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Publication Date

1975-11-01

0 0 4 4 0 4 1 5 0

Submitted to Physical Review Letters

LBL-4343
Preprint **c 1**

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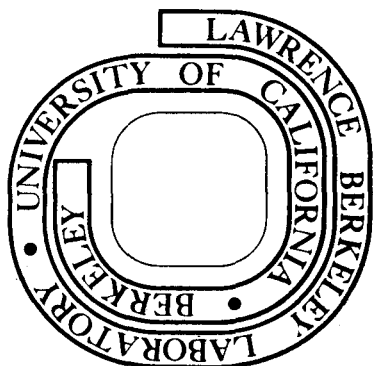
COMPUTING THE FINITE RANGE DWBA

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November 1975

Prepared for the U. S. Energy Research and
Development Administration under Contract W-7405-ENG-48

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COMPUTING THE FINITE RANGE DWBA *

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Abstract:

Two dimensional quadrature formulas with minimal number of points are applied to the computation of the double radial integral in the finite range Distorted waves Born approximation for nuclear reactions. A specific algorithm programmed in the code KUNDRY leads to fast computation, especially in cluster transfer reactions where the area to be integrated is largest. As examples, the reactions $^{11}\text{B}(^{16}\text{O}, ^{15}\text{N})^{12}\text{C}$ and $^{11}\text{B}(^{16}\text{O}, ^{12}\text{C})^{15}\text{N}$ are discussed and their interference investigated.

* Work performed under the auspices of the Energy Research and Development Administration.

The epithet "exact" was introduced by Delic and Robson¹⁾ to describe computation of the Distorted waves Born approximation in a formalism for nuclear reactions which allowed both non-S state stripping from the incident projectile in the transfer reaction $A(a,b)B$, and an interaction of finite range in the DWBA transition matrix amplitude. These computations, performed for d,p reactions, applied an approach suggested by Austern et al²⁾ and did not introduce any further approximations of either a mathematical or numerical nature, other than that of truncating the area integrated in the two-dimensional radial integrals which need to be evaluated. The method requires the numerical evaluation of the Legendre series coefficients³⁾ g_K at each point of the two-dimensional grid of the double radial integral. The largest value of K is a few units larger than L_{\max} , the value of the largest partial wave retained in the DW expansions describing the relative motions of a,A and b,B. Thus, the overall computation time of such calculations is dependent on the number of grid points used, as well as their distribution within the area to be integrated. There are basically three approaches to the problem of evaluating n-dimensional integrals numerically⁴⁻⁶⁾: (1) application of a product rule, either of Newton-Cotes or Gauss type, (2) n-dimensional quadrature formulas based on construction of n-dimensional polynomials, (3) Monte-Carlo, or number theoretic techniques. Monte-Carlo techniques have been applied, with success, to nuclear reaction calculations by Bayman and Feng⁷⁾, and a product Gauss rule appears to be the approach of Ref. ⁸⁾. In the present study approach (2) is taken because the perfectly symmetric two dimensional quadrature formulas of Rabinowitz and Richter⁹⁾ have fewer points than the product Gauss rules of equivalent degree and because such quadratures, based on polynomial interpolation¹⁰⁾, will display rapid convergence with increasing degree (or decreasing area), i.e. with increasing number of grid points.

The algorithm programmed in KUNDRY consists of defining squares of side length

S_c and packing the area to be integrated with these squares. Then squares of side length S_a and S_b are defined so that $S_a = 2S_b = 4S_c$ and each complete group of four squares is replaced by the next square in the hierarchy. The 48, and 28 point formulas of Ref.⁹⁾ are applied to the squares S_a^2 and S_b^2 , respectively, and the 12 point formula of Tyler¹⁰⁾ to the square S_c^2 . These formulas have fewer grid points than the product Gauss rules of equivalent degree which require 64, 36 and 16 points, respectively.

