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UNIVERSITY OF CALIFORNIA

Los Angeles

**Essays on Return Predictability in Financial
Markets**

A dissertation submitted in partial satisfaction
of the requirements for the degree
Doctor of Philosophy in Economics

by

Chan Rathana Mang

2012

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ABSTRACT OF THE DISSERTATION

Essays on Return Predictability in Financial Markets

by

Chan Rathana Mang

Doctor of Philosophy in Economics

University of California, Los Angeles, 2012

Professor Aaron Tornell, Chair

My thesis examines return predictability in government bond markets and currency markets. In Chapter 1, I take the term structure model in Cochrane and Piazzesi (2008) and construct currency market prices. The implied currency market prices are then counterfactually volatile and predictable, at least with respect to commonly used predictor variables. Getting the model closer to currency market data means reducing bond risk compensation but doing so nearly eliminates predictability in bond markets. One way to generate sensible time-variation in bond and currency risk-premia allows the volatility of returns to be time-varying. In Chapter 2, I test to see if alternative forecast rules perform better than the return-forecasting factor of Cochrane and Piazzesi (2008). I compare forecasts assuming all historical data is available to recursively made ones that are revised with the arrival of news. Differences in the two forecast rules systematically move with realized bond risk-premia and forecast mean yield curve levels and short-term interest rates one year ahead not just for the U.S., but also for government bond markets of other industrialized economies. I show that lower long-term rates relative to short-rates in 2004-2005 is consistent with an expected a decline of interest rates by market participants. In Chapter 3, I show that cross-sectional average of the spread in the return-forecasting factor of Cochrane and Piazzesi (2005, 2008)

can forecast currency risk-premia. However, the return-forecasting factor spread consistent with real-time data does not forecast currency risk-premia. I also find that exchange rate changes have a predictable component that is detected by the information gap, what I call the hidden FX market factor, between forecasts that takes as given the full sample of data and those consistent with real-time availability. This information gap also forecasts currency risk-premia. Controlling for large and transitory exchange rate changes using this information gap make interest rate differentials between the average foreign country and the U.S. positively correlated with dollar appreciation rates, delivering the right sign predicted by uncovered interest parity.

The dissertation of Chan Rathana Mang is approved.

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*To my family and Nikki for their patience ...
among so many other things.*

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CHAPTER 1

Uncertain Risk and Return in Bond Markets, I

1.1 Introduction

I take the term structure model in Cochrane and Piazzesi (2008) and construct currency market prices to see if they are consistent with predictable returns in bond and foreign exchange markets. I find that the predictable component in currency markets using bond risk prices from their model is substantially larger than commonly employed predictor variables imply. In doing so, I investigate the foreign exchange market implications of linear factor models that capture a sizable predictable component in bond risk-premia. My results suggest that linear factor models require modification before they are capable of jointly capturing predictable bond and currency risk-premia.

I document several findings in this paper. After running similar regressions as those performed in Cochrane and Piazzesi (2005), I find that bond risk-premia are as predictable in industrialized countries as they are in the U.S. I then determine whether the mentioned authors' term structure model can deliver an empirically consistent view of exchange rates and interest rates across countries. I find that it cannot. The model is unable to do so because the discount factor used to capture time-variation in bond returns lead to incredible currency market prices. I pinpoint the underlying reasons why the model is unable to describe key features of currency market data and highlight how the amount of return predictability can directly be linked to variation in market risk prices. Long story short, there

is strong circumstantial evidence that the volatility of returns is time-varying because admitting that there is not much predictability at all is inconsistent with empirical evidence in international bond markets. Unfortunately, the model specification necessarily rules out time-varying volatility.

Conditionally heteroskedastic returns can generate more sensible movements in both the pricing kernel and conditional Sharpe ratios, but it is also worth considering the possibility that the amount of measured predictability cannot be attributed to the risk factors in the stochastic discount factor alone. Later in the paper, I will discuss other avenues that can generate the amount of measured return predictability.

Amidst the diverse set of approaches, I choose the four-factor affine term structure model of Cochrane and Piazzesi (2008) as the foundation of my analysis because it incorporates key information in forward rates that have been shown to forecast well expected returns in bond markets. Among the industrialized countries under study, the term structure of interest rates displays similar comovement. Thus, international comparisons may uncover useful results and reveal global forces affecting bond and currency risk-premia.

The results of my paper are not necessarily negative. The limitations I highlight in the paper should not discourage us from pursuing a deeper understanding of bond and currency prices. Rather, by acknowledging that the complex nature of bond price movements cannot satisfactorily be approximated with a class of linear models, I hope to direct the literature to search for deeper explanations for predictable returns in bond and currency markets and to help reveal the underlying, macroeconomic reasons for why expected returns vary over time.

1.2 Literature Review

My work builds directly upon the term structure model of Cochrane and Piazzesi (2008) but also upon Fama and Bliss (1987) and Cochrane and Piazzesi (2005) who document predictable variation in expected returns using forward interest rates. I extend their analysis to other bond markets using only a subset of available forward rates, similar to Tang and Xia (2007) who also document predictable bond risk-premia in international markets under a different model specification than my own.

Similar work documenting predictable variation in bond markets within the context of a term structure model have been done by Duffee (2002, 2010) and Dai and Singleton (2002), who model returns as conditionally heteroskedastic. Adopting the more flexible specification for the market price of risk enables them to characterize time-varying risk and in doing so, account for the failure of the expectations hypothesis, making it a leading term structure model of interest rates alongside the model of Cochrane and Piazzesi (2008) my paper carefully examines. In adopting the model of Cochrane and Piazzesi (2008), I make two choices. One, I specify that returns are conditionally homoskedastic, which is in contrast to Dai and Singleton (2002). Two, I adopt a flexible specification for the market price of risk as does Duffee (2002, 2010), although I use an additional variable to do so. Both specification choices allow the model to precisely characterize risk adjusted dynamics, and my paper examines the implications of this specification choice.

Empirical research documenting predictability in currency markets dates back further than that of bond markets. The analysis in my paper tests if we can simultaneously capture predictability in bond and currency markets, especially in light of new findings by Lustig et al. (2009, 2010) who show that there is a large predictable component in currency markets for a portfolio of currencies. My analysis is similar to Backus, Foresi, and Telmer (2001) who also link bond

markets to currency markets using stochastic discount factors implied by an affine model, and determine whether it can account for the forward premium anomaly.

Because the literature is rather deep, I will not be able to satisfactorily discuss the numerous important findings. I take the convenient stance and refer the reader to the references in this paper for further references. Given that my objective for this paper is to critically examine our understanding of bond markets with term structure models, I believe that the brief list of references in this paper is sufficient.

Section 3 covers the empirical findings on predictability in international bond markets. In Section 4, I explicitly link bond markets to currency markets through equilibrium asset pricing conditions and then outline the tension of fitting yields. In section 5, I discuss the affine term structure model along with a description of the implications of the model that are important for the paper. In section 6, the results explore the currency price implications of the model. In section 7, I discuss my findings in relation to the broader literature. Section 8, I conclude. Section 9 is the appendix.

1.3 International Predictability Regressions

1.3.1 Definitions

Let $P_t^{(n)}$ be the price of an n -maturity bond and $p_t^{(n)}$ be the log price. From now on, all lowercase variables are log variables. Zero-coupon bond prices are easily mapped into yields. The interest rate of an n -maturity bond is then $y_t^{(n)} = -\frac{1}{n}p_t^{(n)}$. The log return to holding an n -maturity bond for one period is $hpr_{t+1}^{(n)} = p_{t+1}^{(n-1)} - p_t^{(n)}$. The return in excess of the short rate is the risk-premia on n -maturity bonds, $rx_{t+1}^{(n)} = hpr_{t+1}^{(n)} - y_t^{(1)}$.

1.3.2 Data Description

The zero-coupon yield data are gathered from international bond markets, ranging from January 1973 until May 2009. Only Germany and the U.S. cover this time period. The other countries start dates are within this range and have more than 250 observations. In total, there were ten countries (The U.S., Germany, Japan, Great Britain, Canada, Australia, New Zealand, Norway, and Switzerland, and Sweden) although not all of the countries are highlighted in this paper. The data are of monthly frequency on bond yields of maturities ranging from one year up to ten years, partitioned equally twelve months apart. More simply, the panel of bond market data are for n year-to-maturity interest rates, where $n = 1, \dots, 10$. The forward rates are constructed from bond yield data using the definition given above. Only a subset of forward rates are used in the forecasting regressions, namely the one year forward rate from today to one-year ahead, from year 4 to year 5, and from year 8 to year 9. The subset of forwards is then describe by the set $\zeta := \{0 \rightarrow 1, 4 \rightarrow 5, 8 \rightarrow 9\}$. A more detailed description is relegated to the appendix.

1.3.3 Estimation Procedure and Findings

To construct the return forecasting factor, I stay as close as possible to Cochrane and Piazzesi (2005). To do so, I use country-specific forward rates extracted from zero coupon government bond yield data, although I depart from their approach because I only choose a subset of forward rates. Using the full set of available forward rates generates telltale signs of multicollinearity, and thus exposing the results to the vulnerability of how zero-coupon forward rates were interpolated from the data. The sensitivity of the results to the set of forward rates is a concern, but I mitigate many of the related issues by restricting the set of forwards to ensure that the tent-shape is preserved, a similar to the approach Tang and Xia

(2007). There is no theoretical reason for ensuring that the tent-shape holds, but in practice and as discussed by Singleton (2006), it reduces the possibility that measurement error or the interpolation technique will distort results.

To be clear on notation, let $rx_{t+1,(j)}^{(n)}$ be the realized term-premia for holding onto an n year-to-maturity bond for country j one year from now. Let $\zeta = (\{0 \rightarrow 1, 4 \rightarrow 5, 8 \rightarrow 9\})$ be the subset of forwards. The set of forward rates used in the construction of the return-forecasting factor are given by $\mathbf{f}_{t,(j)} = (f_{t,(j)}^{(0 \rightarrow 1)}, f_{t,(j)}^{(4 \rightarrow 5)}, f_{t,(j)}^{(8 \rightarrow 9)})$ and the coefficients $\boldsymbol{\gamma}_j = (\{\gamma_j^{(0 \rightarrow 1)}, \gamma_j^{(4 \rightarrow 5)}, \gamma_j^{(8 \rightarrow 9)}\})$ where the index $t, (j)$ is meant to describe the date t forward rate for country j . To actually construct the return-forecasting factor, I regress the average term-premia for a cross-section of bonds of maturity n , on the subset of country-specific forward rates:

$$\frac{1}{N} \sum_{n=1}^N rx_{t+1,(j)}^{(n)} = \overline{rx_{t+1,(j)}} = a_{(j)} + \boldsymbol{\gamma}'_{(j)} \cdot \mathbf{f}_{t,(j)} + \varepsilon_{t+1} \quad (1.1)$$

The fitted values are then used as the "local" return-forecasting factor, i.e. the relevant information contained in forward rates given by the expression below

$$\hat{a}_{(j)} + \hat{\boldsymbol{\gamma}}'_{(j)} \cdot \mathbf{f}_{t,(j)} = \hat{a}_{(j)} + \Sigma_{\zeta} \hat{\boldsymbol{\gamma}}_{t,(j)} \cdot \mathbf{f}_{t,(j)} := x_{t,(j)}. \quad (1.2)$$

where Σ_{ζ} denotes that I fit the forwards only for the relevant subset. I should emphasize that I extract the return-forecasting factor using the exact same procedure for each country. In the next step, the following time-series regressions are run to determine if there is bond predictability from holding an n -year maturity bond for one year for each country j

$$rx_{t+1,(j)}^{(n)} = \alpha_j(n) + \beta_j(n) \cdot (x_{t,(j)}) + \varepsilon_{t+1,(j)}^{(n)} \quad (1.3)$$

Before making international comparison, it is worth mentioning the differences in the predictable component that I find using a subset of forwards relative to the benchmark case. I find less predictability using the subset of forward rates, a considerable amount in relation to that found in Cochrane and Piazzesi (2005).

To compare the amount of predictability I find for the U.S. using only a subset of forward rates to Cochrane and Piazzesi (2005, 2008) using the Fama and Bliss forward rates, I plot the two variables along with the realized bond risk-premia for an equally weighted portfolio of returns.

Figure 1.1: Return-Forecasting Factor Comparison

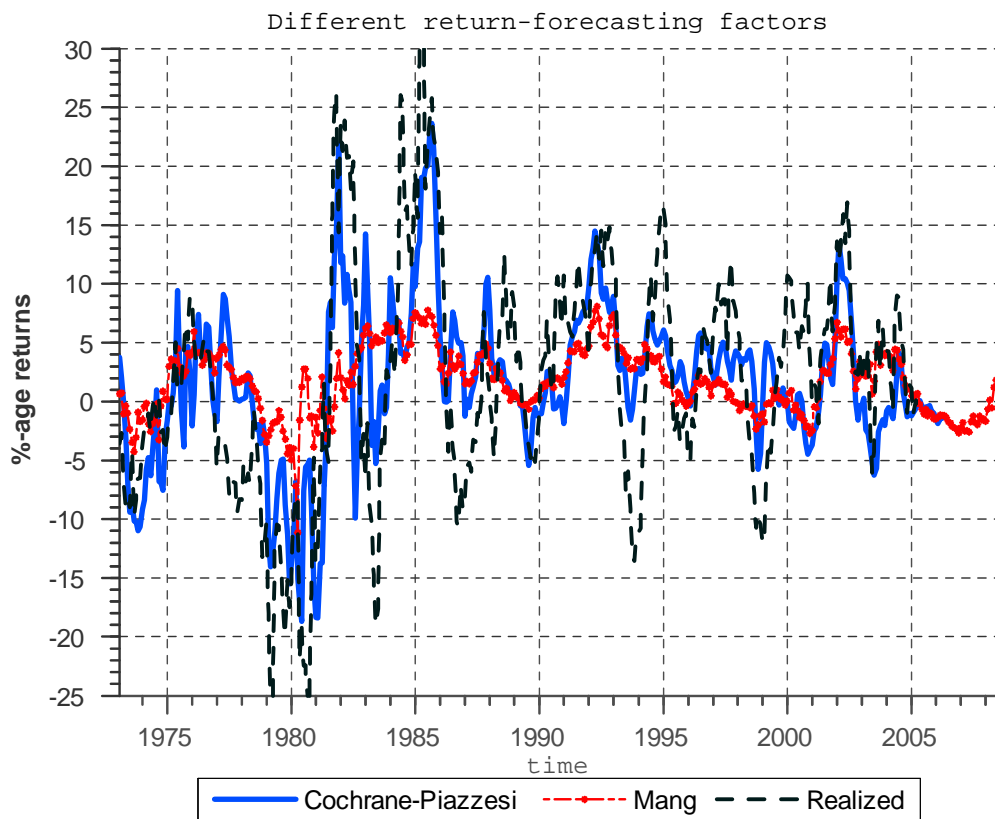


Table 1.1: Comparing Predictable Component Using Different Return-Forecasting Factors

n	2	3	4	5	6	7	8	9	10
	0.17	0.33	0.47	0.60	0.73	0.85	0.97	1.08	1.19
U.S. (CP)	(8.07)	(9.29)	(9.8)	(9.93)	(9.94)	(9.90)	(9.86)	(9.78)	(9.67)
	0.31	0.35	0.39	0.41	0.43	0.43	0.43	0.44	0.44
	0.24	0.65	0.65	1.01	1.01	1.20	1.54	1.70	1.70
U.S. (Mang)	(2.69)	(2.66)	(2.67)	(2.77)	(2.77)	(2.82)	(2.93)	(2.97)	(2.97)
	0.12	0.13	0.14	0.16	0.16	0.17	0.18	0.18	0.18

The estimates are linear GMM estimates with 12 lags to account for overlapping monthly data. The standard errors are Newey-West corrected. Each column n represents the regression of the risk-premia of an n year-to-maturity bond on the subset of forwards. Within each n , the first column displays the intercept term for the respective regressions with t-statistics in parenthesis below. The second column displays the slope coefficients with associated t-statistics below in parenthesis. The last column displays the R-squares of the regression.

The Fama and Bliss forward rates seem to capture more variation in the magnitude changes in excess returns than just a subset of forwards I use, although for the most part my forecasting factor seems to capture variation in the direction of movement in excess returns. The difference in the amount of predictability captured using only a subset of forwards is a reasonable concern, however the reduction in predictability will not materially alter the main results of my paper primarily because it will not significantly affect how the market price of risk parameters are calibrated. These mentioned parameters are chosen to match the amount of variation of returns along the cross-section of bonds, i.e. along the time-to-maturity n dimension. Fortunately, these parameters are not that sensitive to which subset of forward rates I use to extract the return-forecasting factor, at least not when I use the subset ζ . That is, when I compare my estimates using

only a subset of forwards to that of Cochrane and Piazzesi (2008) for the U.S., I find differences in the key market price of risk parameter but the magnitudes are comparable. In short, the main results in my paper are insensitive to the choice to use only a subset of forward rates.

I use linear GMM to estimate (3) to correct for heteroskedasticity and autocorrelation, with 12 lags to account for overlapping monthly data for annualized returns. The displayed estimation results are from unrestricted regressions. The t -statistics and R^2 coefficients are also listed in Table 1.2. Despite some differences in when bond price data are available, the estimates display strikingly similar patterns.

Table 1.2: International Return Predictability Regressions

n	2	3	4	5	6	7	8	9	10
country j									
U.S	0.24	0.45	0.65	0.84	1.01	1.20	1.37	1.54	1.70
	(2.69)	(2.66)	(2.67)	(2.71)	(2.77)	(2.82)	(2.88)	(2.93)	(2.97)
Germany	0.12	0.13	0.14	0.15	0.16	0.17	0.17	0.18	0.18
	(2.38)	(2.50)	(2.58)	(2.67)	(2.77)	(2.88)	(3.00)	(3.12)	(3.23)
England	0.27	0.52	0.76	0.98	1.14	1.26	1.32	1.36	1.39
	(2.36)	(2.43)	(2.50)	(2.58)	(2.66)	(2.71)	(2.67)	(2.56)	(2.39)
Switzerland	0.08	0.09	0.10	0.11	0.11	0.10	0.09	0.08	0.07
	(4.06)	(4.78)	(5.59)	(6.35)	(6.83)	(6.94)	(6.80)	(6.50)	(6.15)
Japan	0.29	0.51	0.71	0.88	1.04	1.19	1.33	1.45	1.57
	(3.45)	(4.35)	(5.08)	(5.52)	(5.70)	(5.71)	(5.64)	(4.78)	(4.72)
Canada	0.17	0.20	0.22	0.23	0.24	0.24	0.25	0.25	0.25
	(1.45)	(1.54)	(1.71)	(1.89)	(2.04)	(2.14)	(2.23)	(2.32)	(2.40)
Australia	0.20	0.38	0.58	0.79	1.00	1.20	1.40	1.61	1.83
	(1.06)	(1.37)	(1.64)	(1.88)	(2.07)	(2.24)	(2.36)	(2.46)	(2.53)
New Zealand	0.05	0.05	0.07	0.09	0.10	0.11	0.12	0.12	0.14
	(5.38)	(6.60)	(7.40)	(7.73)	(7.70)	(7.44)	(7.12)	(6.80)	(6.51)
	0.17	0.38	0.60	0.81	1.01	1.21	1.41	1.61	1.80
	0.05	0.08	0.10	0.12	0.13	0.14	0.15	0.16	0.16
	0.24	0.46	0.67	0.85	1.02	1.19	1.36	1.52	1.68
	(5.38)	(6.60)	(7.40)	(7.73)	(7.70)	(7.44)	(7.12)	(6.80)	(6.51)
	0.30	0.35	0.38	0.39	0.40	0.40	0.40	0.39	0.39

The estimates are obtained using linear GMM. The standard errors are calculated using 12 lags to account for overlapping monthly data and are Newey-West corrected. Each column n represents the regression of the risk-premia of an n year-to-maturity bond on the subset of forwards. Within each n , the first column displays the intercept term for the respective regressions with t-statistics in parenthesis below. The second column displays the slope coefficients with associated t-statistics below in parenthesis. The last column displays the R-squares of the regression.

Predictable returns implies that variation in the return-forecasting factor x_t precede or forecast movements in excess returns one year in the future. Given that the variables are in logs, the regression coefficient reveals that a one percent movement in x_t corresponds to expected movements in the risk-premia to holding onto an n -year treasury by $\beta(n)$ percentage points, or $\beta(n) \cdot 100$ basis points. For example, a one percent movement in the return-forecasting factor will on average precede a 24 basis point movement in the risk-premium for holding a 2-year treasury security in the U.S. over the time period. The empirical results also suggest that larger movements are expected from longer maturity treasuries (e.g. 170 basis point movement in the risk-premium of holding onto 10-year treasury for one year in the U.S. in response to a 1 percent movement in x_t over the sample period). With few exceptions, namely Canada and Australia, the estimated coefficients are statistically significant. Not only that, the large R^2 suggests that a relatively large percentage of movement in country-specific expected returns can be attributed to their country-specific forward rates. In addition, the size of the regression coefficient tell us that even relatively small movements in $x_{t,(j)}$ forecast relatively large movements in $\mathbb{E}_t \left[rx_{t+1,(j)}^{(n)} \right]$, particularly for long-horizon returns. Clearly, there is evidence that suggests bond returns are predictable for this set of industrialized countries, confirming the results found in Cochrane and Piazzesi (2005). One also notices the cross-country pattern of monotone increases in the regression coefficient across maturity, suggesting that the return-forecasting factor

affects bonds of all maturities in a similar fashion. This lends some support to a key identifying restriction made in Cochrane and Piazzesi (2008) that random walk shocks exclusively move risk-premia, one that is used to precisely pin down the dynamics of the model. In all, predictability in bond markets appears to be a robust, global phenomenon.

1.4 Linking Bond and Currency Markets

Before discussing the term structure model, I will make transparent the relationship between bond and currency markets. If markets are complete and there are no riskless arbitrage opportunities, then there exists a unique one-to-one transformation of a domestic stochastic discount factor M_{t+1} to its foreign counterpart M_{t+1}^* . In other words, there exists a unique conversion unit S_t , denominated in foreign currency per dollar, that allows the following equilibrium asset pricing condition to hold simultaneously across two different markets, as shown in the following relation:

$$\mathbb{E}_t [M_{t+1} R_{t+1}] = \mathbb{E}_t \left[M_{t+1}^* R_{t+1} \left(\frac{S_{t+1}}{S_t} \right) \right] \quad (1.4)$$

for some foreign stochastic return process such that that $R_{t+1} \left(\frac{S_{t+1}}{S_t} \right) = R_{t+1}^*$ for any state of the world and any time period. In plain English, we can then express returns in a foreign country as a function of dollar returns using some conversion or exchange rate. In doing so, we can derive an exact relationship between the exchange rate and the relative discount factors for each country. Readers are referred to a proof in Backus, Foresi, and Telmer (2001) for a more complete argument. Under the mentioned conditions, and if exchange rate are in units of foreign currency (Euro) per dollar, then

$$\frac{S_{t+1}}{S_t} = \frac{M_{t+1}}{M_{t+1}^*} \quad (1.5)$$

In logs, we have

$$s_{t+1} - s_t = \Delta s_{t+1} = m_{t+1} - m_{t+1}^* \quad (1.6)$$

where m_{t+1} and m_{t+1}^* is the domestic and foreign (log) stochastic discount factor, respectively. Taking conditional expectations yields

$$\mathbb{E}_t [\Delta s_{t+1}] = \mathbb{E}_t [m_{t+1}] - \mathbb{E}_t [m_{t+1}^*] \quad (1.7)$$

In addition, we can also derive an expression for currency risk-premia using the interest rate differential. If interest rates are well-described by only its first two conditional moments, then $y_t^{(1)} = -\log \mathbb{E}_t [\exp (M_{t+1})] = -\mathbb{E}_t [m_{t+1}] - \frac{1}{2}\sigma_t^2 [m_{t+1}]$. The same is true for foreign interest rates, so $y_t^{(1),*} = -\mathbb{E}_t [m_{t+1}^*] - \frac{1}{2}\sigma_t^2 [m_{t+1}^*]$. Combining the two expressions relates the interest rate differential to the key moments of the stochastic discount factors:

$$y_t^{(1),*} - y_t^{(1)} = \underbrace{(\mathbb{E}_t [m_{t+1}] - \mathbb{E}_t [m_{t+1}^*])}_{\text{dollar appreciation}} + \underbrace{\frac{1}{2} (\sigma_t^2 [m_{t+1}] - \sigma_t^2 [m_{t+1}^*])}_{\text{risk-premium}} \quad (1.8)$$

Thus, the currency risk-premia can be expressed as the following expression:

$$\mathbb{E}_t [rx_{t+1}^*] = y_t^{(1),*} - y_t^{(1)} - \mathbb{E}_t \Delta s_{t+1} \quad (1.9)$$

The exchange rate data is well-approximated by a a random walk, so $\mathbb{E}_t \Delta s_{t+1} \approx 0$, hence we can further simplify the expression for currency risk premia to be equal to one-half the difference in the conditional variance of the pricing kernel between the U.S. and Germany:

$$\mathbb{E}_t [rx_{t+1}^*] = \frac{1}{2} (\sigma_t^2 [m_{t+1}] - \sigma_t^2 [m_{t+1}^*]) \quad (1.10)$$

As one can see from the following expressions, the amount of movement in interest rate differentials is due to either movement in exchange rates and/or currency risk-premia. More importantly, these expressions relate interest differentials explicitly to the first two conditional moments of the pricing kernels for each country. From these expressions, we can see how both moments are affected by the affine model estimation procedure.

1.4.1 The Tension of Fitting Interest Rates

How the model trades off fitting conditional means and conditional variances of the pricing kernel will be critical for the exchange rate implications. Cochrane and Piazzesi (2008) model variation in market risk prices an additional factor, relative to the usual three factors used in term structure models. The model imposes that time variation in the conditional variance of the stochastic discount factor M_{t+1} is *exclusively* driven by the return-forecasting factor. This has implications for how much risk compensation is required to move both bond and currency prices. The large predictable component in bond markets is directly channeled into the conditional variance of the stochastic discount factor through the market price of risk. Because the market price of risk moves around a lot, time-variation in the conditional mean and the conditional variance of the stochastic discount factor must be equally large. This key point can be made without great details about an affine term structure model. According to the affine model, the conditional mean and variance of the stochastic discount factor is directly related the market price of risk in the following set of equations:

$$-\mathbb{E}_t(m_{t+1}) = y_t^{(1)} - \frac{1}{2}\kappa\lambda_t^2 \quad (1.11)$$

$$-\sigma_t^2(m_{t+1}) = \kappa\lambda_t^2 \quad (1.12)$$

The amount of forecastability in bond markets captured by the return-forecasting factor x_t means that the conditional mean of the log pricing kernel has to fluctuate wildly as a consequence of the large required risk compensation for changes in x_t . Although there is some flexibility in choosing how much risk compensation is required over time via the $\kappa\lambda_t^2$ term, the model estimation procedure must trade off which conditional moment to emphasize. It turns out that the conditional mean is given priority *at the expense of* the conditional variance of the discount factor. The severity of this tradeoff can be further examined in currency market prices. Before I do so, let me outline the affine term structure model.

1.5 The Affine Term Structure Model

Given the amount of predictability found in bond market data, using the return-forecasting factor may be useful for characterizing price dynamics within a term structure model. Extracting the standard level, slope and curvature factors follows from a principal component decomposition using the residuals after constructing the return-forecasting Factor, and is thus done using forward rates. The comprehensive procedure for extracting the latent factors can be found in Cochrane and Piazzesi (2008). For the purpose of this paper, I give considerable attention to the predictability characteristics the model implies and what that means for currency markets, so I refer interested readers either to the appendix of this paper or Cochrane and Piazzesi (2008) for more specific details.

To ensure that prices satisfy no-arbitrage, we must ensure that the following condition holds

$$P_t^{(n)} = \mathbb{E}_t \left[P_{t+1}^{(n-1)} M_{t+1} \right] \quad (1.13)$$

In words, this says that a date t bond price with time-to-maturity n is equal to the expected price of the bond appropriately discounted by M_{t+1} . We must also specify how the key risk factors evolve over time. In this model, we impose that the set of factors \mathbf{Z}_t follows a Markovian structure, in particular a first-order VAR, $\underbrace{\mathbf{Z}_{t+1}}_{4 \times 1} = \underbrace{\boldsymbol{\mu}}_{4 \times 1} + \underbrace{\boldsymbol{\phi}}_{4 \times 4} \cdot \mathbf{Z}_t + \underbrace{\boldsymbol{\varepsilon}_{t+1}}_{4 \times 1}$ where $\boldsymbol{\varepsilon}_{t+1}$ is a Gaussian error term with mean zero and variance-covariance matrix $\mathbb{E} \left[\boldsymbol{\varepsilon}_{t+1} \cdot \boldsymbol{\varepsilon}'_{t+1} \right] = \underbrace{\mathbf{V}}_{4 \times 4}$, and how the market price of risk varies over time, in this case, a linear function of the latent factor, $\underbrace{\boldsymbol{\lambda}_t}_{4 \times 1} = \underbrace{\boldsymbol{\lambda}_0}_{4 \times 1} + \underbrace{\boldsymbol{\lambda}_1}_{4 \times 4} \cdot \mathbf{Z}_t$, we can express the stochastic discount factor as a function of both the factors and the price of risk, $-\log(M_{t+1}) = \delta_0 + \boldsymbol{\delta}'_1 \cdot \mathbf{Z}_t + \frac{1}{2} \boldsymbol{\lambda}'_t \cdot \mathbf{V} \cdot \boldsymbol{\lambda}_t + \boldsymbol{\lambda}'_t \cdot \boldsymbol{\varepsilon}_{t+1}$. We then posit an exponentially affine functional form for log bond prices, $\log \left(P_t^{(n)} \right) = p_t^{(n)} = A_n + \mathbf{B}'_n \cdot \mathbf{Z}_t$, and solve the no-arbitrage conditions for any point in time t for any maturity n . The result of doing so is equivalent to estimating the cross-sectional loadings (A_n, \mathbf{B}_n) so that the no-arbitrage pricing

condition is satisfied. It can be gleaned from the expression for bond prices that for $n = 1$, that $-p_t^{(1)} = y^{(1)} = A_1 + \mathbf{B}'_1 \cdot \mathbf{Z}_t = \delta_0 + \boldsymbol{\delta}'_1 \cdot \mathbf{Z}_t$ is the one-year nominal interest rate. The remaining parameters are to be estimated using some method that mimics maximum likelihood where the parameters are chosen to minimize the fit of the data relative to some loss function. For the model under examination, a quadratic loss function was chosen. Alternatively, we could have solved for forward rates since they are a function of prices, $f_t^{(n)} = p_t^{(n)} - p_t^{(n-1)}$. Once we have parameters that well describe these prices, we can derive expressions that precisely characterize expected returns.

