Title
A note on global output regulation of nonlinear systems in the output feedback form

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of Σ; and write $\rho^+(\overline{\rho},\, \mathbf{d},\, t)$ $(\rho^-(\overline{\rho},\, \mathbf{d},\, t))$ for a solution of (4) with $\rho(0) = \overline{\rho}$, $\vartheta(0) = 0$ $(\vartheta(0) = -\pi)$, and disturbance $\mathbf{d}$.

Find an integer K such that $\varepsilon/4 > 2^{-K+1}$ and $2^{-K+1} < \pi/12$. Let $x_0 = e^{(6K-2)\pi}$, so that $\xi_0 = \lambda(e^{(6K-2)\pi})$; and let also $x_1 = e^{(6K+2)\pi}$ and $\xi_1 = \lambda(x_1)$.

Recall that, by property 2), $x_1 \leq r(x_1,\, t) < 1.03x_1$ and $x_0 \leq r(x_0,\, t) < 1.03x_0$ for all $t \in [0,\, \pi/12]$. In particular, for any $t \in [0,\, \pi/12]$, and with $x_0 = e^{(6K-2)\pi}$, $x_1 = e^{(6K+2)\pi}$, both $r(x_0,\, t)$ and $r(x_1,\, t)$ will be in the interval $[e^{(6K-2.5)\pi},\, e^{(6K+2.5)\pi}]$. Then, by construction of λ, both $\rho(\xi_0,\, t) = \lambda(r(x_0,\, t))$ and $\rho(\xi_1,\, t) = \lambda(r(x_1,\, t))$ will belong to the interval $[e^{6K\pi} - 2^{-K},\, e^{6K\pi} + 2^{-K}]$, so that

$$\rho^+(\xi_1,\, t) - \rho^+(\xi_0,\, t) < 2^{-K+1}, \qquad t \in [0,\, \pi/12].$$

Therefore, there must exist a positive $\tau_0 < 2^{-K+1}$ such that if $\mathbf{d}_0 := 1_{[0,\, \tau_0]}$, then

$$\rho^+(\xi_1,\, \tau_0) = \rho^+(\xi_0,\, \mathbf{d}_0,\, \tau_0).$$

So

$$\rho^+(\xi_0,\, \mathbf{d}_0,\, \pi) = \rho^+(\xi_1,\, \pi) = -\mu\left(e^{(6K+1)\pi}\right).$$

Let $\xi_2 := -\mu(e^{(6K+1)\pi})$, $\xi_3 := -\mu(e^{(6K+5)\pi})$.

Next, take a disturbance $\mathbf{d}_1 = 1_{[\pi,\, \pi+\tau_1]}$, with some $\tau_1 < 2^{-K+1}$ such that

$$\rho^-(\xi_3,\, \tau_1) = \rho^-(\xi_2,\, \mathbf{d}_1(\cdot + \pi),\, \tau_1).$$

Then

$$\begin{aligned}\rho^+(\xi_0,\, \mathbf{d}_0 + \mathbf{d}_1,\, 2\pi) &= \rho^-(\xi_2,\, \mathbf{d}_1(\cdot + \pi),\, \pi)\\ &= \rho^-(\xi_3,\, \pi) = \lambda\left(e^{(6K+4)\pi}\right).\end{aligned}$$

Generally, for each $k \geq 0$, we let

$$\begin{aligned}\xi_{4k} &:= \lambda\left(e^{(6(K+k)-2)\pi}\right)\\ \xi_{4k+1} &:= \lambda\left(e^{(6(K+k)+2)\pi}\right)\\ \xi_{4k+2} &:= -\mu\left(e^{(6(K+k)+1)\pi}\right)\\ \xi_{4k+3} &:= -\mu\left(e^{(6(K+k)+5)\pi}\right)\end{aligned}$$

and choose $\tau_{2k} \leq 2^{-K-k+1}$ and $\tau_{2k+1} \leq 2^{-K-k+1}$ so that

$$\rho^+(\xi_{4k+1},\, \tau_{2k}) = \rho^+(\xi_{4k},\, \mathbf{d}_{2k}(\cdot + 2k\pi),\, \tau_{2k})$$

and

$$\rho^-(\xi_{4k+3},\, \tau_{2k+1}) = \rho^-(\xi_{4k+2},\, \mathbf{d}_{2k+1}(\cdot + (2k+1)\pi),\, \tau_{2k+1})$$

with $\mathbf{d}_l := 1_{[0,\, \tau_l]}$.

Finally, let $\mathbf{d} := \sum_l \mathbf{d}_l$. Then

$$\int \mathbf{d}(t)\, dt = \sum \tau_l \leq 4/2^{K-1} < \varepsilon$$

and

$$\lim_{t \to +\infty} \rho^+(\xi_0,\, \mathbf{d},\, t) = \infty.$$

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A Note on Global Output Regulation of Nonlinear Systems in the Output Feedback Form

Qi Gong and Wei Lin

***Abstract*—This note shows how the adaptive control method developed recently for nonlinearly parameterized systems can be used to solve the problem of global output regulation, for nonlinear systems in the so-called output-feedback form with unknown parameters and exogenous signals belonging to a compact set whose bound is also unknown.**

***Index Terms*—Adaptive nonlinear control, global output regulation, output feedback.**

I. Introduction and Preliminaries

In this note, we consider the problem of global output regulation for nonlinear systems of the form

$$\begin{aligned}\dot{x} &= F(\mu)x + G(y,\, \omega,\, \mu) + g(\mu)u\\ \dot{y} &= H(\mu)x + K(y,\, \omega,\, \mu)\\ \dot{\omega} &= S\omega\\ e &= y - q(\omega,\, \mu)\end{aligned} \tag{1.1}$$

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where $\omega \in \mathbb{R}^s$ is the exogenous signal, $x \in \mathbb{R}^n$, $u \in \mathbb{R}$, and $y \in \mathbb{R}$ are the system states, input and output, respectively. The error signal e is the only measurable variable that can be used in feedback design. The unknown constant μ belongs to a compact set $\wp \subset \mathbb{R}^p$ *whose bound is unknown*. $G(y, \omega, \mu)$, $K(y, \omega, \mu)$ and $q(\omega, \mu)$ are smooth functions of their arguments, not necessarily vanishing at $(y, \omega) = (0, 0)$. The exosystem is assumed to be neutrally stable, i.e., all the eigenvalues of S are simple and lie on the imaginary axis.

