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Title
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Permalink
https://escholarship.org/uc/item/7w7634qd

Journal
Physical Review D, 49(5)

ISSN
0556-2821

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Publication Date
1994

DOI
10.1103/physrevd.49.2322

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Peer reviewed
New phases in $CP$-violating $B$ decay asymmetries from mixing to singlet down quarks

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(Received 16 August 1993)

Groups such as $E_6$ with extra SU(2)$_L$ singlet down quarks give rise to flavor-changing neutral currents (FCNC’s) through the mixing of four or more down quarks [1]. These FCNC’s with $Z^0$-mediated exchange may contribute to part of $B^0_d-B^\pm_d$ mixing [2] and of $B^0_s-B^\pm_s$ mixing, giving a range of nonzero values for the fourth quark’s mixing parameters. If these are a large contributor to the $B_d-B_d$ mixing, they introduce new angles into the $CP$-violating $B$ decay asymmetries. The size of the contribution of the FCNC amplitude $|U_{db}|$ to the unitarity triangle is bounded to be less than 0.05 of the unit base, but we find that it can contribute as large an amount to $B_d-B_d$ mixing as does the standard model. We find that the new phases can appear in this mixing as well as in $B_s-B_s$ mixing, and give phases completely different from that of the standard model in $CP$-violating $B$ decay asymmetries.

FCNC experiments put limits on these new mixing angles that constrain the possibility of new physics accounting for $B^0_d-B^\pm_d$ and $B^0_s-B^\pm_s$ mixing. In an earlier paper [3] we analyzed the constraints on the mixing angles analytically but approximately, and formed a plot of the FCNC amplitude $|U_{db}|$ versus $|V_{td}|$ which showed an allowed region for FCNC’s dominating $B^0_d-B^\pm_d$ mixing. In this paper we analyze jointly all constraints on a $4 \times 4$ mixing matrix obtained by assuming only one of the SU(2)$_L$ singlet down quarks mixes appreciably. We use the experiments used on the $3 \times 3$ Cabibbo-Kobayashi-Maskawa (CKM) submatrix elements [4] which include those on the six matrix elements $V_{ub}, V_{cd}, V_{cs}, V_{ub}, V_{cd}, V_{cs}$ of the u and c quark rows, and, in the neutral K system, include $\epsilon$, $K_L \to \mu \bar{\nu}$ (for which there are two experiments), and also $B_d-B_d$ mixing. In this paper studying FCNC’s, we add the $B \to \mu \bar{\nu}X$ bound (which constrains $b \to d$ and $b \to s$), $\Delta m_K$ (which constrains $s \to d$ along with the other K experiments), and $Z^0 \to \mu \bar{\nu}$ (which directly constrains the $V_{ub}$ quark mixing element). We analyze all of these together using a joint $\chi^2$ for fitting all of the 13 experiments in the nine-parameter angle space of the $4 \times 4$ mixing matrix. We include both the standard model and FCNC contributions through effective Hamiltonians. We then make a maximum likelihood plot for $|U_{db}|$ versus the magnitude of the competing standard model amplitude $|V_{td}|$ which is proportional to $|V_{td}|$. Plots are also made for the phase difference of the combined amplitude for $B_d-B_d$ mixing minus the standard model phase, versus the phase of the standard model amplitude alone, which show that almost any phase can occur through new physics to be observed in $CP$-violating $B_d$ decay asymmetries. Similar conclusions follow for $B_s-B_s$ mixing, only with a smaller allowed region of new phases.

There are additional five parameters in the four-quark mixing matrix above those present in the three-quark CKM [5] matrix, namely, three new angles and two new phases, and there are essentially seven experiments on FCNC to set limits on their magnitudes and phases. Experiments limit combined phases in $U_d$ and somewhat in $U_{db}$. If the FCNC does contribute significantly to $B^0_d-B^\pm_d$ mixing, this phase, along with the detection of the other FCNC, will be needed for a complete determination of the $4 \times 4$ matrix. The $B^0_d-B^\pm_d$ mixing sets the best limit on $|U_{db}|$. The $B \to \mu \bar{\nu}X$ limit on $|U_{db}|$ is much weaker, but it sets the best limit on $|U_{ab}|$. The small 2.4% error on $\Gamma(Z^0 \to bb)$ bounds the mixing matrix element $|V_{ub}|$. Also, the very small 0.2% error on $|V_{ub}|$ and similar error on $|V_{td}|$ with the normalization of the u quark row set similar stringent limits on $|V_{ud}|$. Then, $K_L \to \mu \bar{\nu}$ bounds $|\text{Re}(U_{da})|$, the $K_L-K_S$ mass difference bounds $|\text{Re}(U_{da})^2|$ and thereby effectively $|\text{Im}(U_{da})|$, and $\epsilon$ bounds $|\text{Im}(U_{da})^2|$ giving a bound on the product $|\text{Re}(U_{da})| |\text{Im}(U_{da})|$. In first examining each experiment separately, we may give indicative bounds as if the FCNC acted alone, but in the $\chi^2$ and maximum likelihood analysis, we consider all possible contributions of the standard model and FCNC parts.

