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ABSTRACT

This paper describes regression analysis of certain types of experimental data obtained from geotechnical testing. By suitable transformation, nonlinear equations may be converted into linear equations if appropriate weighting factors are applied to the data before performing the regression analysis. Sample problems in BASIC which can be used with desktop computers are described. Typical examples from soil mechanics and rock mechanics are included.

KEYWORDS

regression, nonlinear functions, transformation, weighting factor, experimental data, soil and rock mechanics.

REGRESSION ANALYSIS OF EXPERIMENTAL DATA USING DESKTOP COMPUTERS

Douglas Frink[1] and Panchanatham N. Sundaram[1]

Introduction

Regression in a broad sense connotes the functional relationship between two or more variables. In the case of two variables, say x and y, the relationship may be either linear or nonlinear resulting respectively in a regression line or regression curve. In many experiments, one variable x is the cause, in part at least, of the variation in the other variable y.

In scientific experiments, one or both of the variables x and y may be subject to a certain amount of unpredictable variations often called "scatter." In geotechnical tests, performed either in the laboratory or in the field, scatter in the derived data is more of a rule than exception. If the data is plotted on a graph sheet the problem becomes that of fitting a line or curve to the data points. It has been customary for practicing engineers to draw this curve by "eye judgement." However, engineering organizations are increasingly using desktop computers and computer graphics equipment to perform engineering computations. These devices can assist the engineer to select the mathematical function best suited for the analysis of experimental data, provided suitable software is available. This calls for the use of appropriate statistical procedures for handling the experimental data. The purpose of this paper is to introduce simple programs in BASIC to perform regression analysis of certain types of experimental data. The programs presented were

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written specifically for use in HP9845 series computers. However, with appropriate modifications, the sample programs may be used in other computers which use BASIC.

Theoretical Basis

Linear Functions. Suppose that a function is to be fit to a set of data pairs, x_i and y_i. The method of least-squares is often used to get the "best fit" to the data set. According to this method, if f(x) is the best fit then the quantity "chi squared," should be minimum and may be given as

$$\lambda^2 = \sum_{i=1}^{N} \{y_i - f(x_i)\}^2 w_i, \tag{1}$$

where w_i is the weighting factor to be applied to the values of y_i. In the absence of any information, $w_i = 1$. Normally, w_i is inversely proportional to the variance of y_i. (Note: this approach assumes that most of the scatter occurs in y_i.)

As an example, we assume that

$$f(x) = a + bx . \tag{2}$$

Therefore,

$$\chi^2 = \Sigma\{y_i - (a + bx_i)\}^2 w_i . \tag{3}$$

The condition for the minimum value of χ^2 is satisfied by the differential equations called Normal Equations, viz.,

$$\frac{\partial\chi^2}{\partial a} = \sum_{i=1}^{N} (y_i - a - bx_i)w_i = 0, \tag{4}$$

$$\frac{\partial \chi^2}{\partial b} = \sum_{i=1}^{N} (y_i - a - bx_i)w_i x_i = 0. \tag{5}$$

Equations (4) and (5) are linear in the two unknowns a and b. Matrix formulation of the two equations may be given as:

$$\begin{bmatrix} \Sigma w_i & \Sigma x_i w_i \\ \Sigma x_i w_i & \Sigma x_i^2 w_i \end{bmatrix} \begin{Bmatrix} a \\ b \end{Bmatrix} = \begin{Bmatrix} \Sigma y_i w_i \\ \Sigma x_i y_i w_i \end{Bmatrix} \tag{6}$$

or

$$[A](a) = (c) \tag{7}$$

or

$$(a) = [A]^{-1}(c). \tag{8}$$

Thus equations (4) and (5) may be solved by simple algebraic methods or by matrix inversion using equation (8).

This method of <u>linear least squares</u> can be extended to a polynomial of of the form:

$$y = a_1 + a_2 x + a_3 x^2 + \ldots a_{n+1} x^n. \tag{9}$$

Since there are n + 1 unknowns, the method of linear least squares will result in (n + 1) linear algebraic equations. When n exceeds 3, it is customary to use matrix-inversion techniques to solve for the unkowns. Many desktop computers have built-in functions to perform matrix inversion.