The *robust* output regulation problem is to find a smooth dynamic controller

$$\begin{aligned}\dot{\zeta} &= \alpha(\zeta, e), \qquad \zeta \in \mathbb{R}^r\\ u &= \beta(\zeta, e)\end{aligned} \tag{1.2}$$

such that the closed-loop system (1.1) and (1.2) is *globally bounded* for any initial condition $(x(0), y(0), \zeta(0), \omega(0))$ and any $\mu \in \wp$. Moreover, $\lim_{t\to\infty} e(t) = 0$.

In the absence of ω and when the vector fields $G(\cdot)$ and $K(\cdot)$ vanish at $y = 0$, (1.1) is in the *output feedback form*, whose global stabilization problem by output feedback has been well studied; see, for instance, [9], [5], as well as the references therein. As for the output regulation problem of (1.1), the first global result was reported in [11], under the condition that the exogenous signal ω and the unknown parameter μ belong to *a priori* known compact set. Obviously, this is a restrictive assumption, simply because when information of the bounds of exosignals are changed, the controller should also be changed accordingly. In [13], a universal controller was proposed to remove the restriction for a class of uncertain decentralized systems with polynomial nonlinearities, which covered the system in the output feedback form as a special case.

The purpose of this note is to propose an adaptive output regulator based on the adaptive control method developed recently for nonlinearly parameterized systems [8] and the feedback domination design technique [6], which achieves global output regulation of the nonlinear system (1.1) without requiring the knowledge of the bounds of the unknown parameters and exosignals. Our result provides an interesting alternative solution to the problem considered in [13] (in nondecentralized case) and, thus, complementing the results obtained in [13].

Throughout this note, we make the following assumptions that have been commonly used when dealing with output regulation of nonlinear systems in the output feedback form.

Assumption 1: System (1.1) has a uniform relative degree $r \geq 2$.

Assumption 2: For system (1.1), the sign of the high-frequency gain

$$b(\mu) = H(\mu)F^{r-2}(\mu)g(\mu)$$

is known and satisfies $|b(\mu)| \geq b_0 > 0$, with b_0 being a known constant.

Assumption 3: For every $\mu \in \wp$, the linear system $(F(\mu), g(\mu), H(\mu))$ is minimum phase.

Remark 1.1: By Assumption 3, the linear system $\dot{x} = F(\mu)x + g(\mu)u$, $y = H(\mu)x$, is minimum phase. This, together with Assumption 1, implies that Assumption 3 is essentially equivalent to the condition that the matrix $F(\mu) - (1/b(\mu))g(\mu)H(\mu)F^{r-1}(\mu)$ is Hurwitz with the restriction $H(\mu)F^i(\mu)x = 0$, $i = 0, 1, \ldots, r-2$. This is exactly the assumption used in [9], [5], where the global stabilization problem was studied.

Remark 1.2: System (1.1) is a bit more general than those studied in [11], [13], for the reason that the vector fields $G(y, \omega, \mu)$ and $K(y, \omega, \mu)$ need not to be vanished at $y = 0$. Hence, (1.1) is not necessary to be globally minimum phase with respect to the output y.

To ensure the solvability of the output regulation problem, the following assumption is necessary [4], [11].

Assumption 4: There exists a global defined smooth functions $\pi(\omega, \mu)$ and $c(\omega, \mu)$ satisfying the regulator equations

$$\begin{aligned}\frac{\partial \pi(\omega, \mu)}{\partial \omega} S\omega &= F(\mu)\pi(\omega, \mu) + G(q(\omega, \mu), \omega, \mu) + g(\mu)c(\omega, \mu)\\ \frac{\partial q(\omega, \mu)}{\partial \omega} S\omega &= H(\mu)\pi(\omega, \mu) + K(q(\omega, \mu), \omega, \mu).\end{aligned}$$

Under Assumptions 1–3, it has been shown in [9] that there exists a parameter-dependent filter transformation such that system (1.1) can be put into a lower-triangular form. Indeed, introduce the filter

$$\dot{\xi} = A\xi + Bu := \begin{bmatrix} -\lambda_1 & 1 & 0 & \cdots & 0 & 0\\ 0 & -\lambda_2 & 1 & \cdots & 0 & 0\\ \vdots & \vdots & \vdots & \ldots & \vdots & \vdots\\ 0 & 0 & 0 & \cdots & -\lambda_{r-2} & 1\\ 0 & 0 & 0 & \cdots & 0 & -\lambda_{r-1}\end{bmatrix}\xi + \begin{bmatrix}0\\0\\ \vdots\\0\\1\end{bmatrix}u \tag{1.3}$$

with $\lambda_i > 0$, $1 \leq i \leq r-1$, being real numbers, and a parameter-dependant change of coordinates

$$z = x - D(\mu)\xi - \frac{d(\mu)}{b(\mu)}y \tag{1.4}$$

where $d(\mu) = (F + \lambda_1 I) \cdot (F + \lambda_2 I), \ldots, (F + \lambda_{r-1} I)g(\mu)$ and $D(\mu)$ satisfies

$$\begin{aligned}F(\mu)D(\mu) - D(\mu)A &= [d(\mu), 0, \ldots, 0]\\ g(\mu) &= D(\mu)[0, \ldots, 0, 1]^T\\ H(\mu)D(\mu) &= [b(\mu), 0, \ldots, 0].\end{aligned} \tag{1.5}$$

With the aid of (1.3) and (1.4), (1.1) is transformed into

$$\begin{aligned}\dot{z} &= \overline{F}(\mu)z + \overline{G}(y, \omega, \mu)\\ \dot{y} &= \overline{H}(\mu)z + \overline{K}(y, \omega, \mu) + b(\mu)\xi_1\\ \dot{\xi} &= A\xi + Bu\\ \dot{\omega} &= S\omega\\ e &= y - q(\omega, \mu)\end{aligned} \tag{1.6}$$

where

$$\begin{aligned}\overline{G}(y, \omega, \mu) &= \left(F(\mu) - \frac{d(\mu)}{b(\mu)}H(\mu)\right)\frac{d(\mu)}{b(\mu)}y\\ &\quad + G(y, \omega, \mu) - \frac{d(\mu)}{b(\mu)}K(y, \omega, \mu)\\ \overline{H}(\mu) &= H(\mu)\\ \overline{F}(\mu) &= F(\mu) - \frac{d(\mu)}{b(\mu)}H(\mu)\\ \overline{K}(y, \omega, \mu) &= H(\mu)\frac{d(\mu)}{b(\mu)}y + K(y, \omega, \mu).\end{aligned}$$

By construction, it is easy to see that $\overline{F}(\mu)$ is a Hurwitz matrix for all $\mu \in \wp$.

