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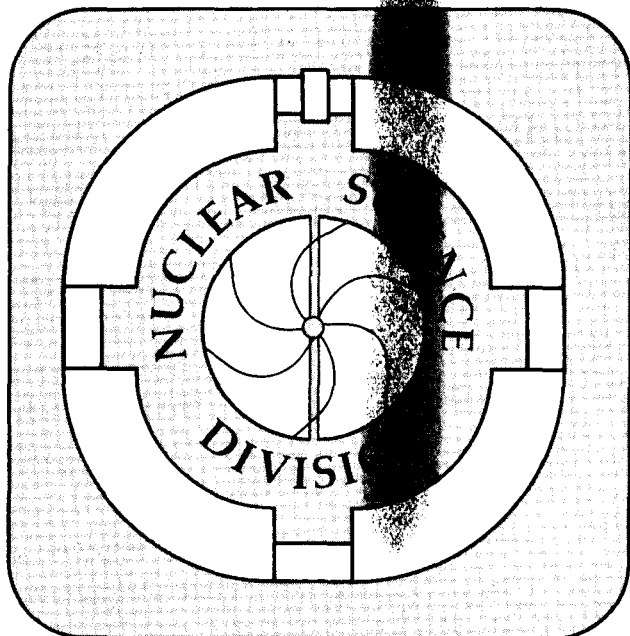October 1981

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Fragment spin correlations in damped nuclear reactions

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Abstract

The correlation between the spins of damped nuclear reaction products are studied theoretically. The dynamical exchange of nucleons preferentially excites positive-signature dinuclear rotational modes and produces significant fragment spin correlations, in contrast with the results of a purely statistical model. Illustrative calculations are made for the angular correlations between fission fragments following the reaction 1768 MeV ^{208}Pb + ^{238}U.

Damped nuclear reactions provide a powerful tool for exploring nuclear dynamical properties. In recent years, a number of reaction models have been developed but the relative importance of the various reaction mechanisms advocated is still not settled. With several different models having comparable success in accounting for the available data, there is a need to consider more exclusive quantities, which are better suited as specific probes. In a recent development of the theory for angular-momentum transport caused by nucleon exchange it was suggested that the spins of the reaction products might exhibit correlations reflecting the basic mechanism by which the spins have been accumulated [1]. In the present paper we pursue this idea quantitatively. This undertaking has been stimulated by a suggestion from Lazzarini [2] that the determination of spin correlations is experimentally feasible by coincidental measurement of fission fragments from both reaction products.

The spin of a nuclear fragment may be probed via the angular distribution of its decay products. In the present paper we shall consider the fission process, but the formulation applies equally well to other decay processes, such as the emission of alpha particles, nucleons, and gamma rays. We consider a damped nuclear reaction leading to the projectile-like fragment A and the target-like fragment B. Let the probability that the ejectile from the nucleus A appears in the direction $\hat{\alpha}$ be given by $P_A(\hat{\alpha}, \vec{S}_A)$, where $\vec{S}_A$ is the spin of A. [The direction $\hat{\alpha}$ is with reference to a right-handed coordinate system that moves along with the ejector A with its z-axis in the direction of motion (as seen in the overall CM system) and its y-axis perpendicular to the reaction plane.] Let analogously $P_B(\hat{\beta}, \vec{S}_B)$ be the probability that the ejectile from the complementary fragment B appears in the direction $\hat{\beta}$. [The direction $\hat{\beta}$ is with reference to a coordinate system that

is aligned with the above one but comoving with the target-like ejector B. These coordinate systems are indicated in fig. 1. The practical problem of transforming to the laboratory frame is elementary and need not be dealt with here.] The object of study is the joint ejection probability which is given by

$$P_{AB}(\hat{\alpha},\hat{\beta}) = \iint P_A(\hat{\alpha},\vec{S}_A)P_B(\hat{\beta},\vec{S}_B) \; f_{AB}(\vec{S}_A,\vec{S}_B)d\vec{S}_Ad\vec{S}_B \qquad (1)$$

Here $f_{AB}(\vec{S}_A,\vec{S}_B)$ is the joint spin distribution for the fragments A and B, just after the reaction has ceased. The normalization is such that $\iint P_{AB}(\hat{\alpha},\hat{\beta})d\hat{\alpha}d\hat{\beta} = 1$.