<u>Nonlinear Functions</u>. There are functions for which the normal equations are not linear functions of the unknown parameters. Let us take for example the hyperbolic equation which is often used to fit axial or deviatoric stress and

axial strain data for soils,

$$y = \frac{x}{a + bx} . \tag{10}$$

Thus,

$$\chi^2 = \Sigma \left\{ y_i - \frac{x_i}{a + bx_i} \right\}^2 w_i \tag{11}$$

The normal equations are:

$$\Sigma \left(y_i - \frac{x_i}{a + bx_i} \right) \frac{x_i w_i}{(a + bx_i)^2} = 0 \tag{12}$$

$$\Sigma \left(y_i - \frac{x_i}{a + bx_i} \right) \frac{x_i^2 w_i}{(a + bx_i)^2} = 0 \tag{13}$$

Equations (12) and (13) cannot be easily solved. However, if we transform equation (10) into

$$Y = aX + b \tag{14}$$

where $Y = 1/y$ and $X = 1/x$, then:

$$\chi^2 = \Sigma(Y_i - b - aX_i)^2 w_i' = 0 \tag{15}$$

where w_i' is the <u>transformed</u> weighting factor. In general, if a transformation function T is applied to y_i, then the weighting factor w_i should be transformed to w_i', using equation (1) and the methods described by Guest [1], so that,

$$w_i' = w_i [1/(\partial T/\partial y_i)^2]. \tag{16}$$

In this case,

$$T = \frac{1}{Y_i} \tag{17}$$

so that

$$w_i' = Y_i^4 w_i \tag{18a}$$

or

$$w_i' = Y_i^4 \tag{18b}$$

since $w_i = 1$ for all i.

The effect of the transformed weighting factor w_i' is to make the values of the unknown parameters obtained by minimizing equation (15) as close as possible to the values obtained by minimizing equation (11). However, the normal equations from equation (15) are linear and thus much simpler to solve than the normal equations (12) and (13). Substituting equation (18a) in (15) and finding the differential coefficients with respect to a and b respectively, we have

$$\frac{\partial \chi^2}{\partial a} = \Sigma(Y_i - b - aX_i)\frac{X_i}{Y_i^4} = 0 \tag{19}$$

and

$$\frac{\partial \chi^2}{\partial b} = \Sigma(Y_i - b - aX_i)\frac{1}{Y_i^4} = 0. \tag{20}$$

Normal equations (19) and (20) are linear in a and b and can be easily solved. However, it is important to remember that whenever a transformation is made of the original function then the weighting factor should also be correspondingly changed.

Another common function used in curve-fitting is the power law of the form,

$$y = ax^n \tag{21}$$

Taking logarithms

$$\log y = \log a + n \log x \tag{22}$$

then

$$\chi^2 = \Sigma\{\log y_i - (\log a + n \log x_i)\}^2 w_i' \tag{23}$$

where

$$w_i' = y_i^2 w_i.$$

Equation (23) results in linear normal equations.

A slight variation of equation (21) is of the form,

$$y = a\ (x - b)^n \tag{24}$$

where a, b and n are the unknown parameters. Proceeding as before;

$$\log y = \log a + n \log (x - b) \tag{25}$$

or

$$Y = A + nX \tag{26}$$

where

$$Y = \log y; \quad A = \log a \quad \text{and} \quad X = \log(x - b) \tag{27}$$

This gives

$$\chi^2 = \Sigma(Y_i - A - nX_i)^2 w_i'. \tag{28}$$

Differentiating the "chi-squared" equation (28) with respect to A and n respectively, yields the following normal equations:

$$\Sigma(Y_i - A - nX_i)w_i' = 0 \qquad (29)$$

and

$$\Sigma(Y_i - A - nX_i)w_i'x_i = 0. \qquad (30)$$

These two equations are linear. However, the third normal equation, i.e., differentiating equation (28) with respect to b, yields

$$\Sigma(Y_i - A - nX_i)w_i'/(x_i - b) = 0 \qquad (31)$$

and is not linear in b. Thus equations (29) through (31) cannot be easily solved for the unknowns A, n and b. An efficient method amenable to computer usage is called the Secant method (or Newton's method), and is explained below.