1.5.1 Expected Returns and the Market Price of Risk

Linear factor models have analytically tractable expressions that can be particularly useful for both our understanding and further analysis. An equilibrium object of interest is the one year ahead expected excess return from holding an n -maturity bond, is a function of bond prices as defined previously and is given below:

$$\mathbb{E}_t \left(rx_{t+1}^{(n)} \right) = \mathbf{cov}_t \left(rx_{t+1}, \underbrace{\varepsilon'_{t+1}}_{\text{shocks to } \mathbf{Z}_t} \right) \cdot \boldsymbol{\lambda}_t - \frac{1}{2} \sigma^2 \left(rx_{t+1}^{(n)} \right) \quad (1.14)$$

In terms of the model parameters, expected excess returns is then

$$\mathbb{E}_t \left(rx_{t+1}^{(n)} \right) = \mathbf{B}'_{n-1} \cdot \mathbf{V} \cdot \boldsymbol{\lambda}_t - \frac{1}{2} \mathbf{B}'_{n-1} \cdot \mathbf{V} \cdot \mathbf{B}_{n-1} \quad (1.15)$$

Finally, in terms of differences in actual and risk-neutral dynamics and the state variables, expected excess returns is given by the following expression:

$$\mathbb{E}_t \left(rx_{t+1}^{(n)} \right) = \mathbf{B}'_{n-1} (\boldsymbol{\mu} - \boldsymbol{\mu}^*) + \mathbf{B}'_{n-1} (\boldsymbol{\phi} - \boldsymbol{\phi}^*) \cdot \mathbf{Z}_t - \frac{1}{2} \mathbf{B}'_{n-1} \mathbf{V} \mathbf{B}_{n-1} \quad (1.16)$$

We can simplify the expression for expected returns once we adopt the identification restrictions from Cochrane and Piazzesi (2008). That is, if we restrict that (1) expected returns only vary with the x -factor and (2) only level

shocks matter for risk-compensation, then risk prices are simply linear in x_t , i.e. $\lambda_t = \lambda_0^{\text{level}} + \lambda_1^{\text{level}} \cdot x_t$. Cochrane and Piazzesi (2008) find that the cross-sectional fit of expected returns along the n time-to-maturity dimension was not affected by restricting returns to only depend on the level shock. I too find evidence documented in the previous section that level shocks play a prominent role in changing expected returns, given that changes in country-specific return-forecasting factors affect returns of all maturities. This is a very useful approximation because it reduces the amount of parameters that need to be estimated. However, given the stylized nature of the model, it is important to recognize how the simplifications made can affect our understanding of the data.

We then have a simple expression for expected excess returns on n -year maturity bonds:

$$\begin{aligned} \mathbb{E}_t \left(r x_{t+1}^{(n)} \right) &= \left(B_{n-1}^{\text{level}'} \text{cov} \left(v_t, \varepsilon_{t+1}^{\text{level}} \right) \lambda_0^{\text{level}} - \frac{1}{2} \mathbf{B}'_{n-1} \mathbf{V} \mathbf{B}_{n-1} \right) + B_{n-1}^{\text{level}'} V_{(2,2)} \lambda_1^{\text{level}} (\mathbf{1} \cdot \mathbf{1} \cdot \mathbf{x}_t) \\ &\approx \alpha(n) + \beta(n) \cdot x_t \end{aligned} \quad (1.18)$$

This equation holds for any n -maturity bond, so we can choose any $\boldsymbol{\lambda} = (\lambda_0^{\text{level}}, \lambda_1^{\text{level}})$ to match the predictability returns along the cross-section, i.e. for bond returns of maturity n . Cochrane and Piazzesi choose $\boldsymbol{\lambda}$ so that the slope coefficient is unity and the intercept term is zero. In other words, $\boldsymbol{\lambda}$ is chosen so that the weighted portfolio of long-maturity bonds will be related to the return-forecasting factor. This is equivalent to a portfolio of bonds weighted by q_r , the dominant eigenvector in the principal component decomposition used to construct the four factors in the term structure model. Implicitly, we are choosing $\boldsymbol{\lambda}$ so that $\mathbb{E}_t \left(\mathbf{q}'_r \cdot \mathbf{r} \mathbf{x}_{t+1}^{(n)} \right) = x_t$. Keep in mind that this portfolio of bonds will have different predictability characteristics than the one that I discuss in section 3 because the choice of a portfolio lets an investor alter their risk exposure, and hence the returns they can achieve. The choice to weight a portfolio of bonds by q_r therefore *maximizes* the fit of expected returns along year-to-maturity n

dimension using the return-forecasting factor x_t as a forecasting variable.

Table 4 in the appendix highlights the calibrated parameters for a subset of countries where the model well-describes the cross-section of bond prices. The estimates of the risk price parameters reveal large magnitude changes. We can then see how some key assumptions made in the estimation procedure affect these parameter estimates. In particular, restricting that expected returns only move with level shocks, that expected returns only move with the return-forecasting factor, and that the volatility of expected returns are constant over time which is implicitly assumed in the model specification, are essential for identification. As a result, the Jensen's term $\frac{1}{2}\mathbf{B}'_{n-1}\mathbf{V}\mathbf{B}_{n-1} = \sigma^2\left(rx_{t+1}^{(n)}\right)$ depend only on the length of maturity n , but not time t . However, the problem is that the estimation procedure is implicitly trading off fitting the mean and covariance of the yields, but under the mentioned restrictions and model assumption, the fitting procedure favors the conditional first moment. The market price of risk is directly proportional to the conditional variance of the pricing kernel, as can be seen in the following relation

$$\sigma_t^2(m_{t+1}) = \boldsymbol{\lambda}'_t \cdot \mathbf{V} \cdot \boldsymbol{\lambda}_t \quad (1.19)$$

which simplifies to the following under the mentioned restrictions:

$$\begin{aligned} \sigma_t^2(m_{t+1}) &= V_{(2,2)}^2 (\lambda_0^{\text{level}} + \lambda_1^{\text{level}} \cdot x_t)^2 \\ &= \kappa x_t^2 \end{aligned} \quad (1.20)$$

What remains to be seen is how variation in the market price of risk quantitatively responds to changes in the return-forecasting factor. Choosing $\boldsymbol{\lambda}_1$ is essential to the identification of risk-neutral dynamics $\boldsymbol{\phi}^*$ since the mapping from risk-neutral and actual dynamics is given by the following equation:

$$\boldsymbol{\phi}^* = \boldsymbol{\phi} - \mathbf{V} \cdot \boldsymbol{\lambda}_1 \quad (1.21)$$

The importance of the conditional homoskedasticity assumption becomes clear in this expression because it allows the risk adjustment term to be independent

of time. It also implies that identification requires large movements in the market price of risk since the relevant entries in \mathbf{V} are small, the entries in \mathbf{V}^{-1} are large so it is necessary for identification that $\lambda_1 = \mathbf{V}^{-1}(\phi^* - \phi)$. In other words, identification of risk-neutral dynamics requires not only that the restriction on how risk compensation varies exclusively with the return-forecasting factor, it also requires time-invariant conditional variance of excess returns.

The framework also links variation in the return-forecasting factor exclusively to the expected excess bond returns and not the conditional variance. However, it could be the case that changes in the return-forecasting factor occur during times of large surprise changes in bond prices. The specification choice renders the model unable to account for the changes in the conditional variance of returns. Although this simplifying approach seems innocuous given that most of the data describes the post-1985 "Great Moderation era," where monetary regimes are relatively stable, it remains to be seen what this simplification means for implied asset prices in other markets, and how the risk-return tradeoff varies over time. Currency markets and Hansen-Jagannathan bounds can serve as additional avenues for robustness checks that I will employ.

1.6 Results

I choose to study the U.S. and Germany because these two countries have the longest time series available, with data that span the time period 1 : 1973 – 5 : 2009. Another reason for choosing these two is the in-sample model fit of the U.S. and German bond yields to the affine model are quite good, with root mean squared errors (RMSE's) of less than 10 basis points. I do not report them here in this paper to focus my discussion although Litterman and Scheinkman (2001) tell us that we only need three factors to get a pretty good understanding of the yield curve. Nonetheless, it is comforting to know that adding an additional factor does

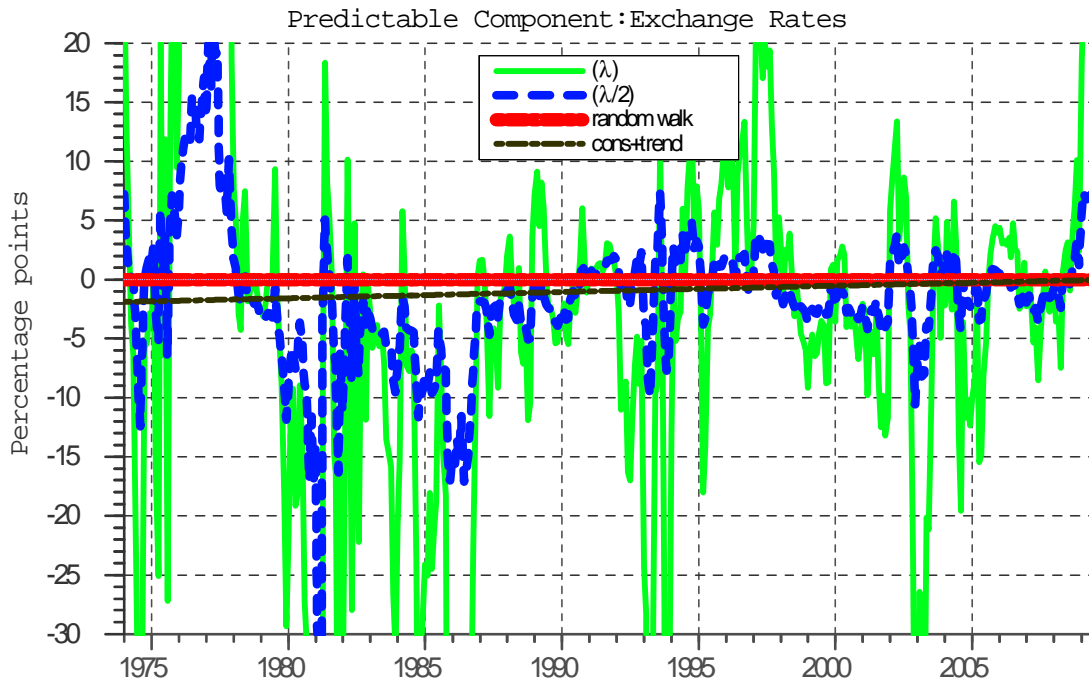
not degrade the ability of the model to capture the data. The small root mean squared errors (RMSE) can only mean that the amount of predictability of bond risk-premia is large given that we use an additional factor to closely track time-variation in expected returns for the given sample. More important, the model will capture a large predictable component in their respective bond markets that are consistent with empirical regressions.

If exchange rates are well-approximated by a random walk, then they are essentially unforecastable. Therefore, differences in the stochastic discount factors should roughly net out on average. Although this benchmark case for currency markets is admittedly strong, it is a useful first pass to ensure that the model-implied exchange rates do not deviate too far from the data. If they do, then exchange rates are said to be excessively volatile or predictable.

After estimating the model for the U.S. and Germany, and extracting the implied discount factors from each country, I construct the theoretically implied exchange rate changes with them. I compare them to several benchmarks. The first benchmark scenario is the random walk (-.- line), the second that allows for a trend (.-. line). Figure 1.2 captures the comparison between the model-implied exchange rate changes, without altering the estimates for the market price of risk $\lambda_{(j)}$ for each country. I also include the scenario in which the estimates are scaled down by two. In total, there are two model-implied scenarios for exchange rates: (1) one case that constructs exchange rate changes using $\lambda_{(j)}$ (2) model generated exchange rate changes with $\lambda'_{(j)} = \lambda_{(j)}/2$. The case where $\lambda_{(j)}$ is reduced by a factor of 2 is acknowledged by Cochrane and Piazzesi (2008) and I highlight this case to show that even while acknowledging that there is estimation uncertainty around this parameter, my results will still hold. As you can see, the model-implied exchange rates deviate substantially from random walk behavior in a rather persistent manner relative to the data. When I scale down the market price of risk λ by one half, the predictable component of exchange rates are reduced

somewhat, but still imply sizeable, persistent deviations. The results suggest that either the size of the predictable component in exchange rates is a lot larger than a random walk specification implies, or alternatively that the magnitude changes in market prices and hence the return-forecasting factors do not cancel out. This is not entirely unbelievable, but it is difficult to accept such a large predictable component without reasons for doing so.

Figure 1.2: Constructing Exchange Rates



The λ case I construct exchange rates without altering risk prices. The $\lambda / 2$ case I reduce risk prices by half. The cons+trend case refers to a random walk with drift case that I construct by estimating realized exchange rate changes on a constant and time. The variables are in annualized percentages.

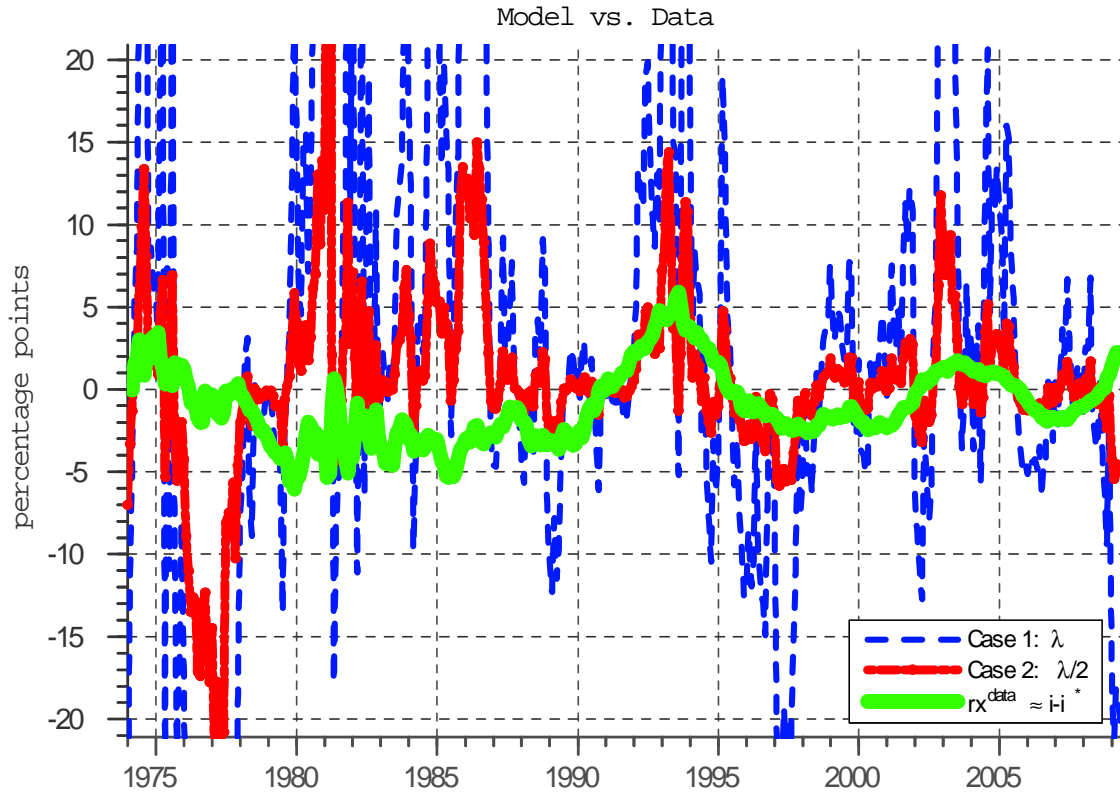
I construct currency risk-premia using the conditional variance of the pricing

kernel between the U.S. and Germany, given by the following expression:

$$\mathbb{E}_t [rx_{t+1}^*] = \frac{1}{2} (\sigma_t^2 [m_{t+1}] - \sigma_t^2 [m_{t+1}^*]) \quad (1.22)$$

This will also allow me to control for the wild movements in the model-implied exchange rates we saw in Figure 1.2. I compare this time-series with currency risk premia to a benchmark in the data, namely the interest rate differential, $rx_{t+1}^{data} \approx y_t^{(1),*} - y_t^{(1)}$. This approximation is consistent with longstanding empirical research on the carry trade. Given the empirical robustness of the forward premium anomaly across a broad set of countries, it seems reasonable to check that the model captures this feature of the data. Figure 1.3 illustrates a puzzlingly volatile model-implied currency risk-premia that is definitively too large to be consistent with a forward premium anomaly. It is not immediately clear why differences in the conditional variance of the discount factor implied by the model are not so smooth as interest rate differentials, however the findings, when taken at face value, are inconsistent with the forward premium anomaly.

Figure 1.3: Constructing Predictable Currency Risk-Premia



Currency risk-premia is constructed by taking the difference in bond risk prices. The λ case I construct exchange rates without altering risk prices. The $\lambda/2$ case I reduce risk prices by half. The benchmark rx -data case is simply the interest rate differential on a 1-year treasury between the U.S. and Germany. Returns are in annualized percentages

1.6.1 Highlighting the Tradeoff

To illustrate the consequence of this modeling procedure, we look no further than the magnitude of λ_1^{level} , the key parameter that determines how risk compensation changes with x_t . As I mentioned earlier, variation in x_t corresponds with variation in the conditional variance of the pricing kernel. But, this variable is *also* important for currency risk-premia. It has been well-documented that interest rate differentials forecast currency risk-premia, a result commonly known as the carry trade. Can the return-forecasting factor differential also forecast currency

risk-premia?

Differences in the return-forecasting factor between the U.S. and Germany should correspond to differences in the market price of currency risk. If so, $x_t - x_t^*$ should signal changes in currency prices. The remaining concern about the plausibility of currency prices is quantitative in nature; does the model-implied currency risk-premia move with a benchmark measure of the predictable component in currency risk-premia? When compared to the benchmark case of the interest rate differentials, it seems that the return-forecasting factor differentials is definitively not consistent with the forward premium puzzle. Interestingly enough, scaling down each country's magnitude market price of risk (i.e. λ_1^{level}) by a factor of 5 delivers results in line with the negative regression coefficient that is consistent with the data.

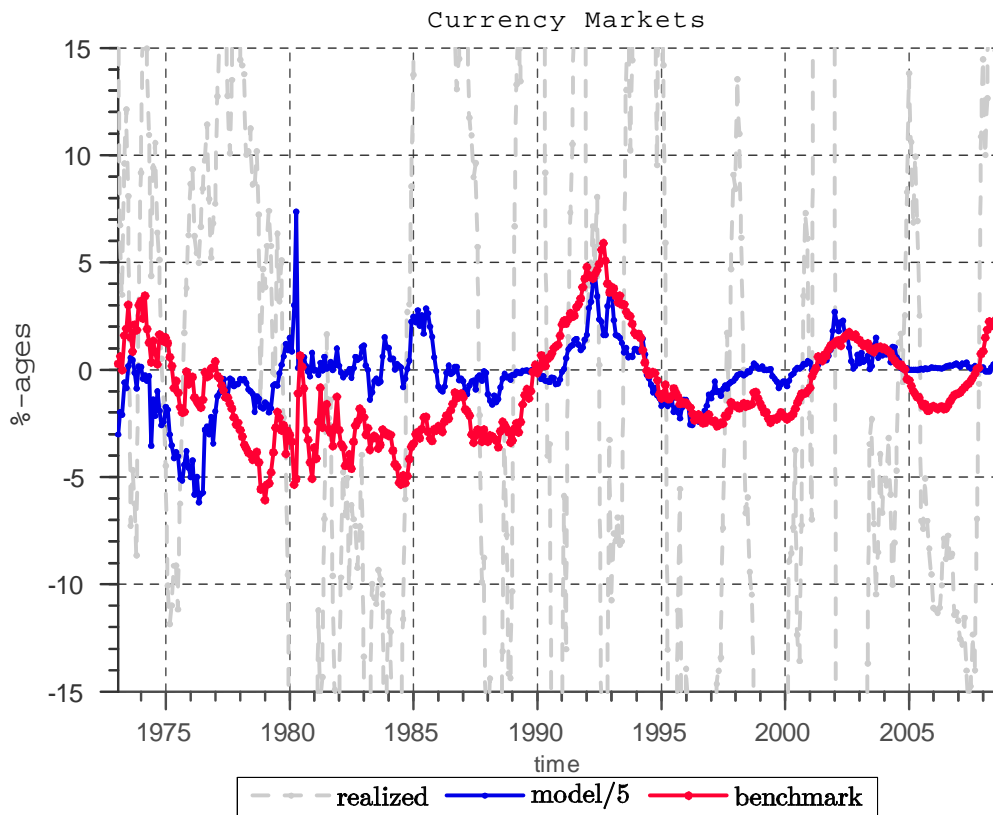
Table 1.3: Forward Premium Regressions

$$\Delta s_{t+1} = \alpha + \beta z_t + \varepsilon_{t+1}$$

z_t	Data ($y_t^{(1),*} - y_t^{(1)}$)	Model ($\mathbb{E}_t [rx_{t+1}^*]$)	Model: $\lambda/5$
β	-0.85	1.53	-0.24
t -stat	(-1.17)	(0.87)	(-0.21)
R^2	0.01	0.02	0.00

The column under the heading Data corresponds to estimates from regressing exchange rate changes on actual interest rate differentials. The Model column reports estimates from regressing exchange rate changes on the implied currency risk-premia, which is one half times the difference in the conditional variance of the discount factor. Model: $\lambda/5$ corresponds to a similar regression but when the bond risk price parameter in the conditional variance of the discount factor is reduced by a factor of 5. The monthly data span 1.1973-5.2009.

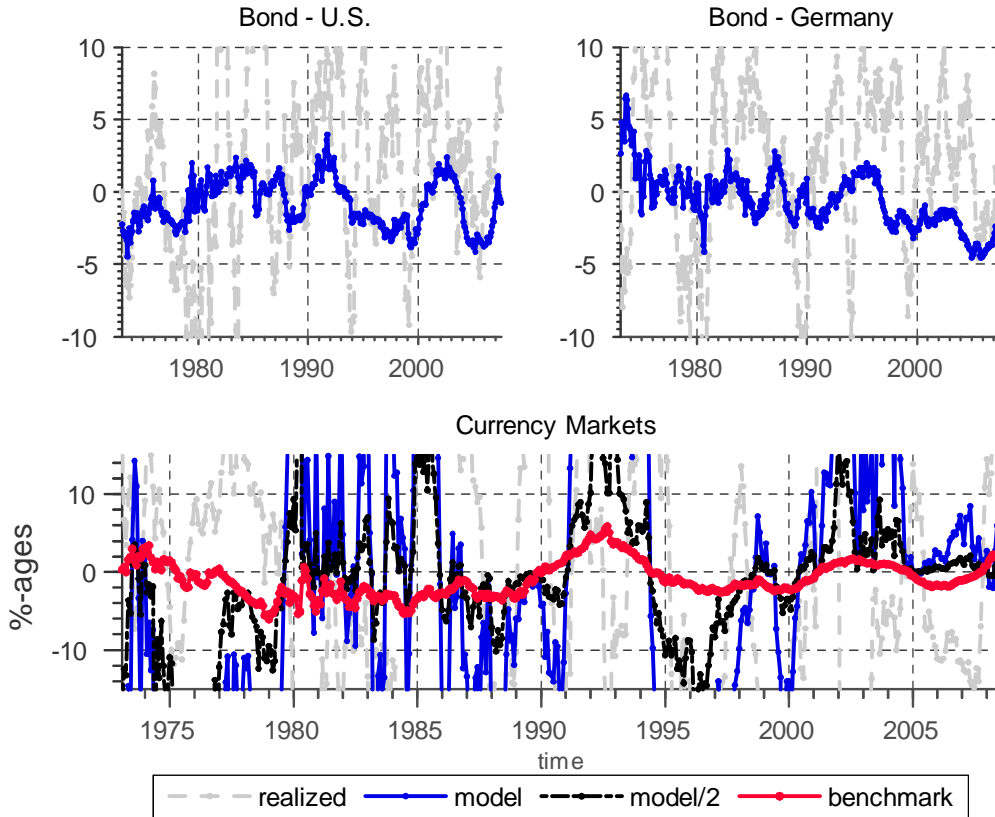
Figure 1.4: Implied Predictable Component in Currency Risk-Premia



Graphically Figure 1.4 illustrates the case in which λ_1^{level} is scaled down by a factor of 5. The difference between the model and the data that is evident once we scale down λ_1^{level} . We see that the model-implied currency risk-premia (the red dotted line) does not co-move with predictable component in currency returns proxied by the interest rate differential $y_t^{(1),*} - y_t^{(1)}$ from 1980-1990 by some level factor. In addition, the model does not capture the subtler movement in the level changes in the predictable component after 1996.

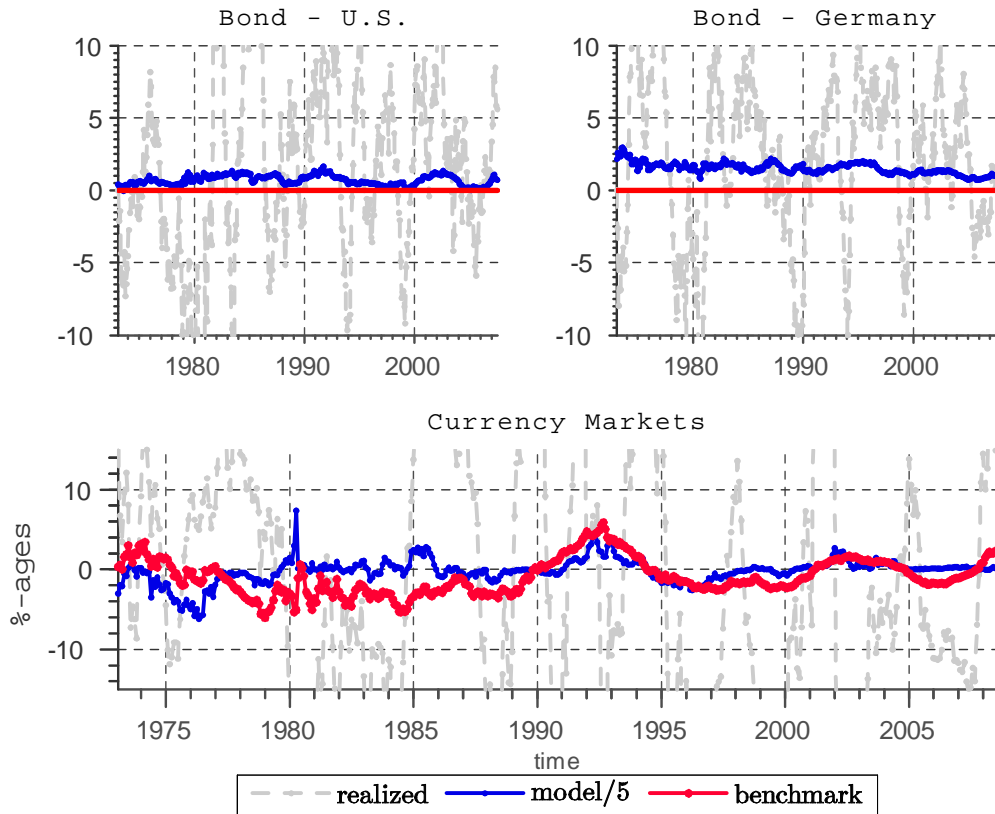
A clearer exposition can be found in time-series graphs that *simultaneously* captures the tension. Figure 1.5 displays exactly this tension, namely the predictable component for U.S. and German government bond markets along with the predictable component in their bilateral currency market.

Figure 1.5: Simultaneous Look At Predictable Bond and Currency Risk-Premia,
I



Now, it is clear that the size of the predictable component in currency markets is a lot larger than the forward premium puzzle would suggest. Reducing the magnitude size of the market price of risk delivers a result more consistent with the forward premium in currency markets, but results in a very small predictable component in either the U.S. or German bond market, as displayed in Figure 6. The set of graphs given in the last two figures illustrates precisely the main result of my paper.

Figure 1.6: Simultaneous Look At Predictable Bond and Currency Risk-Premia,
II



Reducing the market price of risk eliminates bond predictability

1.7 Discussion

1.7.1 Sharpe Ratio Dynamics

In this section, I discuss the underlying tension in characterizing the risk-return tradeoff in bond markets without a formal model. When normalized by a proper unit of risk, the compensation required to hold onto bonds, commonly known as the Sharpe ratio, should vary over time within reasonable limits. The link between predictable returns and market risk prices is very clear in linear factor models that this paper critically evaluates. Using Hansen-Jagannathan bounds to pin down the maximum Sharpe ratios, I come to the same conclusion that allowing

returns to be conditional heteroskedastic is one avenue for this class of models to capture predictable risk premia in bond and currency markets. Indirect support for this conclusion can be found in Duffee (2010) who constrains the Sharpe ratio to not exceed "reasonable" bounds when compared to equity markets or a sensible restriction on unconditional Sharpe ratios while maintaining the time-invariance of the shock structure.

For clarity of exposition, the Sharpe ratio is defined to be the ratio of the conditional volatility of the pricing kernel and the conditional mean of the pricing kernel. We can rewrite the conditional volatility of the pricing kernel as the following expression:

$$[\text{Sharpe Ratio}] = \frac{\mathbb{E}_t [RX_{t+1}]}{\sigma_t [RX_{t+1}]} \leq \frac{\sigma_t [M_{t+1}]}{\mathbb{E}_t [M_{t+1}]} \quad (1.23)$$

We can achieve the maximal Sharpe ratio under some conditions usually satisfied by bonds.¹

$$\max [\text{Sharpe Ratio}] := \Theta_t = R_t^f \sigma_t [M_{t+1}] \quad (1.24)$$

For log-returns, I define $r_{i,t+1} = \log (R_{i,t+1})$ and $rx_{t+1} = \log (RX_{t+1}) = [\log (R_{t+1})] - \log (R_t^f)$. The expression for the Sharpe ratio for log returns is then given by:

$$\Theta_t = \frac{\mathbb{E}_t [r_{i,t+1}] - r_t^f + \frac{1}{2}\sigma_t^2 [r_{i,t+1}]}{\sigma_t [r_{i,t+1}]} \quad (1.25)$$

Under some regularity conditions, we can replace $R_{i,t+1}$ with the excess return, RX_{t+1} .² I obtain the following expression for the conditional variance of the pricing kernel, which for bond markets is the maximal Sharpe ratios that be obtained:

$$\sigma_t [m_{t+1}] = \frac{\mathbb{E}_t [rx_{t+1}] + \frac{1}{2}\sigma_t^2 (rx_{t+1})}{\sigma_t [rx_{t+1}]} \quad (1.26)$$

¹The condition is satisfied for investment strategies where the payouts to holding the security are non-negative.

²We can take a log-linear approximation of excess returns around $\log (RX_{t+1}) = 0$. We also rule out short positions on a portfolio of bonds that is typically used to maximize Sharpe ratios. The portfolio we consider gives positive weights to each of the n year-to-maturity bonds. In all, these conditions are satisfied.

The full implications of the identification restriction used in Cochrane and Piazzesi's (2008) model are included in the appendix, but for the purpose of this discussion, I will only give the essence of the argument. That is, in order for bond returns to be consistent with currency market data, not to mention sensible restrictions on conditional Sharpe ratios, it must be the case that either the conditional volatility of excess returns moves to offset movement in expected excess returns *or* the amount of measured movement in expected returns is an artifact of the data, not nearly moving around as much it is measured to do. Because risk-premia is predictable for broad set of industrialized economies' bond markets, sensible variation in market risk prices would lead one to conclude that the volatility of returns are time-varying.

