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DirectDM: a tool for dark matter direct detection

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Abstract

We provide a `Mathematica` package, `DirectDM`, that takes as input the Wilson coefficients of the relativistic effective theory describing the interactions of dark matter with quarks, gluons and photons, and matches it onto an effective theory describing the interactions of dark matter with neutrons and protons. The nonperturbative matching is performed at leading order in a chiral expansion. The one-loop QCD and QED renormalization-group evolution from the electroweak scale down to the hadronic scale, as well as finite corrections at the heavy quark thresholds are taken into account. We also provide an interface with the package `DMFormFactor` so that, starting from the relativistic effective theory, one can directly obtain the event rates for direct detection experiments.

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I. INTRODUCTION

Dark Matter (DM) scattering on nuclei in direct detection experiments is naturally described by an effective field theory (EFT) [1–19], since the typical momentum exchange for the DM scattering on a nucleus, $q \lesssim 100$ MeV, is much smaller than the mediator mass in most DM models. In Refs. [18, 19] we presented the analytic expressions for the matching, within chiral perturbation theory (ChPT), between two EFTs describing the interactions of DM with the standard model (SM). The first, "relativistic EFT", comprises the partonic interactions of DM with quarks, gluons, and photons, while the second, "nonrelativistic EFT", describes the nonrelativistic interactions of DM with nucleons – neutrons and protons [2–4]. Here, we introduce the `Mathematica` package `DirectDM` which takes as input the Wilson coefficients of the relativistic operators and performs the nonperturbative matching onto the nonrelativistic EFT. An interface with `DMFormFactor` [4] is provided so that, starting from the relativistic theory, one can obtain directly the event rates in the experiment. The `DirectDM` code can be downloaded from

<div align="center">

https://directdm.github.io

</div>

This paper is organized as follows. In Section II we fix our notation and introduce the bases of both the relativistic and nonrelativistic theories. We also include a short discussion of the renormalization-group (RG) evolution of the Wilson coefficients. Section III contains the manual for the `DMFormFactor` package. We conclude in Section IV. Appendix A gives the operator bases for Majorana and real scalar DM, while Appendix B contains the translation to the operator bases of Ref. [20].

II. OPERATOR BASIS AND RENORMALIZATION-GROUP EVOLUTION

A. Fermionic dark matter

The starting point is the interaction Lagrangian between fermionic DM and the SM, which is given in terms of higher dimension operators,

$$\mathcal{L}_\chi = \sum_{a,d} \hat{\mathcal{C}}_a^{(d)} \mathcal{Q}_a^{(d)}, \qquad \text{where} \quad \hat{\mathcal{C}}_a^{(d)} = \frac{\mathcal{C}_a^{(d)}}{\Lambda^{d-4}}\,. \tag{1}$$

Here, the $\mathcal{C}_a^{(d)}$ are dimensionless Wilson coefficients, while Λ can be identified with the mediator mass. The Wilson coefficients depend on the renormalization scale μ (see also the discussion below and in Section II B). The sum runs over the mass dimension of the operators, $d = 5, 6, 7$, as well as the index a of the individual operators. We keep all dimension-five and dimension-six operators, all dimension-seven operators coupling DM to gluons, and the most relevant subset of dimension-seven operators that couple DM to quarks (i.e., we do not keep the operators that are additional suppressed by derivatives – see [21] for the complete basis).

We start with DM that is a Dirac fermion. The operator basis is the same as in [19]. There are two dimension-five operators,

$$\mathcal{Q}_1^{(5)} = \frac{e}{8\pi^2}(\bar{\chi}\sigma^{\mu\nu}\chi)F_{\mu\nu}\,, \qquad \mathcal{Q}_2^{(5)} = \frac{e}{8\pi^2}(\bar{\chi}\sigma^{\mu\nu}i\gamma_5\chi)F_{\mu\nu}\,, \tag{2}$$

where $F_{\mu\nu}$ is the electromagnetic field strength tensor. The magnetic dipole operator $\mathcal{Q}_1^{(5)}$ is CP even, while the electric dipole operator $\mathcal{Q}_2^{(5)}$ is CP odd. The dimension-six operators are

$$\mathcal{Q}_{1,f}^{(6)} = (\bar{\chi}\gamma_\mu\chi)(\bar{f}\gamma^\mu f)\,, \qquad\qquad \mathcal{Q}_{2,f}^{(6)} = (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{f}\gamma^\mu f)\,, \tag{3}$$

$$\mathcal{Q}_{3,f}^{(6)} = (\bar{\chi}\gamma_\mu\chi)(\bar{f}\gamma^\mu\gamma_5 f)\,, \qquad\qquad \mathcal{Q}_{4,f}^{(6)} = (\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{f}\gamma^\mu\gamma_5 f)\,. \qquad (4)$$

The dimension-seven operators that we keep are

$$\mathcal{Q}_1^{(7)} = \frac{\alpha_s}{12\pi}(\bar{\chi}\chi)G^{a\mu\nu}G_{\mu\nu}^a\,, \qquad\qquad \mathcal{Q}_2^{(7)} = \frac{\alpha_s}{12\pi}(\bar{\chi}i\gamma_5\chi)G^{a\mu\nu}G_{\mu\nu}^a\,, \qquad (5)$$

$$\mathcal{Q}_3^{(7)} = \frac{\alpha_s}{8\pi}(\bar{\chi}\chi)G^{a\mu\nu}\widetilde{G}_{\mu\nu}^a\,, \qquad\qquad \mathcal{Q}_4^{(7)} = \frac{\alpha_s}{8\pi}(\bar{\chi}i\gamma_5\chi)G^{a\mu\nu}\widetilde{G}_{\mu\nu}^a\,, \qquad (6)$$

$$\mathcal{Q}_{5,f}^{(7)} = m_f(\bar{\chi}\chi)(\bar{f}f)\,, \qquad\qquad \mathcal{Q}_{6,f}^{(7)} = m_f(\bar{\chi}i\gamma_5\chi)(\bar{f}f)\,, \qquad (7)$$

$$\mathcal{Q}_{7,f}^{(7)} = m_f(\bar{\chi}\chi)(\bar{f}i\gamma_5 f)\,, \qquad\qquad \mathcal{Q}_{8,f}^{(7)} = m_f(\bar{\chi}i\gamma_5\chi)(\bar{f}i\gamma_5 f)\,, \qquad (8)$$

$$\mathcal{Q}_{9,f}^{(7)} = m_f(\bar{\chi}\sigma^{\mu\nu}\chi)(\bar{f}\sigma_{\mu\nu}f)\,, \qquad\qquad \mathcal{Q}_{10,f}^{(7)} = m_f(\bar{\chi}i\sigma^{\mu\nu}\gamma_5\chi)(\bar{f}\sigma_{\mu\nu}f)\,. \qquad (9)$$