A trial value for b, say b_1, is assumed. Using this value, the magnitudes of a and n are obtained by solving equations (29) and (30). Using these values of a, n and b_1, the derivative $\partial\chi^2/\partial b$ (eq. 31) is evaluated as d_1. If b_1 is the exact solution, then d_1 will be zero and no further computation is necessary. Otherwise, the value of b_1 is either increased or decreased by a small amount and a new value of the derivative (eq. 31) is determined as d_2. The secant slope is given as

$$s = \frac{d_2 - d_1}{b_2 - b_1}. \qquad (32)$$

A new value for b_3 may then be estimated by extrapolating the slope to the b axis as

$$b_3 = b_2 - \frac{d_2}{s}. \qquad (33)$$

In general,

$$b(i + 2) = b(i + 1) - d(i + 1)\ \frac{b(i + 1) - b(i)}{d(i + 1) - d(i)}\ . \qquad (34)$$

The process is repeated until the values of a, b and n all satisfy the three normal equations (29) through (31) to the desired accuracy.

There are many other types of functions to which linear regression analysis may be applied after proper transformations. Due to space limitations, only a limited number can be discussed here. Table 1 is a summary of the different steps in the regression analysis for the types of fitting functions discussed in this paper.

Programs in BASIC

The unknowns in the normal functions given in Table 1 can be efficiently solved by matrix-inversion technique. This may be accomplished by the use of Subroutine Poly listed on Table 2. This routine is specifically written for HP9845 series computers; however, with suitable modification it can be incorporated in other computer systems that use BASIC language. In this subroutine, the polynomial function has been described in the most general form as

$$y = \sum_{i=1}^{N} a_{m+i-1} x^{m+i-1} \qquad (35)$$

where m is either a positive or negative integer. This general function can be degraded to specific forms by proper choice of m and N. For example,

$$m = 0 \text{ and } N = 2 \text{ leads to } y = a_o + a_1 x \qquad (36)$$

$$m = 3 \text{ and } N = 1 \text{ leads to } y = a_3 x^3 \qquad (37)$$

and

$$m = -1 \text{ and } N = 2 \text{ leads to } 1/y = a_{-1}x^{-1} + a_o. \quad (38)$$

In the case of logarithmic transformation, Equation (27) can be used and thus the subroutine Poly can be used for the linear regression analysis by specifying $m = 0$ and $N = 2$. The factors m and N are identified as "Omin" and "Ncon" respectively in subroutine Poly. The purpose of the variable "Flg" in the subroutine is to identify the type of transformation done on the weighting factor, w_i.

Tables 3 and 4 give partial listings of two programs named respectively, INVERT and SECANT, that call subroutines Poly. To conserve space, only the portions of the programs that call Poly and illustrate the application of the mathematics are included in the listings. The unlisted positions are graphics-control statements used for the author's specific applications. Program INVERT uses reciprocal transformation procedures in the curve-fitting procedure. In program SECANT, subroutine Poly is used with the Secant method. Users may incorporate the statements on Tables 3 and 4 (or equivalent statements in languages other than HP-BASIC) into their own data analysis programs.

Application Examples

Two examples, one from soil mechanics and the other from rock mechanics are presented to illustrate the use of INVERT and SECANT.

Example from a Soil Mechanics Laboratory Experiment. The nonlinear stress-strain behavior of soils is frequently characterized by a hyperbolic relationship between deviatoric stress and axial strain [2], that has the form of equation (38). The strength parameters from such a fit are used in finite-element analysis of soil behavior under gravity and applied loads [3]. A typical example of the deviatoric stress versus axial strain data from consolidated undrained triaxial compression test of a medium-dense cohesionless sand [4] is plotted on Figure 1. The hyperbolic law (eq. 10 or 38) was fitted to the data points using program INVERT which calls subroutine Poly. The plot of the hyperbolic curve fitted to the data points is shown as Curve 1 on Figure 1. The maximum deviatoric stress, $(\sigma_1 - \sigma_3)_{ult}$, projected by the curve fitting is equal to '1/a' and is 11.19 MPa. Thus, the failure ratio, R_f, [2] is given by

$$R_f = \frac{(\sigma_1 - \sigma_3)_f}{(\sigma_1 - \sigma_3)_{ult}} = \frac{9.50}{11.19} = 0.817 \qquad (39)$$

where $(\sigma_1 - \sigma_3)_f$ is the compressive strength and is obtained by inspection from the data points on Figure 1. When R_f takes the value 1.0 the hyperbolic curve fitted through the data matches the empirical data at failure. However, for the particular data shown on Figure 1, the hyperbolic curve is a relatively poor match of the stress-strain behavior at failure.

