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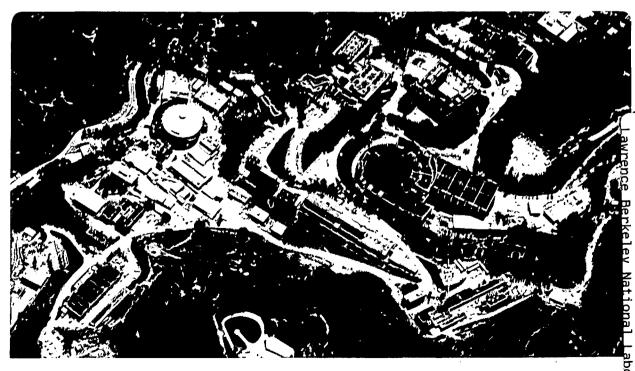
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# VIABLE t-b-au YUKAWA UNIFICATION IN SUSY SO(10)\*

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Abstract: The supersymmetric SO(10) GUT with  $t-b-\tau$  Yukawa coupling unification has problems with correct electroweak symmetry breaking, experimental constraints (especially  $b \to s\gamma$ ) and neutralino abundance, if the scalar masses are universal at the GUT scale. We point out that non-universality of the scalar masses at the GUT scale generated both by (1) renormalization group running from the Planck scale to the GUT scale and (2) D-term contribution induced by the reduction of the rank of the gauge group, has a desirable pattern to make the model phemenologically viable (in fact the only one which is consistent with experimental and cosmological constraints). At the same time the top quark mass has to be either close to its quasi IR-fixed point value or below  $\sim 170$  GeV. We also briefly discuss the spectrum of superpartners which is then obtained.

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Grand Unification (GUT) has been regarded as a serious candidate of physics beyond the weak-scale to explain various puzzling features of the standard model. While the three gauge coupling constants fail to unify in its simplest version, they meet at a scale  $M_{GUT} \simeq 2 \times 10^{16}$  GeV in its supersymmetric (SUSY) extension. SUSY-SO(10) GUT offers further exciting possibility that all three Yukawa coupling constants of top, bottom quarks and tau lepton may also unify at the same scale where the gauge coupling constants unify. This is possible because the supersymmetric standard model contains two Higgs doublets, and large values of  $\tan \beta = v_2/v_1$  (the ratio of the vacuum expectation values for the doublet  $H_2$  and  $H_1$  which couple to the up and down quarks, respectively) lead to a proper bottom top quark mass hierarchy, with approximately equal b and t Yukawa couplings [1]. The consequence of such an exact unification of couplings is that the top quark mass,  $m_t$ , and  $\tan \beta$  are determined, once the bottom quark mass,  $m_b$ , the tau lepton mass,  $m_\tau$ , and the strong gauge coupling,  $\alpha_3$ , are fixed [2], [3], [4].

In this context, an interesting question is the issue of the compatilibity of this exact Yukawa coupling unification with the possibility of breaking the electroweak gauge symmetry through radiative effects. This question has been investigated in a number of papers in the minimal SUSY-SO(10) models with universal [5], [6], [3] and non-universal [7]-[10] soft supersymmetry breaking parameters at the GUT scale. Moreover, it has been recently observed that for these large values of  $\tan \beta$ , potentially large corrections to  $m_b$  may be induced through the supersymmetry breaking sector of the theory [4], [3]. Altogether, the requirement of a physically acceptable value for the  $m_b$  and of the consistency with the recent CLEO result for the decay  $b \to s \gamma$  and with the condition  $\Omega h^2 < 1$  for the relic abundance of the LSP strongly constrain the minimal SUSY-SO(10) with radiative electroweak symmetry breaking. In recent papers [10], [11] it has been shown that those constraints rule out the model with universal soft supersymmetry breaking terms at the GUT scale and select certain class of non-universal boundary conditions which lead to radiative breaking with  $M_2 \gg \mu$ , i.e. with higgsino-like lightest neutralino, as the only acceptable scenario for the minimal SUSY-SO(10).