The fission process is a particularly suitable probe because of the large anisotropy of fission products from nuclei with high spins. The fission direction is determined by $P(K) \sim \exp(-K^2/2K_0^2)$ where K is the component of the nuclear spin along the emission direction. Hence,

$$P_A(\hat{\alpha},\vec{S}_A) \approx \left[(2\pi)^{3/2}\frac{K_A}{S_A}\,\mathrm{erf}\!\left(\frac{S_A}{\sqrt{2}\;K_A}\right)\right]^{-1} e^{-(\vec{S}_A\cdot\hat{\alpha})^2/2K_A^2} \qquad (2)$$

and analogously for $P_B(\hat{\beta},S_B)$. The quantities K_A and K_B are characteristic of the nuclei A and B and depend on their excitation. For the nuclei Pb and U, which we shall consider presently, we use [3,4]

$$[K_{Pb}(\hbar)]^2 = 13.60\;[Q_{Pb}(MeV)]^{1/2}$$

$$[K_U(\hbar)]^2 = 19.37\;[Q_U(MeV)]^{1/2} \qquad (3)$$

where Q_{Pb} and Q_U denote the amounts of intrinsic excitation of the fissioning nuclei (which are calculated by partitioning the total dissipated energy according to mass).

The theoretical spin distribution function $f_{AB}(\vec{S}_A, \vec{S}_B)$ depends on the particular reaction model employed. It is useful to characterize f_{AB} by the mean values $\langle \vec{S}_A \rangle$ and $\langle \vec{S}_B \rangle$ together with the covariance tensor σ, which has the elements $\sigma_{ij}^{FG} = \langle S_i^F S_j^G \rangle - \langle S_i^F \rangle \langle S_j^G \rangle$, where the superscripts F,G denote the fragment labels A,B and the subscripts i,j denote the spatial coordinates x,y,z. The main goal of the present study is to explore the consequences of statistical nucleon exchange on the fragment spin correlations. We therefore employ the model developed in ref. [1]. In that theory, the statistical transfer of individual nucleons between the two reactants produces a dissipative exchange of mass, charge, energy, and angular momentum governed by a Fokker-Planck transport equation. The theory relates the various transport coefficients to the elementary kinematics of quasi-free transfers. The temporal evolution of the associated form factor is estimated by idealizing the reaction complex as two spherical nucleides joined by a small cylindrical neck. The computational problem is solved numerically by integrating the moments of the distribution function f along the average dynamical trajectory. Having thus obtained f_{AB}, the joint emission probability P_{AB} is calculated by statistical (Monte-Carlo) integration over the six-dimensional spin-spin space; the details of this part of the calculation are described in the appendix.

In addition to the four angular variables $\hat{\alpha} = (\theta_\alpha, \varphi_\alpha)$ and $\hat{\beta} = (\theta_\beta, \varphi_\beta)$, P_{AB} depends on the various auxiliary observables, such as the kinetic energy loss, the deflection angle, and the mass partition. In the present study, we shall calculate P_{AB} for the average outcome of a collision with a specified value of the total angular momentum J. This is expected to be sufficient for a first orientation and a preliminary confrontation with experiment; a more refined treatment can be made subsequently, if called for.

In order to judge the significance of the calculated results, we also consider the statistical model developed by Moretto [5]. The statistical model idealizes the dinuclear reaction complex as two rigid spheres (with moments of inertia $\mathcal{J}_A = \frac{2}{5} mAR^2$ and $\mathcal{J}_B = \frac{2}{5} mBR^2$) in statistical equilibrium at a given separation R. The associated temperature τ is related to the dissipated energy Q by $Q = a\tau^2$ with $a = (A+B)/(8 \text{ MeV})$. In the original version [5], the separation was taken as the sum of the two nuclear radii, $R = R_A + R_B$, and the moment of inertia for the relative orbital rotation was taken accordingly as $\mathcal{J}_R = \mu R^2$. However, the observed large energy losses indicate that the two nucleides maintain good communication considerably beyond the touching separation; hence, that assumption may not be reasonable. In order to bracket the effect of this inherent uncertainty in the statistical model, we consider two opposite extremes for the separation used: an expected lower limit given by the smallest separation reached during the particular collision, and an expected upper limit given by the separation at which the neck collapses, both quantities as calculated in the above mentioned dynamical model [1]. The actual expressions employed for calculating the equilibrium values of the moments characterizing f_{AB} are those given in [1], which are in accordance with [5].

