used to purchase foreign oil:

$$Y_t = C_t + \int_0^1 (X_{fit} + X_{git} + X_{nit}) di + P_{ot}^* O_t^*. \quad (2.8)$$

2.2.6 Equilibrium

A *decentralized equilibrium* is given by sequences of wages $(w_{lft}, w_{lgt}, w_{lnt}, w_{sft}, w_{sgt}, w_{snt})$, prices for machines $(P_{fit}^x, P_{git}^x, P_{nit}^x)$, prices for intermediates (P_{ft}, P_{gt}, P_{nt}) , demands for machines $(X_{fit}^d, X_{git}^d, X_{nit}^d)$, demands for intermediates (F_t^d, G_t^d, N_t^d) , demands for labor $(L_{ft}^d, L_{gt}^d, L_{nt}^d)$, demands for scientists $(S_{ft}^d, S_{gt}^d, S_{nt}^d)$, supplies of machines $(X_{fit}^s, X_{git}^s, X_{nit}^s)$, supplies of intermediates, $(F_t^s, G_t^s, N_t^s, (O_t^*)^s)$, supplies of labor $(L_{ft}^s, L_{gt}^s, L_{nt}^s)$, and supplies of scientists $(S_{ft}^s, S_{gt}^s, S_{nt}^s)$ such that:

1. Agents optimize: $(P_{jit}^x, S_{jit}^d, X_{jit}^s)$ maximize the machine producers' profits,
 $j \in \{f, g, n\}$; $(X_{fit}^d, X_{git}^d, X_{nit}^d, L_{ft}^d, L_{gt}^d, L_{nt}^d)$ maximize intermediate-goods producers' profits; $(F_t^d, G_t^d, N_t^d, (O_t^*)^d)$ maximize final-good producer's profits; $(L_{ft}^s, L_{gt}^s, L_{nt}^s, S_{ft}^s, S_{gt}^s, S_{nt}^s)$ maximize the household's utility.
2. Markets clear: $(P_{fit}^x, P_{git}^x, P_{nit}^x)$ clear the machine producer markets;
 (P_{ft}, P_{gt}, P_{nt}) clear the intermediate input markets;
 $(w_{lft}, w_{lgt}, w_{lnt}, w_{sft}, w_{sgt}, w_{snt})$ clear the labor and scientist markets.

2.3 Discussion

The model is designed to endogenize the innovation response to energy price increases triggered by carbon taxes and oil shocks. Both oil shocks and carbon taxes enter the model through the final-good producer's demand for energy inputs. The optimization problem of the representative final-good producer is

$$\max_{F_t, G_t, N_t, O_t^*} \{Y_t - (P_{ft} + \tau_f)F_t - P_{gt}G_t - (P_{ot}^* + \tau_o)O_t^* - P_{nt}N_t\}. \quad (2.9)$$

A carbon tax applies to both the fossil energy and foreign oil prices, while an oil shock only applies to the foreign oil price. Thus, an oil shock increases the demand for both fossil and green energy, while a carbon tax increases only the demand for green energy.

Oil shocks and carbon taxes affect machine production and the accompanying innovation incentives through their general equilibrium effects on energy production and prices. The first-order conditions for the intermediate-goods producers imply the following demand curves for machines (see Appendix A for the full derivation):

$$\begin{aligned} X_{fit} &= \left(\frac{\alpha_f P_{ft}}{P_{fit}^x} \right)^{\frac{1}{1-\alpha_f}} A_{fit} L_{ft}, & X_{git} &= \left(\frac{\alpha_g P_{gt}}{P_{git}^x} \right)^{\frac{1}{1-\alpha_g}} A_{git} L_{gt} \\ X_{nit} &= \left(\frac{\alpha_n P_{nt}}{P_{nit}^x} \right)^{\frac{1}{1-\alpha_n}} A_{nit} L_{nt}. \end{aligned} \quad (2.10)$$

Fossil machine demand is a downward-sloping function of the machine price, P_{fi}^x . Increases in fossil energy demand from an oil shock abroad increase the fossil energy price and market size (measured by fossil employment, L_f), which shifts demand for fossil machines outward. Similarly, decreases in fossil energy demand from a carbon tax reduce fossil energy market size, which shifts demand for fossil machines inward. Analogous mechanisms apply to green energy machines.

Higher technology also shifts machine demand outward. Thus, the machine producer benefits from innovation because he can sell a given quantity of machines at a higher price. The machine producer pays scientists their market wage, and the new technology they develop is embodied in all of the machines he produces. He then sells each machine at a positive, optimally chosen, markup over marginal cost. Therefore, increases in demand for machines increase the machine producer's return from innovation.

The first-order conditions for the machine producer imply the marginal return to innovation in each sector (see Appendix A for the full derivation):

$$\begin{aligned}
 w_{sft} &= \frac{\eta\gamma\alpha_f A_{ft-1} \left(\frac{S_{ft}}{\rho_f}\right)^\eta \left(\frac{A_{t-1}}{A_{ft-1}}\right)^\phi P_{ft} F_t}{\left(\frac{1}{1-\alpha_f}\right) S_{ft} A_{ft}}, & (2.11) \\
 w_{sgt} &= \frac{\eta\gamma\alpha_g A_{gt-1} \left(\frac{S_{gt}}{\rho_g}\right)^\eta \left(\frac{A_{t-1}}{A_{gt-1}}\right)^\phi P_{gt} G_t}{\left(\frac{1}{1-\alpha_g}\right) S_{gt} A_{gt}} \\
 w_{snt} &= \frac{\eta\gamma\alpha_n A_{nt-1} \left(\frac{S_{nt}}{\rho_n}\right)^\eta \left(\frac{A_{t-1}}{A_{nt-1}}\right)^\phi P_{nt} N_t}{\left(\frac{1}{1-\alpha_n}\right) S_{nt} A_{nt}}
 \end{aligned}$$

I discuss the components of the return to fossil innovation. Analogous decompositions apply in the green and non-energy sectors. The marginal return to innovation is increasing in the value of fossil energy production, $P_{ft}F_t$. Increases in either the fossil energy price or quantity raise the demand for fossil machines and the accompanying returns to innovation.

Additionally, the marginal return is increasing in the efficiency with which scientists produce new ideas, γ ; the degree of diminishing returns, η ; and the machine share, α_f . A higher machine share increases machine demand.

The marginal return is decreasing in the price elasticity of machine demand, $\frac{1}{1-\alpha_f}$. Higher demand elasticity reduces the machine producer's markup and the accompanying returns to innovation.

Finally, the technology accumulation process incorporates both path dependence and across-sector technology spillovers. These two drivers of innovation are captured in the marginal return by the term, A_{ft-1} and the catchup ratio, $\left(\frac{A_{t-1}}{A_{ft-1}}\right)^\phi$. Path dependence implies that higher existing technology increases the marginal return to innovation. However, the catchup effect raises the marginal productivity of innovation in sectors that are behind the frontier. The intuition is that if sector J is relatively backward, then there are many ideas from other sectors that have not yet been applied in sector J . This “low-hanging fruit” increases the productivity of research in sector J .

Parameter ϕ measures the strength of the productivity catchup effect. If $\phi = 0$, there are no across-sector spillovers and there is full path dependence, as in AABH and Hemous (2014). Since fossil and green energy are gross substitutes (i.e., $\varepsilon_e > 1$), this strong path dependence implies that innovation in one energy sector raises the relative marginal product of innovation in that sector by so much that the only stable balanced growth paths are corner solutions in which innovation occurs in only one form of energy.³ In contrast, if $\phi = 1$, the marginal return to innovation in a sector is independent of the previous level of technology in that sector, and, hence, there is no path dependence. In this case, there exists a stable interior balanced growth path in which innovation occurs in both forms of energy. The value of ϕ determines the relative strengths of the path dependence and across-sector spillover channels and, thus, governs the stability of the interior balanced growth path. Proposition 1 below proves this result in an analytically tractable version of the model. This result is similar to results in Acemoglu (2002) and Hart (2012), which show the importance of technology spillovers for the stability of interior balanced growth paths in models of directed technical change.

³Innovation will also occur in the non-energy sector since the non-energy and energy sectors are gross complements. The diminishing returns to innovation imply that the corner solution balanced growth paths only exist asymptotically.

Proposition 1. *When the factor shares are equal across the sectors ($\alpha_f = \alpha_g = \alpha_n \equiv \alpha$) and imports of foreign oil are zero, there exists a unique interior balanced growth path with strictly positive innovation in every sector ($S_g > 0, S_f > 0$, and $S_n > 0$). This balanced growth path is stable if and only if $\frac{\phi}{\eta} > (\varepsilon_e - 1)(1 - \alpha)$.*

Proof. See Appendix B. \square

Note that if there are no cross-sector technology spillovers ($\phi = 0$), then the interior balanced growth path is stable if and only if green and fossil energy are gross complements (i.e., $\varepsilon_e \leq 1$) instead of gross substitutes. This is the standard result (e.g., Acemoglu (2002)) that the stability of the interior long-run equilibrium hinges on the elasticity of substitution. However, the potential for technology spillovers across sectors implies that the interior balanced growth path can be stable even when the energy inputs are gross substitutes.

To develop the intuition for Proposition 1, consider the following expression for the relative returns to innovation in fossil and green energy in the special case with equal factor shares (derived in Appendix B)

$$\frac{w_{sgt}}{w_{sft}} = \left(\frac{\rho_f}{\rho_g}\right)^\eta \underbrace{\left(\frac{P_{gt}}{P_{ft}}\right)^{\frac{1}{1-\alpha}}}_{\text{Price}} \underbrace{\left(\frac{L_{gt}}{L_{ft}}\right)}_{\text{Market size}} \underbrace{\left(\frac{A_{gt-1}}{A_{ft-1}}\right)^{1-\phi}}_{\text{Direct productivity}} \left(\frac{S_{ft}}{S_{gt}}\right)^{1-\eta}. \quad (2.12)$$

As discussed in AABH, equation (2.12) decomposes the returns to innovation into a price effect, $\left(\frac{P_{gt}}{P_{ft}}\right)^{\frac{1}{1-\alpha}}$; a market size effect, $\left(\frac{L_{gt}}{L_{ft}}\right)$; and a direct productivity effect, $\left(\frac{A_{gt-1}}{A_{ft-1}}\right)^{1-\phi}$. Increases in prices, market size, and existing technology in a given sector increase the profitability of innovation in that sector. However, unlike in AABH, the inclusion of technology spillovers ($\phi > 0$) dampens the direct productivity effect.

I derive the price and market size effects in terms of the relative levels of technology in each sector (see Appendix B for the derivation):

$$\frac{P_{gt}}{P_{ft}} = \left(\frac{A_{gt}}{A_{ft}}\right)^{-(1-\alpha)} \quad \text{and} \quad \frac{L_{gt}}{L_{ft}} = \left(\frac{A_{gt}}{A_{ft}}\right)^{(\varepsilon_e - 1)(1-\alpha)} \quad (2.13)$$

The price effect implies that relative prices are inversely proportional to relative technology. Since increases in relative technology decrease relative prices, the price effect is a force towards a stable interior balanced growth path. In contrast, the market size effect implies that the relative market size is directly proportional to the relative technology when the two sectors are gross substitutes and, thus, works against stability. Like the market size effect, the direct productivity effect is proportional to the relative technology and, thus, also works against stability. Hence, the interior balanced growth path is stable if the price effect dominates both the market size and direct productivity effects. The magnitude of the direct productivity depends on the size of the cross-sector technology spillovers, parameter ϕ . Larger cross-sector spillovers imply that sector-specific technology is less important, weakening the direct productivity effect. If the direct productivity effect is sufficiently small, (i.e., when $\phi > \eta(\varepsilon_e - 1)(1 - \alpha)$), then the price effect is dominant and the interior balanced growth path is stable.

To better understand the modeling choices and mechanics, consider the following examples of machines. One example of a fossil energy machine is an oil rig. Oil rigs embody technology that enable them to be assembled and to remain stable in rough oceans. An example of a green energy machine is a solar cell. The cell's efficiency, which measures the rate at which it can convert incident sunlight into electricity, is technology embodied in the cell. Cross-sector spillovers affect the development of both oil rigs and solar panels. For example, the first mass commercialization of solar cells was driven by demand from oil companies to light their offshore rigs. (Perlin (2000)).

2.4 Calibration

I begin with a discussion of the time period and the implications for the within-sector technology spillovers. I next describe the data I use to calibrate the model parameters. Finally, following standard procedure (e.g., ?), I calibrate the production and innovation components of the model in two steps. In the first step, I calibrate a group of parameters directly from the data series.

In the second step, I use historical oil price shocks and the accompanying data on energy production and innovation to jointly calibrate the remaining parameters. A growing empirical literature that finds a causal relationship between energy innovation and energy prices supports this approach.⁴

2.4.1 Time period

The time period in the model is five years. This choice implies that technology spillovers within a sector occur in five years. To determine this time period, I examine the rate of technology spillovers experienced in a green and in a fossil industry. In particular, I focus on solar power and offshore drilling. In both cases, within-sector technology spillovers frequently occur in less than five years.

One form of technology embodied in a solar cell is the cell's efficiency. Cell efficiency measures the ratio of the cell's electrical output to incident energy from sunlight. Higher cell efficiency corresponds to higher technology. Figure 2.5 (in Appendix C), from the National Renewable Energy Laboratory, plots advances in cell efficiency from 1970-2010 and the company or research institution that achieved the advance. In most cases, the company with the leading cell efficiency is surpassed by a different company within five years. For example, in 1970, Mobile Solar had the leading efficiency in Single crystal non-concentrator Si cells. In 1978, Renewable Capital Assets (RCA) passed Mobile Solar; in 1980, Sandia National Laboratory passed RCA; and so on. The average length of time that a company or research institution maintains the leading efficiency is 3.84 years. This leapfrogging occurs in less than five years, on average, suggesting that within-sector spillovers over a five-year period are reasonable in the case of solar electricity.

As an example from the fossil energy sector, I consider the development of offshore drilling technology. An early technological advance in the offshore industry occurred in 1954, when the Offshore Drilling and Exploration Company (ODECO) developed the first submersible drilling barge, "Mr. Charlie."

⁴See, for example, Aghion et al. (2015), Crabb and Johnson (2007), Hassler et al. (2012), Lanzi and Sue Wing (2010), Newell et al. (1999), Popp (2002).

Mr. Charlie was designed to drill in what was considered deep water at the time (thirty feet). By 1957, just three years after Mr. Charlie's introduction, there were 23 such units in operation in the Gulf and 14 more under construction by numerous oil companies, including Zapata Offshore Company (founded by George H.W. Bush), ODECO, and others. Thus, in less than five years, the technology embodied in ODECO's Mr. Charlie spilled over to other major players in the industry (Schempf (2007), National Commission on the BP Deepwater Horizon Oil Spill and Offshore Drilling).

A second major development in offshore drilling occurred in 1962, when Shell Oil launched Blue Water 1, a semi-submersible floating drilling platform that was equipped to operate in up to 600 feet of water (previous platforms could not exceed 150 feet). However, when Shell tried to lease the land for drilling, it was the only bidder on some of the deepwater tracts, and the government refused to honor bids without competition. Since no other companies could operate at those depths, "[Shell] realized that the only way [it] could ever have access to those frontier areas was to share [its] knowledge with the rest of the industry, to give them a base of technology from which they could expand" (Ron Geer, Shell mechanical engineer). In 1963, Shell hosted a "school for industries" in which it shared its frontier deep water technology with seven other companies. By 1968, these within-sector spillovers had led to the construction of 23 Blue-Water-like semi-submersibles and opened up deeper and rougher areas of the ocean to oil drilling and exploration (Priest (2007)).

Shell Oil continued its advance into deeper waters and, in 1976, constructed "Cognac," a fixed platform connected to a well in 1000 feet of water in the Gulf. At the time, Cognac was the most costly and technologically advanced fixed platform installation ever completed. But within five years of its construction, other companies innovated on Cognac's design and built similar platforms for much less money. To emphasize its cost savings compared to Cognac, Union Oil named its two 1000-foot platforms constructed from 1980-1981, "Cerveza" and "Cerveza-light." But as energy historian Tyler Priest notes, "these beer-budget projects could not have happened without the deep water precedent established by Cognac" (Priest (2007)). Again, the develop-

ment and diffusion of offshore drilling technology suggests that within-sector spillovers often occur in less than five years.

2.4.2 Data

The National Science Foundation’s (NSF) Survey of Industrial Research and Development reports innovation expenditures by US companies from 1953-2007. The data include both company- and government-funded R&D expenditures. From 1972-2007, the survey reports energy-specific R&D. I split R&D expenditures into fossil, green, and non-energy categories. Fossil innovation corresponds to any R&D expenditures on coal, oil, or natural gas. Green innovation corresponds to any energy R&D expenditures that are not in coal, oil, or natural gas. This category includes renewables and nuclear, as well as energy conservation and efficiency. This mapping reflects the broad definition of green energy to encompass both non-carbon sources of energy and improvements in conservation and efficiency discussed in Section 3.2. Finally, I measure non-energy R&D expenditures as the difference between total and energy R&D expenditures.

Data on fossil energy prices, fossil energy production and consumption, as well as the value of oil imports are from the US Energy Information Administration (EIA). Data on labor, fixed assets, output, and employee compensation are from the US Bureau of Economic Analysis (BEA) industry accounts. Following Mork (1989), I use the refiner acquisition cost of imported crude oil to measure the foreign oil price. This measure captures differences in the foreign and domestic prices of crude oil due to price controls and other policies.

2.4.3 Direct calibration

I calibrate the following six parameters directly from the data series: $\{\alpha_f, \alpha_n, \rho_f, \rho_g, S, \omega\}$, where $\omega = \frac{\omega_o}{\omega_f}$ measures the carbon content of foreign oil relative to that of domestic fossil energy.

I calibrate the labor share in fossil energy, $1 - \alpha_f$, as the cost share of labor in value added in the fossil energy sector. Fossil energy corresponds to

coal, oil, and natural gas extraction, as well as to the production of petroleum and coal products (such as gasoline). I map fossil energy to the mining and the petroleum and coal products industries (NAICS codes 21 and 324) in the BEA accounts. Average labor share in these two industries combined is 0.28 over the period for which the data on labor compensation by industry are available (1987-2013). I use the standard value for labor share in GDP, 0.64 for non-energy labor share, $1 - \alpha_n$, since the non-energy sector comprises most of the economy.

