Maximal Reliability for Unit-weighted Composites

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Although coefficient alpha is the most widely used measure of internal consistency, it does not optimally describe the unidimensional internal consistency of a composite. Coefficients based on a one-factor model have been suggested as improved estimators of internal consistency reliability. When the 1-factor model does not fit the data, however, the meaning of such a coefficient is unclear. A new identification condition for factor analytic models is proposed that assures the composite can be modeled with only one common factor. The associated 1-factor reliability is the maximal internal consistency coefficient for a unit-weighted composite. The coefficient also describes *k*-factor reliability, the greatest lower bound to reliability, and reliability for any composite from a latent variable model with additive errors.

Maximal Reliability for Unit-weighted Composites

Composite scores or scale scores are very frequently used in psychology and related social and behavioral sciences. A composite variable is a sum of other variables. In the typical case, a composite X is a simple sum of p unit-weighted components such as $X = X_1 + X_2 + ... + X_p$.[1] Examples of composites include the total score on a test composed of items, an attitude score based on summed responses to a survey, and so on. An internal consistency reliability coefficient describes the quality of the composite or scale in terms of hypothesized constituents of the components X_i. These might represent true and error parts based on classical test theory ($X_i = T_i + E_i$), common and unique parts based on common factor analysis ($X_i = C_i + U_i$), or the loading of the component on its factor plus residual error ($X_i = \lambda_i F + E_i$).

By far the most widely used measure of internal consistency is Cronbach's (1951) coefficient α (Hogan, Benjamin, & Brezinski, 2000). In the population, it is defined as

$$\alpha = \frac{p}{p-1}\left(1 - \frac{1'D1}{1'\Sigma 1}\right)$$

where D is the diagonal of the covariance matrix Σ of the components X_i, and 1 is a column vector of unit elements which serves as a summing vector. Thus $1'D1$ is the sum of the variances of the p component variables, and $1'\Sigma 1$, the sum of all the elements of the p by p covariance matrix, is the variance of the total score X. In practice, α is applied by substituting the sample covariance matrix S in place of Σ, yielding what we might call $\hat{\alpha}$. The popularity of

[1] Our discussion emphasizes the unit-weighted case, but our results also apply to non-unit equal weights. Generalization to differentially weighted composites is straightforward.

this coefficient stems from several facts: it can easily be computed, it is available in many program packages as a default, it can be applied without fitting or validating any specific model to the components X_i, and, importantly, it is a lower bound to reliability $\alpha \leq \rho_{xx}$ (see e.g., Lord & Novick, 1968). The latter property arises if the components have a decomposition $X_i = T_i + E_i$, where T_i and E_i are uncorrelated with covariance matrices Σ_T and diagonal Ψ_E, so that the component covariance matrix is decomposed into two orthogonal parts $\Sigma = \Sigma_T + \Psi_E$. Then the composite has a similar decomposition $X = T + E$ where $T = \sum_1^p T_i$, $E = \sum_1^p E_i$, and the reliability of the composite is defined as the ratio of var(T)/var(X), or

$$\rho_{xx} = \frac{\sigma_T^2}{\sigma_X^2} = \frac{l'\Sigma_T l}{l'\Sigma l} = 1 - \frac{l'\Psi_E l}{l'\Sigma l}.$$

There are many good recent discussions of α, its problems, and its alternatives (e.g., Barchard & Hakstian, 1997; Becker, 2000; Bonett, 2003; Enders, 2003; Enders & Bandalos, 1999; Feldt & Charter, 2003; Green, 2003; Green & Hershberger, 2000; Hakstian & Barchard, 2000; Komaroff, 1997; Miller, 1995; Osburn, 2000; Raykov, 1997, 1998, 2001, 2004; Raykov & Shrout, 2002; Schmidt, Le, & Ileus, 2003; Schmitt, 1996; Shevlin, Miles, Davies & Walker, 2000; and Vautier & Jmel, 2003). For the purposes of the current paper, two issues are important. First, the lower-bound property $\alpha \leq \rho_{xx}$ has been questioned. When correlated errors are present so that Ψ_E is not diagonal, α can actually overestimate reliability. Second, the size of α provides no information on the degree of unidimensional reliability, sometimes called homogeneity, that is, reliability due to the main or only underlying common true score factor. In order to deal with both of these problems, the recent theoretical literature has suggested abandoning coefficient α and using a coefficient based on a theoretical decomposition of the component covariance matrix.

