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ABSTRACT

The relative probabilities for alternate processes initiated by a nucleon-nucleon collision depend on the dynamics involved and on the volume in phase space accessible to each final state.

The assignment of relative a priori probabilities to the final states proportional to their extension in phase must be consistent with the translational, rotational, and Lorentz invariant properties of the colliding system. The latter in particular implies a conservation law for the center of energy. Its effect is not only to lower the power of the configurational volume by one dimension but also to severely reduce the contributions from high momenta to the phase space integrals.

The limitations on accessibility arising from the controllable constants of motion are not sufficient to insure well defined probabilities. Some additional restriction on the configurational part of the phase space must be imposed. A cutoff factor for each particle is accordingly introduced. The configurational volume accessible to the particle thus decreases with increasing energy, a picture not inconsistent with the uncertainty principle.

A STATISTICAL MODEL FOR HIGH ENERGY EVENTS

Joseph V. Lepore and Maurice Neuman

This note describes a statistical model which differs in some essential respects from the one proposed by Fermi.[1]

Following Fermi we assume that in a high energy collision a state approximating that of equilibrium is established. The probability of disintegration into various possible modes is then taken proportional to their relative extensions in accessible phase space. The limitations on accessibility arise from the assumed controllable constants of motion. In this note we take them to be energy, momentum and isotopic spin. With these, the momentum space projection of the phase space extension is well defined. Galilean invariance of Newtonian dynamics generates an additional configurational constant of motion: the center of mass. In a Lorentz invariant theory with particle coordinates specified on a flat space-like surface this becomes the center of energy, which together with the components of the orbital momenta constitutes the six vector of angular momentum. The effect of conservation of orbital angular momentum is neglected in this note.

The relativistic analogue of the conservation of the center of mass in addition to reducing the power of the configuration volume by one modifies considerably the weight that is being attached to the various parts of the momentum space. It does not restrict however the

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[1] E. Fermi, Prog. of Theor. Physics 5, 4, 570 (1950).

extension of the configuration space sufficiently to give the phase integral a well defined meaning. This difficulty is ordinarily overcome by enclosing the system in a container whose walls are eventually removed to infinity and its volume does not appear in any physically significant context. This procedure cannot be followed, however, in our case. Imagine a light and a heavy particle with center of mass fixed starting out from the origin and moving according to Newtonian mechanics in opposite directions. When the lighter particle reaches the boundary of the container and is reflected, the heavier one must also turn back, even though it is still in the interior of the region, in order that the center of mass may be conserved. Thus, the conservation of center of mass and a finite volume, the same for light and heavy particles, give rise to spatial correlations amounting to interactions. Since these cannot have any physical basis, this trivial example suggests that one enclose the particles in spheres whose radii decrease in some manner with increasing mass. It is clear that in a non-relativistic theory this would have the trivial effect of multiplying the integral over momentum space by a factor dependent on the ratios of masses of the various particles.

In a relativistic theory, however, if different cutoffs in configurational volume for the various particles are introduced and the requirement imposed that the only correlations in momentum space between the various particles are those arising from conservation of energy and momentum, it is found that these cutoffs must be energy dependent. This

is not surprising since it is the center of energy rather than mass that
is now conserved. The result of this new assumption of absence of
fictitious correlations is now, however, far from trivial.

No mention has so far been made of any contraction factor such
as that which occupies a prominent position in Fermi's statistical model.
This indeed is absent here. What one gets instead is something like a
uniform shrinkage of the configurational volume with increasing energy
due to the energy dependent cutoffs. The effect of shrinkage of
configurational volume with energy is of course more significant for a
light particle than for a heavy one. As a result one finds in calculating
the energy spectrum of a single emitted meson in a nucleon-nucleon
collision that low meson energy emissions are favored above the high ones.

A quantitative formulation of the preceding discussion is now
given. The expression for the number of states per unit energy interval
in the accessible phase space as restricted by conservation of energy,
momentum and center of energy may be written as

$$P_n = \frac{1}{(2\pi)^{3(n-1)}} \frac{(2K)^4}{2 c \hbar K} S_n \cdot T_n \int \prod d^3 k_i \; \delta\left(2K - \sqrt{k_i^2 + \chi_i^2}\right) \delta\left(-\sum \vec{k}_i\right)$$

$$\int \prod d^3 x_i \; e^{-x_i^2 \lambda_i^2} \; \delta\left(\sum \vec{r}_i \sqrt{k_i^2 + \chi_i^2}\right).$$

