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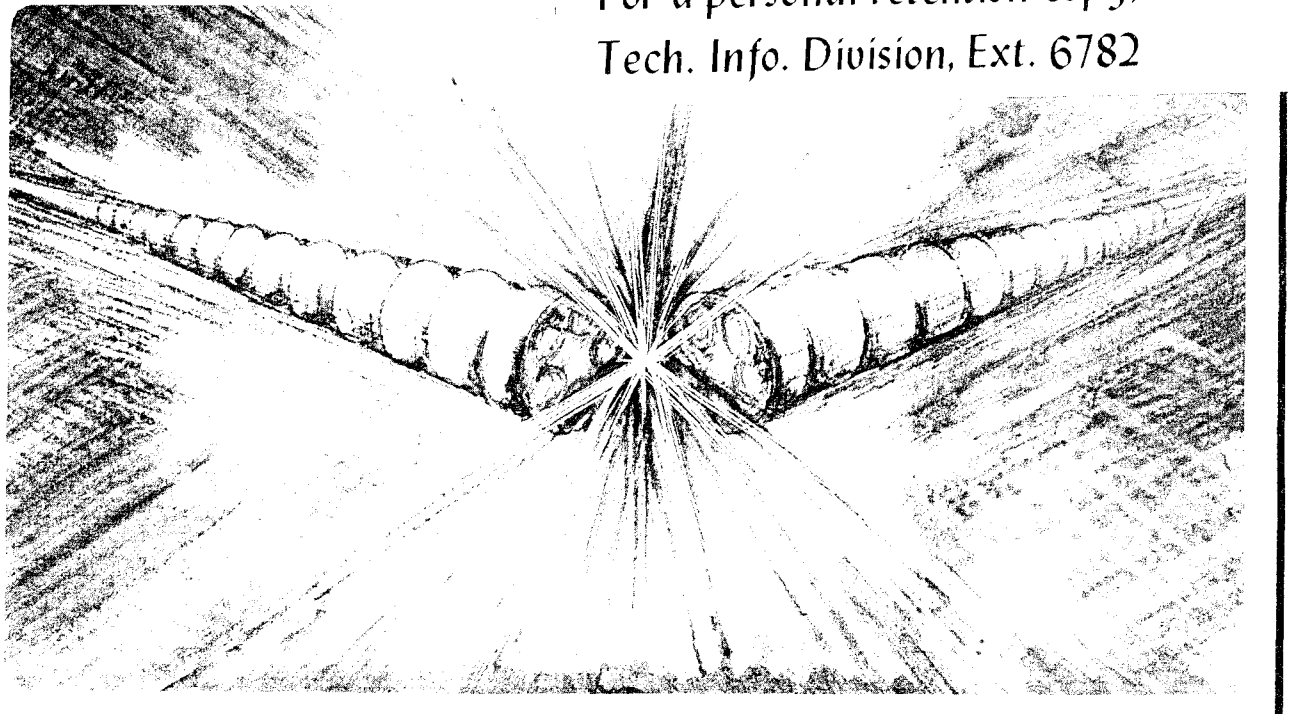

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C. Litwin,* M. C. Vella,** A. Sessler**

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Lawrence Berkeley Laboratory
University of California
Berkeley, CA 94720

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* Present address, Cornell University Ithaca, N.Y.
**Lawrence Berkeley Laboratory, Berkeley, CA.

Abstract

Linear plasma fluid theory is used to study the stability of a cold electron beam in Brillouin equilibrium which passes through a stationary cold ion background, with particular interest in stability for parameters relevant to EBIS devices. Dispersion is studied both analytically and numerically. For $\ell=0$, the usual infinite medium two stream instability condition is shown to correspond to a requirement that beam perveance exceed a minimum value, $P>33$ μpervs; hence, this mode is stable for EBIS ($P \approx 1$μperv). The Brillouin equilibrium rotation is shown to cause an electron-ion rotating stream instability, which is convectively unstable. The $\ell=1$ mode is also found to be unstable. Higher modes numbers, $\ell>1$, are unstable, but have reduced qrowth. Instability is only weakly affected by finite beam radius and boundary conditions.

