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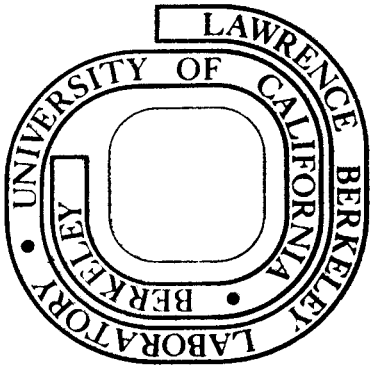

DISCLAIMER

Alpha-Spectroscopic Factors for Light Nuclei[†]

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ABSTRACT

In a recent paper[1)] it was shown that a consistent reaction theory requires a modification of the usual definition for α-spectroscopic factors. In the present paper a simple formula for the new spectroscopic factors is derived in the framework of the harmonic oscillator shell model. These factors are calculated for a number of light nuclei.

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I. Introduction

In two recent papers[1,2] the α-decay theory was reinvestigated. It was shown that the consistency of the reaction theory requires a new formula for the reduced width and the spectroscopic factor. On the basis of these results we recalculate a number of spectroscopic factors for light nuclei. Let us first review the main results of ref.[1,2] briefly.

In a simple physical picture the α-decay constant λ is proportional to the penetrability P through the Coulomb barrier and to the α-particle density ρ_α

$$\lambda \propto P \cdot \rho_\alpha \tag{1.1}$$

Here $\rho_\alpha(\underset{\sim}{R})$ is the probability of finding the α-particle (with wave function ϕ_α) and the daughter nucleus (A nucleons, wave function Φ_A) with a separation $\underset{\sim}{R}$ in the parent nucleus (A+4 nucleons, wave function Φ_{A+4}). The wave functions ϕ_α, Φ_A and Φ_{A+4} are antisymmetric, normalized internal wave functions. We may also define a total probability by

$$S = \int d^3\underset{\sim}{R}\, \rho_\alpha(\underset{\sim}{R}) \tag{1.2}$$

The quantity S is called spectroscopic factor. It may be connected with the decay constant by

$$\lambda = \lambda_{s.p.} \cdot S \tag{1.3}$$

Here $\lambda_{s.p.}$ is the single particle decay constant calculated from a single particle Schrödinger equation with an α-nucleus potential. In the same

sense the spectroscopic factor is used in the standard analysis of α-transfer experiments. The conventional theory gives the following expression for ρ_α

$$\rho_\alpha = |G(\underset{\sim}{R})|^2 = |\langle \mathcal{A}\, \delta(\underset{\sim}{R}-\underset{\sim}{R}_{\alpha A})\ \phi_\alpha\ \phi_A | \phi_{A+4}\rangle|^2 \qquad (1.4)$$

The antisymmetrisation operator $\mathcal{A}$ contains the trivial statistical factors:

$$\mathcal{A} = \binom{A+4}{A}^{\frac{1}{2}} \left[\frac{1}{(A+4)!} \sum_P (-1)^P P\right] \qquad (1.5)$$

The sum in eq. (1.5) runs over all (A+4)! permutations of the (A+4) particles. The vector between the centre of mass of the α-particle and the daughter nucleus is denoted by $\underset{\sim}{R}_{\alpha A}$. We restrict ourselves to the α-particle ground state, and in this section to ϕ_A with total spin zero. More general formulae are given in section 2.

The expression (1.4) is inadequate since a probability density can only be calculated by projecting on states which are normalized to $\delta(\underset{\sim}{R}-\underset{\sim}{R}')$. The normalization of the bra in (1.4) is, however,

$$\langle \mathcal{A}\, \delta(\underset{\sim}{R}-\underset{\sim}{R}_{\alpha A})\ \phi_\alpha\ \phi_A | \mathcal{A}\, \delta(\underset{\sim}{R}'-\underset{\sim}{R}_{\alpha A})\ \phi_\alpha \phi_A\rangle = \delta(\underset{\sim}{R}-\underset{\sim}{R}') - K(\underset{\sim}{R},\underset{\sim}{R}') \qquad (1.6)$$

For a discussion of the properties of K we refer to Feshbach[3]. We introduce basis states $|\mathcal{A}\,((1-\hat{K})^{-\frac{1}{2}})^*\,\delta(\underset{\sim}{R}'-\underset{\sim}{R}_{\alpha A})\ \phi_\alpha\phi_A\rangle$ which are normalized

$$\langle \mathcal{A}\,((1-\hat{K})^{-\frac{1}{2}})^*\delta(\underset{\sim}{R}-\underset{\sim}{R}_{\alpha A})\ \phi_\alpha\ \phi_A | \mathcal{A}((1-\hat{K})^{-\frac{1}{2}})^*\delta(\underset{\sim}{R}'-\underset{\sim}{R}_{\alpha A})\phi_\alpha\phi_A\rangle = \delta(\underset{\sim}{R}-\underset{\sim}{R}') \qquad (1.7)$$

Here we use the short-hand notation $\hat{O}\ f(\underset{\sim}{R})$ for $\int d^3R' O(\underset{\sim}{R},\underset{\sim}{R}') f(\underset{\sim}{R}')$. In the bra and ket in (1.7) $((1-\hat{K})^{-\frac{1}{2}})^*$ operates on the parameter $\underset{\sim}{R}$ and $\underset{\sim}{R}'$, respectively. In Appendix A we give the modification for the case that there are eigenvalues of $\hat{K}$ equal to 1.