II. Problem Transformation

As pointed out in [3], [11], and [12], the problem of output regulation can be transformed into a stabilization problem under suitable conditions. In this section, we use the method introduced in [3] to perform such a transformation under the following hypothesis.

Assumption5: Suppose $c(\omega(t), \mu)$ defined in Assumption 4 is a trigonometric polynomial of the form

$$c(\omega(t), \mu) = \sum_{i=-l}^{l} c_i(\omega(0), \mu)e^{j\hat{\omega}_i t}$$

where l is a fixed finite integer, c_i are unknown complex numbers with $c_i^* = c_{-i}$ for $i = 0, \pm 1, \ldots, \pm l$, in which "$*$" stands for the complex conjugate, and $\hat{\omega}_i = -\hat{\omega}_{-i}$ are known constants.

Remark 2.1: The previous assumption simply says that $c(\omega(t), \mu)$ as a function of time t is a combination of sinusoidal signals and constant signals, with fixed known frequencies and unknown amplitudes that depend on the unknown parameters and the initial condition of exosignals. Assumption 5, in general, allows only polynomial nonlinearities.

As shown in [2] and [3], Assumption 5 implies the existence of a global defined mapping $\tau(\omega(t), \mu)$ and a set of real numbers, $a_i, i = 1, \ldots, v$, for some fixed integer v, satisfying

$$\begin{aligned}\dot{\tau}(\omega(t), \mu) &= \Phi \cdot \tau \\ c(\omega(t), \mu) &= \Psi \cdot \tau \end{aligned} \tag{2.1}$$

where

$$\Phi = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ a_1 & a_2 & a_3 & \cdots & a_\nu \end{bmatrix} \text{ and } \Psi = [1, 0, \ldots, 0].$$

It can be shown that all the eigenvalues of Φ are simple and located on the imaginary axis [2], [3], [12].

We show in the next section that Assumptions 1–5 suffice to solve the global output regulation problem. To begin with, we first introduce a useful lemma.

Lemma 1: Under Assumptions 1–5, there exists a global defined smooth function $\varsigma(\omega, \tau, \mu)$ satisfying

$$\begin{aligned}&\left[\frac{\partial\varsigma(\omega, \tau, \mu)}{\partial\omega}, \frac{\partial\varsigma(\omega, \tau, \mu)}{\partial\tau}\right]\begin{bmatrix} S & 0 \\ 0 & \Phi\end{bmatrix}\begin{bmatrix}\omega \\ \tau\end{bmatrix} \\ &\quad = \overline{F}(\mu)\varsigma(\omega, \tau, \mu) + \overline{G}(q(\omega, \mu), \omega, \mu) \\ &\frac{\partial q(\omega, \mu)}{\partial\omega} S\omega \\ &\quad = \overline{H}(\mu)\varsigma(\omega, \tau, \mu) + \overline{K}(q(\omega, \mu), \omega, \mu) + [b(\mu), 0, \ldots, 0]\Pi\tau \end{aligned} \tag{2.2}$$

where $\Pi \in \mathbb{R}^{(r-1)*\nu}$ is the unique solution of Sylvester equation $\Pi\Phi = A\Pi + [0, \ldots, 0, 1]^T\Psi$.

Lemma 1 can be easily proved by verifying that

$$\varsigma(\omega, \tau, \mu) = \pi(\omega, \mu) - D(\mu)\Pi\tau - \frac{d(\mu)}{b(\mu)} q(\omega, \mu) \tag{2.3}$$

is a solution of (2.2).

Remark 2.2: Note that (2.2) is actually the regulator equation for (1.6), if $\overline{\omega} = [\omega, \tau]^T$ is treated as a new exogenous signal. Therefore, Lemma 1 implies that, under suitable assumptions, if a solution to the regulator equation of the original system (1.1) exists, so does the regulator equation of the transformed system (1.6). Furthermore, it is given by $\varsigma(\omega, \tau, \mu)$, $c(\omega, \mu)$ and $\alpha(\tau, \mu) = [\alpha_1(\tau, \mu), \ldots, \alpha_{r-1}(\tau, \mu)]^T = \Pi\tau$.

Remark 2.3: In [11], a condition is imposed on the transformed system (1.6) requiring that the regulator equation of (1.6) has a global solution. By Lemma 1, the assumption is imposed on the original system directly. The reason for doing this is two-fold: 1) Assumption 4 is necessary for solving the output regulation problem [4]; and 2) it is independent of the controller design procedure.

Now, we are ready to design the internal model and to transform the output regulation problem to a stabilization problem via a global change of coordinates. For simplicity, we give only a sketch of the procedure here. The reader is referred to [10], [11], or [3] for details.