Here $G_{\mu\nu}^a$ is the QCD field strength tensor, while $\widetilde{G}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}G^{\rho\sigma}$ is its dual, and $a = 1,\ldots,8$ are the adjoint color indices. Moreover, χ denotes the DM fields and f the SM fermion fields[1]. The operators can be specified in the three-flavor ($f = u,d,s,e,\mu,\tau$), four-flavor ($f = u,d,s,c,e,\mu,\tau$), and five-flavor scheme ($f = u,d,s,c,b,e,\mu,\tau$). The initial conditions for the Wilson coefficients have then to be specified at the scale $\mu_c = 2\,\text{GeV}$ (three-flavor), $\mu_b = m_b(m_b) = 4.18\,\text{GeV}$ (four-flavor), or $\mu_Z = M_Z = 91.1876\,\text{GeV}$ (five-flavor), respectively. The scheme is set in the code by choosing one of the options `3Flavor`, `4Flavor`, or `5Flavor`, see Section III. In the first case (three-flavor scheme), the Wilson coefficients are directly matched to the nuclear effective theory, see below, while in the latter two cases the code by default performs the QCD and QED RG running down to the hadronic scale $\mu_h = 2\,\text{GeV}$, with the subsequent matching to the nuclear theory. For Majorana DM, the operators $\mathcal{Q}_{1,2}^{(5)}$, $\mathcal{Q}_{1,f}^{(6)}$, $\mathcal{Q}_{3,f}^{(6)}$, $\mathcal{Q}_{9,f}^{(7)}$, and $\mathcal{Q}_{10,f}^{(7)}$ vanish, while the definitions of all the other operators include an additional factor of $1/2$, see Appendix A. Frequently, the operator basis of Ref. [20] is used in phenomenological analyses. We provide the translation to our basis in App. B.

The `DirectDM` code provides the matching between the EFT coupling DM to quarks, gluons and photons, given in Eq. (1), to the EFT where DM interacts with nonrelativistic nucleons, given by the Lagrangian

$$\mathcal{L}_{\text{NR}} = \sum_{i,N} c_i^N(q^2)\mathcal{O}_i^N\,. \qquad (10)$$

---

[1] Although we are primarily interested in the hadronic effects, we keep the SM leptons explicit in our definitions, since the leptonic operators mix into the hadronic ones via QED penguins, see Sec. II B.

We implement the expressions for the coefficients $c_i^N(q^2)$ to leading order (LO) in the chiral expansion, i.e., to LO in an expansion in the momentum transfer $q/(4\pi f)$. At this order, $\mathcal{L}_{\mathrm{NR}}$ contains two momentum-independent nonrelativistic operators,

$$\mathcal{O}_1^N = \mathbb{1}_\chi \mathbb{1}_N\,, \qquad\qquad \mathcal{O}_4^N = \vec{S}_\chi \cdot \vec{S}_N\,, \tag{11}$$

and a set of momentum-dependent operators,

$$\mathcal{O}_5^N = \vec{S}_\chi \cdot \left(\vec{v}_\perp \times \frac{i\vec{q}}{m_N}\right)\mathbb{1}_N\,, \qquad\qquad \mathcal{O}_6^N = \left(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}\right)\left(\vec{S}_N \cdot \frac{\vec{q}}{m_N}\right), \tag{12}$$

$$\mathcal{O}_7^N = \mathbb{1}_\chi \left(\vec{S}_N \cdot \vec{v}_\perp\right), \qquad\qquad \mathcal{O}_8^N = \left(\vec{S}_\chi \cdot \vec{v}_\perp\right)\mathbb{1}_N\,, \tag{13}$$

$$\mathcal{O}_9^N = \vec{S}_\chi \cdot \left(\frac{i\vec{q}}{m_N} \times \vec{S}_N\right), \qquad\qquad \mathcal{O}_{10}^N = -\mathbb{1}_\chi \left(\vec{S}_N \cdot \frac{i\vec{q}}{m_N}\right), \tag{14}$$

$$\mathcal{O}_{11}^N = -\left(\vec{S}_\chi \cdot \frac{i\vec{q}}{m_N}\right)\mathbb{1}_N\,, \qquad\qquad \mathcal{O}_{12}^N = \vec{S}_\chi \cdot \left(\vec{S}_N \times \vec{v}_\perp\right), \tag{15}$$

with $N = p, n$. These operators coincide with the ones defined in [4], while our definition of the momentum exchange differs by a minus sign with respect to the convention used in [4], so that (cf. Fig. 1)

$$\vec{q} = \vec{k}_2 - \vec{k}_1 = \vec{p}_1 - \vec{p}_2\,, \qquad \vec{v}_\perp = \frac{\vec{p}_1 + \vec{p}_2}{2m_\chi} - \frac{\vec{k}_1 + \vec{k}_2}{2m_N}\,. \tag{16}$$

The q^2-dependent coefficients c_i^N in Eq. (10) are given by [19]

$$c_1^p = -\frac{\alpha}{2\pi m_\chi}Q_p\hat{\mathcal{C}}_1^{(5)} + \sum_q \left(F_1^{q/p}\,\hat{\mathcal{C}}_{1,q}^{(6)} + F_S^{q/p}\,\hat{\mathcal{C}}_{5,q}^{(7)}\right) + F_G^p\,\hat{\mathcal{C}}_1^{(7)} \tag{17}$$

$$-\frac{\vec{q}^{\,2}}{2m_\chi m_N}\sum_q \left(F_{T,0}^{q/p} - F_{T,1}^{q/p}\right)\hat{\mathcal{C}}_{9,q}^{(7)}\,, \tag{18}$$

$$c_4^p = -\frac{2\alpha}{\pi}\frac{\mu_p}{m_N}\hat{\mathcal{C}}_1^{(5)} + \sum_q \left(8F_{T,0}^{q/p}\,\hat{\mathcal{C}}_{9,q}^{(7)} - 4F_A^{q/p}\,\hat{\mathcal{C}}_{4,q}^{(6)}\right), \tag{19}$$

$$c_5^p = \frac{2\alpha Q_p m_N}{\pi\vec{q}^{\,2}}\hat{\mathcal{C}}_q^{(5)}, \tag{20}$$

$$c_6^p = \frac{2\alpha}{\pi\vec{q}^{\,2}}\mu_p m_N\hat{\mathcal{C}}_a^{(5)} + \sum_q \left(F_{P'}^{q/p}\,\hat{\mathcal{C}}_{4,q}^{(6)} + \frac{m_N}{m_\chi}F_P^{q/p}\,\hat{\mathcal{C}}_{8,q}^{(7)}\right) + \frac{m_N}{m_\chi}F_{\tilde{G}}^p\,\hat{\mathcal{C}}_4^{(7)}\,, \tag{21}$$

$$c_7^p = -2\sum_q F_A^{q/p}\,\hat{\mathcal{C}}_{3,q}^{(6)}\,, \tag{22}$$

$$c_8^p = 2\sum_q F_1^{q/p}\,\hat{\mathcal{C}}_{2,q}^{(6)}\,, \tag{23}$$

$$c_9^p = 2\sum_q \left[\left(F_1^{q/p} + F_2^{q/p}\right)\hat{\mathcal{C}}_{2,q}^{(6)} + \frac{m_N}{m_\chi}F_A^{q/p}\,\hat{\mathcal{C}}_{3,q}^{(7)}\right], \tag{24}$$

