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ABSTRACT

We employ the hybrid soliton model of the nucleon consisting of a topological meson field and deeply bound quarks to investigate the behavior of the quarks in soliton matter as a function of density. We investigate a particular possible ground state by placing the solitons on a spatial lattice. The model suggests the transition of matter from a color insulator to a color conductor above a critical density of a few times normal nuclear density.

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A great deal of interest has focused recently on solitons as representing non-perturbative solutions of QCD for baryons.[1] A number of authors have shown that at the 30% level, solitons resemble nucleons.[2] What we find particularly appealing in this development is that, having a Lagrangian that describes the internal structure of the nucleon (soliton), one can investigate interesting questions concerning how the internal structure changes when solitons are assembled to form dense matter, and how the properties of matter correspondingly change.

Several of the more interesting questions concern the quark behavior in normal and in dense matter, such as the anomalous muon scattering on nuclei as compared to nucleons (EMC effect)[3], and the onset of deconfinement. For these purposes the Skyrmion[4], which has no quarks, is not interesting. Rather, we would like to have a soliton with quarks that are confined, but not through the mechanism of an impervious bag. In the absence of a known soliton solution possessing true confinement, we opt for a model in which the quarks are deeply bound. The hybrid soliton model fills this requirement.[5,6]

In this note we focus on the behavior of the quark eigenvalues in compressed matter. At the densities that we have in mind, it is not unreasonable to organize the solitons into a crystal lattice because of the short range repulsion. In fact the problem would be exceedingly difficult to solve otherwise. For the soliton, we employ a hybrid model consisting of quarks that are coupled to a topological configuration of scalar and pi meson fields.[5,6] Each soliton has a full Dirac sea of quarks and three which fill a deeply bound color degenerate valence orbital. As the density of matter is increased, we find that the quark eigenvalues shift in response to their neighbors. The valence orbital of each soliton in the crystal contributes to a band of triply degenerate levels, and the band is therefore fully occupied. For low to moderate densities we may say that soliton matter is a color insulator. However, above a certain density, the valence level rises in energy, and the top of the band intersects the lowest eigenvalue of the continuum. At this density, matter ceases to be a color insulator, and becomes a color and electric semi-conductor or conductor.

We now fill in the details of how this result is arrived at.

The Lagrangian of the chiral sigma model[7] is,

$$\mathcal{L}_{\sigma} = \bar{q}(x)\ [i\gamma_{\mu}\partial^{\mu} - g(\sigma(x) + i\gamma_{5}\underset{\sim}{\tau}\cdot\underset{\sim}{\pi}(x))]\ q(x)$$

$$+ \frac{1}{2}\,[\partial_{\mu}\sigma\ \partial^{\mu}\sigma + \partial_{\mu}\underset{\sim}{\pi}\cdot\partial^{\mu}\underset{\sim}{\pi}] \qquad (1)$$

$$+ \frac{1}{4}\,\lambda\ (\sigma^{2} + \underset{\sim}{\pi}^{2} - f_{\pi}^{2}\ )^{2}$$

where q(r) is the quark spinor and σ(r) and π(r) are the scalar and pion fields, which are treated in the mean field approximation. For simplicity, we take λ to be large (large sigma mass), which confines $\sigma^2 + \pi^2$ to the chiral circle. For our purpose, this is an inconsequential restriction. In addition to the usual uniform field solution, the Euler-Lagrange equations of (1) have a lower energy solution for a range of coupling constants which is a static hedgehog soliton possessing a conserved topological quantum number.
This solution has the form,

$$\sigma(\underset{\sim}{r}) = f_\pi \cos\theta(r) \tag{2}$$

$$\underset{\sim}{\pi}(\underset{\sim}{r}) = \hat{\underset{\sim}{r}}\, f_\pi \sin\theta(r)$$

In the presence of this meson field configuration, the Dirac equation,

$$[i\gamma_\mu \partial^\mu - m(\cos\theta(r) + i\gamma_5\, \underset{\sim}{\tau}\cdot\hat{\underset{\sim}{r}} \sin\theta(r))]\, q(\underset{\sim}{r}) = 0 \tag{3}$$

with $m = gf_\pi$, the constituent quark mass, has a lowest energy solution of the form,

$$q(\underset{\sim}{r}) = \begin{pmatrix} F(r) \\ i\underset{\sim}{\sigma}\cdot\hat{\underset{\sim}{r}}\; G(r) \end{pmatrix} |\nu\rangle \tag{4}$$

where $|\nu\rangle$ is a spinor satisfying

$$(\underset{\sim}{\sigma}+\underset{\sim}{\tau})\ |\nu\rangle = 0 \tag{5}$$

Following the usual convention, we call the state described by (4) a positive parity state, after the transformation of the large component.

