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# Detection and Avoidance of Collisions: the REACT Model 

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# Detection and Avoidance of Collisions: The REACT Model Regulation of Speed in Car following 

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#### Abstract

An important perceptual task during driving is the ability to detect and avoid collisions. Failure to accurately perform this task can have serious consequences for the driver and passengers. The present research developed and tested a model of car following by human drivers, as part of a general model under development of a human driver. Unlike other car following models that are based on 3D parameters (e.g., range or distance) the present model is based on the visual information available to the driver. The model uses visual angle and change in visual angle to regulate speed during car following. Human factors experiments in a driving simulator examined performance in car following to speed variations defined by sine wave oscillations in speed, sum of sine wave oscillations, and ramp function. In addition, using real world driving data the model was applied to 6 driving events. The model provided a good fit to car following performance in the driving simulation studies as well as the real-world driving data, accounting for up to $96 \%$ of the variability in speed for the real world driving events.


## Executive Summary

The present study developed and tested a model of car following based on perceptual (visual) information to the human driver. The model specifies that changes in vehicle speed in response to changes in lead vehicle speed is specified by

$$
\text { acceleration }=j \cdot\left(\frac{1}{\alpha}-\frac{1}{\alpha^{\prime}}\right)+k \cdot \frac{d}{d t} \alpha
$$

Where j and k are constants, and $\alpha$ is the visual angle of the lead vehicle. $\alpha^{\prime}$ is defined as
$\alpha^{\prime}=2 \cdot \operatorname{atan}\left(\frac{w}{\text { timegap } \cdot F V_{v}}\right)$
where w is the width of the lead car, timegap is the desired time headway, and FVv is the velocity of the driver. The model was tested using real time data collected in a driving simulator as well as real world driving performance recorded on freeways in the San Francisco area. The driving simulator studies examined car following performance in response to lead vehicle speed changes defined by single sine, sum of sines, and ramp functions. The model successfully tracked human driver performance across variations in frequency and amplitude for sine wave driving functions, and variations in slope for ramp functions. The model was also tested relative to 6 real world driving events that included large variations in speed, straight and curved path lanes, and scenarios in which a second vehicle cut in and became the new lead vehicle. The model successfully tracked realworld driving performance and accounted for up to $96 \%$ of speed variance. These findings suggest that the car following model successfully predicts driver performance across a wide range of speed variation and driving conditions. The utility of the model is for the development of algorithms for semi-autonomous vehicle control, transitions between autonomous and human driver vehicle control, and traffic flow models.

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## Car following model based on vision: Empirical and Computational Studies

## 1. Introduction

A great deal of attention has been given to the subject of car-following (see Brackstone \& McDonald, 1999 for a review) over the years, and if there is any consensus on the matter, it is that it is not a problem easily solved. One significant issue that is often overlooked in the car following literature is that the definition of car following used in a one particular study is often quite different from the definition used in another. The definition of car following that is used in a particular study is usually dependent on the goals of the study. Some studies of car following examine cars driving naturally in traffic. Other studies investigate car following behaviors in regard to particular goals, such as maintaining a safe distance headway or a certain distance headway. Still others look at car following in particular situations, such as coming out of a stopped situation (e.g. at a traffic light), coming to a normal stop, coming to an emergency stop, etc.

So what is an appropriate definition of car following? The definition we will use in the present study is that car following is the way the behavior of a driver is influenced by the car ahead of it. Car following behavior typically does not concern itself with lane changing behavior; we must respond to the car ahead by either accelerating or decelerating. If the car ahead is driving too slowly, the driver might follow a little closer than is safe in order to "encourage" it to accelerate. If the car ahead is going faster than we want to drive, we basically ignore it and try to maintain our preferred speed. But these issues are not really the concern of car following modelers. We assume that drivers are working towards simpler goals, such as matching the velocity of the leading vehicle, or maintaining a constant (safe) distance between our car and the leading vehicle.

Historically, the bulk of car following models (e.g. Chandler, Herman, \& Montroll, 1958; Helly, 1959) have assumed that the inputs to the model are 3D parameters of the situation, namely the distance headway between the leading vehicle (LV) and the following vehicle (FV), the velocities of the LV and the FV, or the accelerations of the LV and the FV. Certainly the goal of drivers who are car following is to manage these 3D parameters. The present research focused on the perceptual information used by a driver for car following (see Michaels; 1963). When considering the perceptual information for car following it is important to note that the visual system does not have direct information for recovering 3D parameters for car following (an exception of course is the speedometer of the driver's vehicle). In order to understand the perceptual information for car following one must consider the projected 2D visual information available to the retina.