The necessary probability density ρ_α can now be defined by

$$\rho_\alpha = |G_N(\underset{\sim}{R})|^2 = |< \mathcal{A}\ (1-\hat{K})^{-\frac{1}{2}*} \delta(\underset{\sim}{R}-\underset{\sim}{R}_{\alpha A})\ \phi_\alpha\ \phi_A |\phi_{A+4}>|^2 . \tag{1.8}$$

with

$$G_N = (1-\hat{K})^{-\frac{1}{2}} G \tag{1.9}$$

and the spectroscopic factor is

$$S = \int d^3\underset{\sim}{R} |G_N(\underset{\sim}{R})|^2 \tag{1.10}$$

Due to equation (1.3) the definition of the spectroscopic factor S is not a matter of semantics. The use of (1.8) instead of (1.4) gives in fact a different prediction for the decay constant. The expression (1.10) fulfills the trivial condition that a probability in quantum mechanics has to be defined as the expectation value of a projection operator[4]. To clarify this aspect let us consider the fictitious case of distinguishable nucleons. In such a case no problem arises: one has

$$\rho_\alpha^{n.a} = |< \delta(\underset{\sim}{R}-\underset{\sim}{R}_{\alpha A})\ \phi_\alpha\ \Phi_A |\ \Phi_{A+4}>|^2 \tag{1.11}$$

$$S^{n.a} = \int d^3\underset{\sim}{R}\ \rho_\alpha = <\Phi_{A+4}|\hat{P}^{n.a}|\Phi_{A+4}> \tag{1.12}$$

with

$$\hat{P}^{n.a} = \int d^3\underset{\sim}{R} |\delta(\underset{\sim}{R}-\underset{\sim}{R}_{\alpha A})\phi_\alpha\Phi_A><\delta(\underset{\sim}{R}-\underset{\sim}{R}_{\alpha A})\phi_\alpha\Phi_A|$$

The operator $\hat{P}^{n.a}$ is the projection operator on the subspace of all wavefunctions of the form $u(\underset{\sim}{R}_{\alpha A})\phi_\alpha\phi_A$ with fixed ϕ_α,ϕ_A. For the actual case of indistinguishable nucleons the effects of antisymmetry cannot be taken into account by simply inserting the operator $\mathcal{A}$ in the r.h.s. of eq. (1.11). This would lead to a spectroscopic factor which is the expectation value of $\int d^3\underset{\sim}{R}|\mathcal{A}\,\delta(\underset{\sim}{R}-\underset{\sim}{R}_{\alpha A})\phi_\alpha\Phi_A><\mathcal{A}\,\delta(\underset{\sim}{R}-\underset{\sim}{R}_{\alpha A})\phi_\alpha\Phi_A|$. This operator is not a projection operator. The correct analogue of (1.12) is

$$S = \int d^3\underset{\sim}{R}|G_N(\underset{\sim}{R})^2| = <\phi_{A+4}|\hat{P}|\phi_{A+4}> \tag{1.13}$$

where

$$\hat{P} = \int d^3\underset{\sim}{R}|\mathcal{A}(1-\hat{K})^{-\frac{1}{2}*}\delta(\underset{\sim}{R}-\underset{\sim}{R}_{\alpha A})\phi_\alpha\Phi_A><\mathcal{A}(1-\hat{K})^{-\frac{1}{2}*}\delta(\underset{\sim}{R}-\underset{\sim}{R}_{\alpha A})\phi_\alpha\Phi_A| \tag{1.14}$$

This projection operator was already introduced by Feshbach[3]. It projects on the subspace of wavefunctions of the form $\mathcal{A}\,u(\underset{\sim}{R}_{\alpha A})\phi_\alpha\phi_A$ with fixed ϕ_α, ϕ_A.

Also a consistent reaction theory[1,2] with one bound state and one open channel leads to the expression (1.8) instead of (1.4). The reaction theory uses an open channel state

$$|\phi_E> = |\mathcal{A}\;u_E(\underset{\sim}{R}_{\alpha A})\phi_\alpha\phi_A> = \int d^3\underset{\sim}{R}\;u_E(\underset{\sim}{R})|\mathcal{A}\,\delta(\underset{\sim}{R}-\underset{\sim}{R}_{\alpha A})\;\phi_\alpha\Phi_A> =$$

$$= \int d^3\underset{\sim}{R}\;\Omega_E(\underset{\sim}{R})|\mathcal{A}\;(1-\hat{K})^{-\frac{1}{2}*}\delta(\underset{\sim}{R}-\underset{\sim}{R}_{\alpha A})\phi_\alpha\Phi_A> \tag{1.15}$$

The inconsistent assumption usually made is that u_E may be approximated by solutions of a single particle Schrodinger equation with a real α-nucleus

potential. This approximation is only reasonable for $\Omega_E = (1-\hat{K})^{\frac{1}{2}} u_E$. This is connected with the fact that u_E may have unusually large amplitudes. This was demonstrated nicely by Saito et al.[5] in the discussion of the case that one eigenvalue of $\hat{K}$ approaches 1.

2. Method of calculating spectroscopic factors

In the following we give the new definition of α-spectroscopic factors for any spin of the daughter nucleus. We describe then a method to calculate these spectroscopic factors under the following simplifying assumptions.

1) The ground state of the α-particle is described by a 0s-state harmonic oscillator internal wavefunction.

2) The relevant states of the parent and daughter nuclei are described by internal wavefunctions corresponding to many particle oscillator shell model configurations with fixed total number of oscillator quanta.

3) The harmonic oscillators for the α-particle and for the other two nuclei involved have all the same frequency.

These assumptions have been used in comparable calculations[6,7,8,9,10] and have been discussed in some detail there.

According to the usual definition the radial part of the reduced amplitude is

$$G^{\ell}(R) = \langle \mathcal{A} \frac{1}{R^2} \delta(R-R_{\alpha A}) [Y_\ell \phi_\alpha \Phi_A{}^{J',\xi'}]^J | \Phi_{A+4}{}^{J,\xi} \rangle \qquad (2.1)$$

We use the short-hand notation $[Y_\ell \phi_\alpha \Phi_A{}^{J',\xi'}]^J$ for the coupling in the α-particle channel. The spherical harmonic Y_ℓ depends on the direction

of the vector $\underset{\sim}{R}_{\alpha A}$. The angular momentum ℓ of the relative motion and the spin J' are coupled to total spin J. The indices ξ', ξ denote all further quantum numbers of the daughter and parent nucleus.

Under the assumptions 1), 2), 3) the radial part of the reduced amplitude (2.1) is proportional to an oscillator function.

$$G^{\ell}(R) = \theta(\ell,J',J;\xi',\xi)\; u_{n\ell}(R;\frac{4A}{A+4}\nu_o) \tag{2.2}$$

The $u_{n\ell}(R,\nu)$ are normalized

$$\int_0^{+\infty} dR R^2 u_{n''\ell}(R,\nu) u_{n'\ell}(R,\nu) = \delta_{n''n'}$$

Here ν denotes the size parameter $\frac{m\omega}{h}$ and $\nu_o = \frac{m_o\omega}{h}$ is the unit for ν, where m_o is the nucleon mass. The number of oscillator quanta $Q = (2n + \ell)$ is equal to the difference of oscillator quanta between the states $\Phi_{A+4}^{J,\xi}$ and $\Phi_A^{J',\xi'}$.

