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Anomalous $U(1)$ and low-energy physics: the power of D-flatness and holomorphy*

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Abstract

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In models with an anomalous abelian symmetry broken at a very large scale, we study which requirements to impose on the anomalous charges in order to prevent standard model fields from acquiring large vacuum expectation values. The use of holomorphic invariants to study D-flat directions for the anomalous symmetry, proves to be a very powerful tool. We find that in order to forbid unphysical vacuum configurations at that scale, the superpotential must contain many interaction terms, including the usual Yukawa terms. Our analysis suggests that the anomalous charge of the μ-term is zero. It is remarkable that, together with the seesaw mechanism, and mass hierarchies, this implies a natural conservation of R-parity.

1 Introduction

Many effective superstring models have an anomalous Abelian gauge symmetry. Its anomalies are cancelled through the four-dimensional field theory remnant [1] of the Green-Schwarz mechanism [2], and it is naturally broken at a scale ξ, a small computable factor times the Planck scale.

This anomalous symmetry may play a role in various domains of relevance for the low energy world [3]-[9]. For example, if one considers an Abelian family symmetry [10, 11] to try and explain the mass hierarchies observed in the quark and charged lepton sector, this symmetry is most probably anomalous [12, 5]. By naturally providing a small expansion parameter, ξ/M_{Pl}, this symmetry is suited to the analysis of lepton and quark mass hierarchies [13]. The same symmetry might also trigger supersymmetry breaking [14, 15, 16]. Its role has also been advocated in predicting neutrino mixing patterns [17, 18, 19], and as possible solutions to the doublet-triplet splitting in grand unified models [20, 21].

In this context, it is especially important to characterize the flat directions along which the breaking of this anomalous symmetry occurs. In this letter, we wish to show that there are simple and powerful methods to undertake this task. Since ξ, the scale of breaking of this $U(1)_X$ symmetry is close to the Planck scale, there are severe constraints from phenomenology: Supersymmetry and the gauge symmetries of the standard model must remain unbroken. This means that all D and F terms must vanish, and that no field of the standard model can get a vacuum expectation value (vev) along these directions, since ξ is much larger than the electroweak scale. This remains true even if the anomalous symmetry plays a role in supersymmetry breaking. Indeed, in the work of Ref. [14] where such a role is emphasized, it is found that $< D_X > \ll \xi^2$ which indicates that the analysis which follows is perfectly relevant to this case.

In a classic paper [22], the correspondance between D-flat directions (*i.e.* field configurations for which D-terms vanish) and extrema of holomorphic invariant polynomials was established. These ideas were recently applied to the anomalous $U(1)$ by Dudas *et al* [23]. Following these authors, we also base our analysis on the construction of holomorphic invariants: invariants of the standard model [24], and then, in order to discuss D_X-flatness (and F-flatness), invariants under the anomalous $U(1)_X$ symmetry.

The requirement that no standard model fields charged under the anomalous symmetry acquire vev's of order ξ give powerful constraints on their anomalous charges, which in turn yield valuable information on several outstanding problems of low energy supersymmetry: the mu-problem, the origin of R-parity [25] or the order of magnitude of its violations (and more generally of baryon and lepton number), and the neutrino mass generation through the seesaw mechanism [26]. About the latter, we will see that right-handed neutrino superfields (in our discussion those are standard model singlets which do not get a vev at the scale ξ) play an important role in the discussion of the $U(1)_X$-breaking directions.

We will start in the next Section by describing the method used and illustrating it on some examples, with only one anomalous $U(1)_X$ broken at the scale ξ. We then turn in Section 3 to a discussion of the possible solutions for the mu-term. In Section 4, we show how these considerations naturally suggest R-parity conservation. Finally in Section 5, we discuss the relevance of this to more complicated situations, such as complete models of mass hierarchies.

2 The power of D-flatness and holomorphy

We study in what follows the directions along which occurs the breaking of an anomalous $U(1)_X$ symmetry. The $U(1)_X$ D-term D_X is of the form:

$$D_X \sim \sum_i X_i |\Phi_i|^2 - \xi^2 \tag{1}$$

where X_i is the X-charge of a generic scalar field Φ_i and ξ is the anomalous Fayet-Iliopoulos term. We consider three types of fields:

