Title
TOPICS IN DUAL MODELS AND EXTENDED SOLUTIONS

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ABSTRACT

Two main topics are explored. The first deals with the infinities arising from the one loop planar string diagram of the standard dual model. It is shown that for the number of dimensions d = 25 or 26, these infinities lead to a renormalization of the slope of the Regge trajectories, in addition to a renormalization of the coupling constant. The second topic deals with the propagator for a confined particle (monopole) in a field theory. When summed to all orders, this propagator is altogether free of singularities in the finite momentum plane, and an attempt is made to illustrate this. We examine the Bethe-Salpeter equation and show that ladder diagrams are not sufficient to obtain this result. However, in a nonrelativistic approximation confinement is obtained and all poles disappear.

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I. INTRODUCTION

The two major approaches to elementary particle physics in recent years have been quantum field theory and S-matrix theory. The former tries to derive physically relevant quantities from interactions of relativistic quantum fields. The latter insists that only the scattering amplitude is physically important. It then tries to derive what it can from properties it demands of the S-matrix. Dual models started out in the radical wing of the S-matrix camp. The original Veneziano formula[1] was proposed as simply an example of a scattering amplitude that satisfied certain criteria. At present dual models stand almost on the borderline between S-matrix and field theories, with many workers trying to obtain the extended structures encountered in dual models from bona fide field theories. In this thesis we present two pieces of work. In Part I, a proof is given that in the simplest dual model, divergences can be renormalized at the one loop level. In Part II, a detailed examination of the monopole propagator is given for a theory with confined monopole-antimonopole pairs. This theory is relevant to dual models in the sense that the string of magnetic flux connecting the monopole-antimonopole pair is a linear extended structure similar to the dual string.

II. SLOPE RENORMALIZATION OF THE ONE LOOP PLANAR STRING DIAGRAM

Numerous excellent review articles on dual models have appeared in the literature.[2] We present here only a brief summary of some of the major developments in the field as an introduction. Veneziano's four point function[1] was soon generalized to n particles by a number of authors.[3] By factorization, this gave the complete S-matrix. This S-matirx has linear Regge trajectories, Regge asymptotic behavior, duality, factorization, and the statistical model density of states. The particles have zero width, but it was hoped that higher order corrections would cure this problem. After this an operator formalism was developed[4] in which factorization is apparent, and ghosts were proved to decouple.[5] Next it was realized that the whole formalism is equivalent to the quantum mechanics of massless relativistic strings.[5,6] Interactions are introduced as the splitting and joining of strings and the amplitude can be determined totally by the topology of the string diagram.[7] In particular, the amplitude equals the functional average of exp (i × Action) for all $X^{\mu}(\sigma,\tau)$ satisfying the boundary conditions of the string diagram. Higher order corrections can then naturally be obtained from string diagrams with loops. The planar loop gives the resonances widths, as had been hoped, but the nonplanar loop generates the Pomeron trajectory, which has twice the intercept and half the slope of the ordinary Regge trajectories.[8] Before going on to a discussion of renormalization, we mention the two major defects of the model that we have so far been sweeping under the rug. First, the theory is only consistent in an unphysical number of dimensions, 26 for the original model, 10 for another (Neven-Schwarz' model[9] with a spinning string. Secondly, the intercepts are too high, 1 for the ordinary Regge trajectories, 2 for the Pomeron trajectory.

In dual models, infrared divergences appear due to the presence of zero mass particles. For the number of dimensions less than 25, Neven and Scherk[10] proved that the infinities of the planar loop can be absorbed into a renormalization of the coupling constant. The case of dimensions 25 or 26 is treated in the following paper, where it is shown that the infinities lead to a renormalization not only of the coupling constant, but also of the slope of the Regge trajectories.

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Slope renormalization of the one-loop planar string diagram*

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It has been previously shown that for the number of dimensions $d < 25$, the one-loop planar string diagram is simply a multiplicative (coupling constant) renormalization of the Born term. It is shown here that for $d = 25$ or 26 the extra divergent term gives, in addition to a further multiplicative renormalization, a renormalization of the slope of the Regge trajectories.

I. INTRODUCTION

The N-point Veneziano amplitude is a tree-diagram approximation to the strong-interaction scattering amplitude. Thus, one is led to examine loop diagrams in the hope of obtaining a more realistic amplitude. As in quantum electrodynamics, these diagrams are found to be divergent. A renormalization procedure must then be found in which all infinities are absorbed into a redefinition of the physical parameters of the theory. This was done for the one-loop planar diagram for $d<25$ by Neveu and Scherk,[1] and the result was found to be a simple multiplicative renormalization of the Born term. In this paper we do this for the one-loop planar diagram in the critical number of dimensions. We show that the divergent part of the amplitude can at the one-loop level be written as

$$c_1\left(B_N(m, s_{ij}) + c_2 \frac{\partial B(m, s_{ij})}{\partial m}\right) = c_1 B_N(m + c_2, s_{ij}),$$

where c_1, c_2 are constants, s_{ij} are the planar subenergies, m is the slope of the Regge trajectories, and B_N is the Veneziano amplitude. Thus, at this level, multiplicative and slope renormalizations are all that are needed to render the integral finite.

II. METHOD

In the interacting-string picture, the single-loop planar amplitude for N scalars is given by the following expression, after a Jacobi transformation has been made on the usual variables of integration[2]:

$$\int_0^1 dq \int_0^{2\pi} d\varphi_1 \int_0^{\varphi_1} d\varphi_2 \cdots \int_0^{\phi_{N-2}} d\phi_{N-1}\, q^{-(d+11)/12} f(\ln q) \prod_{1\le r<s\le N} \exp[-p_r\cdot p_s N(\rho_r, \rho_s)], \tag{1}$$

where

$$\exp[-p_r\cdot p_s N(\rho_r,\rho_s)] = \left\{-\frac{4\pi}{\ln q}\sin\tfrac{1}{2}(\phi_r-\varphi_s)\left[\prod_{n=1}^{\infty}(1-q^{2n}e^{i(\phi_r-\phi_s)})(1-q^{2n}e^{-i(\phi_r-\phi_s)})(1-q^{2n})^{-2}\right]\right\}^{-2p_r\cdot p_s},$$

and where $f(\ln q)$ is a function of $\ln q$ (no powers), $\phi_N = 0$, and the factor $q^{-(d+11)/12}$ should read instead q^{-2} for the special case $d=26$. The integral diverges near $q=0$; that is, the region where the loop shrinks to a point. We note for later use that this point (the loop at $q=0$) is located at $i\infty$ in the φ plane. To examine the integral in the $q=0$ region, we expand the expression in large square brackets in a power series in q^2 and find that for $d<25$, only the constant term leads to a divergent integral. This term was shown by Neveu and Scherk[1] to be simply a multiplicative renormalization of the Veneziano amplitude. For $d\ge 25$, the linear term in q^2 in the power-series expansion also leads to a divergent integral. It is this integral that we examine here. We change to the more convenient variables of integration $u_i = \tan\frac{1}{2}\varphi_i$. In the limit $q\to 0$, these u variables are related to the string-diagram variables ρ by the usual tree-diagram transformation

$$\rho = \sum_{r=1}^{N} \alpha_r \ln(u - u_r), \tag{2}$$

with the cut now at $u = i$. We then obtain as the coefficient of the infinite q integration

$$\sum_{1\le a<b\le N} 2p_a\cdot p_b \int_0^0 du_1 \int_0^{u_1} du_2 \cdots \int_0^{u_{N-2}} du_{N-1} \frac{(u_a-u_b)^2}{(u_a^2+1)(u_b^2+1)} \prod_{1\le i<j\le N} (u_i-u_j)^{-2p_i\cdot p_j}, \tag{3}$$

where the range of integration is restricted only by $u_N \equiv 0 < u_{N-1} < \cdots < u_1 < 0 \equiv u_N$ (which is just a convenient notation for $0 < u_{N-1} < \cdots < u_r < \infty$ and $-\infty < u_{r-1} < \cdots < u_1 < 0$ for some r).

Even after factoring out the infinite q integral, we find that the remaining integral (3) diverges. This remaining divergence cannot still be due to the loop shrinking to a point since the integral (3) diverges only for particular configurations of the u's. In fact, the remaining divergence is due to configurations of the one-loop diagram that correspond to external-line self-energy insertions. Suppose we factor out the self-energy part in one of these configurations. Then we are left with a tree-level diagram with exactly the same incoming states and momenta, or else the contribution is not divergent. Thus, we expect our divergences to be simply an infinite constant multiple of B_N.

In order to evaluate the contribution of (3) to the amplitude, we must first choose some cutoff procedure rendering the integral finite. To this end, we temporarily suspend momentum conservation by introducing a new incoming momentum k (see Fig. 1). We tentatively choose it to enter the string diagram at the position of the loop (remember that we have taken the limit $q\to 0$ which corresponds to the loop shrinking to a point), but we shall see that we will have to modify this slightly. We expect this procedure to eliminate our infinities, since now all self-energy insertions have incoming and outgoing momenta which differ by k. In the limit $k\to 0$, we should then recover a constant multiple of B_N as the divergent part. Thus, instead of the normal energy-momentum conservation equation, we have

$$\sum_{i=1}^{N} p_i + k = 0,$$

where k is the new momentum introduced.

