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Fuzzy Memory Theory and its Use in Cognitive Science

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Abstract

Fuzzy memory theory extends fuzzy set theory to the case of imperfectly performing memory devices. In fuzzy set theory, the key concept is that of graded set membership. The degree to which an item belongs to a set is specified by a continuous function. Fuzzy memory theory is organized around the analogous concept of *graded recall*. Items stored in a fuzzy memory are associated with cues, such that each item is recalled by provision of the corresponding cue. But unlike conventional memory (where cues are typically addresses) the recall process may vary in its degree of error. The item produced may embody missing information. The capacity of a fuzzy memory is then measured in terms of the net information content of recalled items. The theory has potential applications for new forms of technology, but also for the study of cognition. In particular, it can be the means of formalizing the properties of error-prone natural memory mechanisms. It can also supply a non-circular explanation for similarity-based category formation.

Introduction

Since its innovation nearly half a century ago (Gottwald, 2010), fuzzy set theory has become an essential tool of analysis in a wide range of disciplines (cf. Zadeh, 1965, 1976, 1982). In essence, the theory is a generalization of the classical theory of sets, in which set membership becomes a continuous rather than all-or-nothing criterion. The framework is particularly of use in contexts where set membership has a probabilistic character. There have been many applications in cognitive science. For example, the theory has been used for analysis of concept combination and feature emergence (Osherson and Smith, 1981, 1982; Zadeh, 1965, 1976, 1982). In this approach, concepts are deemed to represent fuzzy sets of objects, on which basis combinational concepts can be seen to represent the (fuzzy) intersections of the sets associated with the constituents (Murphy, 2002).

The present paper extends the theory of fuzzy sets to the case of memory. In fuzzy set theory, we assume an item can be a member of a set to a greater or lesser degree. In the proposed extension, it is the degree to which items are recalled that is variable. Information theory (Shannon, 1948; Shannon and Weaver, 1949) provides the means of measurement. The imperfection with which a particular item is recalled can be related to the amount of missing information (i.e., uncertainty) the recalled item exhibits.

Formally, a fuzzy memory device is considered to be a function from cues to items. But the items produced by the memory have to be distinguished from the reference items that are deemed to be stored. The behaviour of a fuzzy memory device is thus characterized as

$$f : C \to X'$$

where C is the set of cues, X is the set of reference items that are considered to be stored and X' is a set that replaces each member of X with the item recalled.

The storage capacity of the device then depends on the information content of the recalled items. This is denoted $I(X')$. But in calculating capacity, we must also take account of C, the set of cues. If this is not done, a function that simply copies its argument has the potential to exhibit an arbitrarily large storage capacity. The capacity of a fuzzy memory device is thus defined to be the *net* information content of recalled items. This is the information content of the reference items less the content of the keys and the information that is lost in recall. Formally, the capacity of fuzzy memory f is

$$I(X') - I(C)$$

where C is the set of cues used to elicit members of X' by f.[1]

In fuzzy set theory, the constituents of a set are characterized using a continuous function. This allows any degree of set membership to be specified. Extending the idea to the case of memory, we require a probabilistic model for the variables on which reference items are constructed. These are termed *base variables* below. A model that imposes a distribution on each variable will not suffice, since it cannot accommodate information losses involving specific variable combinations. A fuzzy memory is therefore considered to be a *composite* of distributions, in which each distribution applies either directly or indirectly to one or more base variables. Information losses arising in the recall of base-variable combinations are then accommodated.

This constituency can be formalized in a recursive way. Letting a distribution be considered *contributory* if it

[1] The word ‘memory’ refers to fuzzy memory in all cases.

applies directly to a base variable, or to a variable whose values themselves designate contributory distributions, we can characterize the constituents of memory f thus:

$$f = \{P \mid \text{contrib}(P, f)\}$$

In this formula, contrib(P, f) is true just in case distribution P applies either directly or indirectly to any base variable of f. Variables that mediate indirectly applying distributions are termed contributory variables. This distinguishes them from the base variables on which reference items are constructed.

A storage criterion for fuzzy memory can then be formalized. Fuzzy memory f is deemed to store some item x if x can be probabilistically reconstructed from f's distribution composite. The informational requirements are as follows. The reconstruction must reconstitute the original item with measurable (but potentially zero) loss of information, and this loss must be less than the information required to specify the cue for the reconstruction. An item is deemed to be stored, then, just in case it can be reconstructed with a net gain of information. The degree of recall is the net gain obtained.

