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Crossing symmetry gives relations between the derivatives of the S- and P-wave amplitudes of the pion-pion system at the symmetry point;[1, 2] these relations are exact if we consider all higher partial waves to be small. At this symmetry point, we have $\nu = \nu_0 = -2/3$ (ν being the square of the c.m. momentum of a pion),[3] the two S amplitudes are given in terms of the pion-pion coupling constant λ, and the first derivatives of the S amplitudes are given by the value of the P amplitude. In addition, there is a single relation connecting the second derivatives of the S waves to the first P-wave derivative. We assume a resonance in the P wave. A two parameter form for this resonance has been given by Frazer and Fulco, the parameters being ν_R, the position, and Γ, the width of the resonance.[4] Such a two-parameter form should be sufficient, we believe, to give a rough first approximation to the P amplitude and its first derivative at ν_0 if ν_R is small and the contribution from the left cut no larger than estimated by Chew and Mandelstam.[2] Recently Ball and Wong have given a four-parameter resonance form which includes a long-range repulsion in the P wave.[5] The strength of this repulsion is, however, an order of magnitude bigger than the estimates given by Chew and Mandelstam.[2] We therefore, consider at present only the two-parameter form and hence calculate at ν_0 the P amplitude and its first derivative in terms of ν_R and Γ. The above crossing relations then largely determine the S-wave amplitudes at low energies in terms of the three parameters, λ, ν_R and Γ. We wish to emphasize, however, that the method described here

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is general, whatever form the P wave may ultimately assume. It is also free from uncertainties such as the arbitrary cutoffs that had to be introduced in the previous P-dominant solutions.[2]

The crossing relations at ν_0 are as follows:[2]

$$\frac{1}{5}\, a_0 = \frac{1}{2}\, a_2 = -\lambda, \tag{1}$$

$$\frac{1}{2}\, a'_0 = -a'_2 = 3a_1, \tag{2}$$

and

$$a''_0 - \frac{5}{2}\, a''_2 = -12\, a'_1, \tag{3}$$

where a_0 and a_2 are the S amplitudes at ν_0 for the isotopic spin 0 and 2, respectively, and a_1 is the P amplitude. The primes indicate derivatives at ν_0.

If we indicate by $A_0{}^I(\nu)$ the S amplitudes at an energy ν for a given isotopic spin I (=0 or 2), we can write it in the familiar form[1]

$$A_0{}^I(\nu) = \frac{N_0{}^I(\nu)}{D_0{}^I(\nu)}\ , \tag{4}$$

where $N_0{}^I(\nu)$ and $D_0{}^I(\nu)$ are the numerator and the denominator functions, respectively. In the usual effective-range approximation in which we replace the left-hand cut by a pole,[2] we obtain

$$N_0{}^I(\nu) = a_I + (\nu - \nu_0)\ \frac{\omega_{SI} + \nu_0}{\omega_{SI} + \nu}\ B_I \tag{5}$$

and

$$D_0{}^I(\nu) = 1-(\nu - \nu_0)\,[K(-\nu, -\nu_0)\,a_I + (\omega_{SI} + \nu_0)\,K\,(\omega_{SI}, -\nu)\,B_I]. \tag{6}$$

where ω_{SI} gives the position of the pole, B_I is proportional to the residue, and K is a known function defined in reference 1.

The corresponding one-pole approximation for $\frac{A_1^1(\nu)}{\nu}$ -- the P amplitude (I = 1) at an energy ν-- was written in the two-parameter resonance form by Frazer and Fulco:[4]

$$\frac{A_1^1(\nu)}{\nu} = \frac{\Gamma}{\nu_R - \nu[1 - \Gamma\,\alpha(\nu)] - i\Gamma\left(\frac{\nu^3}{\nu+1}\right)^{1/2}} , \qquad (7)$$

where $\alpha(\nu)$ is a known function. Given Γ and ν_R, we obviously can calculate the values of a_1 and a_1' needed in Eqs. (2) and (3) above. We have five conditions embodied in the crossing relations (1)... (3) and six parameters to determine in our S-wave effective-range formulas: a_0, a_2, ω_{S0}, ω_{S2}, B_0, and B_2. To achieve a sixth condition, we assume that $\omega_{S0} = \omega_{S2}$ (= β say). Since only second and higher S-wave derivatives are influenced by this assumption, it seems fairly safe. Three different combinations of P-resonance parameters were investigated. Originally Frazer and Fulco proposed $\nu_R = 1.5$ and $\Gamma = 0.4$ as likely values, but recently Bowcock, Cottingham, and Lurié,[6] and Frautschi[7] have suggested that the position of the P resonance should be much higher to be consistent with pion-nucleon scattering. Their suggested values for (ν_R, Γ) are (4.6, 0.2) and (4.6, 0.4), respectively.

