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H.S. Köhler

June 1989

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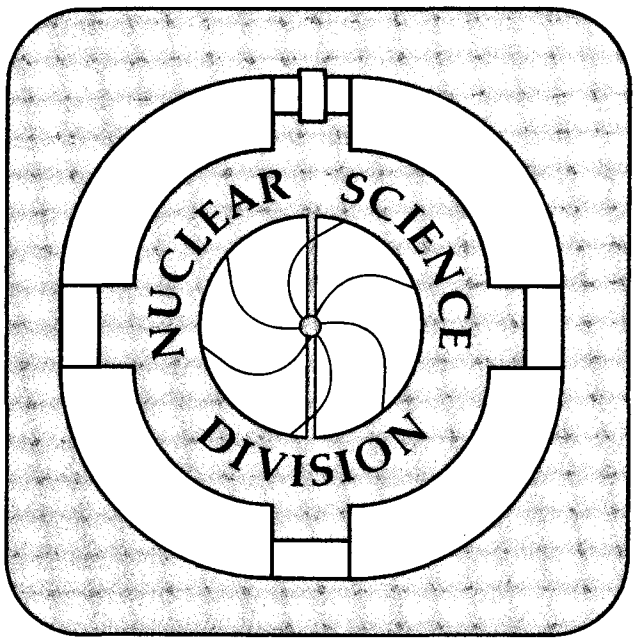

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Mean Field and Collisions in Hot Nuclei

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Collisions between heavy nuclei produce nuclear matter of high density and excitation. Brueckner methods are used to calculate the momentum and temperature dependent mean field for nucleons propagating through nuclear matter during these collisions. The mean field is complex and the imaginary part is related to the "two-body" collisions, while the real part relates to "one-body" collisions. A potential model for the N-N interactions is avoided by calculating the Reaction matrix directly from the T-matrix (i.e. N-N phase-shifts) using a version of Brueckner theory previously published by the author. Results are presented for nuclear matter at normal and twice normal density and for temperatures up to 50 MeV.

1 Introduction

The primary purpose of Heavy-Ion (H.I.) collisions is to explore the properties of hot nuclear matter; both static and dynamic. It is of particular interest to analyse these collisions in order to learn about nucleonic degrees of freedom. But before proceeding to introduce some model of these into a theory of hot nuclei it seems appropriate to investigate the consequence of incorporating important many-body effects in a "nucleons only" theory. This has not been done satisfactorily. A first step is to find the "effective" force V_{eff} in hot nuclear matter from the known "free" N-N interaction. Most "microscopic" calculations have been done with forces that are fitted to zero temperature properties like binding and compressibility and depend on local density only but not on other variables of the medium. This might be adequate for low energies. It is however well known that the effective force in nuclear matter is momentum dependent (non-local). Collisions at higher energy result in strong deformations in momentum space. A force that depends on density only, gives an energy-functional that is independent of such deformations, while a momentum-dependent force leads to a dependence on this deformation. It follows that it may be necessary to incorporate the momentum dependence of the force especially in a theory of high energy H.I. collisions. Furthermore,the strength of the force depends not just on density but also on excitaion; it is "temperature"-dependent. This investigation is a contribution to our understanding of the effective forces in hot nuclei both as regards the mean field and the two-body dissipative collisions.

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Let us go into some more detail as regards the arguments that were given above.

The momentum distribution in the interior of a ground state nucleus is essentially isotropic and with a sharp Fermi-surface as in a Thomas-Fermi approximation. In the initial stage of a high energy collision between two nuclei the momentum distribution at some point where the two nuclei overlap in coordinate space, is strongly deformed. It is roughly that of two Fermi-spheres separated by the relative momentum of the colliding ions. Our earlier calculations [1], incorporating two-body collisions by the relaxation-time method, show this explicitly to be the case for the distribution averaged over coordinate space. These calculations also show that the mean field distorts this averaged distribution only slightly during the course of a collision, while two-body collisions thermalizes it.