Program INVERT can be used to fit the data with a higher order equation of the type

$$y = \frac{x}{a + bx + cx^2} \qquad (40)$$

or as per equation (35), with m = -1 and N = 3, we have

$$\frac{1}{y} = a_{-1}x^{-1} + a_o + a_1x. \qquad (41)$$

Curve 2 on Figure 1 shows the results of such a high-order fit to the triaxial test data described above. Unlike the hyperbolic curve described previously, this fit does not give an asymptotic value for the deviatoric stress but gives a peak value of $(\sigma_1 - \sigma_3)_f$ defined by

$$y_{max} = \frac{1}{b + s\sqrt{ac}} \tag{42}$$

which occurs at $x = \sqrt{ac}$. Using equation (42) the peak or failure deviatoric stress is given as 9.19 MPa. This is very close to the value 9.15 MPa taken from the empirical data. The axial strain at which the deviatoric stress peaks is given as 0.235, which is somewhat greater than the value of 0.15 actually observed during the test. The foregoing example demonstrates how computer techniques can be used to find a function that best fits a set of empirical data.

Example from a rock mechanics laboratory experiment. The relationship between effective normal stress (σ_{eff}) and the normal closure (δ) of a fracture may be expressed by a hyperbolic law of the form [5]

$$\sigma_{eff} = a\left(\frac{\delta}{\delta_{max} - \delta}\right)^n \tag{43}$$

where δ_{max} is the anticipated maximum closure. Since this value of maximum closure cannot be determined directly from the experimental data, it must be estimated as a by-product of the curve-fitting process.

Equation (43) is not amenable to straight forward linear regression even after substituting $y = \sigma$ and $x = \delta/(\delta_{max} - \delta)$ since the term x contains the unknown parameter δ_{max}. However, program SECANT enables the determination of three unknowns a, n and δ_{max}. Figure 2 is a plot of some typical normal stress (σ) versus average fracture deformation (δ) data taken from laboratory tests on granite [6]. As shown on the figure, the curve fitting performed using SECANT gives a value of b, the initial aperture, of 54.919 microns. This permits estimates of absolute fracture closure to be made from the experimental measurements.

Conclusions

Simple programs in BASIC have been presented that enable regression analysis of geotechnical data to be performed using the method of linear least-squares in cases where the functions are nonlinear. In performing the linearization it is important to apply the proper weighting factor to the data. Whenever the original function is transformed to linear form, the weighting factor must also be suitably transformed. The programs described perform these transformations and give results which are statistically consistent.

By applying the mathematical principals discussed in the paper, programs for handling other types of functions can be developed. Programs of this type, that use the appropriate statistical procedures, can be used to rapidly determine the form of the curve that is best suited to the analysis of experimental data used in estimating the properties of geologic materials.

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Table 1. Details of Regression Parameters.

