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## Author

Keaton, G.

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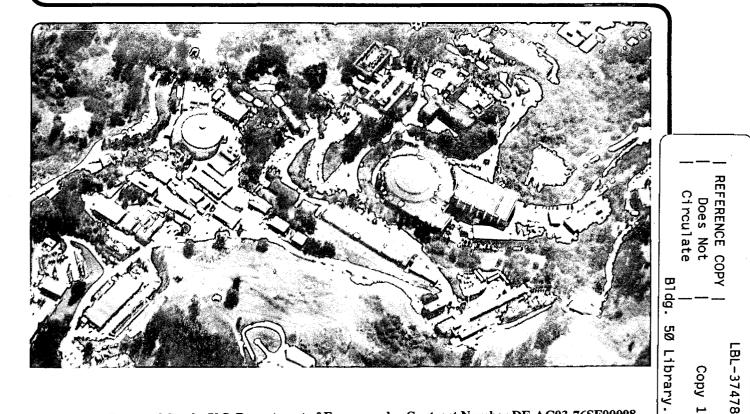
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G.L. Keaton

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## Large-N Baryons, Chiral Loops, and the Emergence of the Constituent Quark<sup>1</sup>

Gregory L. Keaton

Department of Physics, University of California, Berkeley and Theoretical Physics Group, Lawrence Berkeley National Laboratory

Berkeley, CA 94720

#### Abstract

Meson loop corrections to baryon axial currents are computed in the 1/N expansion. It is already known that the one-loop corrections are suppressed by a factor 1/N; here it is shown that the two-loop corrections are suppressed by  $1/N^2$ . To leading order, these corrections are exactly what would be calculated in the constituent quark model. Some applications are discussed.

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## 1 Introduction

How are the baryons' properties renormalized by pion loops? This classic question gains renewed interest with the advent of each new calculational technique.

Pion loop corrections to baryon properties have been studied using the non-linear sigma model with derivative couplings [1]. Later, Jenkins and Manohar [2, 3, 4] simplified the problem by invoking the heavy baryon approximation [5]. Using a Lagrangian that included the baryon octet and Goldstone boson octet, they found that the one-loop correction to the baryon axial current was large—as much as 100% of the tree-level value. However, if the baryon decuplet is also included, the total one-loop corrections are smaller, on the order of 30% of the tree-level value. That is, the loops involving decuplet states tend to cancel the loops involving only octet states. This was good news for perturbation theory, but it left unanswered the question, "What is the loop expansion parameter?" A seemingly coincidental cancellation of large corrections did not leave behind any obvious parameter that could justify, for example, the belief that the two-loop corrections should be any smaller than the one-loop corrections.

This question can be addressed within the framework of large-N techniques for baryons [6, 7], which have recently been rediscovered and greatly expanded [8, 9, 10, 11, 12, 13, 14, 15]. One of the results is that the baryon coupling to the axial current is on the order of the number of colors,

$$g_A \sim N \tag{1}$$

This also means that the baryon-pion coupling is  $\sim g_A k_\mu / f \sim \sqrt{N}$ . However, the renormalization graph of Fig. 1 (a) gives a contribution of order  $N^2$ to  $g_A$ , which if taken alone would violate Eq. (1) and doom perturbation theory (since the one-loop contribution would be much larger than the treelevel value). But there is another graph that must not be forgotten, the wavefunction renormalization of Fig. 1 (b). When both of these diagrams are included, it is found [8, 10, 14] that the leading order behaviour cancels, and the total one-loop correction is  $\mathcal{O}(1)$ , or 1/N times the tree-level value. The one-loop corrections are therefore small in the 1/N expansion and chiral perturbation theory seems to be valid.