1.7.2 A Modeling Tradeoff

The exchange rate implications of the four-factor affine term structure model reveal an inherent modeling tradeoff; either we accept that there is predictability in two separate bond markets or there is predictability in currency markets and almost no predictability in bond markets. To be fair, the inherent issues with the model and the identifying restrictions are natural ones given that they are broadly consistent with a set of government bond markets. However, the problem becomes apparent in a simple extension to currency markets using the same factors. I then establish the first result that highlights a key modeling limitation in characterizing the dynamics of risk and return across bond and currency markets.

Proposition 1 *Within a four-factor affine term structure model, given that variation in risk-premia move exclusively with random walk shocks, if there is predictability in two individual bond markets, then there is a counterfactually large amount of predictability in exchange rates and currency risk-premia. Conversely, if the market risk prices are consistent with currency market data, then risk-premia*

in bond market are nearly unforecastable.

The intuition for this result can be gleaned from the relationship between the risk-neutral dynamics and how the market price of risk parameters matter. Risk-neutral dynamics are equal to the actual dynamics, less an adjustment for risk, i.e. $\phi^* = \phi - \mathbf{V}\lambda$. Reducing the magnitude change in risk prices means that the parameter λ_1^{level} is scaled down. The result is a much smaller risk adjustment in the risk-neutral dynamics, so that is indistinguishable from the actual dynamics, i.e. $\phi^* \approx \phi$. Based on the equation (13), $\mathbb{E}_t \left(rx_{t+1}^{(n)} \right) \approx \mathbf{B}'_{n-1} (\phi - \phi^*) \cdot \mathbf{Z}_t \approx 0$ so that expected excess returns are nearly unforecastable.

1.7.3 An Uncertainty Principle in Bond Markets

The conditional volatility of the stochastic discount factor can directly be linked it to the return-forecasting factor in the following equation:

$$\sigma_t [m_{t+1}] = (V_{(2,2)})^{\frac{1}{2}} \cdot (\lambda_0^{\text{level}} + \lambda_1^{\text{level}} \cdot x_t) \quad (1.27)$$

Hansen-Jagannathan bounds for log returns can also link the stochastic discount factor to expected excess returns, accounting for Jensen's term

$$\sigma_t [m_{t+1}] = \frac{\mathbb{E}_t [rx_{t+1}] + \frac{1}{2}\sigma^2 (rx_{t+1})}{\sigma (rx_{t+1})} \quad (1.28)$$

As I discussed in the previous section, for the Hansen-Jagannathan bounds to be consistent with both bond and currency market data, it must be the case that either (a) expected excess returns be conditionally heteroskedastic (i.e. the $\sigma_t [rx_{t+1}]$ term moves to offset movements in $\mathbb{E}_t [rx_{t+1}]$) or (b) the measured amount of predictability (the $\mathbb{E}_t [rx_{t+1}]$ term) is too large. However, we have evidence from global bond markets that returns are conditionally heteroskedastic. If that is the case, then variation in the return-forecasting factor could also signal changes to either the conditional mean or the conditional variance of returns, thereby highlighting another key proposition.

Proposition 2 *Given the mentioned restrictions on the market price of risk, variation in the return-forecasting factor cannot be exclusively attributed to expected excess returns and can also precede movement in the conditional volatility of returns. Moreover, the amount of measured predictability in bond excess returns is too large to be consistent with empirically consistent bounds on Sharpe ratios.*

The proposition suggests that the amount of variation in the return-forecasting factor implies there is too much forecastability implied by a linear factor model. It is possible that variation in bond risk-premia be driven by serially correlated errors as Hamilton and Wu (2010) find evidence in favor of. What is clear is that the return-forecasting factor is constructed to contain information that varies over time with risk-premia, although it is unclear why its variation does not nearly forecast as well as the factor model would lead you to believe. What can be inferred from the analysis so far is that return volatility is time-varying. If so, then, x_t captures movements in both the conditional mean and variance of excess returns, as shown in the following equation: The model estimation procedure places primary emphasis on conditional mean of the pricing kernel, rendering the model incapable of capturing changes in the conditional variance of risk-premia. The mapping between actual and risk-neutral dynamics is linked together by some risk-adjustment term. If the shocks to the state variable are conditionally heteroskedastic, then this risk-adjustment term is no longer time-invariant. Further, if returns are conditionally heteroskedastic, then the conditional variance of expected excess returns must be time-dependent. Thus, time-variation in market risk prices that are directly linked to the return-forecasting factor, i.e. $\lambda_t(x_t)$, cannot be uniquely determined to either the conditional mean of excess returns or the conditional variance of returns since

$$\mathbb{E}_t \left(rx_{t+1}^{(n)} \right) + \frac{1}{2} \sigma_t^2 \left(rx_{t+1}^{(n)} \right) = \mathbf{B}'_{n-1} \cdot \mathbf{V} \cdot \lambda_t(x_t) \quad (1.29)$$

Taken together, the results highlight a key uncertainty principle.

Corollary 3 *Given the mentioned restrictions on the market price of risk, if there is time-varying volatility, then one cannot track expected returns on bonds without accounting for changes in the conditional variance of excess returns.*

1.7.4 Some Commentary

The results established in this paper highlight a limitation in the class of linear bond pricing models to simultaneously capture the predictable components in bond and currency markets. More broadly, I assess the ability of a framework to help one understand salient features of the data. In doing so, I demonstrate that there are necessary but not entirely innocuous judgment calls that need to be made, making our understanding of phenomena constrained by the model we employ. Different modeling approaches may have different strengths and weaknesses, but their respective importance hinges upon the key insights that can be extracted from them. After all, models are only approximations of reality. The important thing to realize is that they give us an understanding of certain aspects of the data and the world that is not possible with empirics alone.

Using a linear class of models to account for time-varying risk and return in bond and currency markets is, as I have shown, asking too much of the model. An example of a monetary model that can jointly account for time-varying risk bond and currency markets and treats expected returns in individual bond markets as a mean-reverting process is Alvarez, Atkeson, and Kehoe (2009). Directly linking a stochastic discount factor to key economic risk factors in a general equilibrium model is important for our understanding of time-varying risk, but their framework choice requires that they be agnostic about the amount of measured predictability in bond markets. This is neither right nor wrong, but a direct consequence of their modeling choice. Nonetheless, it still may be useful for understanding key features of the data.

At the other end of the spectrum, if one does want to tell a risk-based story of predictable returns in financial markets, then one must account for a large amount of predictability in bond markets as Cochrane and Piazzesi (2005, 2008) find, and in currency markets recently documented by Lustig et. al (2010). Doing so with economic interpretability through key risk-factors in each respective market in relation to macroeconomic aggregates (or at least, the beliefs about future macroeconomic aggregates) would be enormously useful as a guide for thinking and intuition, one that many would welcome.

Another possible source of statistically measured predictability that has not been discussed very extensively in the literature is the importance of the *subjective beliefs of market participants* and how they can reduce the amount of measured predictability that a stochastic discount factor must explain. To show how subjective beliefs can matter, one can decompose measured predictability of returns according to the following expression:

$$\widehat{\mathbb{E}} [R_{t+1}^e] = \underbrace{\left(\widehat{\mathbb{E}} [R_{t+1}^e] - \widetilde{\mathbb{E}} [R_{t+1}^e] \right)}_{(*)} - \frac{\widetilde{\text{Cov}}_t (M_{t+1}, R_{t+1}^e)}{\mathbb{E}_t [M_{t+1}]} \quad (1.30)$$

where $\widehat{\mathbb{E}}$ is expectations from the actual statistical probability distributions and $\widetilde{\mathbb{E}}$ is taken under the subjective beliefs of the market participants as is the covariance operator $\widetilde{\text{Cov}}_t (\cdot)$. Of course, doing so requires that we depart from rational expectations. Assuming rational expectations amount to setting the (*) term to zero, but this means the candidate risk factors that make up the pricing kernel exclusively drive variation expected excess returns. According to Piazzesi and Schneider (2011) and Gourinchas and Tornell (2004) and as the results in this paper indicate, this may be asking too much from the discount factor. One plausible motivating reason for why agents make systematic forecast errors is because of a signal extraction problem, distinguishing transitory disturbances from permanent shocks. As a result, investors are necessarily are making forecasts that minimize how wrong they are and these forecast mistakes show up as measured predictabil-

ity. The full implications of such a departure from rational expectations can help us explain time-series properties of returns not only for bond markets as Piazzesi and Schneider (2011) show, but for any financial market, making it an interesting avenue for future research.

1.8 Conclusion and Future Directions

After documenting that predictable returns in bond markets are a global phenomenon, I determine if the foreign exchange market prices implied by the term structure model of Cochrane and Piazzesi (2008) are consistent with our current understanding of predictable currency risk-premia. I find that currency market prices are substantially more predictable than standard predictor variables suggest. According to my results, even if we wanted to jointly account for the predictable component in bond and currency risk-premia, we could not; linear factor models can account for either bond and currency return predictability, but not both at once. This limitation of a linear class of models is in large part due to the emphasis on fitting the conditional mean of the pricing kernel, among other things.

The results in this paper have implications for the asset pricing implications of dynamic economic models. Existing frameworks such as an affine term structure model and a general equilibrium (New) Keynesian models that form the basis of policy rules generate movement in the conditional mean of the pricing kernel. At business cycle frequencies, changes in the policy instrument, the short-term interest rate, do not correspond to changes in the conditional mean but the conditional variance of the pricing kernel. This result is well-documented in finance and was recently emphasized by Alvarez, Atkeson and Kehoe (2007). By not accounting for changes in risk, their policy response may have unintended consequences. This criticism is especially relevant for rules that may have worked in the past but may

have also outlived their usefulness. One immediate implication of my findings is that inflation expectations management using a Taylor rule alters risk in financial markets. If this is not the intention of central banks, is there a better way to manage expectations about prices?

Macroeconomic policy plays a critical role not only in changing financial risk conditions but also in coordinating future economic activity. Key questions remain unanswered. Does monetary policy cause changes in risk, or is time-variation in risk-premia driven by deeper forces that are outside the direct control of central bank officials? These questions are especially important in light of recent findings that financial market data often move together across countries, suggesting that global forces are at work. How will the coordination of policy affect bond and currency prices? In light of recent events, we may also benefit from addressing alternative methods of managing risk and maintaining financial market stability. These concerns should be, if they are not already, at the frontier of research in the future conduct of monetary policy.

In my future research, I hope to further our understanding of why returns in financial markets move so much. For bond markets, this requires a more profound understand of how expectations are formed by financial market participants. Jointly capturing bond and currency market prices require careful treatment of expectations formation that takes seriously financial market participants' concern about structural change. Ultimately, international financial economists want an integrated understanding of both bond and currency markets that may lead to a macroeconomic understanding of asset prices. I hope this paper provides an impetus for such an undertaking, as well as tantalizing leads for future research.

1.9 Appendix

1.9.1 Data Description

Data on zero-coupon bond yields are taken from Jonathan H. Wright, further details on the construction of the data using Svensson, Spline, or Nelson-Siegel approaches to extracting zero-coupon yields are found on www.econ2.jhu.edu/People/Wright/intyields.

1.9.2 Solving for Bond Market Prices

To begin, we first conjecture an exponentially affine function form of the bond price given by the following

$$P_t^{(n)} = \exp(A_n + \mathbf{B}'_n \cdot \mathbf{Z}_t) \quad (1.31)$$

and in logs, we get:

$$p_t^{(n)} = A_n + \mathbf{B}'_n \cdot \mathbf{Z}_t \quad (1.32)$$

$$\begin{aligned} y_t^{(n)} &= -\frac{1}{n} p_t^{(n)} \\ &= -\frac{A_n}{n} - \frac{1}{n} \mathbf{B}'_n \cdot \mathbf{Z}_t \end{aligned} \quad (1.33)$$

Written in terms of the stochastic discount factor, bond prices are then

$$P_t^{(n)} = \mathbb{E}_t \left[M_{t+1} P_{t+1}^{(n-1)} \right] \quad (1.34)$$

$$= \mathbb{E}_t \left[\exp \left(\sum_{j=1}^{n-1} m_{t+j} \right) \right] \quad (1.35)$$

Suppose that the evolution of the state variable can be characterized as a first-order VAR:

$$\mathbf{Z}_{t+1} = \boldsymbol{\mu} + \boldsymbol{\phi} \cdot \mathbf{Z}_t + \boldsymbol{\varepsilon}_{t+1} \quad (1.36)$$

where $\boldsymbol{\varepsilon}_{t+1}$ is a multivariate normal distribution with mean $\vec{0}$ and variance-covariance matrix $\mathbf{E} [\boldsymbol{\varepsilon}_{t+1}\boldsymbol{\varepsilon}'_{t+1}] = \mathbf{V}$. Also, suppose the (log) discount rate m_{t+1} is an affine function of the state variable along with an additional stochastic term magnified by the market price of risk,

$$m_{t+1} = -\delta_0 - \boldsymbol{\delta}'_1 \cdot \mathbf{Z}_t - \frac{1}{2} \boldsymbol{\lambda}'_t \cdot \mathbf{V} \cdot \boldsymbol{\lambda}_t - \boldsymbol{\lambda}'_t \cdot \boldsymbol{\varepsilon}_{t+1} \quad (1.37)$$

where the market price of risk is also affine in the state variable $\boldsymbol{\lambda}_t = \lambda_0 + \boldsymbol{\lambda}_1 \cdot \mathbf{Z}_t$. Given that shocks to the state variable are conditionally homoskedastic, the conversion of actual dynamics to risk-neutral dynamics involves an adjustment for risk, given by the following relation:

$$\boldsymbol{\mu}^* = \boldsymbol{\mu} - \mathbf{V} \cdot \boldsymbol{\lambda}_0 \quad (1.38)$$

$$\boldsymbol{\phi}^* = \boldsymbol{\phi} - \mathbf{V} \cdot \boldsymbol{\lambda}_1 \quad (1.39)$$

To solve for no-arbitrage prices, we must ensure that the loadings $(A_n, \mathbf{B}_n)_{n=1}^N$ satisfy the equilibrium prices for any t and for all n . We proceed by induction. To ensure that $p_t^{(0)} = 0$, it must be that $A_0 = 0, \mathbf{B}_0 = \vec{0}$. Now, let us solve for the one-year bond price. Then, I assume that the condition holds for some maturity n and then show that it holds for $n + 1$. To start for $n = 1$,

$$P_t^{(1)} = \exp(A_1 + \mathbf{B}'_1 \cdot \mathbf{Z}_t)$$

The one-year price should be equal to negative the one-year interest rate, which is given by the following expression:

$$\begin{aligned} y_t^{(1)} &= \log \mathbb{E}_t [\exp(m_{t+1})] \\ &= \log \left(\exp \left(\mathbb{E}_t m_{t+1} + \frac{1}{2} \mathbb{V}_t m_{t+1} \right) \right) \\ &= (-\delta_0 - \boldsymbol{\delta}'_1 \cdot \mathbf{Z}_t) \\ &= -p_t^{(1)} = A_1 + \mathbf{B}'_1 \cdot \mathbf{Z}_t \end{aligned}$$

which is true if $\delta_0 = -A_1$ and $\boldsymbol{\delta}_1 = -\mathbf{B}_1$. Now, suppose that $P_t^{(n)} = \exp(A_n + \mathbf{B}'_n \cdot \mathbf{Z}_t)$ and show that $P_t^{(n+1)} = \exp(A_{n+1} + \mathbf{B}'_{n+1} \cdot \mathbf{Z}_t)$ by explicitly

solving for $(A_{n+1}, \mathbf{B}_{n+1})$. To do so,

$$P_t^{(n+1)} = \mathbb{E}_t \left[\exp \left(m_{t+1} + p_{t+1}^{(n)} \right) \right] \quad (1.40)$$

$$= \mathbb{E}_t \left[\exp \left(\begin{array}{c} -\delta_0 - \boldsymbol{\delta}'_1 \cdot \mathbf{Z}_t - \frac{1}{2} \boldsymbol{\lambda}'_t \cdot \mathbf{V} \cdot \boldsymbol{\lambda}_t + \boldsymbol{\lambda}'_t \cdot \boldsymbol{\varepsilon}_{t+1} \\ + A_n + \mathbf{B}'_n \cdot \mathbf{Z}_{t+1} \end{array} \right) \right] \quad (1.41)$$

$$= \exp(-\delta_0 + A_n) \underbrace{\mathbb{E}_t \left[\exp \left(\mathbf{B}'_n \cdot \mathbf{Z}_{t+1} \right) \right]}_{\exp(\mathbf{B}'_n (\boldsymbol{\mu}^* + \boldsymbol{\phi}^* \mathbf{Z}_t) + \frac{1}{2} \mathbf{B}'_n \cdot \mathbf{V} \cdot \mathbf{B}_n)} \quad (1.42)$$

$$= \left[\exp \left(\underbrace{-\delta_0 + A_n + \mathbf{B}'_n \cdot \boldsymbol{\mu}^* + \frac{1}{2} \mathbf{B}'_n \cdot \mathbf{V} \cdot \mathbf{B}_n}_{A_{n+1}} + \underbrace{(\boldsymbol{\delta}'_1 + \mathbf{B}'_n \cdot \boldsymbol{\phi}^*)}_{\mathbf{B}'_{n+1}} \cdot \mathbf{Z}_t \right) \right] \quad (1.43)$$

$$= \exp(A_{n+1} + \mathbf{B}'_{n+1} \cdot \mathbf{Z}_t) \quad (1.44)$$

where expectations are taken under the risk-neutral probability distribution. Thus, the expression for the loading coefficients given above satisfies the equilibrium condition. Solving for forward rates is straightforward from the following relation

$$f_t^{(n-1 \rightarrow n)} = p_t^{(n-1)} - p_t^{(n)} \quad (1.45)$$

$$\begin{aligned} &= A_{n-1} - A_n + (\mathbf{B}_{n-1} - \mathbf{B}_n)' \cdot \mathbf{Z}_t \\ &= A_n^f + \mathbf{B}_n^{f'} \cdot \mathbf{Z}_t \end{aligned} \quad (1.46)$$

where $A_n^f = A_{n-1} - A_n = \delta_0 - \mathbf{B}'_{n-1} \cdot \boldsymbol{\mu}^* - \frac{1}{2} \mathbf{B}'_{n-1} \cdot \mathbf{V} \cdot \mathbf{B}_{n-1}$ and

$$\mathbf{B}_n^f = \mathbf{B}_{n-1} - \mathbf{B}_n = \boldsymbol{\delta}'_1 + \mathbf{B}'_{n-2} \boldsymbol{\phi}^* - \boldsymbol{\delta}'_1 - \mathbf{B}'_{n-1} \boldsymbol{\phi}^* \quad (1.47)$$

$$\begin{aligned} &= (\mathbf{B}_{n-2} - \mathbf{B}_{n-1})' \boldsymbol{\phi}^* \\ &= (\mathbf{B}_{n-3} - \mathbf{B}_{n-2})' \boldsymbol{\phi}^{*2} \end{aligned} \quad (1.48)$$

$$\begin{aligned} \dots &= (\mathbf{B}_0 - \mathbf{B}_1)' \cdot \boldsymbol{\phi}^{*n-1} \\ &= \boldsymbol{\delta}'_1 \boldsymbol{\phi}^{*(n-1)} \end{aligned} \quad (1.49)$$

Numerical techniques that solve for $(A_n^f, \mathbf{B}_n^f)_{n=1}^N$ used in Cochrane and Piazzesi (2008) compare the fit of the model to the data with the best linear estimator

of forward interest rates using OLS similar to a maximum likelihood estimator with a quadratic loss function. Not only that, the OLS estimated dynamics has a greater set of parameters to characterize the dynamics of bond prices. The fit of the model is measured by root mean squared error (RMSE) which is sensible given that shocks to the underlying state variable are assumed to be Gaussian with time-invariant volatility.

1.9.3 Cross-country Estimates of the Market Price of Risk

The cross-sectional fitting procedure is critical for the estimates of the market price of risk. The procedure is made even simpler once we only allow random walk shocks to move risk-premia. Given this restriction, we can write the market price of risk to be a function of only the return forecasting factor x_t ,

$$\lambda_t = \lambda_0^{\text{level}} + \lambda_1^{\text{level}} \cdot x_t, \quad (1.50)$$

making the key parameters that govern the time-series properties of the market price of risk only two dimensional instead of twenty if all of the four factors affected the price of risk. For a set of countries where the affine model performs relatively well, I report the estimates of $\lambda = (\lambda_0^{\text{level}}, \lambda_1^{\text{level}})$ given table below

Table 1.4: Bond Market Risk Price Estimates

λ	λ_0^{level}	λ_1^{level}
US	-1.28	-143
DE	-1.45	-215
JP	-1.37	-441
AUS	-2.13	-217

Estimates of the market price of risk for a subset of countries

There are clearly idiosyncratic differences in the parameter estimates that can be attributed to country specific reasons as well as when the bond data become available. However, a common pattern for this set of countries is the sheer magnitude of the estimates for λ_1^{level} . To be sure, the size of λ_1^{level} determines how sensitive a change in risk compensation is to the return-forecasting factor. Based on the large magnitudes in the estimate of $\lambda_{1,t}$, small changes in x_t set off large movements in risk-premia for a set of countries. The interpretation of this parameter estimate is that a 100 basis point increase in the return forecasting factor generates a 143 (.01 * 143 (.01) = 1.43) basis point increase in the amount of compensation required for buying a short-maturity bond.

1.9.4 Sharpe Ratios

We can derive Sharpe ratios for simple returns using the basic asset pricing moment condition

$$\mathbb{E}_t [M_{t+1} R_{t+1}^e] = 0 \quad (1.51)$$

We can expand this expression using some key statistical relations:

$$\begin{aligned} \mathbb{E}_t [M_{t+1} R_{t+1}^e] &= \mathbb{E}_t [M_{t+1}] \mathbb{E}_t [R_{t+1}^e] + \text{Cov}_t (M_{t+1}, R_{t+1}^e) \\ &= \mathbb{E}_t [M_{t+1}] \mathbb{E}_t [R_{t+1}^e] + \text{Corr}_t (M_{t+1}, R_{t+1}^e) \sigma_t (M_{t+1}) \sigma_t (R_{t+1}^e) \end{aligned} \quad (1.52)$$

Rearranging, we get

$$\frac{\sigma_t (M_{t+1})}{\mathbb{E}_t [M_{t+1}]} = - \frac{\mathbb{E}_t [R_{t+1}^e]}{\sigma_t (R_{t+1}^e)} \frac{1}{\text{Corr}_t (M_{t+1}, R_{t+1}^e)} \quad (1.53)$$

Since $\text{Corr}_t (M_{t+1}, R_{t+1}^e) \in [-1, 1]$, if we take the stochastic discount factor to be perfectly negatively correlated with expected excess returns, then

$$\frac{\sigma_t (M_{t+1})}{\mathbb{E}_t [M_{t+1}]} \geq \frac{\mathbb{E}_t [R_{t+1}^e]}{\sigma_t (R_{t+1}^e)} \quad (1.54)$$

For any stochastic return process, these bounds hold. We can then construct Hansen-Jagannathan bounds to understand the properties of the implied stochas-

tic discount factor to see how much our model can explain the variability of expected excess returns.

For log-linear models, we can write down the log Sharpe ratios

$$s_t^i = -\text{Corr}_t(m_{t+1}, rx_{t+1}) \sigma_t(m_{t+1}) \quad (1.55)$$

where lowercase variables are in logs. We also know what the (log) risk-free rate is

$$\begin{aligned} r_{f,t}^{(1)} &= -\mathbb{E}_t[\exp(m_{t+1})] \\ &= -\left(\mathbb{E}_t[m_{t+1}] + \frac{1}{2}\sigma_t^2[m_{t+1}]\right) \end{aligned} \quad (1.56)$$

For a strictly positive cum dividend value process, the log conditional Sharpe ratio can be written as

$$s_t^i = \frac{\mathbb{E}_t[r_{i,t+1}] - r_{f,t} + \frac{1}{2}\sigma_t^2(r_{i,t+1})}{\sigma_t(r_{i,t+1})} \quad (1.57)$$

$$\approx \frac{\mathbb{E}_t[rx_{t+1}] + \frac{1}{2}\sigma_t^2(rx_{t+1})}{\sigma_t(rx_{t+1})} \quad (1.58)$$

for any (log) return process $r_{i,t+1}$. The approximation holds for (63) for values of rx_{t+1} around zero. For now, let us suppose that the unconditional mean of rx_{t+1} is approximately zero. This seems to be a reasonable approximation.

For the restrictions on the market price of risk in Cochrane and Piazzesi (2008) the choice of $(\lambda_0^{\text{level}}, \lambda_1^{\text{level}}) = \lambda$ is equivalent to fitting the cross-sectional regression linking both an unconditional mean expected excess return and the time-variation of expected returns exclusively through movements in the return-forecasting factor (only in relation to random walk shocks). In addition, when the portfolio of bonds is weighted by \mathbf{q}'_r , the magnitude change in expected returns as a function of x_t implies wild swings in the conditional Sharpe ratio, which in turn is linearly related to the conditional volatility of the pricing kernel for the class of linear

affine models. We can see this with the following linear-affine relation between the Sharpe ratios and expected excess returns:

$$\sigma_t(m_{t+1}) = s_t^{(1)} = \frac{\kappa_0 + \kappa_1 \mathbb{E}_t[rx_{t+1}]}{\sigma^2(rx_{t+1})} \quad (1.59)$$

$$= \kappa_0 (\sigma^2(rx_{t+1})^{-1}) + \kappa_1 a(1) + \kappa_1 \beta(1) \sigma^2(rx_{t+1})^{-1} \cdot x_t \quad (1.60)$$

$$= \gamma_0 + \gamma_1 \cdot x_t \quad (1.61)$$

where $(\kappa_0, \kappa_1, \gamma_0, \gamma_1)$ are arbitrary constants, and $\mathbb{E}_t[rx_{t+1}] = a(1) + \beta(1)x_t$ where x_t is the return-forecasting factor constructing from average term-premia not yet weighted by any portfolio. In general, we have the following affine relation between expected returns to the return-forecasting factor $\mathbb{E}[rx_{t+1}^{(n)}] = a(n) + \beta(n) \cdot x_t$. Once we weight the portfolio of bonds by \mathbf{q}'_r , we get the cross-sectional average of returns is given by $\mathbb{E}_t[\mathbf{q}'_r \cdot \mathbf{r}\mathbf{x}_{t+1}^{(n)}] = \tilde{x}_t$, where the $\widetilde{(\cdot)}$ denotes variables that are weighted by the portfolio \mathbf{q}_r . We can then relate the conditional volatility of the pricing kernel $\sigma_t(m_{t+1})$ to the weighted return-forecasting factor with the following weighted regression

$$\begin{aligned} \sigma_t(m_{t+1}) &= \frac{1}{2}a(n)\sigma\left(rx_{t+1}^{(n)}\right) + \underbrace{\beta(n)\left[\sigma\left(rx_{t+1}^{(n)}\right)\right]^{-1}}_{\text{weighted regression coefficient}} \cdot x_t \\ &= \tilde{a}(n) + \theta \cdot x_t \end{aligned} \quad (1.62)$$

so for any n time-to-maturity, $\beta(n)\left[\sigma\left(rx_{t+1}^{(n)}\right)\right]^{-1} = \theta$, a constant that needs to be large to match the large amount of predictability at long horizons. For the U.S. the weighted regression coefficient is $\hat{\theta} = 10.67$, which implies large magnitude changes in the term $\sigma_t(m_{t+1})$ associated with variation in x_t .

The identification restriction allows the model to explain most of the cross-sectional variation in expected returns, but it implies a conditional Sharpe ratio that increases linearly with n years-to-maturity. The problem with this result is that Sharpe ratios are inversely related to maturity as Duffee (2010) argue. Intuitively, large Sharpe ratios imply a large amount of compensation per unit

of risk. This is great for investors, but it is not clear why there needs to be so much risk compensation in bond markets, especially the amount implied by the return-forecasting factor in the linear factor model. Because factor models have no economic content it is unclear what *type* of risk the bondholder is being exposed to and demands compensation.

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CHAPTER 2

Uncertain Risk and Return in Bond Markets, II

2.1 Introduction

Predictable bond risk-premia warrants attention from a broad spectrum of economic actors. Return predictability gives investors the ability to strategically rebalance a portfolio of assets to stabilize unwanted return variability, not to mention helps central bankers conduct policy through financial markets in order to manage expectations where reputation can go a long way toward minimizing undesirable consequences.

In my analysis, I am motivated by the following questions: Are predictable bond excess returns exploitable in real-time? Is variation in predictable risk-premia perfectly captured by a linear combination of forward rates? In order to address these questions, I take the viewpoint of a real-time forecaster who only knows as much as an econometrician at any given point in time.

Taking a different vantage point by placing restrictions on the information set of investors allows us to determine if the size of the predictable component described by Cochrane and Piazzesi (2008) is consistent with real-time data availability. I start with a benchmark forecast nearly identical to Cochrane and Piazzesi (2005, 2008) using forward interest rates to pick up a stable predictable component of bond excess returns using the full-sample of data. I then compare it to forecasts that are consistent with real-time information. Namely, I use recursively updated forecasts that are revised as data becomes available. These forecasts are

constrained by the amount of information available but are flexible in how they are revised with the arrival of news. In taking this approach, I combine the analysis of predictable bond risk-premia by Cochrane and Piazzesi (2005, 2008) with the learning process of an econometrician described by Duffee (2011) using recursive least squares to approximate real-time forecaster behavior.

Ensuring that agents only have information available at the given time period when decisions are made can inform us on the difficulties that real investors face. Parameter uncertainty can play a pivotal role in forecasts of bond risk-premia if the relationship between the predictor and predicted variable have changed, or if there is some omitted term that may not be fully accounted for by the predictor variables. Because agents know that they do not know it all, they behave in ways to guard against the worst thing that can happen. This fear of the unknown forces them to guard against large and persistent declines in asset returns in real-time. In turn, this dynamic forecasting problem can generate forecast differences, relative to forecasts made with the full set of historical data, that make returns predictable in historical data. Therefore, under the mentioned conditions, the exploitability of predictable patterns in risk-premia may not be perceived to be advantageous by real-time forecasters.

If agents have concern that the relationship between risk-premia and the variables that proxy for risk may have changed and if surprise changes cannot be determined to be transitory or long-lived, then real-time forecasters tend to have biased forecasts of bond risk-premia relative to those made with the full set of historical data. The biased forecasts persist and is systematically related to changes to business cycle conditions. I find support of this phenomenon. That is, there is evidence for a cross-section of industrialized economies that realized bond risk-premia tend to move with differences in forecasts between the full-sample return-forecasting factor and its recursively updated counterpart.

Improvements in forecast performance that emphasize more recent data sug-

gest that forecasters are in some ways better off not trusting their model. The lack of trust in the forecasting model means that bond risk-premia must include additional compensation for investors to hold onto a portfolio of bonds. A hidden factor similar to the one described by Duffee (2011) found in the difference between forecasts assuming all historical data is available and those made recursively have predictive power for future short-term interest rates and the mean level of the yield curve. These results are broadly consistent across a cross-section of industrialized government bond markets.