First Step: Picking any controllable pair (M, N) with $M \in \mathbb{R}^{v\times v}$ a Hurwitz matrix and $N \in \mathbb{R}^{v\times 1}$, one can solve the Sylvester equation

$$T\Phi - MT = N\Psi$$

to get a unique nonsingular matrix T [12], [10]. By Assumption 5, $\alpha_i(\tau(t), \mu)$, $i = 1, \ldots, \nu$, are all trigonometric polynomials. Combining this fact with (2.1), we have

$$\begin{aligned}\alpha_1^{(\nu)}(\tau(t), \mu) &= a_1 \cdot \alpha_1(\tau(t), \mu) + a_2 \cdot \dot{\alpha}_1(\tau(t), \mu) \\ &\quad + \cdots + a_\nu \cdot \alpha_1^{(\nu-1)}(\tau(t), \mu).\end{aligned}$$

Let $\hat{\tau} = T\cdot[\alpha_1(\tau(t), \mu), \dot{\alpha}_1(\tau(t), \mu), \ldots, \alpha_1^{(\nu-1)}(\tau(t), \mu)]^T$. Then

$$\begin{aligned}\dot{\hat{\tau}} &= T\Phi T^{-1}\cdot\hat{\tau} \\ \alpha_1(\tau(t), \mu) &= \Psi_1 \cdot \hat{\tau} \\ \alpha_i(\tau(t), \mu) &= \Psi_i \cdot \hat{\tau}\end{aligned}$$

where $\Psi_1 = \Psi T^{-1}$, $\Psi_i = \Psi_{i-1}(\lambda_{i-1}I + T\Phi T^{-1})$, $i = 2, \ldots, r$ and $\alpha_r(\tau(t), \mu) = c(\omega(t), \mu)$. Furthermore, it is shown in [3] that

$$\dot{\eta} = M\eta + N\xi_1$$

is an internal model for (1.6).

Second Step: Use the following change of coordinates:

$$\begin{aligned}\hat{\eta} &= \eta - \hat{\tau}(\omega, \mu) - Nb^{-1}(\mu)e \\ \hat{z} &= z - \varsigma(\overline{\omega}, \mu) \\ e &= y - q(\omega, \mu) \\ \hat{\xi}_i &= \xi_i - \Psi_i\eta, \qquad 1 \le i \le r-1 \\ \hat{u} &= u - \Psi_r\eta\end{aligned}$$

and denote

$$\begin{aligned}Z &= [\hat{\eta}, \hat{z}]^T \\ C_1 &= [I_{v*v}, 0] \\ C_2 &= [0, \ldots, 0, 1]\end{aligned}$$

$$\hat{G}(e, \omega, \mu)\cdot e = \overline{G}(e + q(\omega, \mu), \omega, \mu) - \overline{G}(q(\omega, \mu), \omega, \mu)$$

$$\hat{K}(e, \omega, \mu)\cdot e = \overline{K}(e + q(\omega, \mu), \omega, \mu) - \overline{K}(q(\omega, \mu), \omega, \mu)$$

$$R(\mu) = \begin{bmatrix} M & -b^{-1}(\mu)N\overline{H}(\mu) \\ 0 & \overline{F}(\mu)\end{bmatrix}$$

$$L(e, \omega, \mu) = \begin{bmatrix} b^{-1}(\mu)(MN - N\hat{K}(e, \omega, \mu)) \\ \hat{G}(e, \omega, \mu)\end{bmatrix}.$$

It is easy to check that (1.6) in the new coordinates has the following triangular form:

$$\begin{aligned}\dot{Z} &= R(\mu)Z + L(e, \omega, \mu)e \\ \dot{e} &= \overline{H}(\mu)C_2Z + b(\mu)\Psi_1C_1Z \\ &\quad + \left(\hat{K}(e, \omega, \mu) + \Psi_1N\right)\cdot e + b(\mu)\hat{\xi}_1 \\ \dot{\hat{\xi}}_1 &= -\lambda_1\hat{\xi}_1 - \Psi_1N\hat{\xi}_1 + \hat{\xi}_2 \\ &\vdots \\ \dot{\hat{\xi}}_{r-1} &= -\lambda_{r-1}\hat{\xi}_{r-1} - \Psi_{r-1}N\hat{\xi}_1 + \hat{u}.\end{aligned} \tag{2.4}$$

Clearly, if system (2.4) is globally asymptotically stabilized by measurement feedback $(e, \hat{\xi}_1, \ldots, \hat{\xi}_{r-1})$, the same controller also solves the global output regulation problem for the original system (1.1).

III. Main Result

A main difficulty in stabilizing system (2.4) is due to the unmeasurable exogenous signals and the unknown parameters that enter the system nonlinearly. In this section, we will demonstrate how to utilize the *variable separation technique* [7], combined with the *feedback domination design* method [6], to globally stabilize system (2.4). Note that the result below does not require bounds of the exogenous signals and the unknown parameters to be known, which has been a common condition in the literature such as [11], [3]. The following lemma is useful when dealing with a nonlinear parameterization problem.

Lemma 2 [7], [8]: For any real-valued continuous function $f(x, y)$, where $x \in \mathbb{R}^m$, $y \in \mathbb{R}^n$, there are smooth scalar functions $a(x) \geq 0$, $b(y) \geq 0$, $c(x) \geq 1$ and $d(y) \geq 1$, such that

$$|f(x, y)| \leq a(x) + b(y)$$
$$|f(x, y)| \leq c(x)\, d(y).$$

Using this lemma and the feedback domination design method [8], we can prove the following result.

Theorem 1: Under Assumptions 1–5, global output regulation of the uncertain system (1.1) is achievable by an error feedback of the form (1.2).

Proof: As discussed in the previous section, one needs only to design an adaptive controller for system (2.4) making the closed-loop system globally asymptotically stable. By construction, $R(\mu)$ is a Hurwitz matrix. Thus, there is a matrix $P(\mu) = P^T(\mu) > 0$ satisfying

$$R^T(\mu)P(\mu) + P(\mu)R(\mu) \leq -2I.$$

Recall that both $\omega(t)$ and μ are in compact sets whose bounds are unknown. By Lemma 2

$$\begin{aligned}
\|P(\mu)L(e, \omega, \mu)\|^2 &\leq \overline{\theta}_1(\omega, \mu)\alpha_1(e) \leq \theta_1\alpha_1(e)\\
\|\hat{K}(e, \omega, \mu) + \Psi_1 N\|^2 &\leq \overline{\theta}_2(\omega, \mu)\alpha_2(e) \leq \theta_2\alpha_2(e)\\
\|\overline{H}(\mu)\|^2 &\leq \theta_3\\
|b(\mu)|^2 &\leq \theta_4
\end{aligned}$$