- fields which acquire a vacuum expectation value of order ξ when the $U(1)_X$ symmetry is broken: we denote them generically by θ.
- fields charged under $SU(3) \times SU(2) \times U(1)$, typically the fields found in the MSSM; these fields should not acquire vacuum expectation values. They appear in invariants which form the building blocks used to construct terms in the superpotential. Typically for the superfields in the MSSM:

$$\begin{array}{ll} H_d H_u \ , & \mathbf{Q}_i \overline{\mathbf{d}}_k H_d \ , L_i H_d \bar{e}_k \ , \mathbf{Q}_i \overline{\mathbf{u}}_k H_u \ , \\ L_i H_u \ , & \mathbf{Q}_i \overline{\mathbf{d}}_k L_j \ , L_i L_j \bar{e}_k \ , \overline{\mathbf{u}}_i \overline{\mathbf{d}}_j \overline{\mathbf{d}}_k \ , \end{array} \tag{2}$$

  where i, j, k are family indices. We do not list the higher oder invariants which can be found in the literature [24]. We will denote these invariants generically by S.
- scalar fields, singlet under the standard model gauge group, which do not receive vacuum expectation values of order ξ. These fields are natural candidates for the right-handed neutrinos and we will denote them by $\bar{N}$. Typically, for these fields to be interpreted as right-handed neutrinos, one needs terms in the superpotential to generate Majorana mass terms:

$$W_M \sim M \bar{N}^2 \left(\theta/M\right)^n \tag{3}$$

  and terms to generate Dirac mass terms for the neutrino:

$$W_D \sim S \bar{N} \left(\theta/M\right)^p \tag{4}$$

  where S is the invariant $S = LH_u$. The presence of both terms (3) and (4) is necessary to implement the seesaw mechanism [26].

Finally, we will denote by $< \Phi_1, \Phi_2, \cdots, \Phi_n >$ the direction in scalar field parameter space where the fields $\Phi_1, \Phi_2, \cdots, \Phi_n$ acquire a common vacuum expectation value of order ξ. Our basic requirement is to choose the X-charges and the superpotential so as to forbid all solutions to the vacuum equations except those corresponding to $< \theta_1, \theta_2, \cdots, \theta_m >$.

Since we work in the context of global supersymmetry unbroken at the scale ξ, directions in the scalar field parameter space will be determined by the conditions $D_X = 0$ and $F_i \equiv \partial W / \partial \Phi_i = 0$. For instance, the assumption of D_X-flatness ($D_X = 0$ in (1)) automatically takes care of the directions $< \Phi_1, \Phi_2, \cdots, \Phi_n >$ where $X_i < 0$ for $i \in \{1, \cdots, n\}$.

There is necessarily some gauge symmetry other than the anomalous $U(1)_X$, for example the symmetries of the standard model. D-flatness for these symmetries plays an important role for S invariants: it tends to align the fields present in S. Take for example $S = \Phi_1 \Phi_2$. Invariance under $U(1)_Y$ implies that the hypercharges of Φ_1 and Φ_2 are opposite: $Y_1 = -Y_2$. Then the corresponding D-term reads: $D_Y \sim Y_1(|\Phi_1|^2 - |\Phi_2|^2) + \cdots$ And $D_Y = 0$ implies $< \Phi_1 > = < \Phi_2 > \equiv v_S$. The contribution to D_X from these fields is $x_S |v_S|^2$, where x_S is the total X-charge of S. Hence a positive x_S will allow a vacuum with the flat direction $< \Phi_1 \Phi_2 >$.

On more general grounds, it has been shown [22] that there is a systematic classification of the solutions to vanishing D-terms using holomorphic invariant polynomials. We will assume in what follows that there are enough gauge symmetries to align the fields in each of the invariants S that we consider. We now proceed with different examples with an increasing number of fields.

Models with 2 fields.

Model with θ (X-charge x) and $\overline{N}$ (X-charge $x_{\overline{N}}$). With these fields the $U(1)_X$ D-term is:

$$D_X = x|\theta|^2 + x_{\overline{N}}|\overline{N}|^2 - \xi^2 \; . \tag{5}$$

There are three different flat directions to consider: the desired $< \theta >$, and $< \overline{N} >$, and $< \theta, \overline{N} >$ which we wish to avoid. The direction $< \theta >$ is favored by choosing $x > 0$, and $< \overline{N} >$ is forbidden if $x_{\overline{N}} < 0$. The third direction is allowed by $D_X = 0$. However since $x x_{\overline{N}} < 0$, we can form a holomorphic invariant involving $\overline{N}$ and θ.

The simplest possible invariant in the superpotential is $\overline{N}\theta^n$ but the corresponding F-terms forbid $< \theta >$ and $< \theta, \overline{N} >$. Thus we lose the possibility of a D_X-flat direction with $< \theta > \sim \xi$. Since all possible flat directions are lifted, supersymmetry is spontaneously broken.

We must therefore require the presence of an invariant $\overline{N}^p \theta^n$ with $p \geq 2$ and $n \neq 0 \; mod(p)$ to forbid only the direction $< \overline{N}, \theta >$. The case $p = 2$ corresponds precisely to a Majorana mass term for the right-handed neutrino $\overline{N}$, once θ is allowed a vev. In this case, $n = 2k + 1$ and there must be the following relation

between the X-charges:

$$\frac{x_{\overline{N}}}{x} = -\frac{2k+1}{2} \,, \tag{6}$$

so that $\overline{N}$ is like a spinor, and θ a vector.

