Energetics aspects in Direct Numerical Simulations of a turbulent stratified flow: irreversible mixing

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Abstract

Direct Numerical Simulations of a linearly stratified bounded fluid in decaying turbulence are performed. The abscence of vertical periodicity allows the stratification of the system to evolve in time. We observe that mixing efficiency increases as a function of the Richardson number Ri, and saturates to a maximum value of 0.25. This evolution adjusts quantitatively to the statistical prediction of Venaille et al. (2016).

1 Introduction

Turbulent mixing in the ocean interior plays a crucial role in its global energy budget. This mixing partially drives large scale dynamics, as evidenced in the meridional overturning circulation (Wunsch and Ferrari (2004)). In addition, vertical transport in the ocean is substantial for sequestering large quantities of dissolved greenhouse gases from the atmosphere to the deep ocean. The proportion of energy transferred from turbulent structures to effective mixing is very difficult to estimate through observations (Ivey et al. (2008)), and the details of this energy transfer is yet not fully understood.

Osborn proposed a simple relation between the turbulent kinematic dissipation ϵ_k and the irreversible mixing ϵ_p, $\epsilon_p = \Gamma \cdot \epsilon_k$ with a constant value for the mixing coefficient $\Gamma = 0.2$ (Osborn (1980)). It has been found in experimental studies that the mixing coefficient is far from being constant Barry et al. (2001), Rehmann and Koseff (2004). A turbulent flow in the limit of weak stratification will dissipate more kinetic energy that it will produce irreversible mixing, that is $\Gamma \cdot \epsilon_k \to 0$. We therefore expect a dependency of Γ with respect to the stratification.

Our goal is to study the mixing efficiency and the dynamics of stratified turbulence by means of high resolution Direct Numerical Simulations. We introduce boundaries at the top and bottom of our domain which allows the mean stratification to evolve in time. This differs with the classical approach of homogeneous stratified turbulence (as for example Maffioli et al. (2016)) where the background stratification is fixed. The main interest of our approach is that the irreversible mixing is directly computed from the full density field.

2 Framework

Governing equations

We consider an uncompresible linearly stratified fluid in decaying turbulence. It is modelized by the Navier-Stokes equation under Bousinessq approximation. The dimensionless equations read,

$$\begin{aligned} \frac{\partial \mathbf{u}^*}{\partial t} - \omega^* \times \mathbf{u}^* &= -\nabla p^* - \theta^* \cdot Ri \cdot \mathbf{z}^* + \frac{1}{Re} \cdot \nabla^2 \mathbf{u}^*, & (1) \\ \left(\frac{\partial}{\partial t^*} + \mathbf{u}^* \cdot \nabla\right) \theta^* &= \frac{1}{Re \cdot Sc} \nabla^2 \theta^*, & (2) \\ \nabla \cdot \mathbf{u}^* &= 0, & (3) \end{aligned}$$

where $\theta^* = \frac{1}{\theta_0} \cdot \frac{\rho - \rho_0}{\rho_0}$ is the dimensionless reduced density. ρ is the total density and we introduce $\rho_0 =< \rho >$ the spatial average of ρ (in the following we will use $<>$ for spatial averages over the volume). $\theta_0 = \frac{N_0^2 \cdot H}{g}$ is the initial difference between the top and bottom reduced densities, where H the height of the domain, g the acceleration of the gravity and $N_0^2 = -\frac{g}{\rho_0} \frac{\mathrm{d}\rho}{\mathrm{d}z}$ the initial square buoyancy frequency. ω^* is the vorticity, p^* the total pressure and $\mathbf{u}^* = \mathbf{u}/u_0$ is the dimensionless velocity. The Reynolds, Richardson and Schmidt numbers are respectively,

$$Re = \frac{\ell_0 \cdot u_0}{\nu}, \qquad Ri = \frac{\theta_0 \cdot g \cdot \ell_0}{u_0^2}, \qquad Sc = \frac{\nu}{\kappa}. \qquad (4)$$

where ℓ_0 is the integral lengthscale and u_0 the *r.m.s.* of the initial turbulent velocity field. The Richardson number can also be rewritten as $Ri = \frac{N_0^2 \ell_0 H}{u_0^2}$.