$$\tag{1}$$

In this formula the index n refers to n particles with masses
$\chi_1, \chi_2 \ldots \chi_n$. The letter S_n denotes a constant whose exact value

depends on the number of indistinguishable groups and the population of each occurring among the n particles. The isotopic spin weight factors are denoted by T_n. The symbol K stands for the energy per incident nucleon in the center of energy system. A gaussian cutoff with an, as yet, arbitrary parameter λ_i for the configuration space of each particle has been introduced in the integrand. The quantities $\varkappa_i$, K, $\vec{k}$ have the dimensions of reciprocal length. Thus the integral has the dimensions of the product of four delta functions, or that of $(2\chi)^{-4}$. The weight factors S_n and T_n are of course dimensionless. Carrying out the integration over configuration space we obtain

$$
P_n = \frac{1}{(4\pi)^{3/2(n-1)}} \frac{(2K)^4}{\hbar c\, 2K} S_n T_n \int \prod_{i=1}^{n} \frac{d^3\vec{k}_i}{\lambda_i^3} \frac{1}{\left[\sum_{i=1}^{n} \frac{k_i^2 + \varkappa_i^2}{\lambda_i^2}\right]^{3/2}}
$$

$$
\cdot \delta\left(2K - \sqrt{k_i^2 + \varkappa_i^2}\right) \delta(-\sum k_i) .
$$

(2)

If the system is to represent a collection of essentially independent free particles there can exist in momentum space no other correlation except those arising from conservation of energy and momentum. Stated otherwise: if the four delta functions in integrand of expression (2) were disregarded the integral should factor into a product of integrals in accordance with the law of composition of independent probabilities.

The troublesome factor is the sum function

$$\left[\sum_{i=1}^{n} \frac{k_i^2 + K_i^2}{\lambda_i^3} \right]^{-3/2}$$

It may be eliminated by setting

$$\lambda_i = \frac{\sqrt{k_i^2 + K_i^2}}{\gamma_i^{\frac{1}{2}}} \qquad (3)$$

where the γ_i's are constants, independent of energy. Looking back at (1) we see that the size of the "wave packet" of a particle is determined by its energy and the γ_i's play the role of an intrinsic scaling factors. To simplify the formula we introduce a reduced scaling factor for the system of n particles

$$\frac{1}{\gamma_{(n)}^{n-1}} = \frac{\frac{1}{n} \sum_{i=1}^{n} \gamma_i}{\prod_{i=1}^{n} \gamma_i} \qquad (4a)$$

and define a quantity

$$Q_n = \left(\frac{\gamma_{(n)}}{4\pi} \right)^{\frac{3}{2}(n-1)} \frac{1}{n^{3/2}} . \qquad (4b)$$

Equation (2) then assumes the form

$$P_n = Q_n \, S_n \, T_n \, J_n \qquad (5a)$$

where

$$J_n = \frac{(2K)^4}{\hbar c \; 2\dot{K}} \int \prod_{i=1}^{n} \frac{d^3\vec{k}(i)}{\left[k_i^2 + K_i^2\right]^{3/2}} \; \delta\left(2K - \sum \sqrt{k_i^2 + K_i^2}\right) \delta\left(-\sum \vec{k}_i\right).$$

(5b)

The nonrelativistic limit of expression (5) is not essentially different from that of Fermi's model with the conservation of linear momentum.[2] A short calculation carried out along the lines of reference 2 yields

$$P_n = \gamma_n^{\frac{3}{2}(n-1)} \; S_n \; T_n \; \frac{1}{2} \; \frac{1}{mc^2} \; \frac{1}{n^{3/2}} \; \frac{1}{(\frac{3}{2}n - \frac{5}{2})!} \left(\frac{T}{mc^2}\right)^{3/2}$$

(6)

where

$$\frac{1}{m^{n-1}} = \frac{\sum_{i=1}^{n} m_i}{\prod_{i=1}^{n} m_i} \; .$$

and T is the kinetic energy per colliding nucleon in the center of mass system.

It is readily seen, however, that the factor $\left[k_i^2 + K_i^2\right]^{-3/2}$ will introduce essential modifications in the high energy domain. These will show up primarily in the shape of the energy spectrum of the emitted mesons. For a fixed multiplicity, this is independent of any uncertainties in the factor Q_n. Graphs I, II, and III show the general tendency for

2 J. V. Lepore and R. Stuart, Phys. Rev. (in press).

mesons to be emitted with low rather than high energies. This is a general feature of the model resulting from the fact that high energy mesons have less configurational volume available to them than the low ones. This feature is present in single as well as multiple meson production. An experiment carried out under conditions where single meson production would be expected to dominate would therefore be of some interest.