This new momentum introduces an extra term

$$i\sum_{i=1}^{d-2} k_i X^i(\rho_c)$$

to the exponential of the functional integral for the S matrix. This leads to the extra term in (1)

$$\exp\left[\sum_{i=1}^{d-2}\sum_{r=1}^{N} k_i p_r^i N(\rho_r,\rho_c) + \tfrac{1}{2}N(\rho_c,\rho_c)\sum_{i=1}^{d-2} k_i^2\right].$$

The second term, which in infinite, is similar to an infinite term obtained in the conventional path-integral interacting-string formalism.[2] As in the latter case, it can be absorbed into the volume element since it has no dependence on the integration variables. The first term in the exponent changes (3) to the expression

$$\sum_{1\le a<b\le N} 2p_a\cdot p_b \int_0^0 du_1 \int_0^{u_1} du_2 \cdots \int_0^{u_{N-2}} du_{N-1} \frac{(u_a-u_b)^2}{(u_a^2+1)(u_b^2+1)} \prod_{1\le i<j\le N} (u_i-u_j)^{-2p_i\cdot p_j} \prod_r\left[\frac{(u_r-u_c)^2}{u_r^2+1}\right]^{-p_r\cdot k}$$

$$+\sum_a 2p_a\cdot k \int_0^0 du_1 \int_0^{u_1} du_2 \cdots \int_0^{u_{N-2}} du_{N-1} \frac{(u_a-u_c)^2}{(u_a^2+1)(u_c^2+1)} \prod_{1\le i<j\le N} (u_i-u_j)^{-2p_i\cdot p_j} \prod_r\left[\frac{(u_r-u_c)^2}{u_r^2+1}\right]^{-p_r\cdot k}, \tag{4}$$

where we have defined $k_1=0$ to change the $d-2$ product to a covariant d product, $u_c\equiv i$ is the point to which the loop has been mapped, and we have neglected terms in k^2 since they are second order in a small quantity. The fact that $(u_c^2+1)^{-1} = (i^2+1)^{-1}$ appearing in the second term in undefined is a point we shall deal with later. We can write the last factor of the first term in (4) as

$$\prod_r\left[\frac{(u_r-u_c)^2}{u_r^2+1}\right]^{-p_r\cdot k} = \exp\left[-\sum_r p_r\cdot k \ln\left(\frac{(u_r-u_c)^2}{u_r^2+1}\right)\right]$$
$$= 1-\sum_r p_r\cdot k \ln\left(\frac{(u_r-u_c)^2}{u_r^2+1}\right) + O(k^2). \tag{5}$$

This expansion is valid in the range of integration since the u_r's are real and $u_c=i$, so that the argu-

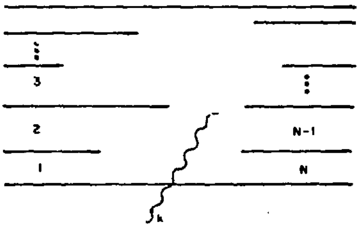

FIG. 1. N-point function with new momentum k entering.

ment of the logarithm never blows up. Also since the logarithm is always well behaved the convergence properties of all the terms in the series are the same. Thus, since we will see that the first term in the series behaves as k^{-1} as $k \to 0$ (this behavior is expected of an *external-line self-energy insertion*), we can neglect the terms of order k^2 and higher in (5). Thus, we have left the first two terms in the expansion (5) in addition to the second term in (4). We refer to these throughout the rest of this paper as terms I, II, and III. We point out that although term I has exactly the same *form* as the original divergent expression (3), it is now well behaved due to the new energy-momentum conservation equation.

III. EVALUATION OF THE N-POINT FUNCTION

Consider term I for the N-point function for a particular choice of a, $b \neq N$ and $u_a < 0$ (just as an illustration). Then we have

$$2p_a\cdot p_b \int_{-\infty}^{0} du_a \int_{0}^{u_a} du_b \int_{u_a}^{0} du_1 \int_{u_a}^{u_1} du_2 \cdots \int_{u_a}^{u_{a-2}} du_{a-1} \int_{u_{a+2}}^{u_a} du_{a+1} \cdots$$

$$\times \int_{u_b}^{u_{b-2}} du_{b-1} \int_{0}^{u_b} du_{b+1} \cdots \int_{0}^{u_{N-2}} du_{N-1} \prod_{i>j} (u_i - u_j)^{-2p_i\cdot p_j} \frac{(u_a - u_b)^2}{(u_a^2+1)(u_b^2+1)}$$

$$= 2p_a\cdot p_b \int_{-\infty}^{0} \frac{du_a}{u_a^2+1} \int_{0}^{-1} \frac{d\alpha_b}{(\alpha_b^2 u_a^2+1)} (1-\alpha_j)^2 \int_{-1}^{0} d\alpha_1 \int_{-1}^{\alpha_1} d\alpha_2 \cdots$$

$$\times \int_{-1}^{\alpha_{a-2}} d\alpha_{a-1} \int_{\alpha_b}^{-1} d\alpha_{a+1} \cdots \int_{\alpha_b}^{\alpha_{b-2}} d\alpha_{b-1} \int_{0}^{\alpha_b} d\alpha_{b+1} \cdots \int_{0}^{\alpha_{N-2}} d\alpha_{N-1} \prod_{i>j} (\alpha_i - \alpha_j)^{-2p_i\cdot p_j} \tag{6a}$$

$$= \pi p_a\cdot p_b \left(\int_{0}^{\eta} + \int_{\eta}^{\eta^{-1}} + \int_{\eta^{-1}}^{\infty} + \int_{-\infty}^{-\eta^{-1}} + \int_{-\eta^{-1}}^{-1} \right) d\alpha_b \frac{(1+\alpha_b)^2}{1+|\alpha_b|}$$

$$\times \left[\int_{-1}^{0} d\alpha_1 \int_{-1}^{\alpha_1} d\alpha_2 \cdots \int_{-1}^{\alpha_{a-2}} d\alpha_{a-1} \int_{\alpha_b}^{-1} d\alpha_{a+1} \cdots \int_{\alpha_b}^{\alpha_{b-2}} d\alpha_{b-1} \int_{0}^{\alpha_b} d\alpha_{b+2} \cdots \int_{0}^{\alpha_{N-2}} d\alpha_{N-1} \times \prod_{i>j} (\alpha_i - \alpha_j)^{-2p_i\cdot p_j} \right]. \tag{6b}$$

In (6a), we have made the substitution $\alpha_i = u_i/u_a$. In (6b), we have done the u_a integration and broken up the α_b integral as shown. If all the ϵ_i were zero, then the quantity in the square brackets would be $[1/\alpha_b(1+\alpha_b)]B_N$, where B_N is the N-point Veneziano (Koba-Nielsen) formula. In the limit $\epsilon_i \to 0$, the second and fifth α_b integrals are still finite. Thus, we are permitted to take the limit before doing the α_b integration, and these two α_b integrals contribute just a constant multiple of B_N. Notice that this result is independent of the value of $\eta > 0$, and we can choose it to be as small as we like. In particular, we can let $\eta \to 0$, as long as this limit is taken *after* the $\epsilon_i \to 0$, and we choose to do so for convenience.

Now let us examine the first α_b integration. Since the range of the α_b integration is infinitesimal, the only possible contribution to this term can arise when the integrand blows up for $\epsilon_i = 0$. This occurs only when all but one of the u's are equal. A detailed calculation for several N confirms this, but we know this must be the case in general since this region corresponds to the configuration where the loop is in one of the strings and far from the interaction region. Since we already have $\alpha_b = u_b/u_a \approx 0$, the only possibility for the u's in which all but one are equal is $u_i \approx 0$, $i \neq a$. Thus, we can restrict the other α's to be less than some number ξ, where we can clearly choose $\eta \ll \xi \ll 1$. In fact, after a little thought, it is clear that, in addition to the above inequality, we can take ξ as small as we like, by simultaneously making η smaller if necessary. Doing this, we obtain

$$\pi p_a\cdot p_b \int_0^\eta d\alpha_a \int_{-\xi}^0 d\alpha_1 \int_{-\xi}^{\alpha_1} d\alpha_2 \cdots \int_{-\xi}^{\alpha_{a-2}} d\alpha_{a-1} \int_{\alpha_b}^{1} d\alpha_{a+1} \cdots \int_{\alpha_b}^{\alpha_{b-2}} d\alpha_{b-1} \int_0^{\alpha_b} d\alpha_{b+1} \cdots \int_0^{\alpha_{N-3}} d\alpha_{N-1} \times \prod_{\substack{i>j\\ i,j\neq a}} (\alpha_i-\alpha_j)^{-2p_i\cdot p_j} \quad (7a)$$

$$=\pi p_a\cdot p_b \int_0^\eta \frac{d\alpha_b}{\alpha_b^{1+\epsilon_a}} \int_{-\xi/\alpha_b}^0 d\beta_1 \int_{-\xi/\alpha_b}^{\beta_1} d\beta_2 \cdots \int_{-\xi/\alpha_b}^{\beta_{a-2}} d\beta_{a-1} \int_1^{1/\alpha_b} d\beta_{a+1} \cdots \int_1^{\beta_{b-2}} d\beta_{b-1} \int_0^1 d\beta_{b+1} \int_0^{\beta_{b+1}} d\beta_{b+2} \cdots \times \int_0^{\beta_{N-3}} d\beta_{N-1} \prod_{\substack{i>j\\ i,j\neq a}} (\beta_i-\beta_j)^{-2p_i\cdot p_j} \quad (7b)$$

$$=\pi p_a\cdot p_b \int_0^\eta \frac{d\alpha_b}{\alpha_b^{1+\epsilon_a}} \int_{-\infty}^{-\alpha_b/\xi} d\gamma_1 \int_{\gamma_1}^{-\alpha_b/\xi} d\gamma_2 \cdots \int_{\gamma_{a-2}}^{-\alpha_b/\xi} d\gamma_{a-1} \int_{\alpha_b/\xi}^1 d\gamma_{a+1} \cdots \int_{\gamma_{b-2}}^1 d\gamma_{b-1} \int_1^\infty d\gamma_{b+1} \int_{\gamma_{b+1}}^\infty d\gamma_{b+2} \cdots \times \int_{\gamma_{N-3}}^\infty d\gamma_{N-1} \prod_{\substack{i>j\\ i,j\neq a,N}} (\gamma_j-\gamma_i)^{-2p_i\cdot p_j} \prod_{l\neq a,b,N} \gamma_l^{-(2p_l\cdot p_a+\epsilon_l)} \quad (7c)$$

$$=\pi p_a\cdot p_b \frac{\eta^{-\epsilon_a}}{-\epsilon_a} \int_{-\infty}^0 d\gamma_1 \int_{\gamma_1}^0 d\gamma_2 \cdots \int_{\gamma_{a-2}}^0 d\gamma_{a-1} \int_0^1 d\gamma_{a+1} \cdots \int_{\gamma_{b-2}}^1 d\gamma_{b-1} \int_1^\infty d\gamma_{b+1} \int_{\gamma_{b+1}}^\infty d\gamma_{b+2} \cdots \int_{\gamma_{N-3}}^\infty d\gamma_{N-1} \times \prod_{\substack{i>j\\ i,j\neq a,N}} (\gamma_j-\gamma_i)^{-2p_i\cdot p_j} \prod_{l\neq a,b,N} (\gamma_l-\gamma_a)^{-(2p_l\cdot p_a+\epsilon_l)}. \quad (7d)$$