Energy transfer equations

The global kinetic and potential energies of the flow are defined respectively as,

$$\begin{aligned} E_k &= \frac{\rho_0}{2} \cdot V \cdot (< \mathbf{u}^2 > - < \mathbf{u} >^2), & (5) \\ E_p &= g \int_V \rho \cdot z \, \mathrm{d}V. & (6) \end{aligned}$$

One can develop the evolution of the energy transfer by using the approach of Winters et al. (1995).

$$\begin{aligned} \frac{\partial E_k}{\partial t} &= -\Phi_z - \epsilon_k, & (7) \\ \frac{\partial E_p}{\partial t} &= \Phi_z + \Phi_i, & (8) \\ \frac{\partial E_b}{\partial t} &= \epsilon_p, & (9) \\ \frac{\partial E_a}{\partial t} &= \Phi_z + \Phi_i - \epsilon_p, & (10) \end{aligned}$$

where the background and available potential energies are defined respectively,

$$E_b = g \int_V \rho_s \cdot z \; \mathrm{d}V, \tag{11}$$

$$E_a = g \int_V (\rho - \rho_s) \cdot z \; \mathrm{d}V \quad = \quad (E_p - E_b), \tag{12}$$

whith ρ_s the 3D vertically sorted density field. This sorted state corresponds to the minimum potential energy of the system, noted E_b. E_a gives a measure of the amount of potential energy available to be transformed in irreversible mixing. The viscous dissipation rate, the reversible vertical buoyancy flux and the conversion rate from internal to potential energy are respectively,

$$\epsilon_k = \int_V \mu \left[2 \left(\frac{\partial u_i}{\partial x_i} \right)^2 + \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)^2 \right] \mathrm{d}V, \tag{13}$$

$$\Phi_z = g \int_V \rho \cdot w \; \mathrm{d}V, \tag{14}$$

$$\Phi_i = -\kappa \cdot g \cdot A \cdot (\rho_{top} - \rho_{bot}), \tag{15}$$

where w is the vertical component of the velocity and ρ_{top} (resp. ρ_{bot}) is the average of ρ over the horizontal surface A at the top (resp. bottom) of the domain. $\frac{\partial E_b}{\partial t} = \epsilon_p$ quantifies the irreversible mixing. This scheme allows to distinguish the increase of potential energy produced by waves and overturns (Φ_z) with respect to the increase produced by the irreversible mixing (ϵ_p) and with respect to the conversion rate from internal to potential energy (Φ_i).

Mixing in stratified turbulence

The mixing produced by a turbulent flow over a stable stratified fluid is usually quantified by the mixing coefficient $\Gamma = \frac{\epsilon_p}{\epsilon_k}$ or the mixing efficiency $\eta = \frac{\epsilon_p}{\epsilon_p + \epsilon_k}$, which are instantaneous, spatially averaged quantities. If it makes sense to compare the instantaneous values of ϵ_p and ϵ_k in stationary turbulence, it is no longer the case in the context of decaying turbulence. We thus define the cumulative mixing coefficient and the cumulative mixing efficiency as,

$$\Gamma_C = \frac{\int_0^t \epsilon_p \cdot \mathrm{d}t}{\int_0^t \epsilon_k \cdot \mathrm{d}t}, \qquad \eta_C = \frac{\int_0^t \epsilon_p \cdot \mathrm{d}t}{\int_0^t \epsilon_p \cdot \mathrm{d}t + \int_0^t \epsilon_k \cdot \mathrm{d}t} \tag{16}$$

3 Numerical method

A set of 3D Direct Numerical Simulations (DNS) of a turbulent stratified flow are performed by solving Navier-Stokes equation under Boussinesq approximation. A classical Fourier pseudo-spectral method is used with 1024^3 grid points. A porous penalization region is introduced to take into account non-flux conditions at the bottom and at the top of the box (see Kadoch et al. (2012)), and we assume periodicity in the horizontal plane. The formulation of the momentum and the mass conservation equation becomes:

$$\frac{\partial \mathbf{u}^*}{\partial t^*} - \omega^* \times \mathbf{u}^* = \nabla p^* - \theta^* \cdot Ri \cdot \mathbf{z}^* \cdot (1-H) + \frac{1}{Re} \cdot \nabla^2 - \frac{1}{\eta_p} \cdot H \cdot (\mathbf{u}^* - \mathbf{u}_s) \quad (17)$$