As encouraging as this result is, it leaves some questions open. If the oneloop results are to be fitted to the data and believed, one should show that the two-loop contribution is small compared to the one-loop result. This is not obvious since, for example, the diagram of Fig. 2(a) is of order  $N^3$  times the one-loop correction. However, once again there are several diagrams to be added together. This paper shows that when all of the two-loop diagrams are taken into account, the largest terms cancel, and the result is of order 1/N times the one-loop contribution. Evidently, the pion loop expansion parameter turns out to be 1/N. Although this has been suspected before [8, 10], it has not been previously demonstrated to two loops.

One result of this analysis is a demonstration that when the pion-baryon vertex is taken to leading order in 1/N, the chiral loop corrections follow exactly the same pattern as would have been calculated in the chiral quark model [16]. This is surprising because the chiral quark model is a constituent quark model where (to leading order) the pions interact with only one quark at a time. In the foregoing analysis, however, the pions interact coherently with all of the quarks in the baryon at once. Nevertheless, when all of the loop diagrams are taken into account, the cross terms where pions connect two or more quarks cancel exactly. All of the pions end up acting on only one quark at a time, and the chiral quark model results. It had already been noted that the constituent quark model fits the data as well as the usual baryon-pion theory [3]; the 1/N expansion sheds some light on why this is so.

The next section presents the two-loop calculation; this is followed by a discussion of possible applications of this formalism.

### 2 **Two-Loop** Corrections

What are the meson loop corrections to the baryon axial current? The spatial components of the axial current are written

$$<\mathcal{B}'|\bar{q}\gamma^i\gamma_5T^aq|\mathcal{B}>_{tree}=X^{ia}_{B'B}$$

where  $T^a$  is a generator of the flavor group. (The magnitude of  $X^{ia}$  is what was loosely called  $g_A$  in the introduction.) The axial current can be expressed in the 1/N expansion using perturbative baryon states |B > [12] (to be contrasted with physical states |B >):

$$|B\rangle = B^{r_1\alpha_1\dots r_N\alpha_N}a^{\dagger}_{r_1\alpha_1}\dots a^{\dagger}_{r_N\alpha_N}|0\rangle$$

Here  $a_{r\alpha}^{\dagger}$  creates a quark with spin  $\alpha$  and isospin r. The quarks are totally antisymmetric with respect to color; their color indices are suppressed, and the operators a and  $a^{\dagger}$  are treated as bosonic, rather than fermionic [12]. Then the axial current can be written in a 1/N expansion

$$X_{B'B}^{ia} = g < B'|G^{ia}|B > + \frac{h}{N} < B'|H^{ia}|B > + \dots$$
(2)

where g and h are constants of order 1 and  $G^{ia}$  and  $H^{ia}$  are the operators [12, 13, 15]

$$G^{ia} = a^{\dagger}_{r\alpha} T^a_{rs} \sigma^i_{\alpha\beta} a_{s\beta}$$
$$H^{ia} = (a^{\dagger}_{r\alpha} T^a_{rs} a_{s\alpha}) (a^{\dagger}_{t\beta} \sigma^i_{\beta\gamma} a_{t\gamma})$$

The mesons are coupled derivatively to the baryon axial current. Some of the Feynman rules for the pion-baryon interactions are given in Fig. 3. The baryons are treated within the heavy fermion approximation [5, 2, 3], and the calculations are performed in the baryon's rest frame. The meson propagator uses the mass matrix  $m_{ab}^2$ , a diagonal matrix that gives the masses of the pions, kaons, and eta under flavor symmetry breaking. For the Npower counting, it is important to keep in mind that the pion decay constant  $f \propto \sqrt{N}$ .