Model with θ (X-charge $x > 0$) and S (X-charge $x_S < 0$). With one field and one invariant, we have

$$D_X = x|\theta|^2 + x_S|v_S|^2 - \xi^2 \,. \tag{7}$$

As previously, D_X-flatness kills the direction $< S >$ and allows $< \theta >$. The main difference is that S being a composite field, $S = \prod_{i=1}^{n} \Phi_i^{n_i}$, the F-terms corresponding to the invariant $S^t\theta^u$ are $F_j = tn_j(\prod_{i\neq j} \Phi_i^{n_i})\Phi_j^{n_j-1}S^{t-1}\theta^u$ and $F_\theta = uS^t\theta^{u-1}$; they therefore only forbid the direction $< S, \theta >$, even for $t = 1$.

One is therefore left with a vev of order ξ along the single direction $< \theta >$. It is certainly encouraging that linear terms in S can appear in the superpotential. Terms such as $\mathbf{Q}_i\overline{\mathbf{u}}_k H_u(\theta/M)^n$ are needed to implement hierarchies among the Yukawa couplings. Conversely, requiring that

$$\frac{x_S}{x} = -n \,, \tag{8}$$

with n integer $\neq 0$, is sufficient to insure the linear appearance of the invariant S. In this case, the vacuum structure is inexorably related to the Yukawa hierarchies. However if $x_S = 0$, there is no danger associated with S, and the above discussion does not apply.

Models with 3 fields.

Model with θ ($x > 0$), $\bar{N}$ ($x_{\overline{N}}$) and S (x_S). Its analysis depends on the sign of the X-charges of $\overline{N}$ and S.

A-) Let us first discuss the case $x_{\overline{N}}, x_S < 0$. The vanishing of the D_X term

$$D_X = x|\theta|^2 + x_{\overline{N}}|\overline{N}|^2 + x_S|v_S|^2 - \xi^2 \tag{9}$$

forbids the directions $< \overline{N} >$, $< S >$ and $< \overline{N}, S >$, but allows the directions $< \theta, \overline{N} >$, $< \theta, S >$ and $< \theta, \overline{N}, S >$. We saw earlier that an invariant $\overline{N}\theta$ forbids the desired direction $< \theta >$. We must require the presence of an invariant $\overline{N}^q\theta^n$ with $q \geq 2$ and $n \neq 0 \; mod(q)$, to disallow the directions $< \theta, \overline{N} >$ and $< \theta, \overline{N}, S >$ ($p = 2$ generates masses for $\overline{N}$). The last direction $< S, \theta >$ is disposed of by adding an invariant of the form $S^t\theta^u$, which is also allowed given the signs of the charges. Consider this model for two different choices of invariants.

a-) Suppose $S = LH_u$ and search for charges which allow for the couplings (3) and (4) of the seesaw mechanism. One can easily show that necessarily:

$$\frac{x_{\overline{N}}}{x} = -\frac{2k+1}{2}\ , \qquad \frac{x_{LH_u}}{x} = -\frac{2k'+1}{2} \tag{10}$$

where k and k' are integers. This assignment automatically forbids a term $S\theta^q$ which would break R-parity. One obtains

$$W \ni \overline{N}^2\theta^{2k+1}\ , \overline{N}LH_u\theta^{k+k'+1}\ , (LH_u)^2\theta^{2k'+1}\ , \cdots \tag{11}$$

where the last term does not break R-parity and gives an extra contribution to neutrino masses.

b-) Take $S = H_dH_u$, the so-called μ-term, the mass term which plays a central role in all supersymmetric extensions of the Standard Model. Phenomenology requires in the low energy theory a term linear in S with a mass of the order of the electroweak scale. If this term comes from a supersymmetric term in the superpotential, one is left with two possibilities:

- the invariant $H_dH_u\theta^p$ which disposes of the unwanted direction $< S, \theta >$. The corresponding μ-parameter is

$$\mu \sim M\left(\frac{\theta}{M}\right)^p , \tag{12}$$

  as long as $x_S/x = -p$, and phenomenology requires p to be an extremely large integer.