Following Chew and Mandelstam,[1] we allow only those λ values that do not give rise to zeros in $D_0^I(\nu)$ on the "nearby" portion of the left cut, and that do not have poles in the S wave in the region $-1 \leq \nu \leq 0$. The latter requirement eliminates large negative values of λ as corresponding to excessively strong attraction, while the former eliminates almost all positive values of λ if the "nearby" portion of the left cut is taken as $-10 \leq \nu \leq -1$.[1] The range of values we get is $-0.25 \leq \lambda \leq +0.04$--much narrower than that originally given by Chew and Mandelstam for S-dominant solutions.[1]

The curves for $[\nu/(\nu+1)]^{1/2} \cot \delta_0^I$ (δ_0^I is the S-wave phase shift for a given isotopic spin I) are given in Figs. 1 and 2 for the three different choices of (ν_R, Γ) and for various values of λ within the allowed range. It is evident that a large value of ν_R gives smaller S phase shifts for a given value of λ. Moreover, the interaction in the $I = 0$ state is attractive and much stronger than in the $I = 2$ state. For positive values of λ, we obtain a resonance in the $I = 0$ state as we approach $\lambda = +0.04$.

Knowing the τ decay into three pions, we can further restrict the S phase shifts.[8] These events show that even though the spectrum of an outgoing pion deviates from the purely statistical one, there are no peaks observed.[9] If the S $\pi\pi$ interaction were strong enough to produce near bound states or resonances, we believe peaks should be observed, as in the reaction, $p + p \rightarrow n + p + \pi^+$, where such a peak is quite striking and corresponds to the near bound state in the singlet (n, p) system.[10] Therefore very large S amplitudes seem to be ruled out, and a rough estimate indicates that the scattering lengths should not be much larger than unity. This estimate has the same order of magnitude as that given by Thomas and Holladay,[11] Khuri and Treiman,[12] and Sawyer and Wali.[13] However, we do not think that any quantitative conclusions can be drawn from τ decay (as these authors have attempted) by considering the problem in terms of two-body interactions only. We have here a case in which the range of interaction, the scattering lengths, and the wavelengths are all of the same order of magnitude, and to resolve such a three-body system according to two-body configurations may be an oversimplification. We are therefore not too concerned over our failure to achieve quantitative accord with the calculations made by the above authors.[11-13] If, however, we use τ decay to exclude large S-scattering lengths, we see that the P resonance, if it exists, probably does not occur at a ν value as low as 1.5. Recent experiments support such a conclusion.[14]

It should be noted that if the ω^0 particle with quantum numbers $J = 1$, $I = 0$ does exist,[15] then the reaction $\pi + \pi \to \omega^0 + \pi$ may compete with the elastic P-wave channel in the resonance region. The form of $\frac{A_1^1(\nu)}{\nu}$ and, therefore, of a_1 and a_1' will then have to be modified.

The author wishes to thank Professor Geoffrey F. Chew for suggesting this problem and for his advice.

FOOTNOTES

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2. G. F. Chew and S. Mandelstam, Lawrence Radiation Laboratory Report UCRL-9126, March 1960 (unpublished) to be published in Nuovo cimento. See also G. F. Chew, Proc. 1960 Annual International Conference on High Energy Physics, Rochester, 1960 (Interscience Publishing Co., New York, 1960). It is assumed that the reader is familiar with the principles discussed here and in reference 1.
3. We employ units with $\hbar = c = \mu = 1$ where μ is the pion mass.
4. W. R. Fraser and J. R. Fulco, Phys. Rev. Letters 2, 364 (1959) and Phys. Rev. 117, 1603 (1960).
5. J. S. Ball and D. Y. Wong, Phys. Rev. Letters 6, 29 (1961).
6. J. Bowcock, W. N. Cottingham, D. Lurié, Nuovo cimento 16, 918 (1960) and Phys. Rev. Letters 5, 386 (1960). These authors use the form

$$\frac{A_1^1(\nu)}{\nu} = \frac{(\nu+1)^{1/2}(\gamma/4)}{\nu_R - \nu - i(\nu+1)^{1/2}(\gamma/4)\,[\nu^3/(\nu+1)]^{1/2}} .$$

Our Γ is $(\nu_R+1)^{1/2}(\gamma/4)$ [≈ 0.2 with $\gamma = 0.376$ and $\nu_R = 4.6$].

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15. A. Abashian, N. E. Booth, and K. M. Crowe, Phys. Rev. Letters 5, 258 (1960) (references to earlier theoretical discussions are given here) and talk presented at the 1960 Conf. on Strong Interactions, University of California, Berkeley, December 1960.