1. General Formalism

The electrostatic stability of the Electron Beam Ion Source (EBIS)[1,2,3] is studied with linear cold plasma fluid theory. Analytic and numerical methods are used to study the linear dispersion of a finite radius electron beam passing through a partially neutralizing ion space charge. A rigid rotor equilibrium, Brillouin flow[4], is assumed for the un-neutralized beam. A two component cold fluid plasma is also assumed. The effect of secondary electrons on stability is neglected.

The plasma is assumed to be very long in the Z-direction, with $\partial_z n_{so}=0$ and $\partial_z v_{so}=0$, where: n, denotes particle density; v, fluid velocity; subscript, s, species; and superscript, o, an equilbrium quantity. The axial component of the electric field, is neglected, $E_{oz}=0$, and axial symmetry of the equilibrium is assumed, $n_{so}(\vec{x})=n_{so}(r)$ and $v_{so}(\vec{x}) = v_{so}(r)$. It follows that neither v_{so} nor B_o have radial components, i.e., $\vec{B}^o = \left[B_o + B_z(r) \right] \hat{z} + B_\theta(r)\hat{\theta}$, where B_o is generated externally, and B_z and B_θ are generated by equilibrium currents. For a low current, non-relativistic electron beam, B_θ may be neglected, i.e., self-pinching does not affect the equilibrium. In cases of interest, the diamagnetic field, $B_z(r)$, is also neglegible. The radial electric field is obtained by solving the Poisson Equation, assuming the equilibrium density,

$$E_{or}(r) = \frac{4\pi}{r} \sum_s \int_0^r dr' \, r' \, n_{os}(r'). \qquad (1)$$

From the radial component of the cold fluid momentum equation, the general equilibrium condition is,[4]

$$\omega_s^2(r) + \sum_q \frac{4\pi\, e_s\, e_q}{m_s\, r^2} \int_0^r dr'\, r'\, n_{os}(r') + \mathrm{sgn}\left[e_s \Omega_s\, {}_s(r)\right] = 0, \qquad (2)$$

where, $\Omega_s = e_s B_o / m_s c$. For a square density profile, we have

$$n_{os}(r) = \left\{ \begin{array}{ll} n_{os}, & o \leq r \leq R_p \\ o\ , & r \geq R_p \end{array} \right\} \quad , \qquad (3)$$

where R_p denotes the nominal plasma radius. The equilibrium condition now becomes algebraic and corresponds to rigid rotation, with,[4]

$$\omega_e(r) \equiv \omega_e^{\pm} = \frac{\Omega_e}{2} \left\{ 1 \pm \left[1 - \frac{2\omega_{pe}^2}{\Omega_e^2} (1-f) \right]^{1/2} \right\} , \qquad (4)$$

and,

$$\omega_i(r) \equiv \omega_i^{\pm} = \frac{\Omega_i}{2} \left\{ 1 \pm \left[1 + \frac{2\omega_{pi}^2}{\Omega_i^2} \frac{(1-f)}{f} \right]^{1/2} \right\} , \qquad (5)$$

For future use, the vortex frequency is now defined, as,

$$\omega_{vs} \equiv -2\omega_s + \Omega_s\ \mathrm{sgn}\ (e_s) = \pm\, (\omega_s^+ - \omega_s^-) \quad . \qquad (6)$$

Electrostatic stability can be studied by the usual first order perturbation theory, with a potential of the form, $\Psi(x,t)=\Psi_o(x)+\delta\Psi(x,t)$, where,

$$\delta\Psi(r,\ \Theta,\ z) = \sum_{\ell=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \delta\Psi^{\ell}(r,k_z)\ e^{i(\ell\Theta + k_z z - \omega t)} \qquad (7)$$

with $k_z=2\pi n/L$, where L represents the system length, and $n=0, \pm1, \pm2, \ldots$.