- the only other invariant linear in S which kills the direction $< S, \theta >$ is $\overline{N}S\theta^p$. One recovers the invariants of the previous example: eq. (11) or more generally if we assume a coupling $\overline{N}^q\theta^{qk+r}$ $(0 < r < q$ and $q \geq 2)$:

$$\overline{N}^q\theta^{qk+r},\ \ \overline{N}H_dH_u\theta^{k+k'+1},\ \ (H_dH_u)^q\theta^{q(k'+1)-r} \tag{13}$$

  where $k + k' + 1 = p$. The μ-parameter reads:

$$\mu = < \overline{N} > \left(\frac{\theta}{M}\right)^{k+k'+1} . \tag{14}$$

  This is somewhat a more hopeful situation since we expect in any case that $< \overline{N} > \ll M$ and $< \theta > /M \ll 1$. But it requires $< \overline{N} > \neq 0$ and the coupling $\overline{N}S\theta^p$ breaks R-invariance if $\overline{N}$ is to be interpreted as a right-handed neutrino (*i.e.* if it has a non-zero Dirac-type coupling). If $q = 2$, then $\overline{N}$ mixes with the right-handed neutrinos and has R-parity equal to -1.

Alternatively if there is no term in the superpotential linear in H_dH_u, there may be a non-holomorphic one in the Kähler potential [27]. The only possible choice is

$$K \ni H_dH_u\overline{N}^{*p} + \text{ h.c. }, \tag{15}$$

but in this case,

$$\mu = m_{3/2}\left(\frac{<\overline{N}^*>}{M}\right)^p, \tag{16}$$

and since we know that $<\overline{N}>\ll M$, we obtain $\mu \ll m_{3/2}$ which is probably too small for phenomenological applications.

B-) We now turn to the case $x_{\overline{N}} < 0$ and $x_S > 0$. The vanishing of D_X avoids only $<\overline{N}>$. The need to dispose of the $<S>$ direction imposes an invariant $\overline{N}S$ (hence $x_{\overline{N}} = -x_S$) which forbids the directions $<\overline{N}>$, $<S>$, $<\overline{N}, S>$, and $<\overline{N}, S, \theta>$. The last direction $<\overline{N}, \theta>$ is taken care of by an invariant $\overline{N}^q\theta^n$ ($n \neq 0\ mod(q)$), which in the simplest case is $\overline{N}^2\theta^{2k+1}$.

a-) When $S = LH_u$, the Dirac neutrino mass term ($\overline{N}S$) appears here with no suppression factor $<\theta>/M$.

b-) When S is the μ-term, it cannot appear by itself in the superpotential, although it is allowed in the Kähler potential [27] as

$$K \ni H_dH_u\theta^{*q} + \text{ h.c. }. \tag{17}$$

But since the X-charge of H_dH_u must be half-odd integer (to forbid the invariant $\overline{N}\theta^m$ that would suppress the direction $<\theta>$), this term cannot appear in the Kähler potential either. Hence the presence of a singlet $\overline{N}$ with charge $x_{\overline{N}} < 0$ is enough to forbid this interesting possibility for generating a low-energy mu-term [6]. One can still generate a μ-term through the invariant $\overline{N}H_dH_u$ in the superpotential, with

$$\mu \sim <\overline{N}>. \tag{18}$$

Model with one field θ ($x > 0$), and two composites, S_1 (x_1) and S_2 (x_2).

If one of the S has a positive charge (say $x_1 > 0$), we are left with the corresponding direction ($<S_1>$) since no F-term can kill a single $<S>$ and the D_X-term only forbids the other $<S>$ ($<S_2>$).

We therefore require both $x_1, x_2 < 0$, in which case D_X-flatness deals with $<S_1>$, $<S_2>$ and $<S_1, S_2>$, the invariant $S_1^t\theta^u$ with $<S_1, \theta>$ and $<S_1, S_2, \theta>$, and the invariant $S_2^r\theta^v$ with the remaining $<S_2, \theta>$. When $|x_1|/x$ and $|x_2|/x$ are integers, this can be used to generate mass hierarchies in the quark sector [4]-[9] with:

$$S_1 = \mathbf{Q}\overline{\mathbf{d}}H_d\ , \quad S_2 = \mathbf{Q}\overline{\mathbf{u}}H_u\ . \tag{19}$$

We come to the important conclusion that the mere determination of the ξ-vacuum requires Yukawa terms in the superpotential. Alternatively, their absence from the superpotential (supersymmetric zeros) can unleash unwanted flat directions in the ξ-vacuum.

Models with θ ($x > 0$), $\overline{N}_1$ (x_1) and $\overline{N}_2$ (x_2).