I normalize the workforce to unity, $L = 1$. Approximately one percent of workers are engaged in R&D in the US (Jones and Vollrath (2013)), and, thus, I set the number of scientists $S = 0.01$. I also normalize ρ_n to unity. Thus, parameters (ρ_f, ρ_g) capture the number of processes in the fossil and green energy sectors relative to the number of processes in non-energy. I measure these relative levels of diversity by the long-run average fractions of fossil R&D to non-energy R&D and green R&D to non-energy R&D. This measure assumes that average R&D is equal across all processes in the long run.

Additionally, I design the model so that the output elasticity of substitution, ε_y , equals zero. This Leontief condition implies that non-energy inputs and the CES composite comprised of the energy inputs (F , G , and O^*) are required in fixed proportions to produce output. Even with this Leontief condition, the amount of fossil energy used to produce a unit of output can vary since agents can substitute green for fossil energy. Empirically, this substitution occurs through increases in renewable energy, nuclear, and/or energy efficiency. As discussed in Section 3.2, green energy includes all of these channels. Thus, any reduction in fossil energy requires an increase in green energy to produce the same quantity of output. Note that when the elasticity of substitution is exactly zero, there are kinks in the equilibrium conditions that are difficult to handle numerically. To avoid these numerical difficulties, I set the elasticity of substitution slightly greater than zero, $\varepsilon_y = 0.05$.

I use a conservative value for the elasticity of substitution between green energy and the composite of fossil energy and foreign oil, $\varepsilon_e = 1.5$. This

parameter is particularly difficult to pin down because of the lack of data on green energy prices and quantities. Values of similar parameters used in integrated assessment and macroeconomic models typically range from around unity to ten (Lanzi and Sue Wing (2010); AABH) while empirical estimates range from 1.6-3 (Lanzi and Sue Wing (2010); Papageorgiou et al. (2013)). The impact of endogenous innovation increases for larger values of this substitution elasticity. Therefore, I take a conservative approach and set $\varepsilon_e = 1.5$. I perform a robustness analysis in Appendix D for different values of ε_e .

Finally, to calibrate ω , I measure the carbon content of fossil energy as the weighted average of the carbon content of coal, oil, and natural gas, where the weights are determined by the average quantities produced in the US in 2012. Parameter ω is the ratio of the carbon content of oil to the carbon content of this fossil energy composite.

2.4.4 A method of moments

I jointly calibrate the remaining parameters $\{\alpha_g, \varepsilon_f, \delta_{\bar{F}}, \delta_y, \eta, \gamma\}$ to capture the relationships between energy prices, production, and innovation. To obtain empirical evidence of these relationships, I analyze the energy price increases triggered by historical oil shocks and the accompanying changes in energy production and innovation. Empirically, these oil shocks led to large increases in the prices of substitute fossil fuels (such as coal and natural gas) in addition to the increases in the price of oil. Thus, like a carbon tax, the oil shocks created a substantial increase in the composite price of fossil energy.

In an ideal setting, to calibrate the model parameters I would use data on energy price increases triggered by climate policy instead of by oil shocks. However, there are not many economy-wide historical examples of climate policies. The closest example is the Emissions Trading System (ETS) in the European Union. However the ETS carbon permit price has been very unstable. In both the pilot period (2005-2007) and the first trading period (2008-2012), the EU over-allocated carbon permits and the price effectively fell to zero. Another alternative to using oil shocks is to use the variation in gas taxes (or

other energy taxes) across countries. However, these taxes are usually specific to a single sector, such as transportation, and, thus, are not necessarily representative of how the aggregate economy would respond to a carbon tax that applies to all carbon-emitting fuels. The oil shocks and the accompanying data on energy production and innovation are a rare historical example of the economic response to an aggregate increase in fossil energy prices. As a robustness check, I compare the elasticity of new green ideas in the model with sector-specific empirical estimates of the price elasticity of green patents. The results, reported in Section 2.5.5, show that the model estimates are within the range spanned by the empirical studies for different sectors.

I focus on the oil shocks triggered by the rise of OPEC in the first half of the 1970s. I use the oil shocks of the early 1970s instead of more recent oil shocks for two reasons. First, because a carbon tax will likely be permanent, it is important to calibrate to an aggregate increase in energy prices that agents at least believe to be permanent. After the rise of OPEC in the early 1970s, there was a sense that the economy had permanently switched from a low-energy-price regime to a high-energy-price regime. Energy price forecasts during the 1970s and early 1980s generally do not predict falling energy prices, suggesting that agents believed that the oil shocks were very long-lived.⁵ However, after oil prices began to fall in the mid-1980s, agents potentially learned that this regime switch was not permanent and that oil shocks could be temporary. The model implicitly assumes that oil price changes are expected to be permanent. This makes using later oil price shocks inappropriate since expectations likely violated this assumption.

Second, a convenient way to introduce an oil shock is to model the economy on a balanced growth path (in which energy prices are constant) and then shock it with an oil shock. The 1970s is the most recent time period that matches these dynamics –that is, a long period of price stability followed by an unexpected jump in the oil price. Energy prices were relatively constant for the 20 years prior to the 1970s.

⁵See, for example, Annual Energy Outlook (EIA, 1979); World Oil (EMF, 1981); Levy, “Beyond the Oil Bonanza: When the Wells Run Down” (NY Times January 4, 1979).

One limitation with using the early 1970s oil shocks to pin down the model parameters is that they happened 40 years ago. It's possible that some of the parameter values could have changed over time. Even so, any meaningful inference from a calibrated growth model requires the assumption of parameter constancy. And the parameters can be constant at values calibrated from any episode along the equilibrium path, whether the episode is early or late. Later, I will describe a robustness check with respect to parameter constancy.

The early 1970s oil shocks coincided with changes in energy and environmental policies. The policy changes likely affected energy innovation incentives and, thus, are important to include in the calibration strategy. In particular, the Environmental Protection Agency (EPA) was initiated on December, 2, 1970, and with it came the authority for the federal government to implement and enforce environmental regulation. This was a major regulatory change which launched the US into a new era of environmental stewardship (Berman and Bui (2001)). Examples of influential environmental regulation from the early 1970s include the Clean Air Act, which limited emissions from coal power plants and oil refineries, and the Clean Water Act and Safe Drinking Water Act, which placed restrictions on fossil energy companies' hazardous waste. Congress also passed a set of long overdue health and safety regulations that reduced labor productivity in underground coal mines (Bohi and Russell (1978)).

In addition to this new era of environmental protection, the government implemented a series of oil price controls and windfall profits taxes on oil companies from 1971 until 1982, when President Reagan deregulated the industry. These price distortions drove a wedge between the prices of imported and domestic oil and led to energy shortages. Additionally, oil import restrictions were relaxed considerably in 1973 (Bohi and Russell (1978)). The share of oil imports increased throughout most of the 1970s despite the rising cost of foreign oil.

I model these policy changes as a negative productivity shock, ν , to

fossil energy production:⁶

$$F_t = \nu_t L_{ft}^{1-\alpha_f} \int_0^1 X_{fit}^{\alpha_f} A_{fit}^{1-\alpha_f} di. \quad (2.14)$$

All of these policies likely reduced the profitability of fossil energy extraction and the accompanying innovation incentives. Since the model is not sufficiently detailed to accurately incorporate each individual regulation change, I use the reduced-form productivity shock to capture the overall effects of the new regulation.

To account for both the oil and policy shocks, I jointly calibrate the parameters to match the data generated by the following experiment in the model with the data generated by the oil and policy shocks of the early 1970s in the US economy.

Initial balanced growth path (pre-1970s): The economy is on a balanced growth path with respect to the foreign oil price and environmental and energy policies.

Period 1971-1975: Two unexpected shocks realize: (1) the foreign oil price increases by 28 percent from its value on the balanced growth path; and (2) a negative productivity shock affects domestic fossil energy production.

Since environmental policy and the foreign oil price were relatively constant prior to the 1970s, I begin the experiment on a balanced growth path. I match this balanced growth path to data from 1961-1970. The value of the foreign oil price on the balanced growth path is 70 percent higher than the domestic fossil energy price –the average empirical relationship from 1961-1970. I begin the shock period in 1971 because the EPA was created in December of 1970, launching the US into a new era of environmental regulation. I measure the size of the oil shock by the observed percentage change in the average foreign oil price from 1971-1975 relative to its average value from 1961-1970.

⁶I could capture these policy changes with a tax on the fossil energy producer instead of a negative productivity shock. For the purpose of calibration, the negative productivity shock ν is isomorphic to a tax on the fossil energy producer of $1 - \nu$.

Both shocks are unexpected by the agents on the balanced growth path since they were unprecedented in the data. Machine production decisions are made prior to the realization of the shocks, while scientist and labor decisions are made after shocks realize.⁷

I construct moments from this experiment so that the model matches the innovation incentives that coincided with the oil price shocks and regulatory changes. Four key moments to capture these innovation incentives are the values of fossil energy production and consumption as share of GDP in both the balanced growth path and the shock period. Equation (2.11) shows that the observed components of the returns to fossil and green energy innovation are the values of fossil and green energy production, respectively ($P_{ft}F_t$ and $P_{gt}G_t$).

Unfortunately, I do not observe the value of green energy production. However, I do observe the value of fossil energy consumption, which is closely related to the value of green energy production. The first-order conditions for the final-good producer imply (see Appendix A for the derivation):

$$\frac{G}{\tilde{F}} = \left(\frac{P_{\tilde{F}}}{P_g} \right)^{\varepsilon_e} \implies P_g G = P_{\tilde{F}} \tilde{F} \left(\frac{P_{\tilde{F}}}{P_g} \right)^{\varepsilon_e - 1}. \quad (2.15)$$

Equation (2.15) implies that the relative demand for green energy is driven by shifts in the value of the composite comprised of fossil energy and foreign oil, $P_{\tilde{F}}\tilde{F}$. Fossil energy consumption includes oil imports and, thus, differs from fossil energy production.

Additionally, I target the long-run growth rate of per capita GDP and research expenditures on fossil and green energy as a fraction of total research expenditures.⁸ The energy research data are not available until 1972, so I construct the empirical averages from 1972-1975. Research expenditures in

⁷The empirical evidence supports these timing assumptions. In the fossil energy sector, the change in the fraction of fixed assets relative to total fixed assets is very small from 1971-1975. In contrast, the fraction of energy research expenditures relative to total research expenditures almost doubles from 1972 to 1975.

⁸I include both government- and company-funded research expenditures. Prior to President Reagan taking office in 1981, a specific goal of federal energy policy was to accelerate new marketable technologies, making federally-funded R&D a potential substitute for company funded R&D. See Popp (2002) for further discussion.

the data correspond to the wage multiplied by the number of scientists in the model. Scientist market clearing implies that the scientists' wages are equated across all sectors. Therefore, the fraction of research expenditures in fossil energy in the data corresponds to the fraction of scientists in fossil energy in the model (and likewise for green research).

This process yields seven moments (listed in Table 2.1) for the six parameters and the policy shock, ν . For each set of parameters, I solve the model, compute the moments, and compare them with the moments in the data. I use the Nelder-Meade simplex algorithm (Nelder and Mead (1965)) to minimize the sum of the square of the residuals between the empirical and model values of the moments. Table 2.1 reports the values of the moments in the model and the data.

Table 2.1: Targeted Moments

	Time Period	Model	Data
Annualized per capita GDP growth rate	1961-1970	2.0	2.0
Foreign oil imports share	1961-1970	0.2	0.2
	1971-1975	0.8	0.8
Fossil energy production share	1961-1970	1.9	1.9
	1971-1975	2.1	2.1
Percent of R&D expenditures in fossil	1971-1975	2.1	2.1
Percent of R&D expenditures in green	1971-1975	3.4	3.4

While all the parameters are jointly determined, the levels of the fossil energy production and consumption shares on the initial balanced growth path are pinned down primarily by the CES distribution parameters, $\delta_{\bar{F}}$ and δ_y . The movements in these shares are largely governed by the policy shock, ν , and the elasticity of substitution between fossil energy and foreign oil, ε_f . For example, if fossil energy and foreign oil are more substitutable, then the oil shock leads to a larger increase in demand for fossil energy, which leads a big increase in the fossil energy price, quantity, or both. Hence, increases in this substitution elasticity result in a larger increase in the fossil energy production share in response to the oil shock.⁹

⁹Note that to pin down the fossil elasticity of substitution, independent of other param-

The research expenditure moments primarily pin down the level of diminishing returns η and the labor share in green energy, $1 - \alpha_g$. Since the price elasticity of demand for green machines is $\frac{1}{1-\alpha_g}$, increases in labor share reduce the price elasticity of demand, reducing the markup and the accompanying returns to innovation. Parameter γ determines the long-run growth rate.

The calibration strategy does not pin down the strength of the cross-sector technology spillovers, ϕ . This parameter is governed by the relative levels of technology on a long-run balanced growth path. For example, stronger technology spillovers imply that the long-run equilibrium levels of fossil and green technology must be closer together. However, data on energy innovation (and, thus, on technology) are not available until 1972, and so I do not observe technology on the balanced growth path of the 1960s.

However, the data do provide suggestive evidence that $\phi > \phi^*$, the cut-off value for which the interior balanced growth path in which agents innovate in both energy sectors is stable. If $\phi < \phi^*$, then the only stable balanced growth paths are corner solutions in which agents innovate in only the fossil or the green energy sector, which would imply that green innovation was zero along the balanced growth path of the 1960s. However, in the early 1970s, green innovation expenditures were over half of all energy innovation expenditures. It is highly unlikely that green innovation would go from nonexistent to over half of all energy innovation in such a short time frame. Thus, the spillovers must be sufficiently strong (i.e., $\phi > \phi^*$) so that positive innovation occurred in both fossil and green energy along the 1960s' balanced growth path. I set $\phi = 0.5$ in the main specification. In Section 2.5.4, I report the main results for a range of values of $\phi > \phi^*$ (where $\phi^* \approx 0.2$).

Table 2.2 reports the parameter values. I perform a robustness analysis for these parameter values in Appendix D. Labor share in green energy is

eters, I would need to use separate data on prices and on quantities, not on price times quantity. However, while not the only determinant, the elasticity of substitution is an important determinant of how price times quantity changes in response to an oil shock. It is crucial for the model to match price times quantity because this value is the observable component of the return to innovation.

0.09, implying that green energy is a very capital-intensive sector. Consistent with this calibration, green energy technologies, such as nuclear, solar, and, particularly energy efficiency, are all very capital-intensive.

Table 2.2: Parameter Values

Parameter	Model Value
Final-good production	
Output elasticity of substitution: ε_y	0.05
Energy elasticity of substitution: ε_e	1.50
Fossil elasticity of substitution: ε_f	6.24
Distribution parameter: $\left(\frac{\delta_y}{1-\delta_y}\right)^{\varepsilon_y}$	0.01
Distribution parameter: $\left(\frac{\delta_{\bar{F}}}{1-\delta_{\bar{F}}}\right)^{\varepsilon_f}$	0.51
Intermediates production	
Labor share in fossil energy: $1 - \alpha_f$	0.28
Labor share in green energy: $1 - \alpha_g$	0.09
Labor share in non-energy: $1 - \alpha_n$	0.64
Research	
Cross-sector technology spillovers: ϕ	0.50
Diminishing returns: η	0.79
Scientist efficiency: γ	3.96
Sector size: ρ_f	0.01
Sector size: ρ_g	0.01
Number of scientists: S	0.01
Climate	
Emissions conversion: ω	1.03

The elasticity of substitution between fossil energy and foreign oil is considerably higher than that between green energy and the fossil energy foreign oil composite, (6.24 compared to 1.5), suggesting that domestically produced fossil energy is a better substitute for foreign oil than green energy. This is intuitive since one component of domestic fossil energy is domestically produced oil, which is a perfect substitute for foreign oil. The diversity of the energy sectors, ρ_f and ρ_g , are both small compared to the non-energy sector. However, ρ_g is slightly larger than ρ_f (0.011 compared to 0.0095), indicating that the green energy sector is more diverse than the fossil sector.

Additionally, the calibrated size of the policy shock, ν , is 0.64, suggesting that the series of environmental, health, and safety regulations, along

with the distortions created by price controls and import policies, reduced productivity in fossil energy. This result relates to the literature linking the productivity slowdown to increased environmental regulation in the 1970's (e.g., ?). To comply with the regulations, firms must divert resources away from output production. The implied effects on fossil energy productivity are much larger than for the aggregate productivity in the economy because the regulation applied to a larger proportion of fossil energy firms.

I have calibrated to the early 1970s' oil shocks because it is the only historical episode that arguably matches the model's assumptions of being on a balanced growth path when there is a large and exogenous change in the oil price that agents perceive to be permanent. However, it's possible that some of the parameter values could have changed over time. As a check on the assumption of parameter constancy in Appendix F, I analyze the effects of the 2003 oil shock in both the model and the data.¹⁰ In particular, I calculate the elasticity of innovation in both the fossil and green energy sectors with respect to a change in the foreign oil price. The model elasticity estimates are very similar to those in the data, suggesting that the parameter values that govern the price elasticity of innovation have not changed substantially over time.

As an additional robustness check, I compare size of the carbon tax necessary to attain a given reduction in emissions in the present model with values from micro-simulation models by the EPA, MIT, and others. Appendix E reports the results. The tax size is within the range of estimates from these other models.

2.5 Results

I perform two exercises to fully explore the interactions between endogenous innovation and climate policy. First, I introduce a constant carbon

¹⁰As discussed earlier, I do not calibrate to the 2003 oil shock because energy prices and energy innovation are not stable for a sustained period preceding the 2003 oil shock, suggesting that the assumption that economy was on a long-run balanced growth path prior to the shock is imperfect. Moreover, after the 1970s, agents learned that energy prices are uncertain, and they formed expectations over future energy prices.

tax into my benchmark model with endogenous innovation. Second, I introduce a constant carbon tax into an alternative model with the endogenous innovation channel shut down. In this alternative model, the machine producers can only choose the quantity of machines they produce; they cannot affect the embodied technology. This model is equivalent to the model described in Section 3.2, except that scientists cannot adjust and, thus, technology growth under the tax is the same as on the baseline path with no tax. I refer to this model as the exogenous-innovation model.