Now look at the vertex renormalization. The momentum integral for Fig. 1(a) is

$$\mathcal{I}_{ab} = i \int \frac{d^4 p}{(2\pi)^4} \frac{1}{p_0^2} \frac{\frac{1}{3} \mathbf{p}^2}{p^2 - m_{ab}^2}$$

For Fig. 2(a) and Fig. 2(c), the integral is

$$\mathcal{J}_{1}^{aa'bb'} = -\int \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \frac{1}{p_0^3} \left(\frac{1}{p_0 + q_0} - \frac{1}{q_0}\right) \frac{\frac{1}{3}\mathbf{p}^2}{p^2 - m_{aa'}^2} \frac{\frac{1}{3}\mathbf{q}^2}{q^2 - m_{bb'}^2}$$

The inner loop includes a counter-term. If this term (the  $1/q_0$  appearing above) is not included, the internal baryon acquires an additional mass, which must then be transformed away by the heavy baryon transformation. It is easier to simply include the counter-term explicitly. The mass differences between the various baryons are proportional to 1/N and/or to the flavor symmetry breaking, and will be ignored.

The integrals for Fig. 2(b),(d),(e), and (f) are (respectively)

$$\begin{aligned} \mathcal{J}_{2}^{aa'bb'} &= -\int \frac{d^{4}p}{(2\pi)^{4}} \frac{d^{4}q}{(2\pi)^{4}} \frac{1}{p_{0}^{2}} \frac{1}{(p_{0}+q_{0})^{2}} \frac{\frac{1}{3}\mathbf{p}^{2}}{p^{2}-m_{aa'}^{2}} \frac{\frac{1}{3}\mathbf{q}^{2}}{q^{2}-m_{bb'}^{2}} \\ \mathcal{K}_{1}^{aa'bb'} &= -\int \frac{d^{4}p}{(2\pi)^{4}} \frac{d^{4}q}{(2\pi)^{4}} \frac{1}{p_{0}^{2}} \frac{1}{p_{0}+q_{0}} \frac{1}{q_{0}} \frac{1}{p^{2}-m_{aa'}^{2}} \frac{\frac{1}{3}\mathbf{q}^{2}}{q^{2}-m_{bb'}^{2}} \\ \mathcal{K}_{2}^{aa'bb'} &= -\int \frac{d^{4}p}{(2\pi)^{4}} \frac{d^{4}q}{(2\pi)^{4}} \frac{1}{p_{0}} \frac{1}{(p_{0}+q_{0})^{2}} \frac{1}{q_{0}} \frac{\frac{1}{3}\mathbf{p}^{2}}{p^{2}-m_{aa'}^{2}} \frac{\frac{1}{3}\mathbf{q}^{2}}{q^{2}-m_{bb'}^{2}} \\ \mathcal{K}_{3}^{aa'bb'} &= -\int \frac{d^{4}p}{(2\pi)^{4}} \frac{d^{4}q}{(2\pi)^{4}} \frac{1}{p_{0}} \frac{1}{p_{0}+q_{0}} \frac{1}{q_{0}^{2}} \frac{1}{p^{2}-m_{aa'}^{2}} \frac{\frac{1}{3}\mathbf{q}^{2}}{q^{2}-m_{bb'}^{2}} \\ \mathcal{K}_{1}^{aa'bb'} &= -\int \frac{d^{4}p}{(2\pi)^{4}} \frac{d^{4}q}{(2\pi)^{4}} \frac{1}{p_{0}} \frac{1}{p_{0}+q_{0}} \frac{1}{q_{0}^{2}} \frac{1}{p^{2}-m_{aa'}^{2}} \frac{1}{q^{2}-m_{bb'}^{2}} \\ \mathcal{K}_{1}^{bb'aa'} &= -\int \frac{d^{4}p}{(2\pi)^{4}} \frac{d^{4}q}{(2\pi)^{4}} \frac{1}{p_{0}} \frac{1}{p_{0}+q_{0}} \frac{1}{q_{0}^{2}} \frac{1}{p^{2}-m_{aa'}^{2}} \frac{1}{q^{2}-m_{bb'}^{2}} \\ \mathcal{K}_{1}^{bb'aa'} &= -\int \frac{d^{4}p}{(2\pi)^{4}} \frac{d^{4}q}{(2\pi)^{4}} \frac{1}{p_{0}} \frac{1}{p_{0}+q_{0}} \frac{1}{q_{0}^{2}} \frac{1}{p^{2}-m_{aa'}^{2}} \frac{1}{q^{2}-m_{bb'}^{2}} \\ \mathcal{K}_{1}^{bb'aa'} &= -\int \frac{d^{4}p}{(2\pi)^{4}} \frac{d^{4}q}{(2\pi)^{4}} \frac{1}{p_{0}} \frac{1}{p_{0}+q_{0}} \frac{1}{q_{0}^{2}} \frac{1}{p^{2}-m_{aa'}^{2}} \frac{1}{q^{2}-m_{bb'}^{2}} \\ \mathcal{K}_{1}^{bb'aa'} &= -\int \frac{d^{4}p}{(2\pi)^{4}} \frac{d^{4}q}{(2\pi)^{4}} \frac{1}{p_{0}} \frac{1}{p_{0}+q_{0}} \frac{1}{q_{0}^{2}} \frac{1}{p^{2}-m_{aa'}^{2}} \frac{1}{q^{2}-m_{bb'}^{2}} \\ \mathcal{K}_{1}^{bb'aa'} &= -\int \frac{d^{4}p}{(2\pi)^{4}} \frac{1}{(2\pi)^{4}} \frac{1}{p_{0}} \frac{1}{p_{0}+q_{0}} \frac{1}{q_{0}^{2}} \frac{1}{q_{0}^{2}-m_{aa'}^{2}} \frac{1}{q^{2}-m_{bb'}^{2}} \frac{1}{q^{2}-m_{bb'}^{2}} \\ \mathcal{K}_{1}^{bb'aa'} &= -\int \frac{d^{4}p}{(2\pi)^{4}} \frac{1}{(2\pi)^{4}} \frac{1}{p_{0}} \frac{1}{p_{0}+q_{0}} \frac{1}{q_{0}^{2}} \frac{1}{q_{0}^{2}} \frac{1}{q^{2}-m_{aa'}^{2}} \frac{1}{q^{2}-m_{bb'}^{2}} \frac{1}{q^{2}-m_{bb'}^{2}} \frac{1}{q^{2}-m_{bb'}^{2}} \frac{1}{q^{2}-m_{bb'}^{2}} \frac{1}{q^{2}-m_{bb'}^{2}} \frac{1}{q^{2}-m_{bb'}^{2}} \frac{1}{q^{2}-m_{bb'}^{2}} \frac{1}{q^{2}-m_{bb'}^{2}} \frac{$$