From the theoretical point of view, non-universal SUSY breaking terms at the GUT scale appear at present as a realistic possibility. In GUT models, even with universal boundary conditions at the Planck scale, the renormalization group (RG) running to the GUT scale generically leads to some non-universality of the scalar masses at that scale [8], [12]. In addition, in models like SO(10), the reduction of the rank of the gauge group by one at  $M_{GUT}$ , together with non-universal scalar masses, generates additional non-universal contributions given by the D-term of the broken U(1) [13], [9].

In this paper, motivated by the phenomenological analysis of ref.[10], we point out that the combination of both types of effects in the minimal SUSY-SO(10) naturally gives the physically desirable non-universal boundary conditions at the GUT scale. This is a non-trivial prediction of the minimal model which depends in a crucial way on the presence of the D-term contribution (but not on its actual value) and on the value of the top quark Yukawa coupling,  $h_t$ . Two branches of correct solutions are obtained: one for  $h_t$  very close to its quasi-infrared fixed point and the other one for lower values of  $h_t$ . With the present uncertainty on the top quark

mass, both solutions may be of phenomenological interest.

The Higgs potential

$$V = m_1^2 H_1^{\dagger} H_1 + m_2^2 H_2^{\dagger} H_2 - m_3^2 \left( H_1^{\dagger} i \tau_2 H_2 + h.c. \right) + \text{quartic terms}$$
 (1)

 $(m_i^2 = m_{H_i}^2 + \mu^2, m_3^2 = B\mu$  where  $\mu$  is the supersymmetric Higgs mixing parameter and B is the corresponding soft term) has for large  $\tan \beta$  values two characteristic features. It follows from the minimization conditions that

$$m_2^2 \simeq -\frac{M_Z^2}{2} \tag{2}$$

and

$$m_3^2 \simeq \frac{M_A^2}{\tan \beta} \simeq 0 \,, \tag{3}$$

with

$$M_A^2 \simeq m_1^2 + m_2^2 > 0. (4)$$

Equations (2) and (3) are the two main constraints on the parameters of the scalar potential, which are characteristic for large  $\tan \beta$  solutions. Combining (2) and (4) we get

$$m_1^2 - m_2^2 > M_Z^2 \ . {5}$$

Let us first discuss the dependence of the low-energy parameters on the GUT-scale boundary values. This discussion clearly shows the need for a special type of non-universality of scalar masses at the GUT scale. After that, we demonstrate such a special type of non-universality can be naturally obtained in SO(10) GUT even with a universal boundary condition at the Planck scale.

The parameters of the low energy potential are given in terms of their boundary values at large scale and the RG running. Here we consider the minimal SUSY-SO(10) model with matter fields in 16 dimensional representations and the two Higgs doublets in one representation of dimension 10. At the GUT scale, after RG running from the reduced Planck scale  $M_{Pl} \simeq 2.4 \cdot 10^{18}$  GeV to the GUT scale with the SO(10) RG equations, all scalar masses depend on three parameters  $\overline{m}_{16}$ ,  $\overline{m}_{10}$  and D (D-term):

$$\overline{m}_{H_1,H_2}^2 = \overline{m}_{10}^2 + \left\{ \begin{array}{c} 2 \\ -2 \end{array} \right\} D,$$
 (6)

$$\overline{m}_{Q,U,D}^2 = \overline{m}_{16}^2 + \left\{ \begin{array}{c} 1\\1\\-3 \end{array} \right\} D.$$
 (7)

The values of the masses at the electroweak scale are obtained by solving the RG equations of the minimal supersymmetric standard model with the initial conditions at the GUT scale given by eqs.(6) and (7). For  $Y_t = Y_b = Y \equiv h^2/4\pi$  the approximate solutions to the 1-loop RG equations read [3]:

$$m_{H_1,H_2}^2 = \left(1 - \frac{3}{7}y_Z\right)\overline{m}_{10}^2 - \frac{6}{7}y_Z\overline{m}_{16}^2 - c_M\overline{M}_{1/2}^2 + \left\{\begin{array}{c} 2\\ -2 \end{array}\right\}D + \dots,$$
 (8)

$$m_{Q,U,D}^2 = \left(1 - \frac{4}{7}y_Z\right)\overline{m}_{16}^2 - \frac{2}{7}y_Z\overline{m}_{10}^2 + c_M^{Q,U,D}\overline{M}_{1/2}^2 + \left\{\begin{array}{c} 1\\1\\-3 \end{array}\right\}D + \dots$$
 (9)