Prior to the introduction of the carbon tax, the economy is on a balanced growth path. The value of the foreign oil price on the balanced growth path is 115 percent higher than the domestic fossil energy price, the average empirical relationship from 2001-2010. The tax is introduced in 2016-2020. Unlike with the oil shocks, the carbon tax is known in advance. In the endogenous-innovation model, machines, workers, and scientists all adjust in response to the tax. In the exogenous-innovation model, only workers and machines adjust; scientists are fixed at their values on the baseline balanced growth path. In both models, I choose the tax to achieve the Power Plan target of a 30-percent reduction in emissions from the baseline balanced growth path by 2030.

2.5.1 Carbon tax: the role of endogenous innovation

I analyze the effects of the carbon tax on different macroeconomic aggregates in 2030 and on the new long-run balanced growth path under the tax. The carbon tax is 30.3 and 24.5 2013 dollars per ton of CO₂ in the exogenous- and endogenous-innovation models, respectively.

The carbon tax is 19.2 percent lower when innovation is endogenous. Regardless of whether innovation is endogenous, the carbon tax operates through prices to shift demand from fossil to green energy, reducing emissions. However, when innovation is endogenous, this shift in demand spurs green innovation. Over time, the increase in green innovation reduces the marginal cost of producing green energy, lowering the equilibrium green energy price and cre-

ating stronger incentives for the final-good producer to switch from fossil to green. Thus endogenous innovation amplifies the price incentives created by the carbon tax, implying that the same reduction in emissions can be achieved with a smaller tax.

An analogous interpretation of this result is that endogenous innovation increases the emissions reduction from a given-sized carbon tax. In particular, if the carbon tax is \$30.3 per ton, then endogenous innovation increases the percent reduction in emissions by close to five percentage points (from 30 percent to 34.6 percent). A policy implication of these results is that if the government designs a cap and trade system to achieve a target permit price (perhaps because a carbon tax is politically infeasible), then endogenous innovation implies that the government should issue fewer permits in order to achieve its price target. This implication is particularly relevant for the EU Emissions Trading System (ETS) where, for several reasons, governments over-allocated permits and the price fell below the desired level.

Table 2.3 provides more details on the mechanisms driving the effects of endogenous innovation. Column 2 of Table 2.3 reports the levels on the baseline balanced growth path. Each row in columns 3-5 reports a measure of the treatment effect; they show the percentage difference from the baseline in each of the variables in 2030 and on the long-run balanced growth path under the carbon tax. For example, the first row of column four implies that when innovation is endogenous, fossil energy scientists are 60.5 percent lower in 2030 under the tax than in the baseline. Note that there are no transitional dynamics when innovation is exogenous, so the values in 2030 equal the values in the long-run balanced growth path under the carbon tax.

The carbon tax leads to large shifts in fossil and green innovation and relatively small movements in non-energy innovation (innovation segment of Table 2.3). The tax reduces fossil innovation by 60.5 percent and increases green innovation by 53.3 percent in 2030. The tax shifts demand from fossil to green energy, since these two inputs are gross substitutes. This demand shift affects the relative market sizes and the accompanying innovation incentives. In contrast, because the energy and non-energy inputs are almost perfect

complements, the effects of the tax on the non-energy market size and the corresponding innovation incentives are small. These movements in innovation affect relative technology. By 2030, the ratio of green to fossil technology is 44.5 percent higher than in the baseline. On the long-run balanced growth path, this ratio is more than double its baseline value.

Table 2.3: Effects of Carbon Tax Which a Achieves the 30% Target

	Levels Baseline BGP	%Δ From the Baseline		
		Exog. Innov. (Yr. 2030)	Endog. Innov. (Yr. 2030)	Endog. Innov. (BGP)
Innovation				
Fossil Scientists: S_f	1.5e-04	0	-60.5	-29.9
Green Scientists: S_g	1.0e-04	0	53.3	23.8
Non-Energy Scientists: S_n	0.010	0	0.4	0.2
Relative Technology				
Green to Fossil: $\frac{A_g}{A_f}$	0.4	0	44.5	144.6
Green to Non energy: $\frac{A_g}{A_n}$	0.9	0	16.9	39.3
Relative Prices				
Green to Fossil: $\frac{P_g}{P_f}$	1.1	0.2	-7.0	-17.1
Green to Non Energy: $\frac{P_g}{P_n}$	1.4	0.6	-1.0	-2.6
Relative Production				
Green to Fossil: $\frac{G}{F}$	1.39	78.0	79.2	112.9
Energy to Non Energy: $\frac{E}{N}$	0.01	-0.7	-0.6	-0.7
Climate				
Emissions	-	-30.0	-30.0	-36.9
Carbon Stock	-	-2.2	-2.1	-36.8

The prices segment of Table 2.3 shows the effects of the tax on relative prices. When innovation is endogenous, the relative price of green compared to fossil energy falls by 7.0 percent in 2030 and by 17.1 percent on the long-run balanced growth path. The fall in the relative price of green energy results from both increases in green innovation and decreases in fossil innovation. Increases in green innovation reduce the marginal cost of green energy production, reducing its price. Decreases in fossil innovation raise the relative marginal cost of fossil energy production, raising its price. In contrast, when innovation is exogenous, there is almost no change in the relative marginal

costs of the different inputs, and relative prices are almost the same as on the baseline balanced growth path.¹¹ Thus, almost all of the change in relative prices in the endogenous-innovation model results from changes in technology.

The production segment of Table 2.3 reports the effects of the carbon tax on the production of the different intermediate inputs. In 2030, the changes in the relative quantities of green compared to fossil energy production are similar between the endogenous- and exogenous- innovation models because the carbon tax achieves the same reduction in emissions. However, the long-run effects are very different; the tax increases the ratio of green to fossil production by 112.9 percent on the new balanced growth path when innovation is endogenous and by only 78.0 percent when innovation is exogenous. This difference arises because green technology relative to fossil keeps growing after 2030, further decreasing the relative price of green energy and, thus, increasing the final-good producer's green energy demand. Unlike fossil and green energy production, the changes in the ratios of non-energy to energy production are almost zero. Since the elasticity of substitution between energy and non-energy inputs is close to zero, the final-good producer must substitute green for fossil energy to reduce emissions, instead of substituting non-energy inputs for energy inputs.

Finally, I calculate the consumption equivalent variation (CEV) to quantify the welfare effects of the policy. The CEV is the uniform percentage increase in an agent's consumption in the baseline that is necessary to make him indifferent between the baseline and the carbon tax scenarios. The CEVs are -0.5 percent and -0.6 percent with endogenous and exogenous innovation, respectively.¹² Total consumption for all individuals in the United States from 2008-2012 was approximately \$53,671 billion (2012 dollars), so the CEVs with and without endogenous innovation equal approximately \$270 and \$320 billion,

¹¹Relative prices under the tax in the exogenous model are not identical to their baseline values because the general equilibrium effects lead to small changes in the wage rate. These changes have different effects on the marginal cost of production in the different sectors, which, in turn, affects relative prices.

¹²To calculate the CEV, I set the annual rate of time preference to 1.5 percent and the intertemporal elasticity of substitution to the standard value of one half, $\theta = \frac{1}{2}$.

respectively.¹³

Endogenous innovation reduces the welfare cost of the policy by 0.1 percentage points. Endogenous innovation affects the welfare costs through three offsetting channels. First, the carbon tax is smaller when innovation is endogenous; hence, the accompanying distortionary cost is smaller. Second, green energy is technologically behind fossil energy when the government implements the tax. Thus, the tax shifts energy production to a less productive sector. Endogenous innovation reduces these productivity losses as green technology catches up to fossil. Third, the shift in innovation from fossil to green energy reduces the aggregate growth rate along the transition path to a new long-run equilibrium. This temporary reduction in growth raises consumption costs and mutes the welfare gain from endogenous innovation.

AABH find that climate policy and endogenous innovation tip the economy to a new long-run equilibrium where green technology grows and fossil technology is constant. The results in the present paper indicate somewhat smaller effects of endogenous innovation on climate policy outcomes than in AABH. These different findings are primarily due to two key parameters: the diminishing returns to innovation, η , and the strength of the across-sector technology spillovers, ϕ . Stronger diminishing returns to innovation (lower η) create incentives to spread scientists across both the fossil and green energy sectors. This spreading reduces the effect of a carbon tax on the direction of technical change. Stronger cross-sector spillovers (higher ϕ) reduce the path dependence in innovation. Green technology accumulates faster than fossil technology in response to the carbon tax. If some of the new green discoveries are applicable to fossil energy, then these spillovers indirectly encourage innovation in fossil energy. The calibration in the present paper uses middle values for both η and ϕ : $\eta = 0.79$, $\phi = 0.5$, while AABH use $\eta = 1$ and $\phi = 0$. This implicit parameter choice increases the role of endogenous innovation in AABH. In Section 2.5.4, I calculate the main results for different values of ϕ .

Additionally, as emphasized in AABH, the elasticity of substitution between green energy and the composite comprised of fossil energy and foreign

¹³See BEA personal consumption expenditures.

oil, ε_e , affects the response of innovation to a carbon tax. Higher substitutability increases the change in the green energy market size from a carbon tax and the corresponding innovation incentives. However, in the present model, reasonably large values of this substitution elasticity will not lead to a tipping if the diminishing returns to innovation and the cross-sector spillovers are both large (i.e., if η is small and ϕ is big).

2.5.2 Dynamics

I discuss the dynamics along the transition to the new balanced growth path, focusing explicitly on the general equilibrium forces driving innovation. Figure 2.1 plots the time paths of the market size and price of green energy relative to fossil in response to a carbon tax. The tax shifts demand from fossil to green energy, leading to an immediate jump in the green energy market size (left panel of Figure 2.1). This jump in market size shifts innovation incentives from fossil to green energy. Figure 2.2 plots the accompanying jumps in fossil and green scientists immediately in response to the tax. The surge in green innovation relative to fossil leads to gradual improvements in the relative level of green technology (right panel of Figure 2.2), which reduce the relative price of green energy over time (right panel of Figure 2.1).

Section 2.3 discussed the stability properties of the interior long-run balanced growth path and their relationship to the price, market size, and direct productivity effects driving innovation (equation (2.12)). The tax leads to an immediate increase in green energy market size, increasing green innovation incentives. This surge in innovation raises the relative levels of green technology, further increasing green innovation incentives. However, the relative price of green energy falls, dampening the green innovation incentives. The cross-sector spillovers are sufficiently strong that the price effect dominates, and the percentage of green energy scientists declines as the economy transitions to a new balanced growth path (middle panel of Figure 2.2).

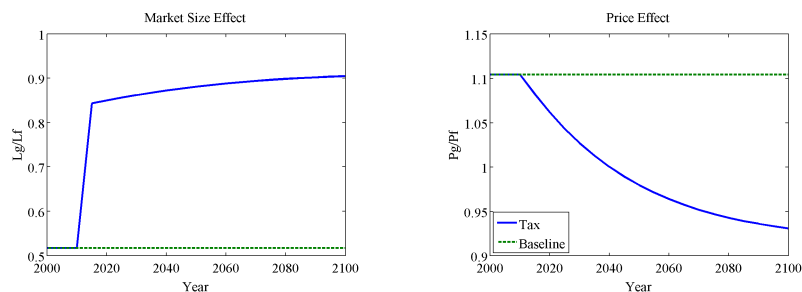


Figure 2.1: Price and Market Size Effects From a Carbon Tax

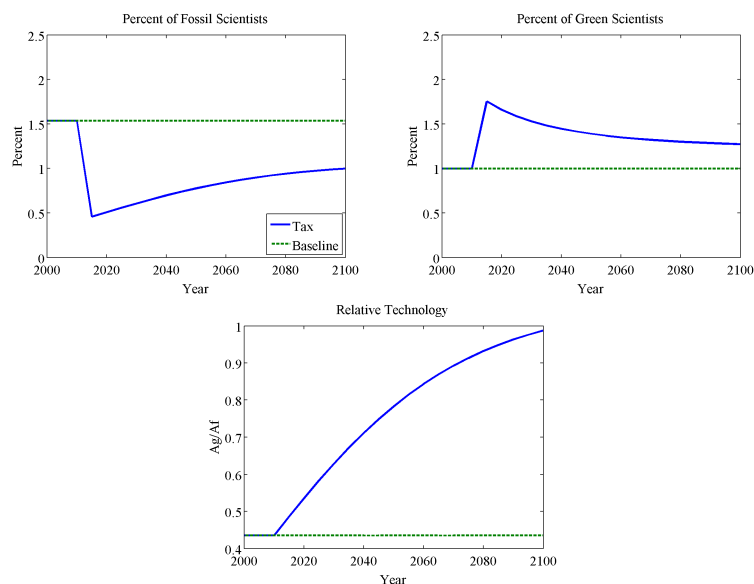


Figure 2.2: Innovation and Technology

2.5.3 Robustness: target stringency and time frame

A key finding is that the carbon tax necessary to achieve the Power Plan target is almost 20 percent lower when innovation is endogenous. However, this result is sensitive to both the size of the targeted reduction in emissions and the time frame in which the reduction must be achieved. In this section, I analyze the effects of innovation for different-sized emissions targets and different time frames.

The left panel of Figure 2.3 plots the percent reduction in the carbon tax from endogenous innovation for different-sized emissions targets. For ex-

ample, the carbon tax required to achieve a 10-percent reduction in emissions by 2030 is 21 percent lower when innovation is endogenous. The effect of endogenous innovation on the size of the carbon tax falls as the stringency of the emissions target increases (e.g., as the target goes from a 10-percent to a 20-percent reduction in emissions). Even with large changes in innovation, the relative technology stocks evolve slowly. A more stringent emissions target forces agents to rely less on technological advances and more on shifts in production factors (i.e., workers and machines) to achieve the target. This switch reduces the role of endogenous innovation and its accompanying effects on the carbon tax.

The right panel of Figure 2.3 plots the percent decrease from endogenous innovation under a carbon tax designed to achieve a 30-percent reduction in emissions by different target years. For example, the carbon tax required to achieve a 30-percent reduction in emissions by 2035 is 22 percent lower when innovation is endogenous. The reduction in the carbon tax from endogenous innovation increases as the target year moves farther into the future. Again, even with large shifts in innovation, changes in the relative technology stocks occur gradually. More-distant target years provide time for technological change to occur and thereby allow agents to rely more heavily on innovation to reduce emissions.

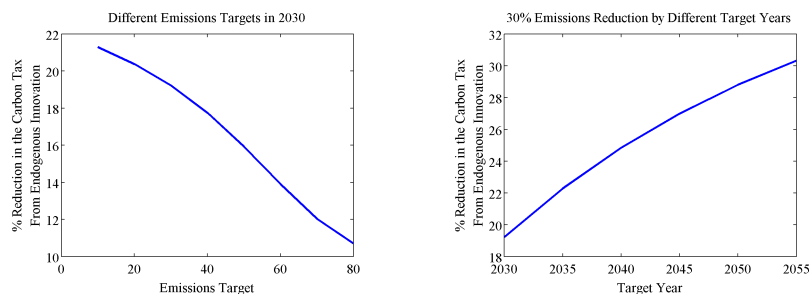


Figure 2.3: Effects of Innovation on the Size of the Carbon Tax

Climate policy simulation models that exogenously assume large advances in green technological progress typically obtain lower carbon tax estimates for a given abatement target. As long as the time horizon for a given emissions target is sufficiently long, the results of this paper suggest that such

technological advances are plausible and could lead to considerable reductions in the carbon tax. However, if policy makers strive to achieve large reductions in emissions quickly, then the potential for innovation to reduce the carbon tax is relatively small.

2.5.4 Robustness: Strength of cross-sector spillovers

I reexamine the main results when ϕ ranges from 0.3 to 0.9, the values such that the interior balanced growth path is stable. For each value of ϕ , I recalibrate the six parameters, $\{\alpha_g, \varepsilon_f, \delta_{\bar{F}}, \delta_y, \eta, \gamma\}$, and the size of the productivity shock that I pinned down from the oil and policy shocks. Table 2.4 in Appendix C reports the alternate calibrations. The effects of ϕ on the calibrated parameter values are relatively small.

The left two panels of Figure 2.4 show the effects of ϕ on the response of relative technologies to a carbon tax that obtains the Power Plan target. The left and middle panels plot the percent increase in $\frac{A_g}{A_f}$ from the baseline in 2030 and on the long-run balanced growth path, respectively. Stronger spillovers dampen the change in the relative technology in response to the carbon tax. The long-run effects exceed the short-run effects because the TFP catchup ratio evolves slowly.

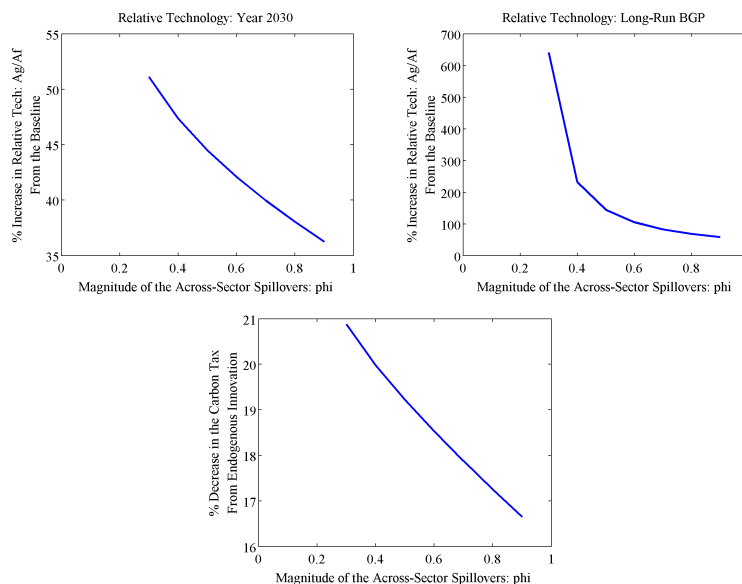


Figure 2.4: Effects of ϕ on Relative Technologies and on the Carbon Tax

The third panel of Figure 2.4 plots the percent reduction in the carbon tax from endogenous innovation for different values of ϕ . Larger spillovers dampen the response of innovation, decreasing the percent reduction in the carbon tax. However, even for very large spillovers, $\phi = 0.9$, endogenous innovation still reduces the carbon tax by over 15 percent.