In Sec. II we formulate the down quark mixing fla-
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vor changing neutral currents. With $Z^0$ exchange to the FCNC we construct effective Hamiltonians in Sec. III. In Secs. IV, V, and VI we apply bounds from $B \to \mu \mu X$, $B_d^+B_u^-$ mixing, and $Z^0 \to \mu\mu$, respectively. In Sec. VII we apply bounds on $U_{ds}, U_{sb}$ from $K$ meson short distance physics. In Sec. VIII we convert to the angles in the $4 \times 4$ mixing matrix, and examine their contribution to $B_d^+B_u^-$ mixing through $U_{ds}$ in Sec. IX. In Sec. X is described the limits on the angles from graphically examining $\chi^2$ surfaces in the nine-parameter space. In Sec. XI the maximum likelihood method is used to construct minimum $\chi^2$ plots in pairs of physical amplitudes and phases for $B_d^+B_d$ and $B_s^+B_s$ mixing. We also examine the maximum likelihood limits on the new mixing matrix elements. In Sec. XII we present maximum likelihood plots for corrections to unitarity triangles. In Sec. XIII we present an estimate of a lower bound on the mass of the first singlet quark and briefly discuss its production mechanisms. In Sec. XV we summarize our conclusions.

II. FLAVOR-CHANGING CURRENTS FROM MIXING

We use a basis where the three up quark states are the mass eigenstates. In the down quark sector we mix in a fourth down quark which is an $SU(2)_L$ singlet, and use the $4 \times 4$ matrix $V$ which diagonalizes the initial down quarks ($d_{ij}^D$) to the mass eigenstates ($d_{ij}$) by $d_{ij}^D = V_{ij}d_{ij}$. The combination of the diagonal matrix in the up quarks and the matrix $V$ give the charged current interactions

$$L = \frac{g}{\sqrt{2}} (W^- J^\mu + W^+_\mu J^-),$$

$$J^\mu = \bar{u}_iL\gamma^\mu d_{ij}^D = V_{ij}\bar{u}_iL\gamma^\mu d_{ij},$$

where here $i = 1,2,3$ and $j = 1,2,3,4$. Here the $4 \times 4$ submatrix of $V$ couples the three up quarks to the four down quarks. The $3 \times 3$ submatrix of $V$ for $i,j = 1,2,3$ takes the role of the CKM matrix for mixing the standard model quarks, but it is not unitary by itself.

The weak isospin part of the three $SU(2)_L$ doublet quarks can be written in terms of all four quarks by using a $4 \times 4 t_3$ matrix such that $(t_3)_{44} = 0$:

$$L = \frac{e}{\sin \theta_W \cos \theta_W} \frac{d_{ij}}{d_{ik}}(t_3)_{km} d_{jL},$$

$$(t_3)_{km} = \frac{1}{2} \delta_{km} + \frac{1}{2} \delta_{kj} \delta_{4m}.$$ (4)

Converting to mass eigenstates by $d_{ik}^0 = V_{ik}d_{ik}$ and $d_{ik}^L = V_{ik}^*d_{ik}$ gives

$$L^Z = \frac{e}{\sin \theta_W \cos \theta_W} V_{ik}V_{mj}d_{ikL}(t_3)_{km}d_{jL}.$$ (5)

The flavor-changing neutral currents ($i \neq j$) are given by the $\delta_{kj} \delta_{4m}$ term in $(t_3)_{km}$:

$$L_{FCNC}^Z = -\frac{e}{2 \sin \theta_W \cos \theta_W} U_{ij}d_{ikL}\gamma^\mu d_{jL}Z_\mu,$$ (6)

with amplitudes [6]

$$-U_{ij} \equiv V_{ij}^*V_{jj} \text{ for } i \neq j.$$ (7)

The diagonal neutral current couplings are reduced in strength by the amplitudes into the FCNC, becoming, when $i = j$,

$$\mathcal{L}_{NC}^Z = -\frac{e}{2 \sin \theta_W \cos \theta_W} \sum_i (1 |V_{ii}|^2) d_{iL}\gamma^\mu d_{iL}Z_\mu.$$ (8)

FCNC experiments will bound the three amplitudes $U_{ds}, U_{sb},$ and $U_{bd}$ which contain three new mixing angles and three phases.

III. FCNC EFFECTIVE HAMILTONIAN

The FCNC processes with $Z^0$ exchange would contribute at the same time as the second-order weak neutral current flavor-changing processes in the standard model. At the FCNC tree level of a four-fermion effective Hamiltonian, for quarks coupling to muon pairs, the FCNC’s couple to a virtual $Z^0$ created in the neutral current annihilation channel. In the four-quark effective Hamiltonian, the FCNC contribute through tree level $Z^0$ exchange as well, and both annihilation and exchange graphs contribute the same. This is the same as in the standard model box diagrams for four-quark couplings. We can then add the FCNC $Z^0$ exchange terms to the standard model second-order weak four-fermion effective Hamiltonian $\mathcal{H}_{eff}$, as computed by Inami and Lim [7], and carry them along in all treatments of $\mathcal{H}_{eff}$ applied to various neutral current processes (see also [8] and [9]).

The four-quark neutral current effective Hamiltonian for $s$ and $d$ quarks from Inami and Lim [7], and from using Eq. (6) in the two tree level $Z$ exchange diagrams of equal magnitude (and dividing by $2 \times 2$ for having two choices for $s$ and $d$ fields),

$$\mathcal{H}_{eff}^Z = \frac{G_F}{\sqrt{2}} \frac{d_{iL}\gamma_\mu sL}{(t_3)_{km}}\left\{ \frac{\alpha}{4 \pi \sin^2 \theta_W} (-\vec{E}^\ast) + U_{ds}^2 \right\}.$$ (9)

For $K$ meson processes, $-\vec{E}^\ast$ becomes [10], with QCD corrections ($y_c = m_c^2/m_W^2$),

$$-\vec{E}^\ast = \bar{q}_i(V_{cs}^*V_{ce})\gamma_\mu y_c + \bar{q}_i(V_{td}^*V_{te})\gamma_\mu y_t,$$

$$+2\bar{q}_i(V_{td}^*V_{ce}V_{te}^*V_{ts})y_c y_t f_5(y_t).$$ (10)