 $\overline{M}_{1/2}$  is the gaugino mass at  $M_{GUT}$ ,  $c_M \sim \mathcal{O}(2)$ ,  $c_M^{Q,U,D} \sim \mathcal{O}(4)$  and for convenience of analytical solutions we have introduced the parameter

$$y_Z = \frac{Y(M_Z)}{Y_{fZ}} \tag{10}$$

where  $Y(M_Z)$  in the well known solution to the MSSM renormalization group equations for the Yukawa coupling in the limit of large tan  $\beta$  [14], [3]:

$$Y\left(M_{Z}\right) = \frac{E_{MSSM}\left(M_{Z}\right)Y\left(M_{GUT}\right)}{1 + 7F_{MSSM}\left(M_{Z}\right)Y\left(M_{GUT}\right)}.$$
(11)

Here,  $Y_{fZ}$  is the auxiliary parameter given by the value of  $Y(M_Z)$  corresponding to the Landau pole in Y at  $M_{GUT}$ 

$$Y_{fZ} = \lim_{Y(M_{GUT}) \to \infty} Y(M_Z) = \frac{E_{MSSM}(M_Z)}{7F_{MSSM}(M_Z)},$$
(12)

and  $E_{MSSM}$  and  $F_{MSSM}$  are functions of the gauge couplings. The dots in eqs.(8) and (9) stand for terms which depend on soft parameter  $\overline{A}_0$  at the GUT scale. In this approximation the condition (5) gives

$$m_1^2 - m_2^2 = 4D > M_Z^2 (13)$$

However, here we have neglected small differences in the running of the two Higgs masses which follow from the different hypercharges of the right top and bottom squarks, from the difference in the running of the bottom and top Yukawa couplings (equal at the GUT scale) and from the effects due to the  $\tau$  lepton Yukawa. After inclusion of those effects we get

$$m_1^2 - m_2^2 = a\overline{M}_{1/2}^2 + c\overline{m}_0^2 + 4D$$
, (14)

where  $\overline{m}_0$  is an average scalar mass at the GUT scale (actually  $\overline{m}_0^2 = (\overline{m}_{10}^2 + 2\overline{m}_{16}^2)/3$ ) and the numerical values of the coefficients are  $a \sim |c| \sim \mathcal{O}(0.1)$  with c < 0. The small effects neglected in eq.(13) but included in eq.(14) are resposible for radiative breaking in the case of universal boundary conditions at the GUT scale  $(D=0\,,\,\,\overline{m}_{16}=\overline{m}_{10}=\overline{m}_0)$  [3]. Then the large  $\tan\beta$  solutions must be driven by large values of  $\overline{M}_{1/2}$ :

$$\overline{M}_{1/2} > \frac{M_Z}{\sqrt{a}} , \qquad \overline{M}_{1/2} > \sqrt{\frac{|c|}{a}} \ \overline{m}_0 , \qquad (15)$$

and as discussed in ref.[11], this scenario is strongly disfavoured for several reasons. It is clear from eq.(14) that in the framework of SUSY-SO(10), with  $D=d\overline{m}_0^2$ , qualitatively new solutions become possible if c+4d>0, with  $\overline{M}_{1/2}\simeq 0$  and

$$\overline{m}_0 > \frac{M_Z}{\sqrt{c+4d}} \,. \tag{16}$$

Thus, with positive D, contrary to the universal case, radiative electroweak breaking can be driven by soft scalar masses and this pattern does not depend on the actual value of the D term as well as on the values of  $\overline{m}_{16}$  and  $\overline{m}_{10}$ . However, as we shall see, there are important properties of the solutions which do depend on those masses.