Function $y = f(x)$	Transformed Function $Y = F(X)$	χ^2	Normal Equations
$y = a + bx$	...	$\Sigma[y_i - (a + bx_i)]^2$	$\Sigma(y_i - a - bx_i) = 0$ $\Sigma(y_i - a - bx_i)x_i = 0$
$y = a_i + a_w x + \ldots + a_n x^n$	...	$\Sigma[y_i - (a_1 + a_2 x + \ldots + a_n x^n)]^2$	$\Sigma(y_i - a_1 - a_2 x_i - \ldots - a_n x_i^n) = 0$ $\Sigma(y_i - a_1 - a_2 x_i - \ldots - a_n x_i^n)x_i = 0$ etc.
$y = \frac{x}{a + bx}$	$Y - aX + b$ where $Y = 1/y$ and $X = 1/x$	$\Sigma(Y_i - b - aX_i)^2(1/Y_i)^4$	$\Sigma(Y_i - b - aX_i)(X_i/Y_i^4) = 0$ $\Sigma(Y_i - b - aX_i)(1/Y_i^4) = 0$
$Y = ax^n$	$Y = A + nX$ where $Y = \log y$ or $\ln y$ $X = \log x$ or $\ln x$ $A = \log a$ or $\ln a$	$\Sigma[Y_i - (A + nX)]^2(y_i^2)$	$\Sigma[Y_i - (A + nX_i)]y_i^2 = 0$ $\Sigma[Y_i - (A + nX_i)]y_i^2 X_i = 0$
$y = a(x - b)^n$	$Y = A + nX$ where $Y = \log y$ or $\ln y$ $X = \log(x - b)$ or $\ln(x - b)$ $A = \log a$ or $\ln a$	$\chi^2 = (Y_i - A - nX_i)^2(1/y_i)$	$\Sigma(Y_i - A - nX)(y_i^2) = 0$ $\Sigma(Y_i - A - nX_i)(X_i y_i^2) = 0$ $\Sigma(y_i - A - nX_i)[y_i^2/(X_i - b)] = 0$

TABLE 2

LISTING OF THE PROGRAM Poly

```
10      ! ******************
20      !
30      SUB Poly(X(*),Y(*),W(*),Npt,Ncon,Omin,Flg,R(*))
40      !
50      ! ******************
60 Poly:   ! SUB-PROGRAM FOR LINEAR LEAST SQUARES FIT FOR POLYNOMIAL
70      ! MAY BE MODIFIED TO A GOSUB ROUTINE.
80      ! INPUT ARRAYS:
90      !                    X(*) - X DATA
100     !                    Y(*) - Y DATA
110     !                    X AND Y ARE EITHER TRANSFORMED OR ORIGINAL DATA
120     !                    W(*) - ASSIGNED WEIGHTS
130     !                    IF W(I)=0 THEN W1 IS SET TO 1
140     ! INPUT VARIABLES:
150     !      Npt - NUMBER OF X,Y PAIRS
160     !      Ncon-NUMBER OF CONSTANTS IN POLYNOMIAL
170     !      Omin-SMALLEST ORDER TO BE USED IN POLYNOMIAL(EG 0,-1,-2,ETC)
180     !      Flg - FLAG FOR TRANSFORM (0=NO TRANSFORM)
190     !      IF ARRAYS X AND Y ARE THE LOGARITHMS OF THE DATA THEN USE
200     !              Flg=1 (BASE 10) OR Flg=2 (BASE e)
210     !      IF ARRAYS X AND Y ARE RECIPROCALS OF THE DATA USE Flg=3
220     ! OUTPUT:
230     !       R(*) - ARRAY HOLDING SOLUTION CONSTANTS
240     !       R(1)=COEFFICIENT OF SMALLEST POWER OF X; R(2) NEXT POWER, ETC
250     !
260     OPTION BASE 1
270     DIM A(Npt,Ncon),B(Npt),At(Ncon,Npt),C(Ncon,Ncon)
280     DIM D(Ncon),E(Ncon,Ncon),F(Ncon)
290     FOR I=1 TO Npt
300       W1=W(I)
310       IF W(I)=0 THEN W1=1  ! IF NO WEIGHTS ASSIGNED THEN ASSUME WEIGHT=1
320       W1=SQR(ABS(W1))                                  SQR
330       IF Flg=1 THEN W1=W1*10^Y(I)
340       IF Flg=2 THEN W1=W1*EXP(Y(I))
350       IF Flg=3 THEN W1=W1*1/(Y(I)*Y(I))
360       B(I)=Y(I)*W1
370       FOR J=1 TO Ncon
380         A(I,J)=X(I)^(Ncon-J+Omin)*W1
390       NEXT J
400     NEXT I
410     MAT At=TRN(A)
420     MAT C=At*A
430     MAT D=At*B
440     MAT E=INV(C)
450     MAT F=E*D
460     FOR I=1 TO Ncon
470       R(I)=F(Ncon+1-I)
480     NEXT I
490     SUBEND
```