We have to calculate

$$G_N^{\ell} = (1-\hat{K}_{\ell})^{-\frac{1}{2}} G^{\ell} \tag{2.3}$$

with

$$\frac{1}{R^2}\,\delta(R-R') - K_{\ell}(R,R') =$$
$$= \langle \mathcal{A}\, \frac{1}{R^2}\,\delta(R-R_{\alpha A})\,[Y_{\ell}\phi_{\alpha}\Phi_A^{J',\xi'}]^J \big| \mathcal{A}\, \frac{1}{R'^2}\,\delta(R'-R_{\alpha A})\,[Y_{\ell}\phi_{\alpha}\Phi_A^{J',\xi'}]^J \rangle \tag{2.4}$$

Since $|\mathcal{A}\, u_{n'\ell}[Y_{\ell}\phi_{\alpha}\Phi_A^{J',\xi'}]^J\rangle$ is an (A+4) particle state with fixed number of oscillator quanta, it is evident that $\langle u_{n''\ell}|(1-\hat{K}_{\ell})|u_{n'\ell}\rangle \sim \delta_{n''n'}$. Moreover, the set $\{u_{n'\ell}|_{n'=0,1\ldots}\}$ is complete:

$\sum_{n'} u_{n'\ell}(R)u_{n'\ell}(R') = \frac{1}{R^2}\delta(R-R')$. Therefore $u_{n\ell}$ are eigenfunctions of $(1-\hat{K}_\ell)$ and (2.3) and (2.4) simplify:

$$G_N^{\ell}(R) = (1 - \lambda_{n\ell})^{-\frac{1}{2}} G^{\ell}(R) \tag{2.5}$$

with

$$1 - \lambda_{n\ell} = \langle \mathcal{A}\, u_{n\ell}[Y_\ell \phi_\alpha \Phi_A^{J',\xi']J} | \mathcal{A} u_{n\ell}[Y_\ell \phi_\alpha \Phi_A^{J',\xi']J}\rangle \tag{2.6}$$

The states in (2.6) are in the space spanned by $\{\Phi_{A+4}^{J,\xi} | \xi = 1,2,\ldots\}$.

Therefore one obtains from (2.6)

$$(1-\lambda_{n\ell}) = \sum_{\xi} \langle \mathcal{A}\, u_{n\ell}[Y_\ell \phi_\alpha \Phi_A^{J',\xi']J} | \Phi_{A+4}^{J,\xi}\rangle \cdot$$

$$\cdot \langle \Phi_{A+4}^{J,\xi} | \mathcal{A}\, u_{n\ell}[Y_\ell \phi_\alpha \Phi_A^{J'\xi']J}\rangle = \sum_{\xi} |\theta(\ell,J',J,\xi',\xi)|^2 \tag{2.7}$$

The final result for the reduced amplitude is

$$G_N^{\ell} = \frac{\theta(\ell,J',J,\xi',\xi)}{(\sum_{\xi''} |\theta(\ell,J',J,\xi',\xi'')|^2)^{\frac{1}{2}}} \quad u_{n\ell} \tag{2.8}$$

Instead of the conventional spectroscopic factor

$$S_G = |\theta(\ell,J',J,\xi',\xi)|^2 \tag{2.9}$$

we obtain according to (2.8) the spectroscopic factor

$$S = |\theta(\ell,J',J,\xi',\xi)|^2 \,/\, \sum_{\xi''} |\theta(\ell,J',J,\xi',\xi'')|^2 \tag{2.10}$$

It should be pointed out that the set of orthonormal states $\{\Phi_{A+4}^{J,\xi}\}$

inserted in (2.7) may be arbitrary as long as it is complete. If harmonic oscillator states are used the sum is finite.

We shall use the SU(3) model for the daughter nuclei. Then the sum contains only a few terms, if we take an SU(3) basis for the (A+4)-particle states. For example in the cases of sd-shell nuclei considered in this paper the sum consisted of less than ten terms.

3. Results

The general trend of the effects of the operator $1-\hat{K}_\ell$ can be seen from Table 1 where S and S_G are given for a number of closed shell daughter nuclei. For comparison we also list some spectroscopic factors for two-particle transfer assuming again a Gaussian wave function for the two nucleons.

In Table 2 we give α-spectroscopic factors for some open shell nuclei of the sd-shell. We describe the parent and daughter nucleus by the leading SU(3) representation. This is a very good approximation for the wave functions of the nuclei considered. Furthermore, detailed calculations have shown[11] that taking into account all sd-shell configurations does not change the conventional spectroscopic factors appreciably.

4. Discussion

4.1 Effects of the operator $1-\hat{K}_\ell$

In Table 1 the spectroscopic factors are given for cluster-model states of the parent nucleus

$$|\Phi^J_{A+4}\rangle = N_\ell \left| \mathcal{A} \; u_{n\ell}\left[Y_\ell \phi_\alpha \Phi_A\right]^J \right\rangle \tag{4.1}$$

N_ℓ is a normalization constant. For these states the new spectroscopic factors are always 1. Both S_G and S are independent of ℓ when the daughter nucleus has a closed major shell configuration as in the examples given in Table 1. Obviously the relative spectroscopic factors are not changed in these cases. They are, however, changed for open shell nuclei.

The general trend of the effects of $(1-\hat{K}_\ell)$ is clear: their importance increases for heavier nuclei. Furthermore the effects of $1-\hat{K}_\ell$ are also more important for the four-particle spectroscopic factors than for the two-particle ones. Restricting ourselves for a moment to the case of lowest possible number of oscillator quanta the conventional spectroscopic amplitude is[6]

$$\theta = \binom{A}{4}^{\frac{1}{2}} \left(\frac{A+4}{A}\right)^{\frac{2n+\ell}{2}} G \quad \langle \psi_A \psi_4 \} \psi_{A+4} \rangle \tag{4.2}$$

The factor $\left(\frac{A+4}{A}\right)^{\frac{2n+\ell}{2}}$ arises from the transformation from shell model to intrinsic wave functions, G is the overlap of the α-particle wave function with the four-particle configuration ψ_4 which appears in the $n \to n-4$ coefficient of fractional parentage $\langle \psi_A \psi_4 \} \psi_{A+4} \rangle$.