The $K^0-\bar{K}^0$ mixing matrix element is

$$\langle K^0|d\bar{\gamma}_\mu(1 - \gamma_5)s|\bar{K}^0\rangle = \frac{4}{3} f_K m_K^2 B_K$$ (11)

and $f_K = \sqrt{2} F_K = 165$ MeV. The value of the coefficient of the standard model result versus $U_{ds}^2$ in Eq. (9) is small (as are the terms in $-\vec{E}^\ast$):

$$\frac{\alpha}{4 \pi \sin^2 \theta_W} = 2.5 \times 10^{-3}.$$ (12)

For the $K_L \to \mu \mu$ semileptonic effective Hamiltonian we have from Inami and Lim [7] and from the FCNC tree level virtual $Z^0$ exchange,
lead to predictions of less than two flavor-changing $b\bar{s}$ or $b\bar{d}$ or their charge conjugate decays out of $Z^0 \times 10^6$ decays.

V. $B_d\bar{B}_d$ MIXING

Mixing may occur by the $b\bar{d}$ quarks in a $\bar{B}_d$ annihilating to a virtual $Z$ through a FCNC with amplitude $U_{db}$, and the $Z$ then creating $b\bar{d}$ quarks through another FCNC, which then becomes a $B_d$ meson.

For $B_d\bar{B}_d$ mixing, the analogous $H_{\text{eff}}$ to Eq. (9) for four-quark $b\bar{d}$ coupling gives

$$x_d = \frac{2G_F}{3\sqrt{2}} B_{\bar{B}B} f_{\bar{B}} m_{\bar{B}} \eta_B \tau_B \left| \frac{\alpha}{4\pi\sin^2\theta_W} y_f(y_f)(V_{td}V_{tb})^2 \right| + (U_{db})^2,$$

where $x_d = \Delta m_{B_d}/\Gamma_{B_d} = \tau_{B_d}\Delta m_{B_d} = \tau_{B_d} M_{B_d}$.

The experimental and calculated parameters are [10]

$$x_d = 0.66 \pm 0.11,$$

$$\eta_B = 0.65,$$

where we have used the new and larger lattice value [13] for $\sqrt{B_{\bar{B}B} f_{\bar{B}}}$ and for consistency, the next to leading order $\eta_B$ [14].

To orient to the magnitude of the terms, requiring that FCNC alone account for the observed value of $B_d^0\bar{B}_d^0$ mixing would result in $|U_{db}/V_{cb}| = 0.016$, which with 1σ limits is

$$0.009 < \frac{|U_{db}|}{|V_{cb}|} < 0.022.$$  

Thus the $B_d^0\bar{B}_d^0$ mixing by $Z$-mediated FCNC, above, is still well allowed by the new bound on $B \to \mu \mu X$, Eq. (18), where $|U_{db}/V_{cb}| < 0.034$. Using the central value of $V_{cb}$ this gives an indicative central value for the FCNC amplitude if acting alone of

$$|U_{db}| = 0.0007.$$  

VI. BOUND FROM $Z^0 \to b\bar{b}$

The reduction in the diagonal neutral current coupling for $Z^0 \to b\bar{b}$ by the factor $(1 - |V_{cb}|^2)$ also imposes a constraint from the CERN $e^+e^-$ collider LEP data. The theoretical decay rate for $Z^0 \to b\bar{b}$ including this FCNC reduction is

$$V^b = -\frac{1}{2} (1 - |V_{cb}|^2) + \frac{3}{2} \sin^2\theta_W + \frac{1}{3} \rho t,$$

$$A^b = -\frac{1}{2} (1 - |V_{cb}|^2) + \frac{1}{3} \rho t,$$

$$C_{\text{QCD}} = 3(1.0385),$$

$$\Gamma^{\text{std+FCNC}}(Z^0 \to b\bar{b}) = C_{\text{QCD}} G_F m_Z^3 \left| (V_{cb})^2 + (A^b)^2 \right|,$$

$$f_{\bar{B}} = 0.220 \pm 0.040 \text{ GeV},$$

$$\eta_B = 0.55.$$
where \( \rho_t = 0.0072 \) for \( m_t = 150 \) GeV. The ratio of \( Z^0 \to b\bar{b} \) to the standard model, for small \( |V_{cb}|^2 \), is

\[
\frac{\Gamma_{b\bar{b}}^{\text{std}+\text{FCNC}}}{\Gamma_{b\bar{b}}^{\text{std}}} = (1 - 2.29|V_{cb}|^2).
\]  

(31)

Recent analysis of LEP data \( [15] \) shows an increase by either about 1\( \sigma \) or 0.3\( \sigma \) for the ratio of experiment to the standard model using either a general analysis of a few experiments or all LEP experiments with standard model assumptions, respectively. With this ambiguity, as well as the fact that a true positive result would be due to a different effect, we use only a zero deviation result with a 2.4\% error, giving a \( \chi^2 \),

\[
\chi^2 = \left( \frac{2.29|V_{cb}|^2}{0.024} \right)^2.
\]  

(32)

This gives the 1\( \sigma \) and 2\( \sigma \) limits

\[
|V_{cb}| \leq 0.10, \quad |V_{cb}| \leq 0.14,
\]  

(33)

respectively. This bound shows up as the maximum \( |V_{cb}| \) below in the minimum \( \chi^2 \) plots (see Fig. 4).