The vertex renormalization to two loops can then be written:

$$V_{B'B}^{ia} = \left( X^{ia} + \frac{1}{f^2} \mathcal{I}^{bb'} X^{jb'} X^{ia} X^{jb} + \frac{1}{f^4} \mathcal{J}_1^{bb'cc'} X^{jb} X^{ia} X^{kc} X^{kc'} X^{jb'} \right. \\ \left. + \frac{1}{f^4} \mathcal{J}_2^{bb'cc'} X^{jb} X^{kc} X^{ia} X^{kc'} X^{jb'} + \frac{1}{f^4} \mathcal{J}_1^{bb'cc'} X^{jb} X^{kc} X^{kc'} X^{ia} X^{jb'} \right. \\ \left. + \frac{1}{f^4} \mathcal{K}_1^{bb'cc'} X^{jb} X^{ia} X^{kc} X^{jb'} X^{kc'} + \frac{1}{f^4} \mathcal{K}_2^{bb'cc'} X^{jb} X^{kc} X^{ia} X^{jb'} X^{kc'} \right. \\ \left. + \frac{1}{f^4} \mathcal{K}_1^{cc'bb'} X^{jb} X^{kc} X^{jb'} X^{ia} X^{kc'} \right)_{B'B}$$
(3)

The operators  $X^{ia}$  are treated as matrices with baryon indices; intermediate baryon states are summed over.