The degree to which an item is recalled by a fuzzy memory is analogous to 'grade of membership' in fuzzy set theory. The evaluation is formalized as follows. The grade of recall for some item x given some cue c is

$$r(x, c) = I(x') - I(c)$$

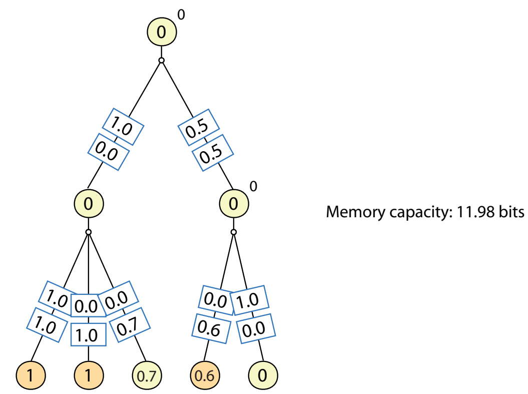
where x' is the device's probabilistic reconstruction of x, and x' satisfies the requirement of being derivable from x by elimination of information. The definition of $I(c)$ depends on how cues are constituted (see below). Given there are k bits of content in each base-variable instantiation, the value of $I(x')$ is

$$I(x') = \sum_i k - H(x'_i)$$

where $H(x'_i)$ is the entropy of the distribution derived for the i'th variable of x'.

Illustrations

To illustrate the use of fuzzy memory, it is convenient to look at the case of 1-bit devices. These are simple assemblies in which base and contributory variables are all binary. Variable values are the digits 1 and 0, each of which has an information content of 1.0 bit. An advantage of this type of device is that it is particularly easy to represent as a tree diagram (cf. Figure 1). It also allows a simple cueing protocol, in which each cue comprises some subsequence of the variable evaluations required to render a fully deterministic reconstruction. In this context, a cue is a sequence of reconstruction constraints.



Reference items	Cues (12 bits)	Recalled items (23.98 bits)					r(x,c)
1 1 0 1 0	00	1	1	0.7	0.6	0	1.26
1 0 0 0 1	11	1	0	0	0	1	3.0
1 1 1 0 1	01	1	1	0.7	0	1	2.18
1 1 1 1 0	00	1	1	0.7	0.6	0	1.26
1 0 0 0 0	10	1	0	0	0.6	0	2.07
1 1 1 0 1	01	1	1	0.7	0	1	2.18

Figure 1: Recall performance of a 1-bit memory.

Consider Figure 1. This is a 1-bit fuzzy memory constructed for the reference items shown in the lower, left part of the figure. The tree structure in the upper part of the figure represents the composite of distributions. Notice this has two levels of construction. These are termed level 1 and level 2. Circles in the bottom row represent base variables. Circles elsewhere represent contributory variables. The digit seen within a circle is the value (or more generally distribution) the variable acquires in recall of the first reference item. Finally, the arcs leading down from each circle show which distributions are designated by which values of the relevant contributory variable.

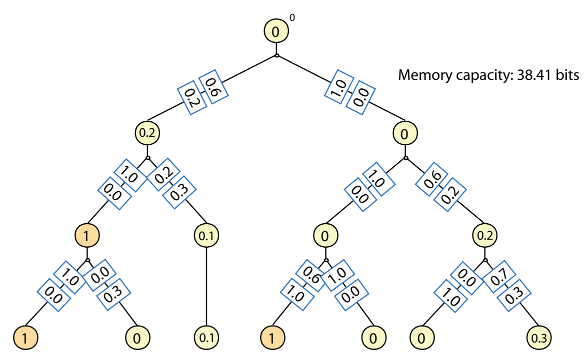
Consider the leftmost contributory variable at the first level of the composite. By following the attached arcs downwards, we see that each value of this variable designates distributions applying to the three leftmost base variables. All distributions being over the values 0 and 1, only the probability applying to 1 is specified. Thus a value such as '0.6' is shorthand for the distribution <0.4, 0.6>, i.e., the distribution that gives probability 0.4 to a value of 0 in the designated variable, and probability 0.6 to a value of 1.

Also of note is the way the distributional values are arranged. Over each set of arcs, we have two rows of boxes. Lower boxes contain distributions designated by a value of 0 in the contributory variable; upper boxes represent distributions designated by a value of 1. Applying these conventions, it should be possible to interpret all the distributions of Figure 1. For example, looking at the bottom, left part of the structure, we see that a value of 0 in the rightmost variable at level 1 designates a distribution on the third base variable which gives a probability

0.7 to the value 0.