TABLE 3

PARTIAL LISTING OF THE PROGRAM INVERT

```
10     ! ****************
20     !
30     ! PROGRAM: INVERT
40     ! PREPARED BY DOUG BRINK 12/15/81
50     ! LAWRENCE BERKELEY LABORATORY
60     ! WRITTEN SPECIFICALLY FOR HP 9845 SERIES
70     ! MAY HAVE TO BE MODIFIED FOR OTHER COMPUTERS
80     !
90     ! ****************
100    ! FITS AN EQUATION OF THE FORM y=x/(a + bx + cx^2 + etc)
110    ! TRANSFORMED TO Y=ax^(-1) + b + cx + etc WHERE Y=1/y
120    ! USES POLYNOMIAL FIT
130    OPTION BASE 1
140    DIM Sig(50),Delf(50),Eu(50),Delm(25),Result(25),A(50),X(50),Y(50),B(6)
150    FOR I=1 TO 50                      ! READ DATA
160      READ Sig(I),Delf(I)
170      IF Sig(I)<0 THEN Out
180    NEXT I
190 Out:  Npt=I-1
200      FOR I=1 TO Npt
210        X(I)=Delf(I)
220        Y(I)=1/Sig(I)
230      NEXT I
240      Ncon=3                ! SPECIFIES NO OF CONSTANTS
250      Omin=-1               ! LOWEST ORDER OF X
260      Flg=3                 ! USE RECIPROCAL TRANSFORM
270      CALL Poly(X(*),Y(*),A(*),Npt,Ncon,Omin,Flg,B(*))
280 Contin:  !
```

TABLE 4

PARTIAL LISTING OF THE PROGRAM SECANT

```
10    ! *************************
20    !
30    ! PROGRAM SECANT
40    ! WRITTEN SPECIFICALLY FOR HP 9845 SERIES
50    ! MAY HAVE TO BE MODIFIED FOR OTHER COMPUTERS
60    ! *************************
70    !
80    !
90    ! FITS AN EQUATION OF THE FORM Y=A(X/(B-X))^N
100   ! LOG TRANSFORMATION IS USED TO LINEARIZE THE FUNCTION.
110   ! SECANT METHOD IS USED TO SOLVE FOR B.
120   ! CONVERGENCE CRITERION FOR B IS BASED ON PERCENTAGE
130   ! DIFFERENCE BETWEEN SUCCESSIVE ITERATIONS AFTER
140   ! NORMAL EQ 3 CHANGES SIGN.
150   OPTION BASE 1
160   DIM Sig(50),Delf(50),B0(25),Result(25)
170   DIM W(50),X(50),Y(50),R(5)
180   Crit=.0001          ! CONVERGENCE CRITERION
190   FOR I=1 TO 50                  ! READ DATA
200     READ Sig(I),Delf(I)
210     IF Sig(I)<0 THEN Out
220     Dels=MAX(Delf(I),Dels)
230   NEXT I
240 Out:  Npt=I-1
250   B0(1)=Dels*1.02
260   Flg=1                    ! SPECIFIES LOG TRANSFORM
270   Ncon=2                   ! NO OF CONSTANTS
280   Omin=0                   ! SMALLEST ORDER OF X
290   FOR M=1 TO 25
300     FOR I=1 TO Npt
310       X(I)=LGT(Delf(I)/(B0(M)-Delf(I)))
320       Y(I)=LGT(Sig(I))
330     NEXT I
340     !
350     CALL Poly(X(*),Y(*),W(*),Npt,Ncon,Omin,Flg,R(*))
360     !
370     A=R(1)
380     N=R(2)
390     GOSUB Deriv         ! CALCULATE DERIVATIVE
400     Result(M)=Der       ! NORMAL EQ 3
410     PRINT "RESULT,DELMX";Result(M);B0(M)
420     IF M=1 THEN Init
430     Ratio=Result(M)/Result(M-1)
440     IF (ABS((B0(M)-B0(M-1))/B0(M-1))<Crit) AND (Ratio<0) THEN Contin
450     Slope=(Result(M-1)-Result(M))/(B0(M-1)-B0(M))
460     B0(M+1)=B0(M)-Result(M)/Slope
470     GOTO Cont
480 Init:  B0(M+1)=1.05*B0(M)
490 Cont:  NEXT M
500   PRINT " NON-CONVERGENT"
510   STOP
520 Deriv:  Der=0
530   FOR I=1 TO Npt
540     W=W(I)
550     IF W=0 THEN W=1
560     IF Flg=1 THEN W=W*10^Y(I)*10^Y(I)
570     Der=Der+(Y(I)-A-N*X(I))/(B0(M)-Delf(I))*W
580   NEXT I
590   RETURN
600 Contin:  Delmx=B0(M)
```