As an instructive case, we consider the reaction 1768 MeV ^{208}Pb + ^{236}U, which has been suggested as suitable for actual experimentation [2]. In the present context the partially damped reactions are of most interest since they produce fragments with high excitation and high spins and hence optimize the fission probability; the simple dinuclear parametrization employed in the dynamical calculations is also expected to be best in this region. Table 1 contains a number of relevant quantities, as calculated for various values of J.

As is evident from eq. (2), the function $P_{AB}(\hat{\alpha},\hat{\beta})$ is invariant under inversions of the emission directions (i.e. $\hat{\alpha} \to -\hat{\alpha}$ and/or $\hat{\beta} \to -\hat{\beta}$). Furthermore, there is symmetry with respect to an overall reflection in the reaction plane (i.e. $\varphi_\alpha \to -\varphi_\alpha$ and $\varphi_\beta \to -\varphi_\beta$). It therefore suffices to explore only the forward hemispheres ($\theta_\alpha < \pi/2$ and $\theta_\beta < \pi/2$) and, furthermore, only upwards emission from the target-like ejector (i.e. $0 < \varphi_\beta < \pi$); values in other domains are determined by the symmetry properties. Figure 2 displays some instructive results calculated for $J = 500$ ℏ, which corresponds to a typical partially damped reaction. The joint emission probability $P_{AB}(\hat{\alpha},\hat{\beta})$ is shown for two different settings of the polar angles ($\theta_\alpha = \theta_\beta = $ either 45° or 90°) as a function of the azimuthal angle φ_α for various fixed values of φ_β.

First, it should be noted that the two limiting calculations with the statistical model generally yield qualitatively similar results, although the absolute magnitudes may differ considerably (which is a consequence of the fact that the respective values of $\mathcal{J}_R$ differ by a factor of two in the case considered). The pronounced preferential emission close to the reaction plane ($\varphi = 0°, 180°$) is a consequence of the significant alignment of the fragment spins with the reaction normal; this feature decreases with the angular momentum J, as discussed in [1].

A striking feature of the statistical results is the approximate symmetry with respect to reflections in the reaction plane (i.e. $\varphi_\alpha \to -\varphi_\alpha$). This simplicity can be understood on the basis of the following discussion: The six normal rotational modes of the disphere can be characterized as either positive or negative according to whether the two spheres turn in the same or in the opposite sense, respectively [1]. In the statistical model, each of these modes carries an average excitation energy equal to half the temperature

τ. By inverting the representation of the normal modes in terms of the individual fragment spins, one obtains the covariance tensor characterizing the statistical spin distribution. This elementary manipulation will demonstrate how the equally excited positive and negative normal modes counteract each other in the coupling tensor σ^{AB} characterizing the correlations between the two fragment spins. As a consequence, σ^{AB} emerges as considerably smaller than σ^{AA} and σ^{BB}, and the fragment spins are largely uncorrelated. Indeed, in the case under study, the elements of σ^{AB} are typically an order of magnitude smaller than those of σ^{AA} and σ^{BB}.