$$c_{10}^p = F_{\tilde{G}}^p \hat{\mathcal{C}}_3^{(7)} + \sum_q \left(F_P^{q/p} \hat{\mathcal{C}}_{7,q}^{(7)} - 2\frac{m_N}{m_\chi} F_{T,0}^{q/p} \hat{\mathcal{C}}_{10,q}^{(7)} \right) , \tag{25}$$

$$c_{11}^p = \frac{2\alpha}{\pi} Q_p \frac{m_N}{\vec{q}^{\,2}} \hat{\mathcal{C}}_2^{(5)} + \sum_q \left[2\big(F_{T,0}^{q/p} - F_{T,1}^{q/p}\big) \hat{\mathcal{C}}_{10,q}^{(7)} - \frac{m_N}{m_\chi} F_S^{q/p} \hat{\mathcal{C}}_{6,q}^{(7)} \right] - \frac{m_N}{m_\chi} F_G^p \hat{\mathcal{C}}_2^{(7)} , \tag{26}$$

$$c_{12}^p = -8 \sum_q F_{T,0}^{q/p} \hat{\mathcal{C}}_{10,q}^{(7)} . \tag{27}$$

Here $Q_{p(n)} = 1(0)$ is the charge of the proton (neutron), while α is the electromagnetic fine structure constant. The sums run over the light quark flavors $q = u, d, s$. The coefficients for neutrons are obtained by replacing $p \to n$, $u \leftrightarrow d$. In the above expressions the nonperturbative effects of the strong interactions is encoded in the form factors for the single-nucleon currents,

$$\langle N' | \bar{q}\gamma^\mu q | N \rangle = \bar{u}_N' \left[F_1^{q/N}(q^2)\gamma^\mu + \frac{i}{2m_N} F_2^{q/N}(q^2)\sigma^{\mu\nu} q_\nu \right] u_N , \tag{28}$$

$$\langle N' | \bar{q}\gamma^\mu\gamma_5 q | N \rangle = \bar{u}_N' \left[F_A^{q/N}(q^2)\gamma^\mu\gamma_5 + \frac{1}{2m_N} F_{P'}^{q/N}(q^2)\gamma_5 q^\mu \right] u_N , \tag{29}$$

$$\langle N' | m_q \bar{q}q | N \rangle = F_S^{q/N}(q^2)\, \bar{u}_N' u_N , \tag{30}$$

$$\langle N' | m_q \bar{q}i\gamma_5 q | N \rangle = F_P^{q/N}(q^2)\, \bar{u}_N' i\gamma_5 u_N , \tag{31}$$

$$\langle N' | \frac{\alpha_s}{12\pi} G^{a\mu\nu} G_{\mu\nu}^a | N \rangle = F_G^N(q^2)\, \bar{u}_N' u_N , \tag{32}$$

$$\langle N' | \frac{\alpha_s}{8\pi} G^{a\mu\nu} \tilde{G}_{\mu\nu}^a | N \rangle = F_{\tilde{G}}^N(q^2)\, \bar{u}_N' i\gamma_5 u_N , \tag{33}$$

$$\langle N' | m_q \bar{q}\sigma^{\mu\nu} q | N \rangle = \bar{u}_N' \left[F_{T,0}^{q/N}(q^2)\, \sigma^{\mu\nu} + \frac{i}{2m_N} \gamma^{[\mu} q^{\nu]} F_{T,1}^{q/N}(q^2) \right.$$
$$\left. + \frac{i}{m_N^2} q^{[\mu} k_{12}^{\nu]} F_{T,2}^{q/N}(q^2) \right] u_N . \tag{34}$$

Here we shortened $\langle N' | = \langle N(k_2)|$, $|N\rangle = |N(k_1)\rangle$, $\bar{u}_N' = u_N(k_2)$, $u_N = u_N(k_1)$ and introduced $q^\mu = k_2^\mu - k_1^\mu$, $k_{12}^\mu = k_1^\mu + k_2^\mu$. Expanding the form factors to LO in chiral counting, the expressions for the axial current, the pseudoscalar current, and the CP-odd gluonic current contain light-meson poles,

$$F_i^{q/N}(q^2) = \frac{m_N^2}{m_\pi^2 - q^2} a_{i,\pi}^{q/N} + \frac{m_N^2}{m_\eta^2 - q^2} a_{i,\eta}^{q/N} + \cdots , \quad i = P, P' , \tag{35}$$

$$F_{\tilde{G}}^N(q^2) = \frac{q^2}{m_\pi^2 - q^2} a_{\tilde{G},\pi}^N + \frac{q^2}{m_\eta^2 - q^2} a_{\tilde{G},\eta}^N + b_{\tilde{G}}^N + \cdots , \tag{36}$$

while all the other form factors can be evaluated at $q^2 = 0$,

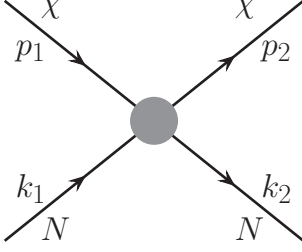$$F_i^{q/N}(q^2) = F_i^{q/N}(0) + \cdots . \tag{37}$$

Figure 1: The kinematics of DM scattering on nucleons, $\chi(p_1)N(k_1) \to \chi(p_2)N(k_2)$.

parameter	value	parameter	value	parameter	value
μ_p	2.793	σ_u^p	(17 ± 5) MeV	g_T^u	$0.794(15)$
μ_n	-1.913	σ_d^p	(32 ± 10) MeV	g_T^d	$-0.204(8)$
g_A	$1.2723(23)$	σ_u^n	(15 ± 5) MeV	g_T^s	$(3.2 \pm 8.6) \cdot 10^{-4}$
$\Delta u + \Delta d$	$0.521(53)$	σ_d^n	(36 ± 10) MeV	$B_{T,10}^{u/p}$	3.0 ± 1.5
Δs	$-0.031(5)$	$\sigma_s^{p,n}$	(41.3 ± 7.7) MeV	$B_{T,10}^{d/p}$	0.24 ± 0.12
m_u/m_d	$0.46(5)$	m_G	$848(14)$ MeV	$B_{T,10}^{s/p}$	0.0 ± 0.2
$2m_s/(m_u + m_d)$	$27.5(3)$				

Table I: The hadronic parameters used in evaluating the form factors in the code (see main text for details).

The ellipses denote terms of higher order in chiral counting. Below we collect the expressions for the proton form factors with further details given in Ref. [19]. We work in the isospin limit, so that $F_i^{u(d,s)/p} = F_i^{d(u,s)/n}$, with the exception of the scalar form factors, where we give the values separately for proton and neutron, and of the tensor form factors, where the isospin relations involve quark masses (see below). The numerical input for the hadronic parameters is collected in Tab. I. In the `DirectDM` package, these parameters are set in the file `DirectDM/inputs.m` in case the user wishes to update the values or set different ones.

