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Publication Date
1972-07-01

Submitted to Nuclear Physics

LBL-987
Preprint

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July 1972

AEC Contract No. W-7405-eng-48

OFF-SHELL BEHAVIOR IN PERIPHERAL DYNAMICS AND INCLUSIVE SUM-RULES*

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July 10, 1972

ABSTRACT

We illustrate, by a specific example, the usefulness of inclusive unitarity for constraining dynamical models. Using inclusive sum-rules together with measurable inclusive processes, an estimate is given for a parameter which determines the scale for the off-shell extrapolation in the ABFST model.

* Work supported by the U. S. Atomic Energy Commission.

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The inclusive formulation of unitarity [1)] has been recently receiving much attention. This approach to unitarity has the important advantage that inclusive processes are relatively easy to investigate both experimentally and theoretically (applying Mueller's [2)] generalized optical like theorem).

Obviously, the inclusive sum-rules [3,4)] do not contain specific dynamics. However, they turn out to be extremely useful for constraining a given dynamical theory. Indeed the attractive possibility, of a nonlinear relation for the strong coupling constant in dual models, has been pointed out by Veneziano [5,6)] using the simplest inclusive sum-rules (derived from energy-momentum conservation). Also, by applying Regge theory to inclusive cross-sections, many properties of Regge couplings have been derived [7,8)]. The usefulness of the inclusive sum-rules, as illustrated by the above examples, is exactly analogous to the ordinary Dolen-Horn-Schmid finite-energy-sum-rules, being powerful <u>only</u> when incorporated with a specific dynamical model, e.g., a saturation with narrow resonances in the s channel and Regge poles in the t channel. The success of such a scheme motivated the celebrated Veneziano representation (in nondiffractive processes).

In this note we would like to study the off-shell extrapolation in a model based on peripheral dynamics [9,10)] (see below). We shall discuss this model in relation to the high-energy inelastic pp collisions. It will be argued later that the ABFST peripheral model is mostly applicable to a process which is difficult to measure, namely $p + p \to n + X$. Therefore the model cannot unambiguously be directly confronted with experimental data. However, as we shall see, one can use the measured processes $p + p \to p + X$ and [11)] $p + p \to \pi + X$ (where $\pi = \pi^{\pm}, \pi^{0}$) in order to study the ABFST model when applied to

$p + p \to n + X$. This will be done by using the energy-momentum conservation sum-rules in which the total pp cross-section is given as a sum of integrals over the single particle distributions of $p + p \to i + X$ where mainly $i = p, \pi, n$. We have here an interesting case in which the discussed theoretical model is naturally suitable for a process which is difficult to measure (i.e., $p + p \to n + X$) and is not directly applicable to the measured processes (i.e., $p + p \to p + X$ and $p + p \to \pi + X$). Such a case shows the importance in applying inclusive sum-rules for relating measurable reactions to an unmeasured process for which a theoretical model is directly applicable.

The kinematics of $p + p \to n + X$ is given in fig. 1. The following invariant variables will be used;

$$s = (p_1 + p_2)^2, \qquad t = (p_1 - q)^2, \qquad M^2 = (p_1 + p_2 - q)^2 . \tag{1}$$

These variables are related to the energy E and longitudinal momentum q_L, of the outgoing neutron in the center of mass, by

$$\frac{2E}{(s)^{\frac{1}{2}}} = 1 + \frac{m^2}{s} - \frac{M^2}{s} \approx 1 - \frac{M^2}{s} \tag{2a}$$

and for large q_L

$$x \equiv \frac{2q_L}{(s)^{\frac{1}{2}}} \approx 1 - \frac{M^2}{s} \tag{2b}$$

where for $0.5 \lesssim x \lesssim 1$

$$t \approx - \frac{m^2(1 - x)^2 + q_T^{\;2}}{x} . \tag{2c}$$

Here m and q_T are, respectively, the nucleon mass and the transverse momentum of the emitted neutron.