In sharp contrast with this scenario, the dynamical nucleon exchange model yields a significant asymmetry with respect to $\varphi_\alpha \rightarrow -\varphi_\alpha$. This feature can be understood as follows. When an individual nucleon is transferred the recoil spins deposited in the two nucleides are nearly parallel [1]. Therefore, the nucleon exchange mechanism preferentially excites the positive rotational modes in the dinucleus. This tendency is further enhanced for nearly symmetric systems (as are relevant in a double-fission experiment). As a consequence, no cancellation occurs in the coupling tensor σ^{AB}. which therefore emerges with elements of similar magnitudes as those of σ^{AA} and σ^{BB} and thus produces an optimal correlation between the two spins. In this situation, the bias introduced by detecting one of the emission processes significantly modifies the angular distribution of ejectiles from the other emitter. The effect can be stated in the following simplistic way: When the two fragment spins are closely correlated the emission of an ejectile in a given direction enhances the probability that the ejectile from the other fragment also appears in that same direction. To exhibit this tendency, the arrows on fig. 2 indicate the various fixed values of φ_β. [It should be remembered, though, that the dominant bias on φ_α is still that introduced by

σ^{AA} itself, as evidenced by the general tendency for in-plane ejection (i.e. $\varphi_\alpha = 0°, 180°$).]

A curious feature of the dynamical results is the tendency for an enhancement of $\varphi_\alpha = 180°$ relative to $\varphi_\alpha = 0°$. This property is a reflection of the calculated in-plane anisotropy of the spin distribution. In the gaussian approximation, the contour surfaces of the spin distribution are ellipsoids. The corresponding principal system is indicated in fig. 1. The symmetry of the situation dictates that the 3-axis be equal to the reaction normal. As was shown in ref. [1], the 2-axis is directed approximately halfway between the beam and the direction of the projectile-like fragment A, as seen in the CM-system. Since usually the principal dispersions (i.e. the eigenvalues of the tensor σ^{AA}) satisfy $\sigma_1 < \sigma_2 < \sigma_3$ it follows that the contour lines in the reaction plane are ellipses directed approximately towards $\theta_{CM}/2$, as illustrated in fig. 1. This feature then favors emission at $\varphi_\alpha = 180°$ relative to $\varphi_\alpha = 0°$.

Calculations for other values of J, as well as other angular settings, yield the same general features as in the illustrative example discussed above. Furthermore, crude attempts to include the blurring effects associated with the finite angular resolution in practicable experiments indicate that a measurable difference between the two models should remain. It would therefore be valuable to actually carry out such an experiment. If successful, the measurement would establish novel information on the dynamics of damped nuclear reactions and contribute to our quantitative understanding of the specific reaction mechanisms acting.

In the course of this work I have received great stimulation and encouragement from numerous discussions with Albert Lazzarini and Robert

Appendix

The joint probability $P_{AB}(\vec{S}_A, \vec{S}_B)$ is given in terms of the six-dimensional integral in eq. (1). The numerical evaluation of such a quantity can be computationally quite demanding and therefore calls for special consideration.

The integration is not suitable for the application of a standard grid-point integration formula, such as a Hermite integration, since the factors P_A and P_B are peaked in the plane perpendicular to the directions $\hat{\alpha}$ and $\hat{\beta}$, respectively, while the distribution function f_{AB} itself is peaked at $(\langle\vec{S}_A\rangle, \langle\vec{S}_B\rangle)$. In order to obtain a reasonably accurate result, it would then be necessary to use a relatively large number of integration points, which in turn would make the calculation prohibitively time consuming.

A much more efficient and reliable method, and usually a superior one for the evaluation of multi-dimensional integrals, is offered by the statistical ("Monte Carlo") method. In this method, the value of the integral is approximated by the average value of the residual factor $P_A P_B$ evaluated at a large number K of points in $(\vec{S}_A, \vec{S}_B)$-space picked at random according to the distribution function $f_{AB}(\vec{S}_A, \vec{S}_B)$.

The covariance tensor σ characterizing f is generally not diagonal in the standard representation employed here. In order to facilitate the numerical calculations, it is useful to bring σ on normal form. The diagonalization with respect to the spatial indices x,y,z is readily accomplished by a

rotation into the principal frame. Due to the symmetry of the distribution f with respect to a reflection in the reaction plane, this rotation is around the reaction normal $\hat{y}$. The three principal directions are then given by

$$\hat{1} = \hat{z} \cos \psi_0 + \hat{x} \sin \psi_0$$

$$\hat{2} = \hat{x} \cos \psi_0 - \hat{z} \sin \psi_0 \tag{4}$$

$$\hat{3} = \hat{y}$$

where the angle ψ_0 is determined by

$$\tan 2\psi_0 = \frac{2\sigma_{xz}^{FG}}{\sigma_{zz}^{FG} - \sigma_{xx}^{FG}} \tag{5}$$