$$\frac{\partial \theta^*}{\partial t^*} = [(1-H) \cdot \mathbf{u}^* \cdot \theta^* + \mathbf{u}_s \cdot H \cdot \theta^*] + \nabla \cdot \left\{ \left[\frac{1}{Re \cdot Sc}(1-H) + \lambda \cdot H \right] \cdot \nabla \theta^* \right\} \quad (18)$$

$$\nabla \cdot \mathbf{u}^* = 0 \quad (19)$$

where H is a step function vanishing in the fluid domain. $\eta_p = 10^{-3}$ is the porous media permeability, $\lambda = 10^{-8} \cdot \eta_p$ is the diffusion term in the porous media, and $\mathbf{u}_s$ is the velocity of the boundaries (in this work $\mathbf{u}_s = 0$). A turbulent velocity field is introduced at $t = 0$ which perturbs the initially stable buoyancy profile. The turbulent flow freely decays over several overturning times ℓ_0/u_0.

The parameters of the DNS runs that are varied are shown in table 1. The Schmidt number is taken constant for all runs, $Sc = 5$. We based our parameters on typical physical experiment of water grid turbulence performed at LMFA, where $u_0 = 0.1$ m/s, $\ell_0 = 0.01$ m , $H = 20 \cdot \ell_0$ and we impose different values of N.

Table 1: Main parameters of the DNS: Re the Reynolds number and Ri the Richardson number.

Run	A_0	A_1	A_2	A_2'	A_2''	A_3	A_4	A_5	A_6
Re	1000	1000	1000	1600	2500	1000	1000	1000	1000
Ri	1	4	16	16	16	64	256	1024	4096

4 Results

Energy budget

A snapshot of a vertical cut of the reduced density field θ^* is shown in figure 1(a). In figure 1(b) is shown a vertical profile of the horizontally averaged reduced density, for the full reduced density and for the 3D vertically sorted reduced density field. The instantaneous reduced density field is used to compute E_p, while the instantaneous sorted reduced density field is used to compute E_b. E_p will contain the energy increase produced by the mixing within the flow in addition to the energy fluctuations associated to the reversible vertical buoyancy flux of waves and overturns. In contrast, the variation of E_b is associated only to irreversible mixing.

The evolution of E_p, E_b, E_a and E_k are shown in figure 2 for run A_2. The response of the buoyancy field to the initially imposed velocity field produces a maximum of E_a at $t \approx \frac{\pi}{N_0}$, and this quantity further oscillates at the frequency close to N_0 while it slowly decays, as predicted by Salhi and Cambon (2007). In contrast, the background potential energy E_b, monotonically increases reaching a final value which represents the amount of irreversible mixing that occurred during the run. This last term and E_p will converge to E_b when the kinetic energy of the system goes to zero. In consequence, at the end of the simulation no available energy (E_a) is left to produce irreversible mixing.