In the case of the lowest possible number of oscillator quanta the factors G and $\left(\frac{A+4}{A}\right)^{\frac{2n+\ell}{2}}$ are the same for all spectroscopic amplitudes

appearing in (2.10). They cancel therefore leading to

$$S = |\langle \psi_A \psi_4\} \quad \psi_{A+4}\rangle|^2 \; / \; \Sigma_\nu |\langle \psi_A \psi_4\} \quad \psi^\nu_{A+4}\rangle|^2 \qquad (4.3)$$

We introduced here the index ν to express the summation over (A+4) particle states. It turns out that the main difference in the numerical value of the conventional spectroscopic factor and the new one arises from the factor $(\frac{A+4}{A})^{2n+\ell}$ G^2. For closed major shell configurations of the daughter nucleus (e.g. $^{20}Ne \to {}^{16}O + \alpha, {}^{44}Ti \to {}^{40}C_a + \alpha$) the new spectroscopic factor differs exactly by the factor $(\frac{A+4}{A})^{2n+\ell}$ G^2 from the old one.

Finally it is interesting to note that the conventional spectroscopic factor for Li + d is greater than 1. This fact demonstrates once more that the conventional spectroscopic factor cannot be interpreted as a probability.

4.2 Centre-of-mass motion

In the calculation of the conventional spectroscopic factor the removal of the centre-of-mass motion (cmm) is a very important step: It results in a factor $(\frac{A+4}{A})^{2n+\ell} \simeq 6$ for the s-d shell. The present formalism for the new spectroscopic factor can easily be written down also for shell model (SM) states ψ_{A+4}, ψ_A instead of internal states ϕ_{A+4}, ϕ_A (ϕ_α is an internal wave function in both cases). For the minimal number of quanta the factor $(\frac{A+4}{A})^{2n+\ell}$ obviously cancels in the expression (2.10) for the spectroscopic factor. For these cases we could as well calculate with SM functions.

For the discussion of the case of more oscillator quanta we take the $(\lambda,\mu) = (9,0)$ states of ^{20}Ne as an example discussed also in Ref. 6. The internal state is

$$|\phi_{A+4}(^{20}Ne,(9,0)\ell)\rangle = N_{(9,0)} |\hat{\mathcal{A}} u_{n\ell}^{(9,0)}(R_{\alpha A}, \frac{16}{5}\nu_o) Y_\ell \phi_\alpha \phi_A(^{16}O_{g.s.})\rangle \qquad (4.4)$$

The index (9,0) means $2n+\ell = 9$. A corresponding cluster-like SM state is

$$|\psi_{A+4}((0s)^4(0p)^{12}(sd)^3(pf)(9,0)\ell) > =$$

$$= \bar{N}_{(9,0)} |\mathcal{A} u_{n\ell}^{(9,0)}(R_\alpha, 4\nu_o) Y_\ell \phi_\alpha \psi_A((0s)^4(0p)^4)> \qquad (4.5)$$

Calculations of $N_{(9,0)}$ and $\bar{N}_{(9,0)}$ are discussed by Ichimura et al.[6]. They calculate the difference between S_G for (4.4) and (4.5). Their result is a factor $\frac{2}{5}$ in addition to $(\frac{A+4}{A})^{2n+\ell}$. The correction $\frac{2}{5}$ is due to the presence of spurious centre of mass motion in (4.5).

The new spectroscopic factors are in both cases, (4.4) and (4.5) equal to 1, provided that the centre of mass motion is treated consistently. This means that in the SM case we have to use the operator $1 - \hat{K}_\ell^{SM}$ defined by

$$\frac{1}{R^2}\delta(R-R') - K_\ell^{SM}(R,R') =$$

$$<\mathcal{A} \frac{1}{R^2}\delta(R-R_\alpha) Y_\ell \phi_\alpha \psi_A((0s)^4(0p)^{12}) | \mathcal{A} \frac{1}{R'^2}\delta(R'-R_\alpha) Y_\ell \phi_\alpha \psi_A((0s)^4(0p)^{12})> \qquad (4.6)$$

We have to be, however, very careful if we use SM wave functions which will be in general not of the form (4.5). A good SM calculation would yield a state which is spurious free. In our case this state is of the form[6]

$$|\frac{1}{\sqrt{4\pi}} u_{oo}(R_{20}, 20\,\nu_o)\ \phi_{A+4}\ (^{20}Ne(9,0)L) > =$$

$$\pm\sqrt{\frac{2}{5}}\,|\psi_{A+4}((0s)^4(0p)^{12}(sd)^3(pf)\ (9,0)\ L)> +$$

$$+\sqrt{\frac{3}{5}}\,|\psi_{A+4}((0s)^4(0p)^{11}(sd)^5(9,0)L)> \qquad (4.7)$$

With the use of (4.6) we would now get the wrong result $S = \frac{2}{5}$. Therefore, for general SM states we should first remove the center of mass motion. The appropriate methods for doing this when calculating $G^{\ell}(R)$ are discussed elsewhere[6]. In addition we have to calculate $1-\hat{K}_{\ell}$. In Appendix B we give general formulae which connect results for $1-\hat{K}_{\ell}$ and $1-\hat{K}_{\ell}^{SM}$.

4.3 Oscillator parameters

One main restriction of the present model is the use of the same oscillator frequency for the α-particle as for the daughter and parent nucleus. In a simple α-decay theory we had instead to calculate with a frequency corresponding to the free α-particle. Preliminary calculations for the $(\lambda, \mu) = (8,0)$ states of ^{20}Ne show a reduction of the spectroscopic factor from 1 to values between .4 and .55, depending on the angular momentum. This effect is somewhat larger than for the conventional spectroscopic factor[12].

For transfer reactions between two approximately equally heavy nuclei it seems, however, not unreasonable to use the larger α-particle of the present model.

5. Conclusion

By deriving a simple formula for the correct α-spectroscopic factor we filled part of the gap between structure calculations and the attempt to extract structure informations from reactions. There is still a good deal of uncertainty in connecting these factors with experimental results.