VII. FCNC BOUNDS FROM K MESON PHYSICS

\( K \) meson physics gives bounds on the real part of \( U_{d\bar{s}} \) and on the real and imaginary parts of \( U_{d\bar{s}}^2 \).

A. Short distance contribution to \( K_L \to \mu \mu \)

Combining the new results for the branching ratio for \( K_L \to \mu \mu \) from BNL \( [16] \) of \( (6.86 \pm 0.37) \times 10^{-9} \) and from KEK \( [17] \) of \( (7.9 \pm 0.7) \times 10^{-9} \) gives

\[
B(K_L \to \mu \mu) = (7.09 \pm 0.33) \times 10^{-9},
\]  

(34)

with an added \( \chi^2 \) for the fit of 1.7. The result from the \( 2\gamma \) intermediate state is \( [18] (6.83 \pm 0.28) \times 10^{-9} \). The \( 2\gamma \) absorptive contribution is purely imaginary, while the \( K_L \) superimposes isolates the real part from the standard model and the FCNC \( Z \)-mediated decays. Bounding the difference of the rates as being due to the real part gives

\[
|A_R|^2 = (0.26 \pm 0.43) \times 10^{-9}.
\]  

(35)

The short distance (SD) contribution to the branching ratio is

\[
\frac{B(K^{+}_L \to \mu^+ \mu)}{B(K^{+}_L \to \mu^+ \mu_{\text{std}})} = \frac{\tau_{K_L}}{\tau_{K^+}} \left| \frac{\alpha}{2\pi \sin^2 \theta_W} \frac{\text{Re}(\bar{C}) + \text{Re}(U_{d\bar{s}}^{2})}{\text{Im}(U_{d\bar{s}}^{2})} \right|^2.
\]  

(36)

If the difference were due to the FCNC amplitude alone, the relation is

\[
|A_R|^2 = 54|\text{Re}(U_{d\bar{s}})|^2.
\]  

(37)

Using the difference plus 1\( \sigma \) error as an upper bound on \( |A_R|^2 \leq 0.69 \times 10^{-9} \) gives the indicative bound

\[
|\text{Re}(U_{d\bar{s}})| \leq 2.6 \times 10^{-5}.
\]  

(38)

B. \( K_L - K_s \) mass difference

The \( Z \)-mediated FCNC also contribute to the neutral \( K \) meson mass difference. The long distance contribution may range from zero to 50\% of the mass difference. The standard model and FCNC short distance contribution is

\[
\Delta m_K (1 - D) = \Delta m_K^{\text{SD}} = 2|M_{12}^{\text{SD}}| = \frac{2G_F m_K f_K^2}{3\sqrt{2}} \left| \frac{\alpha}{4\pi \sin^2 \theta_W} \text{Re}(-\bar{E}^*) + \text{Re}(U_{d\bar{s}}^2) \right|,
\]  

(39)

with the experimental result being

\[
\Delta m_K = 3.52 \times 10^{-15} \text{ GeV}.
\]  

(40)

The first term in

\[
\text{Re}(-\bar{E}^*) = \eta_1 \text{Re}[(V^{*}_{cd}V_{cs})^2] y_c
\]  

(41)

dominates and is essentially fixed. The value of \( B_K = 0.67 \pm 0.20 \) corresponds to a short distance fraction \( (1 - D) = 0.39 \pm 0.12 \). We then use as a 1\( \sigma \) bound on the FCNC the order of the remainder or \( \Delta m_K \). Requiring \( (\Delta m_K)^{\text{FCNC}} \leq \Delta m_K \), the bound from this on the FCNC contribution is

\[
|\text{Re}[(U_{d\bar{s}}^2)]| \leq 8.8 \times 10^{-8}.
\]  

(42)

This effectively gives

\[
|\text{Im}(U_{d\bar{s}})| \leq 3.0 \times 10^{-4},
\]  

(43)

\[
|U_{d\bar{s}}| \leq 3.0 \times 10^{-4}.
\]  

(44)

C. Bounds on FCNC CP violation in \( |\varepsilon| \)

The contributions of the four-quark \( H_{\text{eff}}^{4q} \), Eq. (9), for the standard model and the FCNC \( Z^n \) exchanges to \( |\varepsilon| = 2.27 \times 10^{-3} \) give the contribution

\[
|\varepsilon| = \frac{G_F f_K^2}{12\Delta m_K} \left[ \frac{\alpha}{4\pi \sin^2 \theta_W} \text{Im}(-\bar{E}^*) + \text{Im}(U_{d\bar{s}}^2) \right],
\]  

(45)

If this is applied as an upper bound to the FCNC part alone, the indicative result (for \( B_K = 2/3 \)) is

\[
|\text{Im}[(U_{d\bar{s}}^2)]| \leq 9.2 \times 10^{-10}.
\]  

(46)

This effectively gives

\[
|\text{Re}(U_{d\bar{s}})\text{Im}(U_{d\bar{s}})| \leq 4.6 \times 10^{-10}.
\]  