It is an important property of σ that the same spatial rotation diagonalizes all the tensors σ^{AA}, σ^{BB} and $\sigma^{AB} = \sigma^{BA}$. [1]

The tensor σ^{AB} expresses the covariance between the two fragment spins $\vec{S}_A$ and $\vec{S}_B$. It is possible to bring σ on normal form by rotating in spin-spin space:

$$S_k^+ = S_k^A \cos \psi_k + S_k^B \sin \psi_k$$

$$S_k^- = S_k^B \cos \psi_k - S_k^A \sin \psi_k \tag{6}$$

Here S_k^F, k = 1,2,3 are the spin components in the principal frame while S_k^K, κ = +,- are the spin components in the normal representation. The angles ψ_k are determined by

$$\tan 2 \psi_k = \frac{2 \sigma_{kk}^{AB}}{\sigma_{kk}^{AA} - \sigma_{kk}^{BB}} \tag{7}$$

After the above transformations the distribution f is given on normal form as

$$f = [(2\pi)^6 D]^{-1/2} \exp\left[-\sum_{k\kappa} \frac{(\Delta S_k^\kappa)^2}{2\,\sigma_{kk}^\kappa} \right] \tag{8}$$

where $D = \det\sigma = \prod_{k\kappa} \sigma_{kk}^\kappa$ is the determinant of the covariance tensor and $\Delta S_k^\kappa = S_k^\kappa - \langle S_k^\kappa \rangle$ is the deviation of the actual spin value from its average. Once f is expressed on normal form it is straightforward to sample the values of $(\vec{S}_A, \vec{S}_B)$ by using a standard code for normally distributed random numbers. Ordinarily 10^4 sample points suffice to achieve errors at the level of one percent.

References

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Acknowledgement

Table 1.

Various characteristic reaction quantities as calculated for different values of the total angular momentum J, for the reaction 1768 MeV ^{208}Pb + ^{238}U: The total intrinsic excitation energy Q (approximately equal to the total kinetic energy loss), the corresponding temperature (related to Q by Q = $a\tau^2$ with a = (A+B)/8 MeV, the total spin S, and the normal spin components S_3 for each of the two fragments, and the CM scattering angle θ_{CM}.