In (7a) we have excluded those factors with i or j equal to a since $\alpha_a=1$ and all other $\alpha_i \approx 0$. Equation (7b) is obtained by the substitution $\beta_i=\alpha_i/\alpha_b$ and (7c) by $\gamma_i=\beta_i^{-1}$, and where we have used the altered energy-momentum condition extensively. Equation (7d) follows only if the extra pieces added (by changing the limits of integration) contribute nothing to the integral. This will occur only if $s_{ja}<-2$, $j=1,\ldots,a-1$ and $s_{aj}<-2$, $j=a+1,\ldots,b-1$, where $s_{\alpha\beta}=(p_\alpha+p_{\alpha+1}+\cdots+p_\beta)^2$.

It is quite tempting to identify (7b) immediately as a linear combination of derivatives of B_N with respect to $p_a\cdot p_l$, $l\neq a,b,N$. However, this is quite misleading, since the $p_i\cdot p_j$ are not all independent due to the N relations

$$p_i\cdot\left(\sum_{j=1}^N p_j+k\right)=0. \quad (8)$$

If we were to use these relations to eliminate N of the $p_i\cdot p_j$, then the products $p_a\cdot p_l$, $l\neq a,b,N$ would appear elsewhere, and our simple argument would break down.

In order to see that (7d) does involve the derivative of B_N, it will be convenient to change to the variables $s_{\alpha\beta}$ defined above. These have the advantage that they are all independent (we count $s_{1,a-1}=s_{aN}$ as one, etc.), unlike the $p_i\cdot p_j$ which are restricted by (8). We must however decide where we will put k in the definition of the $s_{\alpha\beta}$. That is, we could choose

$$s_{\alpha\beta}=(p_\alpha+p_{\alpha+1}+\cdots+p_\beta+k)^2$$
$$=(p_{\beta+1}+p_{\beta+2}+\cdots+p_{\alpha-1})^2,$$

or we could put the k in the last expression. We note that using the wrong $s_{\alpha\beta}$ in the divergent term leads to extra finite terms in the final result. Since the divergent part of the term we are dealing with here is proportional to $(1/\epsilon_a)B_N$, it arises from the configuration where the loop is in string a. Thus, the arguments of the B_N should be the kinematic variables with p_a replaced by p_a+k. We therefore use the $s^a_{\alpha\beta}$ defined so that the k appears in that sum of momenta that contains p_a. Then it is not hard to show (working backwards) that

$$\prod_{i>j}(\gamma_j-\gamma_i)^{-2p_i\cdot p_j}=\prod_i(\gamma_i-\gamma_a)^{\epsilon_i}\left[\frac{(\gamma_{a+1}-\gamma_a)(\gamma_a-\gamma_{a-1})}{\gamma_{a+1}-\gamma_{a-1}}\right]^{-\epsilon_a}\prod_{\text{all } i,j}\left(\frac{(\gamma_j-\gamma_i)(\gamma_{j+1}-\gamma_{i+1})}{(\gamma_j-\gamma_{i+1})(\gamma_{j+1}-\gamma_i)}\right)^{-s^a_{ij}-2}. \quad (9)$$

If one of the γ's is infinite (here γ_N), then this formula still holds and all factors with that γ cancel. Also we can write the square bracket in (9) as

$$\left[\frac{(\gamma_{a+1}-\gamma_a)(\gamma_a-\gamma_{a-1})}{\gamma_{a+1}-\gamma_{a-1}}\right]^{\epsilon_a}=\prod_{l\le a\le m<b}\left[\frac{(\gamma_m-\gamma_l)(\gamma_{m+1}-\gamma_{l-1})}{(\gamma_m-\gamma_{l-1})(\gamma_{m+1}-\gamma_l)}\right]^{\epsilon_a}, \tag{10}$$

where the above result depends crucially on the relations $\gamma_a=0,\ \gamma_b=1,\ \gamma_c=\infty$. Then using (9) and (10), (7d) becomes

$$\pi p_a\cdot p_b\frac{\eta^{-\epsilon_a}}{\epsilon_a}\int^0 d\gamma_1\int_{\gamma_1}^0 d\gamma_2\cdots\int_{\gamma_{a-2}}^0 d\gamma_{a-1}\int_0^1 d\gamma_{a+1}\cdots\int_{\gamma_{b-2}}^1 d\gamma_{b-1}\int_1^\infty d\gamma_{b+1}\cdots\int_{\gamma_{N-2}}^\infty d\gamma_{N-1}$$

$$\times\prod_{\text{all } s^a_{ij}}\left[\frac{(\gamma_j-\gamma_i)(\gamma_{j+1}-\gamma_{i-1})}{(\gamma_j-\gamma_{i-1})(\gamma_{j+1}-\gamma_i)}\right]^{-\alpha^a_{ij}-2}\prod_{l\le a\le m<b}\left[\frac{(\gamma_m-\gamma_l)(\gamma_{m+1}-\gamma_{l-1})}{(\gamma_m-\gamma_{l-1})(\gamma_{m+1}-\gamma_l)}\right]^{\epsilon_a}$$

$$=\pi p_a\cdot p_b\left[\frac{1}{\epsilon_a}B_N(s^a_{ij})+\sum_{\substack{l\le a\le m<b\\ l\neq m}}\frac{\partial B(s_{ij})}{\partial s_{lm}}\right] \tag{11}$$

in the limit $\epsilon_a\to 0$. Terms proportional to $\ln\eta$ have been dropped, since it is known that they cancel with terms in other α_b integrals (this is due to the fact that η is an arbitrary division point of an integral). Notice that it is unimportant which s_{ij}'s we use as the argument of the derivatives of B_N in the limit $\epsilon_i\to 0$.

By a similar argument, the third and fourth α_b integrals each lead to an identical expression to (7d) except that ϵ_b replaces ϵ_a in the coefficient, and analogous steps lead to an expression similar to (11). If $u_a>0$ we get, in addition to (11), one more term identical to (11). Although the calculation is slightly different, the result holds over if one of a or b equals N. However, in order to obtain (7d) for all a,b, we must have $s_{\alpha\beta}<-2$ for all planar channels. This poses no problem as the $s_{\alpha\beta}$ are all independent. Later the proof holds also in the physical region by analytic continuation.

Adding up all the contributions, we obtain for term I

$$2\pi\left[\sum_{a<b}\left(\frac{1}{\epsilon_a}+\frac{1}{\epsilon_b}\right)p_a\cdot p_b B_N(s^a_{ij})+\sum_{\substack{a,b,l,m\\ \text{with } l\le a\le m<b\\ \text{or } a<l\le b\le m<N}}p_a\cdot p_b\frac{\partial B(s_{ij})}{\partial s_{lm}}\right]$$

$$\underset{\epsilon_a\to 0}{\sim}-2\pi\left[-\left(N-\sum_a\frac{1}{2\epsilon_a}\right)B_N(s_{ij})+\sum_{1\le l<m<N}s_{lm}\frac{\partial B_N(s_{ij})}{\partial s_{lm}}\right]=-2\pi\left[\kappa B_N(s_{ij})+\frac{\partial}{\partial m}B_N(s_{ij})\right],$$

where κ is a constant, m is the slope, and we have used the relations

$$2p_a\cdot p_b=s_{ab}-s_{a,b-1}-s_{a+1,b}+s_{a+1,b-1}$$

(in which we must use the definitions $s_{aa}=-1$, $s_{a+1,a}=0$). This is the desired result, and thus we have a universal renormalization of the slope of the Regge trajectories.

To complete the proof, we must show that terms II and III do not affect our result. Term II is actually a sum of terms with the term I integrand and the extra pieces

$$-\sum_r\frac{\epsilon_r}{2}\ln\left[\frac{(u_r-i)^2}{u_r^2+1}\right].$$

Since the extra term is well behaved throughout the range of integration and is first order in ϵ_r, we neglect all but the divergent part of the integral. If $r\neq a$ or b, then this always occurs for $u_r=0$, and we can write

$$\frac{\epsilon_r}{2}\ln\left[\frac{(u_r-i)^2}{u_r^2+1}\right]=-\frac{i\pi\epsilon_r}{2},$$

and the result is a multiple of the divergent part for $d<25$. For $r=a$ or b, we write

$$\ln\frac{(u_r-i)^2}{u_r^2+1}=-2i\tan^{-1}\frac{1}{u_r}.$$

Then the u_a integration is modified using

$$\int du_l\frac{\tan^{-1}(1/u_l)}{(u_l^2+1)(u_l^2\alpha_j^2+1)}=\begin{cases}\frac{1}{2}\left(\frac{\pi}{2}\right)^2 & \text{for } \alpha_j\to 0\\ \frac{1}{\alpha_j}\left(\frac{\pi}{2}\right)^2 & \text{for } \alpha_j\to\infty,\end{cases}$$

$$\int du_l\frac{\tan^{-1}(1/u_l\alpha_j)}{(u_l^2+1)(u_l^2\alpha_j^2+1)}=\begin{cases}\left(\frac{\pi}{2}\right)^2 & \text{for } \alpha_j\to 0\\ \frac{1}{2\alpha_j}\left(\frac{\pi}{2}\right)^2 & \text{for } \alpha_j\to\infty.\end{cases}$$