Further properties of the solutions and certain phenomenological classification of non-universal boundary conditions follows from the equation (2) [10]. Since  $m_2^2 = m_{H_2}^2 + \mu^2$ , this equation can be interpreted as an equation for  $\mu^2$ . In the universal case large values of  $\mu^2$  are needed to cancel large negative values of  $m_{H_2}^2$ . Now, with non-universal scalar terms it follows from eqs.(8) and (2) that

$$\mu^2 = c_M \overline{M}_{1/2}^2 + c_m \frac{\overline{m}_{10}^2 + \overline{m}_{16}^2}{2} + 2D - \frac{M_Z^2}{2} + \dots , \qquad (17)$$

where

$$c_m = -\left[\left(1 - \frac{9}{7}y_Z\right) + \left(1 + \frac{3}{7}y_Z\right) \frac{\overline{m}_{10}^2 - \overline{m}_{16}^2}{\overline{m}_{10}^2 + \overline{m}_{16}^2}\right] , \qquad (18)$$

and the second term in eq.(18) is generated by departures from universality. The values of  $\mu^2$  depend, contrary to equation (16), on the pattern of the deviation from universality in  $\overline{m}_{16}$  and  $\overline{m}_{10}$ . We obtain the following classification [10]:

(A) 
$$\mu^2 > c_M \overline{M}_{1/2}^2$$
 for  $c_m > 0$ 

(B) 
$$\mu^2 < c_M \overline{M}_{1/2}^2$$
 for  $c_m < 0$ 

It is clear that in case (A), generically the values of  $\mu$  remain large  $\mu \gg M_Z$  even in the limit  $\overline{M}_{1/2} \simeq 0$ , due to the positive correlation with the scalar masses.

In case (B) the parameter  $\mu$  can be arbitrarily small due to the cancellation betwen the scalar and gaugino contributions in eq.(17) (with c+4d>0 and only then; in the opposite case  $\overline{M}_{1/2}>\overline{m}_0$ , as in eq.(15), and the cancellation is impossible unless  $-c_m>c_M$  which is very difficult to achieve). Note that for phenomenological reasons (experimental bounds) we are actually not interested in the strict limit  $\overline{M}_{1/2}=0$ . Thus, radiative breaking can be driven by  $\overline{m}_0\gtrsim\overline{M}_{1/2}\gtrsim\mu\simeq\mathcal{O}(M_Z)$ . As shown in refs.[10], [11], it is the case (B) which is phenomenologically acceptable, with higgsino-like lightest chargino and neutralino. The non-universalities of type (A) suffer from similar problems as the universal case and are disfavoured by the combinations of constraints from  $\Omega h^2<1$  and  $BR(b\to s\gamma)$ .

The condition  $c_m < 0$  puts non-trivial contraints on the values of  $\overline{m}_{16}$  and  $\overline{m}_{10}$  at the GUT scale and on the Yukawa coupling. In the following we demonstrate that they are satisfied by the values of the masses obtained by RG running in the minimal SO(10) model from the Planck scale, with universal boundary conditions  $M_{1/2}$  and  $m_0$  for the soft gaugino and scalar masses at the Planck scale (the unbarred quantities denote Planck scale parameters) and for interesting range of values of the top quark Yukawa coupling.

The set of the relevant SUSY-SO(10) RG equations is as follows:

$$\frac{d}{dt}\alpha = -b\alpha^2\,, (19)$$

$$\frac{d}{dt}Y = \left(\frac{63}{2}\alpha - 14Y\right)Y,\tag{20}$$

$$\frac{d}{dt}A = \frac{63}{2}\alpha M - 14YA\,, (21)$$

$$\frac{d}{dt}M = -b\alpha M\,, (22)$$

$$\frac{d}{dt}m_{16}^2 = -5Y(m_{10}^2 + 2m_{16}^2 + A^2) + \frac{45}{2}\alpha M^2,$$
 (23)

$$\frac{d}{dt}m_{10}^2 = -4Y(m_{10}^2 + 2m_{16}^2 + A^2) + 18\alpha M^2.$$
 (24)