The baryon wavefunction renormalization constant can be computed from the diagrams of Fig. 1(b) and Fig. 4:

$$\left( Z_{2}^{-1} \right)_{B'B} = \left( 1 + \frac{1}{f^{2}} \mathcal{I}^{bb'} X^{jb} X^{jb'} + \frac{1}{f^{4}} (2\mathcal{J}_{1}^{bb'cc'} + \mathcal{J}_{2}^{bb'cc'}) X^{jb} X^{kc} X^{kc'} X^{jb'} \right. \\ \left. + \frac{1}{f^{4}} (\mathcal{K}_{1}^{bb'cc'} + \mathcal{K}_{2}^{bb'cc'} + \mathcal{K}_{1}^{cc'bb'}) X^{jb} X^{kc} X^{jb'} X^{kc'} \right)_{B'B}$$
(4)

Finally, the renormalized axial current is

$$<\mathcal{B}'|\bar{q}\gamma^{i}\gamma_{5}T^{a}q|\mathcal{B}>=\left(Z_{2}^{\frac{1}{2}}V^{ia}Z_{2}^{\frac{1}{2}}\right)_{B'B}$$
(5)

When Eqs. (3) and (4) are substituted into Eq. (5) and the result is multiplied out to order  $1/f^4$  (i.e. to two loops), the terms do not simplify in any obvious way. Some identities among the integrals must be used,

$$\begin{aligned} \mathcal{J}_2^{aa'bb'} + \mathcal{K}_2^{aa'bb'} &= \mathcal{K}_1^{aa'bb'} \\ \mathcal{K}_1^{aa'bb'} + \mathcal{J}_1^{aa'bb'} &= 0 \\ \mathcal{K}_1^{aa'bb'} + \mathcal{K}_1^{bb'aa'} - \mathcal{I}^{aa'}\mathcal{I}^{bb'} &= 0 \end{aligned}$$

as well as the identity

$$\mathcal{K}_1^{aa'bb'}[X^{ia}X^{ia'},X^{jb}X^{jb'}]=0$$

Then after a few pages of algebra, Eq. (5) becomes

$$< \mathcal{B}' [\bar{q}\gamma^{i}\gamma_{5}T^{a}q|\mathcal{B}> = X^{ia} + \frac{1}{2f^{2}}\mathcal{I}^{bb'}[X^{jb}, [X^{ia}, X^{jb'}]] + \frac{1}{f^{4}}\mathcal{K}_{1}^{bb'cc'} \left\{ \frac{1}{4} \left[ X^{jb}, [[X^{kc}, [X^{ia}, X^{kc'}]], X^{jb'}] \right] + \frac{1}{2} \left[ [X^{jb}, X^{ia}], [X^{kc}, [X^{jb'}, X^{kc'}]] \right] + \frac{1}{4} \left[ X^{ia}, \left[ X^{jb}, [X^{kc}, [X^{jb'}, X^{kc'}]] \right] \right\} + \frac{1}{4f^{4}}\mathcal{K}_{2}^{bb'cc'} \left[ [X^{jb}, X^{kc}], [X^{ia}, [X^{jb'}, X^{kc'}]] \right]$$

$$(6)$$

This is the main result of the paper. From here it is possible to show that the one-loop corrections to the axial current are suppressed by  $\mathcal{O}(1/N)$  times the tree-level, and the two-loop contribution is suppressed by  $\mathcal{O}(1/N^2)$ .

For example, suppose we take  $X^{ia}$  to leading order in 1/N,

$$X^{ia} = g < B'|G^{ia}|B> \tag{7}$$

where  $G^{ia} = a^{\dagger}_{r\alpha} T^a_{rs} \sigma^i_{\alpha\beta} a_{s\beta}$ . Since the operator  $G^{ia}$  has one *a* and one  $a^{\dagger}$ , it can count the number of quarks in the baryon once, and can therefore be of order *N*. No accidental cancellations occur, so the axial current is  $\mathcal{O}(N)$  to tree level. Now using Eq. (7), the one-loop correction can be read off from Eq. (6):

$$\frac{g^3}{2f^2} \mathcal{I}^{bb'} < B' | [G^{jb}, [G^{ia}, G^{jb'}]] | B >$$
(8)