Figure Legends

Fig. 1. Product of the cotangent of δ_0^0 and $-5\lambda[\nu(\nu+1)]^{1/2}$ for three different choices of (ν_R, Γ) and for $I = 0$ with (a) $\lambda = -0.20$, (b) $\lambda = -0.10$, (c) $\lambda = -0.05$, and (d) $\lambda = +0.01$.

Fig. 2. Product of the cotangent of δ_0^2 and $-2\lambda[\nu(\nu+1)]^{1/2}$ for three different choices of (ν_R, Γ) and for $I = 2$ with (a) $\lambda = -0.20$, (b) $\lambda = -0.10$, (c) $\lambda = -0.05$, and (d) $\lambda = +0.01$.

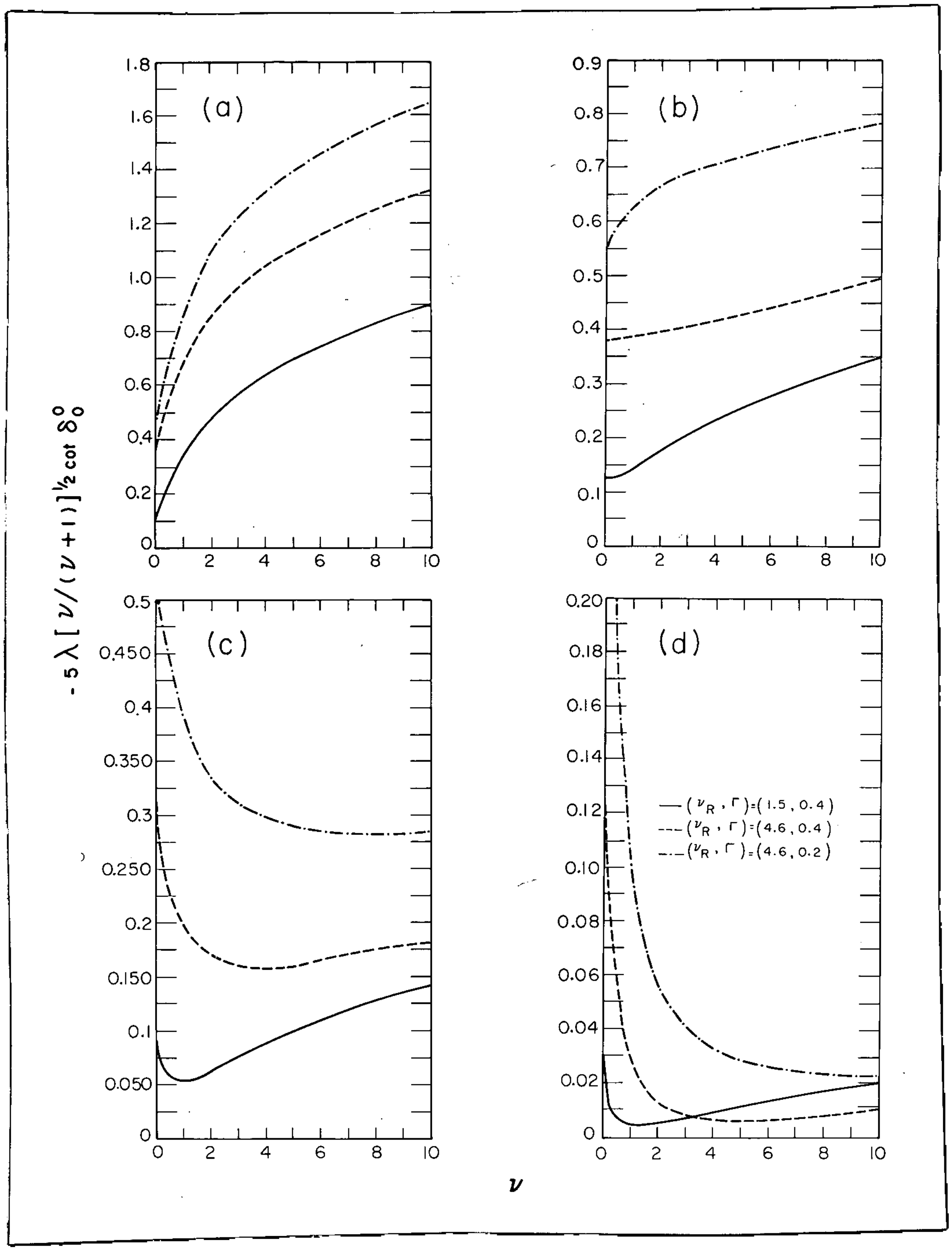
(a)
(b)
(c)
(d)
$-5\lambda[\nu/(\nu+1)]^{1/2}\cot\delta_0^0$
ν
— $(\nu_R, \Gamma) = (1.5, 0.4)$
--- $(\nu_R, \Gamma) = (4.6, 0.4)$
-·- $(\nu_R, \Gamma) = (4.6, 0.2)$