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Appendix A

The orthonormal eigenstates of $\hat{K}$ are called $|\omega_n\rangle$ and the eigenvalues λ_n $(o<\lambda_n \leq 1)$. We assume $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \ldots$. Then

$$\hat{K} = \sum_{n=1}^{N} \lambda_n |\omega_n\rangle\langle\omega_n| \tag{A.1}$$

where N may be finite or not. If N is finite we complete the set $\{\omega_1,\omega_2,\ldots,\omega_N\}$ by an infinite number of functions $\{\omega_{N+1},\omega_{N+2},\ldots\}$ so that $\{\omega_1,\ldots,\omega_N,\omega_{N+1},\ldots\}$ is a complete orthonormal set. The $|\omega_{i>N}\rangle$ may be regarded as eigenstates with the eigenvalue $\lambda_i = \sigma$. Indeed from (A1) follows $\hat{K}|\omega_i\rangle = 0$. We have now

$$\hat{K} = \sum_{n=1}^{\infty} \lambda_n |\omega_n\rangle\langle\omega_n| \tag{A.2}$$

If there are no eigenvalues equal to 1 relation (1.5) can easily be proved. If the first M eigenvalues are equal to 1 the corresponding eigenstates $|\omega_{j\leq M}\rangle$ are forbidden states. We can then restrict ourselves always to the space $\{\omega_{M+1},\omega_{M+2},\ldots\}$ orthogonal to the forbidden states. In particular G is orthogonal to these states. We introduce[3] the operator $\hat{\bar{K}}$ by

$$\hat{\bar{K}} = \sum_{n=M+1}^{\infty} \lambda_n |\omega_n\rangle\langle\omega_n| \tag{A.3}$$

Then $(1-\hat{\bar{K}})^{-\frac{1}{2}*}$ is again well-defined and we have instead of (1.7)

$$\langle \mathcal{A}\,(1-\hat{\bar{K}})^{-\frac{1}{2}*}\delta(\underset{\sim}{R}-\underset{\sim}{R}_{\alpha A})\phi_\alpha\phi_A \,|\, \mathcal{A}\,(1-\bar{K})^{-\frac{1}{2}*}\delta(\underset{\sim}{R}'-\underset{\sim}{R}_{\alpha A})\phi_\alpha\phi_A\rangle =$$

$$\sum_{n=M+1}^{\infty} \omega_n^*(\underset{\sim}{R})\omega_n(\underset{\sim}{R}') = \bar{\delta}(\underset{\sim}{R}-\underset{\sim}{R}') \tag{A.4}$$

This means that our basis functions are normalized to $\bar{\delta}(\underset{\sim}{R}-\underset{\sim}{R}')$ which is in the space in which we work identical to the δ-function $\delta(\underset{\sim}{R}-\underset{\sim}{R}')$.

In the case that there are eigenvalues equal to 1 we have to replace everywhere $\hat{K}$ by $\hat{\bar{K}}$. This has no consequences for our results.

Appendix B

In analogy to the operator $\hat{K}$ introduced in section 1 an operator $\hat{K}^{SM}$ can be defined when shell model states rather than internal states are used. Equations connecting $\hat{K}$ and $\hat{K}^{SM}$ will be derived in the following. These equations provide a method of calculation of $\hat{K}$ when $\hat{K}^{SM}$ is known. For simplicity only the case of spin zero daughter nuclei will be considered.

The operator $\hat{K}$ introduced in section 1 may be equivalently defined by

$$< f_\lambda | (1-\hat{K}) | f_\kappa > = <\mathcal{A} f_\lambda(\underset{\sim}{R}_{\alpha A})\phi_\alpha\phi_A | \mathcal{A} f_\kappa(\underset{\sim}{R}_{\alpha A})\phi_\alpha\phi_A> \qquad (B.1)$$

where $\{ f_\kappa \}$ denotes any complete set of state vectors for relative motion.

The operator K^{SM} is defined by

$$< \tilde{f}_\lambda | (1-\hat{K}^{SM}) | \tilde{f}_\kappa > = <\mathcal{A} \tilde{f}_\lambda(\underset{\sim}{R}_\alpha)\phi_\alpha\Psi_A | \mathcal{A} \tilde{f}_\kappa(\underset{\sim}{R}_\alpha)\phi_\alpha\Psi_A> \qquad (B.2)$$

where $\{ \tilde{f}_\kappa \}$ denotes any complete orthonormal set of state vectors for the motion of the centre of mass of the α-particle. We assume that the shell model state vector Ψ_A for the daughter nucleus factorizes as follows

$$\Psi_A = \Phi_A \cdot \chi_A(\underset{\sim}{R}_A) \qquad (B.3)$$

where χ_A is some normalized wave function of the centre of mass motion.

We denote in the following by $\{\phi^{\nu}_{A+4}\}$ and $\{\chi^{\mu}_{A+4}\}$ orthonormal complete sets of internal wave functions and centre of mass wave functions for the (A+4)-nucleon system respectively. From (B.2) one obtains using eq. (1.5)

$$< \tilde{f}_\lambda | (1-\hat{K}^{SM}) | \tilde{f}_\kappa > = \sum_{\nu,\mu} < A \tilde{f}_\lambda(\underset{\sim}{R}_\alpha)\phi_\alpha \Psi_A | \phi^{\nu}_{A+4} \chi^{\mu}_{A+4} > \cdot$$

$$\cdot < \phi^{\nu}_{A+4} \chi^{\mu}_{A+4} | A \tilde{f}_\kappa(\underset{\sim}{R}_a)\phi_\alpha \psi_A > = \sum_{\nu,\mu} \binom{A+4}{A} < \hat{f}_\lambda(\underset{\sim}{R}_\alpha)\phi_\alpha \Psi_A | \phi^{\nu}_{A+4} \chi^{\mu}_{A+4} >$$

$$\cdot < \phi^{\nu}_{A+4} \chi^{\mu}_{A+4} | \tilde{f}_\kappa(\underset{\sim}{R}_\alpha)\phi_\alpha \Psi_A > \qquad (B.4)$$

since the wave functions ϕ^{ν}_{A+4} are antisymmetric. Similarly one obtains for $\hat{K}$ using eq. (B.1)

$$< f_\lambda | (1-\hat{K}) | f_\kappa > = \sum_{\nu} \binom{A+4}{A} < f_\lambda(\underset{\sim}{R}_{\alpha A})\phi_\alpha \phi_A | \phi^{\nu}_{A+4} > \cdot$$

$$\cdot < \phi^{\nu}_{A+4} | f_\kappa(\underset{\sim}{R}_{\alpha A})\phi_\alpha \phi_A > \qquad (B.5)$$