In a model of H.I. collisions where the effective two-body interaction V_{eff} is local i.e. independent of relative N-N momentum the resultant mean field will also be local and independent of the distribution in momentum space; other than of the zero moment of this distribution, i.e. the local density. Most calculations like VUU [2], BUU [3] or TDHFRX and VRX [4] have used this model. In a calculation of the properties of nuclear matter from realistic nuclear forces the momentum dependence is very important for obtaining the correct saturation density and in calculating the compressibility. In calculations of collisions between heavy ions made by Stöcker et al [5] using a local N-N interaction V_{eff}, it was found necessary to use quite a large compressibility in order to obtain the experimentally observed perpendicular flow.

One can however argue that the increase in compressibility is necessitated by neglecting the (known) momentum dependence. This can be understood as follows. By deforming or heating a zero-temperature Fermi-distribution (while keeping the density constant) the repulsive part of the energy due to the momentum dependence of the force will increase. Collisions between the ions result in such deformations in momentum space. If the energy is assumed to be independent of the deformation this increased repulsion has to be compensated for by increasing the compressibility, making use of the fact that the density also increases in the interaction region.

The calculations by Welke et al [6] (using a momentum dependence resembling the Yukawa force in momentum-space) bear this out, although the simple argument given above may not be correct. It may not just be a question of the energy-functional. Rather it appears that the dynamics especially as regards the perpendicular flow is different in the two separate cases [7].

Increasing the stiffness of the equation of state by incresing the density dependence of V_{eff} results in a larger repulsion in the mean field in the region of overlap between the ions. Nucleons hitting this repulsion in a non-central collision will be reflected out sideways. If the force, and consequently the mean field, is momentum dependent a different mechanism described as a 'coherence' in phase-space seems to enter[7]. A similar effect was observed when displaying the Wigner functions for low energy (head-on) collisions between slabs with a momentum-dependent force of quadratic (Skyrme) form[8]. A density dependent force tended to break up the distribution in phase-space much more than the momentum-dependent although

the compressibility was the same for both.

Whatever the mechanism is, the calculations do indicate that the momentum dependence is important[6] and it is one purpose of this paper to calculate this macroscopically.

We have stressed that in the case of a momentum dependent V_{eff}, the mean field will change if the distribution of nucleons in momentum-space is deformed. There is however another effect of deformation to consider. In a microscopic theory V_{eff} will itself depend on the distribution; not just on density but also on the deformation under constant density. In Brueckner's formulation of the many-body problem this is explicit in that the Reaction-matrix, which in our context here is the same as the effective interaction V_{eff}, is a functional of the Pauli-operator which expresses a dependence on which states are occupied. There is also a dispersion-effect in that the nucleons propagate through a mean field while interacting. All of these effects are included in the Reaction-matrix equation defined below in Sect. 2. They are all important when calculating the Reaction-matrix at zero temperature. There is no reason to beleive they should be less important at higher temperatures.

Although the Reaction-matrix is formally defined for any distribution of occupied states, calculations are in practice restricted to some simple cases. The simplest is the zero-temperature Fermi-distribution for which of course many calculations have been done. Non-zero temperature Brueckner calculations have also been done [9,10] and in relativistic Brueckner theory by Malfliet [11]. While a heated Fermi-sphere would be the appropriate representation for the final stages of the collision, the distribution in the initial stage of a H.I. collision may best be represented by two spheres separated by the relative momentum of the two ions. Calculations for this system has also been done as a step towards calculating an optical model potential for ion collisions [12,13].

In this paper we shall present results of calculations of the mean field potential in hot nuclear matter using a method that was presented in an earlier publication [14]. We have used this method for the two sphere problem as well, but shall reserve those results for later presentation. For completeness a short description of the method is given in sect. 2 and the results are shown in sect. 3. Sect. 4 contains a summary and conclusions.