The differential equations for F and G are,

$$\begin{aligned} -F' + mF \sin\theta(r) &= (E + m\cos\theta(r))\, G \\ G' + \left(\frac{2}{r} + m\sin\theta(r)\right)G &= (E - m\cos\theta(r))\, F \end{aligned} \tag{6}$$

In addition there is the opposite parity state whose equations can be obtained from the above by setting $m \to -m$, where $m=gf_\pi$ is the constituent quark mass. (This peculiar transformation follows by observing that $\gamma_5 q(x)$ generates a quark spinor of opposite parity to $q(x)$, while γ_5 changes the relative sign of the momentum term in (3) with respect to the other terms, which is equivalent to changing the sign of m.)

The topological charge of the Skyrmion[4], and of the meson field configuration of the hybrid model is given, within a sign by,

$$B = \left\{\theta(r) - \frac{1}{2}\sin 2\theta(r)\right\}_{r=0}^{r=\infty} \tag{7}$$

The baryon number and its sign for the hybrid model including the quarks can be established unambiguously by examining the limit in which the soliton radius $R \to \infty$. In this case the method of Goldstone and Wilczek can be employed to calculate the baryon charge.[8] For meson fields satisfying boundary conditions $\theta(0) = -\pi$, and $\theta(\infty) = 0$, one of the quark levels, which for small R lies at positive energy, migrates to the negative energy sea as $R \to \infty$.[5b] This configuration, with a fully occupied negative energy sea, has a topological quantum number which will be used to fix the sign of B as +1.[9] So long as these levels are occupied for any finite R, the baryon charge remains unchanged. The topmost such level is referred to in [5] and below as a valence level, and the baryon charge of the hybrid soliton is therefore +1.

For soliton matter arranged as a crystal, the hedgehog meson configurations are centered at lattice points thus generating a periodic field in which the quarks move. The Bloch theorem requires that the quark spinor obey,

$$q_{\underset{\sim}{k}}(\underset{\sim}{r}) = e^{i\underset{\sim}{k}\cdot\underset{\sim}{r}}\, u_{\underset{\sim}{k}}(\underset{\sim}{r}) \tag{8}$$

where $\underset{\sim}{k}$ is the crystal momentum, and $u_{\underset{\sim}{k}}(r)$ is a periodic function whose period is that of the lattice. We employ the Wigner-Seitz approximation, which replaces the actual problem by a spherically symmetric one and solves for $q_k(r)$, (k=0), the ground state of the band. For convenience we shall make an ansatz for the behavior of the chiral angle, similar in spirit to Kahana et al.[5] Denote by R_s the equilibrium radius of the isolated soliton. When the lattice spacing, 2R, between solitons exceeds the diameter of a soliton,

$$\theta(r) = \begin{cases} \pi(r/R_s - 1), & r < R_s \\ 0, & R_s \leq r \leq R \end{cases}, \quad R > R_s \tag{9a}$$

Otherwise,

$$\theta(r) = \pi(r/R - 1), \qquad ,R \leq R_s \tag{9b}$$

For the ground state of the band, the periodicity of the Schroedinger wave function required by the Bloch theorem imposes the condition that it have zero slope at the Wigner-Seitz boundary. This requirement translates in the case of the Dirac equation to

$$F'(R) = 0, \qquad G(R) = 0 \tag{10}$$

i.e., the large component has zero slope and the small component, zero value at the boundary, as follows from (6) and (9). At the origin, it is evident from (6) that G(0)=0. This in turn requires that F'(0)=0. Therefor the boundary conditions

$$G(0) = G(R) = 0 \tag{11}$$

insure that the Bloch theorem is satisfied, i.e., that both F and G are periodic.

We solve the coupled Dirac equations (6) with the boundary condition in the Wigner-Seitz cell (11), by numerical integration. The eigenvalues for the $0^{+,-}$ states are shown in fig. 1. The lower of these two belongs to the filled sea of quarks, and the other is the valence orbital. This orbital as would be expected from the Schroedinger theory, is lowered in energy from the isolated soliton eigenvalue over a certain range of crystal spacings, and then it rises due to the compression of the solitons by their neighbors. The lower eigenvalue is increased for all crystal spacings, which behavior can be traced to the small component of the Dirac spinor when the eigenvalue is close to -m.

The Wigner-Seitz approximation allows us to calculate the eigenvalue of the ground state of each band (k=0). Denote such an eigenvalue for a particular band by ϵ_0. We need to estimate the band width. In the Schroedinger theory this is done by calculating the energy expectation value of the state, k, which is $\epsilon_0 + k^2/2m$. One could do the same in the Dirac case. Alternately we are motivated by the tight binding approximation of solid state physics.[10] We calculate the eigenvalue for isolated solitons, i.e., with exponentially decaying boundary condition, but with chiral field given by (9). The band width is then estimated as twice the difference between this energy and that computed with the crystal boundary condition, because the band should be spread symmetrically about the unperturbed case. For the valence levels the Wigner-Seitz approximation locates the bottom of the band. However, for the levels belonging to the sea, it locates the top of the band, just as the sea eigenvalues in the free case are $-\sqrt{k^2 + m^2}$.