The listing in the lower part of the figure portrays the behaviour of the device for the six reference items. (Distributions on variables are those obtained during recall of the initial item.) Each row shows the degree to which a particular item is recalled by its cue, with the associated recall grade $r(x, c)$ appearing on the far right of the figure. As noted, each cue in a 1-bit memory is some subsequence of the disambiguating values required to produce a fully determined reconstruction. For the initial reference item <1 1 0 1 0>, we have the cue 00. The initial digit in this sequence resolves the initial ambiguity in the reconstruction: i.e., it supplies the value 0 for the root contributory variable. The second digit resolves the next ambiguity arising. This affects the rightmost contributory variable at level 1. This value is subject to the equiprobable distribution developed in the previous step. The variable acquires the value 0.

With the cue now exhausted, the reconstruction continues in an un-cued way. The first, second and fifth base variables then acquire implicitly deterministic values, i.e., distributions that embody no loss of information. Regarding the third and fourth base variables, we have the distributions <0.3, 0.7> and <0.4, 0.6> respectively. Adopting the previous convention for representing binary distributions, the recalled item is then <1 1 0.7 0.6 0>. There is of recall of 1.26 bits. This is the information obtained from the five bits of the reference item after deducting the two bits of the cue, and the 1.74 bits eliminated by the two information-losing distributions. Summing recall values for all six reference items, the total capacity of the memory is found to be 11.98 bits.



Reference items	Cues (10 bits)	Recalled items (48.41 bits)							r(x,c)
0 0 0 1 0 1 0	0	1	0	0.1	1	0	0	0.3	4.42
1 0 0 1 1 0 1	1	1	0	0.5	0.6	1	0	0.7	3.25
0 1 0 1 0 1 1	0	1	0	0.1	1	0	0	0.3	4.42
1 0 1 1 1 0 0	1	1	0	0.5	0.6	1	0	0.7	3.25
0 0 1 1 0 0 1	0	1	0	0.1	1	0	0	0.3	4.42
1 0 0 1 1 1 1	1	1	0	0.5	0.6	1	0	0.7	3.25
0 1 1 0 1 1 0	1	1	0	0.5	0.6	1	0	0.7	3.25
0 0 0 1 0 1 0	0	1	0	0.1	1	0	0	0.3	4.42
0 0 0 0 1 0 1	1	1	0	0.5	0.6	1	0	0.7	3.25
1 0 0 1 0 1 0	0	1	0	0.1	1	0	0	0.3	4.42

Figure 2: 1-bit memory with a capacity of 38.41 bits.

A more complex illustration of the 1-bit memory is provided by Figure 2. This is a fuzzy memory for the 10 reference items shown on the left. In this example, reference items are defined in terms of seven base variables rather than five. Contributory variables exist at three levels of construction rather than two. The cueing protocol remains the same. But, here, cue sequences comprise a single digit, meaning they supply values for the root variable only. The main part of the construction process thus proceeds in an un-cued way, with the result being a noticeable increase in the uncertainty of what is recalled. However, with the informational cost of cues at a low level, the memory capacity remains relatively substantial.

Applications in cognitive science

Fuzzy memory theory formalizes a statistical form of memory in a way that reflects fuzzy set theory. Applications of a technological nature are one possibility. But might there also be applications in cognitive science? One way the theory could be used involves natural forms of memory. Human memory is notoriously error-prone, perfect recall being more the exception than the rule. There may be potential, then, for using the framework as a way of theoretically modeling human and other biological forms of memory.

Another possible application involves modeling development of categorical and conceptual representations. It is widely believed that such behaviours are at the heart of cognition (e.g. Harnad, 2005) and that categories are constructed so as to group entities by similarity (Machery, 2009). But it remains a considerable challenge to explain why this should be the case (Murphy, 2002). The temptation is to say that categories are formed as a *result* of the ways in which similarities are identified. Unfortunately, this makes no sense if the way we identify similarities depends on the category representations we bring to bear. With similarity being used to explain both why an entity is assigned to a certain category, and also why that category exists in the first place, such theories are placed 'in the perilous position of using explanations which presuppose the very notions that they attempt to explain' (Hahn and Chater, 1997, p. 84).