If $x_1, x_2 < 0$, D_X-flatness excludes $< \overline{N}_1 >$, $< \overline{N}_2 >$ and $< \overline{N}_1, \overline{N}_2 >$. Requiring Majorana mass terms imposes as seen above that $|x_1|/x = (2k+1)/2$ and $|x_2|/x = (2l+1)/2$. The Majorana mass matrix then reads:

$$\mathcal{M}_M \sim \theta \begin{pmatrix} \theta^{2k} & \theta^{k+l} \\ \theta^{k+l} & \theta^{2l} \end{pmatrix}. \tag{20}$$

If $x_1 < 0$ and $x_2 > 0$, only the direction $< \overline{N}_1 >$ is disposed of by D_X. We need an invariant of the form $\overline{N}_1 \overline{N}_2^l$ to kill the direction $< \overline{N}_2 >$ and an invariant $\overline{N}_1^2 \theta^{2k+1}$ to deal with $< \overline{N}_1, \theta >$. This in turn imposes constraints on the X-charges:

$$\frac{x_1}{x} = -\frac{2k+1}{2}, \quad \frac{x_2}{x} = \frac{2k+1}{2l}, \tag{21}$$

If $l = m(2k+1)$, the lowest invariants in the superpotential are

$$W \ni \overline{N}_1^2 \theta^{2k+1}, \ \overline{N}_1 \overline{N}_2^{(2k+1)m}, \ \overline{N}_1 \overline{N}_2^m \theta^k. \tag{22}$$

These three invariants obey a polynomial constraint and do not suffice to eliminate $< \overline{N}_2, \theta >$, along which $F_{\overline{N}_1}$ can vanish, as two invariants are linear in $\overline{N}_1$.

A case of interest is $m = 1$ for which one obtains a mixed Majorana mass term $\overline{N}_1 \overline{N}_2 \theta^k$. For $m \neq 1$, one of the $\overline{N}$ stays massless after ξ-breaking.

3 Mu-term

We will not continue a general study adding fields one by one. The analysis becomes more and more involved as the number of fields increases but one can always restrict to a subset of fields and the previous examples with two or three fields may provide good insights to treat the more complicated cases. Let us illustrate this on an example which makes heavy use of models a-) and b-) discussed above, in which additional constraints on the charges emerge naturally.

It is well-known that the gauge symmetries of the supersymmetric standard model do not make any distinction between the invariants $S_0 \equiv H_d H_u$ and $S_i \equiv L_i H_u$ (i being a family index). The first one appears in the μ-term whereas the others correspond to R-parity violating terms in the superpotential. As discussed in model a-), they also appear in the seesaw mechanism in conjunction with a Standard Model singlet $\overline{N}_i$ which plays the role of a right-handed

neutrino. Similarly, the discussion of model b-) leads to the conclusion that a possible solution to the μ-problem involves another standard model singlet $\overline{N}_0$. We therefore consider the following set of fields and X-charges

$$\begin{array}{cccccc} & \theta & \overline{N}_0 & S_0 & \overline{N}_i & S_i \\ X/x = & 1 & -\frac{3k_0+1}{3} & -\frac{3k_0'+2}{3} & -\frac{2k_i+1}{2} & -\frac{2k_i'+1}{2} \end{array} \tag{23}$$

The superpotential then includes the terms

$$W \ni \overline{N}_i S_j \theta^{k_i+k_j'+1},\ \overline{N}_i \overline{N}_j \theta^{k_i+k_j+1},\ \overline{N}_0 S_0 \theta^{k_0+k_0'+1},\ \overline{N}_0^3 \theta^{3k_0+1}, \cdots \tag{24}$$

(terms mixing $\overline{N}_0$ and $\overline{N}_i$ are highly non-renormalisable). It is invariant under a R_p parity defined as $+1$ for $\overline{N}_0$ and S_0 and -1 for $\overline{N}_i$ and S_i. This is why we have not chosen half-odd charges for $\overline{N}_0$ (and thus S_0): mixed terms $\overline{N}_0\overline{N}_i$ would have led to R_p violations.

There are no supersymmetric mass term for $\overline{N}_0$; it is therefore induced by supersymmetry breaking and it is of the order of the scale of supersymmetry breaking $\tilde{m}$. Assuming that renormalisation group evolution turns the corresponding scalar mass-squared term negative, the potential for the $\overline{N}_0$ scalar field has the form:

$$V = -\tilde{m}^2|\overline{N}_0|^2 + \frac{\lambda}{2}|\overline{N}_0|^4 \left(\frac{\theta}{M}\right)^{2(3k_0+1)} \tag{25}$$

Its minimization leads to the following value for the μ-term:

$$\mu \sim \frac{\tilde{m}}{\sqrt{\lambda}} \left(\frac{\theta}{M}\right)^{k_0'-2k_0}. \tag{26}$$

The terms involving $\overline{N}_i$ and S_i in (24) are precisely the ones necessary to implement the seesaw mechanism.