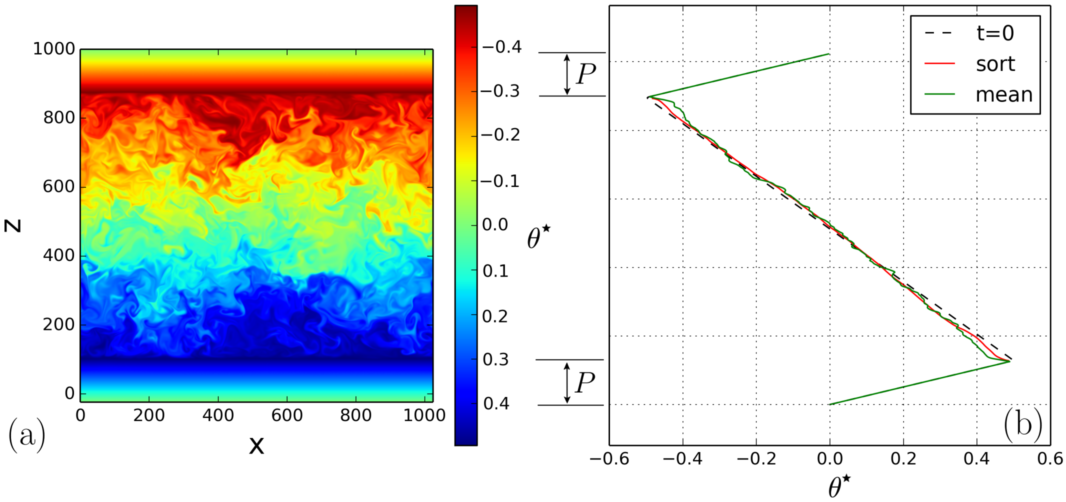

Figure 1: (a) Vertical cut of the instantaneous reduced density field θ^* at $t \cdot N_0/(2\pi) = 0.7$ in run A_2. (b) Vertical profile of the horizontal mean reduced density field (red line) and horizontal mean of the sorted reduced density field (green line). The initial reduced density profile is also indicated (dashed line). The penalization region is indicated by two arrows and the letter P between both figures.

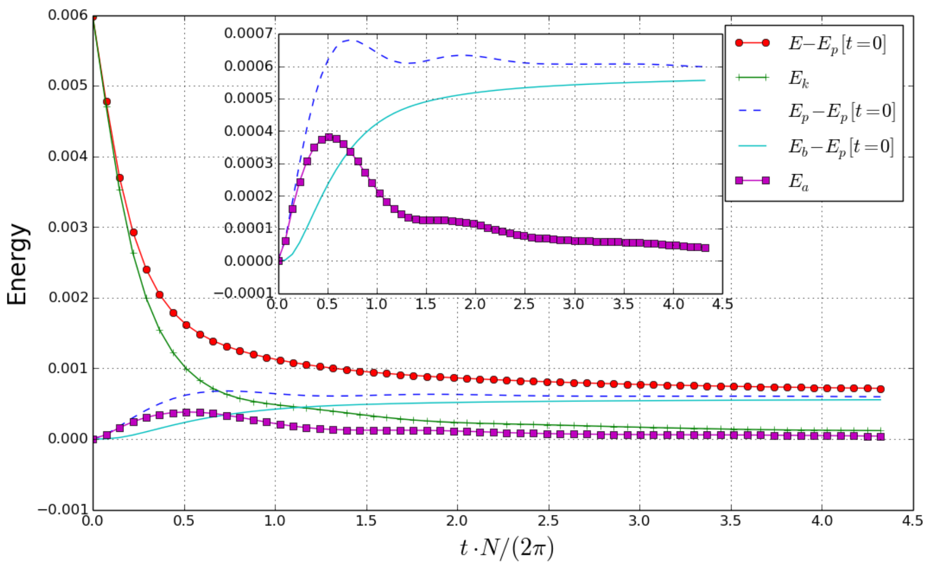

Figure 2: Evolution of the energies in run A_2 as a function of $t \cdot N_0/(2\pi)$. $E = E_p + E_k$ is the total energy. The inset shows a zoom in the lowest values of energy to highlight the evolution of the potential energies.

Energy transfer

In figure 3 are shown the energy transfer terms expressed in equations (7-10) for run A_3. Initially, the kinetic energy transfer is dominant and is compensated by the viscous dissipation term ϵ_k. The energy that is transfered to irreversible mixing ϵ_p presents a maximum at short times, before decaying and reaching the ultimate value ϕ_i. We introduce here the time T_d, at which ϵ_p becomes smaller than $2\Phi_i$. After this time, the irreversible mixing generated by the conversion rate from internal to potential energy is dominant with respect to the irreversible mixing generated by turbulence.