Adding all the terms up, we find that term II is proportional to

$$\sum_{a>b}\frac{\pi}{2}p_a\cdot p_b\left\{-i\pi\left(\frac{1}{\epsilon_a}+\frac{1}{\epsilon_b}\right)\sum_{r\neq a,b}\epsilon_r-2i\left[\epsilon_a\left(\frac{\pi}{4}\frac{1}{\epsilon_a}+\frac{\pi}{2}\frac{1}{\epsilon_b}\right)+\epsilon_b\left(\frac{\pi}{2}\frac{1}{\epsilon_a}+\frac{\pi}{4}\frac{1}{\epsilon_b}\right)\right]\right\}B_N=\frac{N\pi^2 i}{4}B_N.$$

As we remarked earlier, term III contains the explicit factor $(i^2+1)^{-1}$ which must be removed. This can be done by displacing the point of entry of the new momentum k to a fixed point in the string diagram infinitesimally close to the loop. Since we are dealing with the case where the loop has shrunk to a point, we can use the tree-diagram transformation (2) to find the displacement of the point of entry of the loop in the u plane. This gives

$$\Delta u_c=\Delta\rho_c\left(\sum_r\frac{\alpha_r}{i-u_r}\right)^{-1},$$

which changes term III to read

$$\frac{1}{2i(\Delta\rho_c)}\sum_a\epsilon_a\int_0^0 du_1\int_0^{u_1}du_2\cdots\int_0^{u_{N-2}}du_{N-1}\frac{(u_a-i)^2}{u_a^2+1}\left(\sum_r\frac{\alpha_r}{i-u_r}\right)\prod_{1\le i<j\le N}(u_i-u_j)^{-2p_i\cdot p_j},$$

where we have left out the terms with ϵ_r in the exponent, since we already have a factor ϵ_a.

Unfortunately, this expression is still divergent. This divergence, already seen in the old renormalization calculation, occurs when the loop approaches the boundary of the string diagram, or in the region of integration where all the u's are equal. To remedy the situation we introduce another cutoff to eliminate this region of integration, and later take the limit as the cutoff goes away. We do this in the following way. Since the whole term has a coefficient linear in ϵ, the only contributions will come from the region where the integral diverges, i.e., the region where all but one (at least) of the u's, say u_k, are equal to some value u_A. Then we have

$$\sum_r\frac{\alpha_r}{i-u_r}=\frac{\alpha_k}{i-u_k}+\frac{1}{i-u_A}\sum_{r\neq k}\alpha_r=\alpha_k\left(\frac{1}{i-u_k}-\frac{1}{i-u_A}\right).$$

So for the case where $a\neq N$ and $k\neq N$ or a, we can write term III in the above region as

$$\frac{\alpha_k}{2i(\Delta\rho_c)}\sum_{a\neq N}\epsilon_a\int_\xi^{-\xi}du_k\int_{-\eta}^{0}du_a\left(\frac{1}{i-u_k}-\frac{1}{i-u_a}\right)\frac{(u_a-i)^2}{u_a^2+1}\int_{u_a}^{0}du_1\int_{u_a}^{u_1}du_2\cdots\int_{u_a}^{u_{a-2}}du_{a-1}\int_{u_k}^{u_a}du_{a+1}\cdots$$
$$\times\int_{u_k}^{u_{k-2}}du_{k-1}\int_0^{u_k}du_{k+1}\cdots\int_0^{u_{N-2}}du_{N-1}\prod_{1\le i<j\le N}(u_i-u_j)^{-2p_i\cdot p_j},\quad(12)$$

where $\eta\ll\epsilon$. The u_k integral has been restricted so that $|u_k|>\xi$. Since all the other u's are near zero $(=u_N)$, this has the effect of eliminating the region where all the u's are equal. The u_a integral is restricted by $u_a>\eta$ (this should read $u_a<\eta$ if $a<k$) so that we exclude the region $u_k\sim u_a$. The remaining u's actually have been left unrestricted since there will be no contribution anyway unless they are all near zero. Finally, u_A has been set equal to u_a, which is permissible since all u_i, $i\neq k$ are equal. It is clear then from (12) that our result is just a constant multiple of B_N. The terms $a=N$ and $k=N$ or a, although somewhat different, are similar and give the same result. Also we can easily convince ourselves that changing u_c by an infinitesimal amount cannot change our result for term II (since it is finite). Thus, we conclude that both terms II and III simply add to the multiplicative renormalization and do not affect the slope renormalization.

We should point out that we have been using the fact that the s_{ij} are all independent. If the number of particles is greater than 26, the number of dimensions, then this is not strictly true. However, we note that throughout the derivation of the interacting-string amplitude, no use was made of the number of dimensions. Thus, we would have written down exactly the same expression no matter how many dimensions we were working in. We, therefore, calculate always in more dimensions than the number of particles we are dealing with, and are confident that the result will be valid in fewer dimensions.

We have now shown that the single-loop amplitude for N scalar particles is a slope renormalization. By factorization, we trivially obtain the same result for N excited particles.

Added note. In a recent paper by Ademollo *et al.*,[3] the same result has been arrived at. Unlike the above authors, the calculation here is done in the interacting-string picture. The author feels the present work is both shorter and more straightforward. In addition, no explicit use of the appearance of a zero-mass scalar particle in the Pomeron sector is made here.

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III. MONOPOLE PROPAGATOR IN A THEORY WITH CONFINEMENT

A. Theory

Because quarks have not been seen (or if they have they must still be strongly bound), physicists have been interested in field theories with confined particles for some time. If particles are permanently confined, then they cannot appear as asymptotic states. Therefore, no singularities at energies equal to their mass should appear in the S-matrix. One manifestation of this should be in the behavior of the propagator for the confined particles. All singularities at energies equal to their mass should vanish.[11] Thus, for example, the pole that appears in the propagator in the lowest order of perturbation theory must somehow be cancelled by higher order corrections. Also because of the confinement, we would expect that for large spacelike separations, the propagator should fall off very rapidly.

Here we shall examine the propagator in a model theory with confinement. We consider the theory with an electromagnetic field A_μ interacting with a scalar Higgs particle ϕ and a spin-$\frac{1}{2}$ monopole ψ. The Hamiltonian is

$$\mathcal{H} = \frac{1}{2}(\vec{E}^{T2} + \vec{H}^2) + |(\partial_i - ieA_i)\phi|^2 + |\dot{\phi}|^2 + e^2 A_o^2 |\phi|^2 - \mu^2|\phi|^2 +$$

$$+ \lambda|\phi|^4 + \bar{\psi}(\vec{\gamma}\cdot(-i\vec{\nabla} - g\vec{B}) + m)\psi \quad , \qquad (1)$$

where

$$\bar{A} = \bar{A}^T + \bar{A}^g \qquad \bar{B} = \bar{B}^T + \bar{B}^g$$

$$\bar{H} = \nabla \times \bar{A}^T - \nabla \chi_0 \qquad \bar{E} = -\nabla \times \bar{B}^T - \nabla A_0$$

$$A_0(x) = \int d^3x' \; \frac{e\phi^*(x')\overleftrightarrow{\partial}_0\phi(x')}{4\pi|\bar{x} - \bar{x}'|}$$

$$\chi_0(x) = \int d^3x' \; \frac{g\bar{\psi}(x')\gamma_0\psi(x')}{4\pi|\bar{x} - \bar{x}'|}$$

$$\bar{A}^g = g\int d^3x' \; \bar{a}(\bar{x} - \bar{x}')\bar{\psi}(x')\gamma_0\psi(x')$$

$$\bar{B}^g = -e\int d^3x' \; \bar{a}(\bar{x}' - \bar{x})\phi^*(x')\partial_0\phi(x')$$

$$\bar{a}(\bar{x}) = \frac{1}{8\pi|\bar{x}|}\left(\frac{\bar{n} \times \bar{x}}{|\bar{x}| + \bar{n} \cdot \bar{x}} - \frac{\bar{n} \times \bar{x}}{|\bar{x}| - \bar{n} \cdot \bar{x}}\right).$$

Here we have followed the notation of Schwinger.[12] A_μ and B_μ are the vector potentials for the charge and monopole respectively. $A_\mu(B_\mu)$ contains stringlike singularities, Dirac strings, attached to each charge (monopole). We have chosen the monopoles to be fermions because we have quarks in the back of our mind, but the results presented here should be independent of this choice. Since the μ^2 term has the wrong sign for a mass term, the ϕ field will pick up a vacuum expectation value. This leads to confinement of the monopoles by the following argument. For finite energy, the term $|(\partial_\mu - ieA_\mu)\phi|^2$ must vanish far from the monopole. Since

ϕ has acquired a vacuum expectation value, we must have

$$\phi \xrightarrow{r \to \infty} k e^{ie \int_{x_0}^{x} A_\mu \, dx_\mu}$$

where k is a constant and x_o is some arbitrary point. However, in order for ϕ to be well-defined, we must demand that it be independent of the path of integration. Thus the integral in the exponent, taken around a closed path, must equal $2\pi n/e$. To be specific, let us choose a singly charged monopole at the origin with its Dirac string along the z-axis. Then for a path of integration at very large z, circling the z-axis and very far from it, we know the integral equals $2\pi/e$. However, if we translate the coutour to large negative z, the integral equals $-2\pi/e$ (for our two-sided string. In any case, the answer is different). Thus there is no way for ϕ to be well-defined and continuous in a finite-energy single monopole solution. We conclude that for each monopole there must be an antimonopole at which the Dirac string ends. Thus monopole-antimonopole pairs are confined.