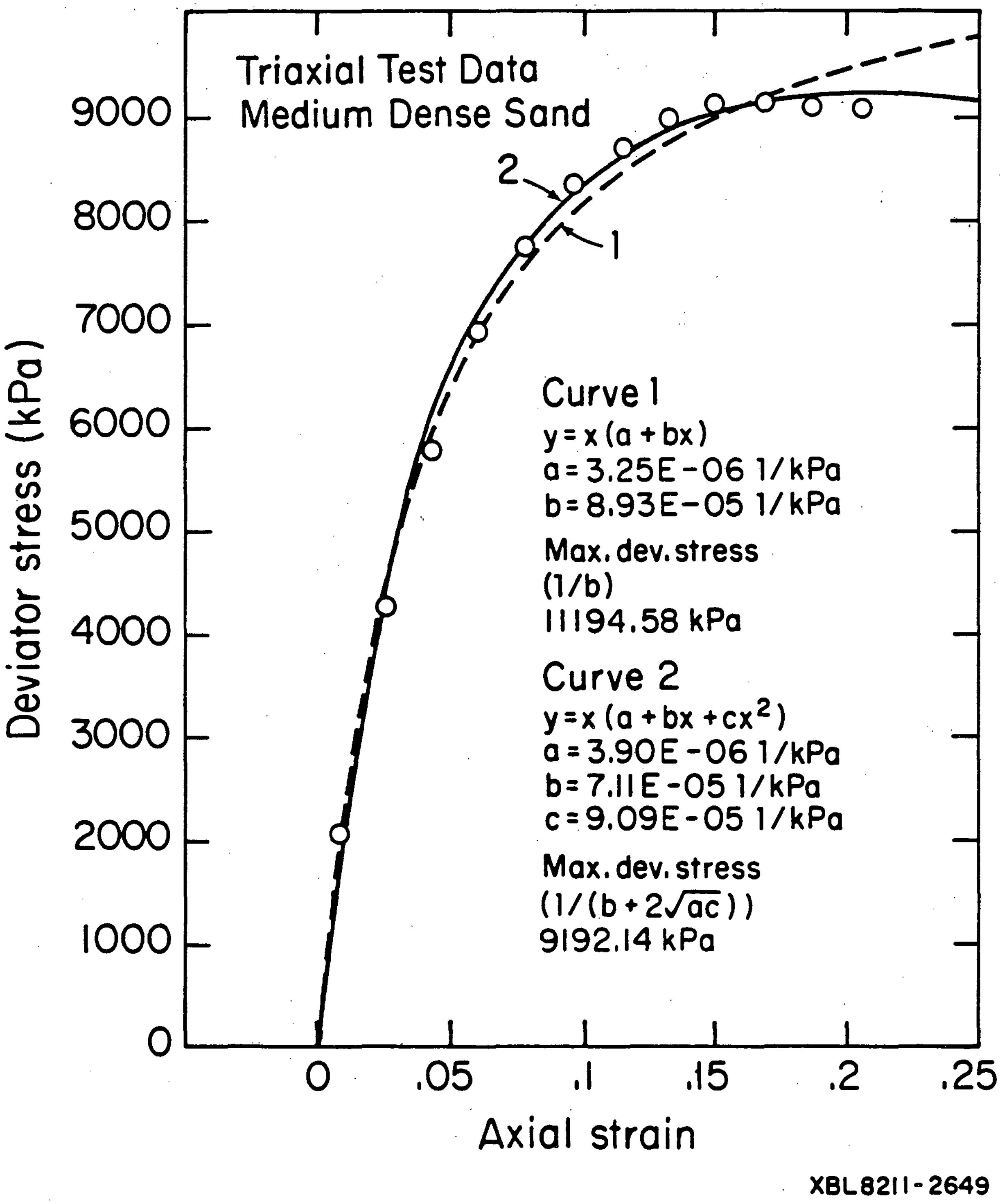

Figure 1. Curves Fit to Triaxial Test Data for Sand.