where $\alpha_1(e) \geq 1$, $\alpha_2(e) \geq 1$ are smooth known functions and $\theta_i \geq 1$, $i = 1, \ldots, 4$, are unknown constants. Denote $\Theta = \max\{\theta_i, i = 1, \ldots, 4\}$ as a new unknown parameter. Without lose of generality, one can assume $b(\mu) > 0$. Now, consider the Lyapunov function

$$V_0(Z, e, \hat{\Theta}) = \frac{r+1}{2} Z^T P(\mu) Z + \frac{1}{2b(\mu)} e^2 + \frac{1}{2}\tilde{\Theta}^2$$

where $\tilde{\Theta} := \Theta - \hat{\Theta}$ and $\hat{\Theta}$ is the estimation of Θ. A direct calculation gives

$$\begin{aligned}
\dot{V}_0 \leq& -(r+1)Z^T Z + (r+1)Z^T P(\mu)L(e, \omega, \mu)e + \frac{e}{b(\mu)}\\
&\cdot [\overline{H}(\mu)C_2 Z + b(\mu)\Psi_1 C_1 Z + (\hat{K}(e, \omega, \mu) + \Psi_1 N)e + b(\mu)\hat{\xi}_1]\\
&- (\Theta - \hat{\Theta})\dot{\hat{\Theta}}.
\end{aligned} \tag{3.1}$$

By the completion of squares, one has

$$\begin{aligned}
(r+1)Z^T P(\mu)L(e, \omega, \mu)e &\leq \frac{1}{3} Z^T Z + \frac{3(r+1)^2}{4}\Theta\alpha_1(e)e^2\\
\frac{e}{b(\mu)}\overline{H}(\mu)C_2 Z &\leq \frac{1}{3} Z^T Z + \frac{3}{4b_0^2}\Theta e^2\\
e\Psi_1 C_1 Z &\leq \frac{1}{3} Z^T Z + \frac{3}{4}\|\Psi_1 C_1\|^2 e^2\\
\frac{e}{b(\mu)}(\hat{K}(e, \omega, \mu) + \Psi_1 N)e &\leq \frac{\Theta}{b_0}\alpha_2(e)e^2.
\end{aligned}$$

From the aforementioned inequalities, it follows that

$$\dot{V}_0 \leq -rZ^T Z + e\hat{\xi}_1 + e^2\left(\rho_1(e)\hat{\Theta} + \frac{3}{4}\|\Phi_1 C_1\|^2\right) + \tilde{\Theta}\left(\rho_1(e)e^2 - \dot{\hat{\Theta}}\right)$$

where

$$\rho_1(e) = \frac{3(r+1)^2}{4}\alpha_1(e) + \frac{1}{b_0}\alpha_2(e) + \frac{3}{4b_0^2}.$$

Clearly, the virtual controller

$$\hat{\xi}_1^* = -\hat{\rho}_1(e, \hat{\Theta})e := -\left[r + \frac{3}{4}\|\Phi_1 C_1\|^2 + \hat{\Theta}\rho_1(e)\right]e$$

is such that

$$\dot{V}_0 \leq -rZ^T Z - re^2 + e\tilde{\xi}_1 + \tilde{\Theta}\left(\rho_1(e)e^2 - \dot{\hat{\Theta}}\right), \qquad \tilde{\xi}_1 = \hat{\xi}_1 - \hat{\xi}_1^*.$$

Step 1: Construct the Lyapunov function $V_1(Z, e, \hat{\xi}_1, \hat{\Theta}) = V_0(Z, e, \hat{\Theta}) + (1/2)\tilde{\xi}_1^2$. Then

$$\begin{aligned}
\dot{V}_1 \leq& -rZ^T Z - re^2 + e\tilde{\xi}_1 + \tilde{\Theta}\left(\rho_1(e)e^2 - \dot{\hat{\Theta}}\right)\\
&+ \tilde{\xi}_1(-\Psi_1 N\hat{\xi}_1 - \lambda_1\hat{\xi}_1 + \hat{\xi}_2) - \tilde{\xi}_1\frac{\partial\hat{\xi}_1^*}{\partial e}[\overline{H}(\mu)C_2 Z\\
&+ b(\mu)\Psi_1 C_1 Z + (\hat{K} + \Psi_1 N)e + b(\mu)\hat{\xi}_1] - \tilde{\xi}_1\frac{\partial\hat{\xi}_1^*}{\partial\hat{\Theta}}\dot{\hat{\Theta}}.
\end{aligned} \tag{3.2}$$

By Lemma 2 and the completion of squares, it is easy to show that

$$-\tilde{\xi}_1\frac{\partial\hat{\xi}_1^*}{\partial e}\overline{H}(\mu)C_2 Z \leq \frac{1}{2} Z^T Z + \frac{1}{2}\left(\frac{\partial\hat{\xi}_1^*}{\partial e}\right)^2\Theta\tilde{\xi}_1^2 \tag{3.3}$$

$$-\tilde{\xi}_1\frac{\partial\hat{\xi}_1^*}{\partial e}b(\mu)\Psi_1 C_1 Z \leq \frac{1}{2} Z^T Z + \frac{1}{2}\left(\frac{\partial\hat{\xi}_1^*}{\partial e}\right)^2\Theta\|\Psi_1 C_1\|^2\tilde{\xi}_1^2 \tag{3.4}$$

$$-\tilde{\xi}_1\frac{\partial\hat{\xi}_1^*}{\partial e}(\hat{K} + \Psi_1 N)e \leq \frac{1}{2}e^2 + \frac{1}{2}\left(\frac{\partial\hat{\xi}_1^*}{\partial e}\right)^2\alpha_2(e)\tilde{\xi}_1^2\Theta \tag{3.5}$$

$$\begin{aligned}
-\tilde{\xi}_1\frac{\partial\hat{\xi}_1^*}{\partial e}b(\mu)\hat{\xi}_1 \leq& \frac{1}{2}e^2 + \frac{1}{2}\tilde{\xi}_1^2\left(\frac{\partial\hat{\xi}_1^*}{\partial e}\right)^2 + \frac{1}{2}\tilde{\xi}_1^2\Theta\\
&+ \frac{1}{2}\tilde{\xi}_1^2\left(\frac{\partial\hat{\xi}_1^*}{\partial e}\right)^2\hat{\rho}_1^2(e, \hat{\Theta})\Theta.
\end{aligned} \tag{3.6}$$