Following Davidson[4], a plasma column of radius, R_p, is considered, surrounded by a conducting wall of radius, R_c, with boundary conditions: (i) electrostatic potential, Ψ, is finite at r=0; (ii) Ψ is continous across the plasma boundary; and (iii) Ψ vanishes on the conducting wall. From the fluid plasma theory, the full dispersion relation is found to be,[4]

$$B(k_z,\ R_p,\ R_c) = \left(1 - \sum_s \frac{\omega_{ps}^2}{\upsilon_s^2}\right) TR_p \frac{J_\ell'(TR_p)}{J_\ell(TR_p)}$$

$$+ \ell \sum_s \left[\frac{\omega_{ps}^2\ \omega_{\upsilon s}}{\upsilon_s^2\ (\omega - k_z V_{sz} - \ell\omega_s)}\right] \quad . \qquad (8)$$

For convenience, the following notation has been adopted:

$(v_{so})_z \equiv V_{sz}$;

$$\upsilon_s^2 \equiv (\omega - kV_{sz} - \ell\omega_s)^2 - \omega_{\upsilon s}^2 \quad ; \qquad (9)$$

and,

$$T^2 \equiv -k_z^2 \frac{\left[1 - \sum_s \omega_{ps}^2 / (\omega - k_z V_{sz} - \ell\omega_s)^2\right]}{\left[1 - \sum_s \omega_{ps}^2 / \upsilon_s^2\right]} \quad . \tag{10}$$

The boundary term is given by,

$$B(k_z, R_p, R_c) = k_z R_p \frac{K_\ell(k_z R_c) I_\ell'(k_z R_p) - K_\ell'(k_z R_p) I_\ell(k_z R_c)}{K_\ell(k_z R_c) I_\ell(k_z R_p) - K_\ell(k_z R_p) I_\ell(k_z R_c)} \quad , \tag{11}$$

where I and K represent modified bessel functions of order ℓ. Stability is determined by solving the dispersion assuming real k, where $Im(\omega)>o$ corresponds to instability.

The nature of unstable growth, whether absolute or convective,[5] is important in an experimental device. For a general dispersion relation, expressed as, $D(\omega,k)=0$, the necessary condition for absolute growth is, $\partial_k D=0$, or, equivalently, $\partial_k \omega=0$. The velocity of propagation of an instability can be determined if one observes that an instability which is convective in the laboratory frame, is absolute in the co-moving frame. Denoting the velocity of this frame by V, the frequency would be $\omega' = \omega - kV$. From the definition of the frame it follows that

$$\partial_k \omega' = o = \partial_k \omega - V = \partial_k \omega_r - V + i\partial_k \omega_i$$

which may be rewritten,

$$V = \partial_k \mathrm{Re}(\omega)\Big|_{k=k_o} \quad \text{and} \quad \partial_k \mathrm{Im}(\omega)\Big|_{k=k_o} = 0.$$

Thus, the velocity of propagation is given by $\partial_k \mathrm{Re}(\omega)$ at the point of the maximal growth. If the velocity of the instability is so large, that it leaves a system of size L before appreciable growth occurs, i.e., if, $\mathrm{Im}(\omega)\ L/V \leq 3$, then the mode is said here to be convectively stable.

In the discussion of beam stability which follows, $l=0$ and $l\neq 0$ are considered separately. All regions of unstable growth have been studied numerically and been found to be convective; hence, the convective criterion is used throughout.

2. $\ell=0$

The following notation is adopted for convenience,

$$T_1 \equiv 1 - \frac{\omega_{pe}^2}{(\omega - kV)^2} - \frac{\omega_{pi}^2}{\omega^2} , \tag{12}$$

and

$$T_2 \equiv 1 - \frac{\omega_{pe}^2}{(\omega - kV)^2 - \omega_{\upsilon e}^2} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{\upsilon_i}^2} \tag{13}$$

where no equilibrium ion drift is assumed, i.e., $V_{zi} = 0$, and $V_{ze} \equiv V$. For $\ell=0$, dispersion relation may now be written,

$$B\Big|_{\ell=o} = - T_2 \; TR_p \; \frac{J_1(TR_p)}{J_o(TR_p)} , \tag{14}$$

where,

$$T^2 = - k^2 \; T_1 / T_2 \tag{15}$$

An alternative form which will also be used is,

$$B = 2 k_z^2 R_p^2 T_1 \sum_{n=1}^{\infty} \frac{1}{Z_{o,n}^2 - T^2 R_p^2} , \qquad (16)$$