The most important feature of the problem is determination of a criterion for truncation of the two dimensional space in the radial coordinates r_{aA}, r_{bB} (the arguments of the DW radial wave functions), over which the double integral is performed. Assuming the bound state radial wave functions to be negligible when they have an asymptotic magnitude smaller than some number ϵ and defining the corresponding radii to be r_{xA}^c and r_{bX}^c , two estimates for the truncation radius in the first DW coordinate may be derived: $r_{aA}^c = r_{xA}^c / \{\alpha(1-\gamma)\}$, and $r_{aA}^c = r_{bX}^c / \{\alpha(1-\delta)\}$, where $\alpha = m_a m_b / \{m_x (m_A + m_a)\}$, $\gamma = m_b / m_a$, $\delta = m_A / m_B$, and m_i is the mass of particle i . For the range Δr_{bB} of the second variable r_{bB} about the value $r_{bB} = r_{aA}$, two estimates can also be made $\Delta r_{bB} = r_{xA}^c / (\alpha \gamma)$ and $\Delta r_{bB} = r_{bX}^c / \alpha$, i.e. $r_{bB} = r_{aA} \pm \Delta r_{bB}$ with only positive values of r_{bB} allowed. The smaller of these two estimates is used in each case. These estimates are conservative and their suitability has been confirmed by computation. Since both ϵ and the side length S_c of the smallest square are input parameters, truncation effects arising from either the physical assumptions (truncation of the bound state wave functions) or the numerical assumptions (size of the area S_c^2) may be explicitly tested by computation.

The FR DWBA code KUNDRY was applied to the study of the proton transfer reaction $^{11}_B(^{16}_O, ^{15}_N)^{12}_C$ and the complementary alpha cluster transfer reaction $^{11}_B(^{16}_O, ^{12}_C)^{15}_N$ measured at $\pi-\theta$ degrees. Previous studies^{11,12)} have analysed some of

the available data ^{13,14)} in the FR DWBA but did not investigate the effects of coherent interference of the two reactions. In a transfer reaction $A(a,b)B$ the DWBA transition matrix amplitude at θ degrees (cm) is $T_{ab}^1(M_A, M_a, M_b, M_B, \theta)$, where M_i is the magnetic quantum number associated with the spin projection of particle i . In the FR DWBA this amplitude contains a summation over the allowed values of the transferred orbital angular momentum ℓ_t . For the complementary transfer reaction $A(a,B)b$ at $\pi-\theta$ degrees (cm), the DWBA amplitude is $T_{aB}^1(M_A, M_a, M_b, M_B, \pi-\theta)$; both FR DWBA amplitudes contain a product of two spectroscopic amplitudes ¹⁵⁾, specific to the transferred particle or cluster x in the initial and final bound state configuration. Furthermore, where more than one type of configuration or cluster is allowed, either (or both) of the DWBA transition matrix amplitudes is a sum over such clusters $T_{ab} = \sum_i T_{ab}^i$ and/or $T_{aB} = \sum_k T_{aB}^k$; an example of the latter is the alpha cluster bound to ^{11}B to form ^{15}N in the 6.3 MeV state, where both 3s and 2d clusters are allowed ¹⁶⁾. The cross section for the coherent interference of the two reactions is the cross section formed from the amplitude $T(M_A, M_a, M_b, M_B, \theta) = T_{ab}(\theta) + T_{aB}(\pi-\theta)$ ¹⁷⁾.

The transitions studied here were for both proton and alpha cluster transfer induced by ^{16}O on ^{11}B for (i) the 6.324 MeV state to ground state, and (ii) g.s. to 4.44 MeV state, and (iii) g.s. to g.s., respectively in ^{15}N and ^{12}C . The first transition was studied at $E^{lab}(^{16}O) = 32.5, 30$ and 27 MeV, the second at 27 MeV and the third at 30 MeV bombarding energy. The proton bound state configurations were the same as those of Ref. ¹³⁾, and the bound state alpha particle was assumed to have the p-shell cluster configurations of Kurath ¹⁶⁾. The bound state parameters for the proton were: radius $r_0 = 1.25$ fm, diffuseness 0.65 fm, spin orbit strength $12 \text{ MeV} \cdot \text{fm}^2$; for the alpha cluster the radius (r_0) and diffuseness parameters were the same as those for the proton. The radius of the Woods-Saxon potential, binding the transferred particle x , was taken as $r_0 A_i^{1/3}$, where A_i is the mass number of particle i to which x is bound. The depth of the bin-