1768 MeV ^{208}Pb + ^{238}U

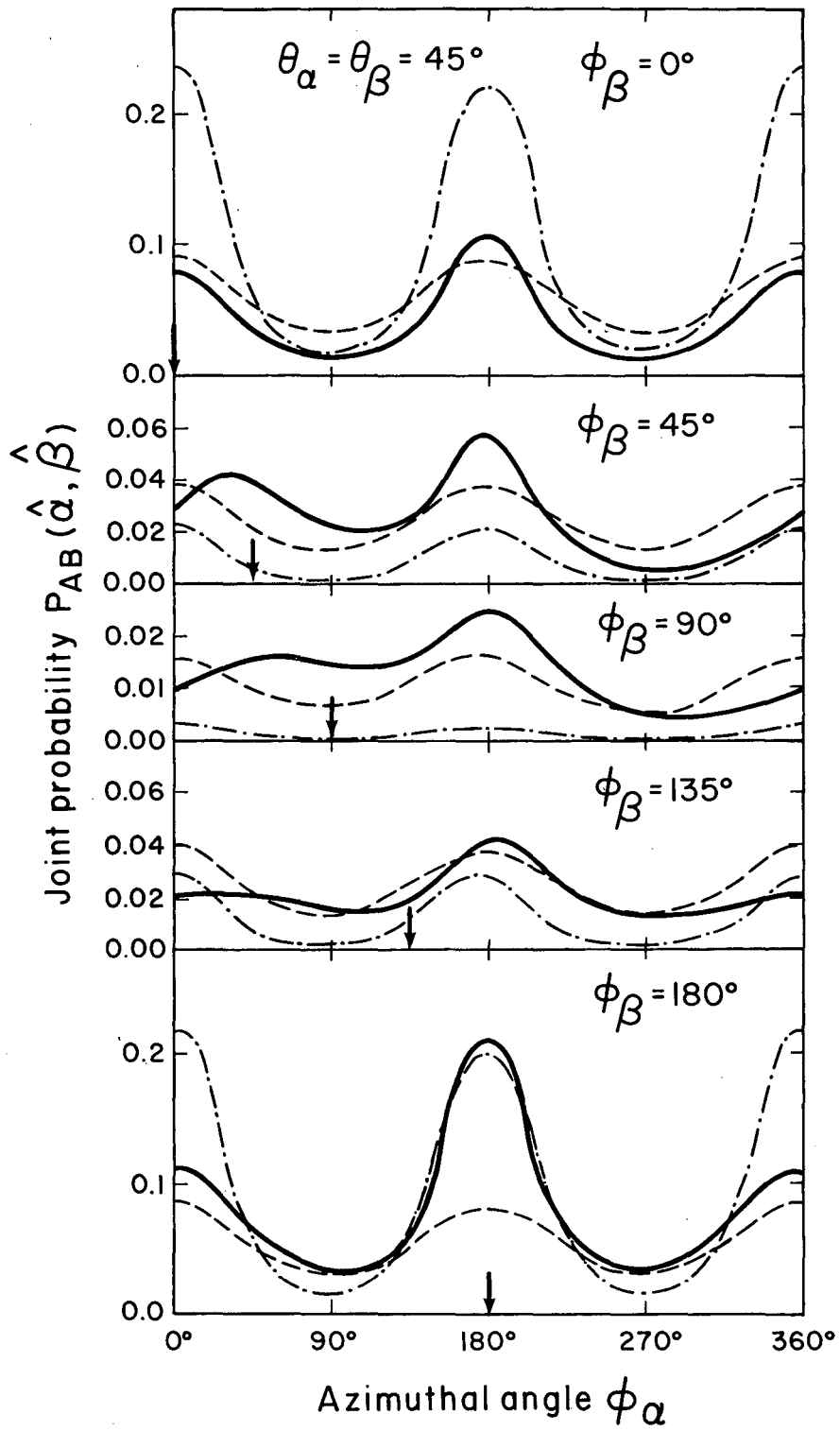| J($\hbar$)	Q(MeV)	τ(MeV)	S^{Pb}($\hbar$)	S_3^{Pb}($\hbar$)	S^U($\hbar$)	S_3^U($\hbar$)	θ_{CM}
600	31	0.8	22	17	24	18	66°.5
550	130	1.5	50	42	53	44	67°.3
500	204	1.9	60	47	64	51	68°.6
450	248	2.1	62	43	67	49	71°.5
400	279	2.2	63	38	68	44	76°.5
350	312	2.4	64	32	69	39	83°.6
300	317	2.4	65	27	69	34	92°.9

Figure Captions

Fig. 1.   Illustration of the geometry of the reaction. The two reaction

products A and B emerge in the reaction plane. The angular

directions of ejectiles from A are referred to a coordinate system

which is comoving with A, has its z-axis in the direction of motion

(as seen in the overall CM system), and its y-axis parallel to the

reaction normal. The directions of ejectiles from B are referred to

a coordinate system with axes parallel to the above one, but

comoving with B. The contours of the spin distribution are

indicated in the reaction plane; the 3-axis points along the

reaction normal, while the major in-plane axis (the 2-axis) points

approximately in between the directions of the beam and A.

Fig. 2.   The joint emission probability $P_{AB}(\hat{\alpha},\hat{\beta})$ for two settings of the

polar angles: $\theta_\alpha = \theta_\beta = 45°$ (a) or $\theta_\alpha = \theta_\beta = 90°$ (b), as a

function of the azimuthal angle φ_α for fixed values of φ_β. The

solid curves indicate the result of the dynamical nucleon-exchange

model, while the others are the results of the statistical model

with the separation R taken as either the minimum separation

encountered (dot-dash) or the scission separation (dash). The

results are calculted for 1768 MeV ^{208}Pb + ^{238}U with J = 500 $\hbar$.