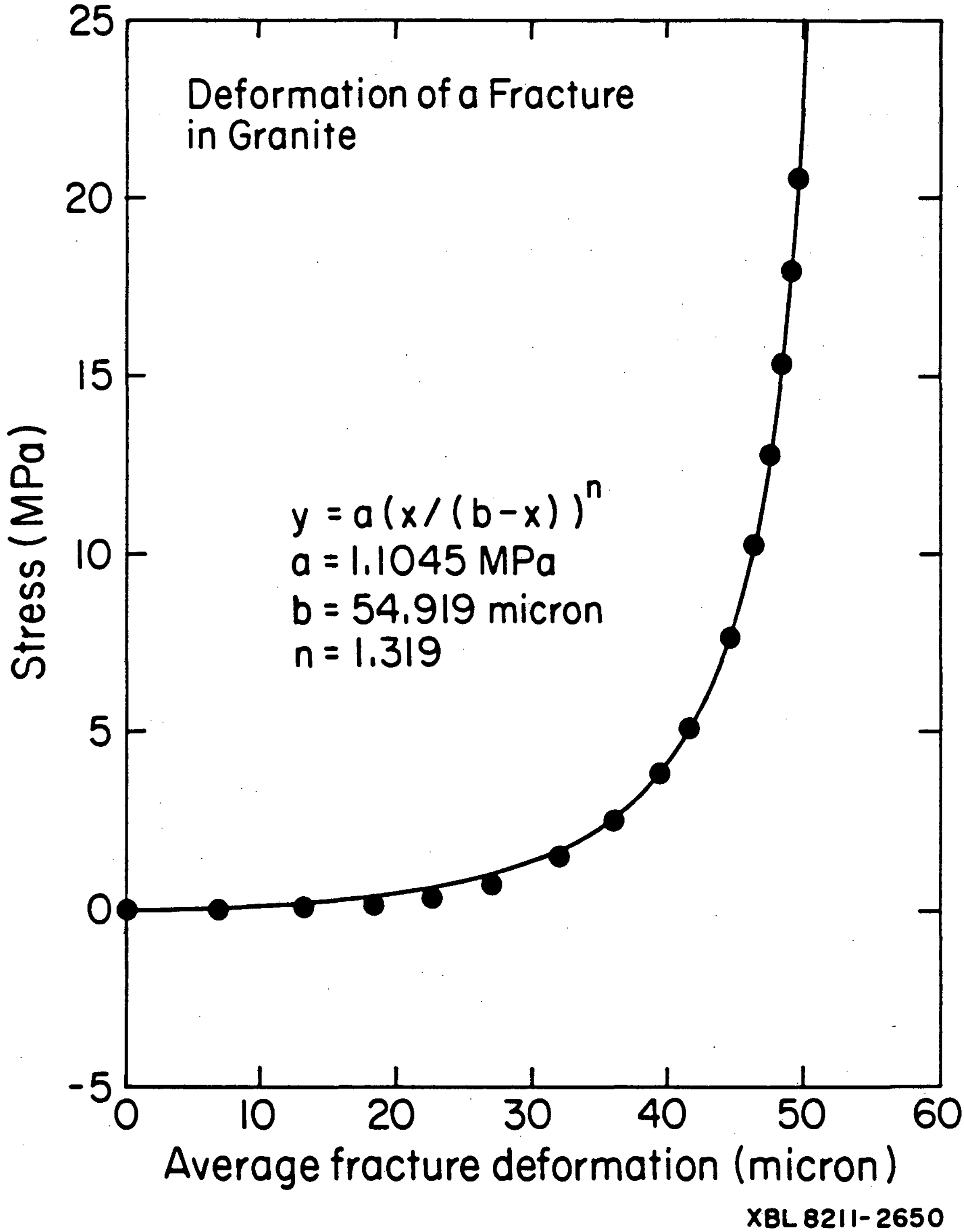

Figure 2. Curve Fit Through Rock Fracture Deformation Data.

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