where, $Z_{o,n}$ is the n-th zero of J_o. Eq. (16) has been derived from the product representation of J_o,

$$J_o(z) = \prod_{n=1}^{\infty} (1 - z^2/Z_{o,n}^2) ,$$

using, $J_o' = -J_1$, to obtain,

$$- \frac{J_1(z)}{J_o(z)} = \frac{d}{dz} \ell n(J_o(z)) = \sum_{n=1}^{\infty} \frac{d}{dz} \ell n(1 - z^2/Z_{o,n}^2)$$

$$= -2z \sum_{n=1}^{\infty} \frac{1}{(Z_{o,n}^2 - z^2)} . \qquad (17)$$

Since we only are concerned with the linear stability of the system, attention is focused on those regions of parameter space in which growth rates can be appreciable. A dimensionless parameter which will be used in the following discussion is, $\chi \equiv \omega_{pe} R_p / V$, which

can be rewritten,

$$\chi^2 = \left(\frac{2m_e}{e}\right)^{1/2} \frac{I}{V_b^{3/2}} = 0.96\ I(A)/V_b^{3/2}(kV)$$

$$= p(\mu pervs)/33. \qquad (18)$$

EBIS beams are typically a few μpervs, which corresponds to $\chi^2 << 1$.

One consequence of the above property is that, for EBIS, instability can occur only in the long wavelength regime. This can be seen by noting that, from the definition of T_1 and T_2, the ions contribute to dispersion only if $|\omega| \lesssim \mathcal{O}(\omega_{pi})$. For a beam of good quality, i.e., sufficiently near Brillouin flow that $|\omega_{ve}| < \omega_{pe}$, this implies that electrons contribute to dispersion only if, $|\omega - kV| \lesssim \mathcal{O}(\omega_{pe})$. Since instability requires that both species contribute, this implies a necessary condition for growth, $|kV| \lesssim \omega_{pe}$. From the definition of χ and equation (18),

$$|kV| \lesssim \omega_{pe} \Rightarrow |kR_p| < |\chi| << 1. \qquad (19)$$

Asymptotic approximations may be used for the boundary term,

$$B(k) \approx 1/\ell n\ (R_p/R_c); \quad |kR_p| << 1, \quad |kR_c| << 1$$

$$\approx 1/\ell n\ (kR_p) \ ; \quad |kR_p| << 1, \quad |kR_c| << 1. \qquad (20)$$

From (16), for $S \cong \sum_{n=1}^{\infty} 1/(1/Z_{o,n}^2 - T^2 R_p^2)$, dispersion implies,

$|T_1 S| = B/2k^2R_p^2 \gg 1$, which allows three possibilities:

$$\text{(i.)} \quad |T_1| < 1, \ |S| \geq 1;$$

$$\text{(ii.)} \quad |T_1| \gg 1, \ |S| \gg 1;$$

$$\text{(iii.)} \quad |T_1| \gg 1, \ |S| < 1.$$

By definition of S, Case (i) implies $T \longrightarrow Z_{o,n}$, and, from (14), $|T_2| \longrightarrow 0$. From the previous discussion, the dispersion relation, given by $T_2 = 0$, can be solved for the fastest growing modes by using $(\omega - kV)^2, \approx k^2V^2$ which gives,

$$\omega \approx \pm \left[\omega_{\nu i}^2 + \omega_{pi}^2 \quad \frac{k^2V^2 - \omega_{\nu e}^2}{k^2V^2 - \omega_{\nu e}^2 - \omega_{pe}^2} \right]^{1/2}, \tag{21}$$

which is purely imaginary for,

$$\omega_{\nu e}^2 + \omega_{pe}^2 \quad \frac{\omega_{\nu i}^2}{\omega_{\nu i}^2 + \omega_{pi}^2} \quad < k^2V_{ez}^2 < \omega_{\nu e}^2 + \omega_{pe}^2. \tag{22}$$