2.5.5 Comparison to empirical studies

It is useful to compare the response of green energy innovation to a change in the composite price of fossil energy and foreign oil with empirical estimates of related elasticities. Both Popp (2002) and Aghion et al. (2015) calculate the elasticity of green energy patents with respect to a change in energy prices. Popp (2002) estimates this elasticity from aggregate US time series data from 1970-1994 on fossil energy prices and green energy patents in 11 energy technologies. Six of these technologies relate to energy supply (such as solar) and five to energy demand (such as the reuse of industrial waste heat). Aghion et al. (2015) focus on the automobile industry. They use a cross-country, firm-level panel on green car patents (e.g., hybrid vehicle technologies) and tax-inclusive gas prices to estimate the price elasticity of

R&D in green car technologies. The five-year price elasticity of green patents is 0.21 in Popp (2002) and is 3.7 in Aghion et al. (2015)¹⁴.

To compare these empirical results with the present paper, I rewrite the technology accumulation equation for green technology (equation (2.5)) as the sum of the existing green technology stock and new green ideas, \mathcal{I}_{gt}

$$A_{gt} = A_{gt-1} + \mathcal{I}_{gt} \quad \text{where} \quad \mathcal{I}_{gt} = \gamma \left(\frac{S_{gt}}{\rho_g} \right)^\eta A_{t-1}^\phi A_{gt-1}^{1-\phi}. \quad (2.16)$$

New green ideas are the flow input into technology and, thus, correspond to green patents in the data. Let $P_{\tilde{F}}$ be the tax-inclusive price of the fossil energy foreign oil composite, \tilde{F} (derived in Appendix A). The (one-period) elasticity of new green ideas with respect to a change in $P_{\tilde{F}}$ from a carbon tax is 1.7.¹⁵

This elasticity is larger than the estimate for the price elasticity of green patents in Popp (2002). One explanation for this difference is that green innovation in the sectors covered in Popp's study is less responsive to changes in $P_{\tilde{F}}$ than is average green innovation. However, a second reason for the different elasticity is the source of the change in $P_{\tilde{F}}$. Popp's calculation uses aggregate variation in fossil energy prices due to oil shocks (or similar macroeconomics events) instead of from a carbon tax. While both oil shocks and carbon taxes increase incentives for green innovation, oil shocks also increase incentives for fossil innovation. If there is crowd-out between fossil and green innovation, then the price elasticity of green innovation will be smaller when the price change is caused by an oil shock than when it is caused by a carbon tax. Consistent with this hypothesis, the model elasticity of green ideas from an increase in $P_{\tilde{F}}$ from an oil shock is 1.3, approximately 25 percent smaller than the elasticity from a carbon tax. In a related empirical patent study, Popp and Newell (2012) find suggestive evidence of this crowd-out within energy supply technologies (such as oil refining and solar).

The elasticity of green ideas is smaller than the value estimated in Aghion et al. (2015), suggesting that innovation in green car technologies is

¹⁴See Table 4 in Popp (2002) and Table 10 Appendix C in Aghion et al. (2015).

¹⁵This elasticity is given by $\epsilon = \left(\frac{\mathcal{I}_{gt^*} - \mathcal{I}_{gt^*-1}}{\mathcal{I}_{gt^*-1}} \right) \left(\frac{P_{\tilde{F}t^*-1}}{P_{\tilde{F}t^*} - P_{\tilde{F}t^*-1}} \right)$, where t^* is the period in which the tax is introduced. Prior to period t^* , the economy is on a balanced growth path.

more responsive than average green innovation to changes in the fossil energy price. Some of the variation in gas prices comes from differences in the gas tax and some comes from oil shocks. However, since the automobile industry does not supply fossil energy, price changes from oil shocks and carbon taxes should create similar incentives for innovation in green car technologies. Thus, differences in the elasticity estimates due to crowd-out are not as likely in this case.

2.6 Conclusion

This paper develops a general equilibrium model to quantify the response of technology, prices, and other macroeconomic aggregates to climate policy. Building on the directed technical change literature, I model an economy in which scarce innovation resources can be allocated towards fossil energy, green energy, or non-energy intermediate inputs. Additionally, agents can import foreign oil from abroad at an exogenously determined price. I calibrate the model parameters using data from the natural experiment on energy prices and innovation from the oil shocks in the first half of the 1970s. I then use this empirically grounded model as a quantitative laboratory in which to study climate policy.

A key result is that endogenous innovation amplifies the price incentives created by the tax. The carbon tax operates through prices to shift innovation from fossil to green energy. This shift in innovation raises green technology compared to the baseline path, reducing the green energy price. Similarly, fossil innovation falls compared to the baseline path, raising the fossil energy price. These additional price movements imply that more abatement can be attained from a given-sized carbon tax when innovation is endogenous. Specifically, endogenous innovation lowers the size of the carbon tax necessary to achieve a 30-percent reduction in emissions by 2030 by close to 20 percent.

Overall, the results imply that endogenous innovation has considerable effects on climate policy outcomes. Shifts in innovation lower the relative price of green energy compared to fossil by approximately 7 percent in the short run

and closer to 20 percent in the long run. Moreover, the relative level of green technology compared to fossil stabilizes at two and a half times its value on the baseline path.

2.7 Appendix

2.7.1 Derivation of the main equations

I derive the main equations in the text. For ease of presentation, some of the equations are repeated. The final goods producer chooses F , G , N , and O^* to maximize profits taking prices as given. His optimization problem is (equation (2.9) in the text)

$$\max_{F_t, G_t, N_t, O_t^*} \{Y_t - (P_{ft} + \tau_f)F_t - P_{gt}G_t - (P_{ot}^* + \tau_o)O_t^* - P_{nt}N_t\}. \quad (2.17)$$

The first order conditions imply the relative demands for the intermediate inputs are inversely related to their prices,

$$\begin{aligned} \frac{G_t^d}{\tilde{F}_t^d} &= \left(\frac{P_{\tilde{F}t}}{P_{gt}}\right)^{\varepsilon_e} \left(\frac{1 - \delta_e}{\delta_e}\right)^{\varepsilon_e} & \frac{F_t^d}{(O_t^*)^d} &= \left(\frac{P_{ot}^* + \tau_o}{P_{ft} + \tau_f}\right)^{\varepsilon_f} \left(\frac{\delta_{\tilde{F}}}{1 - \delta_{\tilde{F}}}\right)^{\varepsilon_f} \\ \frac{E^d}{N^d} &= \left(\frac{P_n}{P_e}\right)^{\varepsilon_y} \left(\frac{\delta_y}{1 - \delta_y}\right)^{\varepsilon_y}. \end{aligned} \quad (2.18)$$

where δ_e is the distribution parameter in the nest between green energy and the composite comprised of fossil energy and foreign oil. δ_e is the weight on the composite comprised of fossil energy and foreign oil and $1 - \delta_e$ is the weight on green energy. This parameter is implicitly set to 0.5 in the main specification in the text. Variables $P_{\tilde{F}}$ and P_e and denote the prices of optimally chosen composites \tilde{F} and E respectively. The first order and zero profit conditions implies that these prices are

$$\begin{aligned} P_{\tilde{F}t} &= (\delta_{\tilde{F}}^{\varepsilon_f} (P_{ft} + \tau_f)^{1 - \varepsilon_f} + (1 - \delta_{\tilde{F}})^{\varepsilon_f} (P_{ot}^* + \tau_o)^{1 - \varepsilon_f})^{\frac{1}{1 - \varepsilon_f}} \\ P_{et} &= (\delta_e^{\varepsilon_e} P_{\tilde{F}t}^{1 - \varepsilon_e} + (1 - \delta_e)^{\varepsilon_e} P_{gt}^{1 - \varepsilon_e})^{\frac{1}{1 - \varepsilon_e}}. \end{aligned}$$

The final good is the numeraire. I normalize its price to unity. This yields the ideal price index

$$\delta_y^{\varepsilon_y} P_{et}^{1 - \varepsilon_y} + (1 - \delta_y)^{\varepsilon_y} P_{nt}^{1 - \varepsilon_y} = P_{yt} \equiv 1. \quad (2.19)$$

The intermediate-goods producers make F , G , and N which they sell to the final-good producer. I discuss the equations with respect to a representative fossil-energy producer, the other sectors are symmetric. The fossil-energy producer chooses labor and machines to maximize profits taking prices as given,

$$\max_{L_{ft}, X_{fit}} P_{ft} L_{ft}^{1-\alpha_f} \int_0^1 X_{fit}^{\alpha_f} A_{fit}^{1-\alpha_f} di - w_{lft} L_{ft} - \int_0^1 P_{fit}^x X_{fit} di. \quad (2.20)$$

Variable P_{fi}^x denotes the price of machine i in sector F . The first order conditions imply the demand for machines (equation (2.10) in the text)

$$X_{fit} = \left(\frac{\alpha_f P_{ft}}{P_{fit}^x} \right)^{\frac{1}{1-\alpha_f}} A_{fit} L_{ft} \quad (2.21)$$

and the wages to workers in sector F ,

$$w_{lft} = (1 - \alpha_f) P_{ft} X_{ft}^{\alpha_f} L_{ft}^{-\alpha_f} A_{ft}^{1-\alpha_f}. \quad (2.22)$$

In equilibrium, labor market clearing requires that workers' wages are equated across all sectors, $w_{lft} = w_{lgt} = w_{lnt}$ and that total labor demand equal the fixed exogenous supply, $L_{ft} + L_{gt} + L_{nt} = L$.

Finally, the machine producers make machines which they sell to the intermediate-goods producers. The machines embody technology. Each machine, regardless of the sector and the level of technology, costs one unit of final good to produce. Each machine producer chooses price, quantity of machines, and the number of scientists to maximize profits subject to the machine demand from the intermediate producer (equation (2.10) in the text). The optimization problem for fossil-energy machine producer i in period t is

$$\max_{X_{fit}, S_{fit}} -X_{fit} - w_{sft} S_{fit} + P_{fit}^x X_{fit} \quad (2.23)$$

subject to

$$P_{fit}^x = \alpha_f P_{ft} L_{ft}^{1-\alpha_f} \left(\frac{A_{fit}}{X_{fit}} \right)^{1-\alpha_f} \quad (2.24)$$

$$S_{fit} = \left[\left(\frac{A_{fit}}{A_{ft-1}} - 1 \right) \frac{\rho_f^\eta}{\gamma} \left(\frac{A_{ft-1}}{A_{t-1}} \right)^\phi \right]^{\frac{1}{\eta}} \quad (2.25)$$

The first order condition for the number of machines imply that the optimal machine price is a constant markup over marginal cost $P_{fit}^x = \frac{1}{\alpha_f}$. This constant markup arises because machine demand is iso-elastic. Increases in the machine share increase the demand elasticity, and decrease the markup.

Finally, the first order conditions for the number of scientists imply that the marginal return to innovation in sector F is

$$w_{sft} = \frac{\eta\gamma A_{ft-1} \left(\frac{A_{t-1}}{A_{ft-1}}\right)^\phi P_{fit}^x X_{fit}}{\rho_f^\eta \left(\frac{1}{1-\alpha_f}\right) S_{ft}^{1-\eta} A_{fit}}. \quad (2.26)$$

The equilibrium is symmetric across all machine producers within a sector, and, hence, $P_{fit}^x X_{fit} = P_{ft}^x X_{ft}$. To derive equation (2.11) in the text, multiply equation (2.24) by X_{ft} to which yields the relationship $P_{ft}^x X_{ft} = \alpha_f P_{ft} F_t$. Substitute this relationship into equation (2.26) to get equation (2.11) in the text.

2.7.2 Proof of Proposition 1

I prove Proposition 1. I divide the proposition into two claims.

Claim 1: *When the factor shares are equal across sectors ($\alpha_f = \alpha_g = \alpha_n \equiv \alpha$) and imports of foreign oil are zero, there exists a unique interior balanced growth path with strictly positive innovation in each sector ($S_f > 0, S_g > 0, S_n > 0$).*

Proof: In equilibrium, the returns to innovation in each sector must be equal. Equation (2.11) in the text thus implies,

$$\frac{w_{sft}}{w_{sgt}} = \left(\frac{\rho_f}{\rho_g}\right)^\eta \left(\frac{X_{ft}}{X_{gt}}\right) \left(\frac{A_{gt-1}}{A_{ft-1}}\right)^{\phi-1} \left(\frac{A_{gt}}{A_{ft}}\right) \left(\frac{S_{gt}}{S_{ft}}\right)^{1-\eta} = 1 \quad (2.27)$$

$$\frac{w_{snt}}{w_{sgt}} = \left(\frac{\rho_n}{\rho_g}\right)^\eta \left(\frac{X_{nt}}{X_{gt}}\right) \left(\frac{A_{gt-1}}{A_{nt-1}}\right)^{\phi-1} \left(\frac{A_{gt}}{A_{nt}}\right) \left(\frac{S_{gt}}{S_{nt}}\right)^{1-\eta} = 1 \quad (2.28)$$

Equation (2.10) in the text implies that the ratio of machines across

the sectors is

$$\frac{X_{ft}}{X_{gt}} = \left(\frac{P_{ft}}{P_{gt}} \right)^{\frac{1}{1-\alpha}} \left(\frac{A_{ft}}{A_{gt}} \right) \left(\frac{L_{ft}}{L_{gt}} \right) \quad \text{and} \quad \frac{X_{nt}}{X_{gt}} = \left(\frac{P_{nt}}{P_{gt}} \right)^{\frac{1}{1-\alpha}} \left(\frac{A_{nt}}{A_{gt}} \right) \left(\frac{L_{nt}}{L_{gt}} \right). \quad (2.29)$$

Labor market clearing requires that wages (equation (2.22) in Appendix A) are equal across sectors. This relationship implies that relative prices are inversely related to the relative levels of technology

$$\frac{P_{ft}}{P_{gt}} = \left(\frac{A_{gt}}{A_{ft}} \right)^{1-\alpha} \quad \text{and} \quad \frac{P_{nt}}{P_{gt}} = \left(\frac{A_{gt}}{A_{nt}} \right)^{1-\alpha}. \quad (2.30)$$

Finally, market clearing in the intermediate goods market combined with equation (2.30) yields the following relationships,

$$\begin{aligned} \frac{L_{ft}}{L_{gt}} &= \left(\frac{\delta_e}{1 - \delta_e} \right)^{\varepsilon_e} \left(\frac{A_{ft}}{A_{gt}} \right)^{(\varepsilon_e - 1)(1 - \alpha)} \\ \frac{L_{nt}}{L_{gt}} &= \left(\frac{1 - \delta_y}{1 - \delta_e} \right)^{\varepsilon_y} \left(\frac{A_{nt}}{A_{gt}} \right)^{(\varepsilon_y - 1)(1 - \alpha)} \\ &\quad \times \left(\delta_e^{\varepsilon_e} \left(\frac{A_{ft}}{A_{gt}} \right)^{(1 - \alpha)(\varepsilon_e - 1)} + (1 - \delta_e)^{\varepsilon_e} \right)^{\frac{\varepsilon_y}{1 - \varepsilon_e}} \\ &\quad \times \left(\delta_e \left(\frac{A_{ft}}{A_{gt}} \right)^{(1 - \alpha)(\varepsilon_e - 1)} + 1 - \delta_e \right)^{\frac{\varepsilon_e}{\varepsilon_e - 1}} \end{aligned} \quad (2.31)$$

Substituting in equations (2.29), (2.30), and (2.31) into equations (2.27) and (2.28) yields the following equilibrium conditions:

$$\frac{w_{sft}}{w_{sgt}} = \left(\frac{\rho_f}{\rho_g} \right)^\eta \left(\frac{\delta_e}{1 - \delta_e} \right)^{\varepsilon_e} \left(\frac{A_{ft}}{A_{gt}} \right)^{(\varepsilon_e - 1)(1 - \alpha) - 1} \left(\frac{A_{gt-1}}{A_{ft-1}} \right)^{\phi - 1} \left(\frac{S_{gt}}{S_{ft}} \right)^{1 - \eta} = 1 \quad (2.32)$$

$$\frac{w_{snt}}{w_{sgt}} = \left(\frac{\rho_n}{\rho_g} \right)^\eta \left(\frac{1 - \delta_y}{1 - \delta_e} \right)^{\varepsilon_y} \left(\frac{A_{nt}}{A_{gt}} \right)^{(\varepsilon_y - 1)(1 - \alpha) - 1} \left(\frac{A_{gt-1}}{A_{nt-1}} \right)^{\phi - 1} \left(\frac{S_{gt}}{S_{nt}} \right)^{1 - \eta} \quad (2.33)$$

$$\begin{aligned} &\times \left(\delta_e^{\varepsilon_e} \left(\frac{A_{ft}}{A_{gt}} \right)^{(1 - \alpha)(\varepsilon_e - 1)} + (1 - \delta_e)^{\varepsilon_e} \right)^{\frac{\varepsilon_y}{1 - \varepsilon_e}} \\ &\times \left(\delta_e \left(\frac{A_{ft}}{A_{gt}} \right)^{(1 - \alpha)(\varepsilon_e - 1)} + 1 - \delta_e \right)^{\frac{\varepsilon_e}{\varepsilon_e - 1}} = 1 \end{aligned}$$

The law of motion for technology (equation (2.5) in the text) implies the following allocations of scientists along the balanced growth path:

$$S_f = \left(\frac{n}{\gamma \rho_f^\eta} \left(\frac{A_f}{A} \right)^\phi \right)^{\frac{1}{\eta}}, \quad S_g = \left(\frac{n}{\gamma \rho_g^\eta} \left(\frac{A_g}{A} \right)^\phi \right)^{\frac{1}{\eta}}, \quad S_n = \left(\frac{n}{\gamma \rho_n^\eta} \left(\frac{A_n}{A} \right)^\phi \right)^{\frac{1}{\eta}},$$

where n is the long-run equilibrium growth rate. Combining these expressions yields:

$$\frac{S_g}{S_f} = \left(\frac{\rho_f}{\rho_g} \right) \left(\frac{A_g}{A_f} \right)^{\frac{\phi}{\eta}} \quad \text{and} \quad \frac{S_g}{S_n} = \left(\frac{\rho_n}{\rho_g} \right) \left(\frac{A_g}{A_n} \right)^{\frac{\phi}{\eta}}. \quad (2.34)$$