2 The Effective Interaction in Nuclei

The calculations in this paper are based upon the Brueckner many body theory of the effective interaction V_{eff}; in this theory referred to as the Reaction matrix K. A difference from the conventional formalism is that we shall calculate the K-matrix directly from the scattering T-matrix. The first order approximation to K will be T rather than a N-N potential as in conventional approaches. In order to calculate K from T one needs the off-diagonal matrix-elements of the related reactance-matrix $\mathcal{K}$ (see below) that are not accessible from N-N scattering. To overcome this problem we simply assume $\mathcal{K}$ to be separable in momentum-space. It is shown below that it

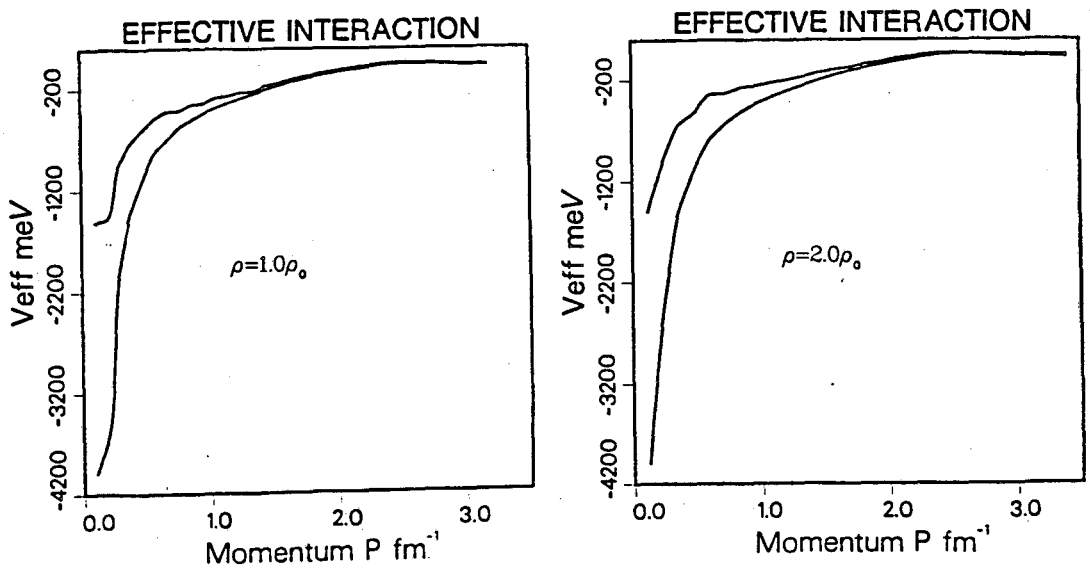

Figure 1: The effective interaction at normal (left curves) and double density (right curves) as a function of relative momentum P. Lower curves are from T- and upper is from K-matrix. See text.

is then rather easy to solve the equations for diagonal elements of K. For nuclear matter calculations we only need these diagonal elements of the effective interaction. The assumption of separability may seem ad hoc. We think it is justified by the agreement with calculations from N-N potentials and by the comparative ease with which calculations can be done for rather complicated distributions.

Our method should not be confused with the method of using a separable potential. We are in fact not using a potential model (at least not explicitly) and we consider this an advantage in itself.

Our approach using the T-matrix as a first approximation is also made more sensible if we note that in the low density limit, and in the limit of large relative momentum, $K \Rightarrow T$. This is shown in fig. 1 at normal (left) and double (right) density. The lower curve in each figure is the T-matrix approximation (in some works referred to as the phase-shift approximation) and the more repulsive curve is the K-matrix effective interaction.

We now describe some details of calculating K following the procedure outlined above. It is defined by

$$K = v + v\frac{Q}{e + i\eta}K. \tag{1}$$

Here Q is the Pauli-operator and the energy-denominator contains the (bubble-)interactions with all other nucleons. The traditional procedure is to calculate K from this equation assuming the interaction v, the 'free' N-N interaction to be known. Instead we proceed by first defining a reactance-matrix $\mathcal{K}$ by

$$\mathcal{K} = v + v\frac{P}{e_0}\mathcal{K} \tag{2}$$

where P denotes that the principal value is to be taken when e_0, which is kinetic energy only, has a pole. This reactance-matrix has the useful property that it is directly related to the phase-shifts. In an angular momentum decomposition one has

$$\mathcal{K} = tan\delta_l(k)/k \tag{3}$$

for free scattering states while for bound states [15]

$$\mathcal{K} = \delta_l(k)/k \tag{4}$$

with $\delta_l(k)$ being the phase-shifts. The relation between K and $\mathcal{K}$ is

$$K = \mathcal{K} + \mathcal{K}\Big(\frac{Q}{e+i\eta} - \frac{P}{e_0}\Big)K \tag{5}$$

The first approximation to this equation is obtained by putting $Q = 1$ and $e = e_0$ in (eq.5) to give

$$K_1 = T$$

which is usually referred to as the impulse approximation. This implies using free cross-sections.