It was argued in ref. 12 that the pion exchange in $p + p \to n + X$ is the dominant mechanism since ρ and A_2 are weakly coupled to nucleons. As a direct result of the pion-pole dominance, prominent features were predicted [12], namely, a strong dip at the end of the spectrum $(x \lesssim 1)$ and a peak at a certain point in the phase-space $(x \approx 0.81$ for $q_T = 0.1$ GeV/c). A recent [13] preliminary data on $p + Be \to n + X$ at 24 GeV/c is in striking agreement with the above predictions.

Detailed [14] calculations, for $p + p \to n + X$, based on the ABFST model with the diagrams in fig. 2 (however, with no off-shell corrections) have confirmed the aforementioned predictions. In particular it has been shown [14] explicitly that the diagram in fig. 2b is not as important as the one in fig. 2a, in accordance with the analysis of ref. 12.

The main purpose of the present work is to show how inclusive sum-rules constrain the off-shell behavior in the ABFST peripheral model. Of course one cannot derive, from inclusive sum-rules only, a specific form for the off-shell extrapolation. Therefore the off-shell behavior will be introduced within the framework of the ABFST model [9,15]. However, as we shall see, the parameter involved [15] in the ABFST off-shell continuation is strongly constrained by the inclusive sum-rules.

We shall show that, although the ABFST model is mostly directly applicable to the process $p + p \to n + X$ (see ref. 14 and footnote in ref. 16), which is very difficult to measure, a constraint on the model can be obtained through the use of inclusive sum-rules and the measured reactions $p + p \to i + X$ $(i = p, \pi)$.

The diagrams relevant to the present model are given in fig. 2. The kinematical variables are given in eqs. (1) and (2) and $s' = (q + q')^2$ is a variable eventually integrated upon.

Consistent with the ABFST [9] model, the pion trajectory is taken to have a zero slope. Then the contribution of the diagram in fig. 2a to the limiting inclusive spectrum of $p + p \to n + X$ is

$$\frac{d^2 \sigma_n}{dt \; d\, M^2/s} = \frac{1}{4\pi} \frac{2g^2}{4\pi} \frac{-t}{(t - \mu^2)^2} \frac{M^2}{s} \sigma_T^{"\pi"p}(t) \qquad (3)$$

where "π" means a pion off the mass shell and $\sigma_T^{"\pi"p}(t)$ is the asymptotic "total cross-section" with the pion having a "mass" t. Also $g^2/4\pi \approx 15$ is the πNN coupling strength. The diagram in fig. 2b is not important relative to the one in fig. 2a and therefore will not be considered here [12,14].

As remarked above, the inclusive sum-rules cannot determine a specific form for the dependence of $\sigma_T^{"\pi"p}(t)$ on t. In order to work within a consistent dynamical framework, we employ a form which is naturally suggested [9,15] by the ABFST model, namely

$$\sigma_T^{"\pi"p}(t) = G(t)\, \sigma_T^{\pi p} \qquad (4)$$

with

$$G(t) = \left(\frac{t_0 - \mu^2}{t_0 - t}\right)^{\bar{\alpha}+1} \qquad (5)$$

and $\sigma_T^{\pi p} \approx 25$ mb is the asymptotic on-shell πp total cross-section. In eq. (5), t_0 is the parameter to be determined from the inclusive sum-rules and $\bar{\alpha}$ is the maximal output trajectory eigenvalue in the forward multiperipheral integral equation. Following the authors of ref. 10, we shall take $\bar{\alpha} = 0.7$.

We shall now give arguments for the importance of the mechanism depicted in fig. 2a in a rather large fraction of phase space. Near the kinematical boundary, where s/M^2 is large (say, $0.9 \lesssim x \lesssim 1$), one might think that the pion exchange is not important due to its low intercept. However, since the $\pi^+ np$ coupling is large [note the factor 2 in eq. (3)] and other exchanges (ρ and A_2) are weakly coupled, the pion will dominate even there (except very near $x = 1$ where also the other contributions are not crucial). Indeed the data [13] shows a pronounced dip at $x \lesssim 1$ which clearly support pion-pole dominance. Now, for $0.5 \lesssim x$, the subenergy between the neutron and its nearest neighbor in X (see fig. 2a) is small and therefore low lying exchanges will dominate. This justifies the pion-pole dominance in the region [19] $x \gtrsim 0.5$ or $s/M^2 \gtrsim 2$. In the region $x < 0.5$, presumably another mechanism is in play. However, since the production of nucleons decreases with decreasing x $(x < 0.5)$, the region $0.5 \leq x \lesssim 1$ will be the most important, at least as long as integrated quantities are concerned. Moreover one can assume that the picture given in fig. 2a will, on the average, describe the events with small x as well.