ding potential was adjusted to give the correct separation energy for particle x . The truncation radii were calculated from the assumption that $\epsilon = 0.0001$ and are given in Table I. The elastic scattering of ^{16}O on ^{11}B has been discussed in Ref.¹⁸⁾ and that of N on C in Ref.¹⁹⁾. The 32.5 and 27 MeV parameter sets of Ref.¹⁸⁾ are labelled here as B0321 and B0271. Two additional parameter sets yielding somewhat improved fits, especially for the 32.5 MeV data, were found with the optical model search code SOPHIE¹⁹⁾ 32.5 MeV (B0322), $r_v = 1.078$ fm, $a_v = 0.5660$ fm, $V = 70.42$ MeV, $r_w = 0.9810$ fm, $a_w = 0.1509$ fm, $W = 77.86$ MeV; 27 MeV (B0272), $r_v = 1.048$ fm, $a_v = 0.6264$ fm, $V = 75.34$ MeV, $r_w = 1.003$ fm, $a_w = 0.1269$ fm, $W = 71.49$ MeV, where the definition of the potential is the same as that of Ref.¹⁸⁾. In the optical model the elastic scattering $A(a, a)A$ of any two systems for the same values of η (Sommerfeld parameter), k (asymptotic wave number) and $R(=A_a^{1/3} + A_A^{1/3})$ are the same. It therefore suffices to select potentials from Table I of Ref.¹⁹⁾ for the b - B channel in the reactions considered here. Since the effect of changes in η and k act on the elastic scattering angular distributions in an opposite sense¹⁹⁾, where either parameter (η or k) in Table I of Ref.¹⁹⁾ is smaller (larger) than that required here, the other is chosen to be larger (smaller) by a similar percentage. The choice of OM parameters for the respective channels together with the values of η and k are given in Table I, the corresponding fits to the transfer reaction data as calculated by KUNDRY are shown in Fig. 1. The unbroken curve corresponds to the coherent sum of the two reactions, the dashed curves are proton transfer (at forward angles) and alpha cluster transfer (at backward angles). The bottom set of curves for the transition from the 6.3 MeV state in ^{15}N to the g.s. in ^{12}C at 27 MeV are calculated with the OM parameters of Ref.¹³⁾ for both channels: these parameters fitted the elastic $^{16}\text{O} + ^{11}\text{B}$ data only at forward angles. All the calculated alpha cluster transfer cross sections contain the corresponding spectroscopic amplitudes of Ref.¹⁶⁾

together with the centre of mass correction factors of eq. (9) of Ref.¹⁶⁾; no other normalization factors are introduced. The proton transfer cross section is normalized to the data at forward angles; the factors by which the respective theoretical cross sections of Fig. 1 have been multiplied are (in order from the top): 1.367, 0.5252, 0.3198, 0.2919, 1.147, 2.230. Given the spectroscopic amplitudes (Ref.²⁰⁾) for $^{11}\text{B} + \text{p} = ^{12}\text{C}$ (g.s.) and ^{12}C (4.4 MeV) respectively as -2.387 and -1.048, the present analysis yields for ^{15}N (g.s.) + p: $-0.4821 \pm 7\%$ and ^{15}N (6.3 MeV) + p: $-0.3634 \pm 35\%$. These estimates ignore discrepancies between Ref.¹³⁾ and Ref.¹⁴⁾ in normalization of the data for the transitions to the excited states. It is seen from Fig. 1 that the OM parameterization of Ref.¹³⁾ provides an inadequate description of these reactions. Calculations for the other two transitions with these parameters (not shown) have forward peaked proton transfer cross sections but underestimate the cross section for alpha cluster transfer by two orders of magnitude: no such discrepancy occurs if OM parameters of the type discussed in Refs.^{18,19)} are used for the analysis of these transfer reactions.

Two conclusions follow from this study. The first is that substantial gains in computational speed, without a corresponding loss in accuracy, result when recent developments in numerical analysis are applied to a problem of nuclear reaction theory. A comparison with the code LOLA⁸⁾, which showed KUNDRY to be typically 4 times faster in execution time, supports this conclusion. The second conclusion is that when two transfer reactions $A(a,b)B$ and $A(a,B)b$ are present simultaneously with comparable cross section, the qualitative features of their interference is successfully predicted by the FR DWBA, if such interference is taken to be coherent.