Substituting equation (2.34) into equations (2.32) and (2.33) and solving for the balanced growth path in which relative technologies grow at the same rate yields:

$$\left(\frac{A_f}{A_g} \right)^{\frac{\phi}{\eta} - (\varepsilon_e - 1)(1 - \alpha)} = \left(\frac{\rho_f}{\rho_g} \right) \left(\frac{\delta_e}{1 - \delta_e} \right)^{\varepsilon_e} \quad (2.35)$$

$$\left(\frac{A_n}{A_g} \right)^{\frac{\phi}{\eta} - (\varepsilon_y - 1)(1 - \alpha)} = \left(\frac{\rho_n}{\rho_g} \right) \left(\frac{1 - \delta_y}{1 - \delta_e} \right)^{\varepsilon_y} \quad (2.36)$$

$$\begin{aligned} & \times \left(\delta_e^{\varepsilon_e} \left(\frac{A_f}{A_g} \right)^{(1 - \alpha)(\varepsilon_e - 1)} + (1 - \delta_e)^{\varepsilon_e} \right)^{\frac{\varepsilon_y}{1 - \varepsilon_e}} \\ & \times \left(\delta_e \left(\frac{A_f}{A_g} \right)^{(1 - \alpha)(\varepsilon_e - 1)} + 1 - \delta_e \right)^{\frac{\varepsilon_e}{\varepsilon_e - 1}} \end{aligned}$$

Equations (2.35) and (2.36) uniquely define the balanced growth path. \square

Claim 2: *The interior balanced growth path is stable if and only if $\frac{\phi}{\eta} > (\varepsilon_e - 1)(1 - \alpha)$.*

Proof: Define n_{ft} and n_{gt} to be the growth rates of A_{ft} and A_{gt} . Then equation (2.32) implies

$$1 = \left(\frac{\rho_f}{\rho_g} \right) \left(\frac{\delta_e}{1 - \delta_e} \right)^{\varepsilon_e} \left(\frac{A_{gt-1}}{A_{ft-1}} \right)^{\frac{\phi}{\eta} - (\varepsilon_e - 1)(1 - \alpha)} \left(\frac{1 + n_{gt}}{1 + n_{ft}} \right)^{1 - (\varepsilon_e - 1)(1 - \alpha)} \left(\frac{n_{gt}}{n_{ft}} \right)^{\frac{1 - \eta}{\eta}}.$$

Define Ω_t as

$$\begin{aligned}\Omega_t &\equiv \left(\frac{1+n_{gt}}{1+n_{ft}}\right)^{1-(\varepsilon_e-1)(1-\alpha)} \left(\frac{n_{gt}}{n_{ft}}\right)^{\frac{1-\eta}{\eta}} \\ &= \left(\frac{\rho_g}{\rho_f}\right) \left(\frac{1-\delta_e}{\delta_e}\right)^{\varepsilon_e} \left(\frac{A_{ft-1}}{A_{gt-1}}\right)^{\frac{\phi}{\eta}-(\varepsilon_e-1)(1-\alpha)}\end{aligned}$$

$\Omega_t = 1$ on the balanced growth path both sectors grow at the same rate. Let A_{fg}^* denote the balanced growth value of $\frac{A_f}{A_g}$. Suppose that $\frac{A_{ft-1}}{A_{gt-1}} < A_{fg}^*$. I show that this implies that $\frac{A_{ft}}{A_{gt}} > \frac{A_{ft-1}}{A_{gt-1}}$ and hence the interior equilibrium is stable.

First note that $\frac{A_{ft-1}}{A_{gt-1}} < A_{fg}^* \iff \Omega_t < 1$. We must show that $\Omega_t < 1 \iff \frac{A_{ft}}{A_{gt}} > \frac{A_{ft-1}}{A_{gt-1}}$. I divide the argument into two cases.

Case 1: Suppose that $1 - (\varepsilon_e - 1)(1 - \alpha) > 0$. Then $\Omega_t < 1 \iff \left(\frac{n_{ft}}{n_{gt}}\right) > 1$ which implies that $\frac{A_{ft}}{A_{gt}} > \frac{A_{ft-1}}{A_{gt-1}}$.

Case 2: Suppose that $1 - (\varepsilon_e - 1)(1 - \alpha) < 0$. Observe that $\left(\frac{1+n_{ft}}{1+n_{gt}}\right) < \frac{n_{ft}}{n_{gt}} \iff \frac{n_{ft}}{n_{gt}} > 1$ and hence $\frac{A_{ft}}{A_{gt}} > \frac{A_{ft-1}}{A_{gt-1}}$. I show that $\left(\frac{1+n_{ft}}{1+n_{gt}}\right) > \frac{n_{ft}}{n_{gt}}$ implies that $\Omega_t > 1$, a contradiction. In this case we have:

$$\begin{aligned}\Omega_t &= \left(\frac{1+n_{ft}}{1+n_{gt}}\right)^{(\varepsilon_e-1)(1-\alpha)-1} \left(\frac{n_{gt}}{n_{ft}}\right)^{\frac{1-\eta}{\eta}} \\ &> \left(\frac{n_{ft}}{n_{gt}}\right)^{(\varepsilon_e-1)(1-\alpha)-1} \left(\frac{n_{gt}}{n_{ft}}\right)^{\frac{1-\eta}{\eta}} \\ &= \left(\frac{n_{gt}}{n_{ft}}\right)^{\frac{1}{\eta}-(\varepsilon_e-1)(1-\alpha)} > 1\end{aligned}\tag{2.37}$$

The last inequality follows since $\left(\frac{1+n_{ft}}{1+n_{gt}}\right) > \frac{n_{ft}}{n_{gt}} \iff \frac{n_{gt}}{n_{ft}} > 1$ and since $\phi < 1$ and $\frac{\phi}{\eta} > (\varepsilon_e - 1)(1 - \alpha)$ we have $\frac{1}{\eta} - (\varepsilon_e - 1)(1 - \alpha) > 0$. Thus equation (2.37) is a contradiction since $\Omega_t < 1$. Therefore, $\frac{n_{ft}}{n_{gt}} > 1$ and hence $\frac{A_{ft}}{A_{gt}} > \frac{A_{ft-1}}{A_{gt-1}}$.

The argument for $\frac{A_{ft-1}}{A_{gt-1}} > A_{fg}^*$ is analogous. Similar reasoning applies for $\frac{A_{nt-1}}{A_{gt-1}} < A_{ng}^*$ and $\frac{A_{nt-1}}{A_{gt-1}} > A_{ng}^*$ where A_{ng}^* is the value of $\frac{A_{nt}}{A_{gt}}$ along the balanced growth path. However, only Case 1 in the above argument is necessary since $\varepsilon_y < 1$ and therefore $1 - (\varepsilon_y - 1)(1 - \alpha) > 0$. Also note that $\varepsilon_y < 1$ implies that $\frac{\phi}{\eta} > (\varepsilon_y - 1)(1 - \alpha)$ for all positive ϕ and η . \square

2.7.3 Additional tables and figures

Table 2.4: Calibrated Parameters for Different Values of ϕ

ϕ	α_g	ε_f	$\left(\frac{\delta \bar{F}}{1 - \delta \bar{F}}\right)^{\varepsilon_f}$	$\left(\frac{\delta_y}{1 - \delta_y}\right)^{\varepsilon_y}$	η	γ
0.30	0.90	6.20	0.52	0.01	0.84	5.11
0.40	0.91	6.17	0.53	0.01	0.84	5.08
0.50	0.91	6.14	0.54	0.01	0.84	5.06
0.60	0.91	6.13	0.54	0.01	0.84	5.05
0.70	0.91	6.12	0.55	0.01	0.84	5.04
0.80	0.91	6.11	0.55	0.01	0.84	5.03
0.90	0.91	6.10	0.55	0.01	0.84	5.03

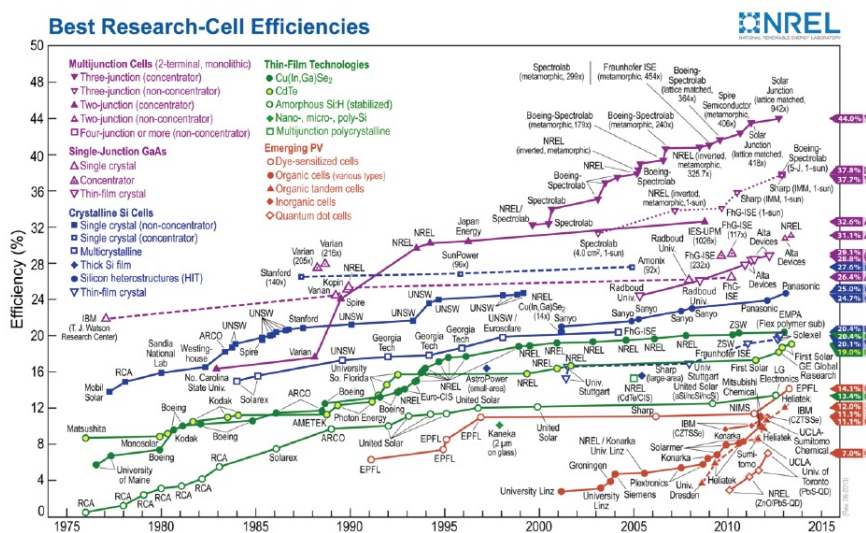


Figure 2.5: Leading Solar Cell Efficiencies

Appendix D: Sensitivity analysis

Table 2.5 reports the sensitivity analysis with respect to the following parameters

$$\{\varepsilon_e, \varepsilon_y, \rho_f, \rho_g, S, \alpha_g, \varepsilon_f, \eta, \gamma\}.$$

Unless otherwise specified, the central value corresponds to the parameter value used in the main specification and the high and low values correspond

to a 50 percent increase and a 50 percent decrease from the central value respectively. I use the percent change in the carbon tax required to obtain the Power Plan target from endogenous innovation as a summary statistic for the robustness analysis.

The first two blocks of Table 2.5 report the robustness analysis for the five parameters that I did not pin down using the 1970s oil shocks, $\{\varepsilon_e, \varepsilon_y, \rho_f, \rho_g, S\}$. I recalibrate the model for each of the different values of these parameters. Since there is general consensus in the literature that green and fossil energy are gross substitutes, i.e., $\varepsilon_e > 1$, I set the low value for ε_e to 1.1 and the high value to 1.9 so that the interval is symmetric. Additionally, the model is designed such that $\varepsilon_y = 0$ but this Leontief condition introduces kinks which are difficult to handle numerically. For $\varepsilon_y < 0.05$, the value used in the main specification, it is very difficult to solve the model. To understand the consequences of setting $\varepsilon_y = 0.05$ instead of $\varepsilon_y = 0$, I look at the results for a small increase in ε_y , $\varepsilon_y = 0.06$.

As in AABH, increases in ε_e increase the role of endogenous innovation. Small changes in ε_y have very little effect, suggesting that the approximation error from setting $\varepsilon_y = 0.05$ instead of zero is relatively small. The number of scientists has no effect on the percent change in the carbon tax. The changes in the number of scientists are completely offset by changes in the efficiency with which scientists produce new ideas, γ .

The last block of Table 2.5 reports robustness analysis for four of the parameters I pinned down using the oil shocks, $\{\alpha_g, \varepsilon_f, \eta, \gamma\}$. I do not recalibrate the model for the different alternative values of these parameters. The diminishing returns to innovation, η , has the largest impact on the results of all of these parameters. If η is large, then agents innovate more in the green energy sector in response to the carbon tax, increasing the effects from endogenous innovation. The reverse reasoning applies if η is small.

Table 2.5: Percent Change in the Carbon Tax Required to Obtain the Target

Parameter	High	Central	Low
Imposed parameters			
Output elasticity of substitution: ε_y	19.1	19.2	-
Direct from data series			
Energy elasticity of substitution: ε_e	21.7	19.2	17.4
Sector diversity: $\{\rho_f, \rho_g\}$	18.7	19.2	19.8
Population of scientists: S	19.2	19.2	19.2
Method of moments			
Green energy labor share: α_g	29.5	19.2	17.3
Fossil elasticity of substitution: ε_f	19.7	19.2	17.8
Diminishing returns to research: η	27.2	19.2	5.9
Scientist efficiency: γ	12.2	19.2	26.3

2.7.4 Comparison to simulation models

It is useful to compare the climate mitigation costs in the present model with existing microeconomic models used for policy analysis (which assume innovation is exogenous). The American Clean Energy and Security Act (H.R. 2454, Waxman-Markey) spawned many analyses of the costs of a cap and trade system in the US. The Waxman-Markey Bill stipulated a 42 percent reduction in emissions by 2030. Table 2.6 compares the implied carbon price change from different model simulations of the Waxman-Markey Bill. The bottom two rows of Table 2.6 compute the carbon tax required to achieve a 42 percent reduction in emissions in the present model for both the exogenous and endogenous innovation cases (with $\phi = 0.5$).

Table 2.6: Model Comparison: 42 percent Reduction in Emissions by 2030

Model	Carbon Price (2013 dollars)
Chamber	\$52.54
EPA	\$30.87
Heritage	\$109.29
MIT	\$15.22
Exogenous innovation	\$49.98
Endogenous innovation	\$41.28

The carbon taxes in the present model are within in the range of those

typically calculated in the micro simulation models. The wide range of results from the micro models stems in part different assumptions about the exogenous rates and directions of technical progress (Pew Center On Global Climate Change (2010)). It is important to point out that the mapping between the present model and the results from the micro models in Table 2.6 is not perfect. These results come from simulations of the Waxman-Markey Bill which includes efficiency standards, performance standards for coal power plants, and R&D subsidies, international carbon offsets, as well as a cap and trade system. While it is reassuring that the carbon tax estimates from the present model are within the range produced by the micro studies, further conclusions beyond this general ball parking are not really justified.

2.7.5 2003 Oil Shock

One limitation of the calibration strategy is that early 1970s oil shocks occurred forty years ago. It's possible that some of the parameter values could have changed over time. To address this concern, I compare the responsiveness of innovation to the 2003 oil shock in the data and in the calibrated model. Note that I do not change the calibration of the model in this exercise; the parameter values are the same as those listed in Table 2.2. The goal of this exercise is to determine if the model calibrated to the OPEC oil shock matches the innovation response to the 2003 oil shock.

As with the early 1970s oil shocks, I begin the simulation on a balanced growth path and then introduce an oil shock. I choose the size of the shock to match the average increase in the foreign oil price during the 2003-2007 time period compared to the previous five years (1998-2002). Table 2.7 reports the elasticities of fossil and green innovation with respect to a change in the foreign oil price in the model and in the data.

Table 2.7: Elasticity of innovation w.r.t the foreign oil price

	Model	Data
Fossil Innovation	0.8	0.7
Green Innovation	0.5	0.7
Total Energy Innovation	0.7	0.7

The empirical and model estimates are similar, suggesting that the parameters governing the responsiveness of energy innovation to price changes have not changed substantially over time. The largest discrepancy between the model and the data is that the price elasticity of green innovation is lower in the model. This difference could be partly explained by policies or expectations of policies which encouraged green innovation during this time. Over this period, an increasing number of states adopted renewable portfolio standards, Europe implemented the pilot phase of its carbon trading system (EU-ETS), and congress and the president began to lay the groundwork for the Energy Independence and Security Act of 2007. All these policy developments would encourage green innovation and lead to a higher empirical price elasticity.

This exercise has two caveats. First, energy prices and energy innovation are not stable for a sustained period preceding the 2003 oil shock, suggesting that the assumption that economy was on a long-run balanced growth path prior to the shock is imperfect. Moreover, following the 1970s, agents learned that energy prices are uncertain and they formed expectations over future energy prices. I do not model expectations in this robustness analysis. The advantage of calibrating the model using the early 1970s' oil shocks instead of the 2003 oil shock is that it avoids both of these caveats. Energy prices were relatively flat for the twenty years prior to the shock, suggesting the economy could plausibly have been on a balanced growth path and that agents did not anticipate the early 1970s' oil shocks.

2.8 Acknowledgement

Chapter 2, in full, has been submitted for publication of the material as it may appear in the American Economic Review. Fried, Stephanie.

Chapter 3

Can Robin Hood Wear Green?

Understanding the

Distributional and Efficiency

Implications of a Carbon Tax

3.1 Introduction

Climate policies, such as carbon taxes and cap-and-trade systems, are designed to increase the price of carbon-based energy and, thus, distort both households' and firms' decisions. The welfare effects of the higher energy prices and the accompanying general equilibrium distortions can vary across different subsets of the population. Understanding these distributional implications is necessary for the government to construct a climate policy that is politically feasible and aligned with its social welfare function. This paper develops a quantitative macroeconomic model to analyze the distribution of welfare effects from a carbon tax. We find that the method through which the government rebates the carbon-tax revenue to the household has substantial implications for the welfare consequences of the tax on different age and income groups.

Following the macroeconomic literature examining distortionary taxation in an overlapping generations framework (e.g., Conesa et al. (2009); Peterman (2013)), we develop a general equilibrium, overlapping generations (OLG) model with within cohort income heterogeneity and distortionary taxation. The welfare consequences of the distortions from the labor, capital, and carbon taxes depend on a household's age, income, and energy consumption. Our OLG framework allows us to analyze the welfare effects across age groups. Additionally, we model energy as an input in both firm production and household utility. Empirically, lower income households spend a larger fraction of their income on energy, which could cause the carbon tax to be regressive (Hassett et al. (2009)). Our model of household energy consumption incorporates this feature of data.

We calibrate the model and use it to analyze the welfare effects from a carbon tax of 35 dollars per ton of CO₂. The method in which the government rebates the carbon-tax revenue to the household can substantially alter the distributional effects of the policy (e.g., Bento et al. (2009); Dinan and Rogers (2002); Metcalf (2007)). We consider four options for the carbon tax revenue: (1) the government does not rebate the revenue and instead “throws it into the ocean”, (2) rebates through lump-sum transfers to the household, (3) rebates

through a reduction in the capital-tax rate, and (4) rebates through a reduction in the labor-tax rate. While the carbon tax introduces a new distortion, the rebate mechanism has the potential to reduce the pre-existing distortions in the economy. The accompanying general equilibrium effects on factor prices and their implications for welfare depend on the interaction between the rebate mechanism and the carbon tax.

We analyze these interactions in both the steady state and during the transition to the steady state. We find that the welfare costs of the carbon tax coupled with the capital or labor-tax rebates are substantially larger over the transition than in the steady state. A key difference between the steady state and the transition is that an agent born in the steady state experiences the policy for his entire life cycle, while an agent undergoing the transition does not. These life cycle factors substantially increase the average welfare cost of the labor-tax rebate during the transition because agents who are retired when the government introduces the policy do not receive any rebate, and, thus, are considerably worse off.¹ Additionally, the welfare cost of the capital-tax rebate is substantially higher over the transition because factor prices take time to adjust. In particular, the wage and risk-free rates rise gradually as the economy accumulates capital, causing welfare to increase over the transition.