We would like to shift the field ϕ by its vacuum expectation value and then examine the higher order corrections to the monopole propagator. However two technical problems stand in our way. First of all, we have the usual problem that arises in all field theories with monopoles. Since the theory only makes sense if $eg = 2\pi n$, a perturbation expansion in both e and g at best carries with it the optimistic hope that after summation of the series with small e and g, a valid analytic continuation can be made to physical values.

However, because we initially expand in both small e and g, to each order in perturbation theory, amplitudes depend on the direction of the Dirac string. We could hope to overcome this difficulty by resumming the series such that, in each step, we add an infinite subset of Feynman diagrams (e. g., Bethe-Salpeter ladders) whose sum is independent of the direction of the Dirac string. To our knowledge, no one (including the present author) has succeeded in finding even a single subset of Feynman diagrams independent of the Dirac string. Another possibility is to average over the direction of the string (see, for example, Rabl[13]), but there is no real reason why this procedure should give correct answers.

The other technical problem is that after shifting the field ϕ, we wind up with a large number of vertices. We might hope that we could generate the properties we expect of the monopole propagator from a few monopole vertices. However, we recall that the proof of confinement depended on the form of the term $|(\partial_\mu - ieA_\mu)\phi|^2$, as well as on ϕ having a vacuum expectation value. This would seem to indicate that most (if not all) of the charged particle vertices need be included too.

Because of all the preceding reasons, the prospects for making progress with this theory are dim. Therefore, we replace the effect of the electromagnetic and charged particle fields by an effective potential for the monopoles. We choose this potential to be linear by analogy with the vortex solutions of Nielsen and Olesen[14] (Landau-Ginzburg type). They exhibited cylind cally symmetric vortex solutions for the Lagrangian of Eq. 1 but without the monopole terms. Since the vortices are cylind cally symmetric, their energy is proportional to their length. In the theory with monopoles, the vortices will be finite, since they will end at the monopoles. However, at least for large distances between the monopoles, the energy should still be proportional to the length of the vortex. Thus we choose an effective potential which is linear.

Our Lagrangian is now

$$\mathcal{L} = \bar{\psi}(x)(i\not{\partial} - m)\psi(x) + \lambda^2 \int d^4y\, j^0(x) j^0(y) |\bar{x} - \bar{y}| \delta(t_x - t_y)$$

where

$$j^0 = \bar{\psi}(x)\gamma^0\psi(x) \quad .$$

Note that this theory is nonlocal and thus nonrenormalizable. However, this should not matter, since the confinement in our theory is a large distance or infrared effect. Thus a simple cutoff in all divergent integrals should suffice for our purposes.

Ordinarily, we could look at the large $|\bar{x}|$ behavior of the propagator in any spacelike direction. However, since we now have a nonlocal interaction, we must confine ourselves to the $t = 0$ direction. Thus in the considerations that follow we shall examine the propagator integrated over p_0.

We are now in a position to begin to calculate the monopole propagator. However, it clearly is not feasible to add up all the higher order diagrams. We must devise some approximation scheme. One attractive choice is the set of ladder-like diagrams in Fig. 1a. Throughout this paper we write the nonlocal four-point interaction as two two-point interactions connected by a dashed line. By crossing symmetry they can be rewritten as in Fig. 1b, even though we are no longer in a physical region of the S-matrix. This is a logical choice because we know that, at least in the nonrelativistic case, repeated exchange has been used to create bound states. Thus we could reasonably hope that these ladder-like diagrams would be sufficient to obtain confinement. We shall see later that, at least in the fully relativistic case, this is not the case, and other diagrams need to be considered.

Before beginning the actual calculations, we first exhibit in a concise way the approximation we have made. We do this by deriving differential equations for the Green's functions. This can be done two ways. In the first method, first written down in a paper by Mandelstam,[15] we apply the operator $(i\partial\!\!\!/ - m)$ on

$$G_{2n} = \langle 0|T(\bar{\psi}(x_1)\ldots\bar{\psi}(x_n)\psi(x_{n+1})\ldots\psi(x_{2n}))|0\rangle$$

and use the equations of motion. In the other method, we use the fact that the functional integral of a total functional derivative equals zero, to derive a differential equation that the generating functional $Z(J)$ satisfies.[16]

$$0 = \int \mathcal{D}\phi \, \frac{\delta}{\delta\phi} \, e^{i(S(\phi)+J\phi)} = \left(\frac{\delta S(\phi)}{\delta\phi} \bigg|_{\phi=\frac{\delta}{\delta iJ}} + J \right) Z(J) \quad .$$

We next operate with $\left(\frac{\delta}{\delta J}\right)^n$ to obtain the same equations as in the first method.

The first two equations that we obtain in our present theory are given diagrammatically in Fig. 2. Substituting Fig. 2b back into a, we get four different equations depending on how we match up the particles (note that in Fig. 2b, we can also switch the in particles with the out particles). These are given by Fig. 3. Note that the second term in each of these four equations involves tadpole diagram corrections, which simply lead to a mass renormalization. Thus we can safely ignore them. If we also ignore the last term in Fig. 3a and d, we obtain the differential equation satisfied by the sum of the ladder diagrams. This can be seen by repeatedly substituting in the whole sum where it appears on the right hand side. Therefore, the last term in either Figure 3a or d (plus its iterations into the third term), which differ only by the inclusion of certain tadpoles, represents all the terms omitted by taking the ladder-like diagram approximation.

As an aside, neglecting the last term in Fig. 3b or c gives us another series of diagrams that we can sum -- multiple iterations of single loops. Using the result A_1 of Eq. A6 in the appendix, we have for the sum

$$\frac{1}{p^2 - m^2 + i\varepsilon} \quad \frac{1}{1 - \dfrac{A_1}{p^2 - m^2 + i\varepsilon}} = \frac{1}{p^2 - m^2 - A_1 + i\varepsilon} \quad .$$

This result, not surprisingly, still has singularities at the mass m, and thus this subset of diagrams fails to exhibit confinement.

B. Calculations

The Bethe-Salpeter equation satisfied by the sum of the ladder diagrams $\psi(p)$ is

$$\psi(p) = \frac{1}{\not p - m + i\varepsilon} + \frac{1}{\not p - m + i\varepsilon} \int \frac{id^4p'}{(2\pi)^4} \left(-8\pi\lambda^2 \frac{(\bar{p} - \bar{p}')^2 - 3\varepsilon^2}{((\bar{p} - \bar{p}')^2 + \varepsilon^2)^3} \right) \times$$

$$\times \quad \gamma_0 \psi(p') \quad \gamma_0 \frac{1}{\not p - m + i\varepsilon} \quad . \qquad (2)$$

The quantity in the brackets in the above equation is the Fourier transform of our potential $\lambda^2 |x| \delta(t)$. Letting $G(p) = (\not p - m + i\varepsilon) \times$ $\times \psi(p)(\not p - m + i\varepsilon)$, we get

$$G(p) = \not p - m + \int \frac{id^4p'}{(2\pi)^4} \left(-8\pi\lambda^2 \frac{(\bar{p} - \bar{p}')^2 - 3\varepsilon^2}{((\bar{p} - \bar{p}')^2 + \varepsilon^2)^3} \right) \times$$

$$\times \quad \gamma_0 \frac{1}{\not p' - m + i\varepsilon} \; G(p') \; \frac{1}{\not p' - m + i\varepsilon} \gamma_0 \; .$$

It is now clear that $G(p)$ must be of the form $G(p) = \gamma_o p_o + H(\bar{p})$ where $H(\bar{p})$ now depends only on the 3-vector $\bar{p}$. We then can do the p_o' integral to obtain

$$H(\bar{p}) = -\bar{\gamma}\cdot\bar{p} - m - \frac{i}{4}\int \frac{i d^3p'}{(2\pi)^3}\left(-8\pi\lambda^2 \frac{(\bar{p}-\bar{p}')^2 - 3\varepsilon^2}{((\bar{p}-\bar{p}') + \varepsilon^2)^3}\right)\times$$

$$\times\left[\frac{2(\bar{\gamma}\cdot\bar{p}' + m)}{(\bar{p}'^2 + m^2)^{\frac{1}{2}}} + \frac{H(\bar{p}')}{(\bar{p}'^2 + m^2)^{\frac{1}{2}}} - \frac{(\bar{\gamma}\cdot\bar{p}'+m)\gamma^o H(\bar{p}')\gamma^o(\bar{\gamma}\cdot\bar{p}'+m)}{(\bar{p}'^2 + m^2)^{3/2}}\right]. \tag{3}$$

We now write $H(\bar{p})$ in the form

$$H(\bar{p}) = J_1(\bar{p}) + \gamma_o J_2(\bar{p}) + \bar{\gamma}\cdot\bar{p}\, J_3(\bar{p}) + \gamma_o\bar{\gamma}\cdot\bar{p}\, J_4(\bar{p}). \tag{4}$$

where the $J's$ are numbers, not matrices. We have not included terms with γ_5 because they do not appear in lowest order and they are not generated in higher orders. After substitution of Eq. 4 into 3, we can separately equate the coefficients of the different γ's. This leads to

$$J_1(\bar{p}) = -m + \frac{1}{2}\int \frac{d^3p'}{(2\pi)^3}\; -8\pi\lambda^2\left(\frac{(\bar{p}-\bar{p}')^2 - 3\varepsilon^2}{((\bar{p}-\bar{p}')^2 + \varepsilon^2)^3}\right)\times$$

$$\times\left[\frac{m}{(\bar{p}'^2 + m^2)^{\frac{1}{2}}} + \frac{\bar{p}'^2(J_1(\bar{p}') - m\, J_3(\bar{p}'))}{(\bar{p}'^2 + m^2)^{3/2}}\right] \tag{5a}$$

$$J_2(\bar{p}) = 0 \tag{5b}$$

$$\bar{p}\, J_3(\bar{p}) = -\bar{\gamma}\cdot\bar{p} + \frac{1}{2}\int \frac{d^3p'}{(2\pi)^3}\left(-8\pi\lambda^2 \frac{(\bar{p}-\bar{p}')^2 - 3\varepsilon^2}{((\bar{p}-\bar{p}')^2+\varepsilon^2)^3}\right)\times$$

$$\times\left[\frac{1}{(\bar{p}'^2+m^2)^{\frac{1}{2}}} + \frac{m(J_1(p') - m\, J_3(\bar{p}'))}{(\bar{p}'^2+m^2)^{3/2}}\right]\bar{p}' \qquad (5c)$$

$$\bar{p}\, J_4(\bar{p}) = \frac{1}{2}\int \frac{d^3p'}{(2\pi)^3}\left(-8\pi\lambda^2 \frac{(\bar{p}-\bar{p}')^2 - 3\varepsilon^2}{((\bar{p}-\bar{p}')^2+\varepsilon^2)^3}\right)\times$$

$$\times \frac{J_4(\bar{p}')}{(\bar{p}'^2+m^2)^{\frac{1}{2}}}\ \bar{p}' \qquad (5d)$$