Using this hidden factor as a proxy for the market's expectation of the level of interest rates, I offer an auxiliary hypothesis for the decline in long-term bond yields and the inversion of the yield curve: the market behaved *as if* it expected a decline in the level of interest rates from 2004 – 2005. I also find support of the claim that the conundrum is consistent with a decline in business cycle related risk as suggested by Cochrane and Piazzesi (2008) albeit under less restrictive information restrictions.

The intuition for the results can be gleaned from the perspective of an investor. If information contained in the return forecasting factor exceeds that of yield spreads, it is possibly related to the long-run average in which interest rates will eventually settle. If that is the case, then it contains turning points, or times where the rates will change direction. The source of movement in long-horizon predictor variables that forecast risk-premia can be linked to market participants' long-horizon expectations of returns that are driven by changes in the level of the yield curve. The crucial issue is whether the turning points are predictable. For one, an investor has difficulties telling apart persistent shocks from transitory ones. Sometimes, the expectations of these mentioned factors change abruptly. This problem is even more pronounced if we acknowledge that the investors are afraid that their model may not generate the optimal forecast. Based on how market participants behave, it is difficult to identify time-variation in bond risk-premia

with the return-forecasting factor because it seems that real-time forecasters must be ready to revise their forecasts so that their portfolio of bonds is minimally impacted by unexpected changes in interest rates.

2.2 Literature Review

My work builds on Cochrane and Piazzesi (2005, 2008) in understanding predictable variation in bond risk-premia. I propose a more flexible framework than theirs that highlights incomplete information on the part of a forecaster. In relation to the inverted yield curve of 2004 – 2005, Cochrane and Piazzesi (2008) suggest that a decline in business cycle related risk explains the inversion of the yield curve, taking a stance that such an episode is no conundrum at all but is characteristic of an economic expansion when risk-compensation is normally low. Rudebusch, Swanson, and Wu (2006) empirically examine and offer several candidate explanations for the inverted yield curve, lending support to the decline in long-term bond yield volatility although they acknowledge that it can only explain a relatively small fraction of the conundrum. I complement existing explanations by providing evidence market participants expected the short-term and level of interest rates will decline during the time. I also discuss reasons for the decline using the central bank decision rule to tie my analysis to key economic aggregates. I posit that a fear of a lower level of interest rates, either due to fears of deflation or a belief that future growth prospects are dim, is also consistent with the inversion of the yield curve.

This paper is similar in spirit to work on robust control, which was given textbook treatment by Hansen and Sargent (2007). Similar in premise to the motivation behind robust control, I allow the forecaster to have doubts about their model and to revise forecasts to guard against being wrong, investigating a set of forecasting models that changes how past data is weighted into updated forecasts.

My analysis specifically highlights a rule that converges to a perfect foresight forecast. In addition, my work is similar to Gourinchas and Tornell (2004) who study how investor’s misperception of the short-lived changes in interest rate differentials can explain key features in foreign exchange markets. Although I also highlight the signal extraction problem faced by investors, I look particularly at forecasting risk-premia in government bond markets for a set of industrialized countries and show that statistically measured predictability can be partly explained by systematic forecast differences among different rules. Piazzesi and Schneider (2011) also study the importance of subjective beliefs using survey data and decompose risk-premia in bond markets into frequency components. I also highlight the role of subjective beliefs on asset prices by approximating adaptive expectations forecasts using recursive least squares. Because I use forecasting rules to approximate subjective expectations formation, I can study a broader set of countries. I also use differences in forecasts to forecast future short-term interest rates and average yield curve levels.

2.3 Forecast Performance

I quantitatively measure the size of the predictable component in bond risk-premia and examine the forecast ability of predictor variables constructed from forward interest rates. In addition to the benchmark forecasting factor of Cochrane and Piazzesi (2005, 2008) I highlight rules that are (a) consistent with real-time availability as well as those that (b) emphasize more recent data in forecasts of the future.

2.3.0.1 Preliminaries

To be clear with notation, let $\mathbb{E}^n \left[rx_{t+1}^{(n)} \right] := (N - 1)^{-1} \sum_{n=2}^N rx_{t+1}^{(n)}$ be the average risk-premia taken over the set years to maturity n , z_t is the predictor variable

that captures a forecastable component in returns, and $\varepsilon_{t+1}^{(n)}$ is some idiosyncratic, unforecastable component. The cross-section of bond risk-premia for a set of long bonds will be denoted by n . One-year ahead forecasts are written as $\mathbb{E}_t [rx_{t+1}]$ with a subscript t denoting expectations formed at date t . In this paper, I often forecast one-year ahead bond risk-premia for a portfolio of long bonds. The notation $\mathbb{E}_t [\mathbb{E} [{}^n rx_{t+1}]]$ to mean the described object is too cumbersome, so I write it as $\mathbb{E}_t^n [rx_{t+1}]$ instead. The predictor variable used for this paper are a subset of forward rates emphasized in Mang (2012), i.e. $\mathbf{z}_t = \left(f_t^{(0 \rightarrow 1)}, f_t^{(4 \rightarrow 5)}, f_t^{(8 \rightarrow 9)} \right)$ where forwards are chosen to capture most of the time-variation of bond risk-premia albeit with less ability to capture the magnitude changes. I will adhere to the choice of these subsets of forward rates in this paper as well.

Predictable returns in bond markets can be characterized by the following estimation equation

$$\mathbb{E}_t^n [rx_{t+1}] = \alpha + \boldsymbol{\beta}' \cdot \mathbf{z}_t + \varepsilon_{t+1} \tag{2.1}$$

where unpredictable returns coincide with $\boldsymbol{\beta} = \vec{0}$, implying that forecasting variables provide no information about future excess returns. However, my analysis along with a plethora of empirical evidence suggests otherwise; one-year ahead excess returns in bond markets have a predictable component.

The more interesting question is the size of the predictable component in bond excess returns. The return-forecasting factor of Cochrane and Piazzesi (2008) captures a predictable component in bond risk-premia that has been proven to be useful for forecasting. They find as much as 44% of the variability in bond excess returns can be captured by using the term structure of (Fama and Bliss) forwards. A subset of forward rates delivers captures about 14–17% of variability in 12-month ahead bond excess returns in the U.S. The estimates of Cochrane and Piazzesi (2008) and Mang (2012) are similar in the amount of time-variation

in returns that is captured, but the magnitude changes of bond excess returns using the predictor variables highlighted in Cochrane and Piazzesi (2005) deliver superior forecasts. However, for the purpose of this paper I will use only a subset of forwards. I do this to reduce the problems associated with multicollinearity in regressions. A discussion of this problem for forecasting bond risk-premia is found in Mang (2012). In the following sections, I describe the benchmark case and outline alternatives and proceed to compare them.

2.3.1 Benchmark Case

Definition 1 *A full-sample forecast rule uses a predictor variable \mathbf{z}_t and all available data to make a forecast about bond risk-premia. This is equivalent to constructing a predictor variable \mathbf{z}_t that captures a predictable component in returns using all available information. First, they construct the predictor variable using forward rates by running the following regression:*

$$\mathbb{E}^n [rx_{t+1}] = \alpha + \boldsymbol{\beta}' \cdot \mathbf{z}_t + \varepsilon_{t+1} \quad (2.2)$$

thereby estimating $(\alpha, \boldsymbol{\beta}) = \theta$. The resulting forecasting factor can be written as

$$\mathbb{E}_t^n [rx_{t+1}] = \tilde{\alpha} + \hat{\boldsymbol{\beta}}' \cdot \mathbf{z}_t := x_t \quad (2.3)$$

For the remainder of the paper, I will describe this forecasting factor as the x -factor.

In the case of bond markets, Cochrane and Piazzesi (2005) outline the process of constructing the return-forecasting factor. The estimated forecast using the full sample of data only requires running the regression once to find $\hat{\theta}$. By using the estimation equation (2), one implicitly assumes that parameter estimates are stable over time and that the dynamics describing the time-variation in bond risk-premia is stable across subsamples. The approach also assumes that forecasting

agents information set is larger than that of an econometrician. The time-invariant parameters describing the time dynamics of bond risk-premia highlight a lack of concern on the part of the forecaster that the model is the optimal one. The degree of certainty in the model specification avoids unnecessary complication and allows for a straightforward construction of a stable, predictable forecasting factor. This more or less describes the construction of the return-forecasting factor highlighted in Cochrane and Piazzesi (2005)

I am also interested in the real-time exploitability of predictable bond risk-premia. Ensuring real-time exploitability requires relaxing the implicit assumptions made to construct the return-forecasting factor. In doing so, I introduce a class of rules that are available to forecasters who have concerns related not only about real-time exploitability of returns, but also about whether they have the correct model. Implicit assumptions and see how the forecast performance of doing so changes. In doing so, I introduce a wider class of forecasting models that is available to forecasters that are also consistent with real-time availability considerations.

2.3.2 Alternative Forecast Rules

The benchmark forecasting rule can potentially be improved upon in several ways by relaxing some of the implicit assumptions. First, constraining the information set of the forecaster and determining the forecast performance relative to the benchmark case. The corresponding estimation equation is given by

$$\mathbb{E}^n [rx_{t+1}] = \alpha_t + \beta_t' \cdot \mathbf{z}_t + \varepsilon_{t+1} \quad (2.4)$$

Notice that the parameters of the forecasting equation now depend on time, distinguishing it from the full-sample forecasts. Recursively estimating the model by incorporating newly released data involves solving for a set of parameters that minimize the errors of the forecasts using a criteria on how to weight the losses.

The equivalent problem of estimating the model requires solving for the set of parameters implicit in running an ordinary least squares regression where the loss function is taken to be the sum of squared errors. In this paper, I highlight a forecast that recursively updates parameter estimates over time as news becomes available where past forecasts are not discounted and new data receive equal weight in forecasts. This alternative return-forecasting factor will be extensively analyzed in this paper *along with and in relation to* the full-sample return-forecasting factor. The set of parameters $\{\hat{\theta}_t\}_{t=\hat{T}}^T$ describes the sequence of forecast revisions where $\hat{T} = 30$ denotes the number of observations used to estimate the initial forecast. The recursively updated forecasts can then be written as the following:

Definition 2 *The recursive return-forecasting factor (henceforth, the rx-factor) characterizes a forecasting rule in which real-time data are used to revise over time estimates of $\theta_t = (\alpha_t, \beta_t)$.*

$$\tilde{\mathbb{E}}_t^n [rx_{t+1}] = \tilde{\alpha}_t + \tilde{\beta}'_t \cdot \mathbf{z}_t := \tilde{x}_t \quad (2.5)$$

This forecast rule coincides with the gain parameter t^{-1} where new data affect parameter estimates, but the importance of new data decline as the forecasts reach the end of the sample period.

2.3.3 Results

I display the relative forecasting rules used to predict one-year ahead average risk-premia of an equally weighted portfolio of long bonds. I compare full-sample forecasts with a forecasting rule that either (a) weight all data equally (recursive) (b) to weight recent observations more heavily, with a discount factor of $\rho = 0.9995$, (c) a forecast rule with discount factor $\rho = 0.9994$, and (d) a rolling regression estimate, which looks only at the latest $\hat{T} = 30$ observations. I include

the average realized risk-premia (grey line) to visually compare the accuracy of the forecasts. Forecasts that heavily discount past data have the lowest root mean squared errors (RMSE) and root mean absolute errors (RMAE), making the rolling regressions the most accurate. I also determine how realized risk-premia co-vary with differences in the alternative forecast rules relative to the full-sample forecasts. I do so by regressing the average realized bond risk-premia on the forecast differences. The percentage of co-variation is simply the R^2 from the regression. In figure 2.1, I compare some alternative forecast rules to that which assumes the full-sample of data is available when making forecasts.

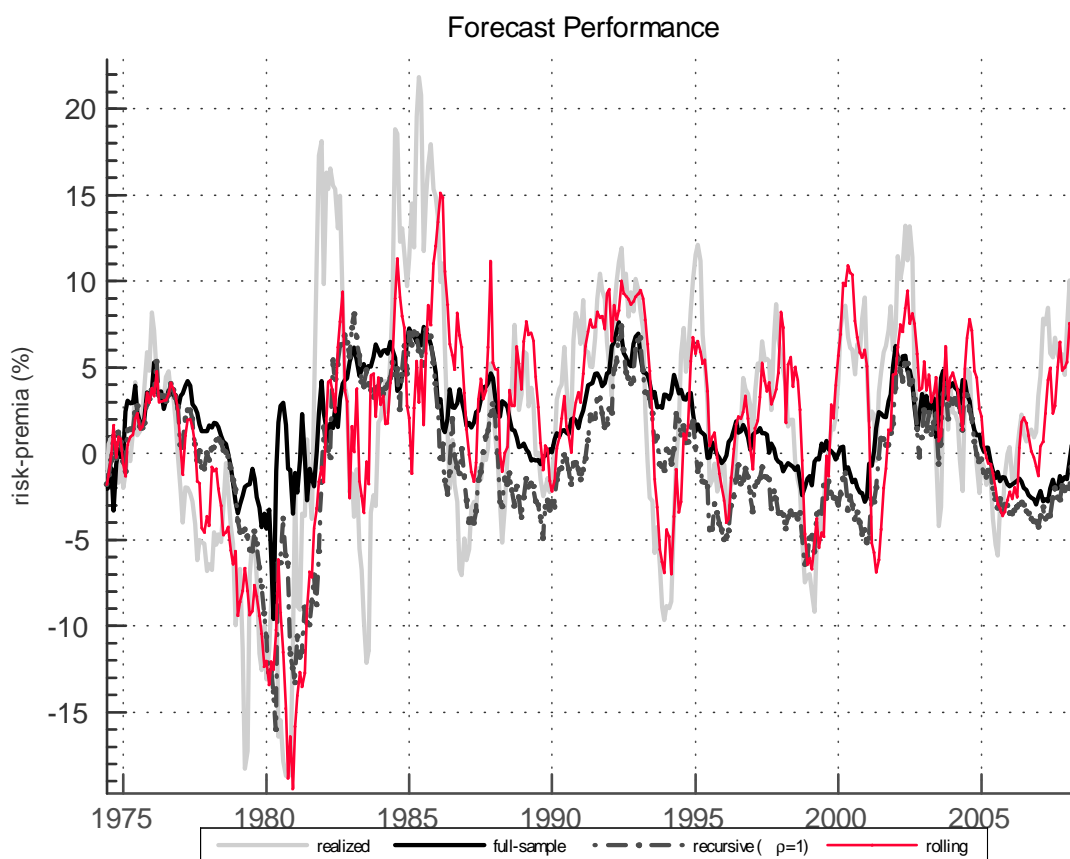
Table 2.1: Forecast Performance under Alternatives

Rule		full- sample	$\rho = 1$	$\rho = 0.9995$	$\rho = 0.9994$	rolling
U.S 12.1971-5.2009	RMSE	640	643	621	619	481
	RMAE	499	468	469	470	353
	*	0	63.9	61.4	60.4	34.9

* percentage of co-variation of forecast errors with realized risk-premia

Comparing forecasting rules with the forecast made with the full-sample of data. The first two rows of the table displays root mean squared errors (RMSE) and mean absolute errors (MAE) in basis points (100 basis points = 1 percentage point) for a set of forecasting rules. The third row is the amount of co-variation the forecast differences, between the full-sample return-forecasting factor and the recursively updated forecasts, have with realized risk-premia, which were calculated by running an GMM estimates of the realized risk-premia on the forecast differences between the associated forecast differences. The standard error estimates include 12 lags to account for overlapping monthly data and are Newey-West corrected.

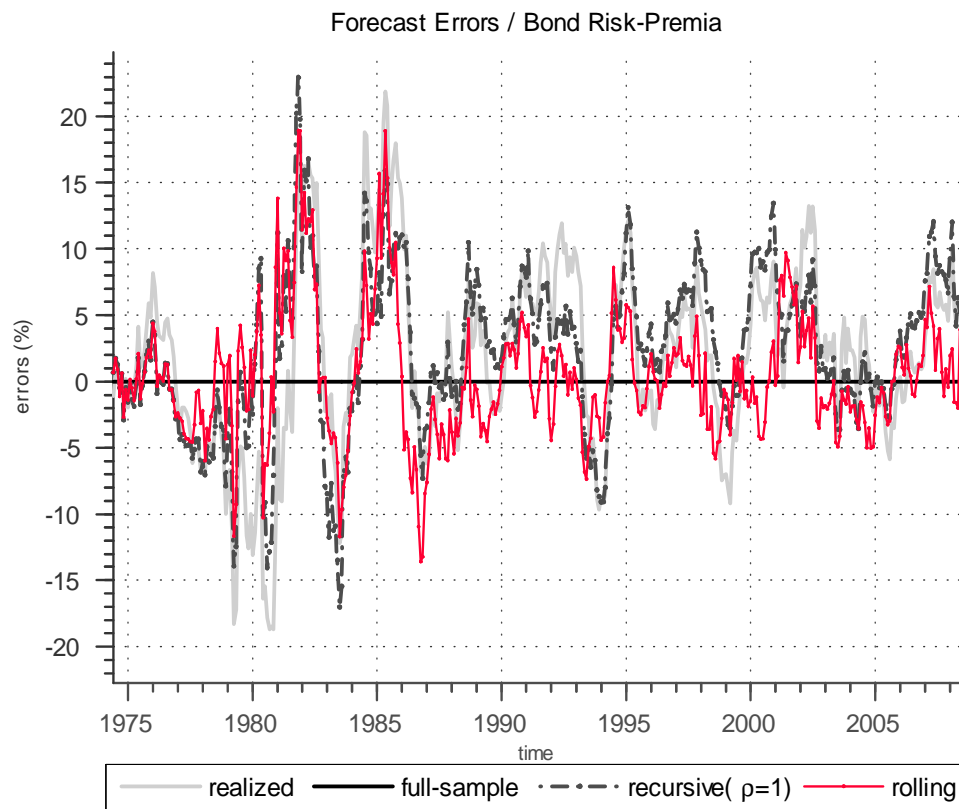
Figure 2.1: Graphical Illustration of Forecast Performance



Comparing forecasts with those made with the full-sample of data. The light grey line is the realized bond risk-premia for an equally weighted portfolio of long bonds. The solid dark line is the forecast from the full-sample of data. The (.-.) line is the recursively made forecasts. The (- - -) line is the rolling regressions. The data are monthly frequency and spans from end-of-month November, 1974 to June, 2008.

Risk-premia are in annualized percentages.

Figure 2.2: Graphical Illustration of Forecast Errors



Realized risk-premia and forecast errors. The light grey line is the realized bond risk-premia for an equally weighted portfolio of long bonds. The solid dark line is the average forecast error made with the full-sample of data. The (.-.-) line is the recursively made forecasts. The (- - -) line is the rolling regressions. The data are monthly frequency and spans from end-of-month November, 1974 to June, 2008.

Risk-premia are in annualized percentages.

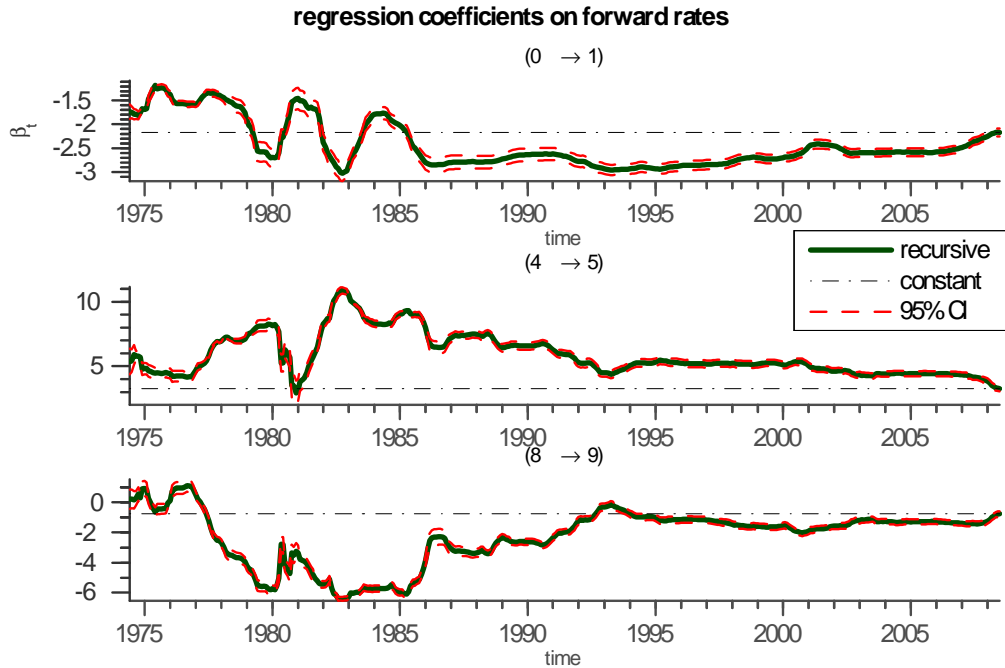
2.4 Comparing Return-Forecasting Factors

The full-sample return-forecasting factor uses all historically available data to capture changes over time in the required risk compensation to hold onto bonds. A real-time forecaster uses forward rates in a similar fashion and re-estimates their forecast as new data arrive. Recursive forecasts acknowledge that their

model is only approximately capturing reality and in updating they are expressing some doubt that they possess the right one. Differences between forecasts, full-sample versus recursive, are apparent in how estimated parameters vary over time. Through parameter variation, recursive forecasts capture changes in real-time expectations about future bond excess returns. Consequently, comparing return-forecasting factors can inform us on sources of disturbances that are hard to anticipate in real-time but are implicitly guarded against. I compare the perfect foresight forecasts with a recursively updated forecast that converge to the benchmark case.

In the figure below, I plot the loadings on β_t when executing this recursive regression approach as a time series, including 95% confidence intervals around the point estimates. The variation in the regression coefficients and the rx -factor may reveal information about real-time forecasts in ways that a full-sample estimation cannot. Interesting dynamics arise in the parameters in comparison to the benchmark constant coefficients case, given by the dash-dot line obtained by using the full-sample of data to come up with a point estimate. There are several noteworthy patterns. The estimates suggest that average risk-premia negatively depends on short-run and long-run changes in forward rates and positively on medium-term forwards. In addition, At every point in time that recursive forecasts are re-estimated, there is a tent-shape in the regression coefficients that characterize the same common factor in pricing bond risk-premia for all maturities, We also see that the regression coefficients fluctuated wildly in the 1980's but have stabilized somewhat after the 1990's.

Figure 2.3: Time-Varying Coefficients on Forward Interest Rates



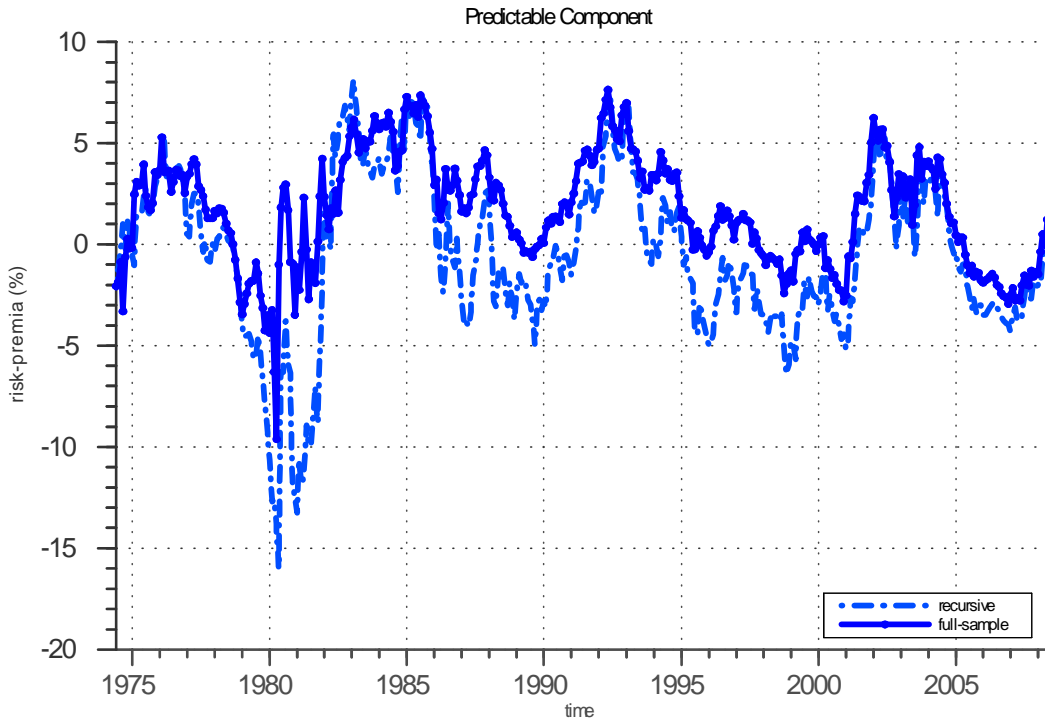
Time-variation in the regression coefficients on forwards. The top panel plots the time-variation in the regression coefficient/loading on the forward rate from year 0 to year 1. The middle plots the time-variation in the coefficient on the forward rate from year 4 to year 5, and the bottom panel plots the loading in the long-horizing forward rate, from years 8 to year 9. The (- - -) line around the time-series is the 95 confidence intervals are also Newey-West adjusted for serial correlation. The (.-.-.) line plots the constant coefficient from the full-sample forecasts. The data are monthly frequency and span from November 1974 to June 2008.

There are potential objections to my approach. Updating forecasts every time period with new data and not discounting past data while increasing the sample size means that new data affect estimated parameters late in the sample period less over time so that the final parameter estimates in the recursively updated forecasts converging to the estimates obtained using the full set of information. This approximates a forecaster's concern about their model specification, but as we approach the end of the sample of data, forecasts are made as if uncertainty

about the model specification becomes less important. Therefore, adaptive expectations made by recursively estimating the model highlights a real-time forecaster who has incomplete information about the future but believes that the model is approximately correct and that the relationship between the predictor and predicted variable is indeed stabilizing over time. This allows for a direct comparison between the forecast rules to focus primarily on the arrival of news and its impact on forecasts under different information restrictions.

The difference in the two forecasting factors is unobservable in real-time, but relate to the variation in the parameter estimates. I compare the recursive return-forecasting r -factor, the full-sample forecasting x -factor graphically in Figure 2.4:

Figure 2.4: Graphical Comparison of Key Return-Forecasting Factors



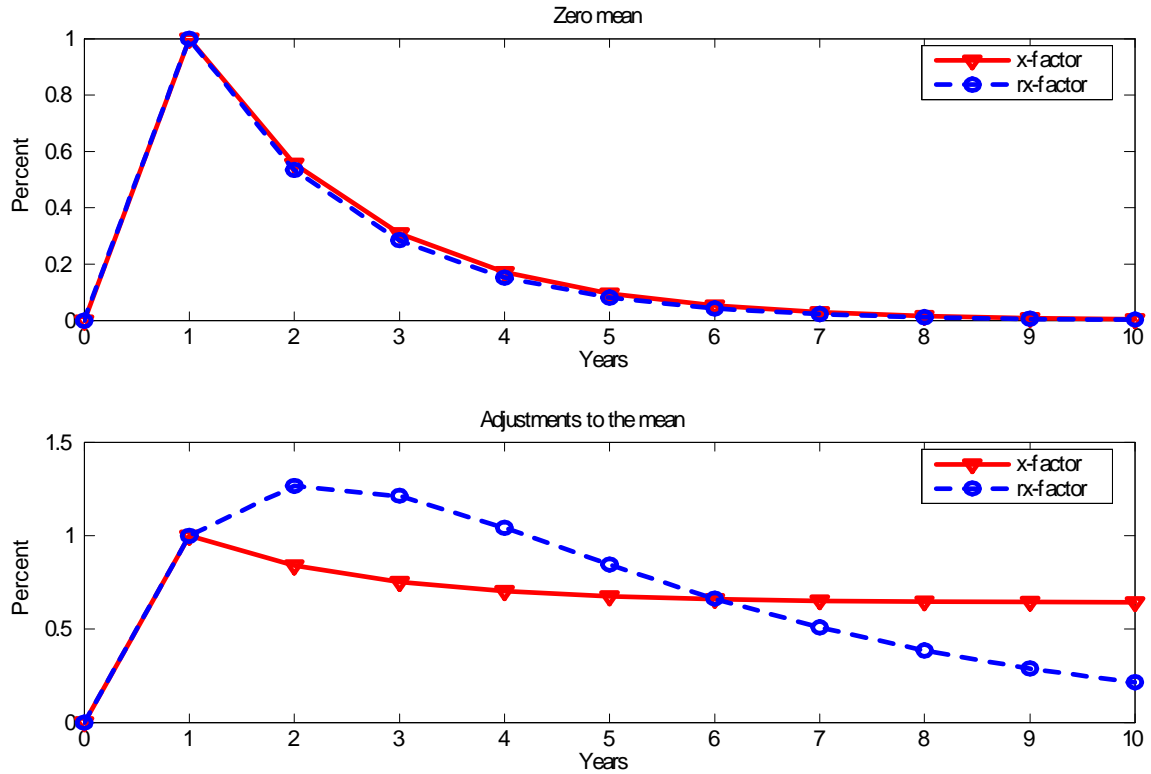
A comparison between the x -factor and the rx -factor. The (-.-.-) line is the recursively updated return-forecasting factor. The solid (blue) line is the full-sample return-forecasting factor. The latter display nearly identical time-variation as in Cochrane and Piazzesi (2005, 2008) with less predictability captured of magnitude changes in realized risk-premia than theirs. See Mang (2012) for more details.

The recursively updated return-forecasting factor, or rx -factor, strongly comoves with the full-sample return-forecasting factor, or x -factor. The forecasting factors are positively correlated and countercyclical. While under less restrictive information assumptions, information contained in the term structure of forward rates is indeed capturing how risk compensation varies along the business-cycle. It is also interesting to note that although the two factors move together, they occasionally differ. Notice that the recursive forecasting factor usually reaches the trough of risk-premia before the full-sample forecasting factor and is slow to adjust

upward. The essential reason for differences in forecasts is how news is treated by recursive forecasts. Adaptive forecasts display additional persistence relative to full-sample forecasts of bond risk-premia since large adjustments to the current forecast pass directly to parameter estimates. Real-time forecasters are unsure if new data changing their estimates are altering the slope of their regression or the intercept term. As a result, the additional persistence is then passed through to the intercept term.

Allowing parameter estimates to vary over time complicates things because the dynamics of the state need not be stationary. For instance, if one were to model the dynamics of bond risk-premia, a standard impulse response for the reaction to the forecasts to a one standard deviation disturbance will likely lead to similar persistence. However, the forecasts have peculiarities that differentiate themselves from one another. For one, recursively updated forecasts have a time-varying intercept term so it is not appropriate to say that the dynamics of bond risk-premia are well-described by a mean zero time-series model. Second, the full-sample forecasting factor has a constant but nonzero mean. Incorporating these features will generate impulse responses that have a more meaningful comparison in how it captures their differences of the bond risk-premia dynamics, one that acknowledges the differences in how the two forecasts are generated.