A substantial body of research has shown that humans do not veridically perceive the 3D geometry of the world from projected 2D information. Many researchers have looked at how the perceived distance to an object varies with its physical distance. Gilinsky (1951) found that perceived distance decreased hyperbolically with physical distance when observers were asked to construct as series of equal intervals in depth. This basic result has been replicated with a variety of methods (see Sedgwick, 1986, for a review). The degree of foreshortening found varies with the method used to measure it, but in general the research supports the idea that humans do not have veridical access to real world distances.

So if we cannot begin with the 3-D parameters of the car following scenario, what projected 2D information is available to the visual system? Michaels (1963) suggested that visual size of the LV be used to model car following. A few researchers have already presented models of car following based on visual parameters (e.g. Lee and Jones, 1967; van Winsum, 1999), but they suffer from other problems that are addressed in the model. Lee (1976) added valuably to this issue by showing that time to contact, the inverse rate of expansion of an approaching object, (TTC) during braking was an optical variable. TTC is of limited usefulness in actual car following, as successful car following entails keeping TTC infinite. Van Winsum's model employs TTC, but only addresses one part of car following, negative accelerations, and so is difficult to evaluate in general terms. Lee and Jones present a model that scales acceleration by the rate of change of the visual angle, a model that does a very good job of matching the velocity of the LV. It suffers though (as do many of the 3D parameter models) by failing to match distance. With these models, when relative velocity between the LV and FV is zero (and consequently so is the rate of change of the visual angle of the LV), the acceleration response of the model is also zero.

An example of when this would be a problem for car following is when the LV enters the lane ahead of the FV at the same velocity as the FV, but at a distance too close for comfort. The velocity matching models have no ability to affect a deceleration to address this problem. Velocity matching models make another mistake in assuming that the acceleration/braking performance of the LV and the FV are equal. If the FV's ability to accelerate was not as great as the LV, quick increases in speed by the LV would be unmatchable by the FV, and it would lose headway, even when the LV stopped accelerating. There must be a component of the model able to change speed in order to achieve desired distance headway.

Helly (1959) presented a model that contains a linear combination of a velocity difference minimizing factor with a distance headway minimizing factor. In models of this nature, the acceleration response is only zero when both velocity difference is zero and when the desired distance headway has been achieved. Helly's model will serve as a basis for the present model, with 3D parameters replaced by appropriate 2D visual parameters.

The visual angle has a near one-to-one relationship (except at close distances) with the distance headway between the cars. Distance between the LV and FV is related to visual angle by:

$$
\tan \left(\frac{\theta}{2}\right)=\frac{\mathrm{w}}{2 \mathrm{~d}}
$$

where $\theta$ is the visual angle, $w$ is the width of the $L V$, and $d$ is the distance between the cars. One can reasonably accurately approximate the tangent of small angles by the angle itself (in radians). By this approximation, we see that the visual angle is related to width and distance by the following:

$$
\theta=\frac{\mathrm{w}}{\mathrm{~d}}
$$

This approximation is very accurate for $\theta<40$ degrees, or for cars at distances greater than about 3 meters. (At 40 degrees, for a car 2 meters in width, the approximation yields a distance of 2.865 m . The exact calculation gives 2.749 m .) This simple relation can also be used in a control system to scale acceleration response in order to maintain a desired headway, by scaling acceleration by distance headway (d), or equivalently by $1 / \theta^{1}$.

$$
\mathrm{d}=\frac{\mathrm{w}}{\theta}
$$

In order for this parameter to function properly, it must be equal to zero when the appropriate distance headway (or visual angle) is reached. This is accomplished by subtracting the desired distance headway from the above expression.

After substituting in visual parameters for 3D parameters, Helly's model becomes:
acceleration $=j \cdot\left(\frac{1}{\alpha}-\frac{1}{\alpha^{\prime}}\right)+\mathrm{k} \cdot \frac{\mathrm{d}}{\mathrm{dt}} \alpha$

Where alpha is the visual angle extent of the lead car, alpha prime is the desired visual angle extent of the lead car, $\mathrm{d} \alpha / \mathrm{dt}$ is the rate of change of $\alpha$, and j and k are scalars.