Vector current. The Dirac form factors at zero recoil count the number of valence quarks in the nucleon, thus $F_1^{u/p}(0) = 2$, $F_1^{d/p}(0) = 1$, and $F_1^{s/p}(0) = 0$. The Pauli form factors for u and d quarks are $F_2^{u/p}(0) = 2(\mu_p - 1) + \mu_n + F_2^{s/p}(0)$, $F_2^{d/p}(0) = 2\mu_n + (\mu_p - 1) + F_2^{s/p}(0)$, where we use as inputs the proton and neutron magnetic moments, $\mu_p \simeq 2.793$, $\mu_n \simeq -1.913$, and the Pauli form factor for the s quark, $F_2^{s/p}(0) = -0.064(17)$ [22].

Axial current. The axial form factor at zero recoil is $F_A^{q/p}(0) = \Delta q$. As numerical inputs we use $g_A = \Delta u - \Delta d = 1.2723(23)$ [23], and, in the $\overline{\text{MS}}$ scheme at $Q = 2$ GeV,

$\Delta u + \Delta d = 0.521(53)$ [24], $\Delta s = -0.031(5)$ [25–28]. The residua of the pion- and eta-pole contributions to $F_{P'}^{q/N}$ are $a_{P',\pi}^{u/p} = -a_{P',\pi}^{d/p} = 2g_A$, $a_{P',\pi}^{s/p} = 0$, and $a_{P',\eta}^{u/p} = a_{P',\eta}^{d/p} = -a_{P',\eta}^{s/p}/2 = 2(\Delta u + \Delta d - 2\Delta s)/3$, respectively.

Scalar current. The scalar form factors at zero recoil are conventionally referred to as nuclear sigma terms, $F_S^{q/N}(0) = \sigma_q^N$. We use $\sigma_u^p = (17 \pm 5)$ MeV, $\sigma_d^p = (32 \pm 10)$ MeV, $\sigma_u^n = (15\pm5)$ MeV, $\sigma_d^n = (36\pm10)$ MeV, obtained from expressions in Ref. [29] using a rather conservative estimate $\sigma_{\pi N} = (50\pm15)$ MeV [19], along with $\sigma_s^p = \sigma_s^n = (41.3\pm7.7)$ MeV [30–32].

Pseudoscalar current. The residua of the light-meson poles for the pseudoscalar form factors, $F_P^{q/N}(q^2)$, are given by $a_{P,\pi}^{u/p}/m_u = -a_{P,\pi}^{d/p}/m_d = g_A B_0/m_N$, $a_{P,\pi}^{s/p} = 0$, $a_{P,\eta}^{u/p}/m_u = a_{P,\eta}^{d/p}/m_d = -a_{P,\eta}^{s/p}/(2m_s) = B_0(\Delta u_p + \Delta d_p - 2\Delta u_s)/(3m_N)$, where B_0 is a ChPT constant, related to the quark condensate, that always appears multiplied by a quark mass, $B_0 m_q$. In the code we re-express these as $B_0 m_u = m_\pi^2/(1 + m_d/m_u) = (6.1 \pm 0.5) \times 10^{-3} \, \mathrm{GeV}^2$, $B_0 m_d = m_\pi^2/(1 + m_u/m_d) = (13.3 \pm 0.5) \times 10^{-3} \, \mathrm{GeV}^2$, and $B_0 m_s = m_\pi^2 m_s/(m_u + m_d) = (268 \pm 3) \times 10^{-3} \, \mathrm{GeV}^2$, using $m_u/m_d = 0.46 \pm 0.05$ and $2m_s/(m_u + m_d) = 27.5 \pm 0.3$ [23].

CP-even gluonic current. The value of the relevant form factor at zero recoil is given by $F_G^N(0) = -2m_G/27$, where $m_G = m_N - \sum_q \sigma_q^N = (848 \pm 14)$ MeV, using the values of nuclear sigma terms in Table I.

CP-odd gluonic current. The parameters describing the CP-odd gluonic form factor in (36) can be expressed in terms of the matrix elements of the axial current, and are given by $2a_{\tilde{G},\pi}^N = -\tilde{m} m_N g_A(1/m_u - 1/m_d)$, $6a_{\tilde{G},\eta}^N = -\tilde{m} m_N (\Delta u + \Delta d - 2\Delta s)(1/m_u + 1/m_d - 2/m_s)$, $b_{\tilde{G}}^N = -\tilde{m} m_N (\sum_q \Delta q/m_q)$, where $1/\tilde{m} = (1/m_u + 1/m_d + 1/m_s)$.

Tensor current. The matrix elements of tensor currents are described by three sets of form factors, but only two enter the chirally leading expressions, $F_{T,0}^{q/p}(0) = m_q g_T^q$, and $F_{T,1}^{q/N}(0) = -m_q B_{T,10}^{q/N}(0)$. In the $\overline{\mathrm{MS}}$ scheme at $Q = 2$ GeV, one has $g_T^u = 0.794 \pm 0.015$, $g_T^d = -0.204\pm0.008$, $g_T^s = (3.2\pm8.6)\cdot10^{-4}$ [33, 34]. Using the results of the constituent quark model in [35] we estimate $B_{T,10}^{u/p}(0) = 3.0\pm1.5$, $B_{T,10}^{d/p}(0) = 0.24\pm0.12$, and $B_{T,10}^{s/p} = 0.0\pm0.2$. For neutrons one has $F_{T,0}^{u(d,s)/n}(0) = m_{u(d,s)} g_T^{d(u,s)}$ and $B_{T,10}^{u(d,s)/n}(0) = B_{T,10}^{d(u,s)/p}(0)$.
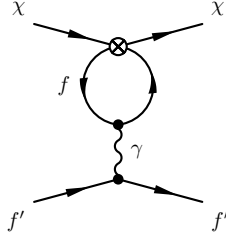
Figure 2: Mixing of dimension-six four-fermion operators into each other via the photon penguin insertion.

B.   QCD and QED running

The Wilson coefficients for the `4Flavor` and `5Flavor` bases are defined at $\mu_b = m_b$ and $\mu_Z = M_Z$, respectively, and need to be evolved down to $\mu_h = 2\,\mathrm{GeV}$, where the matching to the hadronic theory is performed. The RG evolution is achieved by standard methods (see, e.g., Ref. [8]) and involves the running from μ_Z to μ_b in the five-flavor scheme, and from μ_b to μ_h in the four-flavor scheme, integrating out the b quark and the c quark at the two thresholds.