Fig. 1

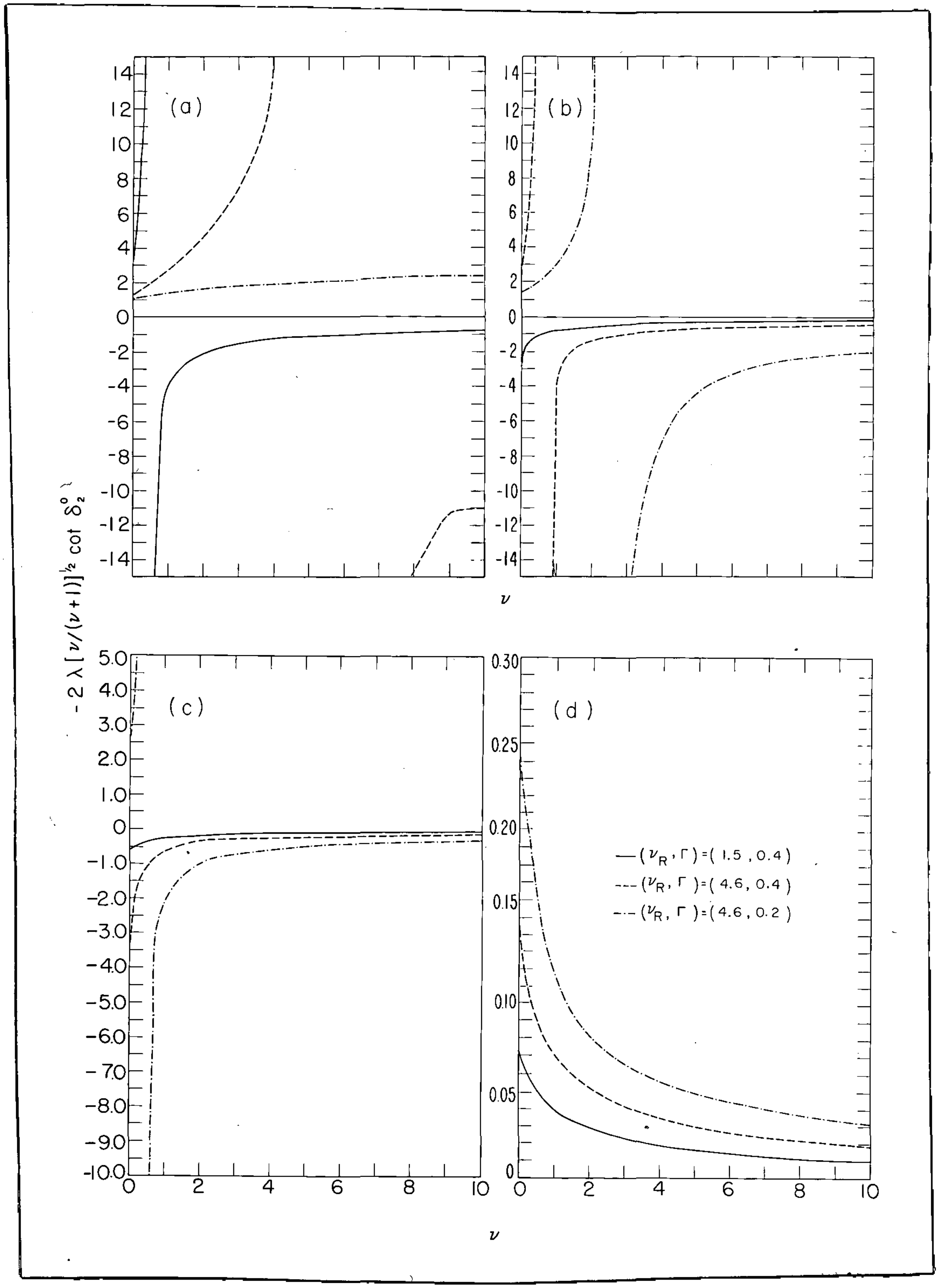
(a)
(b)
(c)
(d)
$-2\lambda[\nu/(\nu+1)]^{1/2} \cot \delta_2^0$
ν
— $(\nu_R, \Gamma) = (1.5, 0.4)$
--- $(\nu_R, \Gamma) = (4.6, 0.4)$
-·- $(\nu_R, \Gamma) = (4.6, 0.2)$

Fig. 2