In this approach, the covariance matrix of the true scores is presumed to be unidimensional, that is $\Sigma_T = \lambda\lambda'$, where λ (px1) is the factor loading vector of the p variables on a single common factor. Hence the covariance matrix of the observed scores is decomposed as

$$\Sigma = \lambda\lambda' + \Psi_u,$$

where Ψ_u is the covariance matrix of the unique variables or residual errors. Then

$$\rho_{11} = \frac{\sigma_T^2}{\sigma_X^2} = \frac{1'\lambda\lambda'1}{1'\Sigma 1} = \frac{(1'\lambda)^2}{1'\Sigma 1} = \frac{(\sum_1^p \lambda_i)^2}{1'\Sigma 1} = 1 - \frac{1'\Psi_u 1}{1'\Sigma 1}$$

defines reliability ρ_{11} ($\leq \rho_{xx}$) based on the hypothesis of a unidimensional latent variable (see, e.g., Jöreskog, 1971, p. 112). Typically, Ψ_u is taken to be a diagonal matrix representing the hypothesis of uncorrelated error components, but in some circumstances correlated errors may be hypothesized. In practice, of course, ρ_{11} is not operational. In order to make it operational, the model $\Sigma = \lambda\lambda' + \Psi_u$ is fit to a sample covariance matrix S, and estimators $\hat{\lambda}$ and $\hat{\Psi}$ are obtained. These are plugged into the defining formula, yielding $\hat{\rho}_{11}$. The approach also provides important information about the contribution of a given component variable to reliability via the factor loading $\hat{\lambda}_i$. Recent discussions of this approach are given by Kano and Azuma (2003) and Raykov (2004).

Although ρ_{11} is certainly an improvement over α, it has a serious and fundamental flaw that has been overlooked. In realistic applications of covariance structure analysis, especially with a large number of variables X_i such as might be used in a reliability study, the null hypothesis $\Sigma = \lambda\lambda' + \Psi_u$ of a single common factor may hardly be tenable. If this null hypothesis is rejected, it is hard to know what ρ_{11} describes. We would argue that if

$\Sigma \neq \lambda\lambda' + \Psi_u$, estimating $\hat{\rho}_{11}$ based on an incorrect 1-factor model is inappropriate. In this paper, we propose an extension of factor-based reliability so that it yields an appropriate coefficient of unidimensional internal consistency for all covariance matrices that can be fit by an exploratory factor analytic model. We show that among coefficients based on unit-weighted composites, it gives the largest reliability.

Proposed Identification Condition for Factor Models

Suppose that an exploratory factor analysis model of the type

$$\Sigma = \Lambda\Lambda' + \Psi$$

holds in the population. Here we allow the factor loading matrix Λ to be (pxk), where the number of factors $k \leq (p-1)$ can be any appropriate number. When standard approaches to estimation of factor models are used, k has to be small enough so that there are positive degrees of freedom when fitting the model to a sample covariance matrix S. We will call this the "small-k" situation. In such a case, it is well-known that without further restrictions this model is not identified (e.g., Jöreskog, 1967). Based on the partition of the factor loading matrix into $\Lambda = [\lambda \mid \bar{\Lambda}]$, where λ is (px1) and $\bar{\Lambda}$ is (px(k-1)), we propose the following identification conditions:

1) λ contains unrestricted free parameters.
2) $I'\bar{\Lambda} = 0$, that is, the k-1 columns of $\bar{\Lambda}$ sum to zero.
3) $\bar{\Lambda}$ contains free parameters subject to (k-1)(k-2)/2 restrictions. Some examples of such restrictions are:
   a. $\bar{\Lambda}'\Psi^{-1}\bar{\Lambda}$ is diagonal. This is similar to the standard identification condition in exploratory maximum likelihood factor analysis, where it is based on all k factors.

b. $\bar{\Lambda}$ contains free parameters except for an upper triangle of fixed zero elements, e.g., if k=5, the first 4 rows (out of p) are given as $\begin{bmatrix} * & 0 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & 0 \\ * & * & * & * \end{bmatrix}$, where "*" represents a free parameter and "0" is a fixed zero. An advantage of this approach is that any structural equation modeling program can be used to estimate the free parameters of the model.

c. Rotational criteria are imposed on $\bar{\Lambda}$ so that it is in some simple structure form. If oblique transformations are considered, the defining model may contain correlated factors, that is, $\Sigma = \Lambda\Phi\Lambda' + \Psi$, where Φ is the covariance matrix of the factors. In this approach, the factor corresponding to λ remains uncorrelated with the remaining factors.