Since the DM fields are QCD and QED singlets, the RG evolution of the operators Eqs. (2)-(9) is due only to their SM fields. Several of the operators have vanishing anomalous dimensions and the associated Wilson coefficients are RG invariant: this is the case for the dipole operators Eq. (2), the operators involving a quark vector current Eq. (3), and the scalar operators Eq. (7). Moreover, there is no one-loop QCD running for the operators involving an axial-vector quark current Eq. (4), the pseudoscalar operators Eq. (8), and the gluonic operators Eqs. (5)-(6) , so the only relevant effect of the RG evolution is a (small) rescaling of the coefficients of the tensor operators Eq. (9), and the mixing of the gluonic operators Eqs. (5)-(6) into the scalar operators Eqs. (7)-(8).

The QED contributions to the RG evolution can, in general, be neglected, due to the smallness of the electromagnetic coupling constant. The only exception is the off-diagonal mixing induced by photonic penguin diagrams (see Fig. 2) of the operators $\mathcal{Q}_{1,f}^{(6)}$ or $\mathcal{Q}_{2,f}^{(6)}$ among themselves, for different fermion flavors f. In this way, scattering on atomic nuclei can be generated even if, at tree level, DM couples only to leptons [36]. Note that the conservation of parity forbids the mixing of $\mathcal{Q}_{1,f}^{(6)}$ into $\mathcal{Q}_{2,f}^{(6)}$, or vice versa. The penguin insertions for all operators other than Eq. (3) vanish.

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Finite corrections arise at each heavy flavor threshold. Beside the usual threshold corrections to α_s (see, e.g., Ref. [37]), there are also finite threshold corrections for the operators Eq. (5)-(6), where at $\mu = \mu_b$,

$$
\begin{aligned}
\hat{\mathcal{C}}_{1(2)}^{(7)}|_{n_f=4}(\mu_b) &= \hat{\mathcal{C}}_{1(2)}^{(7)}|_{n_f=5}(\mu_b) - \hat{\mathcal{C}}_{5,b(6,b)}^{(7)}|_{n_f=5}(\mu_b) , \\
\hat{\mathcal{C}}_{3(4)}^{(7)}|_{n_f=4}(\mu_b) &= \hat{\mathcal{C}}_{3(4)}^{(7)}|_{n_f=5}(\mu_b) + \hat{\mathcal{C}}_{7,b(8,b)}^{(7)}|_{n_f=5}(\mu_b) ,
\end{aligned}
\tag{38}
$$

while at $\mu = \mu_c$,

$$
\begin{aligned}
\hat{\mathcal{C}}_{1(2)}^{(7)}|_{n_f=3}(\mu_c) &= \hat{\mathcal{C}}_{1(2)}^{(7)}|_{n_f=4}(\mu_c) - \hat{\mathcal{C}}_{5,c(6,c)}^{(7)}|_{n_f=4}(\mu_c) , \\
\hat{\mathcal{C}}_{3(4)}^{(7)}|_{n_f=3}(\mu_c) &= \hat{\mathcal{C}}_{3(4)}^{(7)}|_{n_f=4}(\mu_c) + \hat{\mathcal{C}}_{7,c(8,c)}^{(7)}|_{n_f=4}(\mu_c) ,
\end{aligned}
\tag{39}
$$

such that the effects of the heavy quarks appear, at low energies, as additional contributions to the gluonic operators Eq. (5)-(6). All other Wilson coefficients cross the thresholds continuously, $\hat{\mathcal{C}}_i^{(d)}|_{n_f-1} = \hat{\mathcal{C}}_i^{(d)}|_{n_f}$.

C. Scalar dark matter

For scalar DM, the effective interactions with the SM start at dimension six,

$$
\mathcal{L}_\varphi = \hat{\mathcal{C}}_a^{(6)} \mathcal{Q}_a^{(6)} + \cdots , \qquad \text{where} \quad \hat{\mathcal{C}}_a^{(6)} = \frac{\mathcal{C}_a^{(6)}}{\Lambda^2} .
\tag{40}
$$

Again, the $\mathcal{C}_a^{(6)}$ here are the dimensionless Wilson coefficients[2] of the effective interactions between DM and the SM. The operators coupling DM to quarks and gluons are

$$
\mathcal{Q}_{1,f}^{(6)} = \left(\varphi^* i \overset{\leftrightarrow}{\partial_\mu} \varphi\right)(\bar{f}\gamma^\mu f) , \qquad\qquad \mathcal{Q}_{2,f}^{(6)} = \left(\varphi^* i \overset{\leftrightarrow}{\partial_\mu} \varphi\right)(\bar{f}\gamma^\mu \gamma_5 f) , \tag{41}
$$

$$
\mathcal{Q}_{3,f}^{(6)} = m_f(\varphi^* \varphi)(\bar{f}f) , \qquad\qquad \mathcal{Q}_{4,f}^{(6)} = m_f(\varphi^* \varphi)(\bar{f}i\gamma_5 f) , \tag{42}
$$

$$
\mathcal{Q}_5^{(6)} = \frac{\alpha_s}{12\pi}(\varphi^* \varphi)G^{a\mu\nu}G_{\mu\nu}^a , \qquad\qquad \mathcal{Q}_6^{(6)} = \frac{\alpha_s}{8\pi}(\varphi^* \varphi)G^{a\mu\nu}\widetilde{G}_{\mu\nu}^a , \tag{43}
$$

while the couplings to photons are[3]

$$
\mathcal{Q}_7^{(6)} = \frac{\alpha}{\pi}(\varphi^* \varphi)F^{\mu\nu}F_{\mu\nu} , \qquad\qquad \mathcal{Q}_8^{(6)} = \frac{3\alpha}{\pi}(\varphi^* \varphi)F^{\mu\nu}\tilde{F}_{\mu\nu} . \tag{44}
$$

---

[2] For operators and Wilson coefficients we adopt the same notation for scalar DM as for fermionic DM. No confusion should arise as this abuse of notation is restricted to this section. In our code, the user can select either fermionic or scalar DM, see Sec. III.

[3] Note that the operator with one electromagnetic field strength tensor, $\partial_\mu \varphi^* \partial_\nu \varphi F^{\mu\nu}$, can be reduced to $\mathcal{Q}_{1,f}^{(6)}$ by using equations of motion for the photon field.

Here $\overset{\leftrightarrow}{\partial}_\mu$ is defined through $\phi_1 \overset{\leftrightarrow}{\partial}_\mu \phi_2 = \phi_1 \partial_\mu \phi_2 - (\partial_\mu \phi_1)\phi_2$. The operators $\mathcal{Q}_4^{(6)}$, $\mathcal{Q}_6^{(6)}$, and $\mathcal{Q}_8^{(6)}$ are CP-odd, while all the other operators are CP-even. In complete analogy to the case of fermionic DM, the operator basis can be specified in the `3Flavor`, `4Flavor`, or `5Flavor` scheme. The running of the Wilson coefficients and matchings at heavy flavor thresholds proceed along the lines discussed in Sec. II B.