At higher energies where multiple production is also possible the predictions of the model would depend on the assumed values of the scaling factors γ_π and γ_N that enter into the definition of Q_n. Explicitly

$$Q_2(nn) = \left(\frac{\gamma_N}{8\,\pi}\right)^{3/2}$$

$$Q_3(NN\,\pi) = \left(\frac{\gamma_N}{4\,\pi}\right)^3 \frac{1}{\left(1 + 2\,\dfrac{\gamma_N}{\gamma_\pi}\right)^{3/2}} \tag{7}$$

$$Q_4(NN\,2\pi) = \left(\frac{\gamma_N}{4\,\pi}\right)^{9/2} \frac{1}{\left[2\,\dfrac{\gamma_N}{\gamma_\pi}\left(1 + \dfrac{\gamma_N}{\gamma_\pi}\right)\right]^{3/2}}$$

It is seen from these expressions that even if $\gamma_n = \gamma_\pi = \gamma$, the assumed value of this single parameter will still effect the predictions of the model in an essential manner.

In view of the general qualitative agreement of the shapes of the spectra with those observed at the Brookhaven Cosmotron a further attempt

might be made to account for the observation in greater detail by fixing

one or two parameters ($\gamma_{\pi} = \gamma_N = \gamma$, or γ_N, $\dfrac{\gamma_{\pi}}{\gamma_N}$) empirically

on the basis of single meson production. With a view towards such a

program when further data become available we include tables with some

computed values of the expressions appearing in Eq. (5). Table I lists

the values of P_n when $S_n = T_n = \gamma_{(n)} = 1$. Table II those of

$P_n/(\gamma_n)^{\frac{3}{2}(n-1)}$ for a p-p collision, Table III the same quantity for an

n-p collision if the colliding particles are in an isotopic spin state

$T = 0$, and Table IV if they are in a state $T = 1$. Columns are labeled

by the kinetic energy (in Bev) of the incident nucleon in the laboratory

system.

TABLE I

Values of P_n for a nucleon-nucleon collision when $S_n = T_n = \gamma_{(n)} = 1$.
The last column indicates the number of nucleons and pions which the
reaction yields.

E	2.5	6.3	10	Process
P_2	0.030	0.025	0.022	2N
P_3	.012	.022	.022	2N 1π
P_4	.0043	.006	.0089	2N 2π
P_4'	0	.000001	.00022	4N

TABLE II

Values of $P_n / \mathcal{T}_{(n)}^{\frac{3}{2}(n-1)}$ for a p-p collision at three energies.

	1.5	6.3	10
pp	0.015	0.0125	0.011
pp^o	.003	.0055	.0055
pn$^+$	.018	.033	.033
pp^{+-}	.00026	.0036	.0053
pp^{oo}	.000043	.0006	.00089
pn^{+o}	.00077	.0108	.016
nn^{++}	.000065	.0009	.0013

TABLE III

Values of $P_n / \mathcal{T}_n^{\frac{3}{2}(n-1)}$ for an n-p collision in a $T = 0$ isotopic spin state at three energies.

	1.5	6.3	10
pn	0.030	0.025	0.022
pp$^-$	.002	.0037	.0037
pn^o	.004	.0073	.0073
nn$^+$	.002	.0037	.0037
pp^{-o}	.00072	.001	.0015
pn^{oo}	.00072	.001	.0015
pn^{+-}	.00043	.006	.0089
nn^{+o}	.00072	.001	.0015

TABLE IV

Values of $P_n/\mathcal{T}_{(n)}^{\frac{3}{2}(n-1)}$ for an n-p collision in a $T = 1$ isotopic spin state at three energies.

	1.5	6.3	10
pn	0.030	0.025	0.022
pp$^-$	.003	.0055	.0055
pn^o	.012	.022	.022
nn$^+$	.003	.0055	.0055
pp^{-o}	.00017	.0024	.0036
pn^{oo}	.00013	.0018	.0027
pn^{+-}	.00077	.011	.016
nn^{+o}	.00017	.0024	.0036

In constructing Table I we reduced J_4 to a double, J_3 to a single integral. J_2 can be expressed analytically in terms of elementary functions. The remaining integration were carried out numerically on the IBM Card Program Calculator at the Livermore site of this laboratory. Tables II, III, and IV were constructed using Table I and the numerical coefficients T_n calculated by Fermi.[3]

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[3]
    E. Fermi, Phys. Rev. 92, 452 (1953).