Each  $G^{ia}$  has one a and one  $a^{\dagger}$ , but each commutator eliminates an  $a-a^{\dagger}$  pair due to the identity  $[a^{\dagger}_{a\alpha}V^{b\beta}_{a\alpha}a_{b\beta}, a^{\dagger}_{c\gamma}W^{d\delta}_{c\gamma}a_{d\delta}] = a^{\dagger}_{a\alpha}[V,W]^{b\beta}_{a\alpha}a_{b\beta}$ . The resultant operator in Eq. (8) has only one a and one  $a^{\dagger}$ , so the matrix element is at most of order N. Since  $1/f^2 \sim 1/N$ , the total one-loop correction is  $\mathcal{O}(1)$ , or 1/N times the tree-level value.

The quadruple commutators, which give the two-loop corrections, also eliminate all but one  $a-a^{\dagger}$  pair. Therefore these commutators are also of  $\mathcal{O}(N)$ , and when they are multiplied by a coefficient of  $1/f^4$ , the result is  $\mathcal{O}(1/N)$ . That is, the two-loop correction is  $1/N^2$  times the tree-level value.

This result has the following interpretation: since the loop corrections contain only one a and one  $a^{\dagger}$ , the pion vertices and the current operator all act on the same quark; the vertex and wavefunction renormalization are carried out on each quark individually. Those diagrams which involve mesons connecting two different quarks evidently do not contribute. But this is exactly what is assumed to be true in the chiral quark model [16] as developed in Ref. [3]. Therefore the constituent quark emerges from the tangle of meson loops.

There are two details that might complicate the above picture, but they do not turn out to be problematic. The first is that we have left out some diagrams. Figs. 5 (a) - (e) also contribute [2, 3]; they are most easily calculated using an effective Lagrangian [12, 13, 14]. It turns out that these diagrams follow the same pattern as above: *l*-loop diagrams are suppressed by factors of  $(1/N)^l$ , and the result is just what would have been expected from the chiral quark model. One difference of the these diagrams, however, is that they are not necessarily suppressed by powers of the coupling constant g of Eq.(7). For example, Fig. 5(a) and Fig. 5(b) are both proportional to g(rather than  $g^3$  or  $g^5$ ). This point does not affect the present discussion, but is important in the next section.

The second technicality is that Eq. (7) is only an approximation to the axial vertex. When the vertex is expanded to the next order in 1/N (as suggested by Eq. (2)), a new operator  $H^{ia}$  is introduced.  $H^{ia}$  acts on two quarks at a time, so the simplest constituent quark picture receives corrections<sup>2</sup>. However, the identities of Ref. [15] can be used to show that

<sup>&</sup>lt;sup>2</sup>Actually such operators appear in the chiral quark model also; rather than being suppressed by 1/N, though, they are suppressed [16] by a power of the wavefunction at the origin divided by the constituent quark mass,  $|\psi(0)|^{2/3}/m_c$ .

the one- and two-loop corrections are still suppressed by powers of 1/N and  $1/N^2$  respectively.

### 3 Discussion

How does all this formalism apply to the real world? The double commutator in Eq. (6), which gives the one-loop correction, is of the order of the number of flavors  $N_F$ , and the quadruple commutators giving the the twoloop corrections are of order  $N_F^2$ . The momentum integrals should be cut off at the chiral symmetry-breaking scale  $\Lambda$ . Let the pion decay constant f be factorized to show clearly its N-dependence:

$$f = \sqrt{N}\hat{f}$$

where  $\hat{f}$  is  $\mathcal{O}(1)$ . As mentioned previously, some diagrams of Fig. 5 are not suppressed by powers of the axial coupling constants (g and h of Eq. (2)). Therefore the chiral loop expansion parameter is