The `DirectDM` code provides the matching onto the nonrelativistic Lagrangian for interactions with nucleons, $\mathcal{L}_{\mathrm{NR}} = \sum_{i,N} c_i^N(q^2)\mathcal{O}_i^N$, also for scalar DM. For the basis of nonrelativistic operators we use the same basis as for fermionic DM, Eqs. (11)-(15), dropping all operators that involve the DM spin. The coefficients $c_i^N(q^2)$ are [19]

$$c_1^N = \sum_q \left(2m_\varphi F_1^{q/N} \hat{\mathcal{C}}_{1q}^{(6)} + F_S^{q/N} \hat{\mathcal{C}}_{3q}^{(6)} \right) + F_G \hat{\mathcal{C}}_5^{(6)} \,, \tag{45}$$

$$c_7^N = -4m_\varphi \sum_q F_A^{q/N} \hat{\mathcal{C}}_{2q}^{(6)} \,, \tag{46}$$

$$c_{10}^N = \sum_q F_P^{q/N} \hat{\mathcal{C}}_{4q}^{(6)} + F_{\tilde{G}} \hat{\mathcal{C}}_6^{(6)} \,, \tag{47}$$

where the sum again runs over the light quark fields, $q = u, d, s$. The nuclear matrix elements of the electromagnetic operators (44) are rather uncertain and are currently not implemented in `DirectDM`.

III.   THE PROGRAM

The matching and running described above are implemented in a `Mathematica` package available at https://directdm.github.io. The `DirectDM` package has been tested on `Mathematica` versions 10 and 11. The package can be loaded via:

```
$DirectDMDirectory="</path/to/directdm/directory>";
<<DirectDM`
```

By default, the DM is assumed to be a Dirac fermion. This setting can be changed with the function

```
SetDMType["type"]
```

DM Type	SetDMType	Operator Numbering	Basis Definition
Dirac fermion	"D"	$\texttt{Q5}\,,\quad i \in \{1,2\}$ $\texttt{Q6}\,,\ i \in \{1,\ldots,4\}$ $\texttt{Q7}\,,\ i \in \{1,\ldots,10\}$	Eqs. (2)-(9)
Majorana fermion	"M"	$\texttt{Q6}\,,\quad i \in \{2,4\}$ $\texttt{Q7}\,,\ i \in \{1,\ldots,8\}$	Eqs. (A1)-(A5)
Complex scalar	"C"	$\texttt{Q6}\,,\ i \in \{1,\ldots,6\}$	Eqs. (41)-(43)
Real scalar	"R"	$\texttt{Q6}\,,\ i \in \{3,\ldots 6\}$	Eqs. (A6)-(A8)

Table II: Operator dimensions and numbering for the Dirac/Majorana fermion and complex/real scalar bases. Setting operator numbers outside the allowed values shown here will cause the `SetCoeff` function to generate an error message.

where `"type"` can be `"D"` for a Dirac fermion, `"M"` for a Majorana fermion, `"C"` for a complex scalar, and `"S"` for a real scalar.

Once loaded, the user can set the Wilson coefficients in the desired initial basis. The package then performs the running in the intermediate EFTs and the matching at the intermediate thresholds until the user-specified final basis is reached. The available bases are: `3Flavor`, `4Flavor`, `5Flavor`, and `NR`. The syntax to set the Wilson coefficients in the $n_f = \{3,4,5\}$ bases is:

```
SetCoeff["basis",QD[i,f],value]
```

The allowed arguments are `basis` $\in$ {`3Flavor`, `4Flavor`, `5Flavor`}, `QD` $\in$ {`Q5`, `Q6`, `Q7`} is the mass dimension of the operator, `i` is the operator number, and, finally, `f` $\in$ {`"u"`, `"d"`, `"s"`, `"c"`, `"b"`, `"e"`, `"mu"`, `"tau"`} is the flavor index of the operator where the allowed values clearly depend on the basis in question. These operators are defined in Eqs. (2)-(9). Note that in the case that the operator does not include a SM fermion current, the operator name syntax is simply `QD[i]` with no flavor index. The allowed values for the indices `i` depend on the type of DM and the operator dimension. They are given in Tab. II. The

matching scales are:

$$\begin{aligned}
\texttt{5Flavor}: \quad & \mu_Z = M_Z = 91.1876\,\text{GeV}\,, \\
\texttt{4Flavor}: \quad & \mu_b = 4.18\,\text{GeV}\,, \\
\texttt{3Flavor}: \quad & \mu_c = 2\,\text{GeV}\,, \\
\texttt{NR}: \quad & \mu_N = 2\,\text{GeV}\,.
\end{aligned} \tag{48}$$

Consequently there is no running in the `3Flavor` flavor basis.

To perform the running and matching, the user must then issue the command

```
ComputeCoeffs["basis_i", "basis_f"]
```

where `basis_i` is the intial basis and `basis_f` is the final basis. The `ComputeCoeffs` function takes an optional argument `Running -> True/False`. It is set to `True` by default. Setting it to `False` disables the QCD and QED running in the intermediate EFTs. As mentioned above, this option has no effect in the `3Flavor` basis.

Finally, to retrieve the Wilson coefficients in the final basis, the package provides two functions: `GetCoeff` and `CoeffsList`. The former takes the same arguments as the `SetCoeff` function but is only implemented for the `5Flavor`, `4Flavor`, and `3Flavor` bases.

```
GetCoeff["basis",QD[i,f]]
```

This function allows the user to retrieve one coefficient at a time. For the `NR` basis, however, it is more practical to retrieve the entire list of Wilson coefficients. This can be done with:

```
CoeffsList["basis"]
```

where `basis`, in this case, can be `5Flavor`, `4Flavor`, `3Flavor`, `NR_p`, or `NR_n`. Note the syntax for the `NR` basis; here, the user must specify the proton, `"NR_p"`, or the neutron, `"NR_n"`, basis explicitly.

Of course, the user might wish to set all Wilson coefficients to zero and start afresh. This can be done via:

```
ResetBasis["basis"]
```

where `basis` can take any of the allowed values discussed above. For the NR EFT, `basis` can only be `NR` – i.e., not `NR_p` or `NR_n`. If called without an argument, i.e., `ResetBasis[]`, the function resets all bases.

Output of the `DirectDM` code is structured in such a way that it is easy to interface with the `DMFormFactor` package [4]. An example of such an interface is given in the `example.nb` notebook, included in the distribution.

IV. CONCLUSIONS

We presented a `Mathematica` package, `DirectDM`, that performs an important intermediate step in the calculation of event rates in dark matter direct detection experiments. It takes as an input the Wilson coefficients of the EFT coupling DM to quark, gluons, photons, and performs a matching onto an EFT describing DM with nonrelativistic protons and neutrons, at leading order in a chiral expansion of the hadronic form factors. The QCD and QED RG evolution from the electroweak to the hadronic scale, finite matching corrections at the heavy quark thresholds, and tree-level meson exchange contributions in the chiral effective theory are consistently taken into account. The effects of operator mixing above the electroweak scale will be included as part of a future project [38].