It may perhaps be worth-while to point out that the probability for the production of a nucleon pair in a nucleon-nucleon collision is seen from Table I to be exceedingly small even at 10 Bev.

FIGURE CAPTIONS

Figure 1: Momentum spectra of singly and doubly emitted mesons in the laboratory system for 1.5 Bev kinetic energy incident nucleons.

Figure 2: Momentum spectra of singly and doubly emitted mesons in the laboratory system for 6.3 Bev kinetic energy incident nucleons.

Figure 3: Momentum spectra of singly and doubly emitted mesons in the laboratory system for 10 Bev kinetic energy incident nucleons.
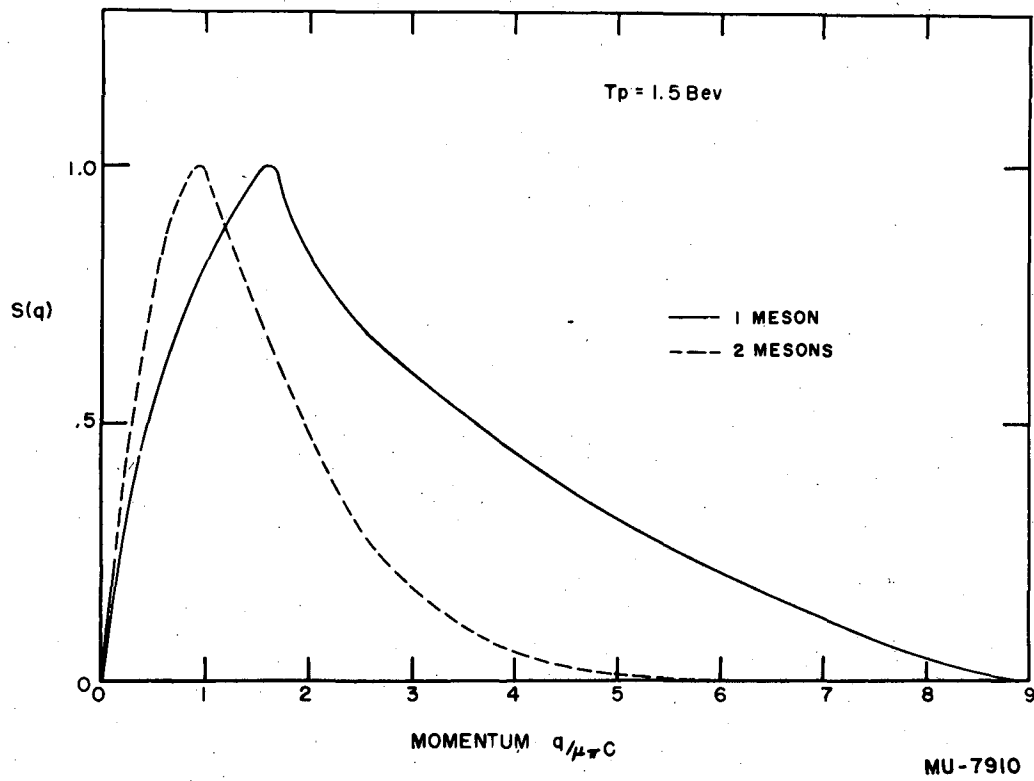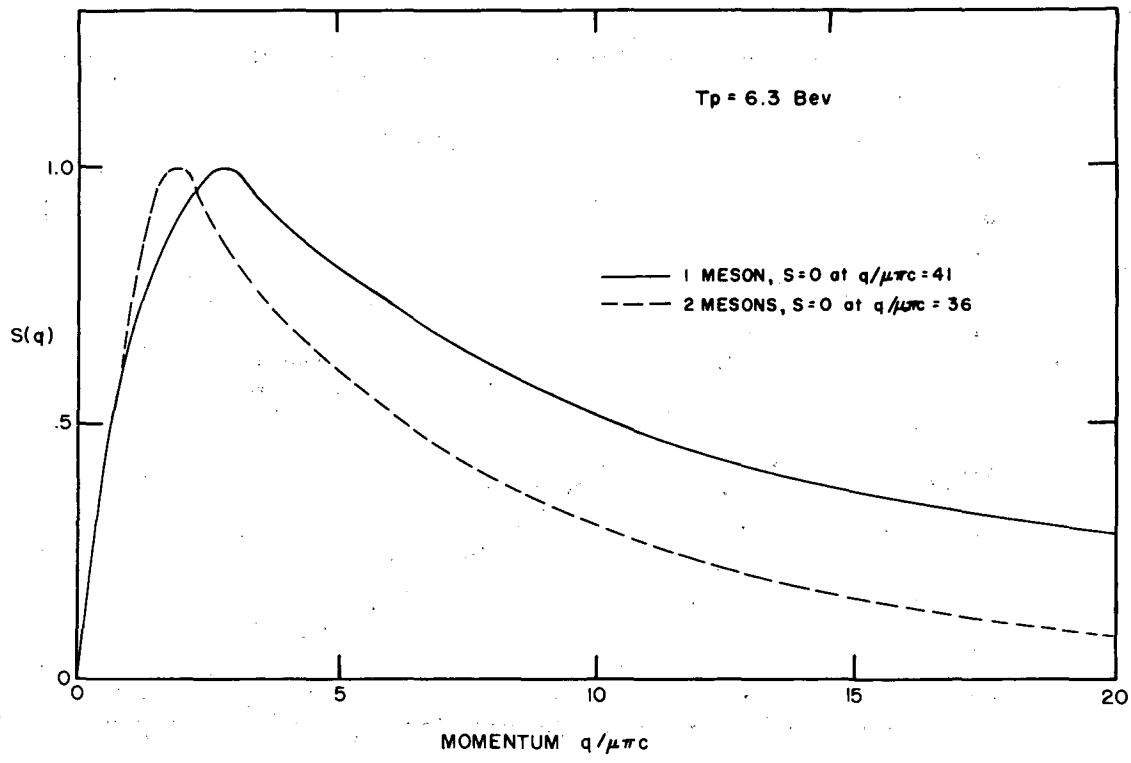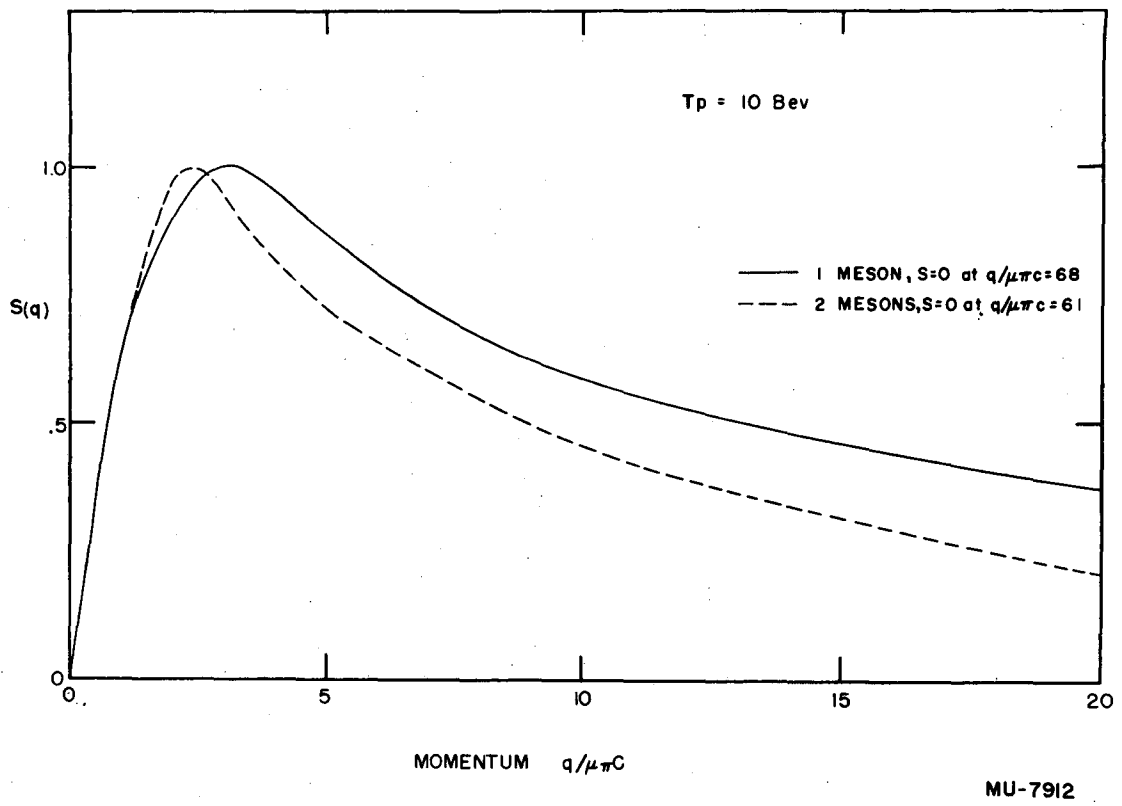
Fig. 1.

Fig. 2.

Fig. 3