The band structure is shown in fig. 1 by the shaded areas and the solid lines are the Wigner-Seitz eigenvalues (k=0). Several points deserve comment. We see a lowering of the valence quark eigenvalue by about 16 MeV at a lattice spacing 2R = 2.45 fm. For smaller spacing the level rises steeply and the top of the band intersects the continuum at a spacing of about 1 fm, which corresponds to a density of 7 times normal nuclear density ($.15/fm^3$). In this model at such a density matter ceases to be a color insulator and becomes increasingly color conducting as the density is further increased.

The above behavior is suggestive of quark deconfinement, although in this model the quarks are not truly confined but only deeply bound in the isolated state. The wave functions of the sea and valence orbitals are shown for a typical lattice spacing in fig. 2, illustrating their periodcity in the crystal. In fig. 3, we compare the quark distribution in soliton matter of several densities, illustrating the increasing concentration at the cell boundary for increasing density. For the pion decay constant we employ the experimental value f_π = 93 MeV, and a coupling constant g = 7.55, which yields a soliton mass M = 966 MeV, and an equilibrium R = 1.22 fm. Only the valence quark level and meson fields are taken into account in calculating M and R, thus neglecting shifts in the negative energy sea.

During the course of this work another paper has been published which investigated Skyrmion matter in a crystal lattice approximation.[11] This model however does not possess quarks. Nevertheless, as these authors point out, the asymptotic behavior of the equation of state is such that the energy density behaves like $n^{4/3}$, just as a relativistic gas of Fermions. This is also the behavior in the model studied here, since the quarks pass into the continuum states of dense matter. We point out however, that for the Skyrmion, this behavior is an artifact of the form chosen to stabilize the Skyrmion, namely a term of fourth order in derivatives and hence in $k \sim n^{1/3}$. This is the lowest order stabilizing term, and can be viewed as the first in a series, the last of which will dominate the momentum dependance of the equation of state at high density.

In summary, we have investigated the behavior of quarks in soliton matter, using the hybrid model consisting of a topological meson field and deeply bound quarks. To organize the calculation, we placed the solitons in a crystal lattice. At a certain critical density, the top of the valence quark band becomes degenerate with the Fermi sea, meaning that the quarks in those states are no longer bound to a lattice site. At still higher densities, additional levels of the band rise into the continuum, suggesting that color conductivity is a gradual function of compression.

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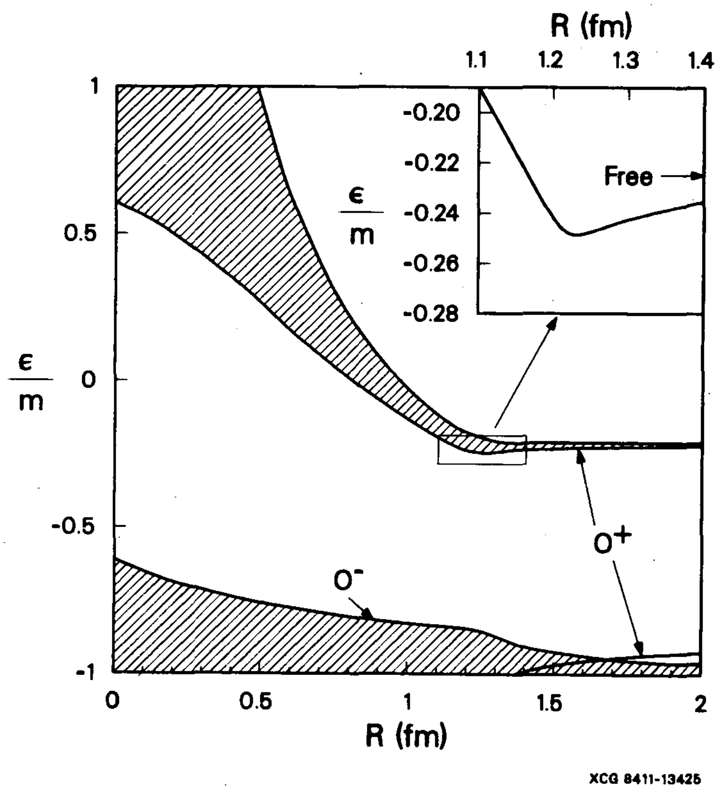
FIGURE CAPTIONS

Figure 1. Eigenvalues of the valance (0+) and sea (0-) orbitals of quarks in soliton matter as a function of Wigner-Seitz cell radius, R. The band of levels that develops as the spacing decreases is shown by the shaded region. In the upper right corner, an enlargement of the region indicated is shown. The eigenvalue of a free soliton is indicated by the arrow.