The simplest inclusive sum-rules are those derived from energy-momentum conservation. In the general case, where the initial particles are not identical, one should take the limit $s \to \infty$ in order to be able to write down separate sum-rules for forward $(x > 0)$ and backward $(x < 0)$ fragments. For definiteness we consider the forward fragments sum-rule which reads

$$\sigma_T(pp) = \sum_i \int \frac{d^2 \sigma_i}{dt \; d \; M^2/s} \left(1 - \frac{M^2}{s}\right) dt \; d \; M^2/s \qquad (6)$$

with the integration over only half of the phase-space $(x > 0)$ and the summation is on the various types of the outgoing stable particles. Also $d^2 \sigma_i/dt \; d \; M^2/s$ is the invariant inclusive distribution function of the particle i produced in $p + p \to i + X$. The most important terms in the right-hand side (r.h.s.) of eq. (6) are for $i = p,n,\pi$. Hence we can write the following approximate relation;

$$\int \frac{d^2 \sigma_n}{dt \; d \; M^2/s} (1 - M^2/s) \; dt \; d \; M^2/s$$

$$\approx \sigma_T(pp) - \sum_{i=p,\pi} \int \frac{d^2 \sigma_i}{dt \; d \; M^2/s} (1 - M^2/s) \; dt \; d \; M^2/s \; . \qquad (7)$$

Note that the elastic peak is included in the r.h.s. of eqs. (6) and (7).

In the left-hand side (l.h.s.) of (7) we have a process which is hard to measure, namely $p + p \to n + X$, for which, however, the theoretical model is mostly applicable. The model cannot be unambiguously applied to the processes appearing in the r.h.s. of (7), but,

however, they are relatively easier to measure [20], as is generally the case for single-particle spectra. Thus eq. (7) provides an important constraint on the ABFST model (applied to $p + p \to n + X$), namely on the only free parameter t_0 [see eqs. (4) and (5)] which determines the scale for the off-shell extrapolation.

The numerical evaluation of t_0 will now be given. The physical meaning, of the integral on the r.h.s. of eq. (7), is the average fractional energy $\left\langle \frac{E}{(s)^{\frac{1}{2}}} \right\rangle$ [in units of $\sigma_T(pp)$] carried by the corresponding species produced in high-energy pp collisions. Such quantities were known a long time ago to cosmic-ray physicists and an independent analysis was performed by Bali et al.[21], using accelerator data, in agreement with the cosmic-ray results. For $i = p$ (i.e., $p + p \to p + X$) the corresponding term in the r.h.s. of (7) is $\sim 0.5 \; \sigma_T(pp)$ and for $i = \pi$ it is $\sim 0.3 \; \sigma_T(pp)$. If in addition other kinds of produced particles (e.g., $\bar{p}$, K, Λ, etc.) are assumed to contribute $\sim 0.05 \; \sigma_T(pp)$, then we can write

$$\int \frac{d^2 \sigma_n}{dt \; d \; M^2/s} (1 - M^2/s) \; dt \; d \; M^2/s \approx 0.15 \; \sigma_T(pp) \approx 5.7 \text{ mb} \qquad (8)$$

for $\sigma_T(pp) \approx 38$ mb. Note that the estimate in (8) is very close to the rough value quoted by Chou and Yang [3].

For the calculation of the l.h.s. of eq. (8), we consider only the diagram in fig. 2a [its contribution is given in eq. (3)]. Presumably other mechanisms are present leading to a small contribution, e.g., the mechanism depicted in fig. 2b. Therefore the value of t_0, to be determined from the inclusive constraint in (8) (see below), must be considered as an <u>upper bound</u>.