$$\frac{N_F}{N} \frac{\Lambda^2}{16\pi^2 \hat{f}^2}$$

This parameter is not small in any estimation [17]. However, one can adopt the following approach: start with the bare coupling g (or for example h), assume that it can be renormalized to all orders in the flavor symmetric limit  $(m_{\eta} = m_K = m_{\pi} \approx 0)$ , resulting in the renormalized constant  $g_R$ . This new constant  $g_R$  is to be used in computations, and the effects of virtual pions can be computed loop-by-loop, keeping only those terms that violate  $SU(N_F)$ symmetry. In this case all terms involving  $\Lambda^2/16\pi^2 \hat{f}^2$  are to be thrown away, since their effects have already been included in  $g_R$ . The new loop expansion parameter then becomes

$$rac{N_F}{N} rac{m_K^2}{16\pi^2 \hat{f}^2} \log rac{\Lambda^2}{m_K^2}$$

This procedure is equivalent to using dimensional regularization for all the integrals of Eq. (6).

Such a program has already been carried out by Ref. [3]. The oneloop corrections to the baryon axial current were computed in the chiral quark model using dimensional regularization. This is exactly equivalent to a leading order 1/N calculation. The model fits the data well; a best fit is obtained for  $g_R = 0.56$ .

So far we have examined the corrections to the octet axial currents. What about the singlet current, the "spin content" of the baryon? When computing the renormalization of the singlet current, fewer diagrams appear than for the octet current: Figs. 5 (a)-(c) do not exist for the singlet current. Therefore the one-loop contribution is suppressed by a factor  $g_R^2(N_F/N)(\Lambda^2/16\pi^2\hat{f}^2)$  compared to the tree value. The two-loop diagrams are suppressed by a factor of  $g_R^2(N_F/N)(\Lambda^2/16\pi^2\hat{f}^2)$  (Figs. 2 and 4) or  $(N_F/N)(m_K^2/16\pi^2\hat{f}^2)(\log \Lambda^2/m_K^2)$  (Figs. 5(d) and (e)) compared to the oneloop diagrams. Therefore, the loop expansion parameter  $\epsilon$  for the singlet current is

$$\epsilon = \max\left(g_R^2 \frac{N_F}{N} \frac{\Lambda^2}{16\pi^2 \hat{f}^2}, \frac{N_F}{N} \frac{m_K^2}{16\pi^2 \hat{f}^2} \log \frac{\Lambda^2}{m_K^2}\right)$$

If  $\epsilon$  is small enough, we do not have to go through the extra step of first doing the flavor-symmetric renormalization and then returning to the integrals using dimensional regularization. Cutoff regularization can be used from the outset.

Using this approach, and the leading 1/N baryon-pion vertex, the spin content of the proton turns out to be

$$= g \left\{ 1 + \frac{g_{R}^{2}}{2f^{2}}\mathcal{I}^{bb'} \right\}$$
$$= g \left\{ 1 - g_{R}^{2} \left[ \frac{32}{9} \frac{\Lambda^{2}}{16\pi^{2}f^{2}} - \frac{22}{9} \frac{m_{K}^{2}}{16\pi^{2}f^{2}} \log \frac{(2\Lambda)^{2}}{m_{K}^{2}} \right] \right\}$$
(9)

(Here I used  $m_{\pi}^2 = 0$  and  $m_{\eta}^2 = \frac{4}{3}m_{K}^2$ .) Unfortunately, since neither g nor  $\Lambda$  is known, this equation has no predictive power. It is comforting, however, that a reasonable choice of the parameters gives a reasonable result. For example, for g = 1 and  $\Lambda = 1$  GeV, Eq. (9) yields a spin content of 0.57, which is within  $\mathcal{O}(\epsilon^2)$  of the experimental value [18] of  $0.27 \pm 0.11$ . In this case the expansion parameter  $\epsilon$  is rather large,  $\epsilon \approx 0.75$ , so the effects of chiral loops are estimated to be very important.