From eq. (B.3) and the completeness of the orthonormal set $\{f_\nu\}$ follows

$$< \tilde{f}_\lambda(\underset{\sim}{R}_\alpha)\phi_\alpha \Psi_A | \phi^{\nu}_{A+4} \chi^{\mu}_{A+4} > = < \tilde{f}_\lambda(\underset{\sim}{R}_\alpha)\chi_A(\underset{\sim}{R}_A)\phi_a \phi_A | \phi^{\nu}_{A+4} \chi^{\mu}_{A+4} > =$$

$$= \sum_{\lambda'} < f_{\lambda'}(\underset{\sim}{R}_{\alpha A})\phi_\alpha \phi_A | \phi^{\nu}_{A+4} > < \tilde{f}_\lambda(\underset{\sim}{R}_\alpha)\ \chi_A(\underset{\sim}{R}_A) |\ f_{\lambda'}(\underset{\sim}{R}_{\alpha A}) \chi^{\mu}_{A+4} > \qquad (B.6)$$

Inserting (B.6) and its complex conjugate in (B.4) one obtains finally

$$< \tilde{f}_\lambda | (1-\hat{K}^{SM}) | \tilde{f}_\kappa > = \sum_{\lambda'\kappa'} C_{\lambda\lambda';\kappa\kappa'} < f_{\lambda'} | (1-\hat{K}) | f_{\kappa'} > \qquad (B.7)$$

where the coefficients $C_{\lambda\lambda';\kappa\kappa'}$ are defined by

$$C_{\lambda\lambda';\kappa\kappa'} = \sum_{\mu} \langle \tilde{f}_{\lambda}(\underset{\sim}{R}_{\alpha})\chi_A(\underset{\sim}{R}_A)|\chi^{\mu}_{A+4}(\underset{\sim}{R}_{A+4}) \quad f_{\lambda'}(\underset{\sim}{R}_{\alpha A})\rangle \cdot$$

$$\cdot \langle \chi^{\mu}_{A+4}(\underset{\sim}{R}_{A+4}) \quad f_{\kappa'}(\underset{\sim}{R}_{\alpha A})| \tilde{f}_{\kappa}(\underset{\sim}{R}_{\alpha}) \chi_A(\underset{\sim}{R}_A)\rangle \qquad (B.8)$$

Equations (B.7) constitute a system of linear equations for the unknowns $\langle f_{\lambda}|(1-\hat{K})|f_{\kappa}\rangle$.

We consider the case of practical interest where the centre of mass wave function χ_A is a harmonic oscillator wave function:

$$\chi_A(\underset{\sim}{R}) = \left(\frac{A\nu_o}{\pi}\right)^{3/4} e^{-\frac{1}{2}A\nu_o R^2} \qquad (B.9)$$

In this case the system of equations (B.7) has a unique solution and we give an explicit expression for the matrix elements of $(1-\hat{K})$.

We chose orthonormal bases $\{f_{\lambda}\}$ $\{\tilde{f}_{\lambda}\}$ and $\{\chi^{\mu}_{A+4}\}$ consisting of harmonic oscillator wave functions with the size parameters $\nu=4A/(A+4)\nu_o$, $\nu=4\nu_o$ and $\nu=(A+4)\nu_o$, respectively

$$f_{\lambda} = (\lambda_1!\lambda_2!\lambda_3!)^{-\frac{1}{2}}(a_x^+)^{\lambda_1}(a_y^+)^{\lambda_2}(a_z^+)^{\lambda_3} f_o$$

$$\tilde{f}_{\lambda} = (\lambda_1!\lambda_2!\lambda_3!)^{-\frac{1}{2}}(\tilde{a}_x^+)^{\lambda_1}(\tilde{a}_y^+)^{\lambda_2}(\tilde{a}_z^+)^{\lambda_3} \tilde{f}_o \qquad (B.10)$$

$$\chi^{\mu}_{A+4} = (\mu_1!\mu_2!\mu_3!)^{-\frac{1}{2}}(A_x^+)^{\mu_1}(A_y^+)^{\mu_2}(A_z^+)^{\mu_3} \chi^o_{A+4}$$

Here is

$$f_o(\underset{\sim}{r}) = \left(\frac{4A}{A+4}\frac{\nu_o}{\pi}\right)^{\frac{3}{4}} e^{-\frac{1}{2}\left(\frac{4A\nu_o}{A+4}\right)\underset{\sim}{r}^2}$$

$$\tilde{f}_o(\underset{\sim}{r}) = \left(\frac{4\nu_o}{\pi}\right)^{\frac{3}{4}} e^{-\frac{1}{2}(4\nu_o)\underset{\sim}{r}^2} \qquad (B.11)$$

$$\chi^o_{A+4}(\underset{\sim}{r}) = \left(\frac{(A+4)\nu_o}{\pi}\right)^{\frac{3}{4}} e^{-\frac{1}{2}(A+4)\nu_o \underset{\sim}{r}^2}$$

and a_x^+, a_y^+, a_z^+, $\tilde{a}_x^+$, $\tilde{a}_y^+$, $\tilde{a}_z^+$, A_x^+, A_y^+, A_z^+ denote creation operators for oscillator quanta in the x, y, for the corresponding oscillators, e.g.,

$$a_x^+ = \frac{1}{\sqrt{2}}\left(\sqrt{\frac{4A\nu_o}{A+4}}\,x - \sqrt{\frac{A+4}{4A\nu_o}}\,\frac{\partial}{\partial x}\right)$$

$$\tilde{a}_x = \frac{1}{\sqrt{2}}\left(\sqrt{4\nu_o}\,x - \frac{1}{\sqrt{4\nu_o}}\,\frac{\partial}{\partial x}\right) \qquad \text{(B.12)}$$

$$A_x^+ = \frac{1}{\sqrt{2}}\left(\sqrt{(A+4)\nu_o}\,x - \frac{1}{\sqrt{(A+4)\nu_o}}\,\frac{\partial}{\partial x}\right)$$