(47)
VIII. BOUNDS ON MIXING ANGLES

We consider the up quark basis as being diagonal in the up quark masses, up to phases. Despite having three up quarks with the four down quarks, the down quarks still mix with a $4 \times 4$ unitary matrix, and the extra phases can be removed by shifting the relative down and up quark phases, and the fourth row can still have its relative phase changed, even though there is no fourth up quark to which it couples. We thus use the standard CKM-like $4 \times 4$ matrix [19]. This holds for one new singlet down quark, but not beyond. For a rigorous derivation, see [20]. All cosine terms in the $4 \times 4$ mixing matrix are near 1. For simplicity we show the leading terms in the matrix elements of $V$, in terms of the sines and cosines of the mixing angles, although in practice the calculations are done with exact elements [19]:

$$V = \begin{pmatrix} c_{12}c_{34} & s_{12}c_{34} & & \s13e^{-i\delta_{13}} & s_{14}e^{-i\delta_{14}} \\ -s_{12} & 1 & & s_{23} & s_{24}e^{-i\delta_{24}} \\ (s_{12}s_{23} - s_{13}e^{i\delta_{13}}) & -s_{23} & V_{4d} & V_{4s} \\ V_{4d} & V_{4s} & & & \end{pmatrix}$$ (48)

The complex conjugates of the mixing matrix elements in the fourth row, keeping only leading terms in the new angles, are [19]

$$V_{4d}^* = -s_{14}e^{-i\delta_{14}} + s_{24}e^{-i\delta_{24}}s_{12}$$
$$V_{4s}^* = s_{24}e^{-i\delta_{24}} + s_{34}(s_{23} + s_{12}s_{13}e^{-i\delta_{13}})$$
$$V_{4b}^* = -s_{14}e^{-i\delta_{14}}s_{12}.$$ (49)

Using the bound on $\text{Re}(U_{ds})$ above, for the four-quark mixing form $-U_{ij} = V_{4i}^*V_{4j}$, we get

$$|\text{Re}(V_{4d}^*V_{4s})| \leq 2.6 \times 10^{-5}. \quad (52)$$

Having tight bounds on $U_{ds}$ and $U_{ub}$ we can ask if it is still consistent to have a value for $U_{db}$ large enough to account for $B_d\rightarrow B_s$ mixing. We combine all three to get a bound on $V_{4s}$ by first substituting in the definitions of the $U$’s as follows:

$$-U_{db} = V_{4d}^*V_{4b} = \frac{U_{db}}{V_{4s}^*V_{4s}^*}. \quad (53)$$

We then substitute on the right the bounds on $|U_{ds}|$ and on $|U_{ub}/V_{cb}|$, and on the left the value of $|U_{db}/|V_{cb}|$ to account for $B_d\rightarrow B_s$ mixing:

$$0.016 = |U_{db}/V_{cb}| \leq 3.0 \times 10^{-4} \times 0.034/|V_{4s}|^2, \quad (54)$$

which gives

$$|V_{4s}|^2 \leq 0.00064 \quad \text{or} \quad |V_{4s}| \leq 0.025. \quad (55)$$

If this restriction is satisfied, the two bounds above are consistent, and the matrix elements $|V_{4d}|$ and $|V_{4b}|$ can be large enough for $U_{db}$ to account for a part of $B_d\rightarrow B_s$ mixing. (From Fig. 4 below, this is indeed the case.)

IX. PHASES IN CP-VIOLATING B DECAY ASYMMETRIES

The $CP$-violating decay asymmetries [21] rely on a relative phase between the $B_d^0 - B_d^0$ mixing and the $b$ quark decay amplitudes into final states of definite $CP$. Since we have found that $Z$-mediated FCNC processes may contribute significantly to $B_d^0 - B_d^0$ mixing, the phases of $U_{db}$ would be important. To leading order in $s_{34}, s_{24}e^{-i\delta_{24}},$ and $s_{14}e^{-i\delta_{14}}$ we have

$$U_{db} = -s_{34}(s_{34}V_{4d}^* + s_{14}e^{-i\delta_{14}} - s_{24}e^{-i\delta_{24}}s_{12}). \quad (56)$$

We note that the first term has the same phase as in the standard model where the box diagram with the $t$ quark dominates the mixing. This requires a numerical analysis to find how much the other terms contribute.

X. ISOSURFACES OF CONSTANT $\chi^2$ IN THE $4 \times 4$ MATRIX PARAMETER SPACE

Here we examine the nine-angle-parameter space of the $4 \times 4$ down quark mixing matrix to find what the relationships between different angles are because of the constraints from all of the experiments. To do this we show the isosurfaces of constant $\chi^2$ in three-dimensional subspace of the nine-dimensional parameter space. (We can also show predictions for the FCNC transition amplitudes or for experiments on these isosurfaces by different colors or gray shadings.)

For the four parameters of the usual $3 \times 3$ CKM, $\theta_{12}(V_{us})$ is very well determined from $V_{us}$ and $V_{ud}$, and we take only its central value here, since its variation has little effect on any of the other experiments. In the $\chi^2$ we take from CLEO II [22,4]

$$|V_{ub}/V_{cb}| = 0.07 \pm 0.01, \quad (57)$$
$$|V_{cb}| = 0.043 \pm 0.007. \quad (58)$$

$\theta_{23}(V_{cb})$ and $\theta_{13}(V_{ub})$ are determined to about 17% and 21%, respectively. We vary these within 2$\sigma$ of their central values. The angle, $\theta_{13}$ and phase $\delta_{13}$ are elements of $V_{ub} = s_{13} \exp(-i\delta_{13})$ and $V_{td}$ and are often shown by $\rho$ and $\eta$, where [23]

$$\rho + i\eta = V_{ub}/(|V_{ub}/V_{cb}|) \approx s_{13} \exp(i\delta)/(s_{12}s_{23}) \quad (59)$$

and

$$V_{td}/(|V_{td}/V_{cb}|) = 1 - \rho - i\eta. \quad (60)$$
\( \delta_{13} \) and therefore \( \rho \) and \( \eta \) are not well determined [10,3] and FCNC limits on angles vary with these. Thus we are predominantly working in a space of three new angles and three phases with important variation. The calculations of \( \chi^2 \) are carried out with a grid of seven points on the mixing angles and phases (sampling new phases at every 60° and \( \delta_{13} \) about every 30°), and over a range of angles encompassing 2\( \sigma \) limits on the old angles and on the enclosing 2\( \sigma \) \( \chi^2 \) range for the new angles.