Appendix A: Operator basis for Majorana and real scalar DM

a. Majorana fermion.   For simplicity we use the same notation for the operators with Majorana fermion DM and for the operators with Dirac fermion DM, if the Lorentz structures of the DM $\otimes$ SM currents coincide. For a Majorana fermion the operators $\mathcal{Q}_{1,2}^{(5)}$, $\mathcal{Q}_{1,f}^{(6)}$, $\mathcal{Q}_{3,f}^{(6)}$, $\mathcal{Q}_{9,f}^{(7)}$, and $\mathcal{Q}_{10,f}^{(7)}$ are absent since in that case the vector and tensor currents vanish. We include an additional factor of 1/2 in the definition of the Majorana DM operators to compensate for the additional Wick contraction in the case of a Majorana fermion. The two

nonzero dimension six operators are,

$$\mathcal{Q}_{2,f}^{(6)} = \frac{1}{2}\left(\bar{\chi}\gamma_\mu\gamma_5\chi\right)(\bar{f}\gamma^\mu f)\,, \qquad\qquad \mathcal{Q}_{4,f}^{(6)} = \frac{1}{2}\left(\bar{\chi}\gamma_\mu\gamma_5\chi\right)(\bar{f}\gamma^\mu\gamma_5 f)\,, \qquad \text{(A1)}$$

and the eight nonzero dimension seven operators,

$$\mathcal{Q}_1^{(7)} = \frac{\alpha_s}{24\pi}(\bar{\chi}\chi)G^{a\mu\nu}G_{\mu\nu}^a\,, \qquad\qquad \mathcal{Q}_2^{(7)} = \frac{\alpha_s}{24\pi}(\bar{\chi}i\gamma_5\chi)G^{a\mu\nu}G_{\mu\nu}^a\,, \qquad \text{(A2)}$$

$$\mathcal{Q}_3^{(7)} = \frac{\alpha_s}{16\pi}(\bar{\chi}\chi)G^{a\mu\nu}\widetilde{G}_{\mu\nu}^a\,, \qquad\qquad \mathcal{Q}_4^{(7)} = \frac{\alpha_s}{16\pi}(\bar{\chi}i\gamma_5\chi)G^{a\mu\nu}\widetilde{G}_{\mu\nu}^a\,, \qquad \text{(A3)}$$

$$\mathcal{Q}_{5,f}^{(7)} = \frac{m_f}{2}(\bar{\chi}\chi)(\bar{f}f)\,, \qquad\qquad \mathcal{Q}_{6,f}^{(7)} = \frac{m_f}{2}(\bar{\chi}i\gamma_5\chi)(\bar{f}f)\,, \qquad \text{(A4)}$$

$$\mathcal{Q}_{7,f}^{(7)} = \frac{m_f}{2}\left(\bar{\chi}\chi\right)(\bar{f}i\gamma_5 f)\,, \qquad\qquad \mathcal{Q}_{8,f}^{(7)} = \frac{m_f}{2}(\bar{\chi}i\gamma_5\chi)(\bar{f}i\gamma_5 f)\,, \qquad \text{(A5)}$$

    b. Real scalar. Similarly, the operators for a real scalar DM are a subset of the operators for complex scalar DM, and carry an additional factor of $1/2$. The relevant dimension six operators are,

$$\mathcal{Q}_{3,f}^{(6)} = \frac{m_f}{2}(\varphi\varphi)(\bar{f}f)\,, \qquad\qquad \mathcal{Q}_{4,f}^{(6)} = \frac{m_f}{2}(\varphi\varphi)(\bar{f}i\gamma_5 f)\,, \qquad \text{(A6)}$$

$$\mathcal{Q}_5^{(6)} = \frac{\alpha_s}{24\pi}(\varphi\varphi)G^{a\mu\nu}G_{\mu\nu}^a\,, \qquad\qquad \mathcal{Q}_6^{(6)} = \frac{\alpha_s}{16\pi}(\varphi\varphi)G^{a\mu\nu}\widetilde{G}_{\mu\nu}^a\,, \qquad \text{(A7)}$$

$$\mathcal{Q}_7^{(6)} = \frac{\alpha}{2\pi}(\varphi\varphi)F^{\mu\nu}F_{\mu\nu}\,, \qquad\qquad \mathcal{Q}_8^{(6)} = \frac{3\alpha}{4\pi}(\varphi\varphi)F^{\mu\nu}\tilde{F}_{\mu\nu}\,. \qquad \text{(A8)}$$

Appendix B: Translation from the basis of Goodman et al.

    Here we provide a translation between our basis for DM interactions, Eqs. (2)-(9) and Eqs. (41)-(44), to the basis used by Goodman et al., Ref. [20]. For *Dirac fermion DM* the EFT interaction Lagrangian in the basis of Ref. [20] is

$$\mathcal{L}_\chi = \sum_i G_{Di}\mathcal{Q}_{Di}, \qquad \text{(B1)}$$

where the operators $\mathcal{Q}_{Di}$, $i = 1,\ldots,16$ (the Wilson coefficients G_{Di}) are listed in the 2nd (3rd) column of Table II, left, in Ref. [20], see also Ref. [39] where G_{Di} were labeled G_χ. The Wilson coefficients in (1) are thus, for Dirac fermion DM, given by

$$\hat{\mathcal{C}}_{1,q}^{(5)} = \frac{8\pi^2}{e}M\,, \qquad\qquad \hat{\mathcal{C}}_{2,q}^{(5)} = -i\frac{8\pi^2}{e}D\,, \qquad \text{(B2)}$$

$$\hat{\mathcal{C}}_{1,q}^{(6)} = \left[M_*^{\{\text{D5},q\}}\right]^{-2}\,, \qquad\qquad \hat{\mathcal{C}}_{2,q}^{(6)} = \left[M_*^{\{\text{D6},q\}}\right]^{-2}\,, \qquad \text{(B3)}$$

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$$\hat{\mathcal{C}}_{3,q}^{(6)} = \left[M_*^{\{D7,q\}}\right]^{-2} , \qquad\qquad \hat{\mathcal{C}}_{4,q}^{(6)} = \left[M_*^{\{D8,q\}}\right]^{-2} , \tag{B4}$$

$$\hat{\mathcal{C}}_1^{(7)} = 3\pi \left[M_*^{\{D11\}}\right]^{-3} , \qquad\qquad \hat{\mathcal{C}}_2^{(7)} = 3\pi \left[M_*^{\{D12\}}\right]^{-3} , \tag{B5}$$

$$\hat{\mathcal{C}}_3^{(7)} = 2\pi i \left[M_*^{\{D13\}}\right]^{-3} , \qquad\qquad \hat{\mathcal{C}}_4^{(7)} = -2\pi i \left[M_*^{\{D14\}}\right]^{-3} , \tag{B6}$$

$$\hat{\mathcal{C}}_{5,q}^{(7)} = \left[M_*^{\{D1,q\}}\right]^{-3} , \qquad\qquad \hat{\mathcal{C}}_{6,q}^{(7)} = \left[M_*^{\{D2,q\}}\right]^{-3} , \tag{B7}$$

$$\hat{\mathcal{C}}_{7,q}^{(7)} = \left[M_*^{\{D3,q\}}\right]^{-3} , \qquad\qquad \hat{\mathcal{C}}_{8,q}^{(7)} = -\left[M_*^{\{D4,q\}}\right]^{-3} , \tag{B8}$$

$$\hat{\mathcal{C}}_{9,q}^{(7)} = \frac{1}{m_q}\left[M_*^{\{D9,q\}}\right]^{-2} , \qquad\qquad \hat{\mathcal{C}}_{10,q}^{(7)} = \frac{1}{m_q}\left[M_*^{\{D10,q\}}\right]^{-2} . \tag{B9}$$