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## **Figure Captions**

Fig. 1 The one loop vertex renormalization (a) and wavefunction renormalization (b). The "x" represents the axial current operator.

Fig. 2 Two loop vertex renormalization diagrams.

Fig. 3 Some of the Feynman rules for baryon-meson interactions.

Fig. 4 Two loop wavefunction renormalization diagrams.

Fig. 5 Additional one and two loop diagrams that contribute to the renormalization of the axial current.

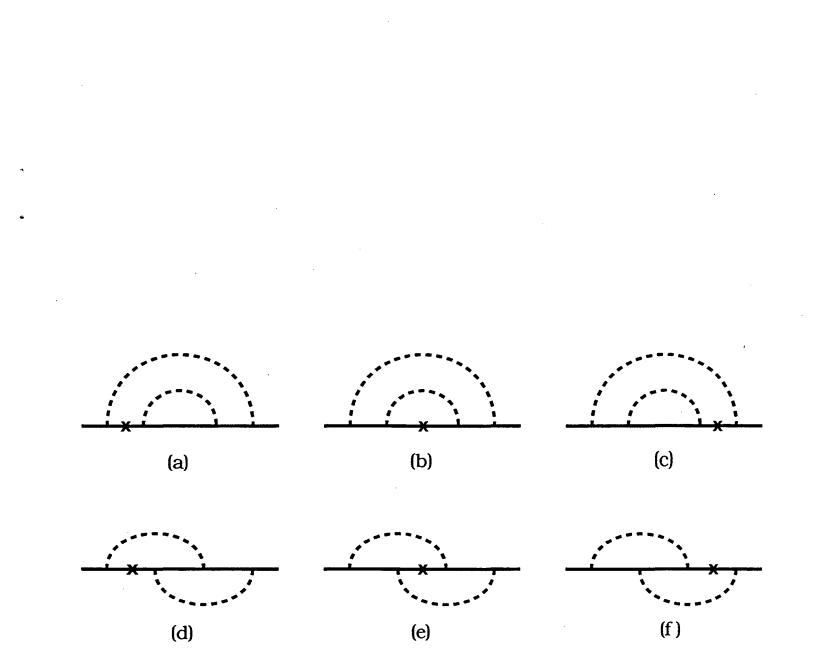


Fig. 2

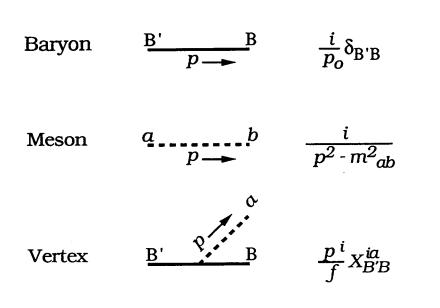
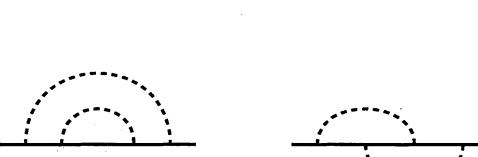


Fig. 3







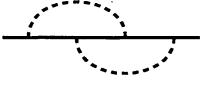
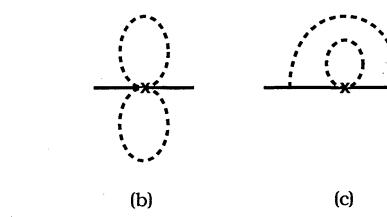




Fig. 4

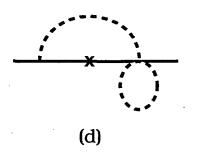


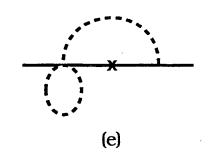














LAWRENCE BERKELEY NATIONAL LABORATORY UNIVERSITY OF CALIFORNIA TECHNICAL & ELECTRONIC INFORMATION DEPARTMENT BERKELEY, CALIFORNIA 94720

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