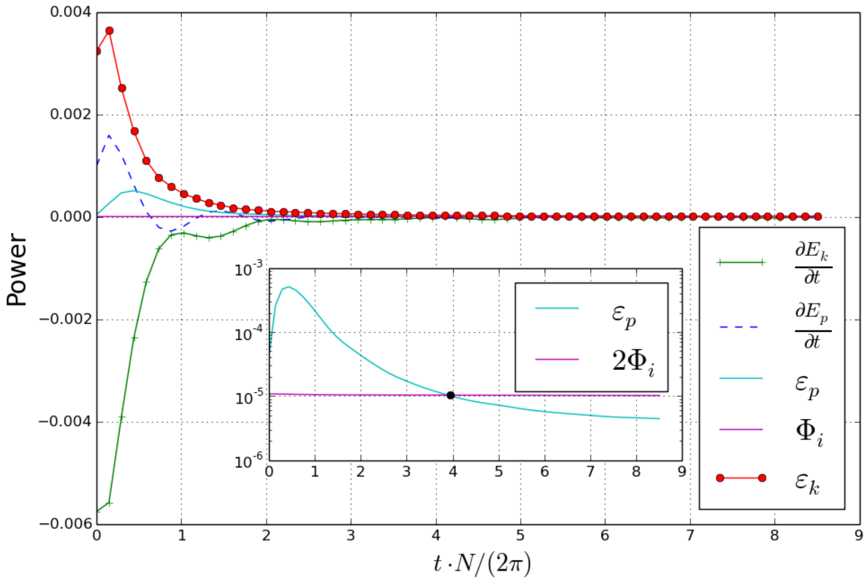

Figure 3: Evolution of the temporal energy transfer terms of run A_3 as a function of $t \cdot N_0/(2\pi)$. The inset compares ϵ_p and Φ_i in order to define the time T_d where $\epsilon_p < 2\Phi_i$.

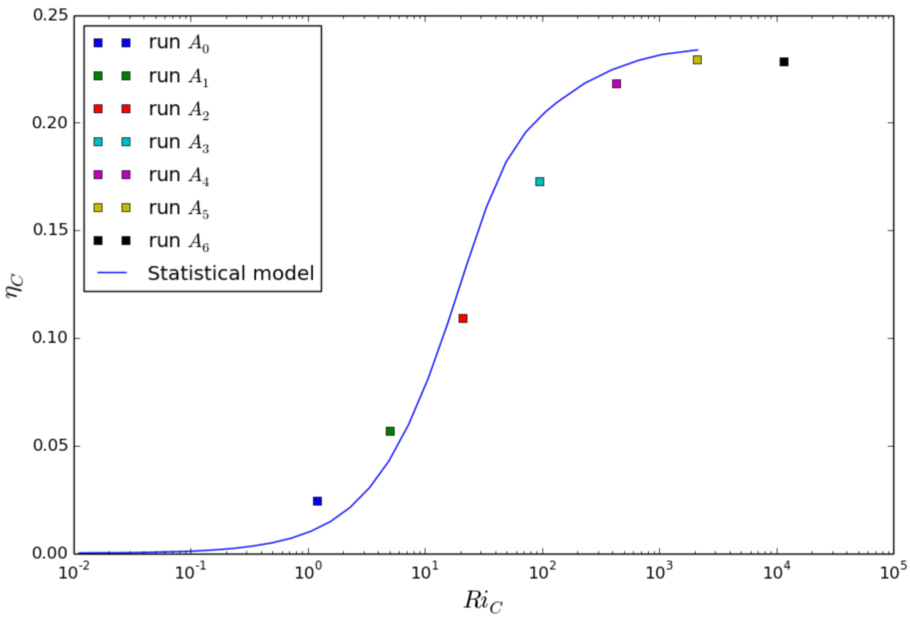

Figure 4: η_C as a function of Ri_C for all DNS runs. Each symbol indicates the value of the mixing efficiency integrated in a interval of time $[0, T_d]$. In blue solid line is shown the prediction of the mixing efficiency adapted from the statistical theory of Venaille et al. (2016).

Mixing efficiency

We will now estimate the cumulative mixing efficiency η_C at time T_d. The pertinent Richardson number at this stage of the DNS is given by,

$$Ri_C = Ri \cdot \frac{E_k(t=0)}{\int_0^{T_d} \epsilon_k \cdot \mathrm{d}t}. \tag{20}$$

In figure 4, η_C is plotted as a function of Ri_C and compared to the prediction of the statistical theory proposed by Venaille et al. (2016). The agreement is remarkably good.

Conclusions and perspectives

We computed the mixing efficiency from high resolution DNS of stratified turbulence in a large range of Richardson numbers. This allowed us to observe the increase of η with Ri, which is in very good agreement with the statistical theory for stratified turbulence proposed by Venaille et al. (2016). This study will be extended to the case of a two layer stratification.

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