In order to solve eq.5 for K we first introduce another matrix K_p by

$$K_p = v + v\frac{QP}{e}K_p \tag{6}$$

related to K by ($\delta(e)$) is here *delta*-function of e)

$$K = K_p(1 - iK_pQ\delta(e))^{-1} \tag{7}$$

and to $\mathcal{K}$ by

$$K_p = \mathcal{K} + \mathcal{K}\Big(\frac{QP}{e} - \frac{P}{e_0}\Big)K_p. \tag{8}$$

This equation is readily solvable in momentum space for the diagonal elements K_p if we assume $\mathcal{K}$ to be separable. In fact we find $K_p = D(k)\mathcal{K}$ where D is a function of relative momentum k only and is related to the integral

$$I(k) = \int_0^\infty |\mathcal{K}|\Big(\frac{QP}{e} - \frac{P}{e_0}\Big)d\mathbf{k}'. \tag{9}$$

by $D(k) = 1/(1 - I(k))$. The Reaction-matrix K is then obtained from Eq. (7). It is in general complex. A non-zero imaginary part is obtained whenever we have a pole, that results in energy-conserving transitions. To first approximation Eq. (7) is solved by

$$K = K_p + i|K_p|^2Q\delta(e). \tag{10}$$

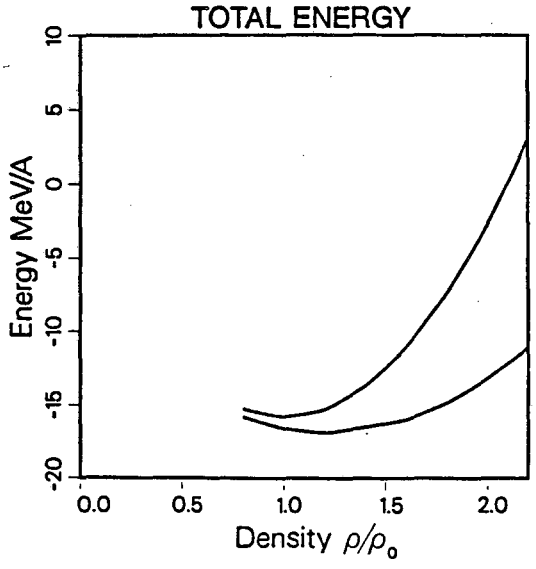

Figure 2: Upper curve is the empirical binding energy curve saturating at ρ_0. The lower is from our Brueckner-calculation.

If the absorption is small i.e. if the imaginary part of K is small compared to the real part of K, this approximation which has often been used is adequate, but we shall find later that for large nucleon momenta with large absorption, corrections can be quite large. It is also to be noted that the total energy now is complex, except in case of a zero-temperature distribution.

The method descibed above has been tested against calculations of the Reaction matrix from the Reid HC potential model and was found quite satisfactory[14]. The ^1S states were reproduced essentially exactly. For some states ($^1D_2$,$^1P_1$,$^3P_0$ and $^3P_1$) the T-matrix approximation is actually already very good and was therefore used. The coupled states ($^3S - ^3D$) require a special treatment. For these states the contribution from the coupling was simply added to the right hand side of Eq. (9) and this procedure was found quite adequate.

The integration in Eq. (9) was cut off at $20fm^{-1}$. The phase-shifts used in the calculations presented below are those of Arndt et al [16] which are for labenergies below 500 MeV only. For higher energies a straight line extrapolation was used with a slope that was considered a parameter. In a potential model this procedure corresponds to the choice of short-range repulsion. With a reasonable value of this parameter a binding energy of about 16 MeV/A is obtained at normal saturation-density.

The phase-shifts, the stipulation of a separable matrix and this parameter constitutes our 'potential' model although the potential itself is not obtainable.