Fig. 1

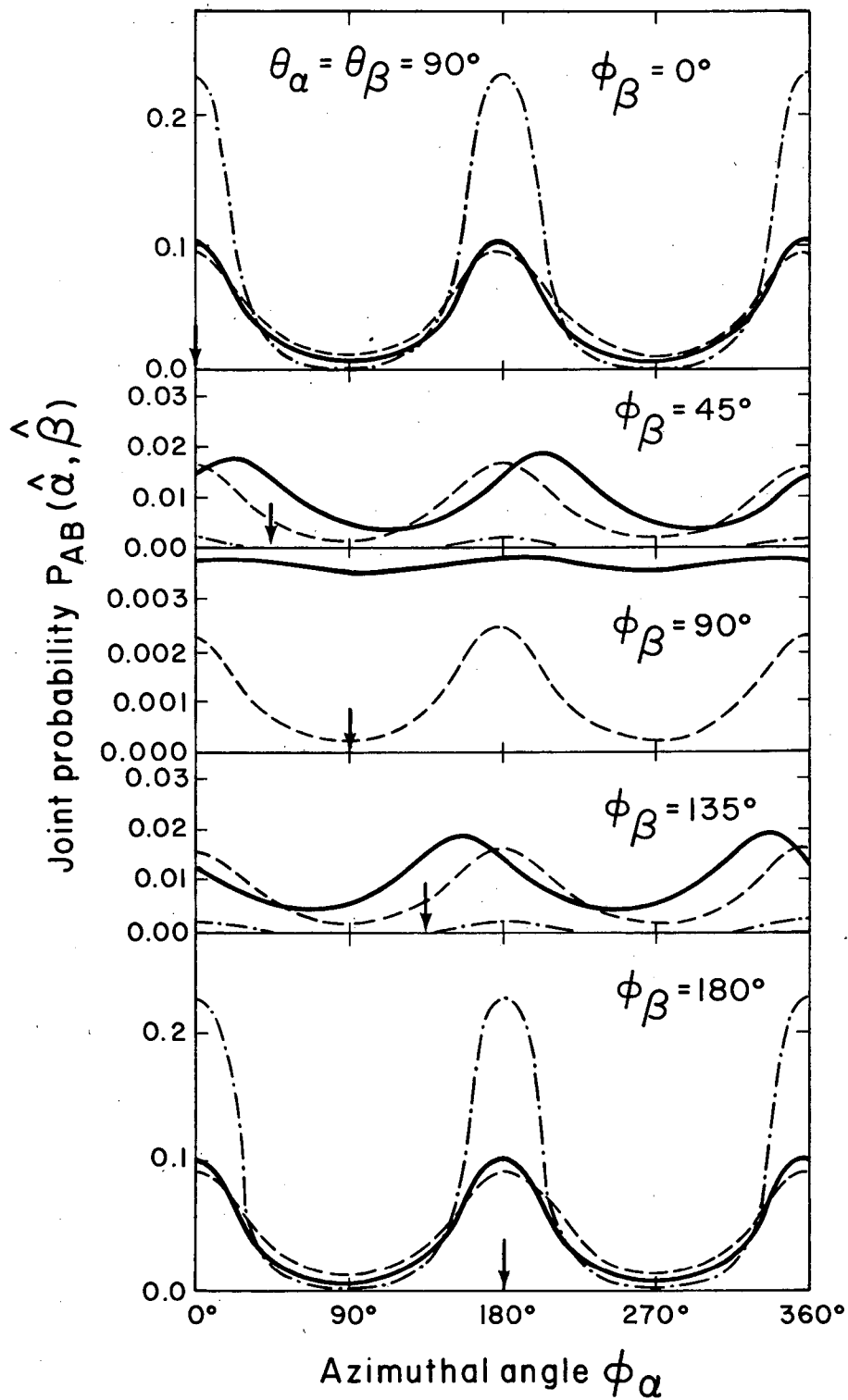Fig. 2a

XBL 8110-1412

Fig. 2b

XBL 8110-1413