The l.h.s. of eq. (8) is evaluated as a function of the parameter t_0 [using eqs. (3), (4), and (5)] with the result displayed in fig. 3. From fig. 3 and the inclusive constraint in (8) we then find $\underline{t_0 \approx 0.7 \text{ GeV}^2}$.

With the same parameter $t_0 \approx 0.7$ GeV^2, we have calculated the quantity [22]

$$\frac{1}{\sigma_{inel}(pp)} \int \frac{d^2 \sigma_n}{dt\, d\, M^2/s} (1 - M^2/s)^{1.7} \, dt \, d\, M^2/s \, ,$$

which appears in the spectra of cosmic-ray nucleons in the atmosphere. The obtained value is ~ 0.14 to be compared with $(0.081 - 0.185)$ as derived from cosmic-ray measurements.

Since the inclusive sum-rules contain integrated quantities, they in fact serve to determine the normalization which depends sensitively on t_0. As a result, a model without [14] off-shell corrections $(t_0 \to \infty)$, although predicting the gross features in the $p + p \to n + X$ spectrum (namely a dip at $x \lesssim 1$ and a peak at $x \approx 0.81$ for $q_T = 0.1$ GeV/c), overestimates the normalization.

When better data for $p + p \to n + X$ are available and a direct test of the model is possible, the inclusive sum-rules will still provide an additional useful and independent constraint on the model.

The author would like to thank C. F. Chan and C. Sorensen for useful discussions on the ABFST model. Thanks also go to M. B. Einhorn and H. J. Yesian for reading the manuscript.

FOOTNOTES AND REFERENCES

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16. If a proton is detected then a complication will arise due to the pomeranchon hidden in the upper blob in fig. 2b (see ref. 14). Moreover, only a P' is generated in that blob but not the ω, which can be a serious drawback of the model [17,18]. Also reflections, from diffractively produced resonating states in the missing mass (with P exchange in fig. 2a, instead of π), are present. All these complications are not present in $p + p \to n + X$. Indeed here there is no pomeranchon exchange in the upper blob in fig. 2b and no reflections from resonating missing-masses in fig. 2a in the limit of infinite incident energy. Also for $p + p \to n + X$ the decoupling of A_2 (from the upper blob in fig. 2b) is not a serious drawback of the model because it couples weakly to nucleons.

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FIGURE CAPTIONS

Fig. 1. Kinematics of $p + p \to n + X$.

Fig. 2. The peripheral diagrams, relevant to the ABFST model, for the process $p + p \to n + X$. The four momentum q' (of the pion emitted from the upper blob in fig. 2b) is eventually integrated upon. Also M^2 is the missing mass squared and $s' = (q + q')^2$.

Fig. 3. The l.h.s. of eq. (8) is plotted for different values of the cut-off parameter t_0 [defined in eq. (5)].