Let us note here that if we had taken the more general charges for $\overline{N}_0$ and S_0:

$$\frac{X_{S_0}}{x} = -\frac{(2p+1)k_0'+r_0}{2p+1}, \quad \frac{X_{\overline{N}_0}}{x} = -\frac{(2p+1)(k_0+1)-r_0}{2p+1}$$

with $p \geq 1$ and $0 < r_0 < 2p+1$, we would have obtained a mu-term of order:

$$\mu \sim \left(\frac{\tilde{m}M^{2(p-1)}}{\sqrt{\lambda}}\right)^{1/(2p-1)} \left(\frac{\theta}{M}\right)^{[k_0'(2p-1)-2k_0+r_0-2]/(2p-1)}. \tag{27}$$

If we take M to be of the order of the Planck scale, the first factor is a scale intermediate between $\tilde{m}$ and M_{Pl} (unless $p=1$) and the suppression due to the second factor can only be minor. We conclude that we must take $p=1$ in order to have a low energy mu-term. We are thus led to the choice (23) of quantum numbers. The corresponding model with the superpotential terms (24) leads at

low energy to the so-called (M+1)SSM model (minimal supersymmetric model plus an extra singlet), with the important difference that the field $\overline{N}_0$ is not a complete gauge singlet: it is charged at least under the anomalous $U(1)_X$ symmetry.

We discussed in detail the previous case because it is illustrative of the power of the method and how far one can take it. Let us now summarize for further use the different scenarios that we have encountered, depending on the value of the charge

$$X^{[\mu]} \equiv X_{H_u} + X_{H_d} \tag{28}$$

and possibly the charge of the associated singlet $X_{\overline{N}_0}$. We only consider scenarios which do not lead to an obvious breaking of R-parity (for example if we need $<\overline{N}_0>\neq 0$, then $\overline{N}_0$ has R-parity $+1$).

- if $X^{[\mu]}/x = -p_0$ with p_0 integer, then

$$\mu = M\left(\frac{\theta}{M}\right)^{p_0} \tag{29}$$

- if $X^{[\mu]}/x = -(3k_0' + r_0)/3$ and $X_{\overline{N}_0}/x = -(3k_0 + 3 - r_0)/3$ with k_0 and k_0' integers and $r_0 = 1$ or 2, then

$$\mu = <\overline{N}_0> \left(\frac{\theta}{M}\right)^{k_0+k_0'+1} . \tag{30}$$

  This is the case just discussed.

- if $X^{[\mu]}/x = -X_{\overline{N}_0}/x = (3k_0 + r_0)/3$ with k_0 integer and $r_0 = 1$ or 2, then

$$\mu = <\overline{N}_0> . \tag{31}$$

- finally, if $X^{[\mu]} = 0$, the H_dH_u term is unseen by the anomalous symmetry and therefore not likely to lead to vevs of order ξ for the Higgs doublets, in the direction where the $U(1)_X$ is broken: indeed the direction $< H_u, H_d >$ is taken care of by the requirement of D_X-flatness. If the μ-term does not appear in the superpotential, supersymmetry breaking can then lift the remaining flat direction ($|<\theta>| = \xi/\sqrt{x}$, $|<H_u>| = |<H_d>| = v$) [23].

We will see below that anomaly cancellation conditions tend to give integer values for $X^{[\mu]}$, which disfavours the second and third possibilities. And the first one requires too large values of p_0 if M is of the order of the Planck scale. The last solution ($X^{[\mu]} = 0$), together with the absence of the μ-term in the superpotential, seems to us favoured. We offer no reason for this absence, except by deferring to string lore according to which there are no mass terms in the superpotential.

4 R-parity

We now show that the constraints discussed above on the $U(1)_X$ quantum numbers of the low-energy fields may naturally lead to conserved R-parity. The presence of standard model singlets $\overline{N}_i$ necessary to implement the seesaw mechanism plays in this respect a key role.

We assume the seesaw mechanism requiring the presence of the invariants:

$$\bar{N}_i \bar{N}_j \theta^{n^0_{ij}} + L_i \bar{N}_j H_u \theta^{n^\nu_{ij}} \tag{32}$$

We saw that, in order not to spoil the ξ-vacuum, the powers n^0_{ii} must be odd integers, or equivalently the X-charges $X_{\overline{N}_i}$ of the fields $\bar{N}_i$ must be, in units of x, half-odd integers:

$$\frac{X_{\overline{N}_i}}{x} = -\frac{2k_i + 1}{2} \tag{33}$$

where k_i is an integer. Henceforth we set $x = 1$. The last term in (32) determines the R-parity of the right-handed neutrino superfields to be negative.

Let us study the X-charges of possible standard model invariant operators made up of the basic fields $\mathbf{Q}_i$, $\bar{\mathbf{u}}_i$, $\bar{\mathbf{d}}_i$, L_i, $\bar{e}_i$ (i being a family index) and of the Higgs fields H_u, and H_d.