Here  $t = \frac{1}{2\pi} \log \frac{M_{Pl}}{Q}$ ,  $\alpha$  is the SO(10) gauge coupling, M is the running gaugino mass, A – the trilinear soft breaking term and as earlier  $Y = h_t^2/4\pi$  is the Yukawa coupling for the third generation. In eqs.(19)–(24) we have explicitly introduced the numerical values for the  $\beta$ -functions which depend only on the representation assignment of the matter and light Higgs fields. The coefficient b in eqs.(19) and (22) depends on the heavy Higgs field content of the model. We do not have to specify it here as our results are stable under varying b in the range from 3 to the maximal value of order 30, corresponding to the Landau pole for the gauge coupling<sup>‡</sup>. The solution to those equations read:

$$\alpha = \frac{\alpha_0}{1 + b\alpha_0 t} , \qquad M = M_{1/2} \frac{\alpha}{\alpha_0} , \qquad Y = Y_0 \frac{E_G}{1 + 14Y_0 F_G} ,$$
 (25)

$$E_G = \left(\frac{\alpha_0}{\alpha}\right)^{63/2b}, \ F_G = \int_0^t E_G(t')dt' = \frac{1}{\frac{63}{2} + b} \frac{1}{\alpha_0} \left[ \left(\frac{\alpha_0}{\alpha}\right)^{(63/2b+1)} - 1 \right]. (26)$$

The parameters  $\alpha_0$ ,  $M_{1/2}$  and  $Y_0$  are the Planck scale boundary values of the gauge coupling, gaugino mass and Yukawa coupling, respectively.

Similarly as for the running from  $M_{GUT}$  to  $M_Z$  it is convenient to define the parameter

$$y_G = \frac{Y(M_{GUT})}{Y_{fG}} \tag{27}$$

where

$$Y_{fG} = \lim_{Y_0 \to \infty} Y(M_{GUT}) = \frac{E_G(M_{GUT})}{14F_G(M_{GUT})}$$
 (28)

We can then express the parameter  $y_Z$  introduced in eqs.(10) and (18) in terms of  $y_G$ :

$$y_Z = \frac{xy_G}{1 + xy_G} \tag{29}$$

where

$$x = 7F_{MSSM}(M_Z)Y_{fG}. (30)$$

The value of x in eq.(30) depends on the scales  $M_{GUT}$  and  $M_{Pl}$ , on the value of the gauge  $\beta$ -function coefficient b and on  $\alpha_3(M_Z)$  (we take here the attitude that  $M_{GUT}$ 

<sup>&</sup>lt;sup>‡</sup>In models where the heavy Higgs sector correctly breaks SO(10) to the standard model gauge group without any additional unwanted massless fields, the beta function is typically  $b \ge +7$ . We use b = +3 for numerical analyses in this letter as a conservative choice. The generated non-universality is larger for larger values of b. However, as we will describe below, the final result is rather insensitive on the value of b.

is determined by the crossing of  $\alpha_1$  and  $\alpha_2$  and we allow for a small mismatch of  $\alpha_3$  at that scale [15]). For  $M_{GUT} = 2 \cdot 10^{16} GeV$ ,  $M_{Pl} = 2.4 \cdot 10^{18} GeV$ , b = 3 and  $\alpha_3(M_Z)$  in the range 0.11–0.13, we get x in the range 22–25. It increases (decreases) by 5 for b = +30 (for  $M_{Pl}/M_{GUT}$  larger by factor 10). The condition  $c_m < 0$ , eq.(18), now reads:

$$\frac{\overline{m}_{10}^2 - \overline{m}_{16}^2}{\overline{m}_{10}^2 + \overline{m}_{16}^2} > \frac{2xy_G - 7}{10xy_G + 7} \,. \tag{31}$$

The solutions  $c_m = 0$  are shown in Fig.1 as the solid curves, for three different values of x = 15, 20, 25. Fig.1 illustrates the interplay between the GUT scale values of the scalar soft masses and the Yukawa coupling which is necessary to assure  $c_m < 0$  for different values of x.