Figure 2.5: Impulse Responses for Bond Risk-Premia



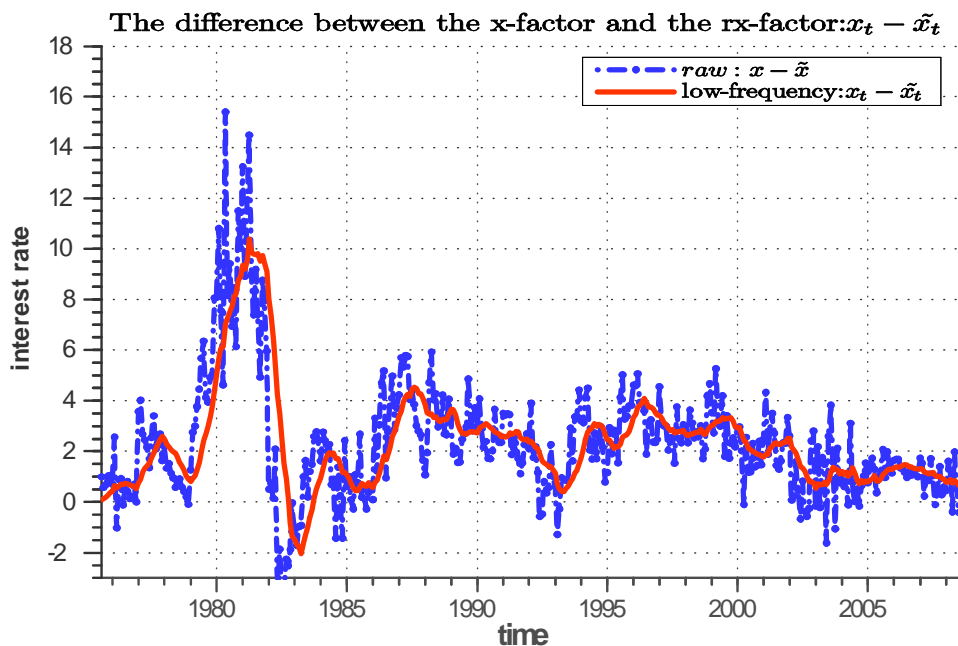
Impulse responses of a dynamic system describing the return-forecasting factors. The top panel is generated by estimating each return-forecasting factor as a first order autoregressive process with zero mean. The bottom panel adjusts for differences to the mean of the factors: the x -factor has a constant but non-zero mean, whereas the rx -factor has a time-varying mean whose dynamics are approximated as an autoregressive process. For the rx -factor, a time-varying mean adds persistence to the dynamics of the rx -factor, which generates a hump-shaped impulse response

2.4.1 Forecasts of Future Interest Rates

The wedge between perfect foresight forecasts and recursively updated forecasts reveal market participant beliefs about the average level of the yield curve. By simply taking the difference between the x -factor and the rx -factor, we will see information captured by the full-sample estimation procedure that is not observ-

able to a real-time forecaster. The wedge between the x -factor and the rx -factor seems to vary over time. This series (the $-$ line) appears to also contain idiosyncratic movements. To mitigate the noise, I take a 12-month moving average to isolate a slow-moving, low-frequency component. The result is a (smooth red) time-series that seems high in the late 1970's and early 1980's, dips below zero from 1980 – 1985 and then slowly moves within a range of 1 – 4%. One can also speculate that the series captures a latent component of risk that is important in bond markets, particularly the market participant's subjective beliefs about where short-term interest rates will eventually settle. This low-frequency component of risk that is important in bond markets can be seen in the graph below:

Figure 2.6: The Difference in Forecasts



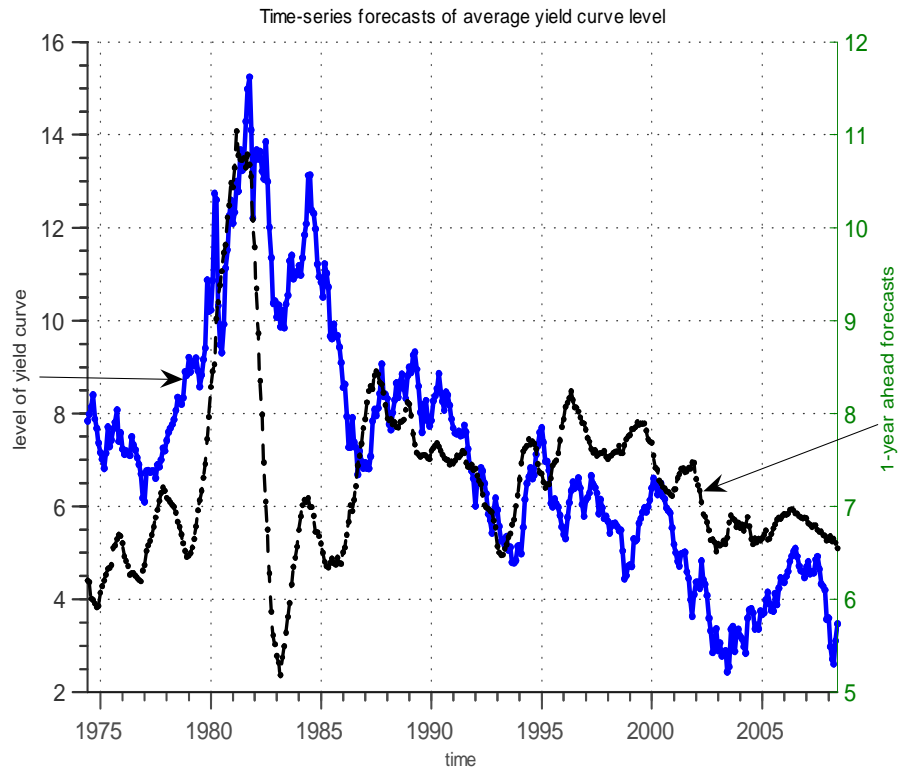
Unobservable component of the return-forecasting factor. The (.-.-) line captures the difference in the full-sample and recursively made forecasts of the risk-premia of an equally weighted portfolio of long bonds. The solid (blue) line looks at the 12-month moving average of the difference. Taking a moving average corrects for the re-estimation procedure updating monthly as opposed to re-estimating every 12-months. Data are monthly frequency and span from November, 1974 to June, 2008.

The y-axis measures annualized risk-premia in percentages

To my knowledge, the procedure that I use to tease out this unobservable risk factor has not been documented in the literature although Duffee (2011) relates a hidden factor to both risk-premia and expected future interest rates that can be extracted using a Kalman smoother. In contrast, I use recursive least squares to mimic forecasts of real-time market participants to uncover their beliefs about future interest rates. The wedge between forecasts, full-sample less recursive, is precisely the "fear factor," or the component of yields that real-time forecasters find difficult to anticipate and thus implicitly guard against. This wedge also

captures the difference in opinion between forecasts made with the full-sample of data and recursive forecasts reveals that market participants forecasted a decline in the level of interest rate.

Figure 2.7: The Difference in Forecasts and The Mean Level of the Yield Curve



Market expectations about the average level of the yield curve. The solid (blue line is the interest rate level, taken to be the equally weighted average of interest rates ranging from maturities of 1 year up to 10 years. The (- - -) green line is the adjusted, smoothed difference between the full-sample and recursively updated forecasts of bond risk-premia. The adjustment is equal to the estimated intercept when regressing interest rate levels on the differences in forecasts to make the level of the graphs comparable. Data are monthly and range from 11.1974-6.2008. Both y-axes are in annualized percentages.

Table 2.2: Difference in Forecasts and Future Interest Rates

	intercept	slope	R^2
level	6.23 (6.45)	0.49 (1.72)	13%
short-rate	6.03 (6.24)	0.52 (1.84)	14.5%

Market forecasts using the difference between the return-forecasting factors to forecast interest rate levels. The estimates describe the regression of interest rates, both the level and the short-rate, on the difference between the full-sample and recursively made forecasts. The estimates are linear GMM estimates with 12 lags to account for overlapping monthly data. The standard errors are Newey-West corrected. The first column displays the intercept term for the respective regressions with t-statistics in parentheses below. The second column displays the slope coefficients with associated t-statistics below in parenthesis. The last column displays the R-squares of the regression.

Economic expansions are generally characterized by negative risk-premia economic expansion, but also by a high average level of the yield curve. Throughout the 1990's, with the exception of a mild recession early in the decade, investors tend to accept lower risk-premia for holding onto a portfolio of bonds that coincide with the market's expectations of high interest rate levels. In other words, they tend to bid away the risk-premium and are zealous about accepting low risk-adjusted returns because they perceive the level of interest rates will persist for some time. In 2004-2005, market investors accepted less risk-adjusted returns than fully rational investors, but they did not bid down bond risk-premia with as much zeal as in previous cycles. Thus, the difference in forecasts reveal that market participants behaved as if they expected future short-term interest rates and average interest rate levels to decline.

This result could be driven by the convergence of adaptive forecasts to perfect foresight forecasts, but the error bands is informative in this case. Large error bands coincide with high average levels of the yield curve precisely because large bands means that there is a large degree of parameter uncertainty in the forecasts. The bands do tighten precisely when the level of interest rates is expected to be low, but they are large enough so that the range of possibilities is within the average level over the sample.

2.4.2 Evidence from Industrialized Government Bond Markets

Extending my analysis to international bond markets reveal interestingly similar patterns to the U.S. experience. Comparing recursively made forecasts with forecasts made with the full-sample of available data reveal the following stylized facts:

1. Forecast performances that discount past data improve upon forecasts made with the full-sample of data.
2. Differences in the forecast are positively correlated with realized bond risk-premia for a portfolio of long bonds.
3. The difference in the mentioned forecasts tends to track 12-month ahead interest rates, both the short-rate and the average level of the yield curve.

The strength of these findings in terms of its statistical significance will vary due to idiosyncratic experiences across countries and to the limited availability of data. Moreover, it is striking to see that the overall level of the yield curve has declined for all the major industrialized economies studied in this paper, notably Japan. Short-term interest rates in Japan has been approximately zero since at least 2001.

The forecasting factors for German bond markets closely resemble the U.S. Issues related to data availability should only affect estimates early the sample, so the general patterns through the entire sample are largely consistent across time. Similar interest rate forecasts hold not only for the U.S., Germany, and Japan, but also Australia, Canada, and England. The experiences of New Zealand, Switzerland, and Sweden are similar with respect to the interest rates forecasted to be low, but limited data availability means that the standard errors of the estimates are large. Error bands are especially large for Norway where data availability is most limited.

2.5 Commentary

In this section, I will discuss my analysis in relation to central bank policy often summarized using the Taylor rule to relate time-varying risk, risk perceptions, and bond excess returns to key aggregate economic data.

2.5.1 Relation to Key Economic Aggregates

The analysis in this paper is relevant for equilibrium models that inform us about the macroeconomic factors that determine asset prices. Forecasts of 12-month ahead bond risk-premia can be improved upon if they discount past data, meaning real-time concerns about changes to the data generating process matter for forecasts about future asset prices. My estimates inform me that the difference in the full-sample and recursively made forecasts not only positively co-move with realized risk-premia but they also forecast future short-term interest rates and the average level of the yield curve. These findings suggest that unobservable factors compel forecasters to react to real-time changes in bond prices tied to future interest rate movements. Interest rates are a key instrument at the disposal of central banks around the world in achieving objectives regarding output growth

and inflation in the aggregate economy. Central bank policy and views about future economic activity is thus a key non-diversifiable risk factor in government bond markets.

In light of my findings, it is possible to interpret the views of a forecaster and how it relates to my analysis on predictable returns in government bond markets. Based on my results, one possible interpretation relates a forecaster's concern about their model specification to economic aggregates: Asset prices reflect expectations about *future* economic aggregates and because this factor is unobservable, forecasting using available data is the best that they can do. In that regard, the agents are acting rationally. Moreover, because the problem implies that model uncertainty never disappears, forecasters never completely trust their model. The permanence of uncertainty could be due to the fear of unanticipated changes in monetary policy, the fear of changes in the growth rate of the aggregate economy, i.e. admitting a stochastic trend in economic growth, or the relative contribution of both. The answer likely depends on feedbacks and linkages between both factors, as central bank authorities interpret the data on macroeconomic aggregates using their own model and make adjustments to policy accordingly. At the same time, general worldwide events are sometimes difficult to incorporate into a parsimonious model. Often, model approximations leave out features that are only later incorporated into analysis. In any case, the next section outlines key historical events that I explain using my framework for analysis that takes seriously the perspective of market participants.

2.5.2 Monetary Policy and Future Economic Activity

Parameter uncertainty in the recursively updated forecasts reveal information about the wedge between full-sample and recursive forecasts of bond risk-premia. Large differences reflect large disagreements in future short-term and average level of the yield curve, evident in the large error bands around the level of this term. If

so, this term reflects not only the market's expectations about the average level of the yield curve, but also the ability of central bank authorities to keep them under control. Therefore, uncertainty in a forecaster's estimates of bond risk-premia will require additional compensation.

If the average level of the yield curve is linked to a central banker's decision rule in the determination of an equilibrium, then it can be said that bond market participants require additional compensation for holding onto bonds if long-run expectations about inflation particularly for bond markets are not well-defined. In other words, if expectations about interest rates can be held reasonably under control by central bankers, then the market will not demand additional compensation for larger interest rate levels. The stochastic properties of the market's expectations of interest rate levels also reveals to us that this uncertainty, while subdued at times especially of late, does not disappear. It is precisely the persistent nature of this term that drives a wedge between full-sample forecasts and those made recursively.

Evidence of changes in monetary policy can be found in my analysis. Time-variation in the regression coefficients in the recursively updated forecasts tells a story of structural breaks in the data generating process. For instance, during the early to mid 1980's the regression coefficients on short, medium, and long-horizon forward rates display erratic fluctuations consistent with my interpretation of a change in monetary policy regimes. Fed Chairman Paul Volcker decided to tighten the money supply, by raising interest rates and by narrowing the definition of money, to curtail the high inflationary expectations during that time. In fact, the difference between the full-sample and recursively made forecasts were particularly large during the tail end of the Great Inflation in the early 1980's right around the time disinflationary policy was implemented.

The importance of medium-term business cycle risk can be seen in Figure 2.3. Notice that the regression coefficient on $f_t^{(4 \rightarrow 5)}$, the medium-term forward rate

used in forecasting excess returns has stabilized since the early to mid 1990's. One interpretation of the stabilization of the regression coefficients is interest rate targeting successfully anchored nominal interest rate movements. Under this scenario, we can interpret the stabilization of fluctuations in the estimated regression parameters as the resolution of uncertainty about medium-term frequency risk usually attributed to the yield spread or the slope factor usually attributed to monetary policy. One can then argue that this is driven by monetary policy credibility building as a result of Volcker disinflation in the early 1980's. The stabilization of regression coefficients after 1990, particularly for medium-term forwards hint that the relationship between risk-premia and its co-movement with predictor variables have stabilized as well.

My analysis can also help shed light on a puzzling time period in history where the yield curve inverted after expansionary monetary policy lowered short-term interest rates from 2004 – 2005. First, both forecasts point to the same conclusion: there was a decline in business cycle required risk compensation around 2004 – 2005. Second, as I have highlighted in the previous section, the differences in the forecast rules highlighted in this paper, full-sample versus recursive, have predictive power for future interest rates. Forecast differences fluctuate from 2003 – 2004, get very close to zero in 2004 – 2005 and tend to be below its unconditional average of about two percent from 2005 – 2006. During this time period, financial market participants were willing to accept low returns on a long position of an equally weighted portfolio of long-term bonds. Thus, during the time period 2004 – 2005, the difference between the recursive and full-sample forecasts declined, suggesting that market participants behaved as if they expected a decline in short-term interest rates.

I can pair my analysis with a central banker's decision rule to link short-term interest rates and the level of the yield curve to economic aggregates. Interest rate targeting by the U.S. central bank can be described using the Taylor rule where

nominal short-term interest rates react more than one-for-one with surprise inflation and less than one-for-one with output surprises relative to their respective inflation and output growth targets. The relevant issue then is how changes in short-term interest rates affect bond risk-premia. From the central bank's decision rule, this can be interpreted as a negative surprise to inflation and/or output growth. The expected decline in the average level of the yield curve agree with the assessment that the inversion of the yield curve not only reflects a decline in business cycle risk, but also an increase in long-run risk, either due to deflationary expectations and/or a decrease in the growth rate relative to trend. Because the distinction between the two cannot be made in my empirical analysis alone however since I have not specified an equilibrium model, I also pair my analysis with relevant research using survey data and macroeconomic data by Wright (2011) suggests that inflation uncertainty has declined and that inflation expectations are well-anchored around the unofficial target rate of two percent. If so, then the decline in the long end of the yield curve relative to the short end reflects a decline in future output growth prospects. Despite the fact that the difference in forecasts loses explanatory power for future interest rates towards the end of the sample given their convergence, my analysis highlights the inherent difficulty in determining the time-variation in expected returns during uncertain times.

2.6 Conclusion

In this paper, I depart from implicit assumptions made in the construction of the return-forecasting factor by Cochrane and Piazzesi (2005, 2008). I compare it to recursively updated forecasts that are constrained by real-time information but flexible in that they are revised with the arrival of news. The hidden factor that is simply the difference in forecasts that assume all historical data is available to those that are recursively updated forecasts future interest rates and in doing so,

suggests that market participants expected a decline in the mean level of the yield curve 2004 – 2005, which may explain why the yield curve inverted during this time. I also find support for a decline in business cycle related risk as Cochrane and Piazzesi (2008) offer using an alternative approach that generalizes theirs while also allowing risk perceptions to vary over time. By taking a different perspective, I find that a decline in business cycle risk partly explains the inversion of the yield curve and is robust to the information restrictions I place on the forecaster. I find that the return-forecasting factor can capture what the market expects short-term interest rate will eventually settle. In all, my analysis lends support to the claim that the inversion of the yield curve in 2004 – 2005 is likely related to a decline in future growth prospects if expectations about inflation are well-anchored.

In this paper, I posit an explanation of why risk-premia in bond markets can move, sometimes abruptly. Distrust of the forecasting model can be linked to changes in prices that are hard to anticipate. For bond markets, it seems that the interplay between central bankers and financial market participants reveal interesting insights about their respective roles in bond price dynamics. Because of the inherent signal extraction problem that investors face under the modeled information restrictions, policy that generates unanticipated movements in asset prices may unintentionally change risk perceptions. Transparency over the path of future interest rates can thus improve the transmission of monetary policy in coordinating future economic activity through financial markets.

A model that jointly captures both central bank policy and macroeconomic dynamics in a general equilibrium framework can help sort out the relative importance of beliefs about the future and how central bankers affect changes in risk and return in financial markets. In addition, there is ample evidence that suggests that bond markets are driven by common forces. How will these global forces affect the coordination of policy for leading industrialized nations? These are issues of theoretical and practical relevance, not to mention fruitful areas of

future research.

2.7 Appendix

2.7.1 Data Description

Data on zero-coupon bond yields are taken from Jonathan H. Wright, further details on the construction of the data using Svensson, Spline, or Nelson-Siegel approaches to extracting zero-coupon yields are found on www.econ2.jhu.edu/People/Wright/intyields.

2.7.2 Alternative Forecast Rules in Detail

The problem of finding the optimal parameters can be defined by the following objective:

$$\min_{\{\gamma_t, \phi_t\}} \mathbb{E}_t [v(\mathbb{E}^n [rx_{t+1}] - \boldsymbol{\beta}'_t \cdot \mathbf{z}_t)] \quad (2.6)$$

where $v(rx, \mathbf{z}; \boldsymbol{\beta})$ is the quadratic loss function $(rx_{t+1} - \boldsymbol{\beta}' \cdot \mathbf{z})^2$. The solution to the posited problem can be described by the following set of equations

$$\boldsymbol{\beta}_{t+1} = \boldsymbol{\beta}_t + \phi_t \Gamma_t^{-1} \cdot z_t (\mathbb{E}^n [rx_{t+1}] - \boldsymbol{\beta}'_t \cdot \mathbf{z}_t) \quad (2.7)$$

$$\Gamma_t = (1 - \phi_t) \Gamma_{t-1} + \phi_t (\mathbf{z}'_{t-1} \cdot \mathbf{z}_{t-1}) \quad (2.8)$$

is then equivalent to $\boldsymbol{\beta}_t$ being described by recursive least squares to re-estimate their parameters and Γ_t is the covariance matrix that controls the direction of comovement between key variables.

The parameter ϕ_t characterizes how recent data is weighted in forecasts. In that regard, it can be viewed as a Kalman gain parameter. It is a free parameter in that it is exogenously chosen. The choice will affect how variables move

together, or the speed of the drift movement in the relationship between key variables. Notice that the full-sample forecast is a special case of the model where the parameters of the model are constant, $(\gamma_t, \phi_t, \Gamma_t) = (\gamma, 0, \Gamma)$. In the special case, there is no need to update since all historical data is available. Alternatively, a recursively updated forecast is akin to using recursive least-squares. As for the parameter controlling how past data is discounted, I choose the parameter history discounting factor $\phi_t = (1/t)$ so that only data available at time period t is used and all data points are equally weighted, thus new data are given decreasing weights relative to the already realized data. I also let (γ_t, Γ_t) vary over time. This characterization is isomorphic to exponentially discounted observations where the set of regressors is discounted by the following

$$\tilde{z}_t = \sum_{j=0}^t \rho^{t-j} z_j \quad (2.9)$$

so choosing $\rho = 1$ is equivalent to choosing $\phi_t = t^{-1}$. How heavily past data is discounted coincides with $\rho < 1$ where $\phi'_t > t^{-1} = \phi_t$. The mapping between ρ and ϕ_t is one-to-one but non-linear. Intuitively, it is a weight that is placed upon the arrival of news. If agents are reasonably certain that there is no structural breaks, then new observations are given less weight over time.

The analysis in this paper will investigate forecasts for $\rho = 1, \rho = 0.9995$ and $\rho = 0.9994$ for comparative purposes. I will also incorporate an extreme case is when agents only use recent data and perform rolling regressions forecasts. This accounts for forecasts that discount past observations quite considerably.

For this paper, I focus on the recursively updated forecasts that weights all new data equally that corresponds to $\rho = 1$.

2.7.3 Results for International Bond Markets

Table 2.3: International Forecast Performance

Rule		full-sample	$\rho = 1$	$\rho = 0.9995$	$\rho = 0.9994$	rolling
U.S. 12.1971–5.2009	<i>RMSE</i>	640	643	621	619	481
	<i>RMAE</i>	499	468	469	470	353
	*	0	63.9	61.4	60.4	34.9
Germany 1.1973–5.2009		520	530	484	480	375
		407	372	370	370	283
		0	68.5	73.2	72.9	33.2
Japan 1.1985–5.2009		368	363	324	319	258
		264	258	242	239	193
		0	60.7	50.0	46.7	22.6
England 1.1979–5.2009		545	570	529	523	395
		416	394	388	387	292
		0	64.2	67.1	66.9	33.5
England 1.1986–5.2009		468	478	420	414	349
		337	317	299	298	268
		0	71.0	74.0	73.0	40.4

* percentage of co-variation of forecast errors with realized risk-premia

Root mean squared errors (RMSE) and root mean absolute errors (RMAE) using different forecasting rules. The first two rows of the table displays root mean squared errors and mean absolute errors in basis points (100 basis points = 1 percentage point) for a set of forecasting rules. The third row is the amount of co-variation the forecast differences, between the full-sample return-forecasting factor and the recursively updated forecasts, have with realized risk-premia, which were calculated by running an GMM estimates of the realized risk-premia on the forecast differences between the associated forecast differences. The standard error estimates include 12 lags to account for overlapping monthly data and are Newey-West corrected.

Table 2.4: International Forecast Performance, Continued

Rule		full-sample	$\rho = 1$	$\rho = 0.9995$	$\rho = 0.9994$	rolling
U.S. 12.1971–5.2009	<i>RMSE</i>	640	643	621	619	481
	<i>RMAE</i>	499	468	469	470	353
	*	0	63.9	61.4	60.4	34.9
Australia 2.1987–5.2009		592	621	599	597	421
		433	448	443	442	320
		0	74.9	72.7	72.1	30.2
Switzerland 1.1988–5.2009		353	379	330	325	257
		277	257	262	263	207
		0	27.4	34.7	33.7	50.1
New Zealand 12.1992–5.2009		453	443	418	415	368
		338	301	304	304	281
		0	68.7	77.8	78.6	44.0
Sweden 12.1992–5.2009		405	576	541	536	438
		331	414	403	401	277
		0	61.3	57.2	56.2	25.1
Norway 1.1998–5.2009		390	404	406	408	338
		309	320	334	337	270
		0	65.2	61.2	60.3	40.0

Table 2.5: Differences in Forecasts and Future Interest Rates - International

		intercept	slope	R^2 (%)
U.S.	level	6.23	0.49	
	t -stat	(6.45)	(1.72)	13
12.1971–5.2009	short-rate	6.03	0.52	14.5
	t -stat	(6.24)	(1.84)	
Germany		4.81	0.58	
		(7.74)	(3.88)	27.5
1.1973–5.2009		4.56	0.61	29.8
		(7.57)	(4.08)	
Japan		2.80	-0.09	
		(3.63)	(-0.42)	0.8
1.1985–5.2009		2.71	-0.12	1.4
		(3.5)	(-0.52)	
England		6.73	0.52	
		(4.99)	(1.37)	19.9
1.1979–5.2009		6.66	0.53	20.2
		(4.99)	1.38	
Canada		6.37	-0.021	
		(5.52)	(-0.07)	0
1.1986–5.2009		6.29	-0.06	0
		(5.23)	(-0.151)	

Using the differences in forecasts of bond risk-premia to forecast interest rates. The estimates describe the regression of interest rates, both the level and the short-rate, on the difference between the full-sample and recursively made forecasts. The estimates are linear GMM estimates with 12 lags to account for overlapping monthly data. The standard errors are Newey-West corrected. The first column displays the intercept term for the respective regressions with t-statistics in parenthesis below. The second

column displays the slope coefficients with associated t-statistics below in parenthesis.
The last column displays the R-squares of the regression.

Table 2.6: Differences in Forecasts and Future Interest Rates - International, continued

	level	6.63	0.46	
Australia	<i>t</i> -stat	(8.86)	(1.31)	13.5
2.1987–5.2009	short-rate	6.50	0.47	14.1
	<i>t</i> -stat	(9.09)	(1.40)	
		3.92	−0.28	
Switzerland		(5.05)	(−1.13)	6.3
1.1988–5.2009		3.86	(−0.29)	6.2
		(4.57)	(−1.09)	
		6.04	0.31	
New Zealand		(22.82)	(3.21)	39
12.1992–5.2009		6.00	0.33	42
		(23.60)	(3.49)	
		4.60	0.22	
Sweden		(9.02)	(2.62)	14
12.1992–5.2009		4.46	0.19	12
		(8.55)	(2.21)	
		4.73	0.51	
Norway		(29.34)	(−38.75)	65.9
1.1998–5.2009		4.61	0.58	68.5
		(22.58)	(53.95)	

Using the difference in forecasts of bond risk-premia to forecast interest rates
continued... The estimates describe the regression of interest rates, both the level and

the short-rate, on the difference between the full-sample and recursively made forecasts. The estimates are linear GMM estimates with 12 lags to account for overlapping monthly data. The standard errors are Newey-West corrected. The first column displays the intercept term for the respective regressions with t-statistics in parenthesis below. The second column displays the slope coefficients with associated t-statistics below in parenthesis. The last column displays the R-squares of the regression.

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CHAPTER 3

On Forecasting Risk-Premia in Currency Markets

3.1 Introduction

Foreign exchange market price movements are puzzling. Short-term interest rate differentials forecast currency risk-premia, moving in the opposite direction that it is expected to move according to basic no arbitrage relations. Exchange rate fluctuations are large, transitory and notoriously hard to forecast. In light of puzzling empirical findings over the years, this paper aims to integrate our understanding of currency market prices by analyzing key features of the data in relation to bond market prices.

For this paper, I build on Mang (2012) to quantitatively measure the size of the predictable component in currency markets for a portfolio of industrialized currencies. I specifically use predictor variables constructed from bond market prices. I start by examining the predictive content of interest rate differentials, the difference between a foreign country's yield on a zero-coupon government bond and the U.S., the dollar carry trade. I then incorporate the return-forecasting factor spread to determine if it can improve forecasts relative to the carry trade. As Cochrane and Piazzesi (2005, 2008) and Mang (2012) demonstrate, the return-forecasting factor spread picks up a sizable portion of predictable variation in bond risk-premia. Variables that generate movement in risk-premia can be linked to movement in risk-prices for either bond or currency markets and hopefully

both, making the return-forecasting factor spreads a candidate predictor variable. I find that they forecast currency risk-premia up to twelve months ahead. I also use recursively updated return-forecasting factors and find that, puzzlingly, it does not seem to forecast currency risk-premia. Some variables forecast currency risk-premia but not exchange rate changes, while some forecast both. The different size in the predictable component captured with each variable is interesting and may help us understand certain anomalies in the data. Further, the analysis highlighted in this paper documents findings relevant for simultaneously understanding predictable variation in bond and currency risk-premia, namely that the amount of predictability depends on information used to make forecasts.

By highlighting forecasts that are made recursively relative to those made with the full-sample of available data, I focus on the information sets of forecasters. Because recursive forecast rules emphasize the signal extraction problem and the subsequent revisions to expectations, real-time projections may differ from those made with the full-sample of data. As in bond markets, one source of predictable variation in currency risk-premia is the forecast revisions using factors from bond markets to news about bond risk-premia, whose reactions to incoming data relative to forecasts made with the full-sample of data can generate counter-cyclical fluctuations in risk-premia. I provide suggestive evidence that differences in currency market forecast rules derived from bond market risk factors can explain the tendency for high interest rate currencies to appreciate.

The reason why information matters for tracking predictable currency risk-premia is the following: Cross-sectional spreads of the full-sample return-forecasting factor treat transitory shocks in currency markets appropriately. That is, exchange rate changes and currency risk-premia using the full-sample return-forecasting factor are forecasted to be large but transitory. On the other hand, expectations made with the recursively updated return-forecasting factor spreads tend to be less volatile and more persistent since recursively updated return-forecasting fac-

tors tend to adjust slowly to shocks to bond risk-premia. As a result, the implied forecasts of currency risk-premia adjust slowly to changes in currency risk-premia and exchange rate changes. The difference between the forecasts made with the full-sample of data and those made recursively with available data removes the persistence in adaptive forecasts. The remaining component is transitory and large. As a result, the difference in the two mentioned forecasts, the FX forecast difference, forecast exchange rate changes. Not only that, adaptive forecasts also move closely with interest rate differentials and do not capture a predictable component in currency risk-premia. In real-time, spreads in the recursively updated return-forecasting factor move with interest rate differentials with a lag, leaving only a small predictable component that is hard to detect. Therefore, currency risk-premia is predictable, but it would be difficult to extract the right signal using information in the return-forecasting factor spread to make risk-adjusted profits in real-time because the currency prices contain a large but transitory component that by nature is hard to forecast.