Of the three-dimensional subspaces we show in Fig. 1 the \( \chi^2 = 9.8 \) isosurface corresponding to 2\( \sigma \) in the space of the three new mixing angles \( \theta_{34} \) (up to 0.2), \( \theta_{14} \) (up to 0.008), and \( \theta_{24} \) (up to 0.03).

Considering the possibility of a nonzero value for \( U_{db} \) from \( B_d \bar{B}_d \) mixing through FCNC, from the form of Eq. (56), \( \theta_{34} \) must be nonzero. Furthermore, for \( U_{db} \) to have a phase distinguishable from that of \( V_{td}^* \) which is in \( U_{db} \)'s first term in Eq. (56), and is the same phase as the standard model's contribution in Eq. (20), then \( \theta_{14} \) or \( \theta_{24} \) must also be nonzero. We find bounds on the new angles graphically by maximizing the extremes of the 2\( \sigma \) isosurface when varying the other angles and phases. The 2\( \sigma \) isosurfaces are all contained within the limits

\[
\theta_{34} \leq 0.15, \quad \theta_{24} \leq 0.02, \quad \text{and} \quad \theta_{14} \leq 0.010. \quad (61)
\]

These limits are then used in the calculations for maximum likelihood plots. While the limit on \( \theta_{24} \) is about twice that on \( \theta_{14} \), the contribution of \( \theta_{24} \) is reduced by \( s_{12} \) in \( U_{db} \), Eq. (56). With these bounds and with \( |V_{td}| \approx 0.01 \) [3], the \( s_{34}V_{td}^* \) term in \( U_{db} \), Eq. (56) is smaller than the other two terms which bring in new phases and are large enough to make \( |U_{db}| = 0.0007 \) to account for \( B_d \bar{B}_d \) mixing.

**XI. MAXIMUM LIKELIHOOD CORRELATION PLOTS BETWEEN STANDARD MODEL AND FCNC AMPLITUDES**

In maximum likelihood correlation plots, we use for axes two output quantities which are dependent on the angles, such as \( |U_{db}| \) and \( |U_{std}| \), and for each possible bin with given values for these, we search through the nine-dimensional angular data set and put in that bin the minimum \( \chi^2 \) that gives the coordinates in the bin. We then draw contours at several \( \chi^2 \) in this plane to present the results. This is a way to reduce the information from the nine-dimensional parameter space, and to give the likelihood for the important physically meaningful functions of the parameters. In general \( \chi^2 \) is not a unique function of the two axis variables, but we do have a unique function to plot in terms of the minimum \( \chi^2 \) or maximum likelihood.

The minimum \( \chi^2 \) is found by computing over a grid of nine points in each angle covering the 2\( \sigma \) ranges for the standard mixing angles, and over the range found in the isosurface plots of the previous section for the new angles. The two new phases are covered in 30° grids, and \( \delta_{13} \) in a 15° grid. The contribution of \( \delta_{12} \) to \( \chi^2 \) for the highly accurate \( |V_{ud}|^2 \) and \( |V_{us}|^2 \) experimental determinations can be treated analytically. The value of \( \theta_{12} \) that gives the minimum \( \chi^2 \) for these two experiments is found analytically, and is effectively a function of \( \theta_{34} \) since

\[
|V_{ud}| \simeq \cos (\theta_{34}) \cos (\theta_{12}), \quad (62)
\]

\[
|V_{us}| \simeq \cos (\theta_{34}) \sin (\theta_{12}). \quad (63)
\]

The other new angles are bounded to be too small to contribute in these two experiments. We use, for the experimental results and limits,

\[
1 - |V_{ud}|^2 = 0.0505 \pm 0.0019, \quad (64)
\]

\[
|V_{us}|^2 = 0.0486 \pm 0.0008, \quad (65)
\]

\[
|V_{ub}|^2 \simeq 10^{-5}, \quad (66)
\]

\[
|V_{ud}|^2 = s_{14}^2 \leq 10^{-4}. \quad (67)
\]

Varying \( \theta_{12} \) to minimize the part of \( \chi^2 \) from these two experiments gives \( s_{12} \) as a function of \( s_{34}^2 \):

\[
s_{12}^2 = (0.0489 - 0.1506s_{34}^2)/s_{34}^2. \quad (68)
\]

This is then put back into \( \chi^2 \) to get the minimum \( \chi^2 \) from these two experiments. The small allowed variation of \( \theta_{12} \) does not affect other experiments which have much larger uncertainties. The maximum likelihood plots are then contoured over a set of 15 \( \times \) 15 bins.

We define the standard model amplitude for \( B_d \bar{B}_d \) mixing from Eq. (20) normalized so that it adds directly to \( U_{db}^2 \):

\[
U_{std}^2 = \frac{\alpha}{4\pi\sin^2\theta_W} y_t f_2(y_t)(V_{td}^*V_{tb})^2, \quad (69)
\]

\[
x_d \propto |U_{std}^2 + U_{db}^2|. \quad (70)
\]

In Fig. 2(a), we show \( |U_{db}| \) on the y axis and the standard model amplitude \( |U_{std}| \) on the x axis with contours at \( \chi^2 = 4.7, \chi^2 = 9.8, \) and \( \chi^2 = 16.0, \) which give the confidence levels corresponding to 1\( \sigma \), 2\( \sigma \), and 3\( \sigma \), respectively. While \( |U_{std}| \) must be nonzero, we see that
$|U_{db}|$ can go from zero up to as large as the magnitude of $|U_{std}|$ at a similar confidence level. In Fig. 2(b) is shown the analogous diagram for $B_s,B_s$ mixing with a similar conclusion. Note the usual ratio of about 20 for $|U_{std-sb}|$ versus $|U_{std}|$ which is about the ratio of $|V_{td}|^2/|V_{td}|^2$.