It follows from the above that the number of identification conditions imposed on the model is (k-1) + (k-1)(k-2)/2, which equals $k(k$-1)/2, the precise number imposed on the standard exploratory factor analysis model. Thus the proposed representation is simply an alternative form of the exploratory factor model.

However, the coefficient is defined more generally. It also holds under conditions where the number of factors generates negative degrees of freedom, i.e., exceeds the Ledermann (1937) bound of $.5(2p+1-\sqrt{8p+1})$ in the standard exploratory factor model. We will call this the "large-k" situation, which may require a number of factors near p. Such a large number of factors does not occur in ordinary exploratory factor analysis, but it occurs in such contexts as minimum trace factor analysis (e.g., Bentler, 1972; Shapiro, 1982; Shapiro & ten Berge, 2000), constrained minimum trace factor analysis (e.g., Bentler & Woodward, 1980, 1983; ten Berge,

Snijders, & Zegers, 1981; Shapiro, 1982), or minimum rank factor analysis (e.g., della Riccia & Shapiro, 1982; ten Berge & Kiers, 1991; Shapiro & ten Berge, 2002). See also ten Berge (2000). In the large-*k* situation, identification condition three is not used since its primary purpose is to enable standard exploratory factor analytic estimation with positive degrees of freedom.

Reliability based on Proposed Parameterization

The internal consistency reliability of the total score *X* under the *k*-factor model is defined as the proportion of common to total variance. Under the model and its proposed parameterization, this is

$$\rho_{kk} = \frac{\sigma_T^2}{\sigma_X^2} = \frac{1'\Lambda\Lambda'1}{1'\Sigma 1} = \frac{(1'\lambda)^2}{1'\Sigma 1} = \frac{(\sum_1^p \lambda_i)^2}{1'\Sigma 1} = 1 - \frac{1'\Psi 1}{1'\Sigma 1}.$$

It is apparent that the reliability coefficient depends only on the size of the factor loadings on the first factor. The remaining *k*-1 factors contribute nothing at all to reliability. This occurs because identification condition two has the consequence

$$1'\Lambda = 1'[\lambda \mid \bar{\Lambda}] = [1'\lambda \mid 1'\bar{\Lambda}] = [1'\lambda \mid 0].$$

Only loadings on the first factor contribute to total common variance. As a result, the reliability coefficient ρ_{kk} based on *k* factors in this parameterization actually describes the internal consistency of the composite score based only on the unidimensional latent factor of interest. While the identification conditions require more than one factor (*k*>1), the actual number of factors *k* is irrelevant.

In practice, the model under the proposed parameterization has to be estimated from the data, and a number *k* chosen so that the model-reproduced covariance matrix $\hat{\Sigma}$ under the given

identification conditions approximates the sample covariance matrix *S* closely enough from a statistical point of view. Then the above formula yields the estimator

$$\hat{\rho}_{kk} = \frac{1'\hat{\Lambda}\hat{\Lambda}'1}{1'\hat{\Sigma}1} = 1 - \frac{1'\hat{\Psi}1}{1'\hat{\Sigma}1} \ .$$

As noted above, in the standard situation of exploratory factor analysis, *k* will be a relatively small number. Also, then $\hat{\Sigma} \neq S$. In contrast, in minimum trace or minimum rank modeling situations, *k* will be quite large and while $\Sigma = \Lambda\Lambda' + \Psi$ as before, also $\hat{\Sigma} = S$. As a result, $\hat{\lambda}$ as well as the estimated total variance $1'\hat{\Sigma}1$ being explained under the types of models (small-*k* vs. large-*k*) are liable to be different, and hence these diverse approaches no doubt will yield different sample estimates $\hat{\rho}_{kk}$.

Properties of the Coefficient

The coefficient ρ_{kk} can be computed without imposing our proposed identification conditions. That is, $\rho_{kk} = 1'\Lambda\Lambda'1 / 1'\Sigma 1$ is invariant to any particular rotation or transformation of the matrix Λ. Only the product $\Lambda\Lambda'$ is required, and this product is invariant to orthogonal or oblique transformations. Any factor solution is good enough. Representation of the latent factors using the proposed set of identification conditions is not needed for defining or computing ρ_{kk}. With an arbitrary Λ, however, the coefficient is interpretable as the proportion of variance attributable to all *k* factors. Thus it actually is a *k*-factor internal consistency coefficient. Furthermore, if Λ is based on minimum trace factor analysis, it is Bentler's (1972) dimension-free coefficient, and when based on constrained minimum trace factor analysis, it will be the greatest lower bound to reliability (Jackson & Agunwamba, 1977).