The binding-energy that we obtain is shown by the lower curve in fig. 2. The upper curve is the empirical saturation curve. Just as in other non-relativistic Brueckner and HNC calculations saturation is not obtained in this model at the empirical point. Following the ideas in ref.[17] it is assumed that the difference between the

calculated saturation curve and the "experimental" is due to 3-body and/or higher order terms. We remedy the situation by adding a density-dependent term E_3 to the Nuclear Hamiltonian. This Hamiltonian will contribute to the mean field. It is repulsive.

3 Calculations

Following the procedure just outlined in the previous section we assume that nuclear matter saturates at -15.8 MeV/A at a saturation density of 0.166 fm^{-3} and with a compressibility of 235 MeV. We achieve this by adding to the microscopic calculation of the energy per particle a function E_3 defined by

$$E_3 = 0.46 - 5.79(\rho/\rho_0) + 5.33(\rho/\rho_0)^2 \tag{11}$$

where ρ_0 is the saturation density. This part of the Hamiltonian will contribute a quantity V_3 to the mean field with

$$V_3 = 0.46 - 11.58(\rho/\rho_0) + 15.99(\rho/\rho_0)^2 \tag{12}$$

including rearrangement terms. It is to be noted that V_3 does not have to be added to the energy-denominator e in the calculation of the Brueckner Reaction-matrix because it is independent of momentum and therefore cancels when taking the difference between particle- and hole-energies.

The Reaction-matrix is calculated with on-energy shell insertions in both hole and particle lines, i.e. with a continuous spectrum. The Pauli operator for $T > 0$ is calculated in the angle averaged approximation [18].

The mean field at normal and double density is shown in fig. 3 at T =0 and 50 MeV. In addition to the energy V_3 the third oder rearrangement energy V_{Rh} has been added. It is well approximated by[19]

$$V_{Rh} = -\rho \mathcal{I}_w V_h \tag{13}$$

where V_h is the first order K-matrix energy. The "wound-integral" $\mathcal{I}_w$ is a model-dependent factor and is given the values 1.2 and 1.0 at normal and double density respectively.

The results (at T=0) shown in fig. 3 have been compared with the results of Wiringa[20] who used HNC methods and various N-N interactions to calculate single particle potentials at zero temperature . The agreement is satisfactory although his potential at double density (interpolating between his 0.3 and 0.4 fm^{-3} results) is about 25 MeV more repulsive than ours at the highest momenta. Part of the difference can be explained by the fact mentioned above after Eq. (10), that we are including propagations in an absorptive medium, even when calculating the real part of the mean field. We have estimated this contribution by calculating K from Eq. (10) instead of from Eq. (7). This increases the repulsion by about 10 MeV at double density and high momenta where the absorption is large. The