Substituting (3.3)–(3.6) into (3.2) yields

$$\begin{aligned}
\dot{V}_1 \leq& -(r-1)Z^T Z - (r-1)e^2 + \tilde{\xi}_1\hat{\xi}_2\\
&+ \tilde{\xi}_1\left[\tilde{\xi}_1\rho_2\hat{\Theta} + e - \Psi_1 N\hat{\xi}_1 - \lambda_1\hat{\xi}_1 + \frac{1}{2}\tilde{\xi}_1\left(\frac{\partial\hat{\xi}_1^*}{\partial e}\right)^2\right.\\
&\left. - \frac{\partial\hat{\xi}_1^*}{\partial\hat{\Theta}}(\rho_2\tilde{\xi}_1^2 + \rho_1 e^2)\right]\\
&+ \left(\tilde{\xi}_1\frac{\partial\hat{\xi}_1^*}{\partial\hat{\Theta}} + \tilde{\Theta}\right)\left(\rho_2\tilde{\xi}_1^2 + \rho_1 e^2 - \dot{\hat{\Theta}}\right)
\end{aligned} \tag{3.7}$$

where

$$\rho_2(e, \hat{\Theta}) = \frac{1}{2}\left(\frac{\partial \hat{\xi}_1^*}{\partial e}\right)^2 + \frac{1}{2}\left(\frac{\partial \hat{\xi}_1^*}{\partial e}\right)^2 \|\Psi_1 C_1\|^2 + \frac{1}{2}\left(\frac{\partial \hat{\xi}_1^*}{\partial e}\right)^2 \alpha_2(e) + \frac{1}{2}\left(\frac{\partial \hat{\xi}_1^*}{\partial e}\right)^2 \hat{\rho}_1^2(e, \hat{\Theta}) + \frac{1}{2}.$$

Observe that the smooth virtual controller

$$\hat{\xi}_2^*(e, \hat{\xi}_1, \hat{\Theta}) = -(r-1)\tilde{\xi}_1 - e + \Psi_1 N\hat{\xi}_1 + \lambda_1\hat{\xi}_1 - \frac{1}{2}\left(\frac{\partial \hat{\xi}_1^*}{\partial e}\right)^2 \tilde{\xi}_1 - \rho_2\hat{\Theta}\tilde{\xi}_1 + \frac{\partial \hat{\xi}_1^*}{\partial \hat{\Theta}}\left(\rho_2\tilde{\xi}_1^2 + \rho_1 e^2\right)$$

renders

$$\dot{V}_1 \le -(r-1)Z^T Z - (r-1)e^2 - (r-1)\tilde{\xi}_1^2 + \tilde{\xi}_1\tilde{\xi}_2 + (\psi_1 + \tilde{\Theta})\left(\phi_1 - \dot{\hat{\Theta}}\right)$$

where $\psi_1(e, \hat{\xi}_1, \hat{\Theta}) = \tilde{\xi}_1(\partial\hat{\xi}_1^*/\partial\hat{\Theta})$, $\phi_1(e, \hat{\xi}_1, \hat{\Theta}) = \rho_2\tilde{\xi}_1^2 + \rho_1 e^2$, $\tilde{\xi}_2 = \hat{\xi}_2 - \hat{\xi}_2^*$. By construction, $\hat{\xi}_2^*(0, 0, \hat{\Theta}) = 0$, $\psi_1(0, 0, \hat{\Theta}) = 0$ and $\phi_1(0, 0, \hat{\Theta}) = 0$.

Inductive Step: Suppose at the ith step that there is a set of smooth virtual controllers

$$\hat{\xi}_j^* = \beta_j(e, \hat{\xi}_1, \ldots, \hat{\xi}_{j-1}, \hat{\Theta})$$

with $\beta_j(0, \ldots, 0, \hat{\Theta}) = 0$, $j = 1, 2, \ldots, i$, and a smooth Lyapunov function $V_{i-1}(Z, e, \hat{\xi}_1, \ldots, \hat{\xi}_{i-1}, \hat{\Theta})$, which is positive definite and proper, such that

$$\dot{V}_{i-1} \le -(r-i+1)Z^T Z - (r-i+1)e^2 - (r-i+1)\tilde{\xi}_1^2 - \tilde{\xi}_2^2 - \cdots - \tilde{\xi}_{i-1}^2 + \tilde{\xi}_{i-1}\tilde{\xi}_i + (\psi_{i-1} + \tilde{\Theta})\left(\phi_{i-1} - \dot{\hat{\Theta}}\right) \quad (3.8)$$

where $\tilde{\xi}_j = \hat{\xi}_j - \hat{\xi}_j^*$, $j = 1, 2, \ldots, i$, and $\psi_{i-1}(0, \ldots, 0, \hat{\Theta}) = \phi_{i-1}(0, \ldots, 0, \hat{\Theta}) = 0, \forall \hat{\Theta} \in \mathbb{R}$.