Nonetheless, if interest centers on *unidimensional* reliability, then a given factor of interest must be chosen so that the proportion of variance due to this particular factor can be determined and interpreted. Under our proposed identification conditions, the factor loading matrix has the structure $\Lambda = [\lambda \mid \overline{\Lambda}]$ and unidimensional internal consistency refers to the proportion of total score variance that is due to the factor whose factor loadings are given by λ. We now show that this variance is maximum. To see this, we start with some arbitrary factor loading matrix $\tilde{\Lambda}$ and finding a maximizing rotation.

Theorem. Let $\Sigma = \tilde{\Lambda}\tilde{\Lambda}' + \Psi$, and let *t* be a normal vector $(t't = 1)$. Then the factor loading vector $\lambda = \tilde{\Lambda}t$ that maximizes $(I'\lambda)^2$ is given by $\lambda = (I'\tilde{\Lambda}\tilde{\Lambda}'I)^{-1/2}\tilde{\Lambda}\tilde{\Lambda}'I$, and the residual factors $\overline{\Lambda}$, where $\overline{\Lambda}\overline{\Lambda}' = \tilde{\Lambda}\tilde{\Lambda}' - \lambda\lambda'$, have zero column sums $(I'\overline{\Lambda} = 0)$.

Proof. Let $\phi = (I'\lambda)^2 - \mu(t't - 1)$. Taking derivatives $\partial\phi / \partial\mu$ and setting to zero establishes $t't = 1$. Then $\partial\phi / \partial t$ yields the eigenequation $(\tilde{\Lambda}'II'\tilde{\Lambda} - \mu I)t = 0$. Solving this yields $\mu = (I'\tilde{\Lambda}\tilde{\Lambda}'I)$ and $t = (I'\tilde{\Lambda}\tilde{\Lambda}'I)^{-1/2}\tilde{\Lambda}'I$. Substituting into $\lambda = \tilde{\Lambda}t$ and simplifying gives $\lambda = (I'\tilde{\Lambda}\tilde{\Lambda}'I)^{-1/2}\tilde{\Lambda}\tilde{\Lambda}'I$. It follows that $\overline{\Lambda}\overline{\Lambda}'I = (\tilde{\Lambda}\tilde{\Lambda}' - \lambda\lambda')I = 0,$ which means that $I'\overline{\Lambda} = 0.$ Finally, $(I'\lambda)^2$ is maximized rather than minimized since the minimum ϕ occurs with $\lambda = 0.$

The proposed identification conditions one and two are thus not arbitrary. They are necessary to finding the maximal unidimensional internal consistency coefficient for a unit-weighted sum of observed variables. The meaning of "unidimensional internal consistency" can be clarified by determining the factor structure of the composite based on the factor structure of its components. Let us make the typical assumption that the covariance structure under our identification conditions $\Sigma = \Lambda\Lambda' + \Psi$ arises from a structure of the underlying variables $x = (X_1, X_2, ..., X_p)'$ as

$$x = \Lambda\xi + \varepsilon$$

where $E(\xi\xi') = I, E(\xi\varepsilon') = 0,$ and $E(\varepsilon\varepsilon') = \Psi.$ Then the composite $X = l'x$ has the simple factor analytic decomposition

$$X = l'\Lambda\xi + l'\varepsilon = (l'\lambda \mid l'\bar{\Lambda})\begin{pmatrix} \xi_\lambda \\ \xi_{\bar{\Lambda}} \end{pmatrix} + l'\varepsilon = \lambda^* \xi_\lambda + \varepsilon^*$$

where $\lambda^* = \sum_1^p \lambda_i$ is a scalar factor loading, ξ_λ is the factor score that corresponds to λ, and $\varepsilon^* = \sum_1^p \varepsilon_i$ is a scalar. In other words, even if the components are multidimensional, the composite can be interpreted to be a function of only a single common factor – the most reliable factor. The theorem also shows that the maximal reliability factor can be obtained as a rotation from any starting factor solution.