The cubic standard model invariants that respect baryon and lepton numbers are, in presence of the gauge singlets $\overline{N}_i$,

$$\mathbf{Q}_i \bar{\mathbf{d}}_j H_d \ , \quad \mathbf{Q}_i \bar{\mathbf{u}}_j H_u \ , \quad L_i \bar{e}_j H_d \ , \quad L_i \overline{N}_j H_u \ , \tag{34}$$

with charges $X^{[d]}_{ij}$, $X^{[u]}_{ij}$, $X^{[e]}_{ij}$ and $X^{[\nu]}_{ij}$ respectively. To avoid undesirable flat directions, all must appear in the superpotential, restricting their X-charges to be of the form

$$X^{[u,d,e,\nu]}_{ij} = -n^{u,d,e,\nu}_{ij} \ , \tag{35}$$

where $n^{u,d,e,\nu}_{ij}$ are all positive integers or zero.

We now turn to the invariants which break R-parity. We have already encountered the quadratic invariants $L_i H_u$ whose charges are determined by the seesaw couplings (32) to be half odd integers

$$X_{L_i H_u} = \frac{2\,(k_j - n^\nu_{ij}) + 1}{2} \ . \tag{36}$$

Consider the cubic R-parity violating operators, $L_i L_j \bar{e}_k$, $L_i \mathbf{Q}_j \bar{\mathbf{d}}_k$ and $\bar{\mathbf{u}}_i \bar{\mathbf{d}}_j \bar{\mathbf{d}}_k$. The charges of the first two, which violate lepton number, satisfy the relations:

$$X_{L_i L_j \bar{e}_k} = X^{[e]}_{jk} + X^{[\nu]}_{il} - X^{[\mu]} - X_{\overline{N}_l} \tag{37}$$

$$X_{L_i Q_j \bar{d}_k} = X^{[d]}_{jk} + X^{[\nu]}_{il} - X^{[\mu]} - X_{\overline{N}_l} \tag{38}$$

where the index l can be chosen arbitrarily.

As a consequence, if $X^{[\mu]}$ is integer, $-p_0$, both charges are half-odd integers and there is no R-parity violation from these operators. However they can still appear as

$$L_i L_j \bar{e}_k \bar{N}_l \ \theta^{(n^e_{jk}+n^\nu_{il}-p_0)} \qquad L_i Q_j \bar{d}_k \bar{N}_l \ \theta^{(n^d_{jk}+n^\nu_{il}-p_0)} \tag{39}$$

in the superpotential. A similar conclusion is reached if $X^{[\mu]}$ is a multiple of one third, in which case one needs to include also apropriate powers of $\overline{N}_0$.

To determine the charges of the operators $\bar{\mathbf{u}}_i\bar{\mathbf{d}}_j\bar{\mathbf{d}}_k$ in terms of the charges of the parity-conserving invariants, one must use the Green-Schwarz condition on mixed anomalies $C_{\rm weak} = C_{\rm color}$ which reads:

$$\sum_i (X_{Q_i} + X_{L_i}) - \sum_i (X_{\bar{u}_i} + X_{\bar{d}_i}) + X^{[\mu]} = 0. \tag{40}$$

One obtains:

$$\begin{aligned} X_{\bar{u}_i\bar{d}_j\bar{d}_k} &= \frac{1}{n_f^2}\sum_{p,q} X^{[d]}_{pq} + \frac{1}{n_f}\sum_l \left[X^{[u]}_{pi} + X^{[d]}_{pj} + X^{[d]}_{pk} - X^{[u]}_{pl} - 2X^{[d]}_{pl} + X^{[\nu]}_{lm}\right] \\ &+ \frac{1-n_f}{n_f} X^{[\mu]} - X_{\overline{N}_m} \ , \end{aligned} \tag{41}$$

true for any two family indices p, m, and where n_f is the number of families which we will take to be three. In a large class of models, the charge $X_{\bar{u}_i\bar{d}_j\bar{d}_k}$ thus obtained will be such as to forbid not only a term $\bar{u}_i\bar{d}_j\bar{d}_k$ in the superpotential but also any term obtained from it by multiplying by any powers of θ, $\overline{N}_i$ or $\overline{N}_0$.

Let us consider for illustrative purpose an anomalous symmetry which is family independent. Then (41) simplifies to:

$$X_{\bar{\mathbf{u}}\bar{\mathbf{d}}\bar{\mathbf{d}}} = X^{[d]} + X^{[\nu]} - (1 - \frac{1}{n_f})X^{[\mu]} - X_{\overline{N}} \ . \tag{42}$$

Remember that $X_{\overline{N}}$ is half-odd integer and $n_f = 3$. If $X^{[\mu]}$ is integer not proportional to $n_f = 3$, then the charge $X_{\bar{\mathbf{u}}\bar{\mathbf{d}}\bar{\mathbf{d}}}$ is such that no term $\bar{\mathbf{u}}\bar{\mathbf{d}}\bar{\mathbf{d}}\theta^m\overline{N}^n$ can be invariant. If $X^{[\mu]}$ is non-integer and a multiple of one third, then similarly no term $\bar{\mathbf{u}}\bar{\mathbf{d}}\bar{\mathbf{d}}\theta^m\overline{N}^n\overline{N}_0^p$ can be made invariant. In the low energy theory, baryon number violation becomes negligible.