3.2 Relevant Literature

I study the properties of currency risk-premia and exchange rate changes for a basket of currencies in a similar vein to Lustig et. al (2010) who find forecastable currency risk-premia with the cross-sectional forward discount rate or short-term interest rate differentials. I determine if variation in the difference in the level of interest rates, above and beyond information in the short-term interest rate differential is informative about currency risk-premia. To do so, I include a novel predictor variable proven to capture a predictable component in government bond markets and only study a set of industrialized countries where detailed information on zero-coupon yields are available. Ang and Chen (2010) also use additional information in the term structure of interest rates to forecast currency risk-premia.

They examine the returns on a portfolio of currencies using cross-sectional factors that price bonds to see if it can outperform carry trade portfolios made using short-term interest rate differentials as the predictor variable. My work is similar to theirs given that I also try to forecast predictable currency risk-premia for a portfolio of countries, but I specifically investigate time-series predictability for a smaller set of industrialized countries.

This paper also investigates how adaptively made forecasts affect the extent to which predictable currency risk-premia are exploitable in real-time. I take the return-forecasting factor of Cochrane and Piazzesi (2005, 2008) that captures a predictable component in bond markets to see if it can forecast currency risk-premia. I also examine how real-time data availability constraints and how revisions to risk-premia forecasts in bond markets affect the size of predictability in currency markets. This approach highlights a departure from forecasts made with the full-sample of data and the investigation of subjective beliefs is also the focus of Froot and Frankel (1989) who use survey data to decompose the forward premium anomaly into a bias in exchange rate forecasts and into a risk-premium component. I perform a similar exercise that highlights differences in expectations formation using least squares estimation procedures that approximate them. This allows me to sidestep the difficulties of finding survey data on exchange rate forecasts. My findings are comparable to Bansal and Dahlquist (2000), who cannot attribute time-varying risk-premia as compensation for systematic risk in their analysis of industrialized economies. In contrast to their results, I find that the forward premium anomaly is consistent with time-varying risk well-proxied by interest rate differentials, but the existence of a second factor complicates things for real-time forecasters. The failure of uncovered interest parity then crucially depends on the information assumptions made in forecasting using information in the term structure of interest rates. High interest rate currencies do tend to appreciate for a cross-section of industrialized economies. However, forecasts of

currency risk-premia using recursive forecasting factors in bond markets imply that high interest currencies tend to have high interest rate levels that coincide with large but transitory exchange rate fluctuations. That is, real-time forecasters have a hard time forecasting using proxies for risk factors in such a random environment under adaptive expectations formation. Bacchetta, Mertons, and van Wincoop (2009) use survey data and find that expectational errors coincide with predictable returns in bond, currency and equity markets. To simultaneously capture predictability in bond and currency markets, I take factors that capture predictable variation in bond markets to assess its predictive ability in currency markets. The inherent difficulty in extracting variation in the level and the spread of interest rates can mean agents must react in ways that forecasts made with the full-sample of data would not. Comparisons between adaptive learning forecasts and full-sample information forecasts can shed light on what type of shocks these agents react to. In using recursive least squares to approximate adaptive expectations, I show that jointly accounting for predictable returns in bond and currency markets is possible if a portion of statistically measured predictability is driven by forecast differences. That is, a sizable fraction of predictable variation in bond and currency risk-premia can be attributed to differences in forecasts using fully rational estimates and adaptively made ones.

Research that highlights a departure from full rationality is Gourinchas and Tornell (2004). In their formulation, investors optimally learn about the persistent factor to guard against large losses due to the possibility that their model is misspecified. As a result, adaptive agents are slow to react to changes in interest rates. The departure from full-rationality can explain anomalous features of foreign exchange market data if the size of the amount of learning done by the agent is sufficiently large. For a set of industrialized economies' currencies, the forward premium anomaly may not require large revisions to beliefs. Exchange rate changes and currency risk-premia forecasts made with the full-sample suggest

large and transitory movements. However, recursively made forecasts are more persistent and slow-moving and as Gourinchas and Tornell (2004) the delayed adjustment tends to move with interest rate differentials.

The use of bond market factors to understand both bond and currency markets is reminiscent of Backus, Foresi, and Telmer (2001). In line with more recent findings in bond and currency markets, I make adjustments to the linear factors that allow for time-varying parameters to account for adaptively made forecasts of bond risk-premia. I do so by employing tractable, ordinary least squares regressions to approximate forecasts under different information restrictions. Finally, I integrate the insights from risk-based explanations with those that investigate departures from full-rationality to understand puzzling features of currency market data. My central contribution in this paper is to provide evidence that accounting for the forward premium anomaly requires accounting for a large and transitory component in exchange rates that tend to average to zero in a longer horizon. One of many explanations for this component is that it reflects forecast differences that depend on the specified information set of investors.

3.3 Definition and Data

Let $y_t^{(k/12)}$ be k -month domestic interest rates and $y_{i,t}^{(k/12)}$ be the k -month foreign interest rates, both of which are denominated in nominal units. Interest rates, as is any variable of interest this paper, are expressed in annualized percentages. Let $s_{j,t}$ be the log spot exchange rate between country j and the domestic country and is denominated in units of foreign currency per unit of domestic currency. For the rest of the paper, I will let the U.S. be the domestic currency. The excess return on buying a foreign currency in country j in a forward market and selling it in the spot market k -months from t will be written as:

$$rx_{j,t+k} = f_{j,t} - s_{j,t+k} \tag{3.1}$$

The expression for the k -month ahead currency excess return can be written in another way

$$rx_{j,t+k} = f_{j,t}^{(k/12)} - s_{j,t} - s_{j,t+k} + s_{j,t} \quad (3.2)$$

$$= y_{j,t}^{(k/12)} - y_t^{(k/12)} - \Delta s_{j,t+k} \quad (3.3)$$

where $f_t^{(k)}$ is a k -period forward contract and $\Delta s_{j,t+k}$ is the exchange rate change between date t and date $t+k$ and under the condition that covered interest parity holds in the data, $f_{j,t}^{(k/12)} - s_{j,t} \approx y_{j,t}^{(k/12)} - y_t^{(k/12)}$. Expected currency excess returns k -months from now can be described by taking conditional expectations $\mathbb{E}_t(\cdot)$:

$$\mathbb{E}_t(rx_{j,t+k}) = y_{j,t}^{(k/12)} - y_t^{(k/12)} - \mathbb{E}_t(\Delta s_{j,t+k}) \quad (3.4)$$

Notice that if unconditional interest parity held, then high interest currencies will depreciate relative to low interest currencies so that $\mathbb{E}_t(rx_{j,t+1}) = 0$. However, among industrialized economies, the opposite tends to hold; high interest rate currencies tend to appreciate in value relative to its low interest rate counterparts, so the regression $\Delta s_{j,t+k} = \alpha + \beta(k) \cdot \left(y_{j,t}^{(k/12)} - y_t^{(k/12)} \right) + \varepsilon_{t+k}$ yields negative estimates of $\beta(k)$. One possibility for the deviation from UIP is that high interest rate currencies are riskier, hence investors are compensated for exposure to the relevant risk factor. Notice that the only stochastic term on the right hand side is the expected dollar appreciation term $\mathbb{E}_t(\Delta s_{j,t+k})$. In my analysis, I will investigate $k = 1, 2, 3, 6$, and 12-month ahead currency risk-premia and expected dollar appreciation rates for a basket of currencies to examine if certain anomalies in foreign exchange markets still remain if I control for short-term distortions or idiosyncratic currency movements. To facilitate analysis, I will introduce the notation for the cross-sectional average to be written as $\mathbb{E}^j(z_t) := J^{-1} \sum_{j=1}^J z_{j,t}$ that will be convenient for writing an expression for an average basket of currencies. For example, to describe the currency risk-premia for a basket of currencies, the expected currency risk-premia k -months from now is expressed as $\mathbb{E}_t[\mathbb{E}^j(rx_{t+k})]$.

However, because this notation is a bit cumbersome, the expression $\mathbb{E}_t [\mathbb{E}^j (z_{t+k})]$ will be written as $\mathbb{E}_t^j (z_{t+k})$.

3.3.1 Data Description

I use a combination of bond and currency market data in my empirical analysis. Forward exchange rates monthly frequency from 1:1988-5:2009 at the 1, 2, 3, 6-month and 12-month horizon. I also use spot exchange rate data covering the same time-span. From bond markets, I take information from the term-structure of interest rates for zero-coupon data from 1:1988-5:2009, ranging from one year to ten year time-to-maturity split by 12-months. The data are monthly frequency. The industrialized countries in which both bond and currency market data overlap are the United States, Canada, Japan, England, Sweden, New Zealand, Norway, and Australia. Though the cross-section of countries is admittedly small, this is primarily due to the limited availability of data on zero-coupon bond yields that are required to construct return-forecasting factors. In any case, I am particularly interested in leading industrialized economies because their tendency to violate uncovered interest parity is pronounced relative to countries with higher inflation rates, as Bansal and Dahlquist (2000) find. In that regard, understanding the experience of industrialized economies will isolate the countries where anomalies in foreign exchange markets are most apparent.

3.3.2 Constructing the Predictor Variables

3.3.2.1 Interest Rate Differentials vs. Return-Forecasting Factor Differentials

In this section, I outline the details of the construction of predictor variables that I use in forecasting in this paper. To summarize this section, I construct predictor variables from bond market prices and use them to forecast in foreign

exchange markets. I start with a commonly used variable, the short-term interest rate differential, and then proceed to use information in longer maturity bonds to construct additional predictor variables. In total, I will use four: interest rate differentials, the full-sample return-forecasting factor spread (x-factor spread), the spread in the recursively updated return-forecasting factors (rx-factor spread) and the difference between the full-sample return-forecasting factor spread and its recursive counterpart. I denote the last forecaster variable the hidden FX market factor.

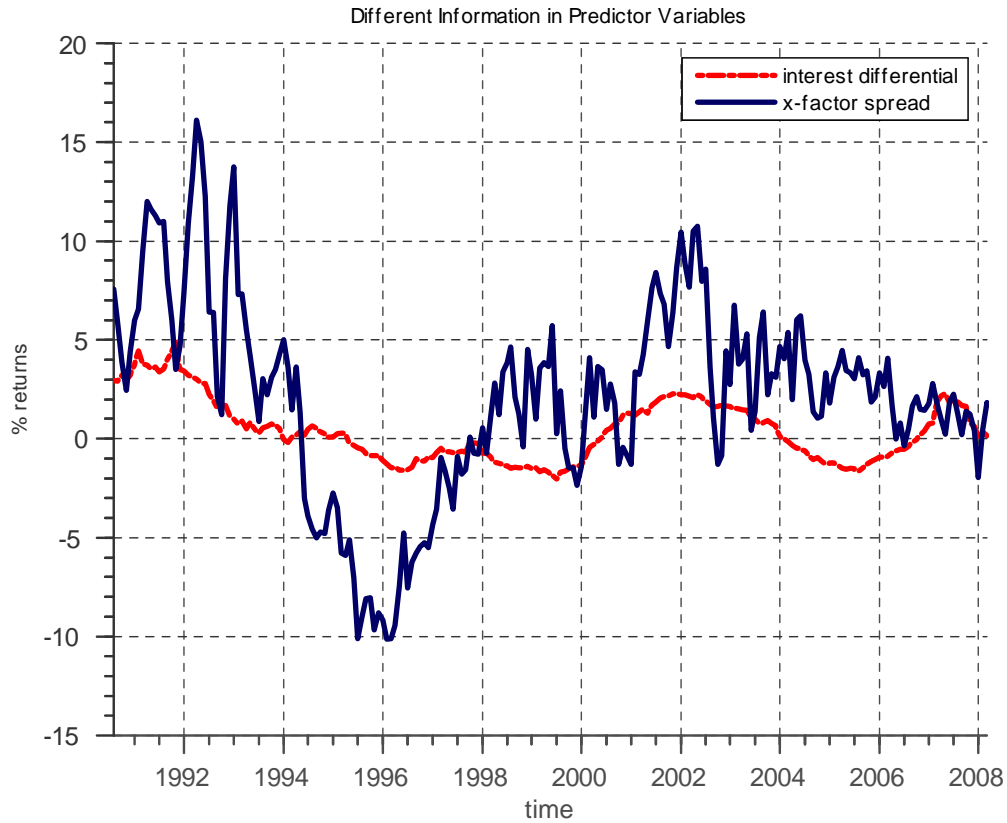
Comparing the quantitative size of the predictable component in foreign exchange markets begin with the benchmark case of the interest rate differential. In line with Lustig et. al (2010), I use the cross-sectional average forward discount or interest rate differential that have proven to capture time-series predictability in currency risk-premia. I write it as the following expression:

$$\mathbb{E}^j \left[f_t^{(k/12)} - s_t \right] := J^{-1} \sum_{j=1}^J \left(f_{j,t}^{(k/12)} - s_{j,t} \right) \quad (3.5)$$

Next, I use a factor that captures a predictable component in bond risk-premia. Let us begin by defining each country specific return-forecasting factor as $x_{j,t}$. I construct the return-forecasting factor taking as given the full sample of data by fitting the estimated coefficients of the average bond risk-premia for a set of maturities on a set of forward rates. The variable derives from the following expression: $x_{j,t} := \mathbb{E}_t^n \left[\widehat{rx}_{j,t+1}^{(n)} \right] = \hat{\alpha}_j + \hat{\gamma}'_j \cdot f_{j,t}$. I then use the collection of country-specific return-forecasting factors $\{x_{j,t}\}_{j=1}^J$ to construct a cross-sectional average of the difference in the return-forecasting factor $\mathbb{E}^j [\Delta(x_t)] := J^{-1} \sum_{j=1}^J (x_t - x_{j,t})$. Notice the switch in the sign. This is because compensation for risk-exposure is positive during recessions when risk is high and negative when risk is low, consistent with empirical findings of countercyclical risk-premia. The two time-series of predictor variables are positively correlated, although the return-forecasting factor tends to have greater variance. The additional variation in the return-forecasting factor

differentials above and beyond the cross-sectional average forward discount rates suggests that the return-forecasting factor has the potential to capture additional, predictable variation in risk prices in currency markets. I graphically display the comparison of the constructed predictor variables below:

Figure 3.1: Comparing Predictor Variables



Cross-sectional average interest rate and return-forecasting factor differentials.

The solid (line) is the cross-sectional average, full-sample return-forecasting factor spread whose construction is described in the previous section. The (-.-) line is the cross-sectional average interest rate differential. The interest rate differential is noticeably smoother than the x-factor spread. The data span 1:1989-5:2009, less the first 30 months of observations used to construct the return-forecasting factor spreads. The variables are expressed in annualized percentage returns

Embedded in the full-sample return-forecasting factor is some flexibility in which information about the future affects forecasts in the past. Recall that the return-forecasting factor involves using a full-sample of data to pin down parameter estimates used to construct the x-factor for the individual bond markets for

each country. An alternative variable that is consistent with real-time availability is the spreads between the recursively updated return-forecasting factors. I construct them using the technique from Mang (2012) where the risk-premia on a portfolio of long bonds are regressed on a subset of forward rates. In essence, the procedure for doing is nearly identical to the construction of the full-sample return-forecasting factor with the exception that the estimates are revised recursively, thereby generating time-variation in the parameter estimates. The recursive return-forecasting factor is then $\tilde{x}_{j,t} := \mathbb{E}_t^n \left[\widehat{rx}_{j,t+1}^{(n)} \right] = \hat{\alpha}_{j,t} + \hat{\gamma}'_{j,t} \cdot f_{j,t}$ where $(\alpha_{j,t}, \gamma_j)$ are now indexed by time.

I can also examine if the difference in the mentioned forecasting factor spreads have predictive power. I do so by comparing the spreads in the x -factor, denoted as $\Delta(x)$ -factor with its recursive return-forecasting factor spread, $\Delta(\tilde{x}_t) := (\tilde{x}_t - \tilde{x}_{j,t})$. I will also take the cross-sectional average of $\Delta(x)$ defined to be $\mathbb{E}^j [\Delta(x_t)] := \mathbb{E}^j (x_t - x_{j,t}) = J^{-1} \sum_{j=1}^J (x_t - x_{j,t})$ and similarly for $\Delta(\tilde{x}) : \mathbb{E}^j [\Delta(\tilde{x}_t)] := \mathbb{E}^j (\tilde{x}_t - \tilde{x}_{j,t}) = J^{-1} \sum_{j=1}^J (\tilde{x}_t - \tilde{x}_{j,t})$. One point of emphasis is that this term is unobservable in real-time because it requires comparison with an object that takes as given the full-sample of data. Let us define this component to be labeled $\tilde{\Delta}(x_t)$ given by the following expression:

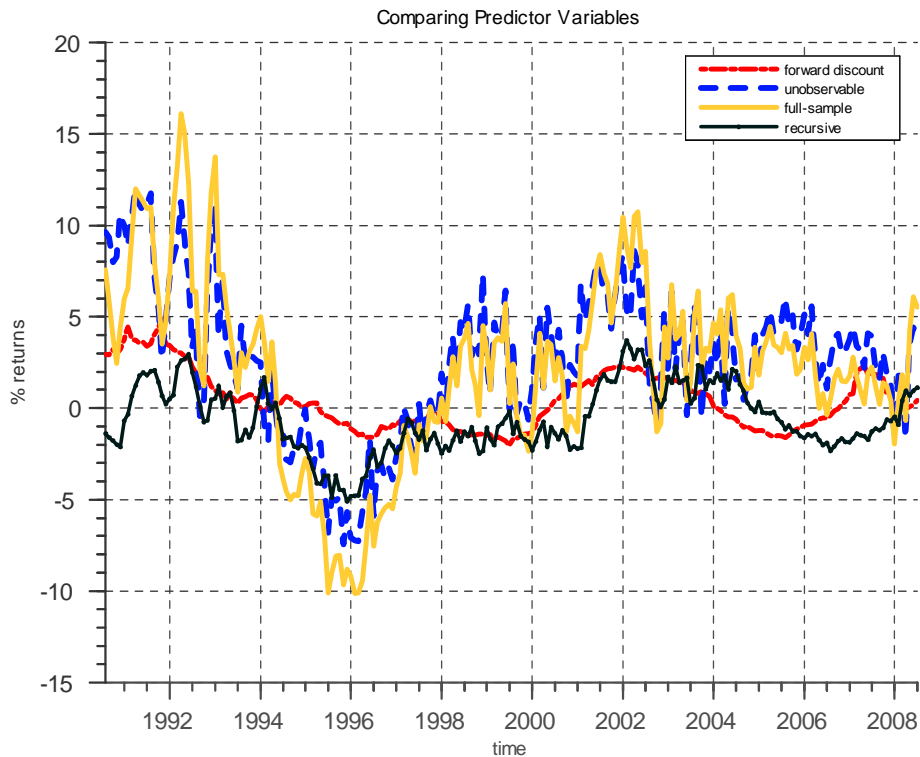
$$\mathbb{E}^j \left[\tilde{\Delta}(x_t) \right] := \mathbb{E}^j [(x_t - \tilde{x}_t - (x_{j,t} - \tilde{x}_{j,t}))] \quad (3.6)$$

Henceforth, this variable shall be named the hidden FX market factor. Notice that the hidden FX market factor is simply the difference between the full-sample x -factor spread $\mathbb{E}^j [\Delta(x_t)]$ and the recursively updated rx -factor spread $\mathbb{E}^j [\Delta(\tilde{x}_t)]$ by the additivity property of expectations. Evidence that this relation holds is given in the appendix.

The four predictor variables tend to move together although the full-sample return-forecasting factor spread although the hidden FX market factor tend to

moves closest with it. The two exhibit large variance relative to the recursive return-forecasting factor spread and the interest rate differential. Despite this, the additional variation in the return-forecasting factor differentials above and beyond the cross-sectional average forward discount rates suggests that the return-forecasting factor has the potential to capture additional, predictable variation in risk prices in currency markets. I graphically display the comparison of the constructed predictor variables below:

Figure 3.2: Comparing Predictor Variables, All



Comparing predictor variables. The thick solid (yellow) line is the cross-sectional average full-sample return-forecasting factor spread, the thick (- - -) line represents the difference in the cross-sectional average x-factor spread and the rx-factor spread. The thin solid (dark) line represents the rx-factor spread. The (-.-.-) line represents the forward discount rate. The data are monthly frequency and all variables are expressed in annualized percentages.

To summarize, I will investigate predictable returns in currency risk-premia for a cross-section of industrialized economies using the combination of the following (cross-sectional) predictor variables: the interest rate differential (or the forward discount rate), the full-sample return-forecasting factor spread, the recursive return-forecasting factor spread, and the difference between the full-sample and recursive return-forecasting factor. Respectively, these predictor variables are

the interest differential, the return-forecasting factor spread that takes the full-sample of data as given, the return-forecasting factor spread that is consistent with real-time data availability, and the difference in the return-forecasting factor spreads, full-sample less recursive, or simply the hidden FX market factor. In all, these four time-series will be used to examine the predictable component in currency market data. The four series are depicted in figure 3 displayed below for the purpose of comparison. Notice that the currency market forecast difference and the return-forecasting x-factor spread move closely together and share a large variance. Forward discount rates are the smoothest series of the four.

3.3.3 Motivating Empirical Regressions

Linking bond risk prices to empirical regressions that inform us about the predictable component in foreign exchange markets require some motivation. The idea is to posit a stochastic discount factor for a set of treasury securities that should also price foreign exchange asset prices with no-arbitrage arguments. Getting the right bond risk factors to price currency risk requires the use of factors in bond markets that capture a predictable component in risk-premia. Currency risk prices then should simply be the difference of the bond factors across countries. We can construct currency risk prices using bond risk prices, particularly variables constructed in the previous section. We start with asset pricing conditions that are satisfied in both countries, given by the following:

$$\mathbb{E}_t [M_{t+1}R_{t+1}] = \mathbb{E}_t [M_{j,t+1}R_{j,t+1}] = 1 \quad (3.7)$$

Suppose that there exists a stochastic process, in units of foreign currency per domestic currency, that transforms returns in a domestic country to returns in a foreign country: $\frac{S_{t+1}}{S_t}R_{t+1} = R_{j,t+1}$. Plugging this expression into the asset pricing condition delivers

$$\mathbb{E}_t [M_{t+1}R_{t+1}] = \mathbb{E}_t \left[M_{j,t+1} \frac{S_{j,t+1}}{S_{j,t}} R_{t+1} \right] \quad (3.8)$$

If there are a complete set of contingent claims, we can take the expression out of the conditional mean operator. The remaining expression is

$$M_{t+1} = M_{j,t+1} \frac{S_{j,t+1}}{S_{j,t}} \quad (3.9)$$

Taking logs and rearranging, we get the following expression:

$$m_{t+1} - m_{j,t+1} = s_{j,t+1} - s_{j,t} = \Delta s_{j,t+1}$$

Suppose that a (log) stochastic discount factor is given by the following expression:

$$-m_{j,t+1} = y_t^{(k/12)} - \frac{1}{2} \lambda_t^2 (\sigma_\varepsilon^2 + \sigma_\eta^2) + \lambda_t \cdot (\varepsilon_{t+1} + \eta_{t+1}) \quad (3.10)$$

For a foreign country j , the (log) stochastic discount factor is given by a similar expression:

$$-m_{j,t+1} = y_t^{(k/12)} - \frac{1}{2} \lambda_{j,t}^2 (\sigma_{j,\varepsilon}^2 + \sigma_\eta^2) + \lambda_{j,t} \cdot (\varepsilon_{j,t+1} + \eta_{t+1}) \quad (3.11)$$

where ε_{t+1} is the domestic, country-specific shock, $\varepsilon_{j,t+1}$ is a country-specific shock and η_{t+1} is a global shock, all assumed to be *i.i.d.* and normally distributed with variance σ_ε^2 , $\sigma_{j,\varepsilon}^2$, and σ_η^2 , respectively. The stochastic discount factor should price assets of any class, especially bond and currency prices. In a linear affine model, bond market price of risk varies with factors that forecast bond risk-premia, namely the return-forecasting factors of Cochrane and Piazzesi (2005, 2008). Let z_t be a relevant predictor variable in bond markets. Bond risk prices can be represented as a linear function of these variables:

$$\lambda_t^z = \lambda_0^z + \lambda_1^z \cdot z_t \quad (3.12)$$

$$\lambda_{j,t}^z = \lambda_{j,0}^z + \lambda_{j,1}^z \cdot z_{j,t} \quad (3.13)$$

The model also implies a link between bond risk prices and the conditional variance of the pricing kernel of the domestic and foreign country, respectively given by

the following expressions:

$$\sigma_t^2(m_{t+1}) = \lambda_t^2 \cdot (\sigma_\varepsilon^2 + \sigma_\eta^2) \quad (3.14)$$

$$\sigma_t^2(m_{j,t+1}) = \lambda_{j,t}^2 \cdot (\sigma_{j,\varepsilon}^2 + \sigma_\eta^2) \quad (3.15)$$

The difference between any factor that prices risk in bond markets should price risk in currency markets. The expression for the excess return in currency markets is then $\mathbb{E}_t[rx_{j,t+1}] = \frac{1}{2} [\lambda_{j,t}^2 (\sigma_{j,\varepsilon}^2 + \sigma_\eta^2) - \lambda_t^2 (\sigma_\varepsilon^2 + \sigma_\eta^2)]$. Keep in mind that currency risk-premia and exchange rate forecasts made with other predictor variables can be constructed in a similar fashion but specify that risk prices will depend on the relevant predictor variable. Details will be relegated to the appendix. For expected exchange rate movements, $\mathbb{E}_t \Delta s_{j,t+1}$, we require the difference in the conditional mean of the stochastic discount factor (sdf), for which expressions are given by the following:

$$\begin{aligned} -\mathbb{E}_t[m_{t+1}] &= y_t^{(k/12)} - \frac{1}{2} \lambda_t^2 \cdot (\sigma_\varepsilon^2 + \sigma_\eta^2) \\ -\mathbb{E}_t[m_{j,t+1}] &= y_{j,t}^{(k/12)} - \frac{1}{2} \lambda_{j,t}^2 \cdot (\sigma_{j,\varepsilon}^2 + \sigma_\eta^2) \end{aligned}$$

After the dust settles and the derivation of relevant expressions are complete, we can write the excess return in currency markets and exchange rate changes as the difference in the bond yields and bond risk prices.

$$\mathbb{E}_t[rx_{j,t+1}] = \kappa \cdot z_t - \kappa_j \cdot z_{j,t} \quad (3.16)$$

$$\begin{aligned} \mathbb{E}_t[\Delta s_{t+1}] &= \mathbb{E}_t[m_{t+1}] - \mathbb{E}_t[m_{j,t+1}] \\ &= y_{j,t}^{(k/12)} - y_t^{(k/12)} - \kappa \cdot z_t + \kappa_j \cdot z_{j,t} \end{aligned} \quad (3.17)$$

To reduce the amount of noise by idiosyncratic factors in pricing bonds in each country, I take an equal-weighted cross-sectional average to isolate the common component. Doing so for expected dollar appreciation and currency risk-premia

delivers the following expressions:

$$\begin{aligned}
\mathbb{E}^j [\mathbb{E}_t [rx_{j,t+1}]] & : = \mathbb{E}_t^j [rx_{t+1}] \\
& = J^{-1} \sum_{j=1}^J (\kappa \cdot z_t - \kappa_j \cdot z_{j,t}) \\
& = \kappa \cdot z_t - J^{-1} \sum_{j=1}^J \kappa_j \cdot z_{j,t} \\
& \approx \kappa \cdot \mathbb{E}^j [z_t - z_{j,t}] .
\end{aligned} \tag{3.18}$$

For a sufficiently large cross-section, $\kappa = \lim_{J \rightarrow \infty} J^{-1} \sum_{j=1}^J \kappa_j$ that will make (26) an exact relation. Exchange rate changes for a basket of currencies is given by the following expression

$$\mathbb{E}_t^j [\Delta s_{t+1}] = \mathbb{E}_t^j \left[y_{j,t}^{(k/12)} - y_t^{(k/12)} \right] - \kappa \cdot \mathbb{E}^j [z_t - z_{j,t}] . \tag{3.19}$$

Expressions are similar for k -period ahead forecasts, with the exception that interest rate differentials are appropriately adjusted for the horizon of the forecast. For instance, a 3-month ahead forecast of the dollar appreciation rate for a basket of currencies is then $\mathbb{E}_t^j [\Delta s_{t+3}] = \mathbb{E}_t^j \left[y_{j,t}^{(3/12)} - y_t^{(3/12)} \right]$ where $y_{j,t}^{(3/12)}$ are 3-month or 90-day interest rates on zero-coupon default-free Treasury securities. This ensures that covered interest parity holds for all horizons k , i.e.

$$f_{j,t}^{(k/12)} - s_t \approx y_{j,t}^{(k/12)} - y_t^{(k/12)} \tag{3.20}$$

for $k = 1, 2, 3, 6, 12$ months ahead. The resulting coefficients describing the size of the predictable component now will be indicated by a k . For instance, the expected currency risk-premia using the x -factor spread can be written as $\mathbb{E}_t^j [rx_{t+k}] = \kappa(k) \cdot \mathbb{E}^j [x_t - x_{j,t}]$.

For further details on the construction of the cross-sectional average predictor variables I have outlined, please see the details that have been relegated to the appendix.

In the next section, I will test to see if these factors are important for capturing predictable currency risk-premia.

3.4 Time-Series Predictability

3.4.1 Currency Risk-Premia

I now use the predictor variables to empirically measure the size of the predictable component in currency risk-premia. To do so, I regress k -month ahead currency risk-premia on each individual predictor variable, described in the following estimation equation:

$$\mathbb{E}_t^j [rx_{t+k}] = \alpha(k) + \kappa(k) \cdot \mathbb{E}^j [z_t] + \varepsilon_{t+k} \quad (3.21)$$

Estimated coefficients can be interpreted as elasticities: a percentage (100 basis point) increase in the predictor variable is consistent with a $\kappa(k)$ percentage point increase in the forecasted variable over the sample. For 3-month ahead forecasts for nearly all regressions yield abnormally large statistical significance and coefficients that differ from those of any other horizon k . This is a concern, but efforts to correct them using traditional computational checks have not resolved the issue.

Beginning with the interest rate differential, the empirical estimates suggest that a 100 basis point increase in the cross-sectional average forward discount rate precedes movement in the cross-sectional average currency risk-premia 12-months from now by about 97 basis points, capturing about 27% of the variation in currency risk-premia for the given sample. Profiting from these patterns in the data requires that investors borrow treasury securities from abroad for a set of industrialized countries and go long on the U.S. dollar using the proceeds, holding the portfolio for 12-months then unwinding your position. If these patterns in the data are robust, it is needless to say that it would not be difficult to construct portfolios to take advantage of these predictable patterns in the data.