We can at this point set $J_4(\bar{p}) = 0$, since it does not appear in lowest order, and from Eq. 5d each succeeding term is zero. Equation 5a and c remain. We do not know how to solve them, but in the Appendix we place an upper limit on the large r behavior of the solution. However, we can show that no solution exists such that the singularity of the propagator at the mass of the monopole has disappeared. Since we are interested in large space like separations at $t = 0$, we examine

$$\int dp_o \psi(p) = \int dp_o \frac{1}{\not{p} - m + i\varepsilon}(\gamma_o p_o + H(\bar{p}))\frac{1}{\not{p} - m + i\varepsilon} =$$

$$= \int \frac{dp_o}{(p^2 - m^2 + i\varepsilon)^2}(\not{p} + m)(\gamma_o p_o + J_1(\bar{p}) + \bar{\gamma}\cdot\bar{p}\, J_3(\bar{p}))(\not{p} + m) =$$

$$= -\frac{\pi i}{(\bar{p}^2 + m^2)^{3/2}}\left\{\left[J_1(\bar{p}) - m\, J_3(\bar{p}) + \frac{m}{\bar{p}^2}(\bar{p}^2 + m^2)\right] + \right.$$

(Equation 6 continued on next page)

$$+ m\bar{\gamma}\cdot\bar{p}\, J_1(\bar{p}) - m\, J_3(\bar{p}) - \frac{1}{m}(\bar{p}^2 + m^2)\Big\}\Big\}. \qquad (6)$$

In doing the p_o integral, we have ignored a term, formally infinite, but odd in p_o. In order for the result in Eq. 6 to be regular at $\bar{p}^2 = -m^2$, we must have both

$$\frac{1}{(\bar{p}^2 + m^2)^{3/2}}\left[J_1(\bar{p}) - m\, J_3(\bar{p}) + \frac{m}{\bar{p}^2}(\bar{p}^2 + m^2)\right]$$

and

$$\frac{1}{(p^2 + m^2)^{3/2}}\left[J_1(\bar{p}) - m\, J_3(\bar{p}) - \frac{1}{m}(\bar{p}^2 + m^2)\right]$$

regular. However, this is impossible because their difference is singular. Therefore, if a solution exists at all, it contains a singularity at the mass of the monopole. Thus we have failed to exhibit confinement. We believe this is due to our inclusion of ladder diagrams with pair production (Fig. 4b), but not the corresponding crossed diagram (Fig. 4c). These diagrams also contribute to the binding "forces" on the monopoles and should be important. Unfortunately, we know of no way to correctly take them into account. However, in the nonrelativistic approximation, there is no pair creation, and the types of diagrams represented by both Figs. 4b and c are absent (remember we have an instantaneous interaction) and only those of Fig. 4a remain. Thus we might hope to obtain confiement in this approximation, and we examine this possibility next.

Non-relativistic approximation

Starting with our Bethe-Salpeter Eq. 2, we can do the $p^{o'}$ integration on the right hand side. Defining

$$\phi(\bar{p}) = \int_{-\infty}^{\infty} dp^o \psi(p)$$

$$H_a = \gamma_o(\bar{\gamma} \cdot \bar{p} + m)$$

$$H_b = \gamma_o(-\bar{\gamma} \cdot \bar{p} + m)$$

we have

$$(p^o - H_a(\bar{p}))\psi(p)(p^o - H_b(\bar{p})) = \gamma^o p^o + \bar{\gamma} \cdot \bar{p} + \int \frac{i\, d^3p'}{(2\pi)^4} \left(-8\pi\lambda^2 \frac{(\bar{p} - p')^2 - 3\epsilon^2}{((\bar{p} - \bar{p}')^2 + \epsilon^2)^3} \phi(\bar{p}') \right) \tag{7}$$

We can now treat the 4×4 matrix $\psi(p)$ as a wave function in the product space of two spinor particles. We proceed according to the method of Salpeter[17] for treating instantaneous interactions and make the following definitions

$$\Lambda_{\pm}^{a} = \frac{E_a(\bar{p}) + H_a(\bar{p})}{2E_a(\bar{p})}$$

where $E_a(\bar{p}) = (\bar{p}^2 + m^2)^{\frac{1}{2}}$

and similarly for particle b. In addition, we define

$$\psi_{++}(p) = \Lambda_+^a(\bar{p})\psi(p)\Lambda_+^b(\bar{p})$$

$$\psi_{+-}(p) = \Lambda_+^a(\bar{p})\psi(p)\Lambda_-^b(\bar{p}), \text{ etc.}$$

Then we arrive at

$$F_{++}(p)\psi_{++}(p) = \Lambda_+^a(\bar{p})\Gamma(p)\Lambda_+^b(\bar{p})$$

$$F_{+-}(p)\psi_{+-}(p) = \Lambda_+^a(\bar{p})\Gamma(p)\Lambda_-^b(\bar{p}), \text{ etc.} \tag{8}$$

where $F_{++}(p) = (p^o - E_a(\bar{p}) + i\varepsilon)(p^o - E_b(\bar{p}) + i\varepsilon)$

$$F_{+-}(p) = (p^o - E_a(\bar{p}) + i\varepsilon)(p^o + E_b(\bar{p}) - i\varepsilon), \text{ etc.}$$

and $\Gamma(p)$ is the right hand side of Eq. 7. We now divide each of Eq. 8 by the appropriate $F(p)$ and integrate over p^o using

$$\int_{-\infty}^{\infty} dp^o (p^o + a \mp i\varepsilon)^{-1}(p^o + b \pm i\varepsilon)^{-1} = \pm 2\pi i (b - a)^{-1}$$

$$\int_{-\infty}^{\infty} dp^o (p^o + a \pm i\varepsilon)^{-1}(p^o + b \pm i\varepsilon)^{-1} = 0$$

$$\int_{-\infty}^{\infty} dp^o\, p^o (p^o - a + i\varepsilon)^{-1}(p^o - b + i\varepsilon)^{-1} = -\pi i$$

$$\int_{-\infty}^{\infty} dp^o\, p^o (p^o + a - i\varepsilon)^{-1}(p^o - b + i\varepsilon)^{-1} = \pi i \frac{a - b}{a + b} .$$

In the last two equations, we have thrown away a term, formally infinite, but odd in p^o. This leads to

$$-(E_a(p) + E_b(\bar{p}))\phi_{-+}(\bar{p}) = \Lambda_-^a(\bar{p})\Big[\pi i(E_b(\bar{p}) - E_a(\bar{p}))\gamma_o + 2\pi i(\bar{\gamma}\cdot\bar{p} - m) - \int \frac{i\,d^3p'}{(2\pi)^4}\left(-8\pi\lambda^2\,\frac{(\bar{p}-\bar{p}')^2 - 3\varepsilon^2}{((\bar{p}-\bar{p}')^2+\varepsilon^2)^3}\right)\phi(\bar{p}')\Big]\times \Lambda_+^b(\bar{p}) ,$$

$$-(E_a(\bar{p}) + E_b(\bar{p}))\phi_{+-}(\bar{p}) = \Lambda_+^a(\bar{p})\Big[\pi i(E_a(\bar{p}) - E_b(\bar{p}))\gamma_o + 2\pi i(\bar{\gamma}\cdot\bar{p} - m) - \int \frac{i\,d^4p'}{(2\pi)^3}\left(-8\pi\lambda^2\,\frac{(\bar{p}-\bar{p}')^2 - 3\varepsilon^2}{((\bar{p}-\bar{p}')^2+\varepsilon^2)^3}\right)\phi(\bar{p}')\Big]\Lambda_-^b(\bar{p}),$$

$$-\phi_{++}(\bar{p}) = \pi i\,\Lambda_+^a(\bar{p})\gamma_o\Lambda_+^b(\bar{p}) ,$$

$$-\phi_{--}(\bar{p}) = \pi i\,\Lambda_-^a(\bar{p})\gamma_o\Lambda_-^b(\bar{p}) .$$

Since in our case, $E_a - E_b = 0$, we have

$$(H_a(\bar{p}) - H_b(\bar{p}))\phi(\bar{p}) = (\Lambda_-^a(\bar{p})\Lambda_+^b(\bar{p}) - \Lambda_+^a(\bar{p})\,\Lambda_-^b(\bar{p}))\Big[2\pi i(\bar{\gamma}\cdot\bar{p} - m) - \int \frac{i\,d^3p'}{(2\pi)^3}\left(-8\pi\lambda^2\,\frac{(p-p')^2 - 3\varepsilon^2}{((p-p')^2+\varepsilon^2)^3}\right)\phi(\bar{p}')\Big], \quad (9)$$

where for convenience we have written the b operators on the left, even though they really should appear on the right. Eq. 9 looks different than the corresponding equation in Salpeter's[17] article. This is due to the fact that our $\phi(\bar{p})$ is a wave function for a particle and antiparticle, whereas his is for two particles. To remedy this we multiply by the charge conjugation operator C on the right to obtain

$$-(H_a(\bar{p}) + H_b^T(\bar{p}))\phi^c(\bar{p}) = (\Lambda_+^a(\bar{p})\Lambda_+^{b\,T}(\bar{p}) - \Lambda_-^a(\bar{p})\Lambda_-^{b\,T}(\bar{p})) \times$$
$$\times \left[2\pi i(\bar{\gamma}\cdot\bar{p} - m)C - \int \frac{d^3k}{(2\pi)^3} \times \right.$$
$$\left. \times \left(-8\pi\lambda^2 \frac{(\bar{p}-\bar{p}')^2 - 3\varepsilon^2}{((\bar{p}-\bar{p}')^2+\varepsilon^2)^3}\right)\phi^c(\bar{p}')\right] \quad (10)$$

where $\phi^c(\bar{p}) = \phi(\bar{p})C$. In the nonrelativistic limit the factor involving the Λ's equals one, and all the homogeneous terms in ϕ reduce to the Schroedinger Hamiltonian operator acting on the "large" part of the wave function $\phi_{++}(\bar{p})$. In coordinate space we then have

$$H_s(\bar{x})\phi^c(\bar{x}) = 2\pi i(-i\bar{\gamma}\cdot\nabla - m)\delta^3(\bar{x})C \quad (11)$$

where $H_s(\bar{x}) = -\frac{1}{2m}\nabla^2 + \lambda^2|\bar{x}|$.