Maximum growth occurs at, $k_o^2V^2 = \omega_{\nu e}^2 + \omega_{pe}^2$. Although the approximation used to obtain (21) breaks down, substituting for k_o into $T_2 = 0$ gives a maximum growth rate,

$$\gamma_{max} = \frac{\sqrt{3}}{2} \left[\frac{\omega_{pe}^2 \ \omega_{pi}^2}{2(\omega_{pe}^2 + \omega_{pi}^2)^{1/2}} \right]^{1/3} \tag{23}$$

The velocity of propagation at maximum growth may be estimated from $k\omega_r \Big|_{k=k_o} \approx V/3$. Since $\gamma_{max} \ 3L/V_{ez} \gg 3$, this mode is

convectively unstable for EBIS. Insight into the nature of the instability can be gained by noting that if the vortex terms, ω_{ve} and ω_{vi}, were not present, the dispersion relation would be the same as for the infinite medium two stream instability (i.e., $T_1=0$), which will be discussed below. Thus, this long wavelength instability corresponds to the two stream instability modified by the equilibrium rotation (Ref. Equations 21-23), and will be referred to as an electron-ion rotating two stream instability.

Case (ii) requires $T_1 \approx \omega_{pe}^2 / (\omega-kV)^2 >> 1$, or $\omega_r \approx kV$ (Note: $T_1 >> 1$ and $\omega_{pi}^2/\omega^2 >> 1 \Rightarrow \gamma << \omega_{pi}$). Since $|S| >> 1$, $TR_p \longrightarrow Z_{o,n}$, using (14), this implies $T_2 \longrightarrow 0$. From (22), this corresponds to $\omega^2 \approx \omega_{pe}^2 + \omega_{ve}^2$, with γ at most of $\mathcal{O}(|\omega-kV| \cdot \omega_{pi}/\omega_{pe})$.

Case (iii) corresponds to $|T_2| \geq 1$. With slight modification, the previous argument applies, i.e., any growth rate is small compared with ω_{pi}.

In the notation used above, the usual infinite medium two stream instability would fall under the category, $|T| < 1$, $|S| \leq 1$, a case which was eliminated <u>a prior</u> by the requirement $|k R_p| << 1$. If the EBIS perveance condition is relaxed to allow $\chi^2 \geq 1$, from (16), the case $|kR_p| >> 1$ corresponds to $T_1 \approx 0$. This is just the usual two stream dispersion relation, with a well known instability condition,[6] $kV^2 < (\omega_{pe}^{2/3} + \omega_{pi}^{2/3})^3$. Multiplying by $(R_p/V)^2$, the usual two stream instability condition corresponds to $\chi^2 \geq (kR_p)^2 >> 1$. From the earlier discussion, EBIS beam perveance is well below the requirement for the two stream instability in this system.

2. First Rotational mode: $\ell=1$

In the long wave-length approximation the $\ell=1$ mode is described by the dispersion relation (Ref. 4, Eq. 2.9.13)

$$\frac{2}{1 - (R_p/R_c)^2} = \frac{\omega_{pe}^2}{(\omega-kV-\omega_e^-)(\omega-kV-\omega_e^+)} + \frac{\omega_{pi}^2}{(\omega-\omega_i^-)(\omega-\omega_i^+)} \quad . \quad (25)$$

Away from the Brillouin limit, i.e., for $2\omega_{pe}^2(1-f)/\Omega_{ce}^2 \ll 1$, an electron-ion rotational instability has been already been found,[4] but this is not relevant to EBIS. Near Brillouin flow, $2\omega_{pe}^2(1-f)/\Omega_{ce}^2 = 1$, we have $\omega_e^+=\omega_e^-= \Omega_{ce}\ z$ and $\omega_i^-=-\omega_i^+= (m_e/m_i)^{1/2}\ \Omega_{ce}/2$.
The dispersion relation becomes,

$$\frac{2}{(1 - R_p/R_c)^2} = \frac{\omega_{pe}^2}{(\omega-kV-\omega_e)^2} + \frac{\omega_{pi}^2}{\omega^2 - \omega_i^2} \quad . \quad (26)$$