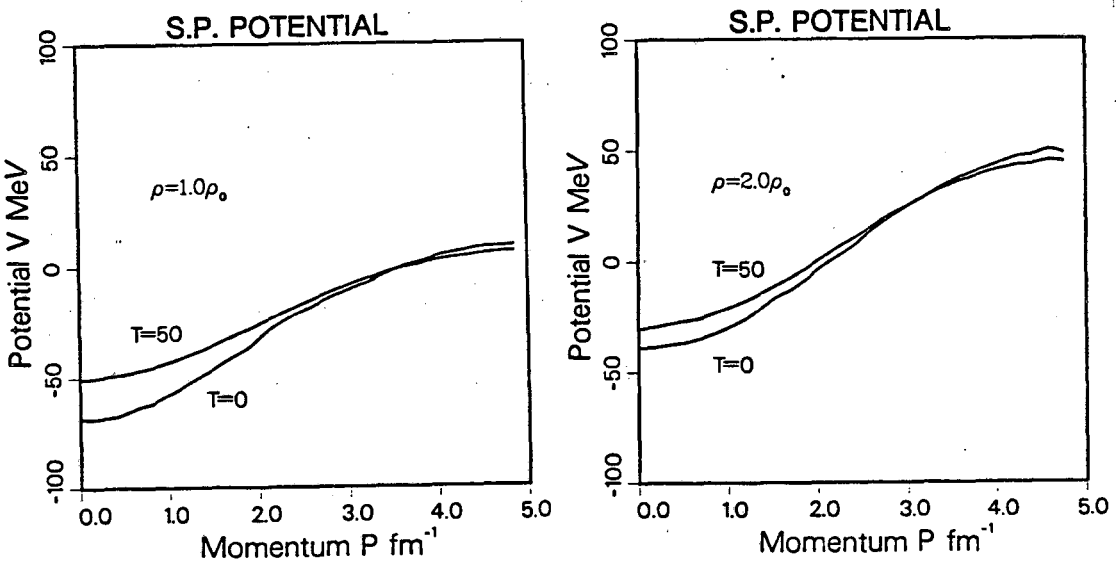

Figure 3: Mean field at normal(left) and double (right) density. at temperatures T=0 and 50 MeV as a function of momentum P.

propagation through an absorptive potential has therefore a non-negligble effect on the real potential , but this effect is not included in ref.[20]. Another point is that Wiringa "normalised" his curves to go through some common point, so that a direct comparison is not really relevant.

In addition to being momentum-dependent the mean field is also temperature-dependent. This is of course at least partly a consequence of the momentum dependence of the two-body interaction V_{eff}. To investigate this further we compare with the predictions of the parametrization used by Welke et al[6] who use a temperature-independent phenomenological interaction. Fig. 4 shows the mean field calculated with their interaction at normal and double density and indicated temperatures. Comparing with fig.3 we see that the overall agreement is good but there is a noticeable difference in temperature dependence especially at the higher density. This is understood as follows. We may consider the ref.[6] effective interaction as the first order contribution in the separation method of Moszkowsi and Scott[21]. This is (essentially) independent of the medium. The second order contribution is of the form $v(Q/e)v$ and depends on temperature (and density) through the Pauli Q-operator which blocks scattering into occupied states. At zero temperature the blocking increases with density. As temperature is increased blocking is decreased and the interaction becomes more attractive. Referring to fig 1, the effective interaction will approach the T-matrix interaction as temperature increases. On the basis of our results one could consider including the temperature dependence by a temperature dependent factor applied to the ref.[6] interaction.

The imaginary part of the mean field relates to dissipative two-body collisions[22]. The left parts of fig. 5 and fig. 6 show the results at normal and double density respectively. The right parts of the same figures show the absorption calculated from

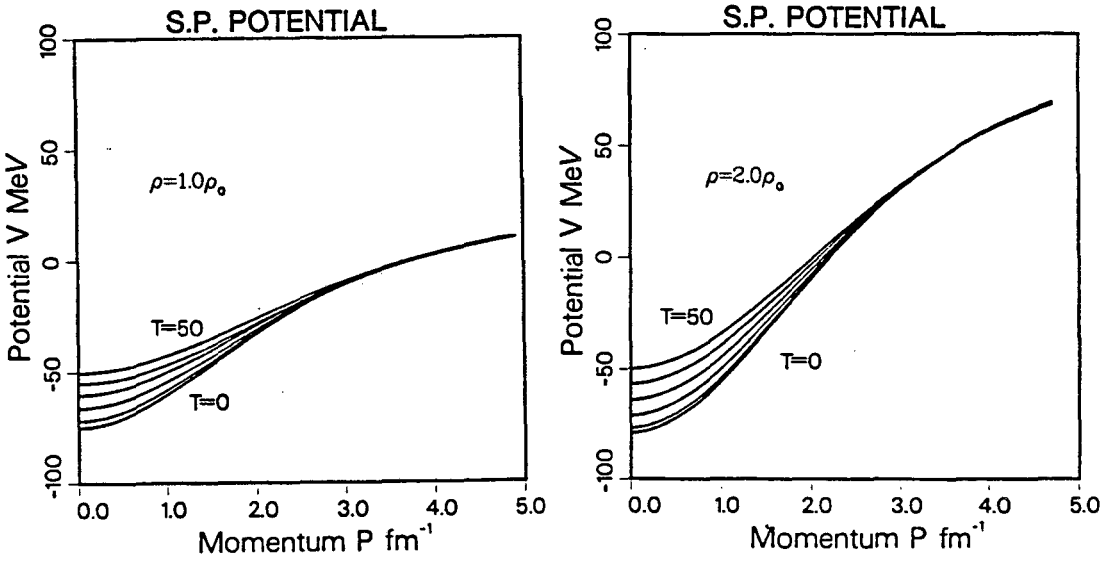

Figure 4: Mean field at normal(left) and double (right) density calculated from the interaction in Welke et al[6]. Temperatures are at 10 MeV interval.