In equation (B.10) and in the following the indices λ, μ are shorthand notations for the triples of indices $(\lambda_1,\lambda_2,\lambda_3)$ and (μ_1,μ_2,μ_3) respectively. With this choice of bases the generalized Talmi-Moshinski brackets appearing in (B.8) are given by

$$\langle \tilde{f}_\lambda(\underset{\sim}{R}_\alpha)\chi_A(\underset{\sim}{R}_A)\,|\,\chi^\mu_{A+4} f_{\lambda'}(\underset{\sim}{R}_{\alpha A})\rangle = \prod_{j=1}^{3}\binom{\lambda_j}{\lambda'_j}^{\frac{1}{2}}\delta_{\mu_j+\lambda'_j,\lambda_j}\left(\frac{4}{A+4}\right)^{\frac{\mu_j}{2}}\left(\frac{A}{A+4}\right)^{\frac{\lambda'_j}{2}} \qquad \text{(B.13)}$$

Therefore, inserting (B.13) in (B.8) one obtains

$$C_{\lambda\lambda',\kappa\kappa'} = \prod_{j=1}^{3}\Big\{\sum_{\kappa_j\geq 0}\binom{\lambda_j}{\lambda'_j}^{\frac{1}{2}}\binom{\kappa_j}{\kappa'_j}^{\frac{1}{2}}\delta_{\mu_j+\lambda'_j,\lambda_j}\,\delta_{\mu_j+\kappa'_j,\kappa_j}\left(\frac{4}{A+4}\right)^{\mu_j}\left(\frac{A}{A+4}\right)^{\frac{\lambda'_j+\kappa'_j}{2}}$$

$$= \prod_{j=1}^{3}\delta_{\lambda_j-\lambda'_j,\kappa_j-\kappa'_j}\,\theta(\lambda_j-\lambda'_j)\binom{\lambda_j}{\lambda'_j}^{\frac{1}{2}}\binom{\kappa_j}{\kappa'_j}\left(\frac{4}{A+4}\right)^{\kappa_j-\kappa'_j}\left(\frac{A}{A+4}\right)^{\frac{\lambda'_j+\kappa'_j}{2}} \qquad \text{(B.14)}$$

where $\theta(x)=1$ for $x\geq 0$ and zero otherwise.

We define coefficients $B_{\lambda\lambda',\kappa\kappa'}$ by

$$B_{\lambda\lambda',\kappa\kappa'} = \left(\frac{A+4}{A}\right)^{\frac{Q_\lambda+Q_\kappa}{2}}\left(-\frac{4}{A+4}\right)^{Q_\kappa-Q_{\kappa'}}\prod_{j=1}^{3}\Big\{\delta_{\kappa_j-\kappa'_j,\lambda_j-\lambda'_j}\,\theta(\kappa_j-\kappa'_j)\binom{\kappa_j}{\kappa'_j}^{\frac{1}{2}}\binom{\lambda_j}{\lambda'_j}^{\frac{1}{2}}\Big\} \qquad \text{(B.15)}$$

where Q_λ, Q_κ, Q'_λ, Q'_κ denote the number of oscillator quanta of the states $\tilde{f}_\lambda$, $\tilde{\tilde{f}}_\kappa$, $f_{\lambda'}$, $f_{\kappa'}$ e.g.,

$$Q_\lambda = \lambda_1+\lambda_2+\lambda_3 \tag{B.16}$$

As it will be proved at the end of this Appendix

$$\sum_{\lambda'',\kappa''} B_{\lambda\lambda'',\kappa\kappa''} C_{\lambda''\lambda',\kappa''\kappa'} = \delta_{\lambda\lambda'}\delta_{\kappa\kappa'} \tag{B.17}$$

From equation (B.7) one obtains therefore,

$$\langle f_\lambda | (1-\hat{K}) | f_\kappa \rangle = \sum_{\lambda',\kappa'} B_{\lambda\lambda',\kappa\kappa'} \langle \tilde{f}_\lambda | (1-\hat{K}^{SM}) | \tilde{\tilde{f}}_\kappa \rangle \tag{B.18}$$

Due to the step functions $\theta(\kappa_j-\kappa_j')$ appearing in (B.15) the sum in (B.14) has only a finite number of nonvanishing terms. In the special case of minimum number of oscillator quanta there is in fact only one non-vanishing term and the (B.14) simplifies to

$$\langle f_\kappa | (1-\hat{K}) | f_\kappa \rangle = \left(\frac{A+4}{A}\right)^{Q_\kappa} \langle \tilde{f}_\kappa | (1-\hat{K}^{SM}) | \tilde{\tilde{f}}_\kappa \rangle \tag{B.19}$$

In the case of closed major shell daughter nuclei (e.g., $^{20}Ne \rightarrow {}^{16}O+\alpha$) $(1-\hat{K}^{SM})$ is diagonal in the basis $\{\tilde{f}_\lambda\}$ and its matrix elements depend only on the total number of oscillator quanta:

$$\langle \tilde{f}_\lambda | (1-\hat{K}^{SM}) | \tilde{\tilde{f}}_\kappa \rangle = \eta^{SM}_{Q_\lambda} \delta_{\lambda\kappa} \tag{B.20}$$

Then one gets from (B.18) and (B.15)

$$\langle f_\lambda | (1-\hat{K}) | f_\kappa \rangle = \eta_{Q_\lambda} \delta_{\lambda\kappa} \tag{B.21}$$

with

$$\eta_Q = \left(\frac{A+4}{A}\right)^Q \sum_{Q' \geq 0} \theta(Q-Q') \left(-\frac{4}{A+4}\right)^{Q-Q'} \binom{Q}{Q'} \eta^{SM}_{Q'} \qquad \text{(B.22)}$$

Proof of Eq. (B.17)

