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Abstract: Sub-Hz drift dominates full-band phase noise of the feed-forward system. This review models its mechanisms through phase-noise integration and transfer functions.

INTRODUCTION

The residual carrier-envelope phase noise reported by Lemons et al. is best understood as a low-frequency noise problem rather than a uniform jitter limit. The reported feed-forward system achieves extremely low short-term phase noise, and excluding the 0.1 – 1Hz decade yields a significant decrease in integrated phase noise. This indicates that within the feed-forward system, the sub-Hz region captures the bulk of the long-term instability.

ANALYSIS AND INTERPRETATION

Keeping a budget for frequency-dependent noise, we must first investigate the integrated phase-noise (IPN) expression utilized by Lemons *et al.* [1]:

$$\mathrm{IPN} = \sqrt{2\int_{f_{\mathrm{low}}}^{f_{\mathrm{high}}} 10^{L(f)/10}\, df}$$

In this definition, L(*f*) is the single-sideband phase-noise density with units of dBc/Hz. Before integration, the logarithmic noise spectrum is converted into linear units. Hence, with high phase-noise density, even a narrow region on a logarithmic frequency plot can dominate the total integrated variance. This is a common characteristic of spectrums that contain low-frequency flicker or drift noise. The paper reports 25 mrad IPN from 0.1 Hz – 3 MHz and 3.5 mrad from 1 Hz – 3 MHz. Given that integrated phase noise scales to the square root of accumulated phase-noise variance, the contribution from 0.1 – 1 Hz can be estimated by subtracting the two regional variances in quadrature:

$$\mathrm{IPN}_{0.1-1\mathrm{Hz}} = \sqrt{\mathrm{IPN}^2_{0.1\mathrm{Hz}-3\mathrm{MHz}} - \mathrm{IPN}^2_{1\mathrm{Hz}-3\mathrm{MHz}}}$$

$$\mathrm{IPN}_{0.1-1\mathrm{Hz}} = \sqrt{25^2 - 3.5^2} \approx 24.75\ \mathrm{mrad}$$

About $(24.75)^2/25^2 \approx 98\%$ of the fullband phase-noise variance is from the 0.1–1Hz decade, identifying the sub-Hz region as the primary contributor to long-term residual CEP noise. This low-frequency phase-noise spectrum can be represented with a power-law model [2, 3]:

$$S_{\phi,\mathrm{subHz}}(f) = \frac{A}{f^{\alpha}}$$

A value near α=1 corresponds to flicker-like phase noise, while larger slopes approach random-walk or drift-dominated behavior. As discussed by Song et al., slow changes in temperature, pump power, cavity length, nonlinear phase, SESAM saturation, fiber coupling, and AOFS operating point all produce noise that scales up in lower Fourier frequency [3]. Overall, the physical origin of the sub-Hz term can be divided into two broad categories. The first is intrinsic laser noise. Paschotta et al. demonstrate that timing jitter, optical phase noise, and carrier-envelope offset (CEO) noise in passively mode-locked lasers are closely related, and that mode-locked lasers may exhibit strong CEO or optical phase noise even when timing jitter is near quantum-limited [2]. The second category is technical drift, such as pump-power drift, thermal drift, and actuator operating-point drift. Lemons et al. report 104 Hz-level free-running drift, necessary manual pump-power adjustments every half hour, and increased jitter when fAOFS deviates from 80 MHz, all of which reflect low-frequency regime perturbances [1]. A deeper model would therefore write the sub-Hz noise as a sum of terms:

$$S_{\phi,\mathrm{subHz}}(f) = S_{\phi,\mathrm{intrinsic}}(f) + S_{\phi,\mathrm{pump}}(f) + S_{\phi,\mathrm{thermal}}(f) + S_{\phi,\mathrm{AOFS}}(f) + S_{\phi,\mathrm{SA}}(f)$$

Each term in this sum can be experimentally separated by a transfer function [4, 5]. For instance, pump-noise coupling and AOFS detuning noise can be respectively described by:

$$S_{\phi,\mathrm{pump}}(f) = \left|\frac{\partial \phi_{\mathrm{CEO}}}{\partial P_{\mathrm{pump}}}\right|^2 S_{P,\mathrm{pump}}(f),\ S_{\phi,\mathrm{AOFS}}(f) = \left|\frac{\partial \phi_{\mathrm{res}}}{\partial f_{\mathrm{AOFS}}}\right|^2 S_{f,\mathrm{AOFS}}(f).$$

These transfer functions can be identified through a follow-up experiment, where pump power, cavity temperature, or AOFS drive frequency is modulated at low Fourier frequencies while the influence on f_{CEO} or out-of-loop phase-noise spectrum is measured. Using this model as the basis, sub-Hz stabilization should therefore be implemented as an auxiliary loop. The most direct solution is a slow pump-power feedback system that locks the free-running f_{CEO} center such that the required AOFS drive is anchored near its optimum value [6]. Mathematically, this adds a low-frequency actuator term to the residual CEO noise,

$$S_{\phi,\mathrm{res}}(f) = |1 - H_{\mathrm{pump}}(f)|^2 S_{\phi,\mathrm{subHz}}(f)$$

where $\mathrm{H}_{\mathrm{pump}}(f)$ is large only below the pump-control bandwidth. This loop would suppress slow drift, while the AOFS feed-forward path continues to remove fast CEO fluctuations [7]. These two actuations are therefore coupled through f_{CEO}: Pump power alters the intracavity phase and group delay, which changes f_{CEO}; the feed-forward electronics then converts it into the AOFS drive frequency. But if the pump loop is too aggressive, it can add relative intensity noise, perturb the SESAM saturation level, change nonlinear phase accumulation, and alter the f-to-2f beat SNR. Hence, the pump loop should operate at a lower bandwidth than the AOFS feed-forward correction. This prevents the two actuators from counteracting each other and is comparable to long-term hybrid CEP stabilization approaches [8].

The unresolved mystery is then what fraction of this sub-Hz error is intrinsic and what fraction is technical. Lemons et al. attribute the below-1-Hz excess partly to 1/f spurious noise associated with quantum noise in the cavity. However, the reported system also contains clear technical drift: free-running f_{CEO} drift is too large for their analyzer to directly measure, the AOFS must be kept near 80 MHz, and manual pump adjustments are required about every half hour. These observations suggest that the measured sub-Hz noise is likely an amalgam of intrinsic noise, pump-induced drift, thermal cavity drift, and AOFS sensitivity. The SESAM mode-locking mechanism is also relevant. Raabe et al. document excess CEP noise generation in saturable absorbers and correlations between amplitude and frequency fluctuations at low Fourier frequencies [9]. This implies a mechanism where slow pulse-energy fluctuations can couple through SESAM reflectivity and intracavity nonlinear phase, converting amplitude noise into CEP noise. Thus, sub-Hz noise in the feed-forward system may arise from both environmental drift and mode-locking dynamics. Also worth noting is that although the feed-forward system can remove the instantaneously measured f_{CEO}, correction quality may degrade when f_{AOFS} drifts away from optimum over time.

CONCLUSIONS

The CEP noise of the feed-forward system is characterized mainly by system dynamics below 1 Hz, and its attosecond-level stability is significantly influenced by the choice of the lower integration boundary. Instead of considering the jitter noise as one single number, differentiating between short-term stabilized CEP behavior and long-term sub-Hz drift behavior paints a more accurate representation, explaining why the feed-forward system yields excellent short-term stabilization while still being dominated by sub-Hz variance in the wider integration band.

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