For $\bar{x} \neq 0$, the right hand side in Eq. 11 equals zero. The problem becomes simply that of finding the large $\bar{x}$ behavior of the

Schrödinger wave function with energy $E = 0$. (Boundary conditions at the origin, which quantize the allowed energies, do not apply here). The angular part of the equation can be separated out in the standard way for a central potential. We are then left with the radial equation

$$\left(\frac{d^2}{dr^2} - 2m\lambda^2 r - \frac{\ell(\ell+1)}{r^2} \right) (rR_\ell(r)) = 0.$$

For large r, we can ignore the angular momentum term compared to the potential. Rescaling r, we arrive at the Airy differential equation. The solution has the asymptotic form

$$R_\ell(r) = \frac{1}{r} \mathrm{Ai}((2m\lambda^2)^{1/3} r) \xrightarrow{r \to \infty} \frac{\sqrt{\pi}}{2(2m\lambda^2)^{1/12}} \frac{\exp(-\frac{2}{3}\sqrt{2m\lambda^2}\, r^{3/2})}{r^{3/2}}$$

where, as usual, we have discarded the exponentially increasing solution. This shows that our wave function, and consequently the propagator, falls off much faster than the free propagator. In fact, since it falls off faster than e^{-mr} for any m, all singularities in momentum space must be absent[19] and we have finally exhibited confinement.

ACKNOWLEDGEMENTS

I would like to thank Professor Stanley Mandelstam for his continual assistance and guidance throughout the period of my graduate research.

Thanks are also due to my parents, without whom I would never achieved what I have.

APPENDIX

As we mentioned earlier, we have not been able to find the large $\bar{x}$ behavior of the solution to the Bethe-Salpeter equation for the propagator. However, at least in the case of bosonic monopoles, and in the context of the approximation scheme below, we can show that any solution must fall off faster than the free propagator.

The idea is to directly calculate the ladder diagrams by iteration in the leading term approximation. Given the amplitude for the n rung diagram, it is easy to write down a single integral expression for the $n + 1$ rung diagram, using the fact that it has an n rung subdiagram. We start at the single loop level and at each level keep only the most singular term at $\bar{p}^2 = -m^2$. We can hope to derive a simple expression for the amplitude as a function of n. The problem with this procedure is that more often than not, the integrals simp‿y become more and more complicated at higher orders and no pattern emerges. We shall see that for bosonic monopoles we are lucky and the procedure works.

The single loop diagram for a bosonic monopole is given by

$$A_1 = \frac{1}{(p^2 - m^2 + i\varepsilon)^2} \int \frac{i\, d^4p'}{(2\pi)^4} \frac{\left[-(2p_0 + p_0')^2\right]}{(p + p')^2 - m^2 + i\varepsilon} \times$$

$$\left(-8\pi\lambda^2 \frac{\bar{p}'^2 - 3\varepsilon^2}{(\bar{p}'^2 + \varepsilon^2)^3}\right) . \qquad \text{(A1)}$$

The quadratic term in p_o is due to the derivative coupling. Introducing Feynman parameters, we have

$$A_1 = -\frac{3i\lambda^2}{\pi^3}\frac{1}{(p^2-m^2+i\epsilon')^2}\int dp_o'(p_o^2+p_o'^2)\int dx_1 \ldots dx_4\,\delta(1-\sum_{n=1}^{4}x_n)\times$$

$$\times\int d^3p'\,\frac{\bar{p}'^2-3\epsilon^2}{\left[\bar{p}'^2+2x_1\bar{p}\cdot\bar{p}'-x_1(p_o'^2-m^2-\bar{p}^2+i\epsilon'+\epsilon^2)+\epsilon^2\right]^4} \quad (A2)$$

$$= -\frac{3\lambda^2}{16\pi}\frac{1}{(p^2-m^2+i\epsilon')^2}\int dp_o'\,(p_o^2+p_o'^2)\left(-\frac{2}{3}\frac{\partial}{\partial\alpha}\right) \quad (A3)$$

$$\times\int_o^1 dx_1\,\frac{(1-x_1)^2}{\left[x_1^2\bar{p}^2+x\alpha(p_o'^2-m^2-\bar{p}^2+i\epsilon'+\epsilon^2)+\epsilon^2(2\alpha-3)\right]^{3/2}}\Bigg|_{\alpha=1}$$

$$= \frac{\lambda^2}{\pi}\frac{1}{(p^2-m^2+i\epsilon)^2}\int_{-\infty}^{\infty}dp_o'\,(p_o^2+p_o'^2)\frac{(p_o'^2-m^2+i\epsilon)^{\frac{1}{2}}}{(p_o'^2-\bar{p}^2-m^2+i\epsilon)^2} \quad (A4)$$

$$= -\frac{2\lambda^2}{\pi}\frac{1}{(p^2-m^2+i\epsilon)^2}\frac{\partial}{\partial\alpha}\int_o^{\Lambda/m}dp_o'\,\frac{(p_o'^2-1+i\epsilon)^{\frac{1}{2}}}{\alpha p_o'^2+(\alpha-1)\left(\frac{p_o}{m}\right)^2-\left(\frac{p}{m}\right)^2-1+i\epsilon}\Bigg|_{\alpha=1} \quad (A5)$$

To obtain Eq. A2, we also shifted the p_o' integral. In arriving at Eq. (A3), we did the $\bar{p}'$ integral using

$$\int\frac{d^3x(\bar{x}^2+a)}{\left[\bar{x}^2+2\bar{b}\cdot\bar{x}+c-i\epsilon\right]^4} = \frac{\pi i}{8}\frac{a+c}{(b^2-c+i\epsilon)^{5/2}}$$

in addition to the x_2, x_3, x_4 integrals. To get Eq. (A4), we used

$$\int \frac{dx}{\sqrt{R^3}} = \frac{2(2cx + b)}{\Delta\sqrt{R}}$$

$$\int \frac{x dx}{\sqrt{R^3}} = -\frac{2(2a + bx)}{\Delta\sqrt{R}}$$

$$\int \frac{x^2 dx}{\sqrt{R^3}} = -\frac{(\Delta - b^2)x - 2ab}{c\ \Delta\sqrt{R}}$$

where $R = a + bx + cx^2$ and $\nabla = 4ac - b^2$, set $\alpha = 1$, and then took the limit $\varepsilon = 0$. Surprisingly, all divergent terms cancel. In Eq. A5, we have introduced a cutoff in the divergent p_o' integral. The integral appearing in Eq. A5 is a mess to do. We must integrate separately the regions above and below one. Each of these can be done by a messy substitution $\left(p_o' = \frac{1 \pm x^2}{1 \mp x^2}\right)$ and a lot of algebra. We thus arrive at the result

$$A_1 = \frac{2\lambda^2}{\pi} \frac{1}{(p^2 - m^2 + i\varepsilon)^2} \left[\ln \frac{2\Lambda}{m} - \frac{\pi i}{2} + \left(\frac{|\bar{p}|}{(\bar{p}^2 + m^2)^{\frac{1}{2}}} + \frac{m^2(p_o^2 + \bar{p}^2 + m^2)}{2|\bar{p}|(\bar{p}^2 + m^2)^{3/2}} \right) \times \right.$$
$$\left. \times \ \ln \left(\frac{(\bar{p}^2 + m^2)^{\frac{1}{2}} - |\bar{p}|}{m} \right) - \frac{p_o^2 + \bar{p}^2 + m^2}{2(\bar{p}^2 + m^2)} \right] . \qquad (A6)$$

Since we are interested only in the large $\bar{x}$ behavior, we need only keep the most singular term at the closest singularity to the real axis, which turns out to be at $|\bar{p}| = \pm im$. This leads to

$$A_1 \longrightarrow \frac{\lambda^2}{\pi} \frac{1}{(p^2 - m^2 + i\epsilon)^2} \frac{m^2 p_o^2}{(\bar{p}^2 + m^2)^{3/2}} \left[\frac{\ell n \left(\frac{(\bar{p}^2 + m^2)^{\frac{1}{2}} - |\bar{p}|}{m} \right)}{|\bar{p}|} \right]$$

The singularity at $|\bar{p}| = \pm$ im inside the logarithm is quite weak compared to that of the $(\bar{p}^2 + m^2)^{-3/2}$ factor, and ignoring it should not affect the answer much. We do this for simplicity. Thus we evaluate the term in the square brackets at $|\bar{p}| =$ im. In either case we get $-\frac{\pi}{2m}$. Our result is now

$$A_1 \longrightarrow -\frac{\lambda^2 m\, p_o^2}{2(\bar{p}^2 + m^2)^{3/2}} \frac{1}{(p^2 - m^2 + i\epsilon)^2} \qquad . \tag{A7}$$

Once we have A_n, we get A_{n+1} as follows:

$$A_{n+1} = -\int \frac{i\, d^4p'}{(2\pi)^4} (p_o + p_o')^2 \left(-8\pi\lambda^2 \frac{(\bar{p} - \bar{p}')^2 - 3\epsilon^2}{((\bar{p} - p')^2 + \epsilon^2)^3} \right) A_n \tag{A8}$$

Now suppose A_n is of the form

$$A_n = \frac{kp_o^2}{(\bar{p}^2 + m^2)^r} \frac{1}{(p^2 - m^2 + i\epsilon)^2} \tag{A9}$$

where k is a constant.