The notation we use above is that the Wilson coefficient $G_{\chi,Di}$, multiplying the operator $\mathcal{Q}_{Di}$, depends on $M_*^{\{Di\}}$. That is, in order for only the operator $\mathcal{Q}_{Dj}$ to contribute one needs to set $M_*^{\{Dj\}}$ to the desired finite value, while taking $M_*^{\{Di\}} \to \infty$ for $i \neq j$ (and setting $D = M = 0$). In the notation of Ref. [20] the superscripts on $M_*^{\{Di\}}$ were suppressed. Note that the above transformation between the two bases involves complex phases, signaling that some the operators in [20] are not Hermitian, but anti-Hermitian. Consequently, the corresponding parameters D, $M_*^{\{D13\}}$, and $M_*^{\{D14\}}$ need to be chosen purely imaginary.

Majorana DM. Similarly, the translation from our basis defined in Eqs. (A1)-(A5) to that of [20] is given by

$$\hat{\mathcal{C}}_{2,q}^{(6)} = \left[M_*^{\{M5,q\}}\right]^{-2} , \qquad\qquad \hat{\mathcal{C}}_{4,q}^{(6)} = \left[M_*^{\{M6,q\}}\right]^{-2} , \tag{B10}$$

$$\hat{\mathcal{C}}_1^{(7)} = 3\pi \left[M_*^{\{M7\}}\right]^{-3} , \qquad\qquad \hat{\mathcal{C}}_2^{(7)} = 3\pi \left[M_*^{\{M8\}}\right]^{-3} , \tag{B11}$$

$$\hat{\mathcal{C}}_3^{(7)} = 2\pi i \left[M_*^{\{M9\}}\right]^{-3} , \qquad\qquad \hat{\mathcal{C}}_4^{(7)} = -2\pi i \left[M_*^{\{M10\}}\right]^{-3} , \tag{B12}$$

$$\hat{\mathcal{C}}_{5,q}^{(7)} = \left[M_*^{\{M1,q\}}\right]^{-3} , \qquad\qquad \hat{\mathcal{C}}_{6,q}^{(7)} = \left[M_*^{\{M2,q\}}\right]^{-3} , \tag{B13}$$

$$\hat{\mathcal{C}}_{7,q}^{(7)} = \left[M_*^{\{M3,q\}}\right]^{-3} , \qquad\qquad \hat{\mathcal{C}}_{8,q}^{(7)} = -\left[M_*^{\{M4,q\}}\right]^{-3} . \tag{B14}$$

For *complex scalar DM* the interaction Lagrangian in the basis of Ref. [20] is given by

$$\mathcal{L}_\varphi = \sum_i G_{Ci} \mathcal{Q}_{Ci}, \tag{B15}$$

with the operators $\mathcal{Q}_{Ci}$, $i = 1, \ldots, 6$, (the Wilson coefficients G_{Ci}) are listed in the 2nd (3rd) column of Table II, right, in Ref. [20]. The translation of the Wilson coefficients to our basis for complex scalar DM is thus,

$$\hat{\mathcal{C}}_{1,q}^{(6)} = -\frac{i}{2}\left[M_*^{\{C3,q\}}\right]^{-2} , \qquad \hat{\mathcal{C}}_{2,q}^{(6)} = -\frac{i}{2}\left[M_*^{\{C4,q\}}\right]^{-2} , \tag{B16}$$

$$\hat{\mathcal{C}}_{3,q}^{(6)} = \left[M_*^{\{C1,q\}}\right]^{-2} , \qquad \hat{\mathcal{C}}_{4,q}^{(6)} = \left[M_*^{\{C2,q\}}\right]^{-2} - \left[M_*^{\{C4,q\}}\right]^{-2} , \tag{B17}$$

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$$\hat{\mathcal{C}}_5^{(6)} = 3\pi \left[M_*^{\{\text{C5}\}} \right]^{-2} , \qquad\qquad \hat{\mathcal{C}}_6^{(6)} = 2\pi i \left[M_*^{\{\text{C6}\}} \right]^{-2} + \sum_q \left[M_*^{\{\text{C4},q\}} \right]^{-2} , \qquad \text{(B18)}$$

where for clarity we display explicitly the operator dependence of each M_*. In deriving the above relations we used equation of motion for the vector current, $\partial_\mu \bar{q} \gamma^\mu q = 0$, and the relation between the chiral QCD anomaly and the axial current, valid for each quark flavor separately,

$$\partial_\mu \bar{q} \gamma^\mu \gamma_5 q = 2 m_q \bar{q} i \gamma_5 q - \frac{\alpha_s}{4\pi} G_{\mu\nu}^a \tilde{G}^{a,\mu\nu} . \qquad \text{(B19)}$$

Note that there is no choice of the scale $M_*^{\{\text{C4},q\}}$ that makes the Lagrangian Eq. (B15) Hermitian.

Real scalar DM. Finally, the translation for the Wilson coefficients of the real scalar operator basis is given by,

$$\hat{\mathcal{C}}_{3,q}^{(6)} = \left[M_*^{\{\text{R1},q\}} \right]^{-2} , \qquad\qquad \hat{\mathcal{C}}_{4,q}^{(6)} = \left[M_*^{\{\text{R2},q\}} \right]^{-2} , \qquad \text{(B20)}$$

$$\hat{\mathcal{C}}_5^{(6)} = 3\pi \left[M_*^{\{\text{R3}\}} \right]^{-2} , \qquad\qquad \hat{\mathcal{C}}_6^{(6)} = 2\pi i \left[M_*^{\{\text{R4}\}} \right]^{-2} . \qquad \text{(B21)}$$

In `DirectDM` the user can directly input the Wilson coefficients in the basis of [20] by setting the value of M_*. To do this, we provide a function, `SetCoeffMstar`, which takes the following arguments

```
SetCoeffMstar["basis",QN[f],value]
```

where $\text{QN} \in \{\text{D1},\dots,\text{D16}; \text{M1},\dots,\text{M10}; \text{C1},\dots, \text{C6}; \text{R1},\dots,\text{R4}\}$ and `"basis"` and `f` are the basis and the quark flavor respectively – see the documentation of `SetCoeff` in Sec. III for further detail.

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