In line with safety recommendations, the desired distance headway can vary with the driver's speed. The present model incorporates a safe distance by replacing the constant $\alpha^{\prime}$ with a functional $\alpha^{\prime}$ :
$\alpha^{\prime}=2 \cdot \operatorname{atan}\left(\frac{\mathrm{w}}{\text { timegap } \cdot \mathrm{FVV}}\right)$
Where $w$ is the width of the lead car, timegap is the desired time headway, and FVv is the velocity of the driver.

The maximum acceleration (or deceleration) of a car is constrained by a number of factors, which aren't addressed here (e.g. engine specifics, weight of the car, braking system). But to avoid allowing the model to brake harder or accelerate faster than the simulator permits, we put the same limits on the model. In the simulator, pressing the brake pedal to its maximum position results in a deceleration of $1.05 \mathrm{~m} / \mathrm{s}^{2}$; pressing the accelerator to its maximum results in an acceleration of $1.05 \mathrm{~m} / \mathrm{s}^{2}$. These limits apply to the model as well: even if the model's calculations for appropriate acceleration exceed $1.05 \mathrm{~m} / \mathrm{s}^{2}$, acceleration/deceleration is capped at $1.05 \mathrm{~m} / \mathrm{s}^{2}$.

[^0]
### 1.1 Evaluation of car following models

Despite our criticism of car following models that lack visual parameters as inputs to existing car-following models, they function effectively in simulation. This begs the question: How to evaluate a given model? The answer to this question lies in the purpose one intends the model to serve. The purpose of the present model is to predict human car-following, with the advantages and limitations inherent to the human visual system. Other models are intended to serve as computer-controlled car following systems, and it is not crucial that they function in the same manner as humans, as long as they perform the task effectively. Computer-based car-following systems may also have access to other sources of information, such as radar- or laser-based headway calculations, or even GPS telemetry giving real-time information of the LV's velocity.

In any case, the most important criterion that a model must satisfy is that it must not crash. But this is a minimal condition, and is not particularly difficult to achieve. Many researchers compare the performance of their models to the behavior of actual drivers in real traffic. This is the height of realism, although it doesn't really address the issue of performance under extreme circumstances. After all, if the model can't deal with an emergency braking of the LV (assuming a human driver could), than it isn't much good. Even matching to a reasonable degree the performance of humans in crashavoidance is not a very good evaluating measure. (Or maybe it is... We could simulate more difficult conditions, ones in which the ability of the driver to avoid a collision is put to the limit. If we could map out the border of crash avoidability by a driver, and match it with the performance of the model, that might actually be interesting.)

What we need is a way to compare data from actual drivers in simulators with the performance of the model in the same simulations. The problem with directly comparing the performance is that the information available at a certain moment during the simulation is dependent on one's performance during the previous time in the simulation.

Brookhuis, de Waard, and Mulder (1994) use car following as part of a battery of tests to evaluate cognitive performance under the influence of external factors (e.g. alcohol, cell phone use). They use coherence, phase shift, and gain as descriptive statistics, and find consistent differences between normal driving and driving under a variety of external factors. We will use phase shift and gain in our analysis of the human drivers' performance and attempt to find model parameters that result in phase and gain results similar to the human drivers.

Two different types of data were analyzed. The first type of data was collected from drivers in a driving simulator at UCR. Drivers were told to maintain their initial distance from the lead car despite changes in its velocity. In this data set the driver was instructed to follow the lead car down a straight road without steering. This controlled situation allows us to examine car-following behavior in situations without the distractions of normal driving, like curves, changes in elevation, amount of daylight, etc. The second set came from one driver data collected in an instrumented vehicle in real traffic. In this set, the driver is just driving normally in traffic. We selected several time periods in which the driver was following another car, and analyzed the driver's behavior during this time. This data represents a real driving situation that allows us to verify that our model of car-following exhibits behavior similar to that of a real driver in real driving situations.

## 2. Experiments

### 2.1 Driving Simulator Study 1: Single sine waves

Drivers were presented with a car following scenario in which the lead vehicle varied its velocity according to a sinusoid. We used 5 levels of frequency ( 0.0513 , $0.05913,0.0711,0.08617$, and 0.1111 Hz ) and 6 levels of amplitude ( $5,10,15,20,25,60$ kph ). Five undergraduates from UCR served as drivers. Drivers were told to maintain their initial separation from the lead car despite changes in velocity of the lead car. At the beginning of each drive, participants were given 5 seconds of driving at a constant speed behind a constant speed lead car to establish the distance to be maintained. Trials lasted 60 seconds. Each combination of frequency and amplitude was presented once during each of two sessions. Driver performance is summarized by gain and phase angle relative to the lead car's velocity profile for each frequency and amplitude presented (Figures 1 and 2).