Fuzzy memory theory has the potential to address this dilemma. The critical factor affecting performance in fuzzy memory is the degree to which recalled items resemble reference items. The more closely each recalled item approximates its referenced counterpart, the less information is lost and the greater the capacity of the memory. But notice that recall loses less information if distributions deploy more extremal probabilities (i.e., probabilities closer to 1.0 or 0.0). At the same time, distributions must fulfil the function of modeling the referenced items; i.e., they must be the means of constructing valid approximations.

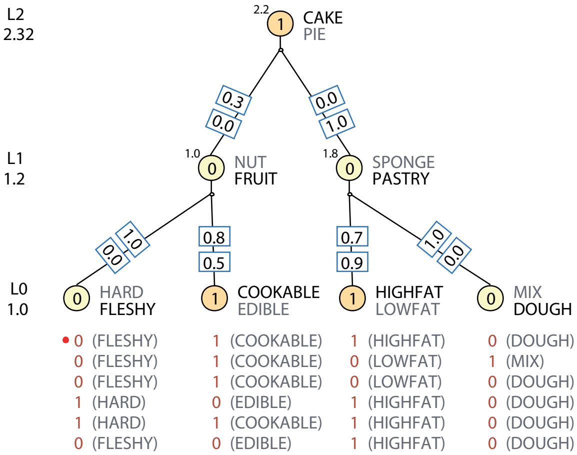


Figure 3: 1-bit fuzzy memory interpreted as a structure of categories.

Reference items	Cues (9 bits)	Recalled items (13.75 bits)				$r(x,c)$
0 1 0 1	0	0	0.5	0.2	1	1.27
0 1 1 0	0	0	0.5	0.2	1	1.27
0 1 1 1	0	0	0.5	0.2	1	1.27
1 0 1 1	11	1	0.4	0.2	1	0.3
1 1 1 1	11	1	0.4	0.2	1	0.3
1 0 1 0	11	1	0.4	0.2	1	0.3

Memory capacity: 4.75 bits

Figure 4: Graded recall using implicit food categories.

Combining these two observations, we see how an implicit similarity preference can arise. The distribution-composite of a fuzzy memory divides the reference data into implicit groupings based on variable subsets. Each of these groupings is then implicitly subdivided into two parts by the (binary) evaluations of the contributory variable involved. More memory capacity is obtained if within-group similarity is maximized, since this is the basis for achieving more extreme probabilities. Distribution composites that group by similarity thus yield relatively greater memory capacity. On the assumption that human memory can be formalized under the fuzzy memory model, the disposition to construct similarity-based categories would then be explained in a non-circular way. It would be seen as a strategy for *increasing* memory capacity.

The schematic of Figure 3 illustrates the idea in more intuitive terms. (The corresponding informational assessments appear in Figure 4.) This memory is constructed to represent the reference items shown in the lower, left part of the figure as usual. But all the variables of the memory are here seen as representing categories. The assumed domain is that of food concepts, such as COOKABLE and DOUGH. Each contributory variable has two labels. These name the categories that are represented by the two values of the variable, with the category represented by a value of 1 appearing above the category represented by a value of 0. The value 0 in the leftmost contributory variable at level 1, for example, represents the FRUIT category, while the value 1 in this variable represents NUT.

The distributions in the composite fulfil their key function of modeling the reference items. But notice how they also mediate similarity-based groupings. FRUIT can be seen to represent a group that includes all FLESHY items, some of which are COOKABLE. SPONGE represents a group that includes all cases of MIX, most of which are HIGHFAT. These fuzzy groupings then become the means of constructing higher-level categories such as PIE. At this level too, the structure can be viewed as exploiting similarities, although here they reflect the way instances of PIE incorporate (manifestations of) PASTRY and FRUIT.

On this interpretation, the distributions of the composite are akin to the prototype representations envisaged by Rosch and others (e.g. Rosch, 1973; Hampton, 2000). The difference is that here they are deployed at multiple levels of description, and in a way that gets around the problem of prototype compositing (Osherson and Smith, 1981; Prinz and Clark, 2004; Connolly et al., 2007). The construction of similarity-based groupings comes to be seen as a way of increasing the capacity to remember certain data. Exploitation of similarity is explained as a way of increasing the capacity of memory.

Concluding comment

Inspiration for the idea of graded recall comes from the idea of graded set membership. This is the key concept in fuzzy set theory. What lies behind the operationalization of the idea is more obviously information-theoretic in nature: assessing degree of recall involves measurement of information loss. Fuzzy memory theory thus combines two distinct areas of theoretical analysis into a hybrid framework. This is found to have applications of a cognitive nature when note is taken of the ways the framework can model error-prone memory, and the development of categorical forms of representation.

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