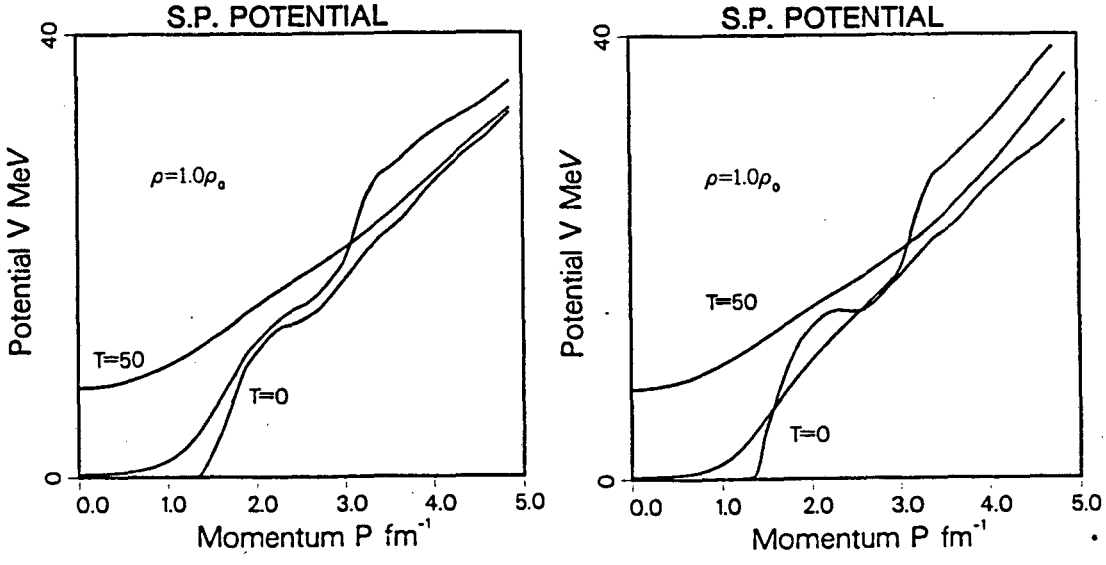

Figure 5: Left curves show the absorptive part of the mean field at normal density as a function of momentum P. Right curves in T-matrix (free cross-section) approximation. Temperatures are 0,10 and 50 MeV.