The higher transitional welfare costs of the capital- and labor-tax rebates lead to opposite welfare rankings for the rebate schemes in the steady state versus in the transition. The least costly rebate options in the steady state are the capital- and labor-tax rebates but over the transition the least costly option is the lump-sum rebate. When the government rebates through either the labor- or capital-tax rates, it reduces the distortions from the pre-existing taxes in the economy, which lowers the overall welfare cost of the policy in steady state. In contrast, the lump-sum rebate does not reduce the pre-existing distortionary taxes, and, thus, results in a higher steady-state welfare cost. However, the welfare cost of the lump-sum rebate over the transition is relatively small because the rebate reduces the agent's need to save to finance

¹However, these age-cohort effects do not impact average steady state welfare because every agent experiences the policy for his entire life cycle.

retirement since it increases government transfer payments in every period of the life cycle, including retirement. Thus, when the government introduces the carbon tax policy, the living population has saved more than they would have, had they known that the government was going to introduce the policy. All else constant, these “extra savings” increase the expected welfare over the remainder of living population’s life cycle, making the lump-sum rebate the least costly mechanism over the transition.

We analyze the distributional effects of a carbon tax across both income and age groups for each of the different rebate mechanisms. We find that the government’s rebate decision substantially affects the carbon tax policy’s distributional implications. In particular, with regards to the distribution of welfare effects across income groups, the policy is regressive when the government does not rebate the revenue because lower income households have larger energy budget shares. However, the government can completely reverse the regressiveness of the policy by rebating the revenue through uniform lump-sum transfers. The policy is progressive under the lump-sum transfers because, since the transfer is the same size for all households, it represents a larger fraction of a low-income household’s lifetime income, and, thus, results in larger benefits for this demographic group.

The distributional effects of the policy across different age cohorts are primarily determined by general equilibrium changes in factor prices, by the agent’s ability to smooth his consumption in response to the policy shock, and by the size of the rebate relative to total remaining lifetime income. All else constant, changes in the wage rate have larger welfare consequences for younger agents because expected future labor income comprises the largest fraction of total remaining lifetime income for this age group. Changes in the risk-free rate have the largest consequences for middle-aged agents because expected future capital income comprises the largest fraction of total remaining lifetime income for this age group. Finally, a retiree is least able to smooth his consumption in response to the increase in energy prices because he has depleted his savings and can no longer adjust his labor supply. We find that the changes in factor-prices are such that the lump-sum rebate is the most the welfare improving for

the older retirees, the capital-tax rebate is the most welfare improving for the middle-aged agents, and the labor-tax rebate is the most welfare improving for the very young agents.

Our paper builds on Carbone et al. (2013) and Williams et al. (2015). Carbone et al. (2013) develop an OLG model with representative age cohorts, in which the revenue from the carbon tax is used to reduce pre-existing distortionary taxes. Williams et al. (2015) link the dynamic output from this OLG model to a static model with income heterogeneity to explore the near-term incidence of the carbon tax across income quintiles. We combine the approaches in these earlier papers by designing an OLG model that includes within cohort income heterogeneity. Thus, our paper contributes to this earlier literature by exploring the distributional impacts across income and age groups over the transition (as opposed to just in the near-term), simultaneously incorporating the five channels: (1) household energy consumption, (2) general equilibrium changes in factor prices, (3) revenue recycling, (4) income heterogeneity, and (5) life cycle dynamics.

The paper precedes as follows: Section 3.2 presents the computational model and Section 3.3 discusses the functional forms and the calibration. Section 3.4 reports the results from different revenue-neutral implementations of a carbon tax. Finally, Section 3.5 concludes.

3.2 Model

3.2.1 Demographics

Time is discrete and there are J overlapping generations. A continuum of new agents is born each period. The population of newborn agents grows at a constant rate, n . Lifetime length is uncertain and mortality risk varies over the lifetime. Parameter Ψ_j denotes the probability an agent lives to age $j + 1$ conditional on being alive at age j . All agents who live to age J die with probability one the following period, $\Psi_J = 0$. Since agents are not certain how long they will live, they could die with positive asset holdings. In this case,

we treat the assets as accidental bequests and redistribute them lump-sum across all living individuals in the form of transfers, T_a . All agents are forced to retire at (exogenous) age j_r . Upon retirement, agents receive social security payments, S .

3.2.2 Households

An individual is endowed with one unit of productive time per period that he divides between labor and leisure. An agent i , age j , earns labor income $y_{i,j}^h \equiv w\mu_{i,j}h_{i,j}$, where w is the market wage-rate, $h_{i,j}$ denotes hours worked, $\mu_{i,j}$ is the agent's idiosyncratic productivity. The log of an agent's idiosyncratic productivity consists of four additively separable components,

$$\log \mu_{i,j} = \epsilon_j + \xi_i + \nu_t + \theta_t. \quad (3.1)$$

This specification is based on the estimates in Kaplan (2012) from the Panel Study of Income Dynamics (PSID). Variable ϵ_j governs age-specific human capital. Variable $\xi_i \sim NID(0, \sigma_\xi^2)$ is an individual-specific fixed effect (or ability) that is observed at birth and is constant for an agent over the life cycle. Variable $\theta_t \sim NID(0, \sigma_\theta^2)$ is an idiosyncratic transitory shock to productivity received every period, and ν_t is an idiosyncratic persistent shock to productivity, which follows a first-order autoregressive process:

$$\nu_t = \rho\nu_{t-1} + \psi_t \text{ with } \psi_t \sim NID(0, \sigma_\psi^2) \text{ and } \nu_1 = 0. \quad (3.2)$$

Thus, agents across cohorts are differentiated along one dimension which affects their labor productivity: their age-specific human capital, ϵ_j . Agents within an age cohort are differentiated along three dimensions which affect their labor productivity: their ability, ξ_i , the realization of the transitory shock, θ_t , and the realization of the persistent shock, ν_t . Different permanent ability types generate an initial productivity distribution within a cohort. As the cohort ages, different realizations of ν_t across individuals over time increase the variance of this distribution.

Agents cannot insure against idiosyncratic productivity shocks by trading explicit insurance contracts, and there are no annuity markets to insure

against mortality risk. However, agents are able to partially self insure against labor-income risk by purchasing risk-free assets, $a_{i,j}$, that have a pre-tax rate of return, r_t .

Agents split their income between saving with a one-period, risk-free asset, $a_{i,j}$, and consumption. Agents can consume both a generic consumption good, $c_{i,j}$, and a carbon emitting energy good, $e_{i,j}^c$. Energy consumption includes expenditures on electricity, gasoline, heating oil, etc. All agents must consume a minimum amount of energy, \bar{e} . Variable \bar{e} represents subsistence energy required for light, transportation, heat, etc. Agents choose labor, savings, generic consumption, and energy consumption to maximize their lifetime utility

$$u(c_{i,1}, h_{i,1}) + \mathbb{E} \left\{ \sum_{s=1}^{J-j-1} \beta^s \prod_{q=1}^s (\Psi_q) u(c_{i,s+1}, e_{i,s+1}^c - \bar{e}, h_{i,s+1}) \right\}. \quad (3.3)$$

We take the expectation in equation (3.3) with respect to the stochastic processes governing the idiosyncratic productivity shocks. Agents discount the next period's utility by the product of Ψ_j and β . Parameter β is the discount factor conditional on surviving. The product, $\beta\Psi_j$, is the unconditional discount rate. Agents do not derive utility from energy consumption required for subsistence, \bar{e} .

3.2.3 Production

Perfectly competitive firms produce a generic final good, Y , from capital, K , labor (measured in efficiency units), N , and carbon-emitting energy, E^p . The final good is the numeraire and can be used for both consumption and investment. The production technology features a constant elasticity of substitution, ϕ , between a capital-labor composite, $K^\zeta N^{1-\zeta}$, and energy,

$$Y = A \left[(K^\zeta N^{1-\zeta})^{\frac{\phi-1}{\phi}} + (E^p)^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}. \quad (3.4)$$

The country behaves as a small open economy with respect to energy; energy is supplied exogenously at price, p_e .

3.2.4 Government Policy

The government performs three activities: (1) it spends resources in an unproductive sector, G , (2) it runs a pay-as-you-go social security system, and (3) it taxes capital and labor income, and energy (i.e., a carbon tax) to finance G .²

The government pays social security benefits, S , to all agents that are retired. The benefits are independent of the agent's lifetime earnings. Instead, retired agents receive an exogenous fraction, b , of the average income of all working individuals. The government finances the social security system with a flat tax on labor income, τ_s . It sets the tax rate to ensure that the social security system has a balanced budget in every period.

The government taxes capital income according to a constant marginal tax rate, τ_k . An agent's period t capital income is the return on his assets plus the return on any assets he receives as accidental bequests, $y^k \equiv r(a + T_a)$. The government taxes labor income according to potentially progressive tax schedule, $T^h(\tilde{y}^h)$, where \tilde{y}^h denotes the agent's taxable labor income. An agent's taxable labor income is his labor income, y^h , net of his employer's contribution to social security which is not taxable under U.S. tax law. Thus, $\tilde{y}^h \equiv y^h(1 - 0.5\tau_s)$, where $0.5\tau_s y^h$ is the employer's social security contribution.

Finally, the government can tax carbon energy to both reduce carbon emissions and to finance its spending. A carbon tax, τ_c , places a price on the externality, carbon. Thus, the government applies the tax per unit of energy consumed, raising the price of energy from p_e to $p_e + \tau_c$. In one of the policy experiments we analyze, the government rebates the carbon-tax revenue through lump-sum transfers to the households, T_c .

²Given that fossil fuel combustion accounts for over 80 percent of GHG emissions, a carbon tax effectively functions as a tax on energy use.

3.2.5 Definition of a Stationary Competitive Equilibrium

We define a stationary competitive equilibrium. The individual state variables, x , are asset holdings, a , idiosyncratic labor productivity, μ , and age j .

Given a social security replacement rate, b , government expenditures, G , demographic parameters, $\{n, \Psi_j\}$, a sequence of age-specific human capital, $\{\epsilon_j\}_{j=1}^{j_r-1}$, a labor-tax function, $T^h : \mathbb{R}_+ \rightarrow \mathbb{R}_+$, a capital-tax rate, τ_k , a social security tax rate, τ_s , a carbon-tax rate, τ_c , transfers from the climate policy, T_c , an energy price, p_e , a utility function $U : \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$, social security benefits, S , and factor prices, $\{w, r, p_e\}$, a stationary competitive equilibrium consists of agents' decisions rules, $\{c, h, e^c, a'\}$, firms' production plans, $\{E^p, K, N\}$, transfers from accidental bequests T_a , and the distribution of individuals, $\Phi(x)$, such that the following holds:

1. Given prices, policies, transfers, benefits, and ν that follows equation (3.2) the agent maximizes equation (3.3) subject to:

$$\begin{aligned} c + (p_e + \tau_c)e^c + a' &= \\ &\mu h w (1 - \tau_s) + (1 + r(1 - \tau_k))(a + T) - T^h(\mu h w (1 - .5\tau_s)) \\ &\quad + T_c \text{ for } j < j_r \\ c + (p_e + \tau_c)e^c + a' &= S + (1 + r(1 - \tau_k))(a + T) + T_c \text{ for } j \geq j_r \\ c \geq 0, e^c \geq 0, 0 \leq h \leq 1, a \geq 0, a_1 &= 0 \end{aligned}$$

2. Firms' demands for E^p , K , and N satisfy:

$$r = \zeta A \left[(K^\zeta N^{1-\zeta})^{\frac{\phi-1}{\phi}} + (E^p)^{\frac{\phi-1}{\phi}} \right]^{\frac{1}{\phi-1}} (K^\zeta N^{1-\zeta})^{-\frac{1}{\phi}} \left(\frac{N}{K} \right)^{1-\zeta} - \delta \quad (3.5)$$

$$w = (1 - \zeta) A \left[(K^\zeta N^{1-\zeta})^{\frac{\phi-1}{\phi}} + (E^p)^{\frac{\phi-1}{\phi}} \right]^{\frac{1}{\phi-1}} (K^\zeta N^{1-\zeta})^{-\frac{1}{\phi}} \left(\frac{K}{N} \right)^\zeta \quad (3.6)$$

$$p_e + \tau_c = A \left[(K^\zeta N^{1-\zeta})^{\frac{\phi-1}{\phi}} + (E^p)^{\frac{\phi-1}{\phi}} \right]^{\frac{1}{\phi-1}} (E^p)^{-\frac{1}{\phi}} \quad (3.7)$$

3. The social security policy satisfies:

$$S = b \left(\frac{wN}{\sum_{j < j_r} \Phi(x)} \right) \quad (3.8)$$

$$\tau_s = \frac{S \sum_{j \geq j_r} \Phi(x)}{wN} \quad (3.9)$$

4. Transfers from accidental bequests satisfy:

$$T_a = \sum (1 - \Psi) a' \Phi(x) \quad (3.10)$$

5. The government budget balances:

$$G = \sum [\tau_k r(a + T_a) + T^h(\mu h w(1 - .5\tau_s)) + \tau_c e^c] \Phi(x) + \tau_c E^p - T_c \quad (3.11)$$

6. Markets clear:

$$K = \sum a \Phi(x), \quad N = \sum \mu h \Phi(x) \quad (3.12)$$

$$\sum (c + p_e e^c + a') \Phi(x) + G + p_e E^p = Y + (1 - \delta)K \quad (3.13)$$

7. The distribution of $\Phi(x)$ is stationary, that is, the law of motion for the distribution of individuals over the state space satisfies $\Phi(x) = Q_\Phi \Phi(x)$ where Q_Φ is the one-period recursive operator on the distribution.

3.3 Calibration and Functional Forms

We calibrate the model in two steps. In the first step, we choose parameter values for which there are direct estimates in the data. In the second step, we calibrate the remaining parameters so that certain targets in the model match the values observed in the U.S. economy. Table 3.1 reports the parameter values.

Table 3.1: Calibration Parameters

Parameter	Value	Target
Demographics		
Retire Age: j_r	66	Assumption
Max Age: J	100	Assumption
Surv. Prob: Ψ_j	-	Data
Pop. Growth: n	1.1%	Data
Firm Parameters		
Capital Share: ζ	0.36	Data
Substitution Elasticity: ϕ	0.5	Van der Werf (2008)
Depreciation: δ	8.33%	$\frac{I}{Y} = 25.5\%$
Productivity: A	1	Normalization
Energy price: p_e	0.0025	$\frac{P_e E}{Y} = 0.05$
Productivity Parameters		
Persistence Shock: σ_ν^2	0.017	Kaplan (2012)
Persistence: ρ	0.958	Kaplan (2012)
Permanent Shock: σ_ξ^2	0.065	Kaplan (2012)
Transitory Shock: σ_θ^2	0.081	Kaplan (2012)
Preference Parameters		
Conditional Discount: β	1.003	$\frac{K}{Y} = 2.7$
Risk Aversion: θ_1	2	Conesa et al. (2009)
Frisch Elasticity: θ_2	0.5	Kaplan (2012)
Disutility of Labor: χ	63	Avg. $h_{i,j} = 0.333$
Subsistence Energy: \bar{e}	9	$\Delta\Omega = -12.8$
Energy exp.: $1 - \gamma$	0.06	Avg. $\Omega = 10.2\%$
Government Parameters		
Labor Tax Function: Υ_0	0.258	GS (1994)
Labor Tax Function: Υ_1	0.768	GS (1994)
Capital Tax Rate: τ_k	0.23	Gravelle (2004)
Government Spending: G	0.140	$\frac{G}{Y} = 0.17$
Replacement Rate: b	0.5	Conesa et al. (2009)

3.3.1 Demographics

In the model, agents are born at the real-world age of 20 that corresponds to a model age of 1. Agents are exogenously forced to retire at real-world age of 66. If an individual survives until age 100, he dies the next period. We choose the conditional survival probabilities based on the estimates in Bell and Miller (2002). We adjust the size of each cohort's share of the population to account for a population growth rate of 1.1 percent.

3.3.2 Preferences

Agents have time-separable preferences over a consumption-energy composite, \tilde{c} , and hours, h . The utility function is given by

$$U(\tilde{c}, h) = \frac{\tilde{c}^{1-\theta_1}}{1-\theta_1} - \chi \frac{h^{1+\frac{1}{\theta_2}}}{1+\frac{1}{\theta_2}} \quad (3.14)$$

where $\tilde{c} = c^\gamma(e - \bar{e})^{1-\gamma}$. This functional form is separable and homothetic in the consumption-energy composite and labor, implying a constant Frisch elasticity of labor supply over the life cycle.

We determine β to match the US capital-output ratio of 2.7. We choose χ such that agents spend an average of one third of their time endowment working. Following Conesa et al. (2009), we use two for the coefficient of relative risk aversion, $\theta_1 = 2$, and following Kaplan (2012), we use 0.5 for the Frisch elasticity, $\theta_2 = 0.5$.

One reason a carbon tax could be regressive is because lower income individuals devote a larger share of their total consumption expenditures to energy. In particular, using data from the Consumer Expenditures Survey (CEX), 1981-2003, we calculate that the energy share for individuals in the top half of the expenditure distribution is approximately two thirds the size of energy share for individuals in the bottom half.

Together, parameters \bar{e} and γ determine a household's energy share of total consumption, and how this share varies with the household's total consumption expenditures. Let \mathcal{M} denote an agent's total consumption expenditures, $\mathcal{M} = c + p_e e^c$. Then energy share of total consumption expenditures,

Ω , is

$$\Omega = (1 - \gamma) + \frac{\gamma p_e \bar{e}}{(1 - \gamma)\mathcal{M}}. \quad (3.15)$$

For $\bar{e} = 0$, energy share is $1 - \gamma$ for all expenditure levels. For $\bar{e} > 0$, energy share decreases with expenditures. Higher \bar{e} increases the responsiveness of energy share to changes in total expenditures. We pin down \bar{e} and γ to match the average energy share in the population and the percent difference in the energy share of the top and bottom halves of the expenditure distribution, $\Delta\Omega = \frac{\Omega_{top} - \Omega_{bottom}}{\Omega_{bottom}} \times 100$, based on the CEX data. The average energy share in the population is 10.2 percent.

In the CEX data, $\Delta\Omega = -33\%$. However, the percent difference in expenditures between the top and bottom halves of the distribution is 142 percent in the CEX, but only 54 percent in our model. Therefore, we adjust for the smaller variance in income in our model and target $\Delta\Omega = -12.8\%$. Table 3.2 reports the values of the energy moments in the model and the data. The model matches the targeted moments well.