As for the full-sample return-forecasting factor spread which constructs the predictable component in bond risk-premia taking the full-sample of data as given,

as the results in table (1) suggest a smaller predictable component relative to the interest rate differential, particularly for 12-month ahead forecasts. For the the difference in the x -factor spread, there is robust evidence of predictive power for currency risk-premia. The difference between the x -factor spread and the rx -factor spread also forecasts risk-premia in currency markets. This is not surprising given that the difference tends to move with the full-sample return-forecasting factor spread as figure (2) suggests. The estimates suggest some predictability for 2-month ahead forecasts and similar patterns of predictability for 6-month and 12-month ahead forecasts when compared to the unobservable component $\tilde{\Delta}(x_t)$. there is the recursive return-forecasting factor spread $\Delta(\tilde{x}_t)$ provides little evidence of predictive power. There is also a positive and significant intercepts in the regressions for 12-month ahead forecasts, hinting that an omitted level that is not explained by the time-variation in the rx -factor spread is important for capturing variation in currency risk-premia. This omitted term does not engender trust in the forecasting relationship since the intercept term is significant.

Despite evidence of predictable currency risk-premia, the three predictor variables tell different stories about the extent to which currency risk-premia are predictable. Because the x -factor spread moves closely with the difference between the full-sample return-forecasting factor spread and the recursive return-forecasting factor spread, or the hidden FX market factor, they capture a quantitatively similar size of the predictable component in currency risk-premia. This suggests that movement in currency risk-premia depend on the relative size of the persistent and transitory components of the *predictor variable*. Both the full-sample return-forecasting x -factor spread and the hidden FX market factor contain a large and transitory component useful for capturing variation in currency risk-premia. The recursive return-forecasting rx -factor spread has smaller variance and is more persistent and does not capture predictable variation in currency risk-premia primarily because it does not vary enough. This point becomes

more clear when we compare the results to the rx -factor spread, where positive α intercept term suggest that an omitted variable is contributing to the amount of predictability *not* captured by the rx -factor spread. In other words, the recursively revised forecasts largely reacts to persistent changes in currency risk-premia but does not seem to precede movements in it. Further analysis using multivariate regressions will demonstrate some reasons for the failure of the rx -factor spread to forecast in currency risk-premia.

Table 3.1: Predictable Currency Risk-Premia Estimates, Univariate

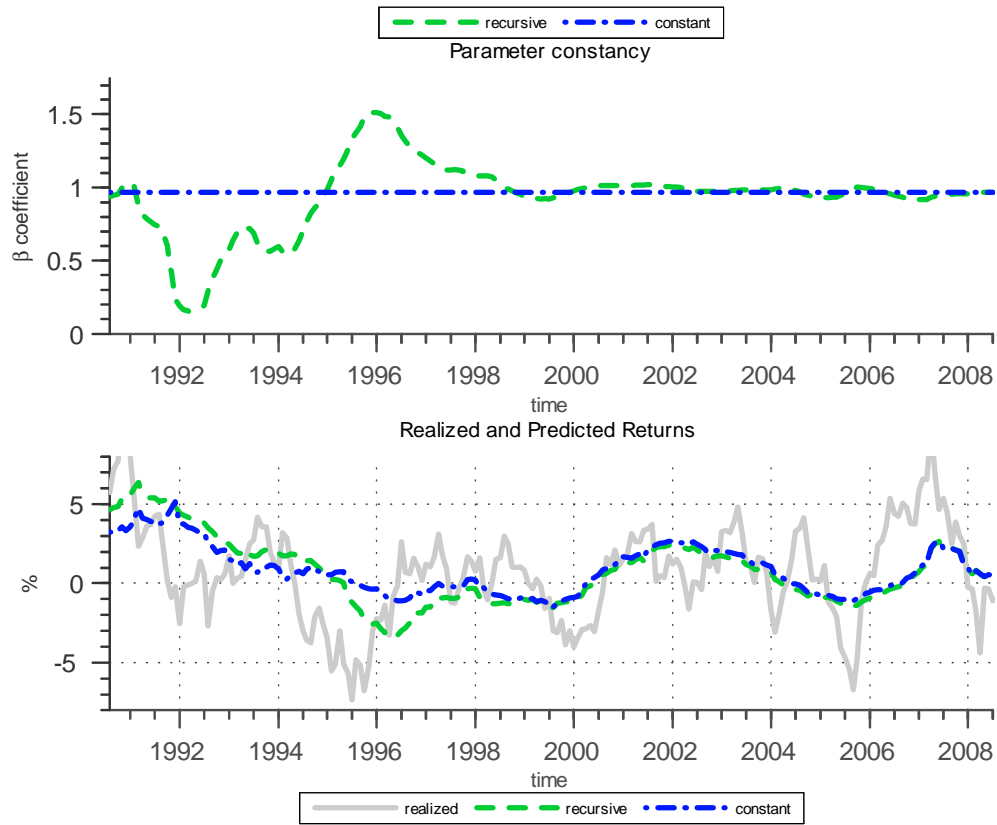
k		1	2	3	6	12	
$\mathbb{E}^j(f_t^{(k/12)} - s_t)$	$\kappa(k)$	0.51	0.97**	1.66	0.94	0.97**	
	t -stat	(0.13)	(3.54)	(5.74)	(1.10)	(3.55)	
	interest different ¹	R^2 (%)	0.00	5.85	47.4	4.12	27.1
$\mathbb{E}^j(\Delta(x_t))$		0.02	0.06**	0.07	0.16	0.19**	
		(0.38)	(3.11)	(0.74)	(1.51)	(2.23)	
	x -factor spread	0.00	6.10	0.00	3.82	12.4	
$\mathbb{E}^j(\tilde{\Delta}(x_t))$		0.07	0.09	0.14	0.27*	0.31**	
		(0.65)	(0.16)	(0.94)	(1.93)	(2.67)	
	FX forecast difference	0.00	6.06	1.22	5.9	17.0	
$\mathbb{E}^j \Delta(\tilde{x}_t)$	$\alpha(k)$	0.0029	0.0023**	0.0064	0.0081	0.0085*	
	t -stat	(0.28)	(1.99)	(0.48)	(0.73)	(1.88*)	
	:	$\kappa(k)$	0.035	0.14**	0.25	0.25	0.32
		t -stat	(0.11)	(2.29)	(0.57)	(0.62)	(1.02)
	rx -factor spread	R^2 (%)	0.00	3.35	0.85	1.0	3.6

Predictability regressions using the cross-sectional average predictor variables. Each row with three quoted statistics represent the following from top to bottom: the regression coefficient, the t-statistic and the R-squares of each regression. Standard errors are corrected for conditional heteroskedasticity with k-lags to account for

overlapping monthly data from $k=1,2,3,6$, and 12 month horizons. The 5 columns represent numbers corresponding to forecasts of k -month ahead risk-premia,. The data span from 6:1991-5:2009 and are at a monthly frequency.

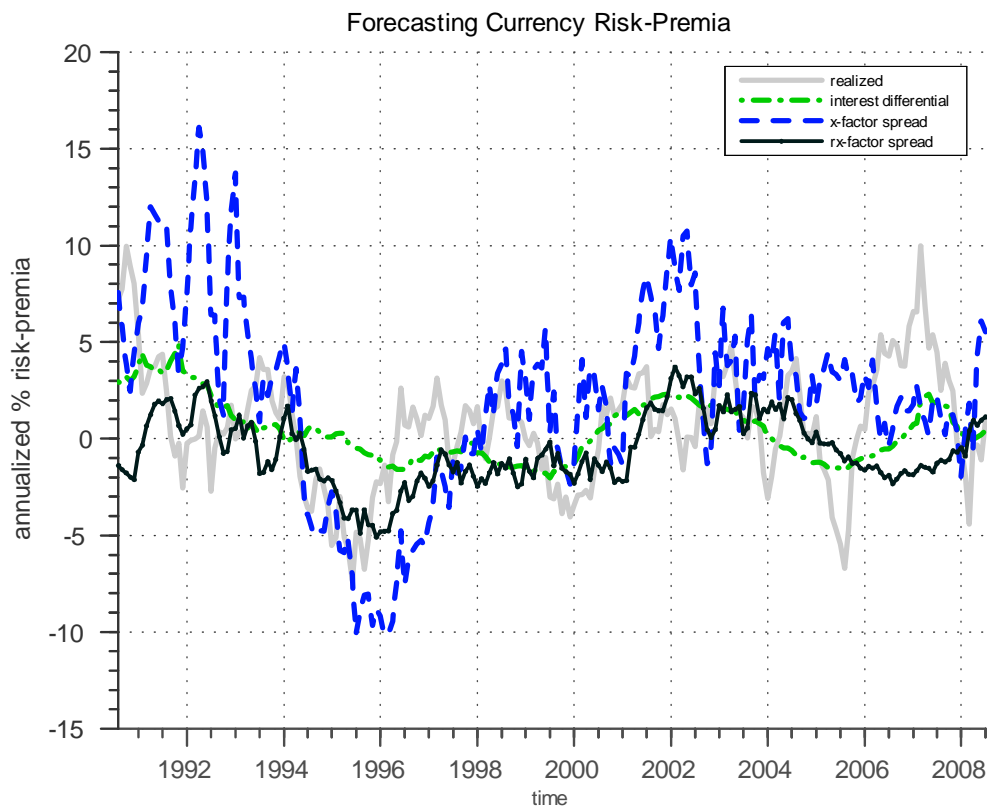
To see if parameter uncertainty is important for measuring the size of the predictable component using interest rate differentials, I recursively regress 6-month ahead currency risk-premia on interest rate differentials, increasing the information set of the forecast each time-period. The results suggest that estimation uncertainty is not important for the results in the predictability regressions given that parameter estimates, while variable, are fairly close to the constant coefficient estimation implied by the use of the full-sample of data. Forecasts of currency risk-premia using variable and constant coefficients tend to agree as well. I plot the regression coefficient over time in the top panel and the forecasts in the bottom panel of the figure below:

Figure 3.3: Parameter Constancy



Accounting for estimation uncertainty. The top panel plots the time-variation in the regression coefficient using recursive least squares. The solid line represents the time-varying coefficients, while the (- - -) line represent the constant coefficient from estimating 6-month ahead currency risk-premia using the forward discount rate. The bottom panel plots the realized risk-premia (solid line) with forecasts of currency risk-premia using the recursive forecasts as well as the forecasts derived from the full-sample of data. The data span from 6:1991-5:2009 and are monthly frequency. Risk-premia are expressed in annualized percentages.

Figure 3.4: Currency Risk-Premia Forecasts



Forecasting Currency Risk-Premia. The solid grey line depicts the realized currency risk-premia for 6-month holding periods. The (.-.-) line represents the interest rate differential, while the (- -) line represent the x-factor spread. Finally, the dark solid line depicts the rx-factor spread. The data span from 6:1991-5:2009 and are monthly frequency. Risk-premia are expressed in annualized percentages.

The forward discount rate forecasts currency risk-premia, capturing the countercyclical patterns that are consistent with business-cycle related risk. I now see if forecastability is strengthened when additional variables are used together. I do so by running the following regressions, always using the forward discount rate $\mathbb{E}^j \left(f_t^{(k/12)} - s_t \right)$ and using an additional predictor variable in conjunction with it, either the spread in the x -factor, the spread in the rx -factor, or the currency

market forecast difference. I obtain the following estimates.

$$\mathbb{E}_t^j (rx_{t+k}) = \alpha(k) + \kappa(k) \cdot \mathbb{E}^j \left(f_t^{(k/12)} - s_t \right) + \kappa_A(k) \cdot \mathbb{E}^j (A_t) + \varepsilon_{t+k} \quad (3.22)$$

Table 3.2: Predictable Currency Risk-Premia Estimates, Multivariate

k		1	2	3	6	12
$\mathbb{E}^j (f_t^{(k/12)} - s_t)$	$\kappa(k)$	-1.07	0.42	1.79	0.31	0.57
	t-stats	(-0.18)	(0.94)	(5.80)	(0.10)	(1.56)
interest differential						
$\mathbb{E}^j (\Delta(x_t))$		0.05	0.04**	0.02	0.14	0.57
x -factor spread		(0.41)	(1.96)	(0.26)	(1.10)	(0.74)
R^2 (%)		0.00	0.067	49.55	3.9	17.73
p -value		0.95	0.02**	0.00	0.33	0.025**
$\mathbb{E}^j (f_t^{(k/12)} - s_t)$	$\kappa(k)$	-0.50	0.72	1.83	0.60	0.80**
	t-stats	(-0.09)	(1.42)	(5.88)	(0.45)	(2.32)
interest differential						
$\mathbb{E}^j (\Delta(\tilde{x}_t))$		0.06	0.077	-0.16	0.10	-0.04
rx -factor spread		(0.15)	(0.91)	(-0.44)	(0.18)	(-0.09)
R^2 (%)		0.00	0.05	49.81	2.0	15.91
p -value		0.46	0.02**	0.00	0.072*	0.012**
$\mathbb{E}^j (f_t^{(k/12)} - s_t)$	$\kappa(k)$	-1.50	0.47	1.78	-0.01	0.48
	t-stats	(0.10)	(1.16)	(5.79)	(-0.01)	(1.43)
interest differential						
$\mathbb{E}^j (\tilde{\Delta}(x_t))$		0.10	0.065*	0.09	0.27*	0.21
FX forecast difference		0.59	1.91	0.82	1.63	1.44
R^2 (%)		0.01	0.068	49.97	5.91	20.99
p -value		0.96	0.06*	0.00	0.68	0.013**

Multivariate predictability regressions using the cross-sectional average predictor variables. Each row has four statistics and represent the following from top to bottom:

the regression coefficient, the t-statistic, the R-squares of each regression, and the p-values testing for joint significance predictors. Standard errors are corrected for conditional heteroskedasticity with 12 lags to account for overlapping monthly data. The 5 columns represent numbers corresponding to forecasts of k-month ahead risk-premia, from k=1,2,3,6, and 12. The data span from 6:1991-5:2009 and are at a monthly frequency.

Positive correlation between the predictor variables reduces the significance when multivariate regressions are run. In fact, the omission of the forward discount rate in the univariate case explains why there are positive α intercept term when the rx -factor spreads are used. The improvement in predictability jumps from 3.6% to almost 21% as a result of the inclusion of the forward discount rate variable. In addition, the inclusion of the full-sample and unobservable factors along with the forward discount rate attenuates the significance when compared to the univariate case. Despite the reduction in statistical significance, the large variability of currency risk-premia captured with the set of forecasting variables suggest that the magnitude and direction that returns are heading are captured by these relevant variables.

3.4.2 Exchange Rate Predictability

In this section, I examine if the same predictor variables can forecast exchange rate changes. To begin, I run the following univariate regressions using the same set of predictor variables, the full-sample return-forecasting (x -)factor spread, the rx -factor spread, and the hidden FX market factor. The exchange rate estimation equation is given by the following:

$$\mathbb{E}_t^j [\Delta s_{t+k}] = \alpha(k) + \kappa(k) \cdot \mathbb{E}^j [z_t] + \varepsilon_{t+k} \quad (3.23)$$

Table 3.3: Exchange Rate Forecasts, Univariate

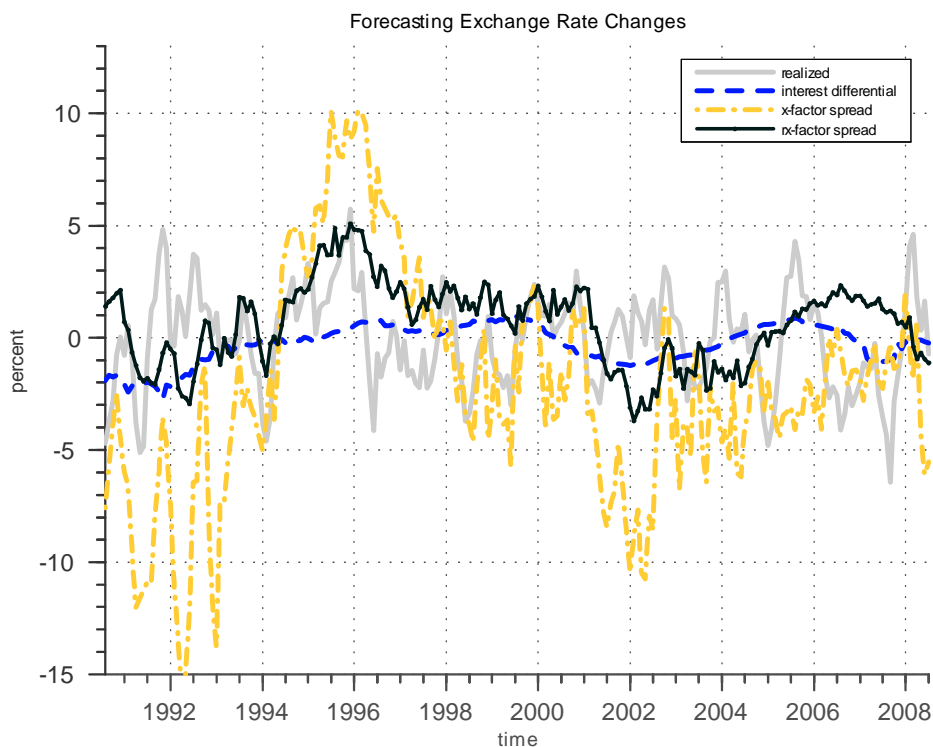
k		1	2	3	6	12
$\mathbb{E}^j(f_t^{(k/12)} - s_t)$ interest different ¹	$\kappa(k)$	0.12	-0.00	-0.08	-0.054	-0.01
	t-stats	(1.34)	(0.01)	(-1.81)	(-0.20)	(-0.03)
	R^2 (%)	0.00	0.00	1.03	0.00	0.00
$\mathbb{E}^j(\Delta(x_t))$ x -factor spread		-0.01	-0.03	-0.05*	-0.07	-0.03
		(1.02)	(-1.47)	(-1.81)	(-1.26)	(-0.27)
		0.00	1.51	2.77	3.45	0.00
$\mathbb{E}^j\Delta(\tilde{x}_t)$ rx -factor spread		-0.02	-0.05	-0.09	-0.08	-0.11
		(-0.61)	(-0.80)	(-0.93)	(-0.38)	(-0.31)
		0.00	0.5	1.0	0.44	0.05
$\mathbb{E}^j(\tilde{\Delta}(x_t))$ FX forecast difference		-0.01	-0.05*	-0.08	-0.13*	-0.11
		(-1.14)	(-1.75)	(-2.22)	(-1.89)	(-0.79)
		0.00	1.87	3.45	5.54	2.35

Exchange rate predictability regressions using the cross-sectional average predictor variables. Each row with three quoted statistics represent the following from top to bottom: the regression coefficient, the t-statistic and the R-squares of each regression. Standard errors are corrected for conditional heteroskedasticity with 12 lags to account for overlapping monthly data. The 5 columns represent numbers corresponding to forecasts of k-month ahead risk-premia, from k=1,2,3,6, and 12. The data span from 6:1991-5:2009 and are at a monthly frequency.

There is great disagreement on the size of the predictable component of exchange rates using these variables, ranging from essentially unpredictable to a small but marginally significant predictable component. For an equally weighted basket of industrialized currencies, the forward discount rate does not forecast future exchange rate changes. In fact, the results suggest that even after averaging out the idiosyncratic noise by constructing a basket of currencies, the empirical

estimates captures the tendency high interest currencies appreciate in the short-term. The x -factor spread and the difference in the x -factor spread and the rx -factor spread does provide evidence of short-to-medium horizon predictability with significant regression coefficients for 2-month, 3-month and 6-month ahead exchange rate changes. Although the amount of variation explained is rather modest, with no more than 7% of exchange rate fluctuations explained, the significance cannot be ignored. Figure 5 graphically demonstrates the co-movement between realized values and the predictor variables for for 6-month ahead forecast of exchange rate changes. Surprisingly, the x -factor spread that captures a predictable component in exchange rates has greater variance than the variable it is supposed to forecast! Similar results are obtained with the hidden FX market factor, though given that the two series move closely together, I choose to omit the time-series in the figure, displayed below:

Figure 3.5: Forecasting Exchange Rates



Exchange Rate Forecasts. The (.-.-) line represent the realized currency risk-premia, the (- - -) represent the forward discount rate, the solid, volatile time-series represent the full-sample return-forecasting factor spread. The solid, less volatile line represents the recursive return-forecasting factor spread. The time-series are in annualized percentages, span 6:1991-5:2009.

In all, the empirical results provide evidence that exchange rate changes and currency risk-premia can be linked to bond market risk factors. Interest rate factors extracted from the term structure bond data suggests that there is additional information in the term structure of interest rates that is useful for forecasting currency risk-premia and exchange rates. Large spreads in the x -factor coincide with differences between the x -factor spread and the rx -factor spread. The problem of overfitting with the x -factor likely carries over from bond markets.

As Mang (2012) finds, the x -factor overstates the amount of bond market predictability and as a result of the large time-variation in risk-premia captured, likely overstates the amount of real-time predictability in currency markets precisely because it correctly assesses when exchange rate fluctuations are transitory. The recursive return-forecasting (rx)-factor spreads move around less because they are constructed using bond market factors that react to persistent changes and as a consequence, are not susceptible to the same concerns about using *ex-post* information in its forecasts. The results suggest that understanding the tradeoff between risk and return in currency markets, the size of the predictable component implied by the studied factors depends critically on the information set of forecasters in their projections of currency risk-premia.

As in forecasts of currency risk-premia, I extend my analysis to multivariate regressions of exchange rate changes. The degree of predictability in exchange rates using real-time information is small with at best weak statistical significance for certain horizons. The fact that exchange rate and currency risk-premia are forecasted by these factors extracted from the term structure data suggests that there is information in the yield curve that is useful for forecasting currency risk-premia and exchange rates, hence in understanding the tradeoff between risk and return in currency markets.

$$\mathbb{E}_t^j(\Delta s_{t+k}) = \alpha(k) + \kappa(k) \cdot \mathbb{E}^j\left(f_t^{(k/12)} - s_t\right) + \kappa_A(k) \cdot \mathbb{E}^j(A_t) \dots + \varepsilon_{t+k} \quad (3.24)$$

Table 3.4: Exchange Rate Forecasts, Multivariate

k		1	2	3	6	12
$\mathbb{E}^j(f_t^{(k/12)} - s_t)$	$\kappa(k)$	0.12	-0.00	-0.08	-0.054	-0.01
	t-stats	(1.34)	(0.01)	(-1.81)	(-0.20)	(-0.03)
	interest differential	R^2 (%)	0.00	0.00	1.03	0.00
$\mathbb{E}^j(f_t^{(k/12)} - s_t)$	$\kappa(k)$	0.48	0.53	-0.10	0.48	0.38
	t-stats	(1.21)	(1.38)	(-1.37)	(0.99)	(0.91)
	interest differential					
$\mathbb{E}^j(\Delta(x_t))$		-0.02	-0.05	-0.05*	-0.12*	-0.10
x -factor spread		(-1.26)	(-1.59)	(-1.76)	(-1.69)	(-0.79)
R^2 (%)		1.00	2.44	4.1	5.64	3.22
p -value		0.62	0.33	0.13	0.23	0.54
$\mathbb{E}^j(f_t^{(k/12)} - s_t)$	$\kappa(k)$	0.27	0.22	-0.10	0.12	0.14
	t-stats	(1.21)	(1.38)	(-1.37)	(0.99)	(0.91)
	interest differential					
$\mathbb{E}^j(\tilde{\Delta}(x_t))$		-0.04	-0.07	-0.07*	-0.11*	-0.05
rx -factor spread		(-1.26)	(-1.59)	(-1.76)	(-1.69)	(-0.79)
R^2 (%)		0.03	0.01	2.2	0.01	1.00
p -value		0.70	0.68	0.31	0.90	0.80
$\mathbb{E}^j(f_t^{(k/12)} - s_t)$	$\kappa(k)$	0.43	0.48	-0.10	0.50	0.48
	t-stats	(1.47)	(1.42)	(-1.81)	(1.12)	(1.28)
	interest differential					
$\mathbb{E}^j(\tilde{\Delta}(x_t))$		-0.02	-0.07*	-0.39	-0.19**	-0.21
FX forecast difference		(-1.40)	(-1.72)	(-2.59)	(-2.11)	(-1.50)
R^2 (%)		1.03	2.73	4.84	8.20	7.3
p -value		0.39	0.23	0.082	0.099*	0.24

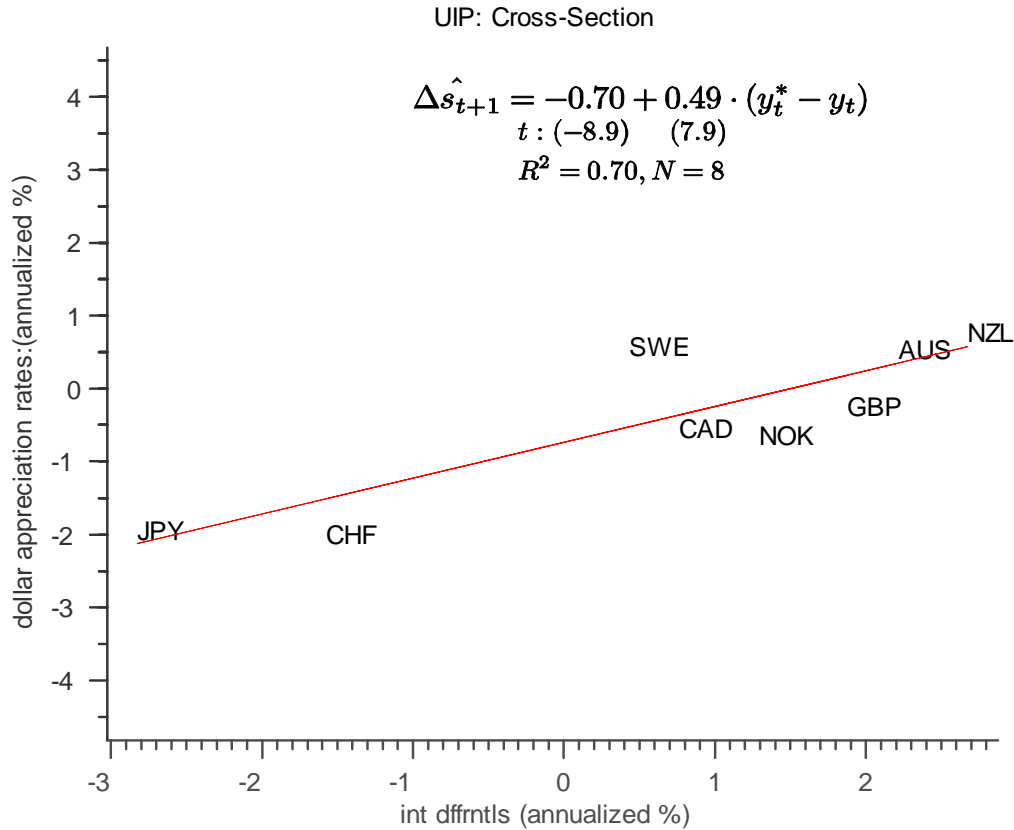
Forecasting exchange rates with two variables. Each column represents k-month ahead forecasts using the two variables, with the forward discount rate being common in all forecasts. Each row associated with a predictor variable and a k-period ahead forecast is contains four numbers, in descending order, the estimated regression coefficient, the

associated t-statistic corrected for serial correlation using the Newey-West procedure, and the R-squared term, and the p-value testing for joint significance. * - indicates significance at the 10 percent level, ** - indicates significance at the 5 percent level.

Once we use x -factor spread and the hidden FX market factor, we find that the degree of predictability is relatively large. Exchange rates are well approximated by a random walk at short-horizons, but looking several months ahead, we see that the subtle predictability components is captured by additional information in the x -factor spread and the hidden FX market factor, i.e. the difference in the x -factor and rx -factor spread. The negative coefficient estimates on the unobservable component suggests that exchange rates (in units of foreign currency per dollar) captures a factor that negatively moves with the dollar, coinciding with dollar appreciation k -months into the future.

The loading coefficients on the forward discount rate are all positive for each multivariate regression but insignificant. The sign is consistent the tendency for high interest rate currencies tend to depreciate if we control for the FX market forecast difference, consistent with Engel (2011) who suggests that at least two risk factors are required to explain patterns of delayed overshooting found in currency markets. In fact, given the cross-section of countries, there is evidence unconditional interest parity for a longer horizon. Looking at the unconditional average dollar appreciation rates relative to an unconditional average interest rate differential for each individual country, I find a slope coefficient of about 0.5 for the cross-section of industrialized economies under study, strikingly close to the estimated regression coefficient of k -month ahead dollar appreciation rates on the interest rate differential once we control for the large and transitory component of exchange rates proxied for by either the x -factor spread or the difference in the x -factor spread and the rx -factor spread. Figure 6 displays the tendency for uncovered interest parity to hold in the long-run.

Figure 3.6: Long-Run Exchange Rates and Interest Rate Differentials Along the Cross-Section



Unconditional UIP. The figure displays the unconditional average dollar appreciation rate (12-months) for each foreign currency relative to its unconditional average interest rate differential for holding onto a one-year government treasury security. Variables are in units of annualized returns. The data span from 1.1989-10.2010. AUD = Australia, NZL = New Zealand, NOK = Norway, SWE = Sweden, CAD = Canada, GBP = England, JPY = Japan

To summarize the paper so far, I find that differences in the factors that are useful for forecasting bond risk-premia are also useful for forecasting in currency markets. Forward discount rates forecast currency as does the full-sample

return-forecasting factor spread as well as the difference in the x -factor spread and rx -factor spread. At best, these predictor variables can also forecast currency risk-premia and can deliver an R^2 of up to 27%. I also find that there is predictability in exchange rates with the difference in the x -factor spread and rx -factor spread. Multivariate regressions reveal that exchange rates are forecastable using the average forward discount rate if we control for large and transitory changes in exchange rates that either the x -factor spread or the difference in the x -factor spread and rx -factor spread proxy for. Controlling for large and transitory fluctuations, I find evidence that exchange rates are consistent with uncovered interest parity, at least in terms of having the right sign.

3.4.3 Discussion

In this section, I discuss my findings in relation to the broader literature on return predictability in foreign exchange markets. The essence of this section is that if you take seriously the role of expectations formation by real-time forecasters who intermittently update their forecasts, then the information gap between those who know the future and those who do not can forecast returns in foreign exchange markets. This information gap is consistent with many phenomena, one of which is the learning dynamics of market participants; uncertainty about future returns are positively correlated with this information gap.

So far, I have documented various results on the size of the predictable component in currency markets using predictor variables constructed using bond market prices. The results are statistically significant, providing evidence of predictable returns in currency markets. However, I have yet to discuss my findings with respect to the broader literature on currency markets. For this section, I will take seriously the role of information in asset prices. I do so by highlighting forecasts that make restrictions on information to be consistent with real-time data and compare them to forecasts made assuming all historical data is available.

To begin, the positive correlation between predictor variables confound their statistical significance. Although Engel (2011) provides evidence that there are at least two factors pricing currency market returns, for the period under study, from 1.1989 – 5.2009, the correlation between interest rate differentials and the full-sample return-forecasting factor spreads is around 0.57. Another worry is that the return-forecasting (x -)factor spread and the FX market forecast difference have large variance. Persistent predictor variables weaken statistical significance in forecasting as Stambaugh (1999) points out, but one that is more variable than the predicted variable leaves one with significance suspicions. Despite their shortcomings, the (x -)factor spread and the hidden FX market factor contain information about the turning points in foreign exchange markets that is essential for understanding the changes in currency market risk conditions because foreign exchange markets are especially volatile. This is where the two predictor variables may have an advantage over the interest differentials.