This is similar to the previously considered case of $T_2=0$ in the Brillouin limit ($\omega_{ve}=0$). For simplicity, we take $R_p/R_c \ll 1$.
Then, instabiltiy can only appear for, $|\omega| \sim \omega_i \ll \omega_e$, in which case,

$$\omega^2 \approx \omega_i^2 + \omega_{pi}^2 \frac{(\omega_e + kV)^2}{2(\omega_e + kV)^2 - \omega_e^2} \quad . \quad (27)$$

The condition for instability, $\omega^2<0$ is now,

$$\omega_i^2 < \omega_{pi}^2 \frac{(\omega_e+kV)^2}{\omega_{pe}^2 - 2\ (kV+\omega_e)^2} \quad . \quad (28)$$

Using the Brillouin condition, this is equivalent to $kV>0$. Maximum growth is found in the long wavelenght approximation, $\gamma_{max} \approx \sqrt{3}\,(2\,\omega_{pi}^2\omega_{pe})^{1/3}/4$, moving with a group velocity of V/3.

3. Higher Order Rotational Modes, $\ell > 1$

For $\ell > 1$, the dispersion relation becomes (Ref. 4, Eq. 2.9.3),

$$\frac{2}{1 - (R_p/R_c)^{2\ell}} = \frac{\omega_{pe}^2}{(\omega - \ell\omega_e - kV)(\omega - \ell\omega_e - k_z V - \omega_{\nu e})}$$

$$+ \frac{\omega_{pi}^2}{(\omega - \ell\omega_i)(\omega - \ell\omega_i + \omega_{\nu i})} \quad . \tag{29}$$

For parameters relevant to EBIS, $\omega_e \approx \mathcal{O}(\omega_{pe})$ and $R_p/R_c << 1$, this leads to,

$$(\omega - \ell\omega_i)^2 + \omega_{\nu i}(\omega - \ell\omega_i) - \frac{\omega_{pi}^2(\ell\omega_e + kV)(\ell\omega_e + kV - \omega_{\nu e})}{2(\ell\omega_e + kV)(\ell\omega + kV - \omega_{\nu e}) - \omega_{pe}^2}$$

$$= 0 \quad . \tag{30}$$

The condition for instability becomes,

$$\omega_{\nu e}^2 + 2\omega_{pe}^2 \frac{\omega_{\nu i}^2}{\omega_{\nu i}^2 + 2\omega_{pi}^2} < (\ell\omega_e + kV)^2 < \omega_{\nu e}^2 + 2\omega_{pe}^2$$

or,

$$\Omega_{ce}^2 \leq (\ell\omega_e + kV)^2 < 2\omega_{pe}^2 f + \Omega_{ce}^2 \tag{31}$$

This implies that growth rates are reduced for large ℓ, and that only a finite number of modes can be unstable.

5. Conclusions

Linear cold plasm fluid theory has been used to study the electrostatic stability of EBIS devices by considering a finite radius electron beam in a Brillouin-type equilibrium which passes through a stationary ion background and is bounded by a finite radius conductor. For $\ell=0$, the usual infinite medium two stream instability has been found to have a critical perveance for onset, $P>33$ (μpervs), and is, therefore, stable for EBIS ($P\approx 1\mu$perv). However, a convectively unstable electron-ion rotating stream instability has been found, this is driven by a combination of streaming plus the natural rotation of the Brillouin equilibrium. It has been shown that the conditions for the onset of convective growth can be satisfied in EBIS. These results are only weakly affected by the boundary conditions. A non-axisymmetric instability, $\ell=1$ was also found to be convectively unstable.

The existence of the predicted modes in EBIS devices has yet to be confirmed by detailed experimental measurement, although unidentified electrostatic oscillations have been obseved.[7] Because growth is convective, the $\ell=o$ mode may grow without necessarily disrupting beam propagation. The $\ell=1$ modes could conceivably introduce $\vec{E}x\vec{B}$ effects, but would also not be disruptive for a convective instability, since any motion across $\vec{B}$ must include the ions and can only occur at the relatively slow Alfven speed. The primary concern about the effect of convective electrostatic modes is the possibility of ion heating, which could conceivable hinder the collapse of the beam to high current density[3] and is generally undesirable in an accelerator ion source.

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