Figure 1. Averaged subject phase shifts for the amplitudes and frequencies presented in Study 1.


Figure 2. Averaged subject gains for the amplitudes and frequencies presented in Study 1.

The model was presented with the same scenarios that were presented to the subjects. Its parameters were found through a series of Monte Carlo simulations designed to produce similar parameters to those found by averaging over the subjects. The desired timegap used by the model was 1.35 sec , which was the initial timegap at the beginning of each trial. A model with $\mathrm{j}=.3$ and $\mathrm{k}=-5$ produces the gains and phase angles shown in Figures 3 and 4.


Figure 3. Model phase angles for the amplitudes and frequencies presented in Study 1.


Figure 4. Model gains for the amplitudes and frequencies presented in Study 1.

Figure 5 shows a velocity profile over time for a representative driver, the model, and the lead car for the scenario with frequency $=0.0711 \mathrm{~Hz}$ and amplitude $=20 \mathrm{kph}$.


Figure 5. A representative velocity profile for a human subject, leading car, and model in Study 1.
Figure 6 shows the headway profile over time for the same driver and the model for the same scenario. As you can see, the human driver shows an asymmetry in distance headway that is not present in the model's performance. This would be a good feature to incorporate into our model: it is more important to avoid small headways than large headways.


Figure 6. A representative headway profile for a human subject, leading car, and model in Study 1.

### 2.2 Driving Simulator Study 2: Sums of sine waves

Drivers were presented with a car following scenario in which the lead vehicle varied its velocity according to a sum of 3 equal-energy sinusoids. The 3 sinusoids used had the following frequencies: . $033, .083$, and .117 Hz . The corresponding amplitudes for these sinusoids were: $9.722,3.889$, and 2.778 kph . The three sinusoids were out of
phase with one another: the initial phase of the high and middle frequency wave was selected randomly and the low frequency wave had the phase value to produce the negative of the sum of the initial values of the other two waves. This ensured that the starting velocity of the lead vehicle would always be 40 kph , yet the velocity profile of the lead car would vary from trial to trial. The procedure was the same as in the previous data set, except that trials lasted 10 minutes. Four drivers participated in this experiment.

The performance of both the drivers and the model is summarized again by gain and phase angle relative to the 3 component sine waves. Figure 7 shows the gains of the 4 drivers and the model; Figure 8 shows the phases of the 4 drivers and the model.


Figure 7. Gains of the 4 drivers and the model in Study 2.


Figure 8. Phase shifts for the 4 drivers and the model in Study 2.
Looking at the velocity profiles for this scenario (Figure 9, only the total frames are shown, for simplicity in presentation) shows that the drivers found this control task more difficult than the singles sines, in general. There is a lot of overcorrecting evident in the velocity profile, and there are also more frequent dramatic accelerations and decelerations. The higher gains showed by the driver are also apparent.


Figure 9. Velocity profile for a driver, the model, and the lead car in Study 2.


Figure 10. Distance headway profile for a driver, the model, and the lead car in Study 2.

### 2.3 Driving Simulator Study 3: Ramp Functions

Drivers were presented with a car following scenario in which the lead vehicle varied its velocity according to a ramp function. The lead vehicle started at $30 \mathrm{~km} / \mathrm{h}$ and increased velocity linearly to $50 \mathrm{~km} / \mathrm{h}$ over $2.5,5$, or 7.5 seconds. Initial modeling attempts were poor, apparently due to the timegap aspect of the model. When the model was configured to maintain timegap rather than distance headway, the velocity profile did not show the subjects' overshoot and correction. The drivers in the simulator experiments were in fact told to maintain distance headway rather than a safe timegap, so this makes sense. The model was reconfigured for this data set to maintain initial distance headway rather than the timegap used in data sets 1 and 2. Figures 11, 12, and 13 shows the velocity profiles for the lead car, the following car, and the model for these scenarios.


Figure 11. Velocity profiles for a representative driver, lead vehicle, and model for the 2.5 second ramp in Study 3.


Figure 12. Velocity profiles for a representative driver, lead vehicle, and model for the 5 second ramp in Study 3.


Figure 13. Velocity profiles for a representative driver, lead vehicle, and model for the 7.5 second ramp in Study 3.