Table 3.2: Energy Moments

Moment	Model	Data
Avg. Share	10.5%	10.2%
$\Delta\Omega$	-12.4%	-12.8%

Our calibration strategy does not have enough degrees of freedom to directly target the variation in energy share over the life cycle. Figure 3.1 compares energy share over the life cycle in the bottom and top halves of the expenditure distribution in the model and from the CEX data. Energy share is relatively flat over the life cycle in the data and but it exhibits a U-shape in the model. This U-shape arises because consumption expenditures are humped-shaped over the life cycle and energy share is inversely related to consumption expenditures. Therefore, while our model estimate of energy share fits the average data for the high and low income groups, it misses some of the dynamics over the life cycle.³

³The difference in energy share is larger in the data than in the model because the

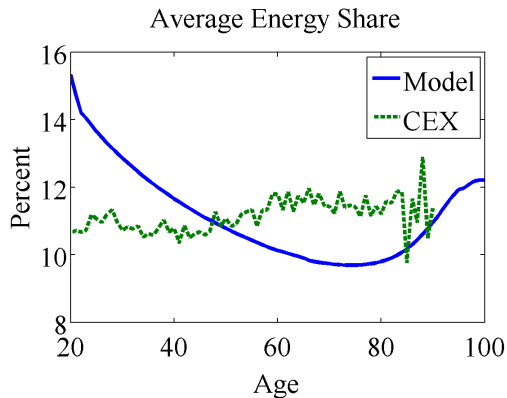


Figure 3.1: Energy Share Over the Life Cycle: Model and Data

3.3.3 Idiosyncratic Productivity

We calibrate the idiosyncratic labor productivity shocks based on the estimates from the PSID data in Kaplan (2012).⁴ These permanent, persistent, and transitory idiosyncratic shocks to individuals' productivity are normally distributed with a mean of zero. We set the remaining shock parameters in accordance with the estimates in Kaplan (2012): $\rho = 0.958$, $\sigma_{\xi}^2 = 0.065$, $\sigma_{\nu}^2 = 0.017$ and $\sigma_{\theta}^2 = 0.081$. We discretize all three of the shocks in order to solve the model, using two states to represent the transitory and permanent shocks and five states for the persistent shock.

3.3.4 Age-Specific Human Capital

We set $\{\epsilon_j\}_{j=0}^{j_r-1}$ to match the values estimated in Kaplan (2012). These values are based off the average hourly earnings by age in the Panel Survey of Income Dynamics.

3.3.5 Production

We use 0.5 for the elasticity of substitution between the capital-labor composite and energy, ϕ . This parameter choice is within the range of estimation in consumption expenditures is larger in the data.

⁴For details on estimation of this process, see Appendix E in Kaplan (2012).

mates reported in Van der Werf (2008). We use $\zeta = 0.36$ for capital's share in the capital-labor composite. We calibrate the price of energy, p_e , so that energy's share of production is five percent.

3.3.6 Government Policies and Tax Functions

We begin our policy experiments in a baseline equilibrium that mimics the U.S. tax code. We follow the quantitative public finance literature (e.g., Castaneda et al. (2003); Conesa and Krueger (2006); Conesa et al. (2009); Peterman (2013)) and use estimates of the U.S. tax code from Gouveia and Strauss (1994). Gouveia and Strauss (1994) match the U.S. tax code to the data using a three parameter functional form,

$$T^h(y_h; \Upsilon_0, \Upsilon_1, \Upsilon_2) = \Upsilon_0 \left(y_h - \left(y_h^{-\Upsilon_1} + \Upsilon_2 \right)^{\frac{-1}{\Upsilon_1}} \right) \quad (3.16)$$

Parameter Υ_0 governs the average tax rate and parameter Υ_1 controls the progressivity of the tax policy. To ensure that taxes satisfy the budget constraint, we leave parameter Υ_2 free. Gouveia and Strauss (1994) estimate that $\Upsilon_0 = 0.258$ and $\Upsilon_1 = 0.768$.

We determine government spending, G so that it equals 17 percent of output, its average empirical value in the U.S data. We set the tax rate on capital income, τ_k , to 23 percent, based on estimates in Gravelle (2004). Following Conesa et al. (2009), the replacement rate for the social security system, b , is 50 percent. We choose the payroll tax, τ_s , to ensure that the social security system has a balanced budget in every period.

3.4 Results

We simulate a baseline economy with no carbon tax and we conduct a series of counterfactual simulations in which we introduce a constant carbon tax set at 35 dollars per ton CO₂. We consider four different rebate options for the carbon tax revenue: (1) the government does not rebate the revenue and instead “throws it into the ocean,” (2) rebates through equal, lump-sum

transfers to the household, (3) rebates through a reduction in the capital-tax rate, and (4) rebates through a reduction in the labor-tax rate.

To implement changes in the labor-tax, we fix Υ_2 at its value in the baseline and reduce the labor-tax rate by lowering parameter Υ_0 to minimize changes in the progressivity of the labor-tax function. It is necessary to adjust the labor-tax rate not only in the labor-tax rebate case but also in the other three counterfactual simulations to ensure that government spending under the carbon tax equals its baseline value. This is because in the no rebate and lump-sum rebate cases, the carbon tax leads to changes in aggregate labor and capital supplies, which affect aggregate tax-revenue. In the capital-tax rebate case, the carbon-tax revenue exceeds the capital-tax revenue when the capital-tax is at its baseline level. Therefore, we set the capital-tax rate to zero and lower the labor-tax rate to finance the same level of government spending as in the baseline. Table 3.3 reports the tax parameters in the baseline and in each of the four simulations.

Table 3.3: Tax Parameters

	No Carbon Tax	Carbon Tax			
		No Rebate	Lump-sum Rebate	Capital Rebate	Labor Rebate
Labor tax: Υ_0	0.26	0.26	0.27	0.25	0.21
Labor tax: Υ_1	0.77	0.77	0.77	0.77	0.77
Labor tax: Υ_2	17.51	17.51	17.51	17.51	17.51
Capital tax: τ_k	0.23	0.23	0.23	0.00	0.23
Carbon tax: $\frac{\tau_c}{p_e}$	0.00	0.33	0.33	0.33	0.33

Table 3.4 reports the fraction of government revenue from the labor, capital, and carbon taxes in the baseline and in each of the four counterfactual simulations. Note that in the no-rebate simulation, total tax revenue exceeds the level of government spending, G . Since the government throws the carbon-tax revenue into the ocean, it does not contribute to financing G in this case. Section 3.4.1 compares the steady states in each of the counterfactual economies with the baseline and Section 3.4.2 reports the results over the tran-

sition. In both sections, we analyze the results for different income quintiles. We form the income quintiles from agents' realized lifetime expenditures.

Table 3.4: Percent of Government Revenue

	No carbon Tax	Carbon Tax			
		No Rebate	Lump-sum Rebate	Capital Rebate	Labor Rebate
Labor Tax	84.10	84.10	84.63	81.87	67.03
Capital Tax	15.91	15.90	15.37	0.00	15.03
Carbon Tax	0.00	17.74	17.60	18.15	17.96
Lump-Sum	-	-	-17.60	-	-

3.4.1 Steady State

We compare the baseline and counterfactual economies in the steady state. Column 2 of Table 3.5 reports the baseline values of the aggregate variables and columns 3-6 report the percent change in the aggregate variables from their baseline values in each of the three simulations.⁵

Regardless of the rebate mechanism, the carbon tax alters both the firms' and the households' decisions. On the firm side, the carbon tax reduces the firm energy use, which lowers the marginal products of both capital and labor. On the household side, the carbon tax raises the relative price of the consumption-energy composite, \tilde{c} . This price change increases the cost of retirement, which raises agents' incentives to save. Additionally, the price change distorts the household's intratemporal allocation between \tilde{c} and leisure, generating both income and substitution effects. The income effect causes households to increase their hours, because the higher cost of \tilde{c} makes them relatively poorer. However, the substitution effect causes households to reduce their hours and substitute leisure for consumption, since the cost of \tilde{c} relative to leisure is higher.

The general equilibrium interactions among these different distortions lead to changes in factor prices. For example, all else constant, the decline in

⁵We define the average net (after-tax) wage as $(1 - \tau_l)w$ where τ_l is the average labor-tax rate.

the marginal product of capital reduces the risk-free rate. Like the carbon tax, these factor price changes impact household decisions through both income and substitution effects. The government's rebate mechanism determines which channels dominate and the corresponding implications for the steady-state aggregates.

In the no-rebate case, the government increases its total tax levy, reducing each agent's disposable income and his corresponding ability to save. This negative income shock dominates the agent's need to increase savings to finance the more expensive retirement under the carbon tax, and the aggregate capital stock falls by 0.92 percent. All else constant, the lower capital stock and energy use reduce the marginal product of labor and the market wage falls by 2.86 percent. Moreover, hours rise because the income effects from the increased tax levy and from the carbon tax dominate the substitution effect from the carbon tax.

In the lump-sum rebate case, each agent receives a uniform transfer over his entire life cycle. This transfer reduces the agent's need to save, since it increases government transfer payments during retirement. This effect dominates the agent's need to increase savings to finance the more expensive retirement, and the aggregate capital stock falls by 3.56 percent. The lower capital stock and energy use reduce the marginal product of labor and the market wage falls by 4.00 percent. Moreover, both hours and consumption fall because the substitution effects from the lower wage rate and from the carbon tax dominate the corresponding income effects and agents substitute leisure for consumption.

In the capital-tax rebate case, each agent receives a rebate proportional to his capital income. All else constant, the lower capital-tax rate increases the after-tax risk-free rate, raising the agent's incentives to save. This effect complements the agent's need to increase retirement savings and the aggregate capital stock rises by 8.62 percent. The higher capital stock dominates the decline energy use, causing the marginal product of labor and the accompanying market wage to rise. Moreover, both hours and consumption rise because the substitution effect from the higher wage dominates and agents substitute

consumption for leisure.

In the labor-tax rebate case, each agent receives a rebate proportional to his labor income. All else constant, the lower labor-tax rate raises the after-tax wage, increasing each agent's disposable income and his corresponding ability to save. This positive income shock complements the agent's need to increase retirement savings and the aggregate capital stock rises by 2.07 percent. The higher capital stock and lower labor-tax rate dominate the decline energy use, causing after-tax market wage to rise. Moreover, both hours and consumption rise because the substitution effect from the higher after-tax wage dominates and agents substitute consumption for leisure.

Table 3.5: Steady State Aggregates

	Base- line	% Δ From Baseline: Carbon Tax			
		No Rebate	Lump-sum Rebate	Capital Rebate	Labor Rebate
Aggregates					
Y	0.84	0.18	-2.72	2.31	0.28
N	0.52	2.07	-1.01	0.17	0.55
K	2.27	-0.92	-3.56	8.62	2.07
C	0.39	-2.47	-0.25	2.84	2.31
Energy					
E^p	16.73	-13.02	-15.53	-11.15	-12.91
$E^c - \bar{E}$	10.05	-26.46	-24.78	-22.45	-22.85
$E^p + E^c$	35.78	-13.52	-14.22	-11.52	-12.45
Prices, Transfers					
$(1 - \tau_l)w$	0.73	-2.86	-4.00	2.78	5.77
$(1 - \tau_k)r$	0.03	0.88	0.19	4.77	-7.45
$T_a + T_p$	0.03	-0.71	79.89	10.29	2.96

Figure 3.2 shows the percent change in the life-cycle profiles induced by the policies. For example, the dashed red line in the top left panel plots the average percent difference in total savings between the baseline and the capital-tax rebate for each age. Savings are lower in every period of the life cycle under the lump sum rebate (relative to the baseline) because the lump-sum rebate reduces agents' need to save to finance retirement. In contrast, savings are higher in every period of the life cycle under the capital-tax rebate

because the rise in the after-tax risk-free rate increases the return to savings.

Additionally, the higher after-tax risk-free rate encourages agents to delay consumption until later in life since an additional unit of consumption for a young agent costs more in terms of forgone future consumption (bottom left panel of Figure 3.2). Agents also shift hours to earlier in life because the increase in the after-tax risk-free rate raises the the return to working more for younger agents than for older agents (top right panel of Figure 3.2). Analogous reasoning reveals that the fall in the risk-free rate under the labor-tax rebate causes agents to shift consumption to earlier in life and hours to later in life. Finally, energy use is lower in every period of the life cycle for all the rebate options because the carbon tax raises the relative price of energy (bottom right panel of Figure 3.2). The change in energy consumption is largest for the middle-aged agents. The level of energy consumption is highest for this age group, implying that their energy demand is the most elastic.

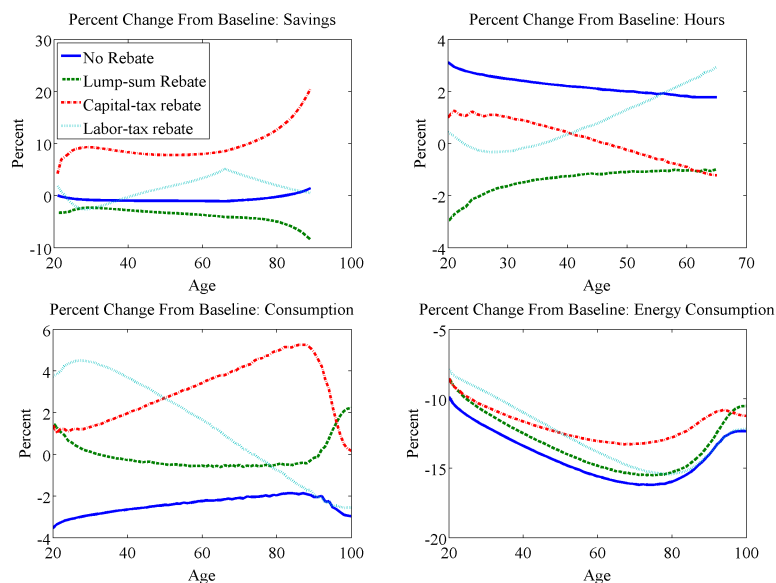


Figure 3.2: Lifecycle Profiles: Percent Change From Baseline

Welfare: Double Dividend

Table 3.6 reports the consumption equivalent variation (CEV) of the tax in each of the four counterfactual economies. We define the CEV as the

expected percent increase in consumption an agent would need in every period of his life in the baseline to make him indifferent between the baseline and the policy. Table 3.6 reports the population CEV and the CEV within each of the five income quintiles. The population CEV measures the change in the ex ante (i.e., before ability is realized), expected (with respect to the idiosyncratic shocks) welfare of a newborn individual in the long-run equilibrium. The CEV in a given income quintile measures the ex-ante change in expected welfare conditional on being born in that particular income quintile.

The population CEV is negative when the government does not rebate the carbon-tax revenue. In this case, the government raises additional revenue which reduces each agent's disposable income and does not contribute to his utility, making him considerably worse off. However, when the government does return the revenue to the household, it can more than undo the negative welfare effects from the carbon tax. The population CEV is positive under the lump-sum, capital, and labor-tax rebates. This result provides evidence of a small double dividend where the carbon tax reduces the overall welfare cost of the tax system in addition to improving environmental quality.

Under the lump-sum rebate, the second dividend occurs because the rebate partially ensures agents against negative income shocks. Agents receive the same transfer regardless of whether they have a good or bad shock. However, the transfer results in a larger percentage increase in income, and hence in consumption, when the agents has a bad shock because he has lower income in this state. This insurance improves expected welfare because agents are risk adverse.

The second dividend under the capital and labor-tax rebates implies that the carbon tax is less distortionary than the labor and capital taxes in our framework. The second dividend occurs because the household's energy demand is relatively inelastic compared to its demands for leisure and capital. Thus, under the labor and capital-tax rebates, the carbon tax shifts the tax system toward less elastic factors, reducing the accompanying distortions and deadweight loss. Additionally, with no rebate, the carbon tax increases labor supply, broadening the labor-tax base which reduces the distortions from the

labor tax.⁶ The reduced labor-market distortions from this income effect (and also from the lower labor or capital tax rates) outweigh the distortions created by the carbon tax, leading to a double dividend. Finally, by placing a tax on energy, the carbon tax reduces the distortionary effects from the pre-existing taxes on the relative marginal products of capital, labor, and energy, moving the economy closer to the optimal tax system.

The double-dividend is larger in the labor-tax case than in the capital-tax case, implying that the labor-tax is more distortionary than the capital-tax in our framework. This result is in line with previous work which finds that the optimal capital-tax in a lifecycle model is large and positive (Conesa et al. (2009)). A positive tax on capital can be optimal for several reasons. Examples include the government's inability to separately tax accidental bequests (which are perfectly inelastic), and liquidity constraints for younger households (which a labor tax makes more binding because it reduces each household's disposable income) (Peterman (2013)).

Welfare: Distribution Across Income Quintiles

Table 3.6: Steady State CEV (percent)

	No Rebate	Lump-sum Rebate	Capital Rebate	Labor Rebate
Quintile 1	-7.14	1.79	-0.40	-0.46
Quintile 2	-6.75	0.48	-0.00	0.14
Quintile 3	-6.50	-0.32	0.28	0.47
Quintile 4	-6.29	-1.08	0.53	0.81
Quintile 5	-6.15	-1.83	0.72	1.29
Population	-6.65	0.09	0.13	0.30

The carbon-tax policy is regressive when the government does not rebate the tax revenue. This regressivity arises because lower income households

⁶The rise in labor supply is primarily driven by an income effect on the household side. By increasing the cost of energy consumption, the carbon-tax reduces the consumption-energy composite, \tilde{c} , raising marginal utility, and, thus, increasing the returns to working. There is also a substitution effect which works in the opposite direction. The carbon tax increases the price of \tilde{c} , which encourages agents to substitute leisure for \tilde{c} , reducing labor supply. However, the income effect dominates the substitution effect for $\theta_1 > 1$.

devote larger fractions of their budgets to energy consumption. Therefore, the carbon tax taxes a larger portion of lower income households' total expenditures than it does for higher income households, making them considerably worse off.

The government can reverse the regressiveness of the tax policy by rebating the revenue in a progressive manner. For example, when the government rebates the revenue through equal, lump-sum transfers, the carbon tax policy increases welfare for the lower income quintiles and decreases it for the higher income quintiles. This rebate mechanism redistributes income from the high to low income households because the wealthier agents fund a larger portion of the transfer since they consume more energy (in absolute terms).