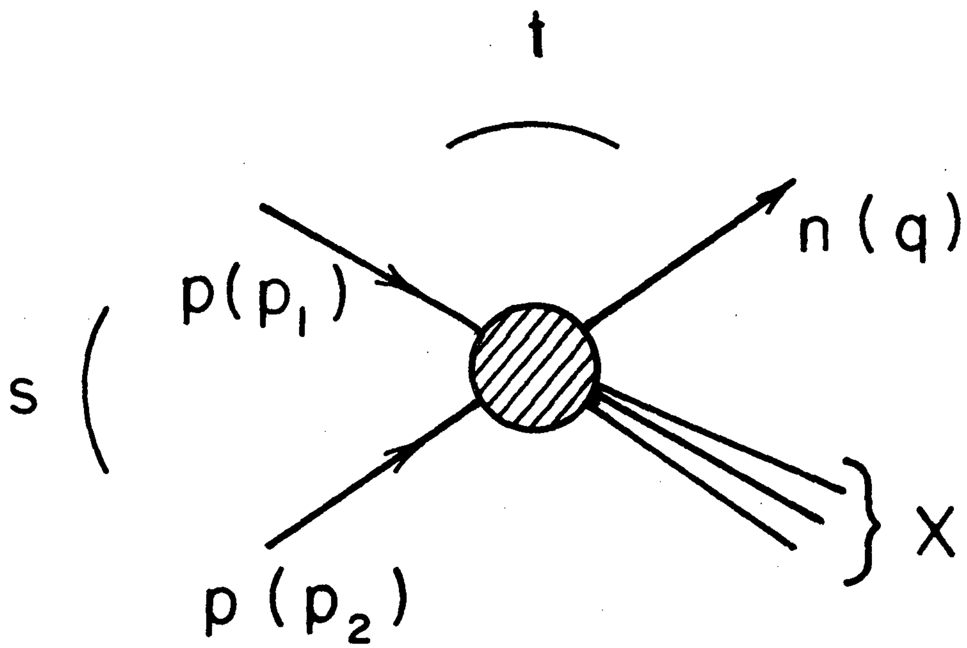

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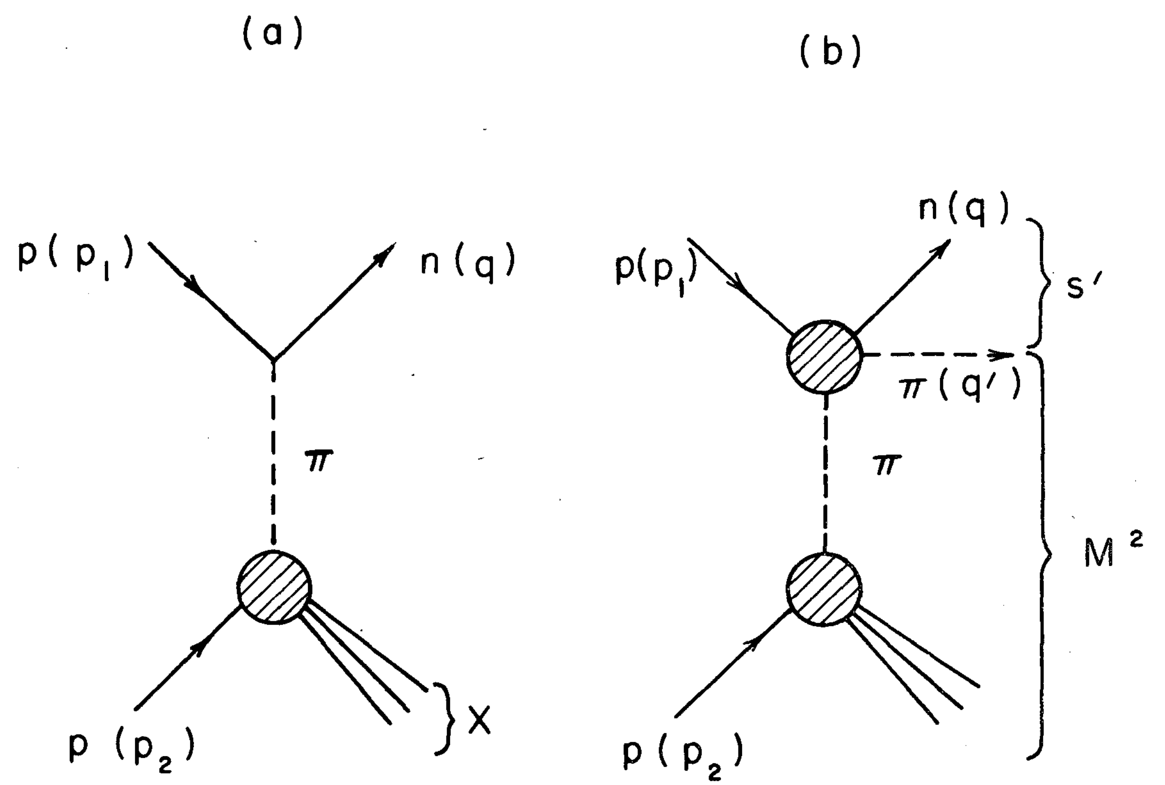
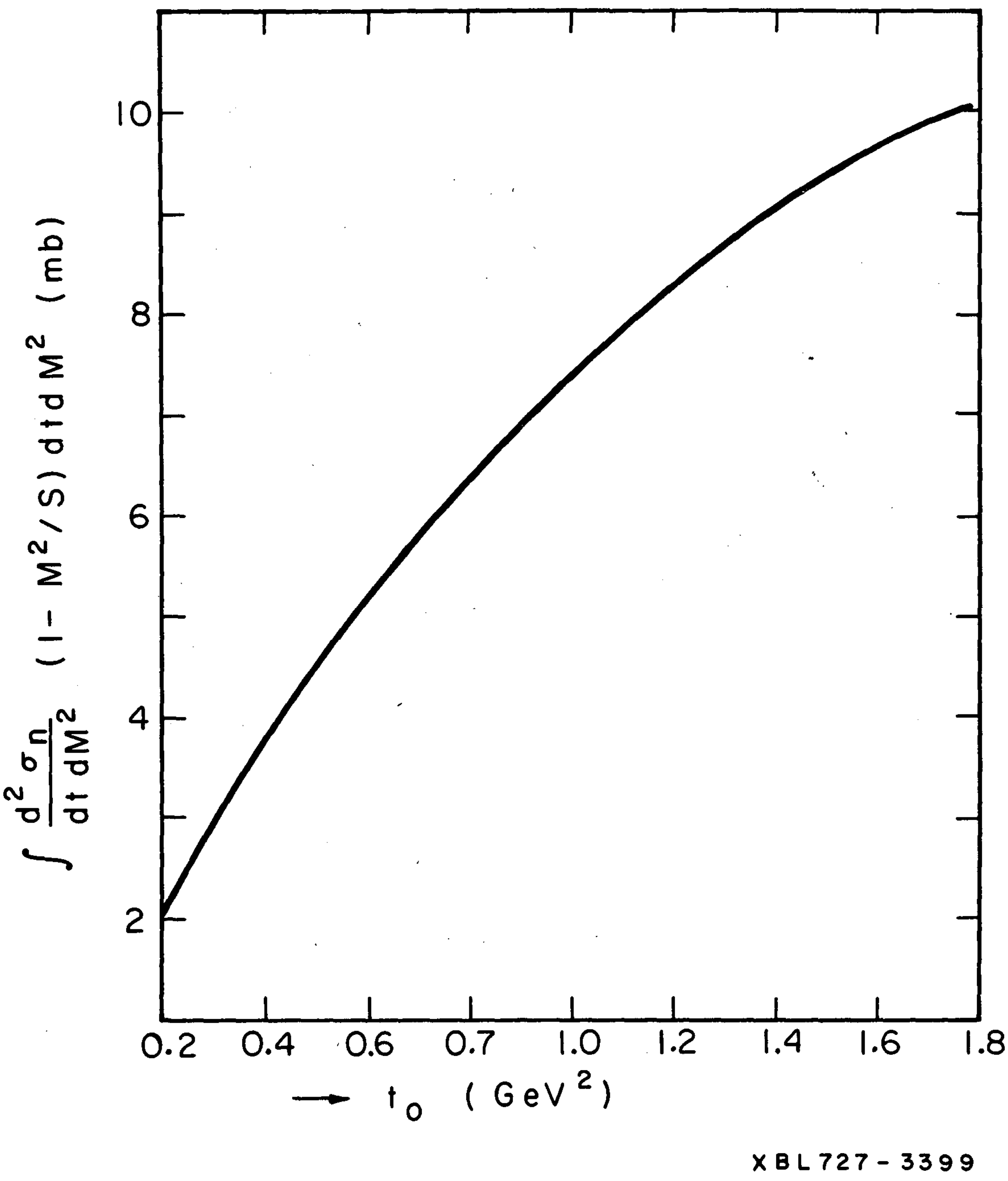
Fig. 1



XBL727 - 3398

Fig. 2



$\int \frac{d^2\sigma_n}{dt\,dM^2}\ (1 - M^2/S)\ dt\,dM^2$ (mb)
10
8
6
4
2
0.2
0.4
0.6
0.7
1.0
1.2
1.4
1.6
1.8
→ t_o (GeV^2)
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Fig. 3

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