One can see that the model doesn't manage to match the end velocity of the lead vehicle very well. It ends up undershooting it in the 3 plots. This is probably because the scalar k used to scale the "rate of change of theta" portion of the model is rather small, and thus has little ability to accelerate the model when the velocity difference between the lead vehicle and the driver is small. When larger k's were used in the Monte Carlo simulations in data set 1 , gains were reduced relative to the human drivers'. This may indicate that the model needs to be redesigned with a higher value of $k$, but possibly to only accelerate based on "rate of change of theta" when acceleration based on theta itself is minimal.

### 2.4 Real Driving Data and Modeling

The raw data for this section was provided by Delphine Delorme from another project for California PATH. It comes from an instrumented vehicle being driven by a normal driver on a normal commute over a period of time. Quite a variety of data is collected by the vehicle's systems; the information we used was primarily the radarmeasured velocity and acceleration of lead vehicle, as well as velocity and acceleration of the instrumented car itself. Data points were recorded at 14 hertz.

The car was equipped with several video cameras (see Figure 14); the camera of the driver's forward view was used to identify portions of the drive during which the driver was following a lead vehicle. Six such sections were selected for analysis. The radar and telemetry data were used to determine the velocity and acceleration of both the lead car and the following car, the distance between them, and consequently the visual parameters that the model would use as input.


Figure 14. Captured frame from the 3 video cameras in the instrumented vehicle. Top left is front-view, top right is rear-view, and bottom left is the driver's face, distorted here to provide anonymity.

The model was then presented with the same driving scenarios as the real driver. The velocity profile of the lead car was used to generate the input parameters for the model at every moment, and a velocity profile for the model was consequently generated. Figures 15 shows the velocity and distance headway profiles of the real lead car, the following car, and the model car for each of the 6 sections of the drive. Gain and phase are not useful descriptors of this data set, given that the lead car is not following a sinusoidally modulated velocity profile. But it should be visually apparent that the model does an adequate job of following the lead car.


Figure 15. Velocity profiles for section 1.


Figure 16. Distance headway profiles for section 1.


Figure 17. Velocity profiles for section 2.


Figure 18. Distance headway profiles for section 2.


Figure 19. Velocity profiles for section 3.


Figure 20. Distance headway profiles for section 3.


Figure 21. Velocity profiles for section 4.


Figure 22. Distance headway profiles for section 4.


Figure 23. Velocity profiles for section 5.


Figure 24. Distance headway profiles for section 5.

Figure 25. Velocity profiles for section 6.


Figure 26. Distance headway profiles for section 6.

A particularly interesting event in car-following is when the lead vehicle changes lanes, effectively removing itself from the model. In this case, the next car ahead of the lead vehicle becomes the new lead vehicle for the model. The inverse situation is also interesting: a car from an adjacent lane moves between the lead vehicle and the following vehicle. In both these cases, the velocity difference between the old lead vehicle and the new lead vehicle can be minimal, although the headway (or timegap) difference is substantial. A model that was accelerating entirely based on velocity difference would not be able to adjust its velocity to either catch up to or back away from the new lead vehicle in this situation. We selected several portions of the drive in which these situations occur. You can see that the model's response to this situation is very similar to that of the human driver.

## 3. Conclusions

In general, the performance of the model successfully matched the performance of human drivers. This shows that visual information is sufficient to perform car following. Most other models of car following behavior use 3D environmental variables (e.g. distance between cars and velocities of cars) as inputs to the model. This approach is not parsimonious: it adds an unnecessary level of assumptions into the control system. Namely, that drivers use visual information to estimate the 3D environmental variables, and then use those estimates to perform car following. Numerous studies show that human judgments of such variables are not veridical. It is also the case that there is little consensus on the geometry of visual space, so assumptions of veridicality are unjustified.

The lower gains and phase shifts found in the model's performance are probably due to the variable attention of the human drivers. The model has a one-track mind in its
present incarnation, unlike human drivers. It never looks away from the windshield to change the radio, checks the rearview mirrors, or even steers. To more completely model human performance, aspects such as multiple simultaneous task demands need to be included in future versions.

The human driver also shows an asymmetry in distance headway that is not present in the model's performance. This would be a good feature to incorporate into future versions of the model: it is more important to avoid small headways than large headways. This could be possibly be captured in a model by incorporating a control parameter based on brake lights, since this would increase the response of the model to braking behavior by the lead car, but leave the response of the model to acceleration behavior by the lead car unchanged. This would be a great feature to incorporate in the model, although it is somewhat difficult to accurately simulate (brake lights come on only during braking, not just any deceleration such as occurs during coasting). It could also be a challenge to accurately detect brake lights in the videos from the real driving scenarios.