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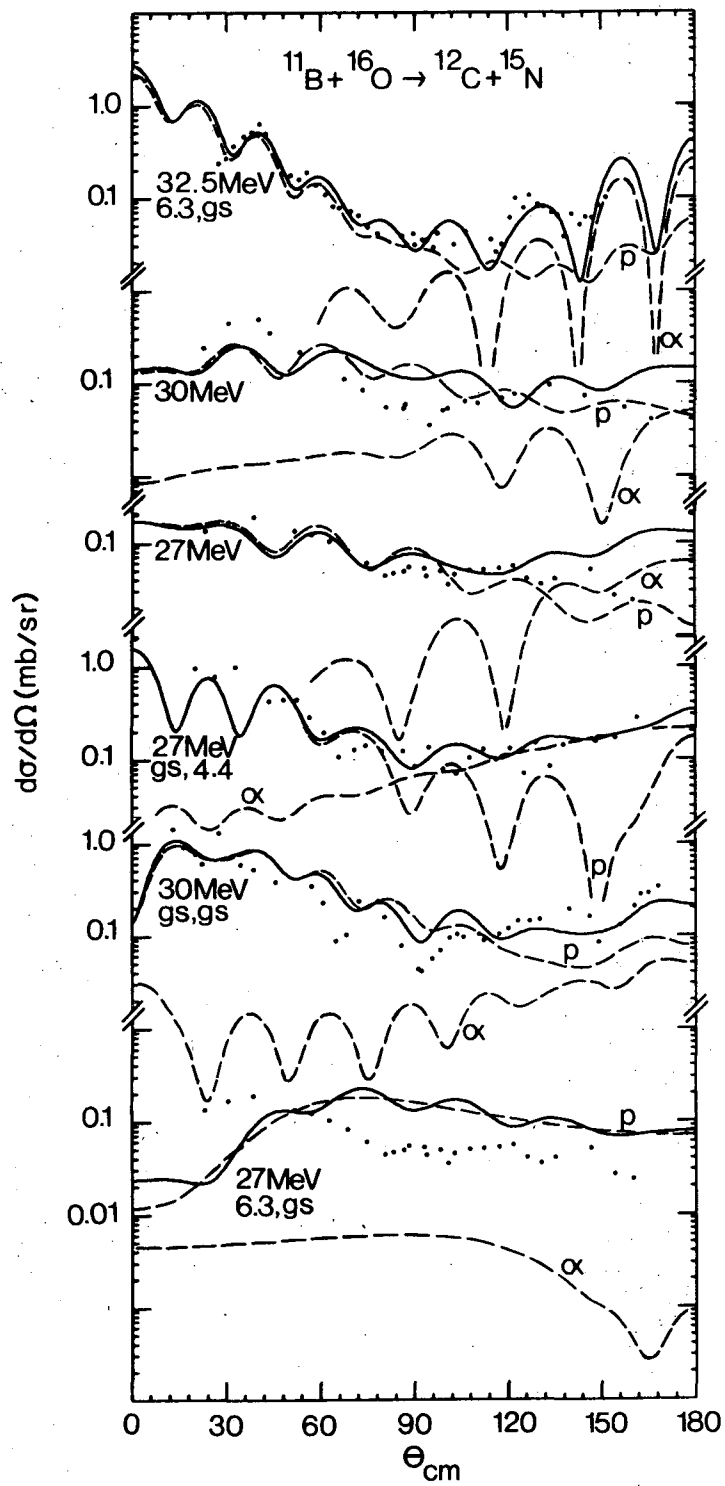
TABLE I. Proton and alpha stripping for ^{16}O on ^{11}B

E_{aA} (MeV)	^{15}N (MeV)	^{12}C (MeV)	η_{aA}	k_{aA} (fm^{-1})	a+A OM ^c	η_{bB}	k_{bB} (fm^{-1})	b+B OM ^c	r_{xA}^{c} (fm)	r_{bX}^{c} (fm)	r_{aA}^{c} (fm)	Δr_{bB} (fm)	l_{t}
32.5	6.324	g.s.	4.419	2.033	B0321	5.207	1.852	C3N1	10.4/10.1	10.0/ 9.5	17.2/16.1	1.42/4.28	0,1,2/0 or 2 ^c
30.0			4.599	1.953	B0322	5.473	1.762	C2N4					
27.0			4.848	1.853	B0272	5.853	1.648	C3N1					
27.0	g.s.	4.44			B0271	5.296	1.821	C3N1	11.4/ 8.4	11.2/ 8.2	19.1/13.9	1.59/3.69	0,1 /0,1,2,3,4
30.0	g.s.	g.s.	4.599	1.953	B0271	4.261	2.263	C2N4	10.4/ 8.4	11.2/ 9.5	19.2/15.2	1.57/4.28	1,2 /2

^a Lab energy ^b Excitation ^c See text ^d For proton and alpha, respectively

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Fig. 1 Exact finite range DWBA calculations with KUNDRY for stripping of protons and alpha clusters (dashed curves denoted by p and α , respectively), and for the coherent sum of the two reactions (unbroken curves). Lab energies of ^{16}O and excitation energies of ^{15}N and ^{12}C , respectively, are as shown. The bottom set of curves are for the parameters of table 2 of ref¹³). The data are those of ref¹⁴); error bars are not shown.



XBL 7511-9395

Fig. 1

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