Unlike in the lump-sum rebate case, the government exacerbates the regressiveness of the carbon tax policy when it rebates the revenues through the capital-tax rate. The left panel of Figure 3.3 plots capital income as a fraction of total income in each of the quintiles. The policy is regressive under the capital-tax rebate because capital income represents a larger fraction of total lifetime income for higher income individuals. Thus, a reduction in the capital-tax rate leads to a larger percentage increase in income for wealthier households.

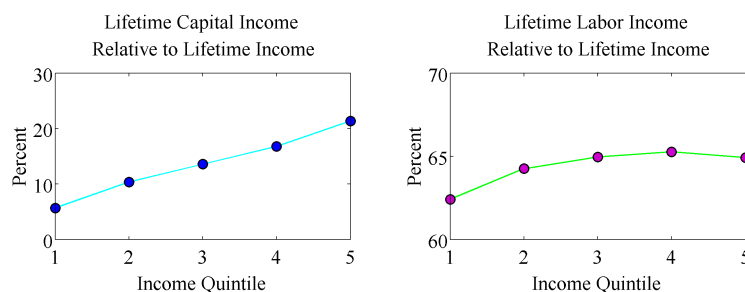


Figure 3.3: Labor and Capital Income as a Fraction of Total Lifetime Income

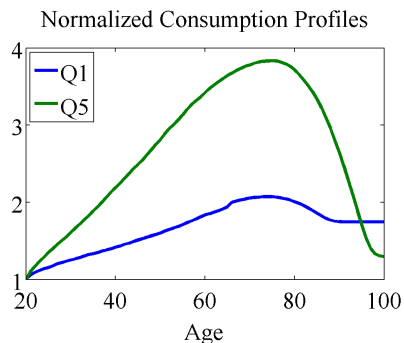


Figure 3.4: Consumption Profiles For First and Fifth Income Quintiles

The policy is also regressive when the government rebates the revenues through the labor-tax rate. The right panel of Figure 3.3 plots the labor income as a fraction of total income in each of the income quintiles. Unlike in the capital-tax case, the fraction of total income that comes from labor income is relatively constant across the income quintiles. However, when the government reduces the labor income tax, agents move consumption from later in life towards earlier in life (see bottom left panel of Figure 3.2). This shift smooths the consumption profile over the life cycle. Figure 3.4 plots the average consumption profiles for agents in the first and fifth income quintiles. The consumption profile is much steeper for the wealthier agents, implying that these agents benefit more from the consumption smoothing, making the labor-tax rebate regressive.

3.4.2 Transition

We report the results along the transition path to the new long-run equilibrium with the policy in place. Figure 3.5 shows the evolution of capital, labor, the after-tax risk-free rate, and the average after-tax wage. As a measure of welfare, we calculate the CEV in terms of an agent's expected future consumption over the remainder of his life cycle. For example, to calculate the welfare effect of the policy for an agent who is 25 when the tax is introduced, we compute the expected, uniform percent change in consumption the agent would need in every remaining period of his life to make him indifferent between living the rest of his life in the baseline versus under the policy. We

begin by discussing the welfare effects for the living population.^{7,8} We next examine how the welfare implications vary across income quintiles. Then we analyze the welfare effects for agents who are different ages when the government introduces the carbon tax policy. Finally, we study the interaction between the age and income welfare effects.

Welfare: No Double Dividend

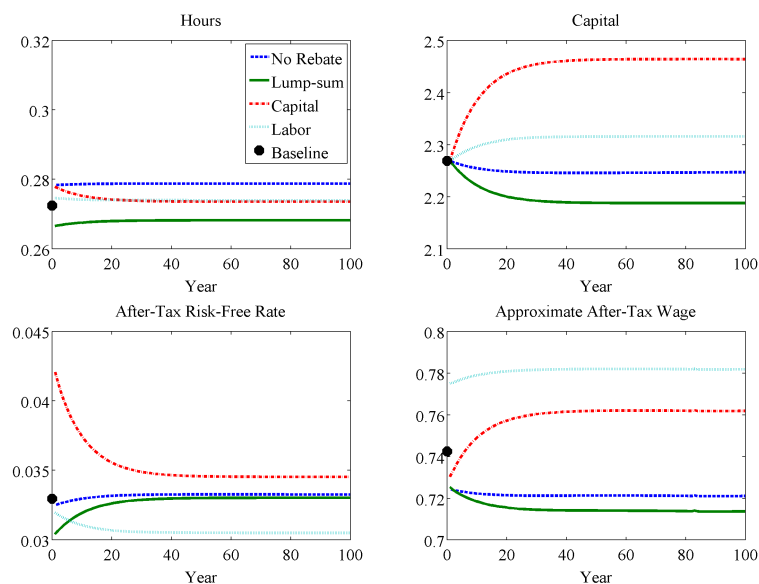


Figure 3.5: Transition Dynamics: Aggregates

The bottom row of Table 3.7 reports the CEV for the living population under each rebate mechanism. Unlike in the steady state, there is no double dividend over the transition and carbon tax reduces welfare for all the rebate options we consider. A key difference between the steady state and the transition is that an agent born in the steady state experiences the policy for his entire life cycle, while an agent undergoing the transition does not. These life cycle factors substantially increase the average welfare cost of the labor-tax rebate during the transition because agents who are retired when the government introduces the policy do not receive any rebate, and, thus,

⁷We use the term living population to refer to the population who is alive when the government introduces the carbon tax.

⁸Appendix A reports to welfare effects for agents born during the transition.

are considerably worse off. Additionally, the welfare cost of the capital-tax rebate is substantially higher over the transition because factor prices take time to adjust (see the bottom two panels of Figure 3.5). In particular, the wage and risk-free rates rise gradually as the economy accumulates capital, causing welfare to increase over the transition.

In the steady state, the welfare gain from the revenue-neutral carbon tax policy is largest under the capital and labor-tax rebates and smallest under the lump-sum rebate. However, over the transition we again obtain the opposite result; the welfare cost of the policy is smallest under the lump-sum rebate and largest under the capital and labor-tax rebates. The cost of the policy over the transition is relatively high under the capital- and labor-tax rebates for the reasons discussed above. The cost of the policy is relatively small under the lump-sum rebate because, as discussed in Section 3.4.1, the lump-sum rebate incentivizes agents to reduce their savings relative to the baseline. Thus, when government introduces the carbon tax policy, the living population has saved more than they would have, had they known that the government was going to introduce the policy. All else constant, these “extra savings” increase the expected welfare over the remainder of living population’s life cycle, making the lump-sum rebate the least costly rebate option over the transition.

Table 3.7: Transition CEV (percent)

	No Rebate	Lump-sum Rebate	Capital Rebate	Labor Rebate
Quintile 1	-5.57	1.35	-1.99	-2.65
Quintile 2	-5.09	0.18	-1.04	-2.34
Quintile 3	-4.81	-0.45	-0.41	-2.21
Quintile 4	-4.56	-1.06	0.20	-2.08
Quintile 5	-4.26	-1.75	0.95	-1.94
Population	-4.99	-0.07	-0.75	-2.30

Welfare: Distribution Across Income Quintiles

The welfare effects across quintiles are qualitatively the same over the transition as in the steady state. The carbon tax policy is regressive then

the government does not rebate the tax revenue and also when it rebates the revenue through reductions in either the capital- or labor-tax rates. The policy is progressive then the government rebates the revenue through lump-sum transfers. The intuition is the same as in Section 3.4.1.

Welfare: Distribution Across Age Cohorts

Regardless of the rebate option, the carbon tax decreases the average expected welfare for the living population. However, the government's rebate choice can make the carbon tax policy welfare improving for some age cohorts over the transition. Figure 3.7 plots the CEV for each age cohort for the four rebate options. The lump-sum rebate is the most welfare improving for the older retirees, the capital-tax rebate is the most welfare improving for the middle-aged agents, and the labor-tax rebate is most welfare improving for the very young agents.

Four factors primarily determine the relative welfare effects across age groups: (1) changes in the wage rate, (2), changes in the risk-free rate, (3) the size of the rebate relative to expected remaining lifetime income, and (4) the agent's ability to smooth consumption in response to the shock. Figure 3.6 plots remaining lifetime labor and capital income relative to total remaining lifetime income for each age cohort.

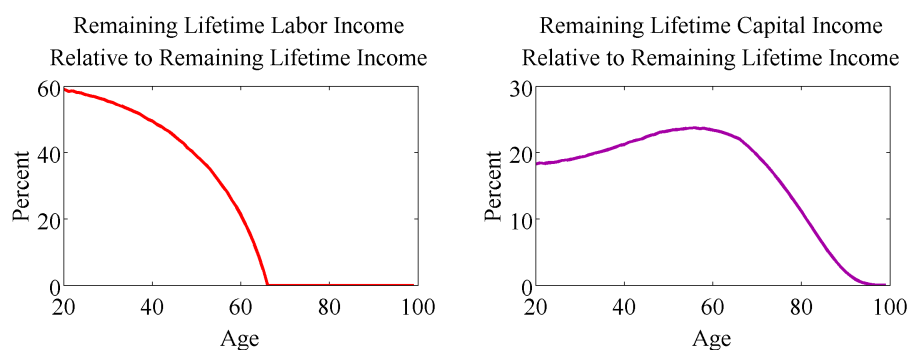


Figure 3.6: Remaining Factor Incomes Relative to Remaining Income

All else constant, changes in the wage rate have larger welfare consequences for younger agents because expected future labor income comprises the

largest fraction of total remaining lifetime income for this age group. Changes in the risk-free rate have the largest consequences for middle-aged agents because expected future capital income comprises the largest fraction of total remaining lifetime income for this age group. Finally, the retirees are least able to smooth their consumption in response to the increase in energy prices because they can no longer adjust their labor supply decisions and they have depleted their savings. Thus, all else constant, the carbon tax is particularly costly for this age group.

Under the lump-sum rebate, both the wage- and the risk-free rates fall. The strengths of these factor price changes for the young and the middle-aged roughly cancel out and as a result, the welfare effects are relatively constant across these age cohorts. The CEV is considerably higher for the retirees because the size of the lump-sum rebate relative to total remaining lifetime income is largest for this age group. This is because the retirees are depleting their savings and, thus, have low expected lifetime incomes relative to their younger counterparts.

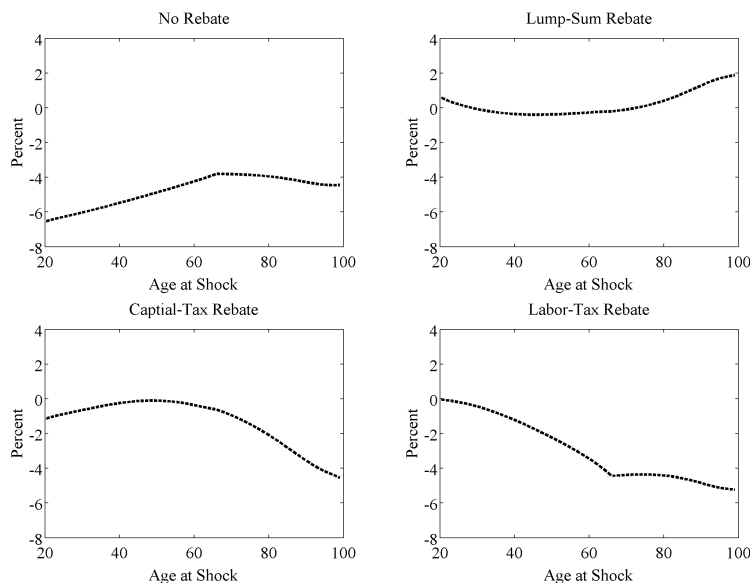


Figure 3.7: CEV: Agents Alive At Time of Shock

Under the capital-tax rebate, the wage rate falls at the start of the transition, which is particularly costly for younger agents. The after-tax risk-

free rate rises, which is particularly beneficial for middle-aged agents, and hence the rebate generates the largest increase in welfare for this age group.

Under the labor-tax rebate, the average after-tax wage rate rises, which is more beneficial for younger agents. The risk-free rate falls which is particularly costly for the middle-aged agents and also for the retired agents who have difficulty smoothing their consumption in response to this negative income shock.⁹ Thus, the welfare costs are lowest for the very young.

The no-rebate case differs from the three rebate options because the government increases its total tax levy, reducing each agent's disposable income. This negative wealth effect reduces welfare for all agents. The rebate is particularly costly for the younger agents because it leads to a drop in the market wage. Additionally, the policy is more costly for the retirees relative to the older working agents because the retirees are less able to adjust to the shock.

Welfare: Distribution Across Age and Income Groups

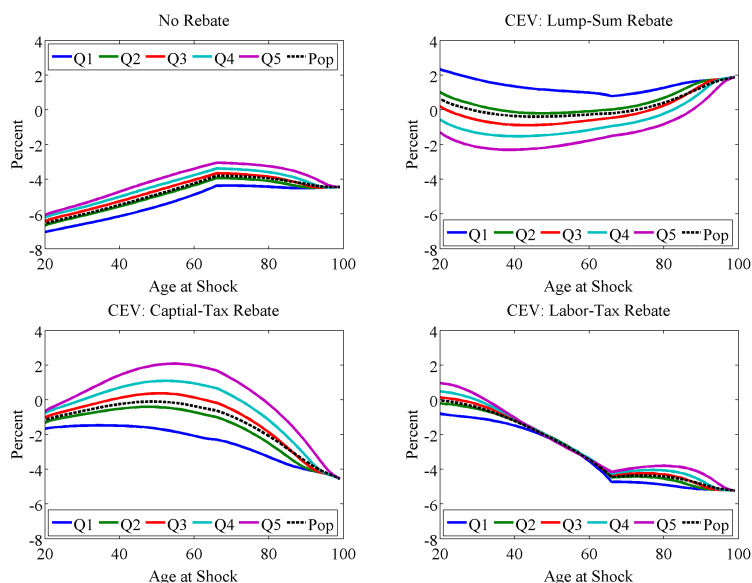


Figure 3.8: CEV: Agents Alive At Time of Shock

⁹Average welfare for the retirees exhibits an inverted-U shape. This shape results from two opposing mechanisms; younger retirees are better off than older retirees because they have more time to adjust their savings decisions in response to the shock, but they are worse off because they experience the higher energy prices for more years.

Finally, we analyze the interaction between age and income over the transition. Figure 3.8 plots the CEV by age and income quintile for each of the four rebate options. We find that the tax policy usually does not have uniform effects for all agents in a given income quintile or age cohort.

The lump-sum rebate is welfare improving older agents and for low-income agents on average. These results continue to hold when we interact age and income. However, the variance in the CEV across the income quintiles is substantially larger for the younger agents because the variance in lifetime income is greatest for this group.

The capital-tax rebate case is welfare improving for middle-aged agents and for high-income agents on average. However, the effects are not uniform across either of these demographic groups. In particular, the capital tax is only welfare improving for households who are both high-income and middle-aged. The CEV is negative for young and old high-income households, and also for middle-aged low-income households. The variance in the CEV across the income quintiles is substantially larger for the middle-aged agents because the variance in capital income as a fraction of remaining lifetime income is greatest for this group.

The labor-tax rebate case does not improve welfare for any income quintile or age group on average. Moreover, the distributional effects of the labor-tax rebate averaged over the age cohorts are regressive (see Table 3.7). However, the bottom right panel of Figure 3.8 shows that the rebate is not regressive for the middle-aged. This change occurs because the labor tax leads to a fall in the risk-free rate which is most costly for the middle-aged, and, thus, undoes the regressiveness of the policy for this age group.

3.5 Conclusion

We develop an overlapping generations model to quantify the distributional implications of a carbon tax across both income and age groups for different revenue-recycling schemes. We find that the welfare effects vary considerably between the steady state and the transition. In particular, in the

steady state, the welfare *gain* from the revenue-neutral carbon-tax policy is largest under the labor-tax rebate and smallest under the lump-sum rebate, but over the transition, the welfare *cost* is largest under the labor tax rebate and smallest under the lump-sum rebate.

Additionally, we find that rebate mechanism has substantial implications for the distribution of welfare effects across income and age groups over the transition. In particular, the lump-sum rebate *improves* welfare for low income households and also for older households at all income levels, the capital-tax rebate *reduces* welfare for all households except those who are middle-aged and wealthy, and the labor-tax rebate *reduces* welfare for all households except those who are very young and wealthy.

Appendix: Transitional Welfare for New Born Agents

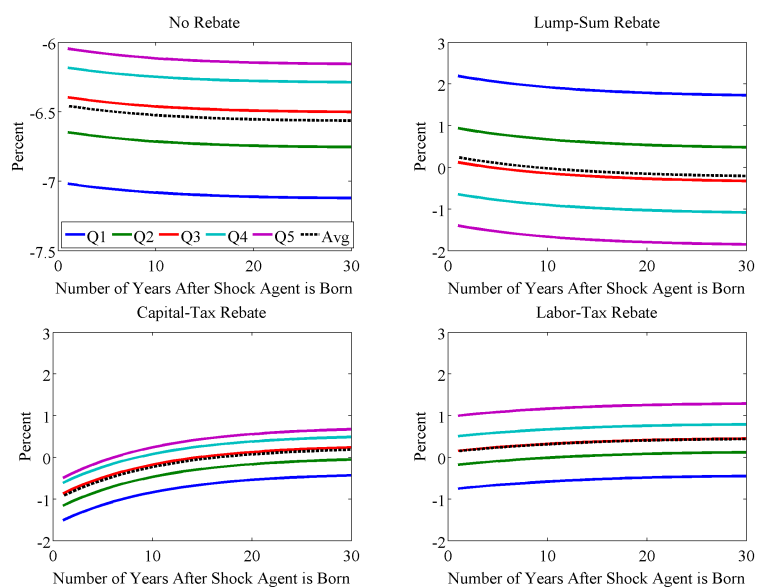


Figure 3.9: CEV: Agents Born After the Shock

The discussion in the text focused on the welfare effects of the different rebate options for agents who are alive when the carbon tax is introduced.

We now turn to the welfare effects of agents who are born after the shock is introduced, but before the economy has fully transitioned to its new long-run equilibrium. Figure 3.9 reports the CEV for these agents. Under the no rebate, lump-sum, and labor-tax rebates, welfare changes relatively little as the economy approaches its new long-run equilibrium. However, under the capital-tax rebate, welfare increases gradually as the economy transitions. The capital-tax rebate leads to a comparatively large accumulation of capital, increasing the wage rate and, thus, welfare over the transition.

3.6 Acknowledgement

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