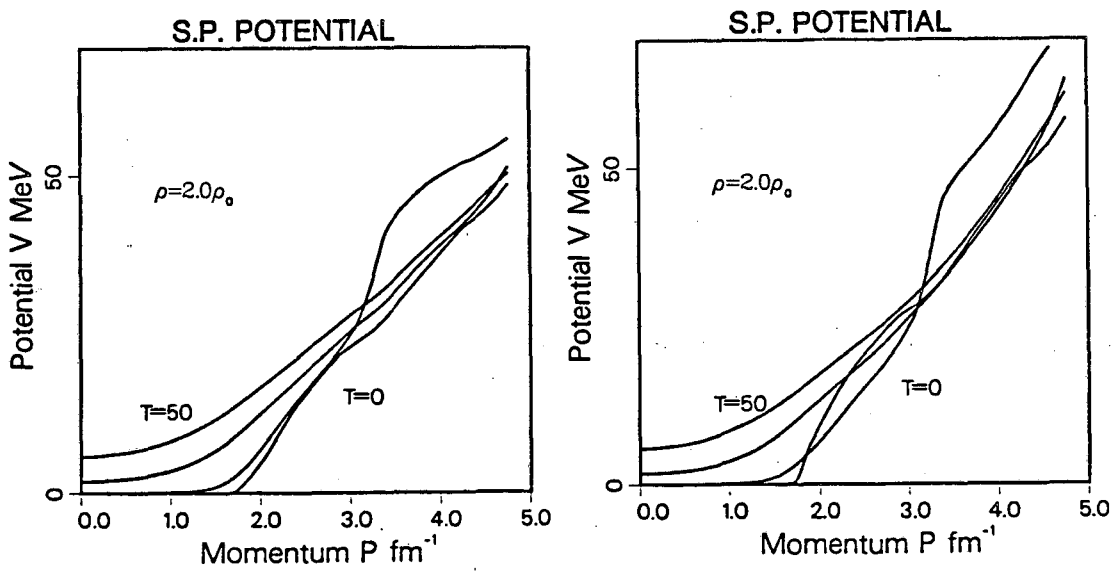

Figure 6: Same as fig. 5 but at double density and temperatures 0,10,30 and 50 meV.

Eq. (10) and with K_p replaced by T. This is equivalent to using the "free" cross-section in the collision term. We find that it is in fact a quite acceptable approximation, especially at the higher temperatures that we are concerned with in H.I. collisions. This approximation has also been used in optical model calculations[13]. It is of course also used in practically all calculations of the Uehling-Uhlenbeck term. There is however a very important note to make here. The mean field enters through the delta function in Eq. (10). In all our calculations above, this is calculated with the selfconsistent mean field. To a first approximation one may consider using the effective mass approximation and the absorption is then proportional to m^*. The result of such an approximation is shown in fig 7. The effective mass is here calculated from [13] to get $m^* = 0.71$.

4 Summary and Conclusions

The mean field in nuclear matter has been calculated as a function of momentum and temperature. At zero temperature the momentum dependence of the mean field is well approximated by deriving it from an effective interaction such as used in ref. [6]. It is suggested that the interaction should be allowed to increase in strength with temperature. Our results justify the use of free cross-sections in the collision term for hot nuclear matter. We like however to stress the more important issue of choosing the correct propagator (e in Eq. (10)) in the collision operator. The simplest choice is an effective mass m^*, as in fig. 7, but the problem is then to find the correct value of m^*. Another matter is that this propagator is complex[23].

The results presented here are for a thermally equilibrated system while the initial un-equilibrated state is really the more interesting because this is when im-

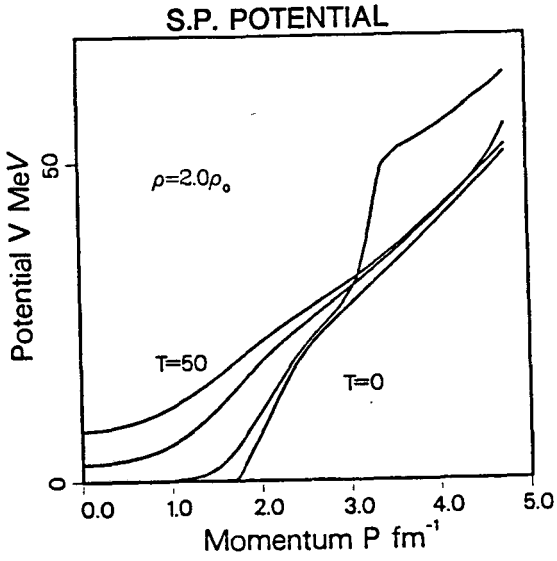

Figure 7: Same as fig. 6 but in the effective mass approximation.

portant dynamics takes place. We have however also made calculations by the same method for this case, namely for the system of two Fermi spheres. The general conclusions appear to be the same however. The main difference is really only in complexity; in the case of two spheres the mean field is not isotropic in momentum-space and the results are more complicated to display.

The results presented here are obtained by a somewhat unconventional method. We beleive however that the essential ingredients of the many-body effects are included. Density and temperature-dependence is included by the Pauli and selfconsistency effects. We do of course not imply that this method supersedes the more conventional potential-model approaches. It has an advantage because of its relative simplicity which allows us to estimate effects otherwise very hard to investigate.

I wish to express my thanks to LBL and its Nuclear Theory group with special thanks to Jørgen Randrup for their hospitality. This work has evolved from discussions with Janusz Dabrowski in Tucson and in Warsaw. I wish to thank him. This work was supported in part by the National Science Foundation, under grant PHY86-04602; and in part by the Director, Office of High Energy Research, Office of High Energy Nuclear Physics, Division of High Energy Physics, of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

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