As the next step, we solve the eqs.(23) and (24) and get for the scalar masses:

$$\overline{m}_{10}^2 = \left(1 - \frac{12}{14}y_G\right)m_0^2 + \frac{4}{14}I, \qquad (32)$$

$$\overline{m}_{16}^2 = \left(1 - \frac{15}{14}y_G\right)m_0^2 + \frac{5}{14}I\tag{33}$$

where

$$I = a_1 A_0^2 + a_2 A_0 M_{1/2} + a_3 M_{1/2}^2 (34)$$

and

$$a_{1} = -y_{G}(1 - y_{G}),$$

$$a_{2} = +y_{G}(1 - y_{G}) \left[ \left( 2 + \frac{63}{b} \right) \frac{1 - \left( \frac{\alpha_{0}}{\alpha} \right)^{(63/2b)}}{1 - \left( \frac{\alpha_{0}}{\alpha} \right)^{(63/2b+1)}} - \frac{63}{b} \right],$$

$$a_{3} = -\frac{63}{2b} \left[ \left( \frac{\alpha_{0}}{\alpha} \right)^{2} - 1 + y_{G} \right] - y_{G} \frac{63}{2b} \left( 1 + \frac{63}{2b} \right) \frac{1 - \left( \frac{\alpha_{0}}{\alpha} \right)^{(63/2b-1)}}{1 - \left( \frac{\alpha_{0}}{\alpha} \right)^{(63/2b+1)}}$$

$$-y_{G}(1 - y_{G}) \left[ \left( \frac{62}{2b} \right)^{2} - \frac{63}{b} \left( 1 + \frac{63}{2b} \right) \frac{1 - \left( \frac{\alpha_{0}}{\alpha} \right)^{(63/2b)}}{1 - \left( \frac{\alpha_{0}}{\alpha} \right)^{(63/2b+1)}} \right]$$

$$+y_{G}^{2} \left( 1 + \frac{63}{2b} \right)^{2} \left( \frac{1 - \left( \frac{\alpha_{0}}{\alpha} \right)^{(63/2b+1)}}{1 - \left( \frac{\alpha_{0}}{\alpha} \right)^{(63/2b+1)}} \right)^{2}.$$

$$(35)$$

With explicit solutions eqs. (32) and (33) we can check if the relation (31) is indeed fulfilled in the model. The dashed lines in Fig.1 show the solutions (32) and (33) for three different values of the ratio  $M_{1/2}/m_0$  at the Planck scale and with the values of the other relevant parameters as specified above eq. (31), with  $\alpha_3(M_Z) = 0.11$  and  $A_0 = 0$ . It is clear that the solutions to the RG running from the Planck scale to the GUT scale satisfy the constraint  $c_m < 0$  (solid lines) for values of  $y_G$  close to the quasi-IR fixed point or lower than about 0.2.

This discussion nicely illustrates the role of the boundary values at  $M_{GUT}$  for the scalar masses in obtaining solutions to radiative breaking in the MSSM with small  $\mu$  values. However, since those values depend on both Planck scale parameters  $m_0$ 

and  $M_{1/2}$ , it is more convenient to rewrite eq.(17) directly in terms of the Planck scale parameters for effective study of the parameter space. Using eqs.(32)-(35) we get:

$$\mu^{2} = \left[c_{M} + \frac{1}{7} (3y_{Z} - 2) a_{3}\right] M_{1/2}^{2} + \frac{1}{7} \left[(2 - 3y_{G}) - 9 (1 - y_{G}) (1 - y_{Z})\right] m_{0}^{2} + 2D + \dots$$
(36)

where the dots stand for  $A_0$  dependent terms. The  $M_{1/2}$  coefficient remains always positive. The D term must be positive (see eq.(13) and (14)) and in principle, with  $D = dm_0^2$ , it should be included into the  $m_0^2$  coefficient. However acceptable solutions to radiative breaking are obtained already with very small values of d, of order  $\mathcal{O}(0.01)^\S$ . At this point it is worth noting that the negative numerical coefficient c in eq.(14) (which is obtained from numerical integration of the 1-loop RG equations) goes strictly to zero for  $y_G \to 1$ . This result follows from the structure of the RG equations and explains why very small positive d is sufficient to change the pattern of solutions into those of eq.(16) (the  $A_0^2$  contribution to eq.(14) is small but also positive).