We define the phases of the complete $B_d,B_d$ amplitude $\phi_{tot}$, the standard model amplitude $\phi_{std} = 2\beta$, and their difference $\phi_{diff}$ by

$$\phi_{tot} = \arg(U_{std}^* + U_{db}^*),$$

$$\phi_{std} = \arg(U_{std}^2) = \arg((V^*_{td}V_{tb})^2) = 2\beta,$$

$$\phi_{diff} = \phi_{tot} - \phi_{std}. \tag{73}$$

The maximum likelihood plot for $\phi_{tot}$ on the $y$ axis and $\phi_{std} = 2\beta$ on the $x$ axis (using $20 \times 20$ bins) is shown in Fig. 3(a) for $m_t = 150$ GeV. The phase of $B_d,B_d$ mixing enters into the $CP$-violating $B$ decay asymmetries, such as in $B_d \rightarrow J/\psi K_S$. We see that the phase of the total amplitude including FCNC and the standard model amplitude for $B_d,B_d$ mixing at $1\sigma$ can range from $-90^\circ$ to $180^\circ$ while the standard model phases $2\beta$ ranges from $0^\circ$ to $50^\circ$. At $2\sigma$ the total amplitude can have any phase, while the standard model phase ranges from $-8^\circ$ to $60^\circ$. The analogous plot for $B_s,B_s$ mixing, using analogous phases $\phi_{tot-sb}$ and $\phi_{std-sb} = \arg((V^*_{td}V_{tb})^2) = 2\beta$, is shown in Fig. 3(b). Here the phase of the total amplitude ranges over about $\pm 13^\circ$ at $1\sigma$ while the standard model phase $2\beta$ is limited to about $\pm 5^\circ$. At $2\sigma$ the total phase ranges to $\pm 26^\circ$, while the standard model phase ranges up to about $\pm 6^\circ$. $\beta_s = \arg((V^*_{td}V_{tb})^2)$ is the small angle from the $b$s unitarity triangle which occurs in $b \rightarrow c\bar{c}s$ decays as $B_s \rightarrow \psi$. In the standard model $\beta_s = \eta \lambda^2$ and is positive and about $1^\circ$. Here with four down quarks, however, $V_{ts} \simeq -s_{33} - s_{13}s_{13} \exp i\delta_{13} - s_{34}s_{24} \exp i\delta_{24}$ gets extra phases from the mixing angles to the fourth down quark and gives a larger and possibly negative phase to $U_{std-sb}$ corresponding to about $\pm 5^\circ$ at $1\sigma$.

We can also find a minimum $\chi^2$ isosurface in a three-dimensional plot to elicit information on the interdependence and bounds on the three matrix elements at once, which we do in three orthogonal views in Fig. 4 to show the $1\sigma$ or $\chi^2 = 4.7$ isosurface in the space with axes $|V_{td}|, |V_{ts}|, |V_{td}|$, and $|V_{tb}|$, that run from 0 to 0.02, 0.02, and 0.30, respectively. We see that at $1\sigma$, the maximum likelihood

![FIG. 2. (a) The maximum likelihood correlation plot between the FCNC amplitude for $B_d,B_d$ mixing, $|U_{db}|$, and the standard model amplitude for the mixing, $|U_{std}|$. The contours are at $\chi^2 = 4.7$, $\chi^2 = 9.8$, and $\chi^2 = 16.0$, corresponding to $1\sigma$, $2\sigma$, and $3\sigma$, respectively. (b) The maximum likelihood plot for $|U_{sb}|$ versus $|U_{std-sb}|$ for $B_s,B_s$ mixing.](image)

![FIG. 3. Maximum likelihood correlation plot between the phase of the total amplitude $\phi_{tot}$ and the standard model phase $\phi_{std} = 2\beta$. Contours are at $1\sigma$, $2\sigma$, and $3\sigma$ for (a) $B_d,B_d$ phases, and (b) $B_s,B_s$ phases with a smaller range for $\phi_{tot-sb}$ and $\phi_{std-sb} = 2\beta$.](image)
FIG. 4. Maximum likelihood correlation plot between the three new mixing matrix elements in the fourth row, \( |V_{4d}| \), \( |V_{4s}| \), and \( |V_{4b}| \), showing the 1\( \sigma \) or \( \chi^2 = 4.7 \) isosurface in three orthogonal views. At the upper right is the top view with axes \( |V_{4d}| \) and \( |V_{4b}| \), on the left is the left side view with base \( |V_{4d}| \) and height \( |V_{4s}| \), and on the lower right is the right side view with base \( |V_{4b}| \) and height \( |V_{4s}| \).

bounds, searched over all angles, are

\[
|V_{4d}| \leq 0.01, \quad |V_{4s}| \leq 0.02, \quad \text{and} \quad |V_{4b}| \leq 0.1. \quad (74)
\]