At step $i+1$, we prove that (3.8) holds as well. For, consider the Lyapunov function

$$V_i(Z, e, \hat{\xi}_1, \ldots, \hat{\xi}_i, \hat{\Theta}) = V_{i-1}(Z, e, \hat{\xi}_1, \ldots, \hat{\xi}_{i-1}, \hat{\Theta}) + \frac{1}{2}\tilde{\xi}_i^2.$$

Then

$$\begin{aligned}\dot{V}_i \le& -(r-i+1)Z^T Z - (r-i+1)e^2 - (r-i+1)\tilde{\xi}_1^2 - \tilde{\xi}_2^2 \\ &- \cdots - \tilde{\xi}_{i-1}^2 + \tilde{\xi}_{i-1}\tilde{\xi}_i + (\psi_{i-1} + \tilde{\Theta})\left(\phi_{i-1} - \dot{\hat{\Theta}}\right) \\ &+ \tilde{\xi}_i\hat{\xi}_{i+1} - \tilde{\xi}_i\lambda_i\hat{\xi}_i - \tilde{\xi}_i\Psi_i N\hat{\xi}_1 \\ &- \tilde{\xi}_i\left(\frac{\partial\beta_i}{\partial e}\dot{e} + \frac{\partial\beta_i}{\partial\hat{\xi}_1}\dot{\hat{\xi}}_1 + \cdots + \frac{\partial\beta_i}{\partial\hat{\xi}_{i-1}}\dot{\hat{\xi}}_{i-1} + \frac{\partial\beta_i}{\partial\hat{\Theta}}\dot{\hat{\Theta}}\right).\end{aligned}$$

Using an argument similar to the ones in the previous step, we have

$$\begin{aligned}\dot{V}_i \le& -(r-i)Z^T Z - (r-i)e^2 - (r-i)\tilde{\xi}_1^2 \\ &- \tilde{\xi}_2^2 - \cdots - \tilde{\xi}_{i-1}^2 + \tilde{\xi}_i\hat{\xi}_{i+1} \\ &+ \tilde{\xi}_i\left[\tilde{\xi}_{i-1} - \Psi_1 N\hat{\xi}_1 - \lambda_i\hat{\xi}_i - \sum_{j=1}^{i-1}\frac{\partial\beta_i}{\partial\hat{\xi}_j}\dot{\hat{\xi}}_j + \rho_{i+1}\tilde{\xi}_i\hat{\Theta}\right. \\ &\left. - \frac{\partial\beta_i}{\partial\hat{\Theta}}\rho_{i+1}\tilde{\xi}_i^2 - \frac{\partial\beta_i}{\partial\hat{\Theta}}\phi_{i-1} - \rho_{i+1}\tilde{\xi}_i\psi_{i-1}\right] \\ &+ \left(\tilde{\xi}_i\frac{\partial\beta_i}{\partial\hat{\Theta}} + \psi_{i-1} + \tilde{\Theta}\right)\left(\rho_{i+1}\tilde{\xi}_i^2 + \phi_{i-1} - \dot{\hat{\Theta}}\right)\end{aligned}$$

where

$$\rho_{i+1} = \left(\frac{\partial\beta_i}{\partial e}\right)^2\left(\frac{3}{4} + \frac{1}{2}\|\Psi_1 C_1\|^2 + \frac{1}{2}\alpha_2(e) + \frac{1}{2}\hat{\rho}_1^2(e, \hat{\Theta})\right).$$

Therefore, the smooth virtual controller

$$\hat{\xi}_{i+1}^* = -\tilde{\xi}_i - \tilde{\xi}_{i-1} + \Psi_1 N\hat{\xi}_1 + \lambda_i\hat{\xi}_i + \sum_{j=1}^{i-1}\frac{\partial\beta_i}{\partial\hat{\xi}_j}\dot{\hat{\xi}}_j - \rho_{i+1}\tilde{\xi}_i\hat{\Theta} + \frac{\partial\beta_i}{\partial\hat{\Theta}}\rho_{i+1}\tilde{\xi}_i^2 + \frac{\partial\beta_i}{\partial\hat{\Theta}}\phi_{i-1} + \rho_{i+1}\tilde{\xi}_i\psi_{i-1}$$

renders

$$\dot{V}_i \le -(r-i)Z^T Z - (r-i)e^2 - (r-i)\tilde{\xi}_1^2 - \tilde{\xi}_2^2 - \cdots - \tilde{\xi}_i^2 + \tilde{\xi}_i\tilde{\xi}_{i+1} + (\psi_i + \tilde{\Theta})\left(\phi_i - \dot{\hat{\Theta}}\right)$$

where $\psi_i(e, \hat{\xi}_1, \ldots, \hat{\xi}_i, \hat{\Theta}) = \psi_{i-1} + \tilde{\xi}_i(\partial\beta_i/\partial\hat{\Theta})$ and $\phi_i(e, \hat{\xi}_1, \ldots, \hat{\xi}_i, \hat{\Theta}) = \phi_{i-1} + \rho_{i+1}\tilde{\xi}_i^2$.

Using this inductive argument, we conclude that at step $r-1$, there are a smooth controller

$$\hat{u} = \beta_r(e, \hat{\xi}_1, \ldots, \hat{\xi}_{r-1}, \hat{\Theta}) \quad \text{with} \quad \beta_r(0, \ldots, 0, \hat{\Theta}) = 0 \quad (3.9)$$

and a smooth Lyapunov function $V_{r-1}(Z, e, \hat{\xi}_1, \ldots, \hat{\xi}_{r-1}, \hat{\Theta})$, which is positive definite and proper, such that

$$\dot{V}_{r-1} \le -Z^T Z - e^2 - \tilde{\xi}_1^2 - \tilde{\xi}_2^2 - \cdots - \tilde{\xi}_{r-1}^2 + (\psi_{r-1} + \tilde{\Theta})\left(\phi_{r-1} - \dot{\hat{\Theta}}\right).$$

Therefore, the controller (3.9), together with the adaptive law

$$\dot{\hat{\Theta}} = \phi_{r-1}(e, \hat{\xi}_1, \ldots, \hat{\xi}_{r-1}, \hat{\Theta}) \quad (3.10)$$

is such that

$$\dot{V}_{r-1} \le -Z^T Z - e^2 - \tilde{\xi}_1^2 - \tilde{\xi}_2^2 - \cdots - \tilde{\xi}_{r-1}^2.$$

In other words, the closed-loop system (2.4)—(3.10) is globally stable. Furthermore, by LaSalle's invariance principle and the properties of the virtual controllers $\hat{\xi}_j^*$, it is easy to see that

$$\lim_{t\to\infty}(\|Z\|^2 + e^2 + \hat{\xi}_1^2 + \cdots + \hat{\xi}_{r-1}^2) = 0.$$