Thus, the necessary and sufficient condition for cancellations in eq.(36) to be possible is the negative sign of the  $m_0^2$  coefficient:

$$0 > [(2 - 3y_G) - 9(1 - y_G)(1 - y_Z)]. (37)$$

By using eq.(29) we easily see that this coefficient is always negative for x in the range (0.6-14.4). However, for the values of x generic for the minimal SUSY-SO(10) we obtain non-trivial constraints on the value of the Yukawa coupling. For x = 22 the eq.(37) is satisfied for  $y_G < 0.2$  or  $y_G > 0.6$ , in agreement with the results presented in Fig.1.

To study in more detail the x dependence of our result (or equivalently, for fixed values of  $M_{GUT}$  and  $M_{Pl}$ , its dependence on  $\alpha_3(M_Z)$ ) and its sensitivity to two-loop corrections in the RG running of the gauge and Yukawa couplings below  $M_{GUT}$ , we plot in Fig.2 our two-loop numerical results as a function of  $\alpha_3(M_Z)$ . Values of  $y_G$  above and below the band depicted by solid lines satisfy eq.(37). For easy interpretation we also plot the curves of constant top quark pole masses. One can see that for  $\alpha_3(M_Z) = 0.11$  the top quark has to be heavier than 181 GeV or lighter than 170 GeV, both regions being of phenomenological interest. For  $\alpha_3(M_Z) = 0.12$  both bounds move up by about 5 GeV.

Finally, we comment on the prameter space which gives correct radiative breaking and on the sfermion masses. For instance, for  $m_t = 182$  GeV,  $\alpha_3(M_Z) = 0.11$  and  $d = \mathcal{O}(0.01)$  we get  $\mu^2 = \mathcal{O}(2)M_{1/2}^2 - \mathcal{O}(0.05)m_0^2 - \mathcal{O}(0.01)A_0^2$  and e.g. for solutions with  $\mu \simeq 100$  GeV and with  $\mu/M_1 \sim 1$  we need  $m_0/M_{1/2} = \mathcal{O}(5)^{\parallel}$ . In this simple example the particle spectrum contains a light pseudoscalar and a higgsino-like

<sup>§</sup>The maximum possible value of d can be calculated if one specifies the model completely. For instance in the models where the rank is reduced by Higgs fields in 16 and  $\overline{16}$  representations, one calculates the difference in their soft masses  $m^2$  and  $\overline{m}^2$  using RG equations and obtains  $D = (m^2 - \overline{m}^2)/10$ .  $m^2$  and  $\overline{m}^2$  can differ easily by  $\mathcal{O}(1)$  because of large group theory factors in SO(10), and hence d as large as  $\mathcal{O}(0.1)$  is possible.

<sup>§</sup>Small values of  $\alpha_3(M_Z)$  are obtained from the fits to the electroweak data in the MSSM [16] Contribution from the  $A_0^2$  term can lower  $m_0$  somewhat.

chargino, both with masses below  $M_Z$  and within the reach of the Tevatron and LEP2. It is interesting to note that similar spectrum is predicted from the best fit to the electroweak data in the framework of the MSSM [16].

The sfermion masses for the third generation tend to remain relatively small, too. Combining eqs. (32), (33) with (8) and (9) we get:

$$m_{Q,U,D}^{2} = \frac{1}{14} \left[ (2 - 3y_{G}) + 12 (1 - y_{G}) (1 - y_{Z}) \right] m_{0}^{2} + \tilde{c}_{M}^{Q,U,D} M_{1/2}^{2} + \dots$$

$$\simeq \mathcal{O}(0.01) m_{0}^{2} + \mathcal{O}(4) M_{1/2}^{2} + \dots$$
(38)

and analogously for the sleptons, with  $\tilde{c}_M^{L,E} \sim \mathcal{O}(0.2)$ . In our example, the masses of the third generation squarks are in the range 200-300 GeV and for sleptons they are  $\mathcal{O}(100 \text{ GeV})$ . A more detailed study is necessary to check if their contribution to the breaking of the custodial SU(2) symmetry is consistent with the electroweak data.