Figure 2. Upper (F) and lower (G) components of the Dirac spinor are shown for the valence and sea orbitals for a cell radius of 0.6 fm.

Figure 3. Probability distribution for the valence quarks for several cell radii, illustrating the increasing concentration of the quarks at the cell boundary as the compression of soliton matter increases.



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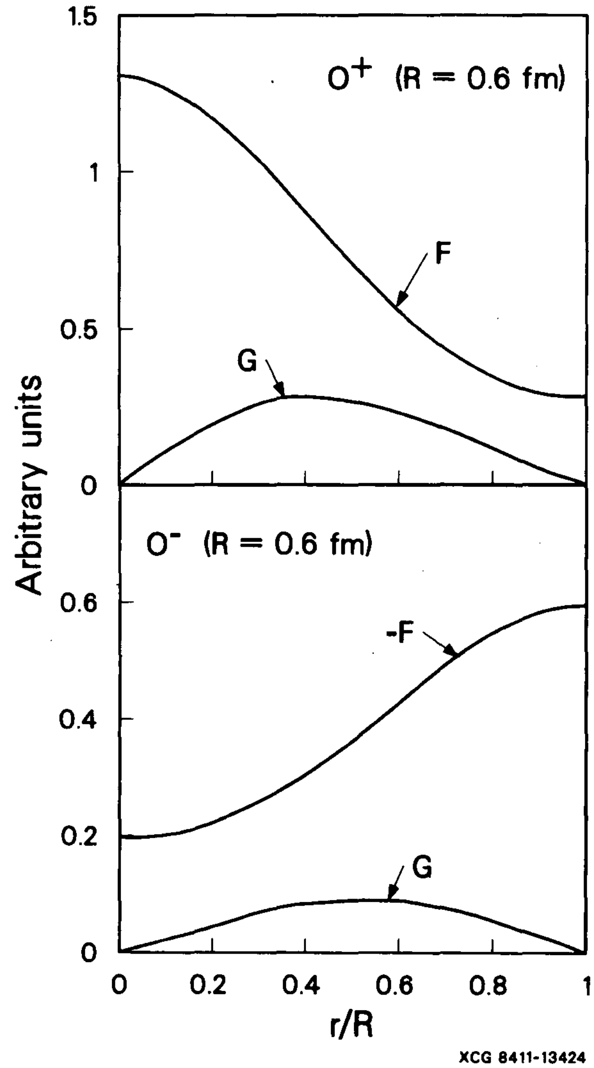

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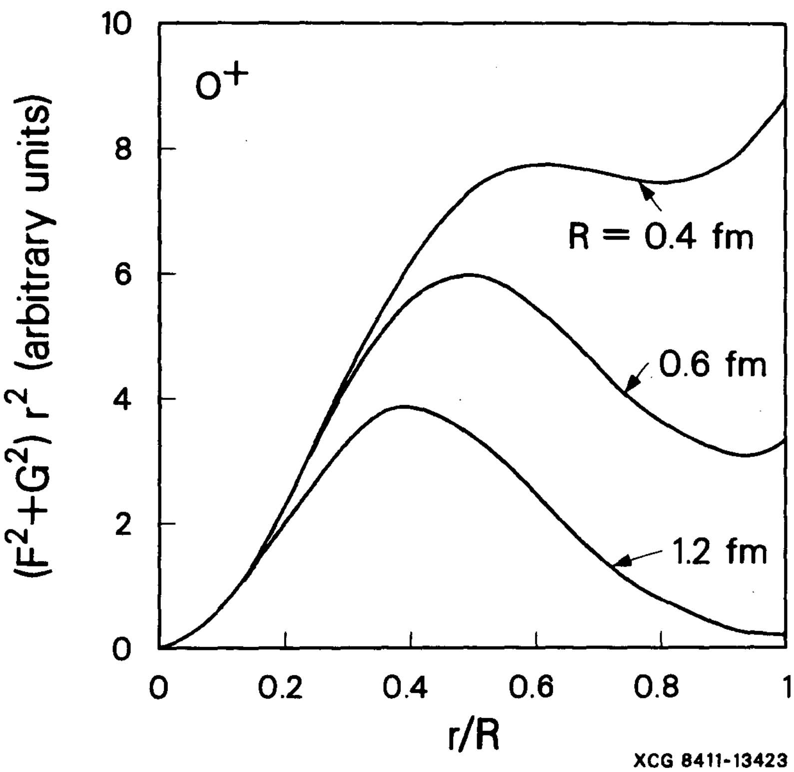

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