In the results of the ramp study data, we indicated that the model with the parameters from the previous studies didn't end the ramp at the same velocity as the leading car. The instructions the subjects received emphasized that the task was to maintain distance headway, which may explain why we had to weight the parts of the model as we did to match subject performance. This may not be representative of real driving, where precise distance headway maintenance is not necessarily the top priority at all times. In the ramp simulations, our intention was to see how long it took the subjects and the model to recover from the ramp back to the new resting velocity. But the data revealed a weakness in the model: the model is not a sensitive to small differences in velocity as we would like it to be.

The present model demonstrates that car following behavior can be successfully modeled using visual parameters as inputs to the control system. We have compared the performance of our model to human drivers in a series of carefully controlled driving simulator studies, as well as to the performance of a carefully monitored but unconstrained human driver in real driving situations. These studies have generated a wealth of data about car following that supports our model of how human drivers perform the task.

## Appendix

```
#include "stdafx.h"
#include <math.h>
double amplitude, frequency; //parameters for sinusoidal velocity
profile
double timegap = 1.1; //timegap maintained by the model (in seconds)
double width = .001; //width of back of leading car in kilometers
double alpha_prime;
double j = .01;
double k = -1000; //j and k are multiplicative constants used in
calculating acceleration
double range[1200]; //distance between lead and following car
double v[1200];
double deltav[1200];
double alpha[1200];
double d_alpha[1200];
double
lead_car_vel(int t) {
    return 40 + amplitude*sin(t*(frequency/20)*(2*3.14));
    //lead car modulates its velocity around 40 kph
}
double
acceleration(double alpha, double d_alpha, double j, double k, double
v) {
    return j*((1/alpha)-(1/alpha_prime*v)) + k*d_alpha;
}
int main(int argc, char* argv[])
{
    int frame;
    v[0] = 40; //kilometers/hour
    range[0] = .018; //kilometers
        alpha[0] = 2*atan(.001/range[0]);
        for(frame=1; frame<1200;frame++) {
            deltav[frame] = lead_car_vel(frame)-v[frame-1];
            alpha_prime = 2*atan(width/(timegap*v[frame-1]));
            //computes the appropriate visual angle for the current
velocity of the following car
                    range[frame] = range[frame-1] + deltav[frame]*.05/3600;
                    alpha[frame] = 2*atan(.001/range[frame]);
                    d_alpha[frame] = alpha[frame] - alpha[frame-1];
                    v[frame] = v[frame-1] + acceleration(alpha[frame],
d_alpha[frame], j, k, v[frame-1]);
    }
    return 0;
}
```


## References

Aycin, M.F. and Benekohal, R.F. 2001. Stability and performance of car-following models in congested traffic. Journal of Transportation Engineering-ASCE, 127, pp. 2-12.

Brackstone, M. and McDonald, M. 1999. Car-following: a historical review.
Transportation Research Record F, 2, pp. 181-196.
Brookhuis, K., deWaard, D., and Mulder, B. 1994. Measuring driving performance by car-following in traffic. Ergonomics, 37, pp. 427-434.

Chandler, R.E, Herman, R., and Montroll, E.W. 1958. Traffic dynamics: studies in car following. Operations Research, 6, pp. 165-184.

Gilinsky, A.S. (1951) Perceived size and distance in visual space. Psychological Review, 58, pp. 460-482.

Helly, W. 1959. Simulation of bottlenecks in single lane traffic flow. In Proceedings of the Symposium on Theory of Traffic Flow. , Research Laboratories, General Motors (pp. 207-238). New York: Elsevier.

Lee, D.N. 1976. A theory of visual control of braking based on information about time to collision. Perception, 5, pp. 437-459.

Lee, J, and Jones, J.H. 1967. Traffic dynamics: visual angle car following models. Traffic Engineering and Control, pp. 348-350.

Michaels, R.M. 1963. Perceptual factors in car following. In Proceedings of the Second International Symposium on the Theory of Road Traffic Flow (pp. 44-59). Paris; OECD.

Van Winsum, W. 1999. The human element in car following models. Transportation Research Part F, pp. 207-211.

Zhang, X. \& Jarrett, D.F. 1997. Stability analysis of the classical car-following model. Transportation Research B, 31, pp. 441-462.


[^0]:    ${ }^{1}$ This expression for $d$ also contains $w$, the width of the LV. Our model does not require that the driver know the veridical 3D width of the LV, only that it is constant.