Summary: In SO(10) SUSY-GUT, a specific pattern of non-universality in the scalar masses at the GUT-scale is generated by the RGE evolution from the Planck scale to the GUT scale and the D-term contribution induced by SO(10) breaking. This particular pattern of non-universality can make the t-b- $\tau$  Yukawa unification phenomenologically viable, consistent with correct electroweak symmetry breaking, experimental and cosmological constraints. The top quark mass is either below 170 GeV or above 181 GeV if  $\alpha_3(M_Z) = 0.11$ .

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### FIGURE CAPTIONS

- Fig. 1. Ratio of masess  $(\overline{m}_{10}^2 \overline{m}_{16}^2) / (\overline{m}_{10}^2 + \overline{m}_{16}^2)$  at the GUT scale as a function of the parameter  $y_G$  defined in eq.(27). The solid curves A, B and C are the solutions to the condition  $c_m = 0$  for three different values of the parameter x = 15, 20 and 25, respectively. The dashed curves represent results in the SO(10) model for three fixed values of the ratio  $M_{1/2}/m_0 = 0$ , 0.5 and 1.0 (curves a, b and c, respectively) at the GUT scale.
- Fig. 2. The region in the  $y_G-\alpha_3(M_Z)$  plane (outside the band between the solid curves) in which the condition (37) can be satisfied in the minimal SUSY-SO(10) model. The deshed curves correspond to the fixed values of the top quark pole mass in GeV. The results were obtained by integrating numerically the two-loop RG equations below  $M_{GUT}$ .

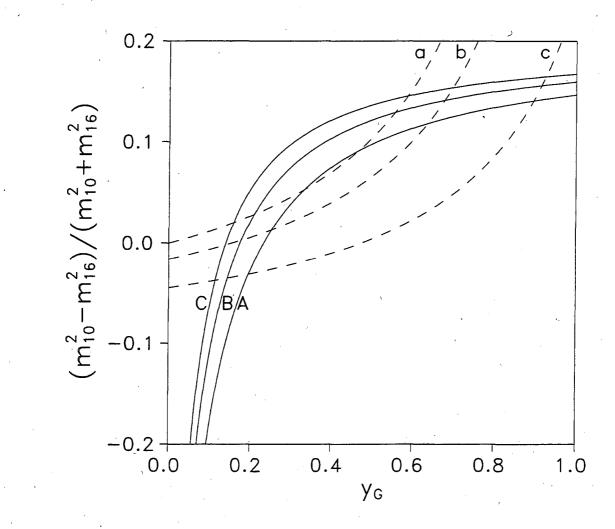


Fig.1

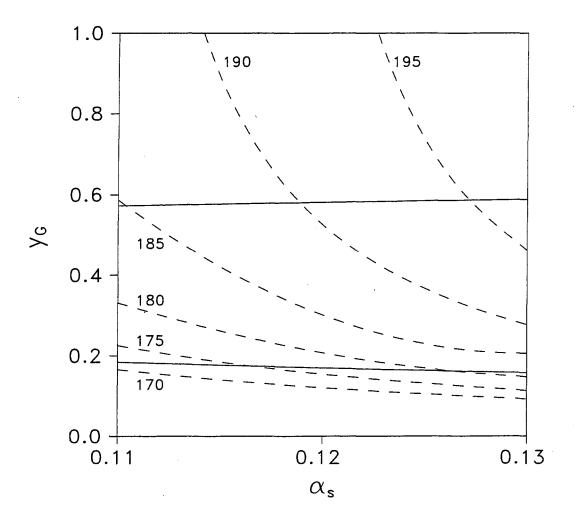


Fig.2

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