If these predictor variables are indeed useful for investment strategies, an analysis of the risk return dynamics will provide information about the real-time exploitability of these forecasting relationships. Quantitatively, full-sample forecasts capture a large predictable component in exchange rate changes as does the hidden FX market factor. However, the large, predictable but transitory changes in currency risk-premia and exchange rate changes imply volatile Sharpe ratios as Duffee (2010) and Mang (2012) has emphasized, likely violating sensible restrictions on Sharpe ratios found in the data. Thus, the size of the predictable component is likely not as large as these predictor variables would suggest.

An interpretation of the recursive return-forecasting (rx -)factor spread is that it mimics the expectations formation of agents who have the same information set as an econometrician. Adaptively made forecasts of exchange rate changes, captured by the rx -factor spread do not move as much as its full-sample counterpart, the x -factor spreads. Adjusting for real-time data availability as the rx -factor

spreads do, the size of the predictable component becomes small and insignificant. The heart of the matter is that real-time forecasters have difficulty disentangling persistent shocks in risk-premia from transitory ones in currency markets. Real-time forecasts react to persistent changes in returns, but often delay adjustment in their forecasts because of the degree of randomness in returns. The reason for the inertia in forecasts is that the recursive return-forecasting, rx -factor spread only reacts to persistent changes in bond risk-premia.

In fact, comparisons of real-time forecasts for currency risk-premia relative to its full-sample counterpart are precisely what the hidden FX market factor captures. As for exchange rates, short-horizon predictability exists using the hidden FX market factor, but not with contemporaneous variables such as the (rx -)factor spread not the interest differentials. As Mang (2012) and the current paper find, the difference in the adaptive forecasts and the full-sample forecasts then represent an information gap that contains predictive ability in both bond and currency markets. It turns out that the hidden FX market factor is precisely the information that predicts exchange rate changes. Accounting for this term delivers the right sign that is consistent with the uncovered interest parity condition. Over long horizons these forecast differences average out.

The results are relevant for the long standing debate about the origin of the forward premium anomaly. For a set of industrialized economies during the time period under study, we find evidence of not only a stable predictable component in currency risk-premia but also evidence that exchange rate changes are forecastable using factors that capture forecast differences using bond risk factors. There is no clear cut resolution to the puzzle because the answer critically depends on the information set that is endowed to the investor. Forecasts that take the full-sample of data as given treat transitory and persistent shocks appropriately, but adaptive forecasts under-react to transitory changes in exchange rates precisely when the interest rate differential widens, consistent with Gourinchas and Tornell

(2004) and Froot and Frankel (1989). This behavior is consistent with the sluggish adjustment of forecasts of changes in bond risk-premia highlighted in Mang (2012). Therefore, the large implied predictable component in bond and currency markets using the full-sample return-forecasting factors is consistent with the lack of information restrictions imposed. With those information restrictions, adaptive forecasts imply a sizably smaller predictable component in real-time.

3.5 Conclusions

In this paper, I document the quantitative size of the predictable component in currency markets using variables constructed from bond market prices. From about 1989 – 2009 for an average over a broad set of industrialized economies currencies interest rate differentials precede movement in currency risk-premia as does the spread in the return-forecasting (x -)factor. Differences in the full-sample return-forecasting (x -)factor spread and the recursive return-forecasting rx -factor spread, the hidden FX market factor, not only forecasts currency risk-premia but also exchange rates. On the other hand, spreads in the recursive return-forecasting rx -factor forecast neither risk-premia or exchange rate changes.

The results from this paper, along with those found in Mang (2012) and Mang (2012), seem to suggest that the quantitative size of the predictable component captured by the predictor variables that take the full-sample of data as given likely overstates the amount of actual predictability in real-time because large risk-premia in both bond and currency markets coincide with forecast differences that *ex-post* turn out to vary with realized values of risk-premia in each market. A plausible reason for these forecast differences is adaptive learners who have model specification uncertainty guard against the worst possible realization and in so doing, make sophisticated "mistakes." Adaptive agents overstate the amount of risk an investor is exposed to because they require additional compensation for

model uncertainty. This additional compensation can be considerably large and move rapidly if agents are concerned that past data does not serve as a useful guide for the future.

In relation to the broader literature, particularly linear factor models often employed in finance, large market price of risk spreads between countries can reflect either time-varying risk or perceptions of risk. In real-time, it is difficult to disentangle the relative contributions of the two. It would be interesting to see if perceptions of risk are more pronounced for emerging market government bond markets where investor confidence in central bankers is low relative to industrialized economies. Concern for structural change will likely be even greater for these countries. For this paper, I have only investigated how real-time revisions to expectations in relation to full-sample forecasts and have not posed the problem for agents to have this concern for structural change. Doing so may give us a better understanding of how actual forecasters behave during tumultuous times in rapidly emerging markets, making it a worthwhile topic for future research.

3.6 Appendix

3.6.1 List of Variables

I describe the relevant list of variables utilized in this paper. This portion should serve as a reference for the notation used in the paper. Notice that the variables without a j -subscript are domestic, U.S. variables.

Table 3.5: List of Relevant Variables

Variable	Description
$y_{j,t}^{(k/12)}$	k -month interest rate on a zero-coupon bond for country j
$f_{j,t}^{(k/12)} - s_t$	k -month forward discount rate for country j
$rx_{j,t+k}$	k -period currency risk-premia
$\Delta s_{j,t+k}$	k -period ahead dollar appreciation rate
$x_{j,t}$	full-sample (FS) return-forecasting factor of country j
$\tilde{x}_{j,t}$	recursive return-forecasting factor of country j
$x_t - x_{j,t}$	bilateral return-forecasting factor spread "x-factor spread"
$\tilde{x}_t - \tilde{x}_{j,t}$	bilateral recursive x -factor spread "rx-factor spread"
$x_t - x_{j,t} - (\tilde{x}_t - \tilde{x}_{j,t})$	difference in difference of x and rx -factor spread "hidden FX market factor"

Table 6: List of relevant variables

3.6.2 Constructing Predictor Variables in Detail

Let z_t be a relevant predictor variable in bond markets. Risk prices can be represented as a linear function of these variables:

$$\lambda_t^z = \lambda_0^z + \lambda_1^z \cdot z_t \quad (3.25)$$

$$\lambda_{j,t}^z = \lambda_{j,0}^z + \lambda_{j,1}^z \cdot z_{j,t} \quad (3.26)$$

Deriving currency risk prices from bond risk factors amounts to taking the difference in the bond risk factors and, to mitigate the effect of idiosyncratic noise, take the average over the cross-section of countries. A common empirical finding is that asset price movements tend to be correlated, hinting of a global risk factor. An alternate formulation will have the market price of risk to be linearly related to both

a country-specific and a global factor of the form $\lambda_{j,t}^z = \lambda_{0,j}^z + \lambda_{1,j} \cdot z_{j,t} + \lambda_G \cdot x_{G,t}$. Constructing cross-sectional averages, or a portfolio of currencies will work to isolate the common factor since the market price of risk for a portfolio of currencies is $\mathbb{E}^j [\lambda_{j,t}^z] = J^{-1} \sum_{j=1}^J (\lambda_{0,j}^z + \lambda_{1,j} \cdot z_{j,t} + \lambda_{G,j} \cdot x_{G,t}) = \mathbb{E}^j [\lambda_0^z] + \mathbb{E}^j [\lambda_{1,j} \cdot z_{j,t}] + \mathbb{E}^j [\lambda_{G,j}] \cdot x_{G,t}$. Taking the difference between the market price of risk of the domestic currency and the cross-sectional average of foreign currencies delivers

$$\mathbb{E}^j [\lambda_{j,t}^z - \lambda_t^z] = \lambda_0^z - \mathbb{E}^j [\lambda_0^z] + \lambda_1^z \cdot z_t - \mathbb{E}^j [\lambda_{1,j} \cdot z_{j,t}] + (\lambda_G - \mathbb{E}^j [\lambda_{G,j}]) \cdot x_{G,t}. \quad (3.27)$$

If $\lambda_G = \lim_{J \rightarrow \infty} J^{-1} \sum_{j=1}^J \lambda_{G,j}$, $\lambda_0^z = \lim_{J \rightarrow \infty} J^{-1} \sum_{j=1}^J \lambda_{0,j}^z$ then $\mathbb{E}^j [\lambda_{j,t}^z - \lambda_t^z] \approx \nu \cdot \mathbb{E}_t^j [z_t - z_{j,t}]$ since $\lambda_1, \lambda_{1,j} < 0$ provided $\lambda_1^z = \lim_{J \rightarrow \infty} J^{-1} \sum_{j=1}^J \lambda_{1,j}^z$. That is, time-variation in the market price of risk spreads are equal to the cross-sectional average spread of z_t relative to $z_{j,t}$ if (a) the domestic currency and the portfolio of foreign currencies has equal exposure to global shocks and if (b) the portfolio of currencies eliminates idiosyncratic risk unrelated to the domestic risk factor. Thus, we can proceed as if the market price of risk depends only on country-specific factors with the idea in mind that the portfolio approximately satisfies (a) and (b). For this paper, I proceed under the assumption that (a) and (b) approximately holds.

Exchange rate changes can then be written as

$$\Delta s_{j,t+1} \quad (3.28)$$

$$\begin{aligned} &= m_{t+1} - m_{j,t+1} \\ &= y_{j,t}^{(k/12)} - y_t^{(k/12)} + \frac{1}{2} [(\sigma_{j,\varepsilon}^2 + \sigma_\eta^2) \lambda_{j,t}^2 - (\sigma_\varepsilon^2 + \sigma_\eta^2) \lambda_t^2] \end{aligned} \quad (3.29)$$

$$+ (\lambda_{j,t} \varepsilon_{j,t+1} - \lambda_t \varepsilon_{t+1}) + (\lambda_{j,t} - \lambda_t) \eta_{t+1} \quad (3.30)$$

Expected exchange rate changes (dollar appreciation rate) is then

$$\begin{aligned}\mathbb{E}_t [\Delta s_{j,t+1}] &= y_{j,t}^{(k/12)} - y_t^{(k/12)} + \frac{1}{2} [(\sigma_{j,\varepsilon}^2 + \sigma_\eta^2) \lambda_{j,t}^2 - (\sigma_\varepsilon^2 + \sigma_\eta^2) \lambda_t^2] \\ &= y_{j,t}^{(k/12)} - y_t^{(k/12)} - \underbrace{\frac{1}{2} [\sigma_t^2 (m_{t+1}) - \sigma_t^2 (m_{j,t+1})]}_{\mathbb{E}[rx_{j,t+1}]}\end{aligned}\quad (3.31)$$

The term $\mathbb{E}_t [rx_{j,t+1}]$ is the currency risk-premia. The link between bond and currency markets is clear with expression (22) because the interest rate differential can be decomposed into an expected dollar appreciation term and a risk-premium component.

In terms of the bond market risk prices, the expression is then

$$\mathbb{E}_t [rx_{j,t+1}] = \frac{1}{2} [\lambda_{j,t}^2 (\sigma_{j,\varepsilon}^2 + \sigma_\eta^2) - \lambda_t^2 (\sigma_\varepsilon^2 + \sigma_\eta^2)] \quad (3.32)$$

$$= \frac{1}{2} \begin{bmatrix} (\lambda_{j,0} + \lambda_{j,1} \cdot z_{j,t})^2 (\sigma_{j,\varepsilon}^2 + \sigma_\eta^2) \\ - (\lambda_0 + \lambda_1 \cdot z_t)^2 (\sigma_\varepsilon^2 + \sigma_\eta^2) \end{bmatrix} \quad (3.33)$$

$$\approx \kappa \cdot z_t - \kappa_j \cdot z_{j,t} \quad (3.34)$$

since $\lambda_0, \lambda_1, \lambda_{0,j}, \lambda_{1,j} < 0$ where the previous line relies on a first-order Taylor approximation of a quadratic expression around $x_t = 0$.

$$\begin{aligned}\lambda_t^2 (\sigma_\varepsilon^2 + \sigma_\eta^2) &= (\sigma_\varepsilon^2 + \sigma_\eta^2) (\lambda_0 + \lambda_1 \cdot z_t)^2 \\ &\approx (\sigma_\varepsilon^2 + \sigma_\eta^2) (\lambda_0 + 2\lambda_1 \cdot z_t) \\ &= -\kappa_0 - 2\kappa \cdot z_t\end{aligned}$$

where $\kappa_0 = -(\sigma_{j,\varepsilon}^2 + \sigma_\eta^2) \lambda_0 > 0$ and $\kappa = -(\sigma_\varepsilon^2 + \sigma_\eta^2) \lambda_1 > 0$. For the foreign country, the approximation around $x_{j,t} = 0$ delivers the following expression:

$$\begin{aligned}\lambda_{j,t}^2 (\sigma_{\varepsilon,j}^2 + \sigma_\eta^2) &= (\sigma_{j,\varepsilon}^2 + \sigma_\eta^2) (\lambda_{j,0} + \lambda_{j,1} \cdot x_{j,t})^2 \\ &\approx (\sigma_{j,\varepsilon}^2 + \sigma_\eta^2) (\lambda_{j,0} + 2\lambda_{j,1} \cdot x_{j,t}) \\ &\approx -\kappa_{j,0} - 2\kappa_j \cdot x_{j,t}\end{aligned}$$

where $\kappa_{j,0} = -(\sigma_{j,\varepsilon}^2 + \sigma_\eta^2) \lambda_{j,0} > 0$ and $\kappa_j = -(\sigma_{j,\varepsilon}^2 + \sigma_\eta^2) \lambda_{j,1} > 0$. The difference between the market price of risk $\lambda_t^2 (\sigma_\varepsilon^2 + \sigma_\eta^2) - \lambda_t^2 (\sigma_{\varepsilon,j}^2 + \sigma_\eta^2)$, then is

$\kappa_0 + 2\kappa \cdot z_t - \kappa_{j,0} + 2\kappa_j \cdot z_{j,t}$, but given the comparable level factors across countries, $\lambda_{j,0} \approx \lambda_0$ and exposure to global shocks, i.e. a common σ_η^2 , the constant term in the market risk prices are roughly the same, i.e. $\kappa_{j,0} \approx \kappa_0$. As a result, equation (25) describes currency risk-premia as a function of the bond risk-factor (to a first-order approximation). Besides, the intercept term will not materially affect our understanding of the topic chosen to be the focus of this the paper, namely characterizing the factors that explain the time-variation in currency risk-premia. Further, the expression for the expected dollar appreciation rate is given by:

$$\mathbb{E}_t [\Delta s_{j,t+1}] = y_{j,t}^{(1)} - y_t^{(1)} - (\kappa \cdot z_t - \kappa_j \cdot z_{j,t}). \quad (3.35)$$

An often used predictor for currency risk-premia is the interest rate differential $y_t^{(1)} - y_{j,t}^{(1)}$. The expression for currency risk-premia is then $\mathbb{E}_t [rx_{j,t+1}] = \theta \cdot y_t^{(1)} - \theta_j \cdot y_{j,t}^{(1)}$. From the perspective of a linear factor model, this occurs if the difference in the bond market price of risk $\lambda_t^2 (\sigma_\varepsilon^2 + \sigma_\eta^2) - \lambda_{j,t}^2 (\sigma_{j,\varepsilon}^2 + \sigma_\eta^2)$ is sufficiently large, which coincides with large spreads in bond market risk factors. Taking a cross-sectional average mitigates the importance of idiosyncratic bond market factors and isolates the common component. Doing so for expected dollar appreciation and currency risk-premia delivers the following expressions:

$$\begin{aligned} \mathbb{E}^j [\mathbb{E}_t [rx_{j,t+1}]] & : = \mathbb{E}_t^j [rx_{t+1}] \\ & = J^{-1} \sum_{j=1}^J (\kappa \cdot z_t - \kappa_j \cdot z_{j,t}) \\ & = \kappa \cdot z_t - J^{-1} \sum_{j=1}^J \kappa_j \cdot z_{j,t} \\ & \approx \kappa \cdot \mathbb{E}^j [z_t - z_{j,t}] \end{aligned} \quad (3.36)$$

For a sufficiently large cross-section, $\kappa = \lim_{J \rightarrow \infty} J^{-1} \sum_{j=1}^J \kappa_j$ that will make (26) an exact relation. Exchange rate changes for a basket of currencies is given by the following expression

$$\mathbb{E}_t^j [\Delta s_{t+1}] = \mathbb{E}^j \left[y_{j,t}^{(k/12)} - y_t^{(k/12)} \right] - \kappa \cdot \mathbb{E}^j [z_t - z_{j,t}]$$

3.6.2.1 Other Predictor Variables

Cochrane and Piazzesi (2005, 2008) look specifically at the return-forecasting factor as does Mang (2012). In their specification, all variation in the market price of risk is driven by variation in the return-forecasting factor.

$$\lambda_t = \lambda_0 + \lambda_1 \cdot x_t \quad (3.37)$$

$$\lambda_{j,t} = \lambda_{j,0} + \lambda_{j,1} \cdot x_{j,t} \quad (3.38)$$

where $x_t := \mathbb{E}_t^n \left[\widehat{rx}_{t+1}^{(n)} \right] = \hat{\alpha} + \hat{\gamma}' \cdot f_t$ and $x_{j,t} := \mathbb{E}_t^n \left[\widehat{rx}_{j,t+1}^{(n)} \right] = \hat{\alpha}_j + \hat{\gamma}'_j \cdot f_{j,t}$ obtained by fitting the estimated coefficients of the average bond risk-premia for a set of maturities on a set of forward rates.

For the interest rate differential, let risk prices be specified as:

$$\lambda_t^y = \lambda_0 + \lambda_1 \cdot y_t \quad (3.39)$$

$$\lambda_{j,t}^y = \lambda_{j,0}^{\tilde{x}} + \lambda_{j,1}^{\tilde{x}} \cdot y_{j,t} \quad (3.40)$$

currency risk-premia is given by the following expression

$$\begin{aligned} \mathbb{E}_t^j [rx_{t+1}] &= \theta \cdot y_t^{(k/12)} - J^{-1} \sum_{j=1}^J \theta_j \cdot y_{j,t}^{(k/12)} \\ &\approx \theta \cdot \mathbb{E}^j \left[y_t^{(k/12)} - y_{j,t}^{(k/12)} \right] \end{aligned}$$

$$\begin{aligned} \mathbb{E}_t^j [\Delta s_{t+1}] &\approx \mathbb{E}^j \left[y_{j,t}^{(k/12)} - y_t^{(k/12)} \right] - \theta \cdot \mathbb{E}^j \left[y_t^{(k/12)} - y_{j,t}^{(k/12)} \right] \\ &\approx \mu \cdot \mathbb{E}^j \left[y_{j,t}^{(k/12)} - y_t^{(k/12)} \right] \text{ where } \mu = 1 + \theta \end{aligned}$$

I want to investigate not only the full-sample return-forecasting factor spreads, or more succinctly the x -factor spread, and the interest rate differential for capturing a predictable component in currency markets, but also the recursive return-forecasting factor spread, the rx -factor spread, as well as the difference between the x -factor spread and the rx -factor spread. The market price of risk expressions

for the rx -factor is then

$$\lambda_t^{\tilde{x}} = \lambda_0 + \lambda_1 \cdot \tilde{x}_t \quad (3.41)$$

$$\lambda_{j,t}^{\tilde{x}} = \lambda_{j,0}^{\tilde{x}} + \lambda_{j,1}^{\tilde{x}} \cdot \tilde{x}_{j,t} \quad (3.42)$$

where $\tilde{x}_t := \widetilde{\mathbb{E}}_t^n [\widehat{rx}_{t+1}] = \alpha_t + \gamma'_t \cdot f_t$ and corresponding expression for the currency risk-premia is then

$$\mathbb{E}_t [rx_{j,t+1}] = \kappa^{\tilde{x}} \cdot \tilde{x}_t - \kappa_j^{\tilde{x}} \cdot \tilde{x}_{j,t} \quad (3.43)$$

For an equally weighted basket of currencies, the expression would then be

$$\mathbb{E}_t^j [rx_{t+1}] \approx \kappa^{\tilde{x}} \cdot \mathbb{E}_t^j (\tilde{x}_t - \tilde{x}_{j,t}) \quad (3.44)$$

if $\kappa^{\tilde{x}} \approx J^{-1} \sum_{j=1}^J \kappa_j^{\tilde{x}}$. The expected dollar appreciation rate is then

$$\mathbb{E}_t^j [\Delta s_{t+1}] = \mathbb{E}^j \left[y_{j,t}^{(k/12)} - y_t^{(k/12)} \right] - \kappa^{\tilde{x}} \cdot \mathbb{E}^j (\tilde{x}_t - \tilde{x}_{j,t})$$

Finally, the expression for the market price of risk for the difference in the x -factor and rx -factor spread

$$\lambda_t^\Delta = \lambda_0^\Delta + \lambda_1^\Delta \cdot (x_t - \tilde{x}_t) \quad (3.45)$$

$$\lambda_{j,t}^\Delta = \lambda_{j,0}^\Delta + \lambda_{j,1}^\Delta \cdot (x_{j,t} - \tilde{x}_{j,t}) \quad (3.46)$$

The corresponding expression for currency risk-premia is then

$$\mathbb{E}_t [rx_{j,t+1}] = \kappa^\Delta \cdot (x_t - \tilde{x}_t) - \kappa_j^\Delta \cdot (x_{j,t} - \tilde{x}_{j,t}) \quad (3.47)$$

and for an equally weighted basket of currencies

$$\mathbb{E}_t^j [rx_{t+1}] \approx \kappa^\Delta \cdot \mathbb{E}^j [(x_t - \tilde{x}_t) - (x_{j,t} - \tilde{x}_{j,t})] \quad (3.48)$$

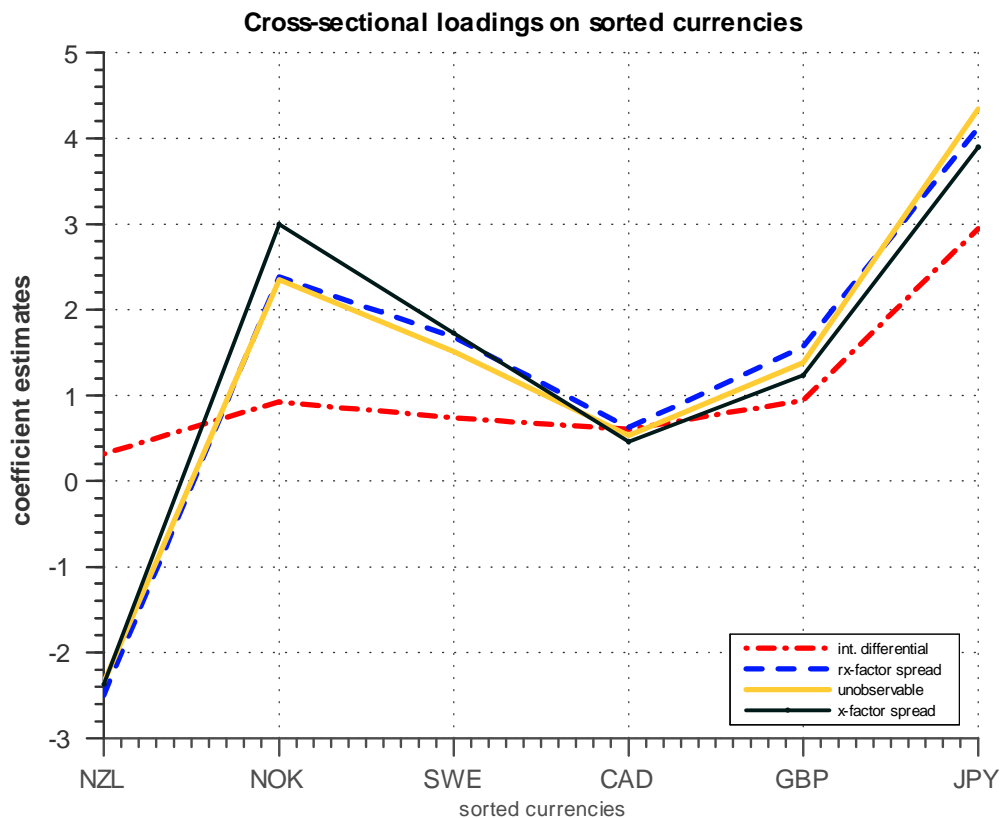
The expected dollar appreciation rate for this factor is then

$$\mathbb{E}_t^j [\Delta s_{t+1}] = \mathbb{E}^j \left[y_{j,t}^{(k/12)} - y_t^{(k/12)} \right] - \kappa^\Delta \cdot \mathbb{E}^j [(x_t - \tilde{x}_t) - (x_{j,t} - \tilde{x}_{j,t})]$$

3.6.3 Cross-Sectional Exposure to Risk

To see if the constructed variables capture the relevant risk-factors in currency markets, I look at the cross-sectional loadings to see if the compensation for exposure to the proxied for risk factors increases with exposure to the mentioned risk factor. Since Lustig et al (2011) find evidence of a risk factor that is well-proxied by interest rate differentials, I use the forward discount rate along with the other constructed risk factor. In a similar vein, I perform a cross-sectional regression of average currency risk-premia on a set of sorted currencies, from high interest rate differentials to low interest rate differentials relative to the U.S. For instance, the interest rate differential tends to be largest between the U.S. and Japan since Japanese currencies have relative low yields. On the other hand, New Zealand tends to offer higher yields relative to the U.S. so they have negative differentials. Below is a comparison of the cross-sectional loadings for six industrialized countries where international bond market data are available in conjunction with currency market data for the sample period under study (1988-2009); New Zealand, Norway, Sweden, Canada, Great Britain, and Japan. The cross-sectional pattern suggests that the constructed variables capture the same pattern of risk-exposure as the interest rate differential. Despite the non-monotonic relation, which is to be expected because interest rate differential reversals occur and also due to the small set of currencies that comprise the cross-section of industrial economies, there is evidence to support to the notion that the predictor variables are capturing *at least one* systematic risk factor. The figure below gives loadings on countries for each predictor variable when I regress $\mathbb{E}^j [rx_{t+6}]$ on $\left[y_t^{(6/12)} - y_{NZL,t}^{(6/12)}, \dots, y_t^{(6/12)} - y_{JPY,t}^{(6/12)} \right]$.

Figure 3.7: Cross-Sectional Risk Exposure



Cross-sectional loadings on six sorted industrialized countries. The two-stage least squares regression is performed to determine the cross-sectional loadings: Average currency risk-premia is regressed on each individual interest rate differential sorted on the magnitude change in interest rate differential. I do so for all relevant predictor variables, the forward discount rate (.-.-.), the rx-factor spread (- - -), the x-spread and the difference in the rx-factor spread and the x-factor spread. NZL = New Zealand, NOK = Norway, SWE = Sweden, CAD = Canada, GBP = England, JPY = Japan.

3.6.4 Equivalence Between Return-Forecasting Factor Spreads

The cross-sectional average of the difference between the full-sample forecasting factor spread and the recursive return-forecasting factor spread is approximately equal to the difference in the cross-sectional average full-sample return-forecasting

factor spread and the cross-sectional average recursive return-forecasting factor spread. More simply, in math:

$$\begin{aligned}
\mathbb{E}^j \left[\tilde{\Delta}(x_t) \right] & : = \mathbb{E}^j (x_t - x_t^j - (\tilde{x}_t - \tilde{x}_{j,t})) \\
& = J^{-1} \sum_{j=1}^J [(x_t - \tilde{x}_t) - (x_{j,t} - x_{j,t})] \\
& = J^{-1} \sum_{j=1}^J [x_t - x_{j,t}] - J^{-1} \sum_{j=1}^J (\tilde{x}_t - \tilde{x}_{j,t}) \\
& \approx \mathbb{E}^j [\Delta(x)] - \mathbb{E}^j [\Delta(\tilde{x})]
\end{aligned}$$

We can verify this property of the forecaster variables in the figure below.

Figure 3.8: Equivalence between Hidden FX Factor and the Difference in Full-Sample and Recursive Return-Forecasting Factor Spread

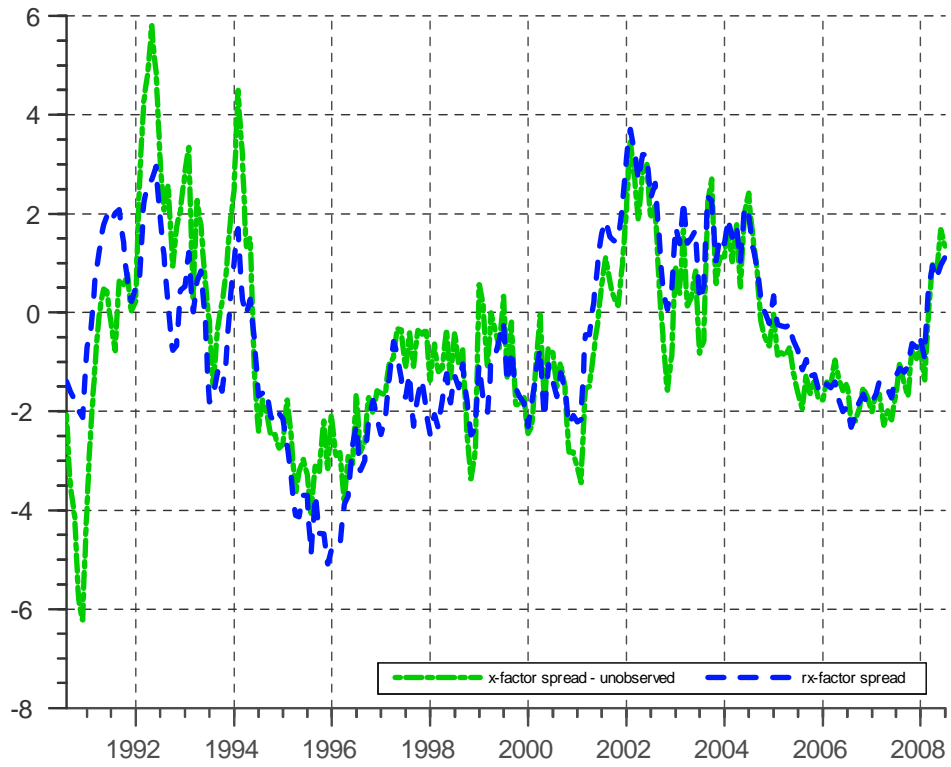


Figure 8: Graphical proof that the cross-sectional average of the difference in the x-factor spread and the rx-factor spread is approximately equal to the cross-sectional average x-factor spread and the cross-sectional average rx-factor spread

$$\mathbb{E}^j \left[\tilde{\Delta}(x_t) \right] \approx \mathbb{E}^j [\Delta(x)] - \mathbb{E}^j [\Delta(\tilde{x})]$$

Deviations could be driven by the small sample of industrialized countries where yield curve data are available and span a sufficiently long time horizon. As the cross-sectional size increases, the two terms should become equal.

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