XII. CORRECTIONS TO CKM UNITARITY TRIANGLES

The unitarity of the \( 4 \times 4 \) mixing matrix requires orthogonality of the different rows. Instead of the three terms in the CKM orthogonality relation, which give a triangle in the complex plane, we now have four terms which give a quadrangle. The fourth terms are the \( U_{ij} = -V^*_{4i}V_{4j} \) and the orthogonality relations are

\[
U_{ds} = V^*_{us}V_{ub} + V^*_{cs}V_{cb} + V^*_{ts}V_{tb}, \quad (75)
\]

\[
U_{se} = V^*_{us}V_{ub} + V^*_{cs}V_{cb} + V^*_{ts}V_{tb}, \quad (76)
\]

\[
U_{db} = V^*_{ud}V_{ub} + V^*_{cd}V_{cb} + V^*_{td}V_{tb}. \quad (77)
\]

In the unitarity triangle

\[
\left| \frac{V_{us}V^*_{ub}}{V_{cd}V^*_{cb}} \right| \approx 0.32 \quad (78)
\]

is the upper left-hand side. To this must be added the fourth side of

\[
\left| \frac{U_{db}}{V_{cd}V^*_{cb}} \right| \leq 0.1, \quad (79)
\]

which is at a barely detectable size. However, the upper right-hand side proportional to \( |V^*_{ub}V_{ub}/V^*_{cb}V_{cb}| \) is heavily weighted against in the formula for \( B_d \to \bar{B}_d \) mixing, Eq. (20), and so \( U_{db} \) can still have an important contribution to the mixing and to its phase. In Fig. 5 is the maximum likelihood plot for the contribution of the scaled FCNC amplitude \( U_{db}/|V^*_{cb}V_{cb}| \) as the fourth side of the unitarity quadrangle in the complex plane. The contours are at 1\( \sigma \), 2\( \sigma \), and 3\( \sigma \). We see that the bound on \( |U_{db}/V^*_{cb}V_{cb}| \) is roughly independent of direction, and bounded by 0.1.

FIG. 5. Maximum likelihood plot for the contribution of the scaled FCNC amplitude \( U_{db}/|V^*_{cb}V_{cb}| \) to the unitarity quadrangle, about the origin in the \( \rho-\eta \) plane, showing 1\( \sigma \), 2\( \sigma \), and 3\( \sigma \) contours. The dimensions are only 0.1 of the unit base of the unitarity quadrangle.

XIII. D QUARK MASS AND PRODUCTION MECHANISMS

To estimate the new \( D \) quark mass, we use a Fritzsch ansatz relation between down quark mass ratios and mixing angles:

\[
\theta_{54}^2 \simeq m_b/m_D. \quad (80)
\]

The \( |V_{4b}| \) mixing matrix element has the limit from Eq. (33) and from the \( \chi^2 \) maximum likelihood isosurface plot, Fig. 4:

\[
|V_{4b}| \simeq \theta_{54} \leq 0.1. \quad (81)
\]

This gives a lower bound for \( m_D \):

\[
m_D \geq 100 \times m_u = 500 \text{ GeV}. \quad (82)
\]

For production in an \( e^+e^- \) collider, we note from Eq. (8) that the isovector coupling of the \( D \) quark to the \( Z^0 \) is suppressed by

\[
(1 - |V_{4b}|^2) \simeq |V_{4b}|^2 \leq 0.01, \quad (83)
\]

and so the \( D \) singlet quark would be mainly produced by the photon and the electromagnetic part of the \( Z^0 \) current. At the 500 GeV mass it will have little influence on \( Z^0 \) parameters. Hadronically, it would be producible by gluon fusion, and the \( W \) fusion amplitude [from Eq. (2)] would be suppressed by \( |V_{4b}|^2 = \theta_{54}^2 \leq 10^{-2} \).
XIV. CONCLUSIONS

Having formed a $\chi^2$ for 13 experiments that constrain the 9 angles in the $4 \times 4$ down quark mixing matrix, we found ranges of the new mixing angles allowed by the constraints. We then studied the ranges and correlations of the magnitudes and phases of standard model amplitudes for $B_d^\pm -\bar{B}_d^\mp$ mixing with amplitudes including the flavor-changing neutral currents. We found that FCNC can contribute a significant part of $B_d^\pm -\bar{B}_d^\mp$ mixing and also $B_s^\pm -\bar{B}_s^\mp$ mixing. We also found that the phases of the total mixing amplitudes can differ from that of the standard model amplitudes by a large range of angles. These phases are those that are detected in $B$ decay $CP$-violating asymmetries.

ACKNOWLEDGMENTS

This research was supported in part by the U.S. Department of Energy under Contract No. DE-FG03-91ER40679. We thank Professor David Cline, Professor William Molzon, and Professor Myron Bander for discussions, and Dr. Allen Schiano.

FIG. 1. The $\chi^2 = 9.8$ isosurface corresponding to $2\sigma$ in the space of the three new mixing angles to the fourth down quark, namely, $\theta_{34}$, $\theta_{14}$, and $\theta_{24}$. The other mixing angles are at their central values, $\delta_{13} = 90^\circ$, and the new phases are at zero.
FIG. 4. Maximum likelihood correlation plot between the three new mixing matrix elements in the fourth row, $|V_{4a}|$, $|V_{4b}|$, and $|V_{4c}|$, showing the 1σ or $\chi^2 = 4.7$ isosurface in three orthogonal views. At the upper right is the top view with axes $|V_{4d}|$ and $|V_{4b}|$, on the left is the left side view with base $|V_{4d}|$ and height $|V_{4a}|$, and on the lower right is the right side view with base $|V_{4a}|$ and height $|V_{4c}|$. 