If we restrict our attention to models which yield $\sin^2\theta_W = 3/8$, the Green-Schwarz condition $5C_{\rm weak} = 3C_Y$ reads:

$$\sum_i (7X_{Q_i} + X_{L_i}) - \sum_i (4X_{\bar{u}_i} + X_{\bar{d}_i} + 3X_{\bar{e}_i}) + X^{[\mu]} = 0. \tag{43}$$

One infers from (40) and (43) the following relation:

$$X^{[\mu]} = \sum_i \left(X^{[d]}_{ii} - X^{[e]}_{ii}\right), \tag{44}$$

which tends to favor models with integer $X^{[\mu]}$ (proportional to n_f in the case of a family-independent symmetry).

If $X^{[\mu]} = 0$ or more generally if $X^{[\mu]}$ is proportional to n_f ($X^{[\mu]} = n_f z_\mu$), the charge in (42) is half-odd integer; it can only be compensated by odd powers of $\bar{N}$: invariance under X means conservation of R-parity. For instance, the above allows the interaction:

$$\bar{\mathbf{u}}\bar{\mathbf{d}}\bar{\mathbf{d}}\bar{N}\ \theta^{[n_d+n_\nu+z_\mu(n_f-1)]}\ . \tag{45}$$

This term allows baryon number violation, but preserves both $B-L$ and R-parity.

A very similar discussion can obviously be given for the general case of a family dependent anomalous symmetry.

To conclude, in a large class of models, there are no R-parity violating operators, whatever their dimensions: through the right-handed neutrinos for example, R-parity is linked to half-odd integer charges, so that $U(1)_X$ charge invariance results in R-parity invariance. Thus none of the operators that violate R-parity can appear in holomorphic invariants: even after breaking of the anomalous X symmetry, the remaining interactions all respect R-parity, leading to an absolutely stable superpartner.

5 Conclusion

The purpose of this work was to show that the breaking of an anomalous Abelian gauge symmetry at a very large scale imposes some very stringent constraints on the anomalous charges of the low energy fields. This is indeed the situation in many superstring models (see for example refs. [28, 29]). Holomorphy and D-flatness, especially for the anomalous symmetry, are the tools that we used to derive these constraints. This in turn puts some restrictions on the dynamics of the model. We illustrated this fact on the generation of a mu-term. If we consider a model with a single extra $U(1)$ and a single field (θ) which acquires a *vev* when the anomalous symmetry is broken, it tends to exclude most of the scenarios proposed to generate a mu-term. The preferred scenario seems to be a mu-term invariant under the anomalous symmetry. It is an open question why such a term does not appear in the superpotential but rather in the Kähler potential. In our discussion, we tried to impose conditions which lift most of the dangerous flat directions. Some of them might however require supersymmetry breaking and it remains to be seen whether, there also, the anomalous $U(1)$ symmetry plays a role [14, 15].

The most interesting result is that, once these constraints are taken into account, the couplings which respect the anomalous symmetry also respect R-parity. This remains true at low energy since we made the supposition that the field θ which breaks this symmetry has R-parity $+1$. We derived this result in the restricted class of models discussed in this paper but we believe that

it is rather general. It makes use of the right-handed neutrinos necessary to generate neutrino masses through the seesaw mechanism. This should lead to an interesting phenomenology of lepton number or baryon number violations.

Of course, although we tried to be more general in this paper, it is tempting to apply these constraints to models of quark and lepton mass hierarchies. It was found [5] that, among models using an Abelian gauge symmetry, the observed hierarchies favor a model with non-zero mixed anomalies satisfying the relation $5C_{weak} = 3C_1$ (if we assume $C_{weak} = C_{color}$ and the charge of the anomalous mu-term to be zero [6]). From the discussion of the last section, this seems in complete agreement with what is required by the constraints discussed in this paper. Of course, if we want to obtain a realistic quark and lepton mass spectrum, we need to introduce several extra $U(1)$ (a single combination of which is anomalous) and several θ fields [9]. The discussion then becomes more involved but the simple case discussed in the present work leads us to expect possible rewards such as R-parity. And it is a fact, often neglected, that any theory of mass has to come up with solutions for R-parity or to explain why its violations are mild.

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