Then we can do the p_o' integral

$$\int_{-\Lambda}^{\Lambda} \frac{dp_o'(p_o + p_o')^2 p_o'^2}{(p_o'^2 - \bar{p}^2 - m^2 + i\epsilon)^2} = 2\Lambda - \frac{\pi i}{2} \frac{3(\bar{p}^2 + m^2) + p_o^2}{(\bar{p}^2 + m^2)^{\frac{1}{2}}} \longrightarrow -\frac{\pi i}{2} \frac{p_o^2}{(\bar{p}^2 + m^2)^{\frac{1}{2}}}$$

where in the last step we have kept only the most singular term at $|\bar{p}| = \pm im$. Therefore

$$A_{n+1} = \frac{\lambda^2 k\, p_o^2}{4\pi^2}\ \frac{1}{(p^2 - m^2 + i\varepsilon)^2} \int \frac{d^3p'}{(\bar{p}'^2 + m^2)^{r+\frac{1}{2}}} \left(\frac{(\bar{p} - \bar{p}')^2 - 3\varepsilon^2}{((\bar{p} - \bar{p}')^2 + \varepsilon^2)^3}\right)$$

$$\longrightarrow -\frac{\lambda^2 k(r + \frac{1}{2}) m\, p_o^2}{2(\bar{p}^2 + m^2)^{r+3/2}}\ \frac{1}{(p^2 - m^2 + i\varepsilon)^2} \qquad \text{(A10)}$$

where we again have kept only the most singular term, this time by Fourier transforming to position space, taking the limit $|\bar{x}| \to \infty$, and then Fourier transforming back. We have been fortunate. The result of Eq. (A10) is again of the form of (A9). Since (A1) is also of this form (Eq. A7), all A_n are of this form. In fact it is not hard to see that

$$r = \frac{3}{2} n$$

$$A_n = -\frac{3\lambda^2(n - \frac{2}{3})m}{4(\bar{p}^2 + m^2)^{3/2}}\ A_{n-1}$$

$$= \left(-\frac{3\lambda^2 m}{4(\bar{p}^2 + m^2)^{3/2}}\right)^{n-1} \frac{\Gamma(n + \frac{1}{3})}{\Gamma(\frac{4}{3})}\ A_{n-1}$$

$$= \frac{2}{3}\ \frac{p_o^2}{(p^2 - m^2 + i\varepsilon)^2} \left(-\frac{3\lambda^2 m}{4(\bar{p}^2 + m^2)^{3/2}}\right)^n \frac{\Gamma(n + \frac{1}{3})}{\Gamma(\frac{4}{3})}\ .$$

If we try to sum up the A_n we have obtained, the series diverges. Instead we Fourier transform to position space, keeping only the leading term for large distance (and time $t = 0$, for simplicity).

This leads to

$$\sum_{n=1}^{\infty} A_n \xrightarrow{r\to\infty} \sum_{n=1}^{\infty} \frac{2}{3}\left(-\frac{i}{4}\right)\left(-\frac{3\lambda^2 m}{4}\right)^n \frac{e^{-mr}}{4\pi r\Gamma(\frac{3}{2}n+\frac{1}{2})}\left(\frac{r}{2m}\right)^{\frac{3}{2}n-\frac{1}{2}} \times$$

$$\times \frac{\Gamma(n+\frac{1}{3})}{\Gamma(\frac{4}{3})}$$

$$= \frac{i\sqrt{2m}\, e^{-mr}}{12 r^{3/2}\Gamma(\frac{4}{3})}\left(\frac{3\lambda^2 m}{4}\left(\frac{r}{2m}\right)^{3/2}\right)\sum_{n=0}^{\infty}\frac{\Gamma(n+\frac{4}{3})}{\Gamma(\frac{3}{2}n+2)} \times \left(-\frac{3\lambda^2 m}{4}\left(\frac{r}{2m}\right)^{3/2}\right)^n$$

$$\xrightarrow{r\to\infty} \frac{i}{8\pi}\sqrt{\frac{2m}{\pi}}\,\frac{e^{-mr}}{r^{3/2}}\ , \qquad \text{(A11)}$$

where, in the last steps, we have used a theorem of Wright.[18] To the result of Eq. (A11) we must add the Fourier transform of the free propagator (at $t = 0$)

$$\int \frac{d^4p}{(2\pi)^4}\ \frac{e^{i\bar{p}\cdot\bar{x}}}{p^2 - m^2 + i\varepsilon} \xrightarrow{r\to\infty} -\frac{i}{8\pi}\sqrt{\frac{2m}{\pi}}\,\frac{e^{-mr}}{r^{3/2}}\ .$$

As we can see, the sum equals zero, and the leading terms cancel. This would suggest that we should look at nonleading terms. This cannot be done in the context of this calculation without the algebra becoming unmanageable. Therefore all we can say is that our propagator should fall off faster than the free propagator.

FOOTNOTES AND REFERENCES

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FIGURE CAPTIONS

Fig. 1: Ladder diagrams.

Fig. 2: Differential equations satisfied by the Green's functions.

Fig. 3: Substitutions of Figure 2b into 2a

Fig. 4: Diagrams neglected in various approximations.

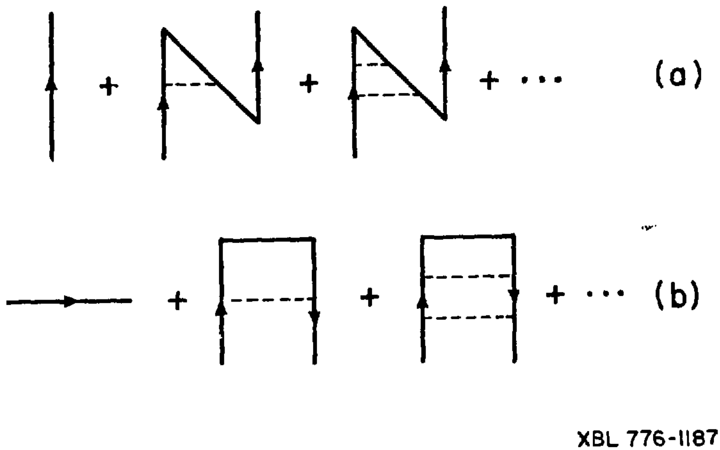

Fig. 1

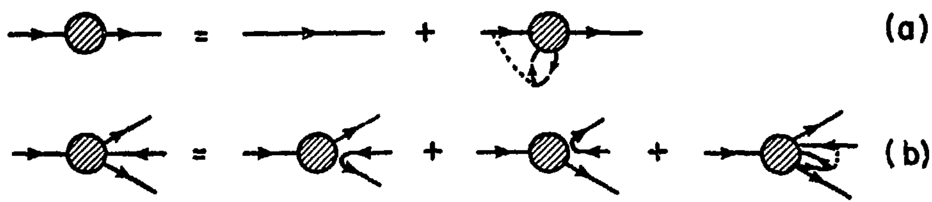

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Fig. 2

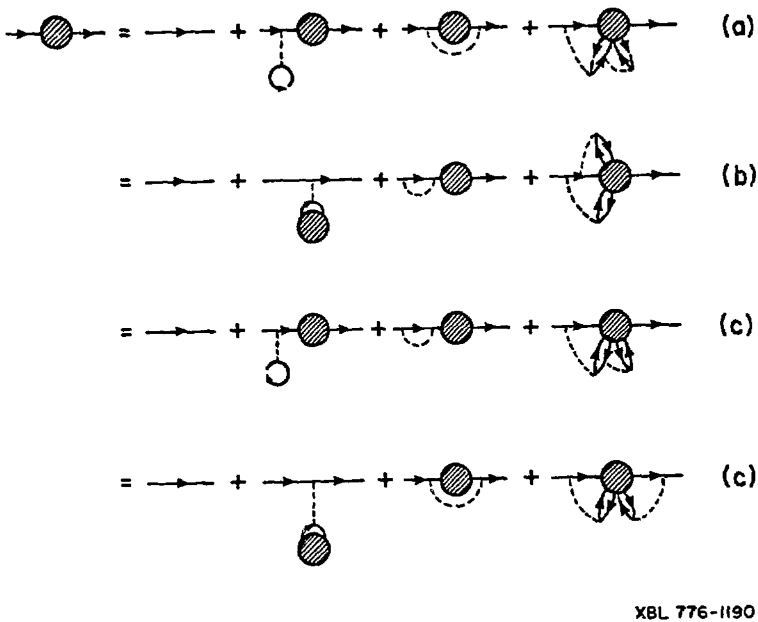

Fig. 3

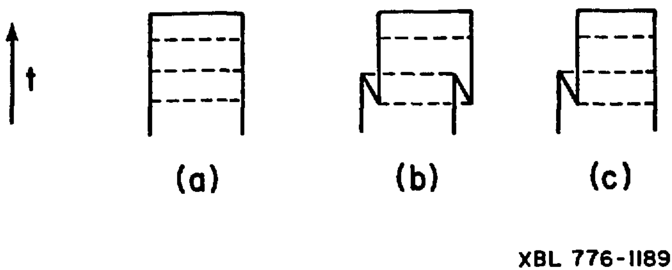

Fig. 4