Using the definition of Q_λ, Q_κ, $Q_{\kappa''}$, one obtains from (B.14) and (B.15) the relation

$$\sum_{\lambda'',\kappa''} B_{\lambda\lambda'',\kappa\kappa''} C_{\lambda''\lambda',\kappa''\kappa} =$$

$$= \sum_{\lambda'',\kappa''} \Big\{ \prod_{j=1}^{3} \left(\frac{A+4}{A}\right)^{\frac{\lambda_j+\kappa_j}{2}} \left(-\frac{4}{A+4}\right)^{\kappa_j-\kappa_j''} \delta_{\kappa_j-\kappa_j'',\lambda_j-\lambda_j''} \theta(\kappa_j-\kappa_j'') \binom{\kappa_j}{\kappa_j''}^{\frac{1}{2}} \binom{\lambda_j}{\lambda_j''}^{\frac{1}{2}} \Big\}$$

$$\cdot \Big\{ \prod_{j=1}^{3} \left(\frac{A}{A+4}\right)^{\frac{\kappa_j'+\lambda_j'}{2}} \left(\frac{4}{A+4}\right)^{\kappa_j''-\kappa_j'} (\kappa_j''-\kappa_j') \delta_{\kappa_j''-\kappa_j',\lambda_j''-\lambda_j'} \binom{\kappa_j''}{\kappa_j'}^{\frac{1}{2}} \binom{\lambda_j''}{\lambda_j'}^{\frac{1}{2}}$$

$$= \sum_{\kappa'',\lambda''} \Big\{ \prod_{j=1}^{3} \left(\frac{A+4}{A}\right)^{\lambda_j-\lambda_j'} \left(\frac{4}{A+4}\right)^{\kappa_j-\kappa_j'} (-1)^{\kappa_j-\kappa_j''} \theta(\kappa_j-\kappa_j'') \theta(\kappa_j''-\kappa_j')$$

$$\cdot \binom{\kappa_j}{\kappa_j''}^{\frac{1}{2}} \binom{\kappa_j''}{\kappa_j'}^{\frac{1}{2}} \binom{\lambda_j}{\lambda_j''}^{\frac{1}{2}} \binom{\lambda_j''}{\lambda_j'}^{\frac{1}{2}} \delta_{\kappa_j-\kappa_j'',\lambda_j-\lambda_j''} \delta_{\kappa_j''-\kappa_j',\lambda_j''-\lambda_j'} \Big\} \qquad \text{(B.23)}$$

From (B.23) one obtains further performing the summation over λ''

$$\sum_{\lambda'',\kappa''} B_{\lambda\lambda'',\kappa\kappa''} C_{\lambda''\lambda',\kappa''\kappa'} =$$

$$= \prod_{j=1}^{3} \Big\{ \left(\frac{A+4}{A}\right)^{\lambda_j-\lambda_j'} \left(\frac{4}{A+4}\right)^{\kappa_j-\kappa_j'} \left(\frac{\kappa_j!\lambda_j!}{\kappa_j'!\lambda_j'!}\right)^{\frac{1}{2}} \delta_{\kappa_j-\kappa_j',\lambda_j-\lambda_j'}$$

$$\cdot \Big[\sum_{\kappa_j''} (-1)^{\kappa_j-\kappa_j''} \frac{1}{(\kappa_j-\kappa_j'')(\kappa_j''-\kappa_j')} \theta(\kappa_j-\kappa_j'') \theta(\kappa_j''-\kappa_j') \Big] \Big\} \qquad \text{(B.24)}$$

However the term in square brackets in (B.24) is equal to $\delta_{\kappa_j,\kappa_j'}$ according to the binomial theorem. Taking this into account (B.15) follows from (B.24) immediately.

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Table Captions

Table 1

The spectroscopic factor S compared to the conventional factor S_G for closed shell daughter nuclei ($J' = 0$). We have $\ell = J$ and the factors are independent of ℓ.

Table 2

The spectroscopic factor S compared to the conventional factor S_G for sd-shell nuclei. The last two columns show the relative spectroscopic factors (ground state transition normalized to 1). The daughter nucleus is in the ground state, J and K refer to the parent nucleus.

Table 1

System	SU3-classification Parent Nucleus	S_G	S	S/S_G
α-α	(4,0)	0.75	1	1.3
α-^{16}O	(8,0)	0.23	1	4
α-^{16}O	(9,0)	0.34	1	3
α-^{16}O	(10,0)	0.51	1	2
α-^{40}Ca	(12,0)	0.07	1	14
α-$^{70}X^{35}_{35}$	(16,0)	0.036	1	28
α-$^{140}X^{70}_{70}$	(20,0)	0.019	1	53
d-α	(2,0)	1.125	1	0.9
d-^{16}O	(4,0)	0.60	1	1.7
d-^{40}Ca	(6,0)	0.42	1	2.4

Table 2

	J	K	S	S_G	S/S_G	S_{rel}	S_{Grel}
$^{16}O+\alpha \rightarrow {}^{20}Ne$	0	0	1.0	0.23	4.3	1.0	1.0
(0,0)→(8,0)	2	0	1.0	0.23	4.3	1.0	1.0
	4	0	1.0	0.23	4.3	1.0	1.0
	6	0	1.0	0.23	4.3	1.0	1.0
	8	0	1.0	0.23	4.3	1.0	1.0
$^{20}Ne+\alpha \rightarrow {}^{24}Mg$	0	0	0.41	0.077	5.3	1.00	1.00
(8,0)→(8,4)	2	0	0.45	0.029	16	1.10	0.37
	4	0	0.00	0.0	-	0.00	0.00
	6	0	0.11	0.012	9.6	0.27	0.15
	8	0	0.00	0.0	-	0.00	0.00
	2	2	0.06	0.0038	16	0.14	0.05
	4	2	0.34	0.035	9.7	0.84	0.46
	6	2	0.52	0.055	9.6	1.28	0.71
	8	2	0.04	0.0038	10	0.10	0.05
$^{24}Mg+\alpha \rightarrow {}^{28}Si$	0	0	0.54	0.087	6.3	1.00	1.00
(8,4)→(0,12) oblate	2	0	0.38	0.020	19	0.70	0.22
	4	0	0.16	0.0088	18	0.30	0.10
	6	0	0.081	0.0039	21	0.15	0.04
	8	0	0.038	0.0012	32	0.07	0.01
(8,4)→(12,0) prolate	0	0	0.15	0.025	6.2	0.98	0.28
	2	0	0.11	0.0056	19	0.20	0.06
	4	0	0.034	0.0018	18	0.06	0.02
	6	0	0.098	0.0047	21	0.18	0.05
	8	0	0.096	0.0031	32	0.18	0.03

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