It might be noticed that that the factor loading matrix $\Lambda = [\lambda \mid \bar{\Lambda}]$ is in a form very similar to that obtained with centroid factor analysis. For a recent discussion of certain aspects of this very old method and relevant references, see Choulakian (2003). However, centroid factor analysis was developed as a method of factor extraction. In the above defining formulae, nothing has been stated about what method of estimation is used to provide estimates of the factor loadings. The theoretical coefficient ρ_{kk} is independent of any estimation method, while the estimator $\hat{\rho}_{kk}$ can be obtained from a factor solution obtained by any appropriate method of factor analysis. In the small-*k* situation, it may be based on maximum likelihood, generalized least squares, or least squares estimation, while in the atypical large-*k* situation, it will be based on minimum trace, minimum rank, or a similar methodology.

Illustration

Table 1 gives the correlation matrix for the widely known nine psychological variables (Harman, 1976, p. 244), which we take as a covariance matrix for purposes of illustration. A one-factor maximum likelihood solution is presented in the left part of Table 2. This solution does not fit

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Insert Tables 1 and 2 about here

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the data. It has a likelihood ratio $\chi^2_{27} = 190.6$. Nonetheless, ρ_{11} was estimated, yielding $\hat{\rho}_{11} = .880$. Actually, this coefficient for the factor that does not explain the data is not an improvement over $\hat{\alpha} = .886$. A three-factor model fits these data extremely well, with $\chi^2_{12} = 1.6$. The maximal reliability factor $\hat{\lambda}$ for this model is given in the right part of Table 2. It will be seen that the sum of factor loadings is larger than was the case for the 1-factor model. The corresponding $\hat{\rho}_{kk} = .939$.

Discussion

Like the 1-factor based internal consistency reliability coefficients, the proposed approach requires modeling the sample covariance matrix. This must be a successful enterprise, as estimation of reliability only makes sense when the model does an acceptable job of reproducing the sample covariances. Of course, since a *k*-factor model rather than a 1-factor model would typically be used in the proposed approach, the odds of adequately modeling the sample covariance matrix are greatly improved. The selected model can be a member of a much wider class of models. Whatever the resulting dimensionality *k*, and whether the typical "small-*k*" or theoretical alternative "large-*k*" approach is used, the proposed identification conditions assure that the resulting internal consistency coefficient $\hat{\rho}_{kk}$ represents the proportion of

variance in the unit-weighted composite score that is attributable to the common factor with maximal internal consistency. The proposed coefficient can be interpreted as representing unidimensional reliability even when the instrument under study is multifactorial, since the composite score can be modeled by a single factor as $X = \lambda^* \xi_\lambda + \varepsilon^*$. Nonetheless, we remind that it equally well has an interpretation as summarizing the internal consistency of the *k* dimensional composite.

When the large-*k* approach is used to model the covariance matrix, the theory of dimension-free and greatest lower bound coefficients can be applied. This is based on a tautological model that defines factors that precisely reproduce the covariance matrix. As these theories have been developed in the past, reliability is defined on scores that are explicitly multidimensional. However, we have shown here that the dimension-free and greatest lower-bound coefficients can equivalently be defined for the single most reliable dimension among the many that are extracted. Based on this observation, new approaches to these coefficients may be possible.

The coefficient developed here also applies to composites obtained from any latent variable covariance structure model with *p*-dimensional additive errors. We may write such a model in the form $\Sigma = \Sigma(\theta) + \Psi$, where $\Sigma(\theta)$ and Ψ are nonnegative definite matrices. Then we can decompose $\Sigma(\theta) = \Lambda\Lambda'$ and proceed as described previously. The maximal reliability coefficient for a composite from such a model, as well as from those models previously described in this paper, has been available in EQS 6 for several years.

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Table 1

Correlation Matrix of Nine Psychological Variables

1.00								
.75	1.00							
.78	.72	1.00						
.44	.52	.47	1.00					
.45	.53	.48	.82	1.00				
.51	.58	.54	.82	.74	1.00			
.21	.23	.28	.33	.37	.35	1.00		
.30	.32	.37	.33	.36	.38	.45	1.00	
.31	.30	.37	.31	.36	.38	.52	.67	1.00

Table 2

Factor Loadings for First Factor in Two Solutions

Variable Number	1-Factor Model	3-Factor Model
1	.636	.727
2	.697	.738
3	.667	.754
4	.867	.789
5	.844	.767
6	.879	.803
7	.424	.492
8	.466	.597
9	.462	.635
Sum	5.942	6.302