This, in turn, implies that in the original coordinates, all the states $x, y, \xi_i, i = 1, \ldots, r-1$, and $\hat{\Theta}$ are all globally bounded and $\lim_{t\to\infty} e(t) = 0$, i.e., global output regulation of (1.1) is achieved. ■

We conclude this section with a simple example that demonstrates the application of Theorem 1. Consider the planar system with the three-dimensional exosystem

$$\begin{aligned}\dot{x}_1 &= -x_1 + (x_2 + \mu_1)e^{x_2} + u & & \dot{\omega}_1 = 0 \\ \dot{x}_2 &= x_1 + \mu_2 x_2 + \omega_2^2 & \text{with} \quad & \dot{\omega}_2 = \omega_3 \\ e &= x_2 - \omega_1 & & \dot{\omega}_3 = -\omega_2.\end{aligned} \quad (3.11)$$

Note that this system is not minimum-phase with respect to the output $y = x_2$, and involves a nonpolynomial nonlinearity. It is easy to check that (3.11) satisfies Assumptions 1–5 with

$$\begin{aligned}\pi(\omega, \mu) &= [-\mu_2\omega_1 - \omega_2^2, \ \omega_1]^\top \\ c(\omega, \mu) &= -\mu_2\omega_1 - (\mu_1 + \omega_1)e^{\omega_1} - 2\omega_2\omega_3 - \omega_2^2.\end{aligned}$$

Following the procedure described in Sections II and III, we can design the dynamic controller

$$\begin{aligned}\dot{\eta} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -3 \end{bmatrix}\eta + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\xi \\ \dot{\xi} &= -\xi + u \\ \dot{\hat{\Theta}} &= \rho_1 y^2 + \rho_2(\hat{\xi} - \hat{\xi}^*)^2\end{aligned}$$

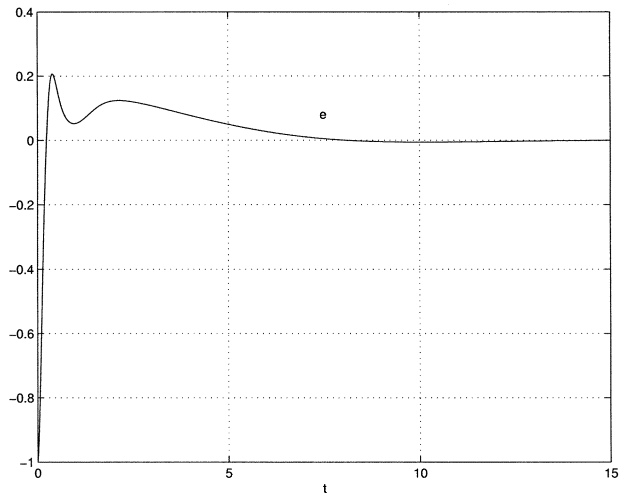

Fig. 1. Transiwnt response of the error signal of (3.11).

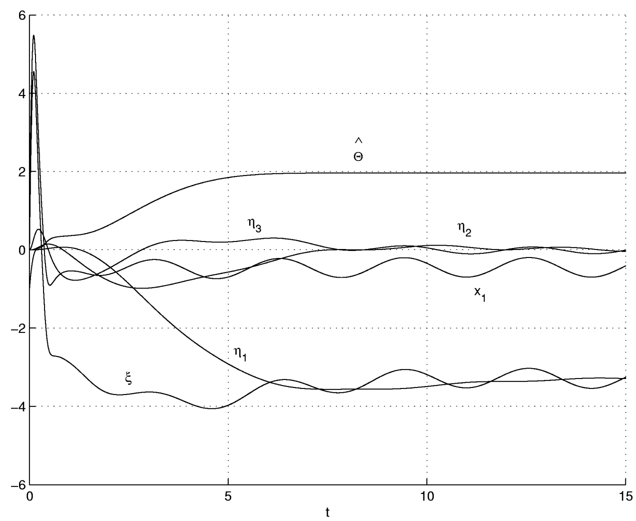

Fig. 2. Transient response of the states of (3.11).

where

$$\begin{aligned} u =& \hat{\xi}^* - y + 3(\xi - \eta_1 + \eta_2 - 3\eta_3) \\ &- (12 + \hat{\Theta})(\xi - \eta_1 + \eta_2 - 3\eta_3 - \hat{\xi}^*)\rho_2 \\ &+ \frac{\partial \hat{\xi}^*}{\partial y}(\xi - \eta_1 + \eta_2 - 3\eta_3) - \rho_1 y(\rho_1 y^2 + \rho_2(\hat{\xi} - \hat{\xi}^*)^2) \\ &+ \eta_1 - \eta_2 + 3\eta_3 \\ \rho_1 =& 1 + 4.5(e^y + 1)^2 \qquad \hat{\xi}^* = -8y - \rho_1 y \hat{\Theta} \\ \frac{\partial \hat{\xi}^*}{\partial y} =& -8 - \rho_1\hat{\Theta} - 9y\hat{\Theta}(e^y + e^{2y}) \qquad \rho_2 = 0.25\left(\frac{\partial \hat{\xi}^*}{\partial y}\right)^2. \end{aligned}$$

Figs. 1 and 2 show simulation results when $\mu_1 = 0.05$, $\mu_2 = 0.2$, the disturbance signal $\omega_2(t) = 0.5\sin(t)$ and the reference signal $\omega_1(t) = 1$. The initial conditions are $(x_1(0), x_2(0)) = (1, 0)$, $\eta(0) = 0$, $\xi(0) = 0$ and $\hat{\Theta}(0) = -1$. Fig. 1 is the error signal which converges to zero and Fig. 2 shows that all the states of the closed-loop system, i.e., $(x_1, x_2, \eta, \xi, \hat{\Theta})$, are bounded.

IV. Conclusion

Using the variable separation technique (Lemma 2) and the nonlinear adaptive control method proposed in [7] and [8], we have presented a solution to the problem of global robust output regulation for a class of uncertain nonlinear systems driven by a linear, neutrally stable exosystem. The merit of our method is that it can deal with the case where the bounds of the exogenous signals and parameters are unknown, and thus removing the common requirement in the literature, i.e., the unknown parameters and exogenous signals must be in a *known* compact set.

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